Conditional Probability

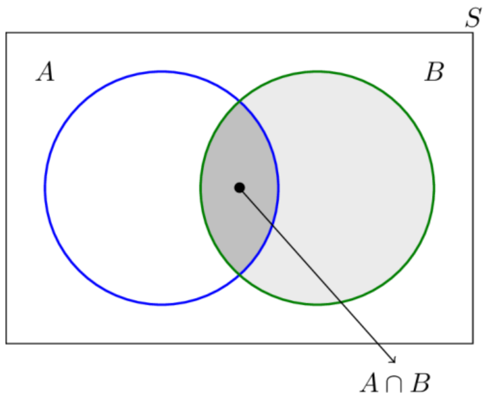
Find the probability of event **A** given that event **B** has already been occurred.

Probability of *A* and *B*

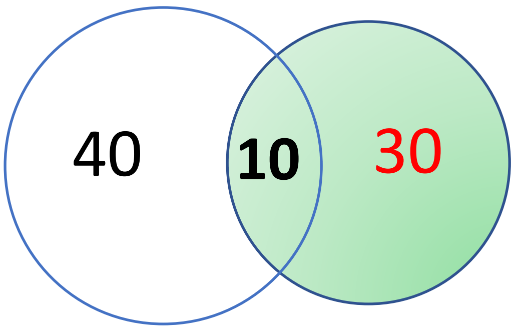
$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

Probability of *A* given *B*

Probability of *B*



$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$



$$P(B) = \frac{30}{80} = 0.375$$

$$P(A \cap B) = \frac{10}{80} = 0.125$$

$$P(A|B) = \frac{0.125}{0.375} = 0.333$$

Bayes Theorem

Statement of theorem

Bayes's theorem is stated mathematically as the following equation:

$$\boxed{P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B)}}$$

where A and B are events and $P(B) \neq 0$.

- $P(A \mid B)$ is a conditional probability: the likelihood of event A occurring given that B is true.
- $P(B \mid A)$ is also a conditional probability: the likelihood of event B occurring given that A is true.
- $P(A)$ and $P(B)$ are the probabilities of observing A and B respectively; they are known as the marginal probability.

Step 01

Find the probability of event **A** given that event **B** has already been occurred.

$$\boldsymbol{P\ (A\ |B) = \frac{P(A \cap B)}{P(B)}}$$

Find the probability of event **B** given that event **A** has already been occurred.

$$\boldsymbol{P\ (B|A) = \frac{P(B \cap A)}{P(A)}}$$

Step 02 **Shifting Marginal Probability to Left Hand Size**

$$\boldsymbol{P\ (A\ |B)\ .\ P(B) = \frac{P(A \cap B)}{}}$$

$$\boldsymbol{P\ (B|A)\ .\ P(A) = \frac{P(B \cap A)}{}}$$

Step 03 **Since, $\boldsymbol{P(A \cap B)\ =\ P(B \cap A)}$.**

Therefore, we can say that

$$\boldsymbol{P\ (A \cap B) = P\ (A\ |B)\ .\ P(B) =\ P\ (B|A)\ .\ P(A)}$$

So, we can say like this as well.

$$\boldsymbol{P\ (A\ |B)\ .\ P(B) =\ P\ (B|A)\ .\ P(A)}$$

Or

$$\boldsymbol{P\ (A|B) = \frac{P(B|A)\ .\ P(A)}{P(B)}}$$

And for the probability of event **B** given that event **A** has already been occurred.

$$\boldsymbol{P\ (B|A) = \frac{P(A|B)\ .\ P(B)}{P(A)}}$$

This **|** means **"Given That" …**

Let's take an Example

Likelihood   Prior Probability

$$P(spam \backslash lottery) = \frac{P(lottery \backslash spam) * P(spam)}{P(lottery)}$$

Posterior Probability   Marginal Likelihood