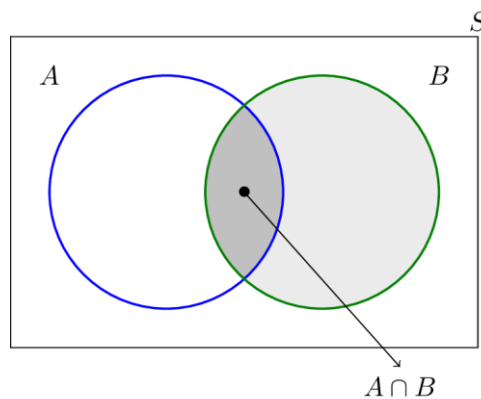


Conditional Probability

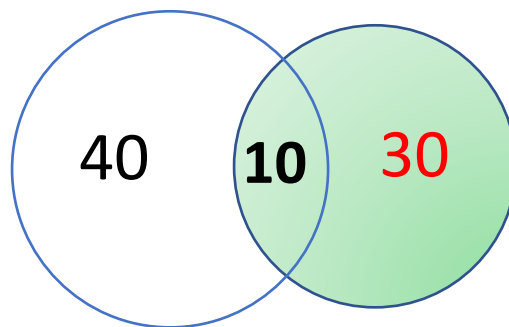
Find the probability of event **A** given that event **B** has already been occurred.

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Probability of A and B
Probability of A given B



$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$



$$P(B) = \frac{30}{80} = 0.375$$

$$P(A \cap B) = \frac{10}{80} = 0.125$$

$$P(A|B) = \frac{0.125}{0.375} = 0.333$$

Bayes Theorem

Statement of theorem

Bayes's theorem is stated mathematically as the following equation:

$$P(A | B) = \frac{P(B | A)P(A)}{P(B)}$$

where A and B are **events** and $P(B) \neq 0$.

- $P(A | B)$ is a **conditional probability**: the likelihood of event A occurring given that B is true.
- $P(B | A)$ is also a conditional probability: the likelihood of event B occurring given that A is true.
- $P(A)$ and $P(B)$ are the probabilities of observing A and B respectively; they are known as the **marginal probability**.

Step 01

Find the probability of event **A** given that event **B** has already been occurred.

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

Find the probability of event **B** given that event **A** has already been occurred.

$$P(B | A) = \frac{P(B \cap A)}{P(A)}$$

Step 02

Shifting Marginal Probability to Left Hand Side

$$P(A | B) \cdot P(B) = \frac{P(A \cap B)}{1}$$

$$P(B | A) \cdot P(A) = \frac{P(B \cap A)}{1}$$

Step 03

Since, $P(A \cap B) = P(B \cap A)$.

Therefore, we can say that

$$P(A \cap B) = P(A | B) \cdot P(B) = P(B | A) \cdot P(A)$$

So, we can say like this as well.

$$P(A | B) \cdot P(B) = P(B | A) \cdot P(A)$$

Or

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$

And for the probability of event **B** given that event **A** has already been occurred.

$$P(B|A) = \frac{P(A|B) \cdot P(B)}{P(A)}$$

This | means “**Given That**” ...

Let’s take an Example

The diagram shows the Naive Bayes formula for calculating the posterior probability of an email being spam given it is a lottery. The formula is:
$$P(\text{spam} \setminus \text{lottery}) = \frac{P(\text{lottery} \setminus \text{spam}) \cdot P(\text{spam})}{P(\text{lottery})}$$
 Four arrows point to different parts of the formula: 'Likelihood' points to $P(\text{lottery} \setminus \text{spam})$, 'Prior Probability' points to $P(\text{spam})$, 'Marginal Likelihood' points to $P(\text{lottery})$, and 'Posterior Probability' points to the entire left side of the equation, $P(\text{spam} \setminus \text{lottery})$.

$$P(\text{spam} \setminus \text{lottery}) = \frac{P(\text{lottery} \setminus \text{spam}) \cdot P(\text{spam})}{P(\text{lottery})}$$

Labels with arrows:

- Likelihood (points to $P(\text{lottery} \setminus \text{spam})$)
- Prior Probability (points to $P(\text{spam})$)
- Marginal Likelihood (points to $P(\text{lottery})$)
- Posterior Probability (points to $P(\text{spam} \setminus \text{lottery})$)