

Homework II

① Find A^{-1} of the following A matrix.

1)

$$A = \begin{bmatrix} 1 & -1 \\ 3 & 4 \end{bmatrix}; [A: I_2] = \left[\begin{array}{cc|cc} 1 & -1 & 1 & 0 \\ 3 & 4 & 0 & 1 \end{array} \right] \xrightarrow{+(-3)}$$

$$\text{divide } \frac{7}{7} \left[\begin{array}{cc|cc} 1 & -1 & 1 & 0 \\ 0 & 7 & -3 & 1 \end{array} \right] \sim \left[\begin{array}{cc|cc} 1 & -1 & 1 & 0 \\ 0 & 1 & -\frac{3}{7} & \frac{1}{7} \end{array} \right] \xrightarrow{+} \sim \left[\begin{array}{cc|cc} 1 & 0 & \frac{4}{7} & \frac{1}{7} \\ 0 & 1 & -\frac{3}{7} & \frac{1}{7} \end{array} \right] \xrightarrow{A^{-1}}$$

$$2) A = \begin{bmatrix} 2 & -1 \\ 5 & 0 \end{bmatrix}; [A: I_2] = \left[\begin{array}{cc|cc} 2 & -1 & 1 & 0 \\ 5 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\text{divide 2}}$$

$$\xrightarrow{(-5)} \left[\begin{array}{cc|cc} 1 & -\frac{1}{2} & \frac{1}{2} & 0 \\ 5 & 0 & 0 & 1 \end{array} \right] \sim \left[\begin{array}{cc|cc} 1 & -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \frac{5}{2} & -\frac{5}{2} & 1 \end{array} \right] \xrightarrow{\text{multiply } (\frac{2}{5})}$$

$$\xrightarrow{(\frac{1}{5})} \left[\begin{array}{cc|cc} 1 & -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 1 & -1 & \frac{2}{5} \end{array} \right] \sim \left[\begin{array}{cc|cc} 1 & 0 & 0 & \frac{1}{5} \\ 0 & 1 & -1 & \frac{2}{5} \end{array} \right] \xrightarrow{A^{-1}}$$

$$3) A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 4 & 1 \\ 3 & 7 & -3 \end{bmatrix}; [A: I_3] = \left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 1 & 0 & 0 \\ 2 & 4 & 1 & 0 & 1 & 0 \\ 3 & 7 & -3 & 0 & 0 & 1 \end{array} \right] \xrightarrow{(-2)} \xrightarrow{(-3)}$$

$$\text{divide } (4) \left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 1 & 0 & 0 \\ 0 & 4 & -5 & -2 & 1 & 0 \\ 0 & 7 & -12 & -3 & 0 & 1 \end{array} \right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 1 & 0 & 0 \\ 0 & 1 & -\frac{5}{4} & -\frac{1}{2} & \frac{1}{4} & 0 \\ 0 & 7 & -12 & -3 & 0 & 1 \end{array} \right] \xrightarrow{(-7)}$$

$$\text{multiply } (-\frac{4}{13}) \left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 1 & 0 & 0 \\ 0 & 1 & -\frac{5}{4} & -\frac{1}{2} & \frac{1}{4} & 0 \\ 0 & 0 & -\frac{13}{4} & \frac{1}{2} & -\frac{7}{4} & 1 \end{array} \right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 1 & 0 & 0 \\ 0 & 1 & -\frac{5}{4} & -\frac{1}{2} & \frac{1}{4} & 0 \\ 0 & 0 & 1 & \frac{2}{13} & \frac{7}{13} & -\frac{4}{13} \end{array} \right] \xrightarrow{(-\frac{5}{4})} \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{19}{13} & -\frac{21}{13} & \frac{12}{13} \\ 0 & 1 & 0 & -\frac{9}{13} & \frac{12}{13} & -\frac{5}{13} \\ 0 & 0 & 1 & \frac{2}{13} & \frac{7}{13} & -\frac{4}{13} \end{array} \right] \xrightarrow{A^{-1}}$$

$$\textcircled{+} A = \begin{bmatrix} 2 & 0 & 4 & 1 & 3 \\ 1 & 1 & 0 & 2 & 3 \\ 3 & -2 & 1 & 0 & 0 \end{bmatrix}$$

rank(A) = ?

row echelon matrix ?

normal form ?

multiply

$$\begin{array}{l} (-2) \rightarrow \\ \text{md} \rightarrow \\ \text{change} \\ \text{rows} \end{array} \begin{bmatrix} 2 & 0 & 4 & 1 & 3 \\ 1 & 1 & 0 & 2 & 3 \\ 3 & -2 & 1 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 0 & 2 & 3 \\ 0 & -2 & 4 & -3 & -3 \\ 3 & -2 & 1 & 0 & 0 \end{bmatrix} \xrightarrow{(-3)} \begin{bmatrix} 1 & 1 & 0 & 2 & 3 \\ 0 & -2 & 4 & -3 & -3 \\ 0 & -5 & 1 & -6 & -9 \end{bmatrix} \xrightarrow{\text{divide } (-2)}$$

$$(5) \begin{bmatrix} 1 & 1 & 0 & 2 & 3 \\ 0 & 1 & -2 & \frac{3}{2} & \frac{3}{2} \\ 0 & -5 & 1 & -6 & -9 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 0 & 2 & 3 \\ 0 & 1 & -2 & \frac{3}{2} & \frac{3}{2} \\ 0 & 0 & -9 & \frac{3}{2} & -\frac{3}{2} \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 0 & 2 & 3 \\ 0 & 1 & -2 & \frac{3}{2} & \frac{3}{2} \\ 0 & 0 & 1 & -\frac{1}{6} & \frac{1}{6} \end{bmatrix} \text{ row echelon matrix}$$

divide
(-9)

$(-\frac{7}{6})$

rank(A) = 3

$$\begin{array}{l} \sim \\ 2 \end{array} \begin{bmatrix} 1 & 1 & 0 & 2 & 3 \\ 0 & 1 & -2 & \frac{3}{2} & \frac{3}{2} \\ 0 & 0 & 1 & -\frac{1}{6} & \frac{1}{6} \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 0 & 2 & 3 \\ 0 & 1 & 0 & \frac{7}{6} & \frac{11}{6} \\ 0 & 0 & 1 & -\frac{1}{6} & \frac{1}{6} \end{bmatrix} \xrightarrow{(-1)} \begin{bmatrix} 1 & 0 & 0 & \frac{5}{6} & \frac{7}{6} \\ 0 & 1 & 0 & \frac{7}{6} & \frac{11}{6} \\ 0 & 0 & 1 & -\frac{1}{6} & \frac{1}{6} \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & \frac{7}{6} & \frac{11}{6} \\ 0 & 0 & 1 & -\frac{1}{6} & \frac{1}{6} \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -\frac{1}{6} & \frac{1}{6} \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} = [I_k \ 0]$$

$(-\frac{7}{6})$
 $(-\frac{11}{6})$

$(\frac{1}{6})$
 $(-\frac{1}{6})$

Normal
form

$$4) A = \begin{bmatrix} 1 & 0 & -1 & 2 \\ 0 & 2 & 4 & 3 \\ -1 & 1 & 1 & 2 \\ 2 & 0 & 1 & 1 \end{bmatrix}, [A: I_4] = \left[\begin{array}{cccc|cccc} 1 & 0 & -1 & 2 & 1 & 0 & 0 & 0 \\ 0 & 2 & 4 & 3 & 0 & 1 & 0 & 0 \\ -1 & 1 & 1 & 2 & 0 & 0 & 1 & 0 \\ 2 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \end{array} \right] \begin{matrix} \\ \\ (-2) \\ \end{matrix}$$

$$\sim \left[\begin{array}{cccc|cccc} 1 & 0 & -1 & 2 & 1 & 0 & 0 & 0 \\ 0 & 2 & 4 & 3 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 4 & 1 & 0 & 1 & 0 \\ 0 & 0 & 3 & -3 & -2 & 0 & 0 & 1 \end{array} \right] \begin{matrix} \\ \\ (-2) \\ \end{matrix} \sim \left[\begin{array}{cccc|cccc} 1 & 0 & -1 & 2 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 4 & 1 & 0 & 1 & 0 \\ 0 & 0 & 4 & -5 & -2 & 1 & -2 & 0 \\ 0 & 0 & 3 & -3 & -2 & 0 & 0 & 1 \end{array} \right] \begin{matrix} \\ \\ \text{divide } 4 \\ \end{matrix}$$

and change 2. and 3. row

$$\sim \left[\begin{array}{cccc|cccc} 1 & 0 & -1 & 2 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 4 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & -\frac{5}{4} & -\frac{1}{2} & \frac{1}{4} & -\frac{1}{2} & 0 \\ 0 & 0 & 3 & -3 & -2 & 0 & 0 & 1 \end{array} \right] \begin{matrix} \\ \\ (-3) \\ \end{matrix} \sim \left[\begin{array}{cccc|cccc} 1 & 0 & -1 & 2 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 4 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & -\frac{5}{4} & -\frac{1}{2} & \frac{1}{4} & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{3}{4} & -\frac{1}{2} & -\frac{3}{4} & \frac{3}{2} & 1 \end{array} \right] \begin{matrix} \\ \\ \text{multiply } \frac{4}{3} \\ \end{matrix}$$

$$\sim \left[\begin{array}{cccc|cccc} 1 & 0 & -1 & 2 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 4 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & -\frac{5}{4} & -\frac{1}{2} & \frac{1}{4} & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 & -\frac{2}{3} & -1 & 2 & \frac{4}{3} \end{array} \right] \begin{matrix} \\ \\ + \\ \end{matrix} \sim \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & \frac{3}{4} & \frac{1}{2} & \frac{1}{4} & -\frac{1}{2} & 0 \\ 0 & 1 & 0 & 4 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & -\frac{5}{4} & -\frac{1}{2} & \frac{1}{4} & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 & -\frac{2}{3} & -1 & 2 & \frac{4}{3} \end{array} \right] \begin{matrix} \\ \\ (-4) \\ \end{matrix}$$

$\frac{5}{4}$ $(-\frac{3}{4})$

$$\sim \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 1 & -2 & -1 \\ 0 & 1 & 0 & 0 & \frac{11}{3} & 4 & -7 & -\frac{16}{3} \\ 0 & 0 & 1 & 0 & -\frac{4}{3} & -1 & 2 & \frac{5}{3} \\ 0 & 0 & 0 & 1 & -\frac{2}{3} & -1 & 2 & \frac{4}{3} \end{array} \right]$$

A^{-1}