Ters Trigonometrik Fonksiyonlar.

$y = \arcsin x = \sin^{-1}x$ TK: $[-1,1]$ GK: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

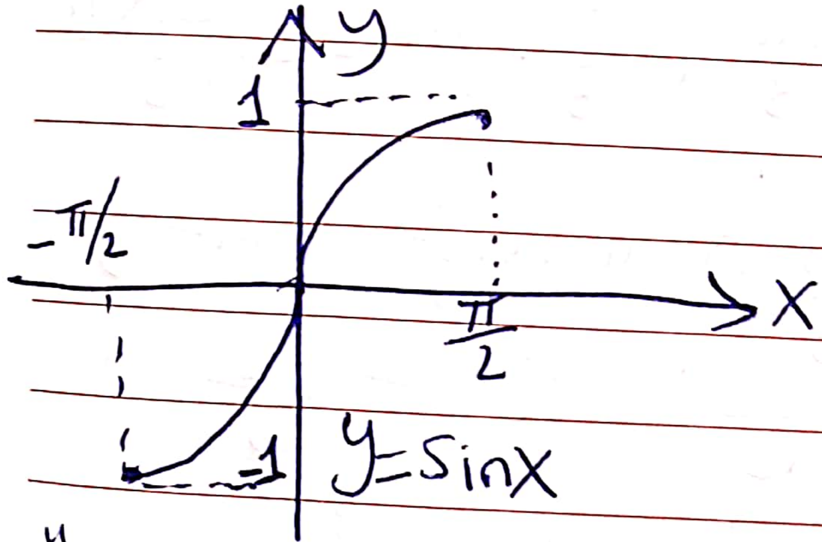
her iki tarafın sinüsü alınırsa.

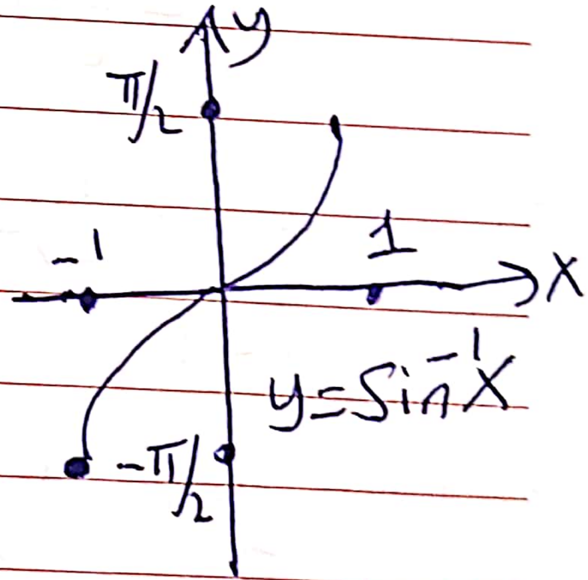
$\sin y = \sin(\arcsin x)$ $f \circ f^{-1}(x) = x$ den.

$\sin y = x$ $f^{-1}(x) = \sin x$ yani $\sin^{-1}x$'in

ters fonksiyonu $f^{-1}(x) = \sin x$ olur.

$y = \sin x$ fonksiyonunda $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \to [-1, +1]$ olur.





Örnekler:

1-) $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) = ?$ 2) $\sin^{-1}\left(\frac{1}{2}\right) = ?$ 3) $\sin^{-1}(-1) = ?$

C: $-\frac{\pi}{4}$ C: $\frac{\pi}{6}$ C: $-\pi/2$

2) $\cos\left(\arcsin\frac{1}{2} - \arcsin 0\right) = ?$ $\arcsin 0 = 0$

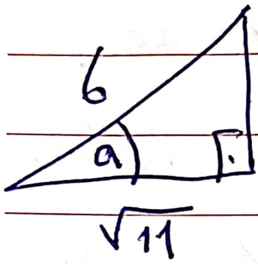
$\arcsin\frac{1}{2} = \frac{\pi}{6}$

$\cos\left(\frac{\pi}{6} - 0\right) = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

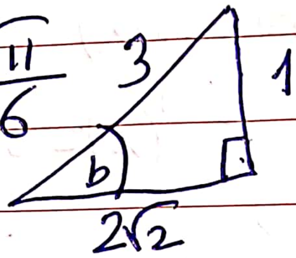
3-) $\sin\left(\arcsin\frac{5}{6}+\arcsin\frac{1}{3}\right)=?$ $b=\arcsin\frac{1}{3}$

$\sin b=\frac{1}{3}$

$a=\arcsin\frac{5}{6}$ $\sin a=\frac{5}{6}$



$\cos a=\frac{\sqrt{11}}{6}$



$\cos b=\frac{2\sqrt{2}}{3}$

$\sin(a+b)=\sin a\cos b+\sin b\cos a$

$=\frac{5}{6}\cdot\frac{2\sqrt{2}}{3}+\frac{1}{3}\cdot\frac{\sqrt{11}}{6}=\frac{10\sqrt{2}+\sqrt{11}}{18}$ //

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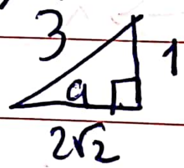
4) $\cos\left(\arcsin\frac{1}{3}\right)=?$ $a=\arcsin\frac{1}{3}$ $\sin a=\frac{1}{3}$

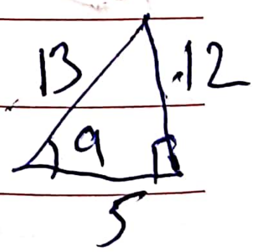
$\cos a=?$



$\cos a=\frac{2\sqrt{2}}{3}$ //

5) $\tan\left(\frac{3\pi}{2}-\cos^{-1}\frac{5}{13}\right)=?$ $a=\cos^{-1}5/13$

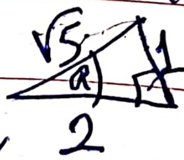
$\cos a=5/13$



$\tan\left(\frac{3\pi}{2}-a\right)=\cot a=\frac{12}{5}$ //

6) $\sin(2\,\mathrm{arccot}\,2)=?$ $a=\mathrm{arccot}\,2$ $\sin 2a=?$

$2\sin a\cos a$



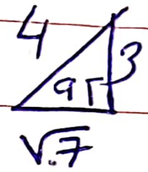
$\cot a=2$ $\sin a=\frac{1}{\sqrt{5}}$

$\cos a=\frac{2}{\sqrt{5}}$

$2\cdot\frac{1}{\sqrt{5}}\cdot\frac{2}{\sqrt{5}}=\frac{4}{5}$ //

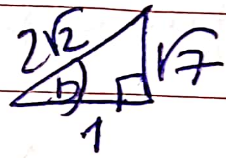
7-) $\tan\left(\sin^{-1}\frac{3}{4} - \cos^{-1}\frac{1}{2\sqrt{2}}\right) = ?$

$a = \sin^{-1}\frac{3}{4}$ $\quad \sin a = \frac{3}{4}$



$\cos a = \frac{\sqrt{7}}{4}$

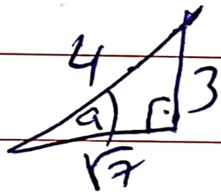
$b = \cos^{-1}\frac{1}{2\sqrt{2}}$ $\quad \cos b = \frac{1}{2\sqrt{2}}$

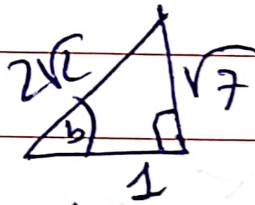


$\sin b = \frac{\sqrt{7}}{2\sqrt{2}}$

$\tan(a-b) = \frac{\tan a - \tan b}{1 + \tan a \cdot \tan b}$ $\qquad \tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \cdot \tan b}$

$\tan 2a = \frac{2\tan a}{1 - \tan^2 a}$ $\qquad \tan a = ?\quad \tan b = ?$





$\tan(a-b) = ?$

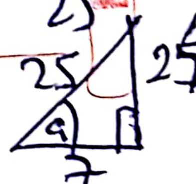
$\tan a = \frac{3}{\sqrt{7}}$ $\quad \tan b = \frac{\sqrt{7}}{1}$

$\tan(a-b) = \frac{\frac{3}{\sqrt{7}} - \sqrt{7}}{1 + \frac{3}{\sqrt{7}} \cdot \sqrt{7}} = \frac{\frac{-4}{\sqrt{7}}}{4} = -\frac{\sqrt{7}}{7}$ ✓//

8) $\arcsin\left(\sin\frac{4\pi}{3}\right) = ?$ $\quad \arcsin\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$

9) $\tan\left(\arcsin\frac{24}{25}\right) = ?$ $\quad a = \arcsin\frac{24}{25}$

$\sin a = \frac{24}{25}$



$\tan a = ?$ $\quad \cos a = 7/25$ $\quad \tan a = \frac{24}{7}$ yedi

$y = \arccos x = \cos^{-1} x$ fonksiyonu

TK: $[-1, 1]$ G.K: $[0, \pi]$ GK: Görüntü Kümesi

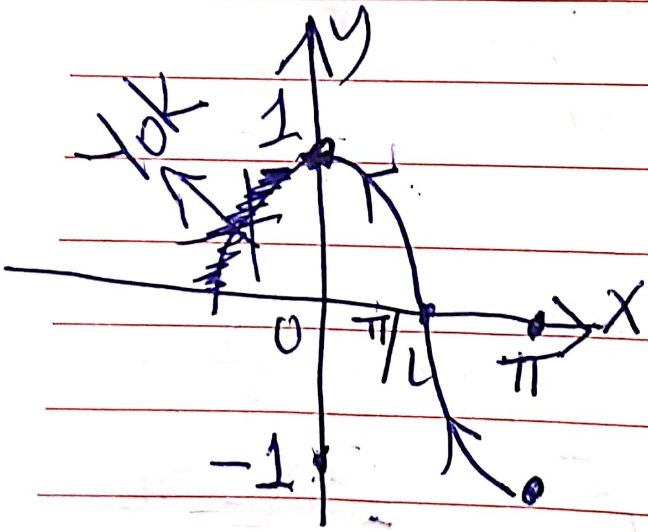
TK: Tanım Kümesi

her iki tarafın Cosinüsü alınırsa

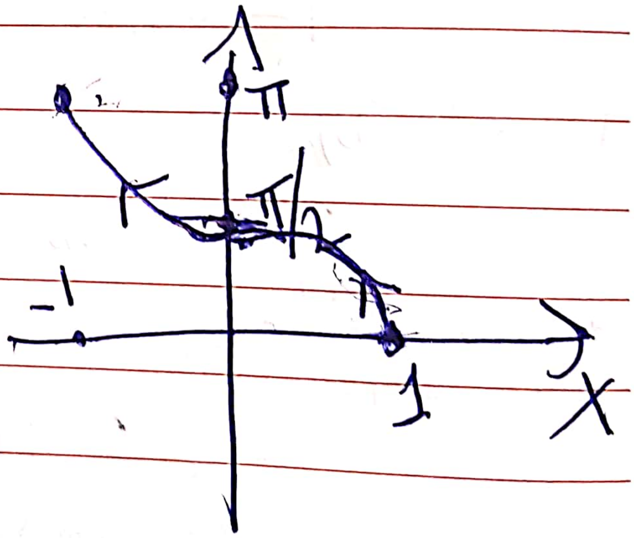
$\cos y = x$ $f^{-1}(x) = \cos x$ ($\cos^{-1} x$'in Ters fonksiyonu

($\cos x$'in ters fonksiyonu $\cos^{-1} x = \arccos x$ dir.

$y = \cos x$ $[0, \pi] \rightarrow [-1, 1]$



$y = \cos x$ fonksiyonunun grafiği



$y = \cos^{-1} x$ fonksiyonunun grafiği

$y = \arctan x = \tan^{-1} x$ fonksiyonu

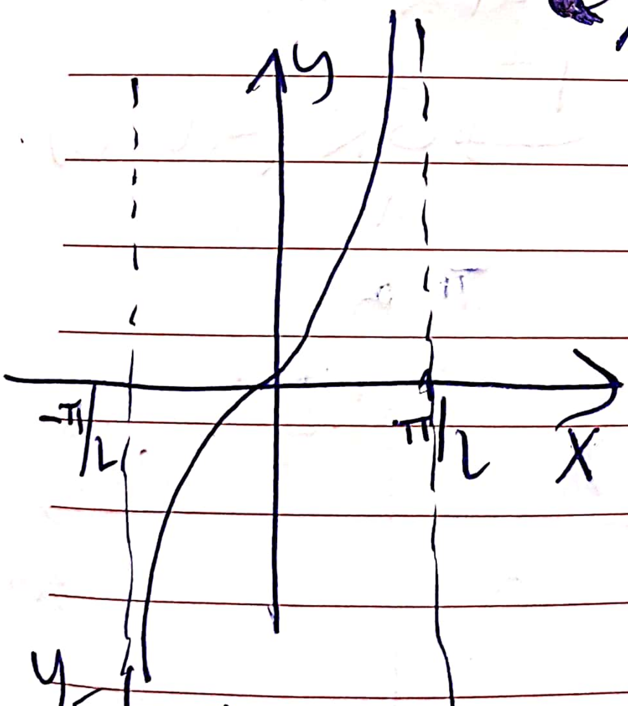
TK: $(-\infty, \infty) \xrightarrow{f}$ G.K. $(-\pi/2, \pi/2)$

her iki tarafın tanjantı alınırsa.

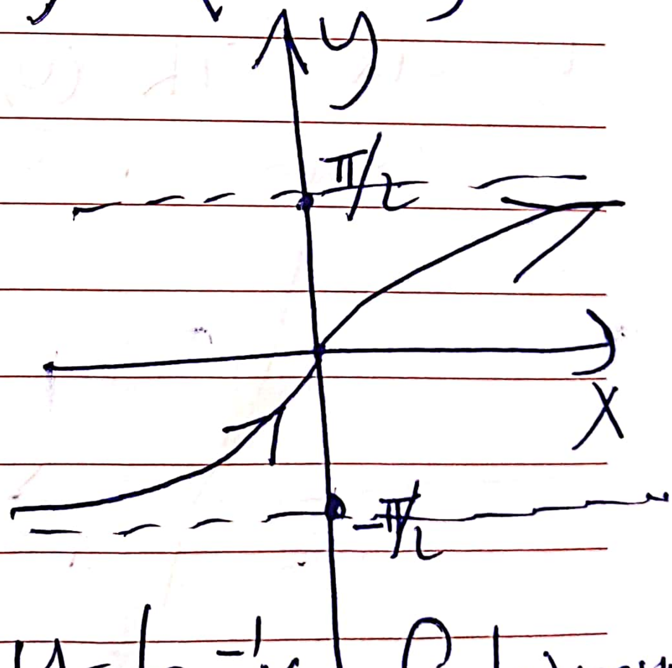
$\tan y = x$ $\quad f^{-1}(x) = \tan x$ ($\tan^{-1}x$'in ters fonksiyonu)

$\tan x$'in ters fonksiyonu ise $\tan^{-1}x$ dir.

$y = \tan x \quad \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \xrightarrow{f} (-\infty, \infty)$



$y = \tan x$ fonksiyonun grafiği



$y = \tan^{-1} x$ fonksiyonunun grafiği

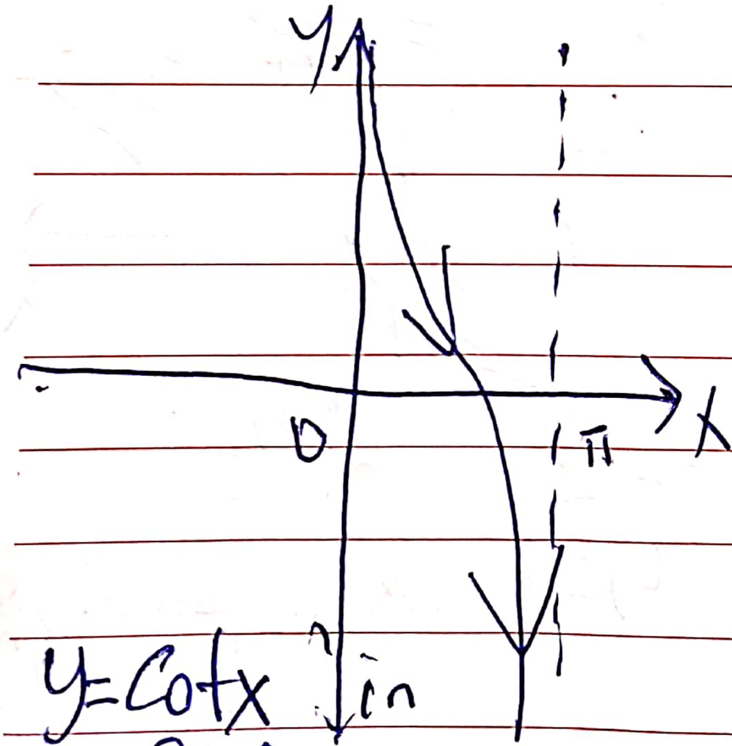
$y = \cot^{-1}x = \text{arccot}\,x$ fonksiyonu

T.K. $(-\infty, \infty) \xrightarrow{f}$ GK $(0, \pi)$

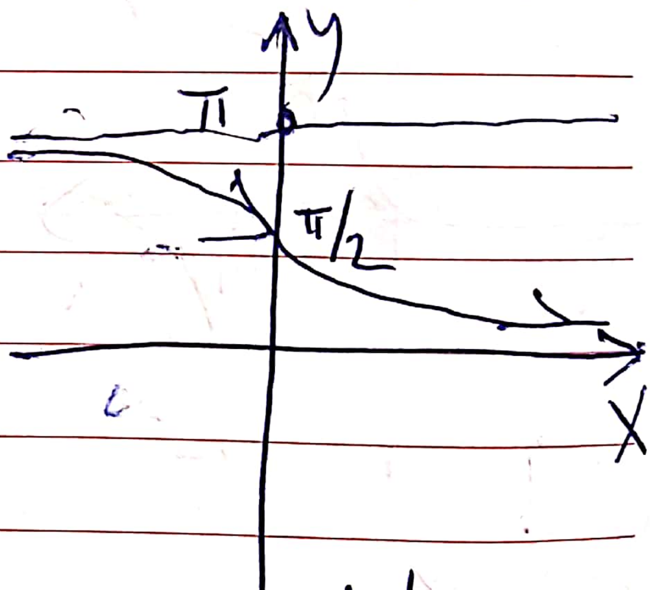
her iki tarafın Cotanjantı alınırsa,

$\cot y = x$, $f^{-1}(x) = \cot x$ elde edilir. Yani $y = \cot^{-1}x$ in ters fonksiyonu $y = \cot x$, $y = \cot x$'in ters fonksiyonu $\cot^{-1}x$ dir.

$y = \cot x$ TK $(0, \pi) \xrightarrow{f}$ GK $(-\infty, \infty)$



$y = \cot x$'in grafiği



$y = \cot^{-1}x$ in grafiği

$y = \sec^{-1}x = \operatorname{arcsec} x$ fonksiyonu

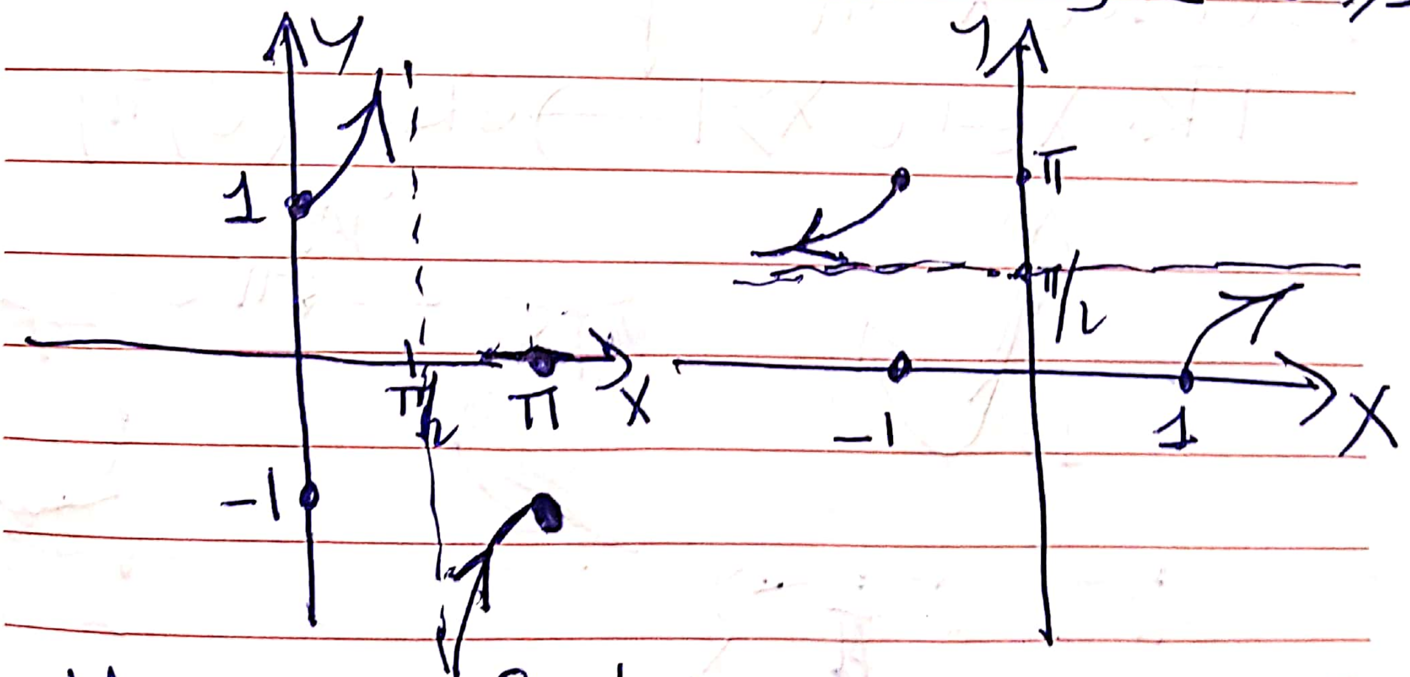
TK: $x \leq -1 \cup x \geq 1 \xrightarrow{f} GK: [0, \pi] \setminus \{\frac{\pi}{2}\}$

$y = \operatorname{arcsec} x$'in her iki tarafının sekantı alınırsa.

$\sec y = x$ $\quad f^{-1}(x) = \sec x$ elde edilir.

$\sec^{-1}x$'in ters fonksiyonu $\sec x$, $\sec x$'in ters fonksiyonu $\sec^{-1}x$ dir.

$y = \sec x$ TK $[0, \pi] \setminus \{\frac{\pi}{2}\} \xrightarrow{f} x \leq -1 \cup x \geq 1$



$y = \sec x$ fonksiyonunun grafiği

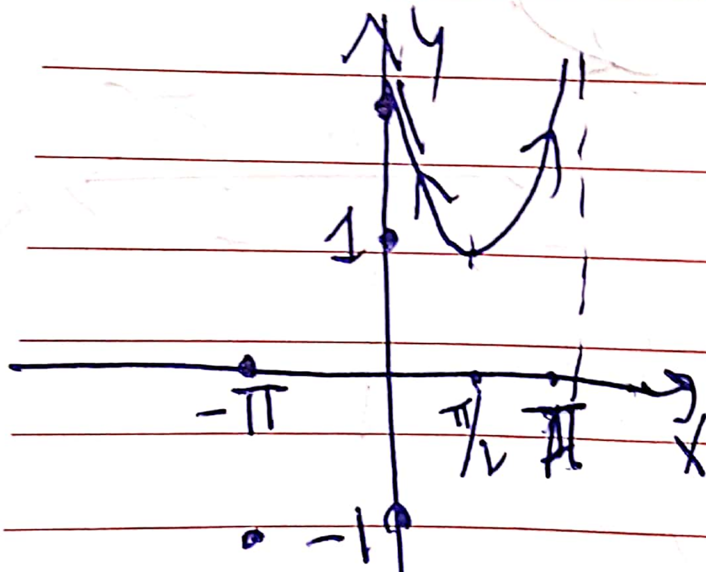
$y=Cosec^{-1}x=arccosecx$ fonksiyonu
her iki tarafın kosecantı alınırsa
$Cosecy=x$, $f^{-1}(x)=Cosecx$
$Cosecx$'in ters fonksiyonu $Cosec^{-1}x$
$Cosec^{-1}x$ in " " $Cosecx$ dir.

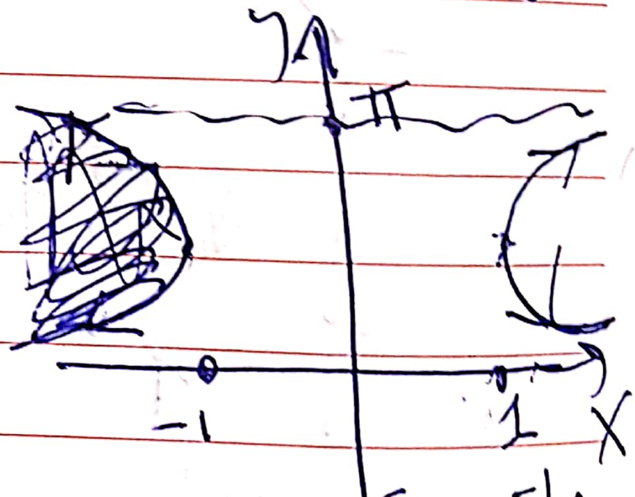
$y=Cosec^{-1}x$ in

TK: $(0,\pi) \xrightarrow{f}$ GK $x\leq-1 \cup x\geq1$

$y=Cosec^{-1}x$ in

TK: $x\leq-1 \cup x\geq1 \xrightarrow{f}$ GK $(0,\pi)$





$y=Cosecx$ fonksiyonunun grafiği

$y=Cosec^{-1}x$ fonksiyonunun grafiği.

$y = \arcsin x \qquad y' = \frac{1}{\sqrt{1-x^2}}$

$y = \arccos x \qquad y' = -\frac{1}{\sqrt{1-x^2}}$

$y = \arctan x \qquad y' = \frac{1}{1+x^2}$

$y = \mathrm{arccot}\, x \qquad y' = -\frac{1}{1+x^2}$

$y = \mathrm{arcsec}\, x \qquad y' = \frac{1}{|x|\sqrt{x^2-1}}$

$y = \mathrm{arccsec}\, x \qquad y' = \frac{-1}{|x|\sqrt{x^2+1}}$