Örnek: P_2 polinomlar uzayının T sıralı bazından

$S=\{\underbrace{x^2+x}_{v_1},\ \underbrace{x-2}_{v_2},\ \underbrace{x}_{v_3}\}$ sıralı bazına geçiş matrisi

$P=\begin{bmatrix}2&0&1\\1&5&4\\3&-2&6\end{bmatrix}$ olduğuna göre T sıralı bazını bulunuz.

$\downarrow$

$[M]_T^S$

$$[\underbrace{\quad}_{S} \mid \underbrace{\quad}_{T}] \sim \cdots [\underbrace{\quad}_{I} \mid \underbrace{\quad}_{[M]_T^S}] \qquad S=\begin{bmatrix}1&0&0\\1&1&1\\0&-2&0\end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc}1&0&0&2&0&1\\0&1&0&1&5&4\\0&0&1&3&-2&6\end{array}\right] \sim \left[\begin{array}{ccc|ccc}1&0&0&2&0&1\\0&0&1&3&-2&6\\0&1&0&1&5&4\end{array}\right] \sim \left[\begin{array}{ccc|ccc}1&0&0&2&0&1\\1&0&1&5&-2&7\\0&1&0&1&5&4\end{array}\right]$$

H_{23} $\qquad$ $H_{21}(1)$ $\qquad$ $H_{23}(1)$

$$\sim \left[\begin{array}{ccc|ccc}1&0&0&2&0&1\\1&1&1&6&3&11\\0&1&0&1&5&4\end{array}\right] \sim \left[\begin{array}{ccc|ccc}1&0&0&2&0&1\\1&1&1&6&3&11\\0&-2&0&-2&-10&-8\end{array}\right]$$

$H_3(-2)$ $\qquad$ (S $\mid$ T)

$$T=\{2x^2+6x-2,\ 3x-10,\ x^2+11x-8\}$$

b-) $[D]_T=\begin{bmatrix}3\\2\\1\end{bmatrix}$ ise P matrisinden yararlanarak $[D]_S$ değerini bulunuz.

$P=[M]_T^S$

$$[D]_S=[M]_T^S\,[D]_T$$

$$=\begin{bmatrix}2&0&1\\1&5&4\\3&-2&6\end{bmatrix}\begin{bmatrix}3\\2\\1\end{bmatrix}=\begin{bmatrix}7\\17\\11\end{bmatrix}$$

4-) **2-)** $v_1 = \begin{bmatrix}1\\2\end{bmatrix}, v_2 = \begin{bmatrix}0\\1\end{bmatrix}$ olmak üzere $\mathcal{M}_{2x1}$ in sıralı tabanları $S = \{v_1, v_2\}$ ve $T \doteq \{w_1, w_2\}$ olsun. S den T ye geçiş matrisi $M_S^T = \begin{bmatrix}2 & 1\\1 & 1\end{bmatrix}$ olduğuna göre T tabanını belirleyiniz. ($\mathcal{M}_{2x1}$: 2x1 mertebeli ve reel elemanlı matrisler uzayıdır)

$$\left[\begin{matrix}1 & 0\\0 & 1\end{matrix}\middle|\underbrace{\begin{matrix}2 & 1\\1 & 1\end{matrix}}_{[M]^T_S}\right] \sim \text{--} \left[\underbrace{\quad}_{T}\middle|\underbrace{\quad}_{S}\right] \qquad S = \begin{bmatrix}1 & 0\\2 & 1\end{bmatrix}$$

$$\left[\begin{matrix}1 & 0\\0 & 1\end{matrix}\middle|\begin{matrix}2 & 1\\1 & 1\end{matrix}\right] \sim \left[\begin{matrix}1 & -1\\0 & 1\end{matrix}\middle|\begin{matrix}1 & 0\\1 & 1\end{matrix}\right] \sim \left[\underbrace{\begin{matrix}1 & -1\\1 & 0\end{matrix}}_{T}\middle|\underbrace{\begin{matrix}1 & 0\\2 & 1\end{matrix}}_{S}\right]$$

$$T = \{w_1, w_2\} \qquad w_1 = \begin{bmatrix}1\\1\end{bmatrix}, w_2 = \begin{bmatrix}-1\\0\end{bmatrix}$$

$$T = \left\{\begin{bmatrix}1\\1\end{bmatrix}, \begin{bmatrix}-1\\0\end{bmatrix}\right\}$$

Başarılar...

Örnek: R^2 uzayının $S=\left\{\begin{bmatrix}1\\2\end{bmatrix},\begin{bmatrix}-1\\0\end{bmatrix}\right\}$ sıralı tabanı veriliyor.

a-) S tabanından T tabanına geçiş matrisi $\begin{bmatrix}3&-1\\1&0\end{bmatrix}$ ise T tabanını bulunuz.

b-) T tabanından S tabanına geçiş matrisini bulunuz.

a-) $$\left[\underbrace{\quad\quad}_{T}\;\middle|\;\underbrace{\begin{matrix}1&-1\\2&0\end{matrix}}_{S}\right]\sim\cdots\sim\left[\underbrace{\quad\quad}_{I}\;\middle|\;\underbrace{\begin{matrix}3&-1\\1&0\end{matrix}}_{[M]_S^T}\right]$$

$$\left[\begin{matrix}1&0\\0&1\end{matrix}\;\middle|\;\begin{matrix}3&-1\\1&0\end{matrix}\right]\sim\left[\begin{matrix}1&-2\\0&1\end{matrix}\;\middle|\;\begin{matrix}1&-1\\1&0\end{matrix}\right]\sim\left[\underbrace{\begin{matrix}1&-2\\0&2\end{matrix}}_{T}\;\middle|\;\underbrace{\begin{matrix}1&-1\\2&0\end{matrix}}_{S}\right]$$

$H_{12}(-2)$ $\qquad$ $H_2(2)$

$$T=\begin{bmatrix}1&-2\\0&2\end{bmatrix}\rightarrow T=\left\{\begin{bmatrix}1\\0\end{bmatrix},\begin{bmatrix}-2\\2\end{bmatrix}\right\}$$

b-) $$\left[\underbrace{\begin{matrix}1&-1\\2&0\end{matrix}}_{S}\;\middle|\;\underbrace{\begin{matrix}1&-2\\0&2\end{matrix}}_{T}\right]\sim\left[\begin{matrix}1&-1\\0&2\end{matrix}\;\middle|\;\begin{matrix}1&-2\\-2&6\end{matrix}\right]\sim\left[\begin{matrix}1&-1\\0&1\end{matrix}\;\middle|\;\begin{matrix}1&-2\\-1&3\end{matrix}\right]$$

$H_{21}(-2)$ $\qquad$ $H_2(1/2)$ $\qquad$ $H_{12}(1)$

$$\sim\left[\underbrace{\begin{matrix}1&0\\0&1\end{matrix}}_{I}\;\middle|\;\underbrace{\begin{matrix}0&1\\-1&3\end{matrix}}_{[M]_T^S}\right]\qquad [M]_T^S=\begin{bmatrix}0&1\\-1&3\end{bmatrix}$$

Örnek: 2) R^3 uzayının $S=\left\{\begin{bmatrix}1\\0\\0\end{bmatrix},\begin{bmatrix}0\\2\\1\end{bmatrix},\begin{bmatrix}0\\1\\3\end{bmatrix}\right\}$ sıralı tabanı veriliyor. T tabanından S tabanına geçiş matrisi $\begin{bmatrix}1&3&-2\\2&-1&1\\3&1&0\end{bmatrix}$ ise T tabanını bulunuz.

$$\left[\underbrace{\quad}_{S}\ \middle|\ \underbrace{?}_{T}\right] \sim \cdots \left[\underbrace{\quad}_{I}\ \middle|\ \underbrace{\quad}_{[M]_T^S}\right] \qquad S=\begin{bmatrix}1&0&0\\0&2&1\\0&1&3\end{bmatrix}$$

$$[M]_T^S=\begin{bmatrix}1&3&-2\\2&-1&1\\3&1&0\end{bmatrix} \qquad \underset{H_{23}(3)}{\left[\begin{array}{ccc|ccc}1&0&0&1&3&-2\\0&1&0&2&-1&1\\0&0&1&3&1&0\end{array}\right]} \sim \underset{H_3(5)}{\left[\begin{array}{ccc|ccc}1&0&0&1&3&-2\\0&1&3&11&2&1\\0&0&1&3&1&0\end{array}\right]}$$

$$\sim \underset{H_{32}(-2)}{\left[\begin{array}{ccc|ccc}1&0&0&1&3&-2\\0&1&3&11&2&1\\0&0&5&15&5&0\end{array}\right]} \sim \underset{H_3(-1)}{\left[\begin{array}{ccc|ccc}1&0&0&1&3&-2\\0&1&3&11&2&1\\0&-2&-1&-7&1&-2\end{array}\right]} \sim \underset{H_{23}}{\left[\begin{array}{ccc|ccc}1&0&0&1&3&-2\\0&1&3&11&2&1\\0&2&1&7&-1&2\end{array}\right]}$$

$$\sim \left[\underbrace{\begin{array}{ccc}1&0&0\\0&2&1\\0&1&3\end{array}}_{S}\ \middle|\ \underbrace{\begin{array}{ccc}1&3&-2\\7&-1&2\\11&2&1\end{array}}_{T}\right] \qquad T=\left\{\begin{bmatrix}1\\7\\11\end{bmatrix},\begin{bmatrix}3\\-1\\2\end{bmatrix},\begin{bmatrix}-2\\2\\1\end{bmatrix}\right\}$$

b-) P_2 nin $S = \{\underbrace{x^2+1}_{v_1}, \underbrace{x+1}_{v_2}, \underbrace{x^2+x}_{v_3}\}$ sıralı tabanı veriliyor. $v = x^2+3$ ise $[v]_S = ?$

$$v = c_1 v_1 + c_2 v_2 + c_3 v_3$$

$$x^2+3 = c_1(x^2+1) + c_2(x+1) + c_3(x^2+x)$$

$$x^2+3 = x^2(c_1+c_3) + x(c_2+c_3) + c_1 + c_2$$

$$\left.\begin{aligned} c_1 + c_3 &= 1 \\ c_2 + c_3 &= 0 \\ c_1 + c_2 &= 3 \end{aligned}\right\} \rightsquigarrow c_2 = -c_3$$

$$\begin{aligned} c_1 + c_3 &= 1 \\ c_1 - c_3 &= 3 \\ \hline 2c_1 &= 4 \end{aligned}$$

$c_1 = 2 \quad c_3 = -1 \quad c_2 = 1$

$$[v]_S = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}_{//}$$