YILDIZ TECHNICAL UNIVERSITY
FACULTY OF ELECTRIAL & ELECTRONICS / DEPARTMENT OF COMPUTER ENGINEERING

BLM 2502: Theory of Computation — Spring 2018-2019

Midterm Exam 1

Print first name: Solution Manual

Print family last name: ______

Print student number: ______

I have read and understand all of the instructions below, and I will obey the Academic Integrity Code of Yıldız Technical University.

______
Signature and Date

- Note the number written on the upper right-hand corner of the first page. On the sign-up sheet being passed around, sign your name next to this number.
- This exam will be 1 hour and 20 minutes in length.
- This exam has 6 pages in total, numbered 1 to 6. Make sure your exam has all the pages.
- There are 5 questions.
- This is a closed-book and closed-note exam. Electronic devices (e.g., cellphone, smart watch, calculator) are not allowed.
- For all problems, follow these instructions:
  - Give only your answers in the spaces provided. Only what you put in the answer space will be graded, and for any scratch work in the answer space, the points will be taken off. Use the scratch-work area or the backs of the sheets to work out your answers before filling in the answer space.
  - DFA stands for deterministic finite automaton; NFA stands for nondeterministic finite automaton; ε denotes the empty string; Σ denotes the alphabet, $*$ denotes Kleene's star operator; R denotes the reverse operator.
  - For any proofs, be sure to provide a step-by-step argument, with justifications for every step.

Problem	1	2	3	4	5	Total
	20	20	20	20	20	100
Points	20	20	20	20	20	100

1. **[20 points]** For each of the following, circle TRUE if the statement is correct. Otherwise, circle FALSE. Each FALSE answer should have to be at most a few sentences. *Your FALSE answers that have no explanations will not be graded.*

(a) TRUE FALSE — If A is a non-empty set and B is an empty set, then $A \cdot B = A$.

Since $A \cdot B = \emptyset$

(b) TRUE FALSE — If $A \subseteq B$ and B is a regular language, then A must be regular.

For example, let A be $A = \{a^n b^n \mid n \geq 0\}$ and $A \subset L(a^* b^*)$.

(c) TRUE FALSE — If A is a finite language, then $\bar{A}$ is a context-free language (CFL).

(d) TRUE FALSE — If the language A has a deterministic finite automaton (DFA) that has at least one loop, then the language A is finite.

Loop results in infinite language

(e) TRUE FALSE — If the language A is infinite and the language B is finite, then the language $A \cap B$ is not regular.

Since $A \cap B$ is regular because of $|A \cap B| \leq |B| < \infty$.

(f) TRUE FALSE — The language $A = \{10^n 0^{2n} 1^m \mid m, n \geq 0\}$ over $\Sigma = \{0,1\}$ is a non-regular language.

since it has a regular expression, that is $r = 1(000)^* 1^*$.

(g) TRUE FALSE — The language defined by the regular expression $r = a(a \cup b)^* a$ is the set of all strings that begin and end with a over $\Sigma = \{a, b\}$.

since the string "a" starts and ends with "a" but it is not the element of $L(a(a \cup b)^* a)$.

(h) TRUE FALSE — The deterministic finite automaton (DFA) has at least one transition loop if the language it recognizes has the infinite number of strings.

(i) TRUE FALSE — A context-free grammar is not ambiguous if it has more than one parsing tree at least for one string of its language.

It is ambiguous if it has more than one parsing tree exist for at least one string.

(j) TRUE FALSE — For a state in a non-deterministic finite automaton (DFA), the number of incoming transitions from all states is always equal to $|\Sigma|$, where Σ denotes the alphabet.

The sentence is wrong. The correc is that the number of outgoing tansitions from a state is $|\Sigma|$.

2. **[20 points]** In the proofs of the following theorems, you are expected to use the contradiction and induction proof techniques, respectively.

(a) Suppose $n \in \mathbb{Z}$. n is odd if and only if $n^2 - 2n + 7$ is even. Prove this statement by using contradiction proof technique.

We will use the method of contradiction.

Let us assume n is even, then n^2-2n+7 is even.

if n is even, we have $n=2k$, accordingly

$$\begin{aligned} n^2-2n+7 &= (2k)^2-2(2k)+7 \\ &= 4k^2-4k+7 \\ &= 4k^2-4k+8-1 \quad \text{or} \quad 4k^2-4k+6+1 \\ &= 2(2k^2-2k+4)-1 \quad \text{or} \quad 2(2k^2-2k+3)+1 \\ &= 2m-1 \quad \text{or} \quad 2\hat{m}+1 \end{aligned}$$

so n^2-2n+7 is odd

which contradicts with the assumption so Theorem is true. $\square$

(b) Suppose $m \in \mathbb{R}$ and $n \in \mathbb{N}$. Prove the following equality by using induction proof technique;

$$m^0 + m^1 + m^2 + m^3 + \cdots + m^n = \frac{1 - m^{n+1}}{1 - m}$$

Let us follow the induction step by step.

for $n=0 \Rightarrow m^0 \Leftrightarrow \frac{1-m^{0+1}}{1-m} = 1$ ✓

for $n=1 \Rightarrow m^0+m^1 = 1+m \Leftrightarrow \frac{1-m^{1+1}}{1-m} = \frac{1-m^2}{1-m} = 1+m$ ✓

for $n=2 \Rightarrow m^0+m^1+m^2 = 1+m+m^2 \Leftrightarrow \frac{1-m^{2+1}}{1-m} = \frac{1-m^3}{1-m} = 1+m+m^2$ ✓

$\vdots$

for $n=n \Rightarrow m^0+m^1+m^2+\dots+m^n \Leftrightarrow \frac{1-m^{n+1}}{1-m}$ ✓ (Assumption)

for $n+1 \Rightarrow m^0+m^1+\dots+m^n+m^{n+1} \Leftrightarrow \frac{1-m^{n+1}}{1-m} + m^{n+1}$

$= 1+m+m^2+\dots+m^n+m^{n+1}$ ✓

So Theorem is correct $\square$

3. **[20 points]** Give short answers to each of the following parts for the alphabet $\Sigma = \{a, b\}$. Each answer should be at most a few sentences. Be sure to define any notation that you use.

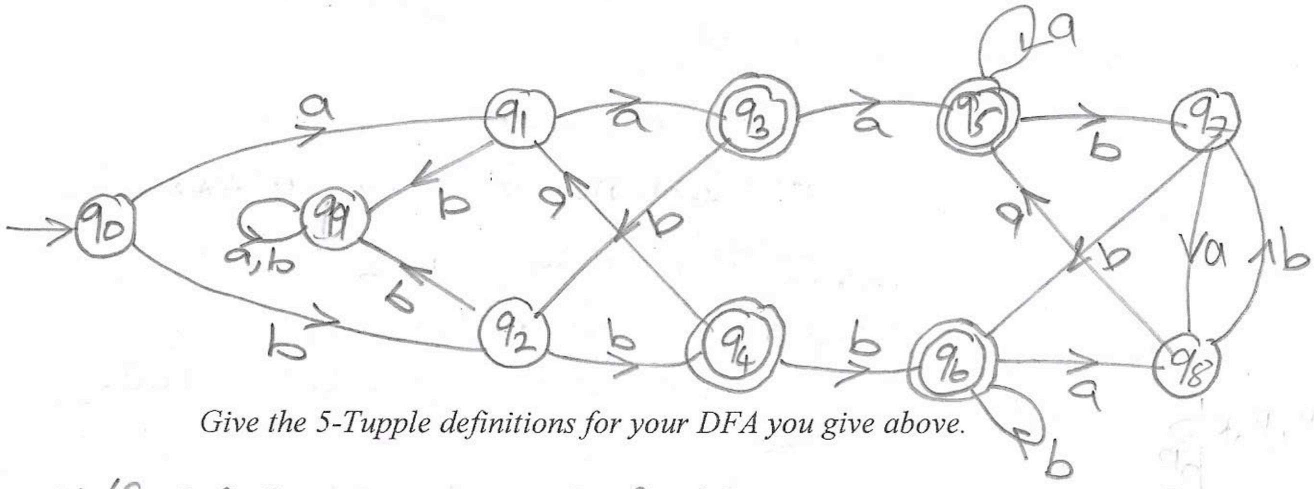
It is said that a string $w \in \Sigma^*$ starts with a double letter if its first two symbols are aa or bb.

(a) Let $A = \{\, w \in \Sigma^* \mid w$ starts and ends with a double letter $\}$.

Give a regular expression for the language A.

$r = (aa+bb)(a+b)^*(aa+bb) + aaa + bbb + aa + bb$

Draw your DFA for the language A.



Give the 5-Tupple definitions for your DFA you give above.

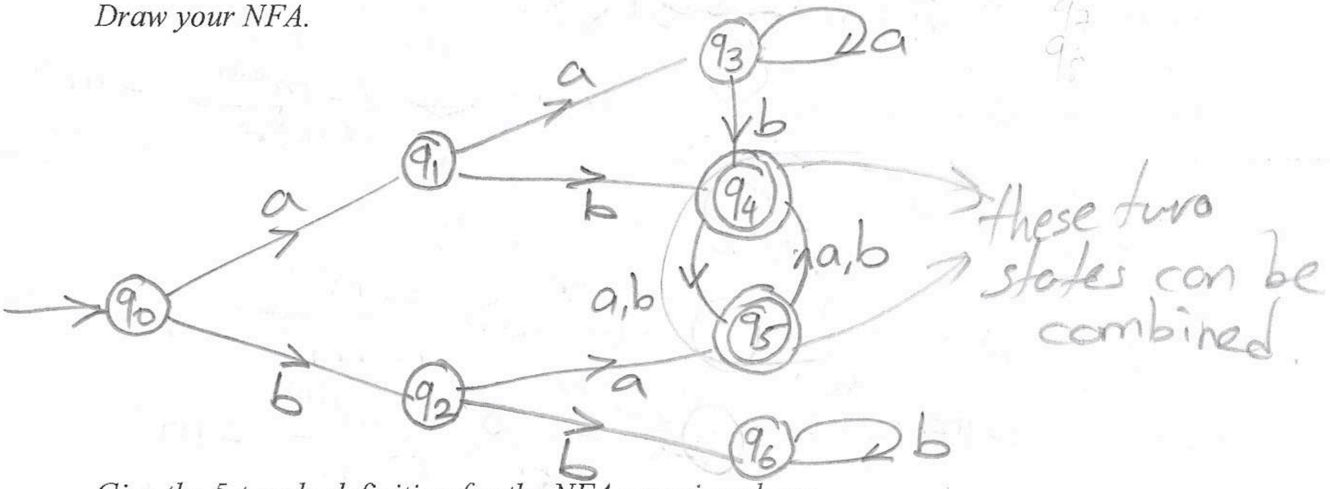
$M(\{q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8, q_9\}, \{a, b\}, \delta, q_0, \{q_3, q_4, q_5, q_6\})$

(b) Let $B = \{\, w \in \Sigma^* \mid w$ contains at least one a and at least one b $\}$.

Give your regular expression.

$r = (a+b)^* a (a+b)^* b (a+b)^* + (a+b)^* b (a+b)^* a (a+b)^*$

Draw your NFA.



Give the 5-tupple definition for the NFA you give above.

$M(\{q_0, q_1, q_2, q_3, q_4, q_5, q_6\}, \{a, b\}, \delta, q_0, \{q_4, q_5\})$

4. **[20 points]** Recall the pumping lemma for regular languages:

Theorem: *If L is a regular language, then there exists a pumping length m where, if $w \in L$ with $|w| \geq m$, then w can be split into three pieces $w = xyz$ such that*

i) $xy^{\ell}z \in L$ *for each* $\ell \geq 0$,

ii) $|y| \geq 1$,

iii) $|xy| \leq m$.

Consider the language $A = \{a^{m \bmod n} b^{n \bmod m} | m > 0 \text{ and } n > 0\}$ over the alphabet $\Sigma = \{a, b\}$. Is the language A regular or non-regular? If the language A is regular, give its regular expression; otherwise prove that it is non-regular.

Let us assum $m = 3$ and $n = 5$ (two primes).

$m \bmod n = 3 \bmod 5 = 3$

$n \bmod m = 5 \bmod 3 = 2$

$w = aaaaabb$ can be written as $w = xyz \in L$

$\Rightarrow$ we also have $xy^0z = xz \in L$

With these conditions let us depart $w = aaaaabb$

$w = \underbrace{aaa}_{x}\underbrace{b}_{y}\underbrace{b}_{z}$

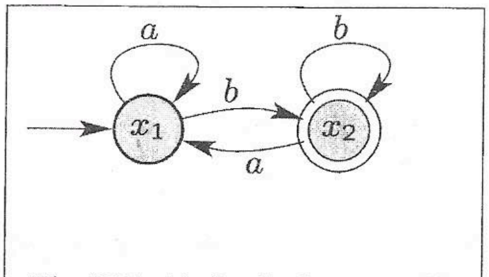
$xy^0z = xz = aaab \in L$ for $m = 3$ an $n = 4$

$xyz \in L$ by selection.

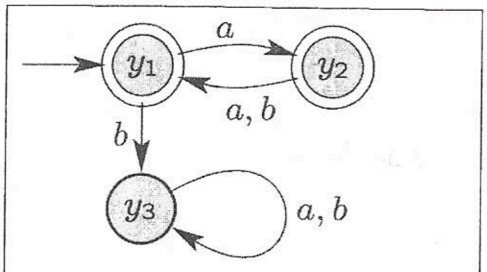
$xy^2z = \underbrace{aaa}_{x}\underbrace{bb}_{y^2}\underbrace{b}_{z} \notin L$

Therefore the language is not reguler. $\square$

5. **[20 points]** Consider the following DFA machines M_1 and M_2, each of which is designed over the alphabet $\Sigma = \{a, b\}$ such that the DFA M_1 recognizes the language $L_1 = L(M_1)$ and the DFA M_2 recognizes the language $L_2 = L(M_2)$.



The DFA M_1 for the language L_1



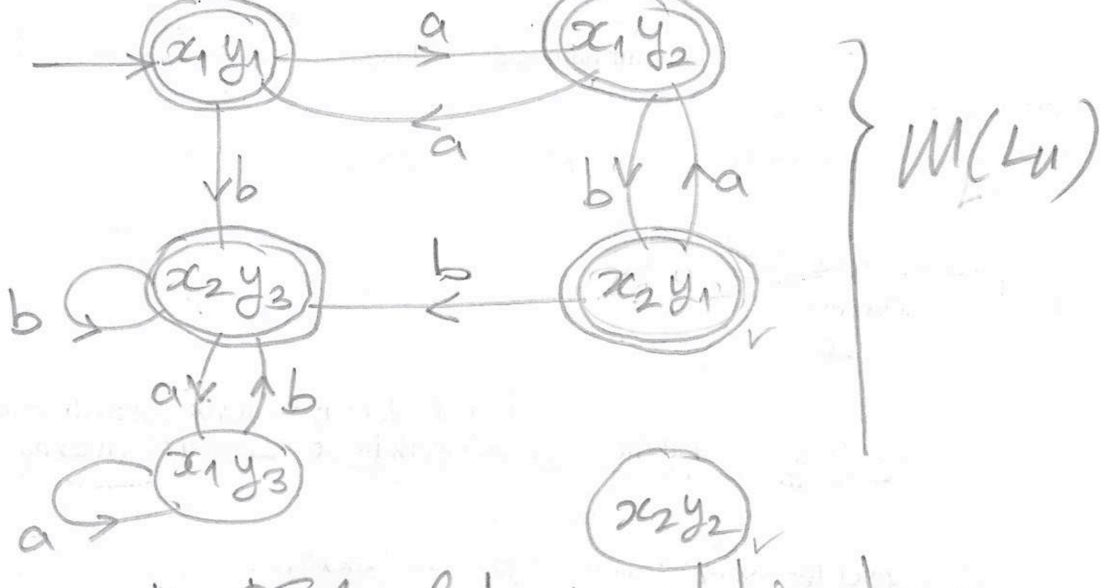
The DFA M_2 for the language L_2

(a) Design a DFA that recognizes the language $L = \overline{L_1} \cap \overline{L_2}$.

Let us use DeMorgan's rule.

$L = \overline{L_1} \cap \overline{L_2} = \overline{(L_1 \cup L_2)} = \overline{L_u}$

where $L_u = L_1 \cup L_2$. First, let us design L_u.



Thus, the DFA of L is obtained as