week 5 lesson 4 application part1

linear alg Ebru Das

Application

1) If $A = \begin{bmatrix} 1 & 0 & 3 \\ 1 & 1 & -2 \\ 0 & 4 & 1 \end{bmatrix} \Rightarrow \mathrm{adj}(A) = ?$

$$\begin{vmatrix} 1 & -2 \\ 0 & 1 \end{vmatrix}$$

$1 + 8 = 9$

$$\mathrm{adj}\, A = \begin{bmatrix} 9 & -1 & 4 \\ +12 & 1 & -4 \\ -3 & +5 & 1 \end{bmatrix}^t$$

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$\Rightarrow A^{-1} = ?$

$$A^{-1} = \frac{\mathrm{adj}(A)}{|A|}$$

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$$\operatorname{adj} A = \begin{bmatrix} 9 & -1 & 4 \\ +12 & 1 & -4 \\ -3 & +5 & 1 \end{bmatrix} = \begin{bmatrix} 9 & 12 & -3 \\ -1 & 1 & 5 \\ 4 & -4 & 1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} \frac{3}{7} & \frac{4}{7} & -\frac{1}{7} \\ -\frac{1}{21} & \frac{1}{21} & \frac{5}{21} \\ \frac{4}{21} & -\frac{4}{21} & \frac{1}{21} \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 0 & 3 \\ 1 & 1 & -2 \\ 0 & 4 & 1 \\ 1 & 0 & 3 \\ 1 & 1 & -2 \end{vmatrix} = 1 + 12 + 0 - 0 + 8 - 0 = 21$$

$$A_{33} = (-1)^6 \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} = 1$$

2) $$A = \begin{bmatrix} 4 & 1 & 1 & -1 \\ 0 & -1 & 2 & 0 \\ 1 & 1 & 1 & 2 \\ 0 & 2 & 1 & 0 \end{bmatrix}$$

$$|A| = 4 \underbrace{\begin{vmatrix} -1 & 2 & 0 \\ 1 & -1 & 2 \\ 2 & 1 & 0 \end{vmatrix}}_{} + \begin{vmatrix} 1 & 1 & 1 \\ -1 & 2 & 0 \\ 2 & 1 & 0 \end{vmatrix}$$

$$\underbrace{-2 \begin{vmatrix} -1 & 2 \\ 2 & 1 \end{vmatrix}}_{} \cdot 4 + (-1) \begin{vmatrix} -1 & 2 \\ 2 & 1 \end{vmatrix}$$

$$(-1-4)(-8) - (-1-4) = 40 + 5 = 45$$

ex

$$A = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix} \Rightarrow A^{-1} = ?$$

$$\begin{vmatrix} -1 & 0 & 1 \\ 0 & 0 & 2 \\ 0 & 1 & 1 \end{vmatrix} = (-1)(0-2) = 2 \neq 0$$

$$\operatorname{Adj} A = \begin{bmatrix} 2 & 0 & \\ & & \\ & & \end{bmatrix}$$

$$\begin{matrix} 0 & 0 & 1 \\ 0 & 0 & 2 \\ 0 & 1 & 1 \end{matrix}$$

ex

$$A = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix} \Rightarrow A^{-1} = ?$$

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$$\operatorname{Adj} A = \begin{bmatrix} 2 & 0 & 0 \\ & & \\ & & \end{bmatrix}$$

$$\begin{matrix} 0 & -1 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 1 \end{matrix}$$

ex

$$A = \begin{bmatrix} \cancel{1} & \cancel{0} & \cancel{0} & \cancel{1} \\ 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix} \Rightarrow A^{-1} = ?$$

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$$\operatorname{Adj} A = \begin{bmatrix} 2 & 0 & 0 \\ & & \\ & & \end{bmatrix}$$

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ex

$$A = \begin{bmatrix} 1 & 0 & 0 & 1 \\ \cancel{0} & \cancel{-1} & \cancel{0} & \cancel{1} \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix} \Rightarrow A^{-1} = ?$$

$$\begin{matrix} 1 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 1 \end{matrix}$$

$$\begin{vmatrix} -1 & 0 & 1 \\ 0 & 0 & 2 \\ 0 & 1 & 1 \end{vmatrix} = (-1)(0-2) = 2 \neq 0$$

$$\operatorname{Adj} A = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ -1 & 1 & -1 & -1 \\ 0 & 0 & -2 & 0 \end{bmatrix}^t \Rightarrow A^{-1} = \begin{bmatrix} 1 & 0 & -\frac{1}{2} & 0 \\ 0 & -1 & 1/2 & 0 \\ 0 & 0 & -1/2 & -1 \\ 0 & 0 & -1/2 & 0 \end{bmatrix}$$

ex $A = \begin{bmatrix} 0 & 2 & 3 \\ -1 & 0 & 2 \\ 1 & 1 & 0 \end{bmatrix}$

$A_{13} = ? \Rightarrow (-1)^4 \begin{vmatrix} -1 & 0 \\ 1 & 1 \end{vmatrix} = -1$

$M_{23} = ? \Rightarrow M_{23} = \begin{bmatrix} 0 & 2 \\ 1 & 1 \end{bmatrix}$

ex If $|A| = \frac{3}{4}$ with 4×4, then $|4A|, \left|\frac{A}{2}\right|, |A^{-1}|, |A^3| = ?$

$|4A| = 4^4 \cdot |A| = 4^{\cancel{4}\,3} \cdot \frac{3}{\cancel{4}} = 64 \cdot 3 = 192$ $\quad \left|\frac{A}{2}\right| = \left(\frac{1}{2}\right)^4 |A| = \frac{1}{16} \cdot \frac{3}{4} = \frac{3}{64}$

$|A||A^{-1}| = 1 \Rightarrow |A^{-1}| = \frac{4}{3}$ $\quad |A^3| = |A||A||A| = \frac{27}{64}$

Break

ex If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 9 & c \\ 1 & c & 3 \end{bmatrix}$ is a singular matrix, then $\rightarrow |A| = 0$
what is c? $(c \geq 0)$

a) 0 b) 2 c) 3 d) 5 e) 6

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a) 0 b) 2 c) 3 d) 5 e) 6

$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 9 & c \\ 1 & c & 3 \end{vmatrix} = 27 - c^2 - (3 - c) + (c - 9) = 0$

$= 27 - c^2 - 3 + c + c - 9 = 0 \Rightarrow +c^2 - 2c - 15 = 0 \rightarrow c_1 = 5$, $c_2 = -3$

$-5 \quad +3$