BLM2502 Theory of Computation

Spring 2016

BLM2502 Theory of Computation

- **Course Outline**

Week	Content
1	Introduction to Course
2	Computability Theory, Complexity Theory, Automata Theory, Set Theory, Relations, Proofs, Pigeonhole Principle
3	Regular Expressions
4	Finite Automata
5	Deterministic and Nondeterministic Finite Automata
6	Epsilon Transition, Equivalence of Automata
7	Pumping Theorem
8	April 10 - 14 week is the first midterm week
9	Context Free Grammars
10	Parse Tree, Ambiguity,
11	**Pumping Theorem**
12	Turing Machines, Recognition and Computation, Church-Turing Hypothesis
13	Turing Machines, Recognition and Computation, Church-Turing Hypothesis
14	May 22 – 27 week is the second midterm week
15	Review
16	Final Exam date will be announced

The Pumping Lemma for CFL's

Simplifications of Context-Free Grammars

A Substitution Rule

$$S \rightarrow \cancel{aB} \mid ab \mid aaA$$

$$A \rightarrow aaA \mid$$

$$A \rightarrow ab\cancel{Bc} \mid abbc \mid abaAc$$

$$\cancel{B \rightarrow aA}$$

$$\cancel{B \rightarrow b}$$

Substitute

$$B \rightarrow b$$

Equivalent grammar

$$S \rightarrow aB \mid ab$$

$$A \rightarrow aaA$$

$$A \rightarrow abBc \mid abbc$$

$$B \rightarrow aA$$

$$S \rightarrow aB \mid ab$$

$$A \rightarrow aaA$$

$$A \rightarrow abBc \mid abbc$$

$$B \rightarrow aA$$

Substitute

$$B \rightarrow aA$$

$$S \rightarrow \cancel{aB} \mid ab \mid aaA$$

$$A \rightarrow aaA$$

$$A \rightarrow \cancel{abBc} \mid abbc \mid abaAc$$

Equivalent grammar

In general: $A \to xBz$

$$B \to y_1$$

Substitute

$$B \to y_1$$

$$A \to xBz \mid xy_1z$$

equivalent grammar

Nullable Variables

$\varepsilon-$production :

$$X \rightarrow \varepsilon$$

Nullable Variable:

$$Y \Rightarrow \ldots \Rightarrow \varepsilon$$

Example:

$$S \rightarrow aMb \mid ab$$

$$M \rightarrow aMb \mid ab$$

$$\cancel{M \rightarrow \varepsilon}$$

Nullable variable

$\varepsilon-$production

Removing ε – productions

$$S \to aMb \mid ab$$

$$M \to aMb \mid ab$$

$$\cancel{M \to \varepsilon}$$

Substitute $M \to \varepsilon$

$$S \to aMb \mid ab$$

$$M \to aMb \mid ab$$

After we remove all the ε - productions
all the nullable variables disappear
(except for the start variable)

Unit-Productions

Unit Production: $X \rightarrow Y$

(a single variable in both sides)

Example:

$$S \rightarrow aA$$

$$A \rightarrow a$$

$$\boxed{A \rightarrow B}$$

$$\boxed{B \rightarrow A}$$

$$B \rightarrow bb$$

Unit Productions

Removal of unit productions:

$$S \rightarrow a\cancel{A} \mid \cancel{aB} \mid abb \mid aa \mid aa$$

$$\cancel{A \rightarrow a}$$

$$\cancel{A \rightarrow B}$$

$$\cancel{B \rightarrow A} \mid a$$

$$\cancel{B \rightarrow bb}$$

Substitute

$$A \rightarrow B$$

$$S \rightarrow aA \mid aB$$

$$A \rightarrow a$$

$$B \rightarrow A \mid B$$

$$B \rightarrow bb$$

Unit productions of form $X \rightarrow X$

can be removed immediately

$$S \rightarrow aA \mid aB$$
$$A \rightarrow a$$
$$B \rightarrow A \mid \cancel{B}$$
$$B \rightarrow bb$$

Remove

$$B \rightarrow B$$

$$S \rightarrow aA \mid aB$$
$$A \rightarrow a$$
$$B \rightarrow A$$
$$B \rightarrow bb$$

$$S \to aA \mid aB$$

$$A \to a$$

→ B gördüğüm yere A yazarım

$$\cancel{B \to A}$$

$$B \to bb$$

Substitute

$$B \to A$$

$$S \to \cancel{aA} \mid aB \mid aA$$

$$A \to a$$

$$B \to bb$$

Remove repeated productions

Final grammar

$$S \to aA \mid aB \mid \cancel{aA}$$

$$A \to a$$

$$B \to bb$$

$$S \to aA \mid aB$$

$$A \to a$$

$$B \to bb$$

Useless Productions

$$S \rightarrow aSb$$

$$S \rightarrow \lambda$$

$$S \rightarrow A$$

sonuçlanamaz

$$\cancel{A \rightarrow aA}$$ Useless Production

Some derivations never terminate...

$$S \Rightarrow A \Rightarrow aA \Rightarrow aaA \Rightarrow \ldots \Rightarrow aa\ldots aA \Rightarrow \ldots$$

Another grammar:

$$S \to A$$

$$A \to aA$$

$$A \to \lambda$$

$$\cancel{B \to bA}$$ → B'ye hiç gelmeyeceğiz

Useless Production

Not reachable from S

In general:

If there is a derivation

$$S \Rightarrow \ldots \Rightarrow xAy \Rightarrow \ldots \Rightarrow w \quad \in L(G)$$

consists of terminals

Then variable A is useful

Otherwise, variable A is useless

A production $A \rightarrow x$ is useless
if any of its variables is useless

$$S \rightarrow aSb$$

$$S \rightarrow \varepsilon$$

Productions

Variables

$$S \rightarrow A$$ useless

useless $$A \rightarrow aA$$ useless

useless $$B \rightarrow C$$ useless

useless $$C \rightarrow D$$ useless

Removing Useless Variables and Productions

Example Grammar:

$$S \rightarrow aS \mid A \mid \cancel{C}$$

$$A \rightarrow a$$

$$\cancel{B \rightarrow aa}$$

$$\cancel{C \rightarrow aCb}$$

First: find all variables that can produce strings with only terminals or ε (possible useful variables)

$$S \to aS \mid A \mid C$$
$$A \to a$$
$$B \to aa$$
$$C \to aCb$$

Round 1: $\{A, B\}$

(the right hand side of production that has only terminals)

Round 2: $\{A, B, S\}$

(the right hand side of a production has terminals and variables of previous round)

This process can be generalized

Then, remove productions that use variables other than $\{A, B, S\}$

$$S \rightarrow aS \mid A \mid \cancel{C}$$

$$A \rightarrow a$$

$$B \rightarrow aa$$

$$\cancel{C \rightarrow aCb}$$

$$\Rightarrow$$

$$S \rightarrow aS \mid A$$

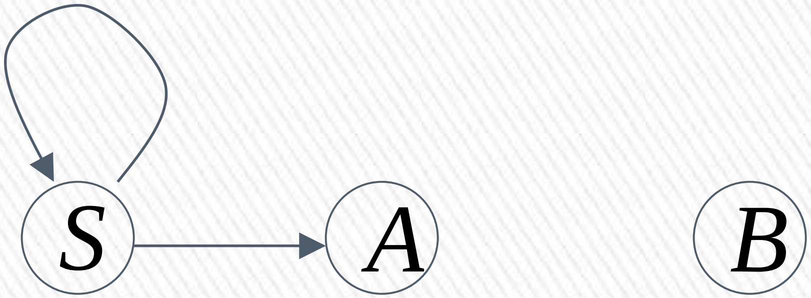
$$A \rightarrow a$$

$$B \rightarrow aa$$

Second: Find all variables reachable from S

Use a Dependency Graph where nodes are variables

$$S \to aS \mid A$$
$$A \to a$$
$$B \to aa$$



unreachable

Keep only the variables reachable from S

$$S \rightarrow aS \mid A$$

$$A \rightarrow a$$

$$\cancel{B \rightarrow aa}$$

$$\Rightarrow$$

Final Grammar

$$S \rightarrow aS \mid A$$

$$A \rightarrow a$$

Contains only useful variables

Removing All

» **Step 1:** Remove Nullable Variables

$\hookrightarrow A \to \varepsilon$

» **Step 2:** Remove Unit-Productions

$\hookrightarrow A \to B$

» **Step 3:** Remove Useless Variables

$\hookrightarrow A \to Aa$

This sequence guarantees that unwanted variables and productions are removed

$S \to ABAC$

$A \to aA \mid \varepsilon$

$B \to bB \mid \varepsilon$

$C \to c$

① Remove null productions

$A \to \varepsilon, B \to \varepsilon$

$S \to ABAC \mid ABC \mid BAC \mid BC \mid C \mid AAC \mid AC$

$A \to aA \mid a$

$B \to bB \mid b$

$\cancel{C \to c}$

$S \to ABAc \mid ABc \mid BAc \mid Bc \mid c \mid AAc \mid Ac$

$A \to aA \mid a$

$B \to bB \mid b$

$S \to ABAC \mid ABC \mid BAC \mid BC \mid AAC \mid AC \mid C$

$A \to aA \mid a$

$B \to aB \mid b$

$\cancel{C \to c}$

Normal Forms
for
Context-free Grammars

Chomsky Normal Form

Each productions has form:

$$A \to BC \qquad \text{or} \qquad A \to a$$

variable variable terminal

Examples:

$$S \to AS$$
$$S \to a$$
$$A \to SA$$
$$A \to b$$

Chomsky Normal Form

$S \to AV_1$

$V_1 \to AS$

$T_1 \to a$

$$S \to AS$$
$$S \to AAS$$
$$A \to SA$$
$$A \to aa$$

$A \to T_1 T_1$

Not Chomsky Normal Form

Conversion to Chomsky Normal Form

» Example:

$$S \to A\cancel{Ba}$$

V_2 V_1

$$A \to \cancel{aa}\cancel{b}$$

$V_1 V_1$ V_3 V_4

$$B \to A\cancel{c}$$

V_5

Not in Chomsky Normal Form

$V_1 \to a$

$V_2 \to BV_1$

We will convert it to Chomsky Normal Form

$V_3 \to b$

$V_4 \to V_1 V_3$

$V_5 \to c$

Introduce new variables for the terminals:

$$T_a, T_b, T_c$$

$$S \rightarrow ABa$$
$$A \rightarrow aab$$
$$B \rightarrow Ac$$

$$S \rightarrow ABT_a$$
$$A \rightarrow T_a T_a T_b$$
$$B \rightarrow AT_c$$
$$\boxed{\begin{array}{l} T_a \rightarrow a \\ T_b \rightarrow b \\ T_c \rightarrow c \end{array}}$$

Introduce new intermediate variable V_1 to break first production:

$$S \to ABT_a$$
$$A \to T_a T_a T_b$$
$$B \to AT_c$$
$$T_a \to a$$
$$T_b \to b$$
$$T_c \to c$$

$\Rightarrow$

$$S \to AV_1$$
$$V_1 \to BT_a$$
$$A \to T_a T_a T_b$$
$$B \to AT_c$$
$$T_a \to a$$
$$T_b \to b$$
$$T_c \to c$$

Introduce intermediate variable: V_2

$$S \to AV_1$$
$$V_1 \to BT_a$$
$$\boxed{A \to T_aT_aT_b}$$
$$B \to AT_c$$
$$T_a \to a$$
$$T_b \to b$$
$$T_c \to c$$

$\Rightarrow$

$$S \to AV_1$$
$$V_1 \to BT_a$$
$$\boxed{\begin{array}{l} A \to T_aV_2 \\ V_2 \to T_aT_b \end{array}}$$
$$B \to AT_c$$
$$T_a \to a$$
$$T_b \to b$$
$$T_c \to c$$

Final grammar in Chomsky Normal Form:

$$S \to AV_1$$

$$V_1 \to BT_a$$

$$A \to T_a V_2$$

$$V_2 \to T_a T_b$$

$$B \to AT_c$$

$$T_a \to a$$

$$T_b \to b$$

$$T_c \to c$$

Initial grammar

$$S \to ABa$$

$$A \to aab$$

$$B \to Ac$$

In general:

From any context-free grammar

(which doesn't produce ε)

not in Chomsky Normal Form

we can obtain:

an equivalent grammar

in Chomsky Normal Form

The Procedure

First remove:

Nullable variables

Unit productions

(Useless variables optional)

Then, for every symbol a:

New variable: T_a

Add production $T_a \rightarrow a$

In productions with length at least 2 replace a with T_a

Productions of form $A \rightarrow a$ do not need to change!

Replace any production $A \rightarrow C_1 C_2 \cdots C_n$

with
$$A \rightarrow C_1 V_1$$
$$V_1 \rightarrow C_2 V_2$$
$$\cdots$$
$$V_{n-2} \rightarrow C_{n-1} C_n$$

New intermediate variables: $V_1, V_2, \ldots, V_{n-2}$

Observations

- Chomsky normal forms are good for parsing and proving theorems

- It is easy to find the Chomsky normal form for any context-free grammar

Greinbach Normal Form

All productions have form:

$$A \to a\,V_1 V_2 \cdots V_k \qquad k \geq 0$$

symbol

variables

Examples:

$T_1 = b$

$S \to aT_1ST_1$

$$S \to cAB$$

$$A \to aA \mid bB \mid b$$

$$B \to b$$

$$S \to abSb$$

$$S \to aa$$

$T_2 = a$

$S \to aT_2$

Greinbach Normal Form

b ✓

Not Greinbach Normal Form

Conversion to Greinbach Normal Form:

V_1 V_1

$$S \to abSb$$

V_2

$$S \to a\cancel{a}$$

$V_1 \Rightarrow b$

$V_2 \to a$

S

$$S \to aT_bST_b$$

$$S \to aT_a$$

$$T_a \to a$$

$$T_b \to b$$

Greinbach

Normal Form

Observations

- Greinbach normal forms are very good for parsing strings (better than Chomsky Normal Forms)
- However, it is difficult to find the Greinbach normal of a grammar

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