Bayesian regression modeling: Theory & practice

Part X: Beyond regression modeling | Theory-driven Bayesian modeling

Michael Franke

Case study: natural use of quantifier *some*

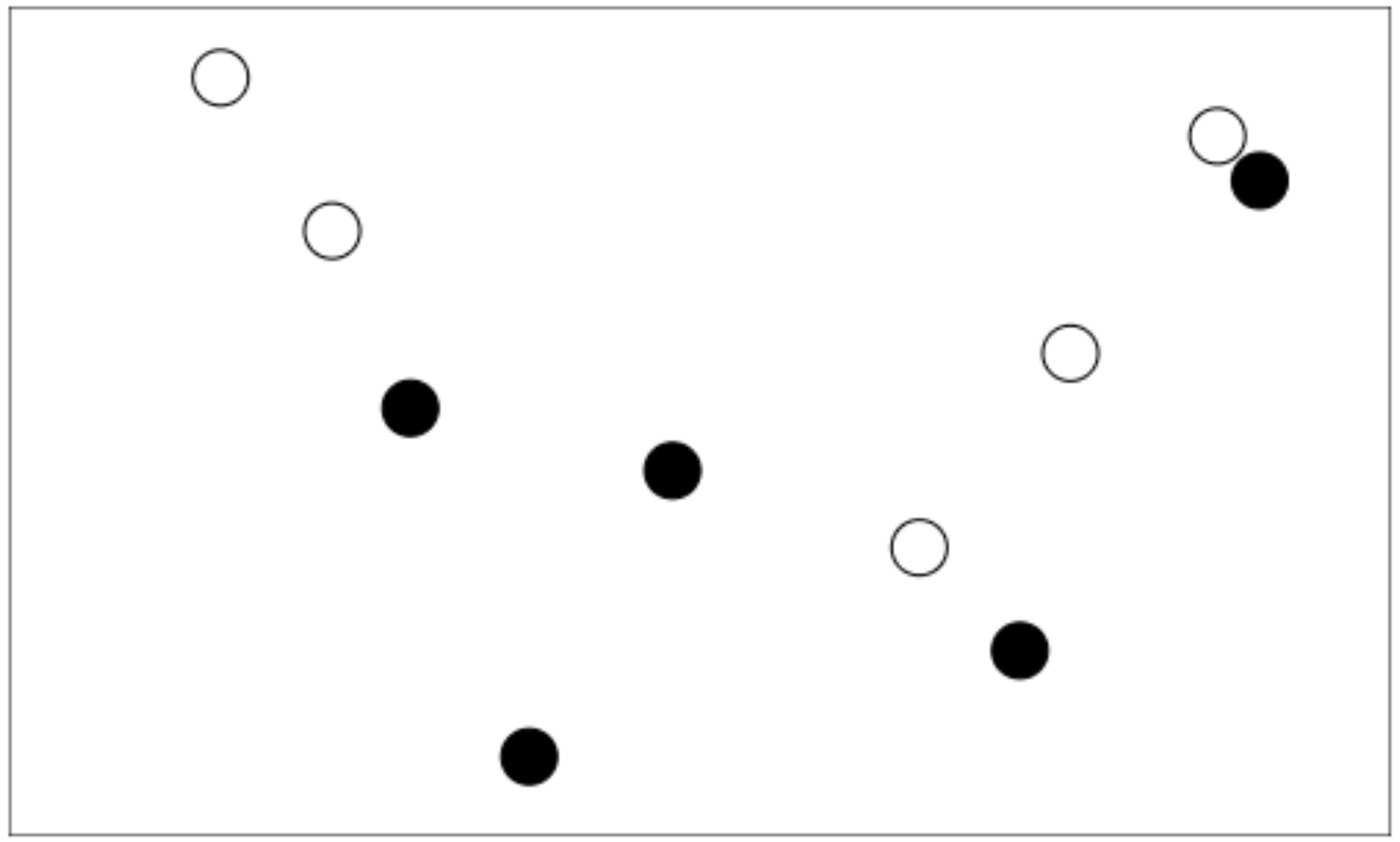
motivation

- inform debate of **what to infer from experimental data**
  1. what does a task measure?
  2. how is that related to established theoretical notions?

Test your intuitions

Truth-value judgement task

"Some of the circles are black"



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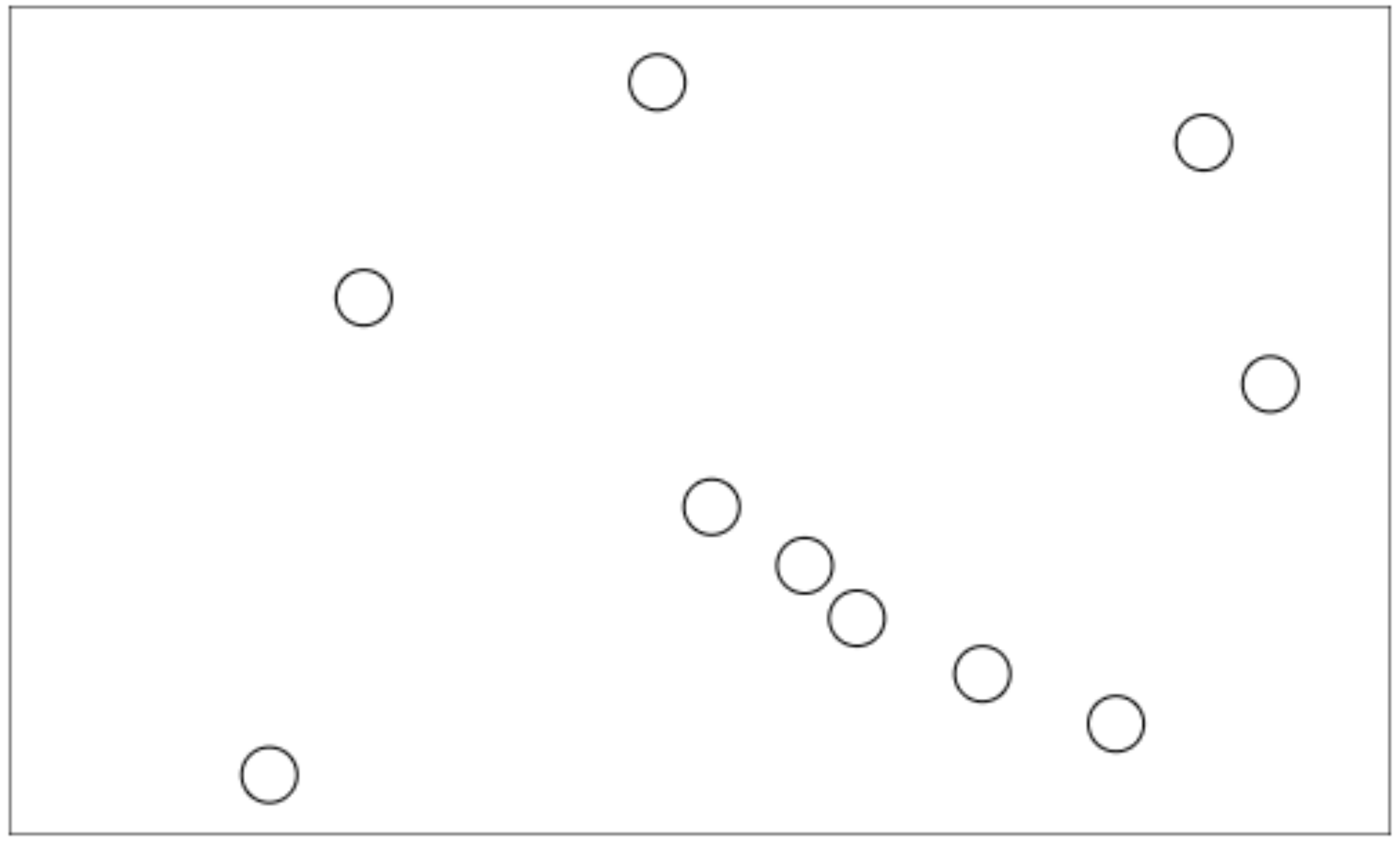

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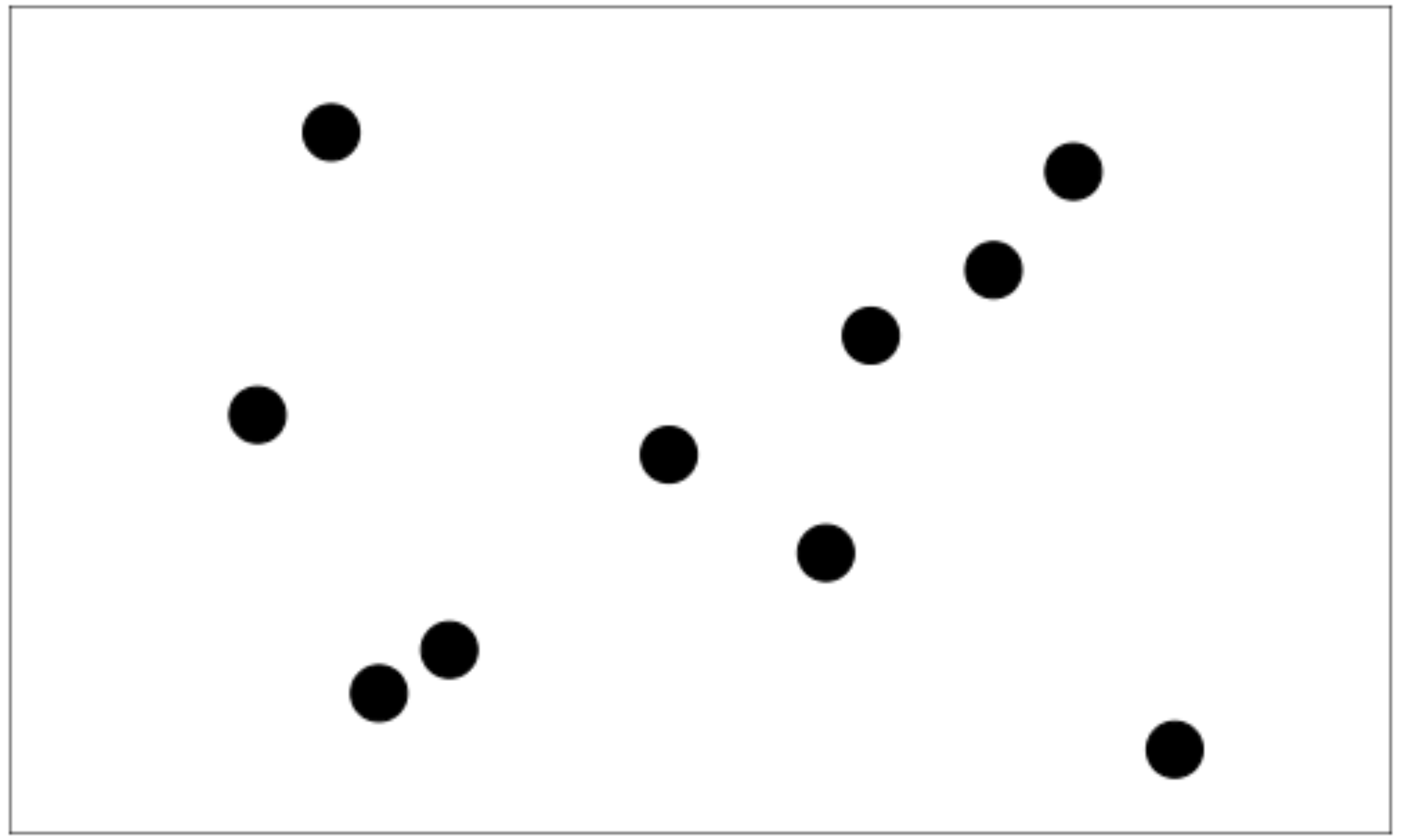

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Felicity rating task

“Some of the circles are black”

How well does the sentence describe the picture?

bad 1 2 3 4 5 6 7 good

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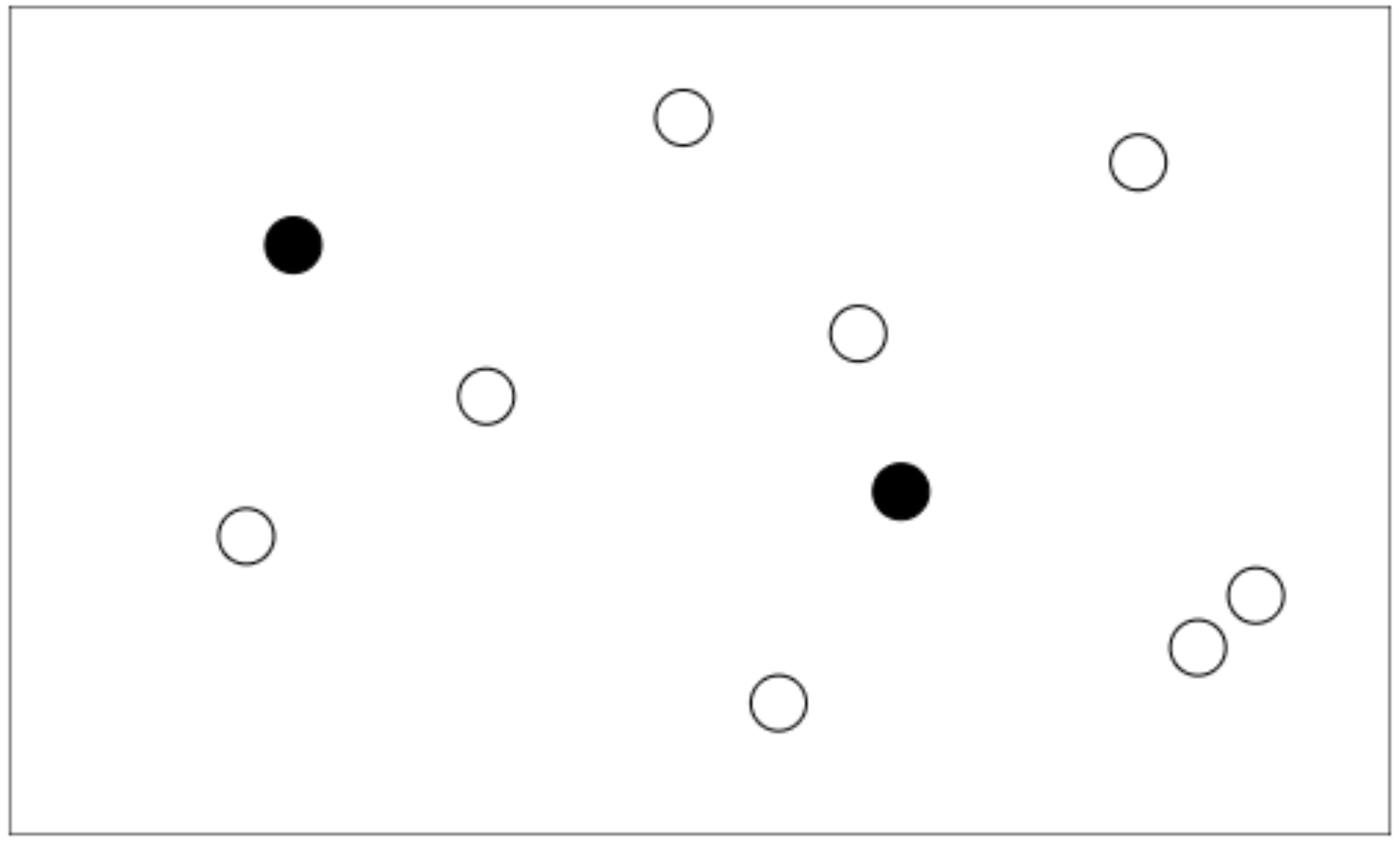

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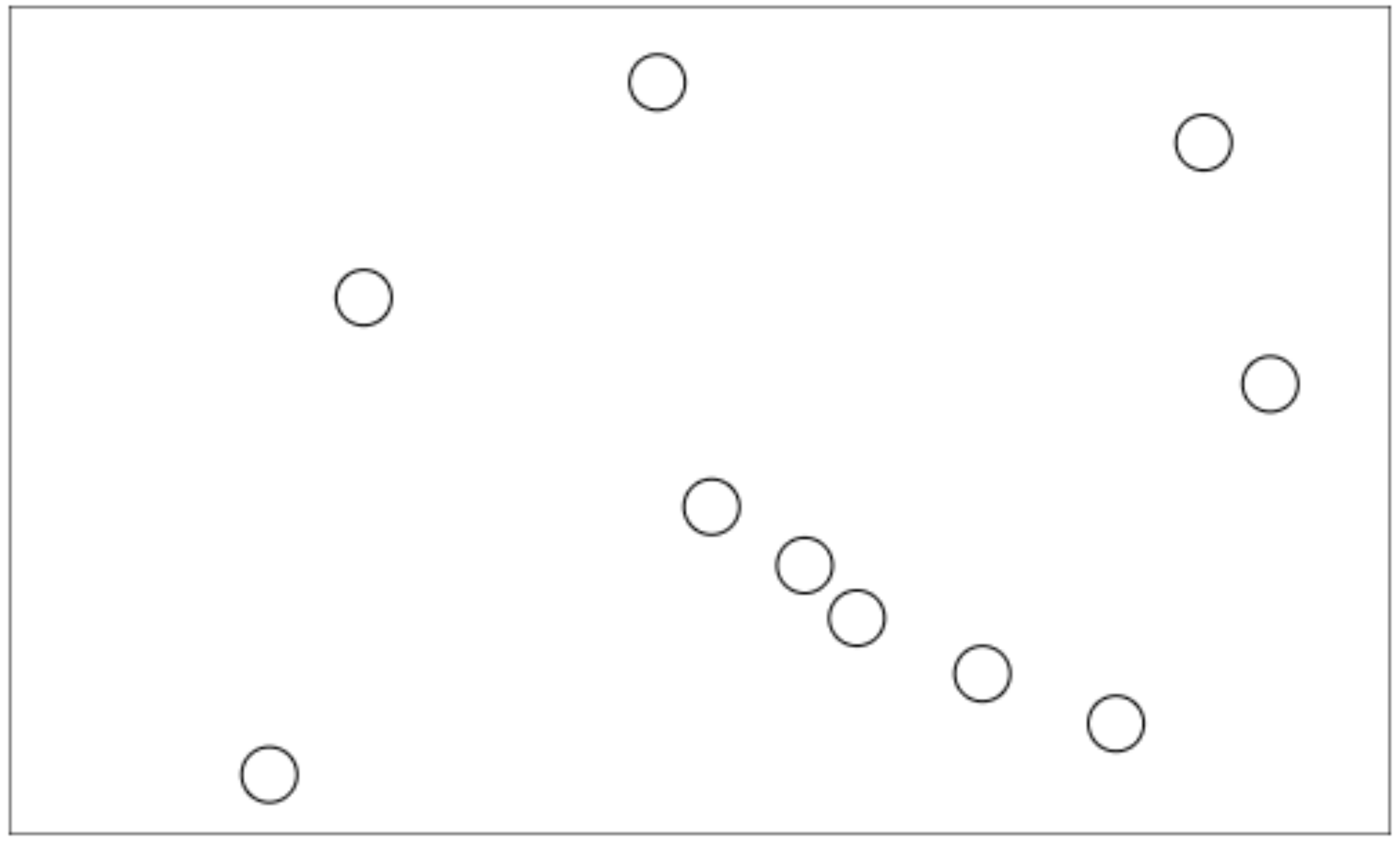

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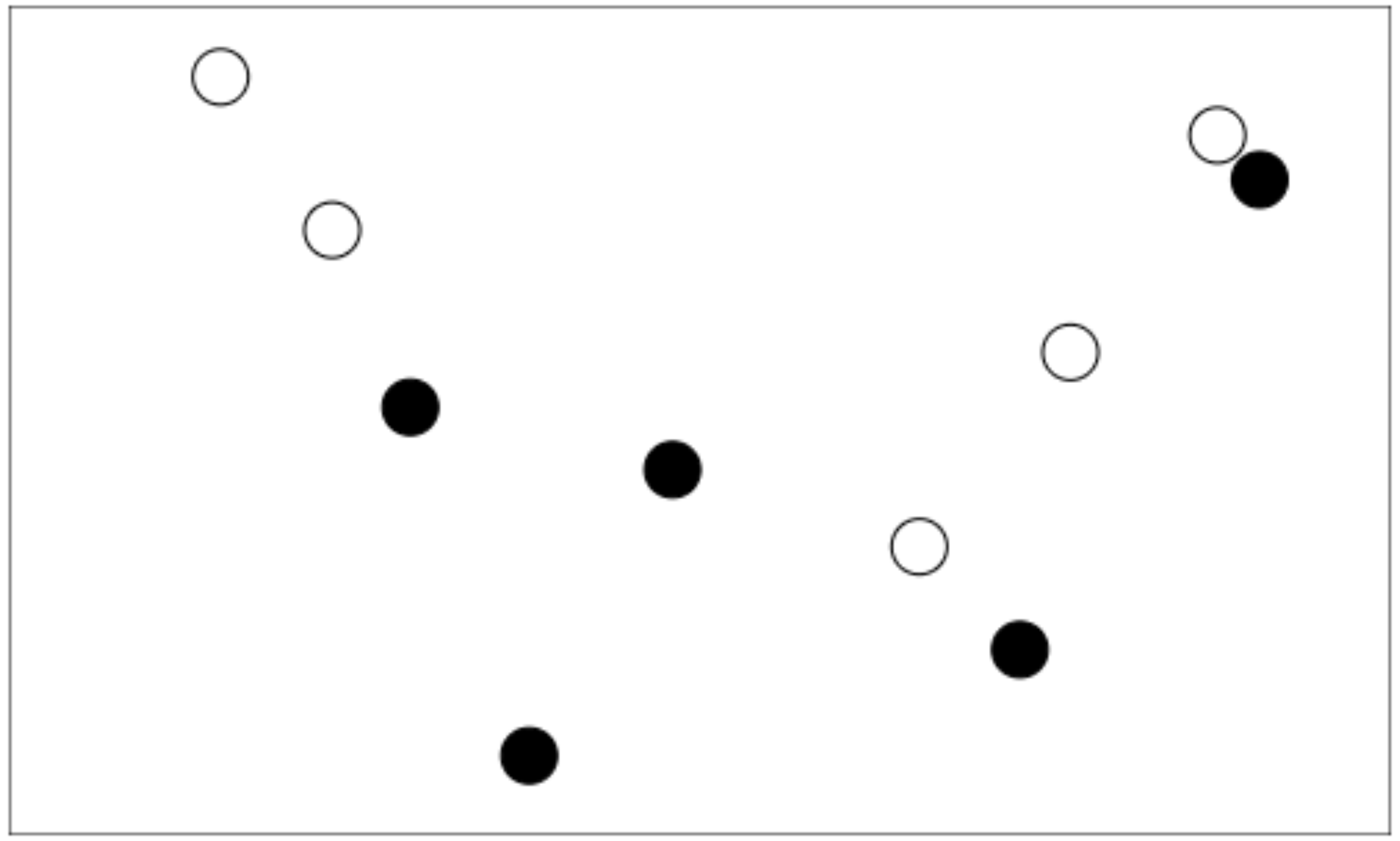

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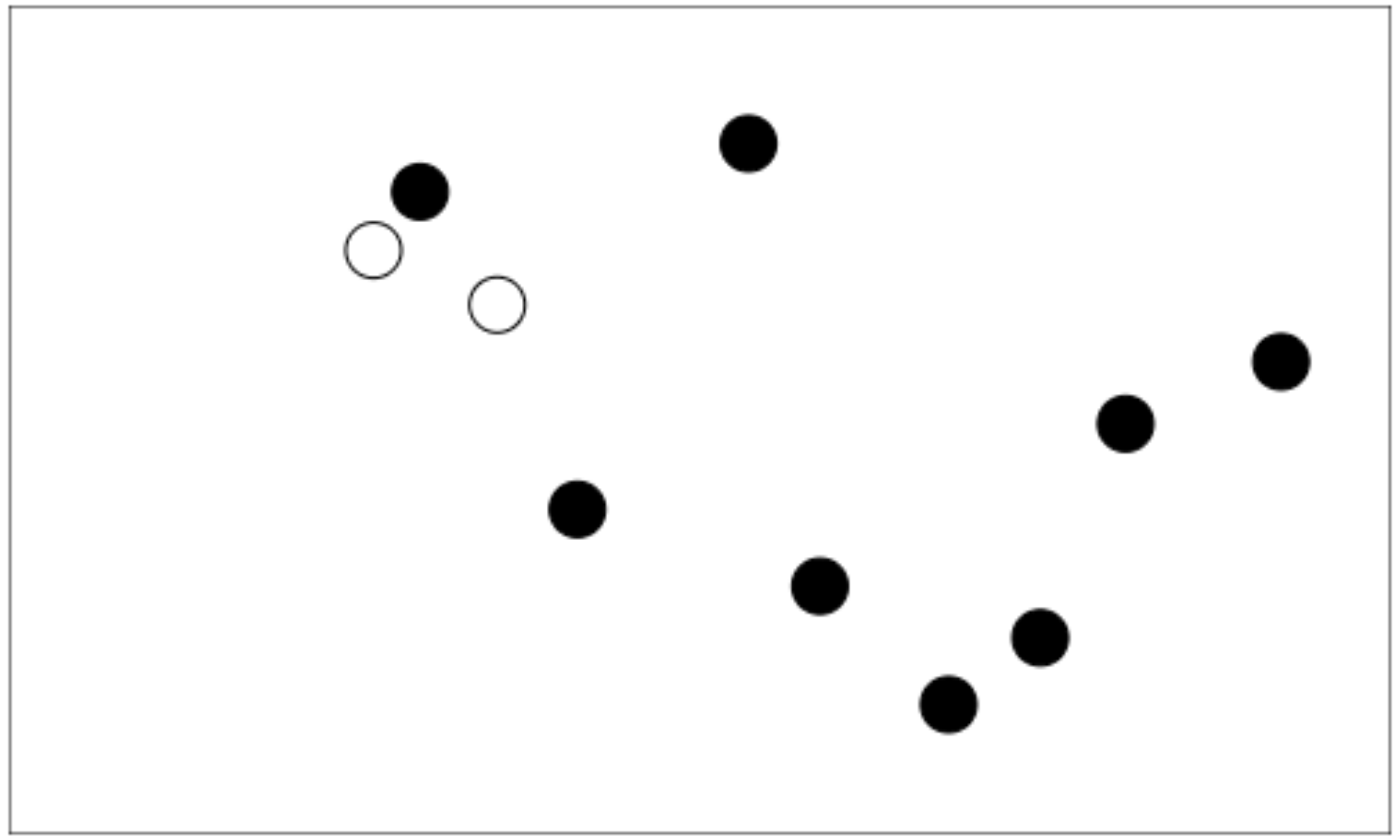

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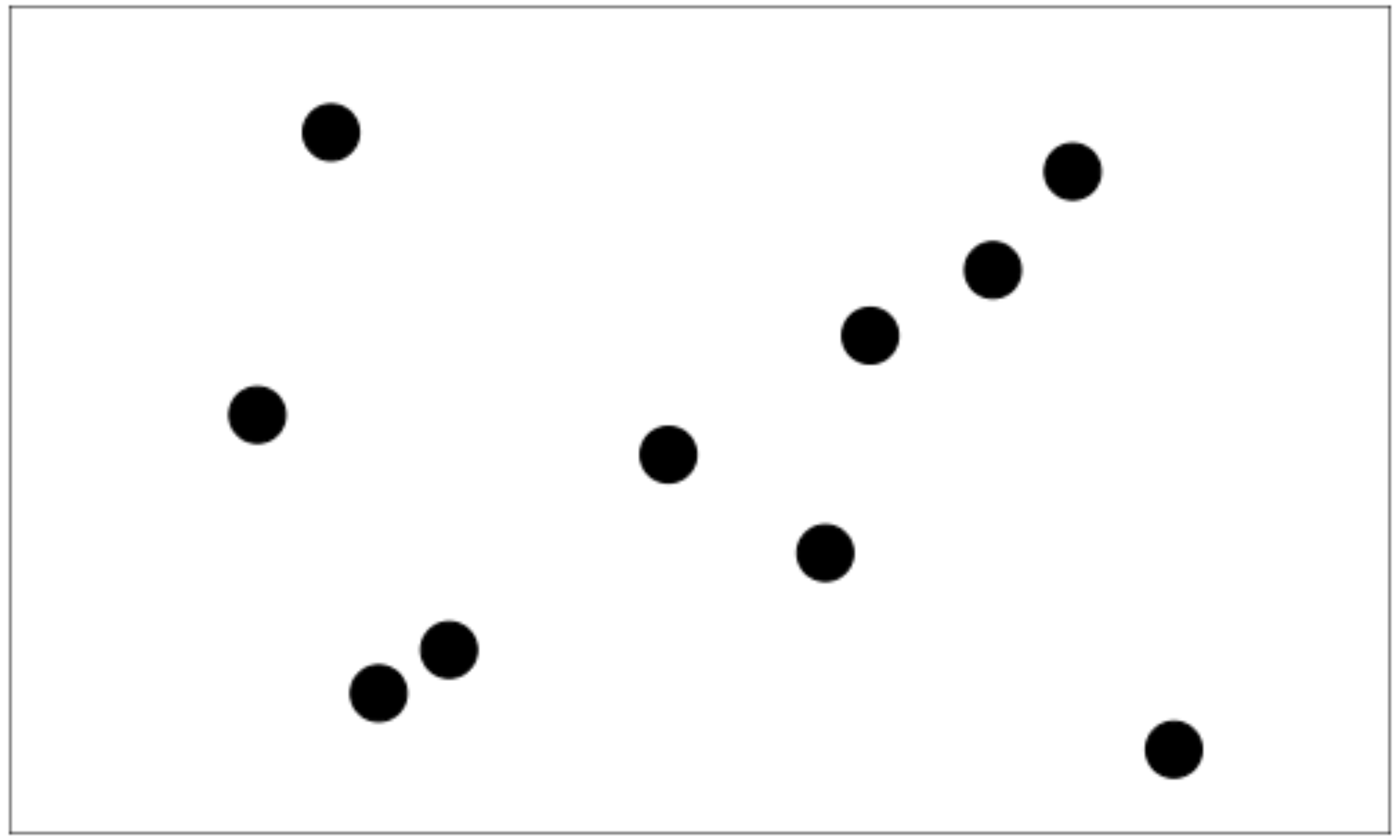

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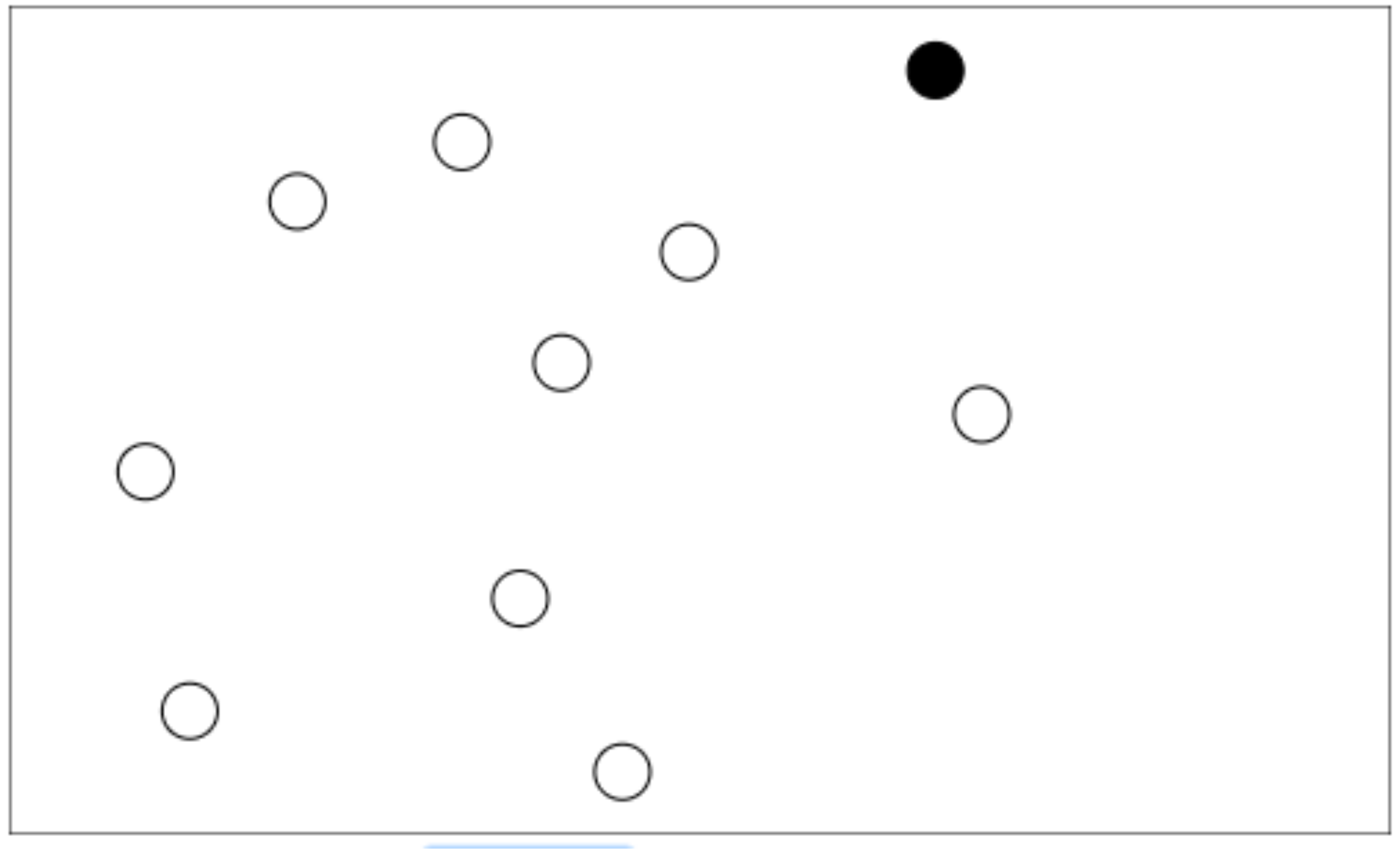

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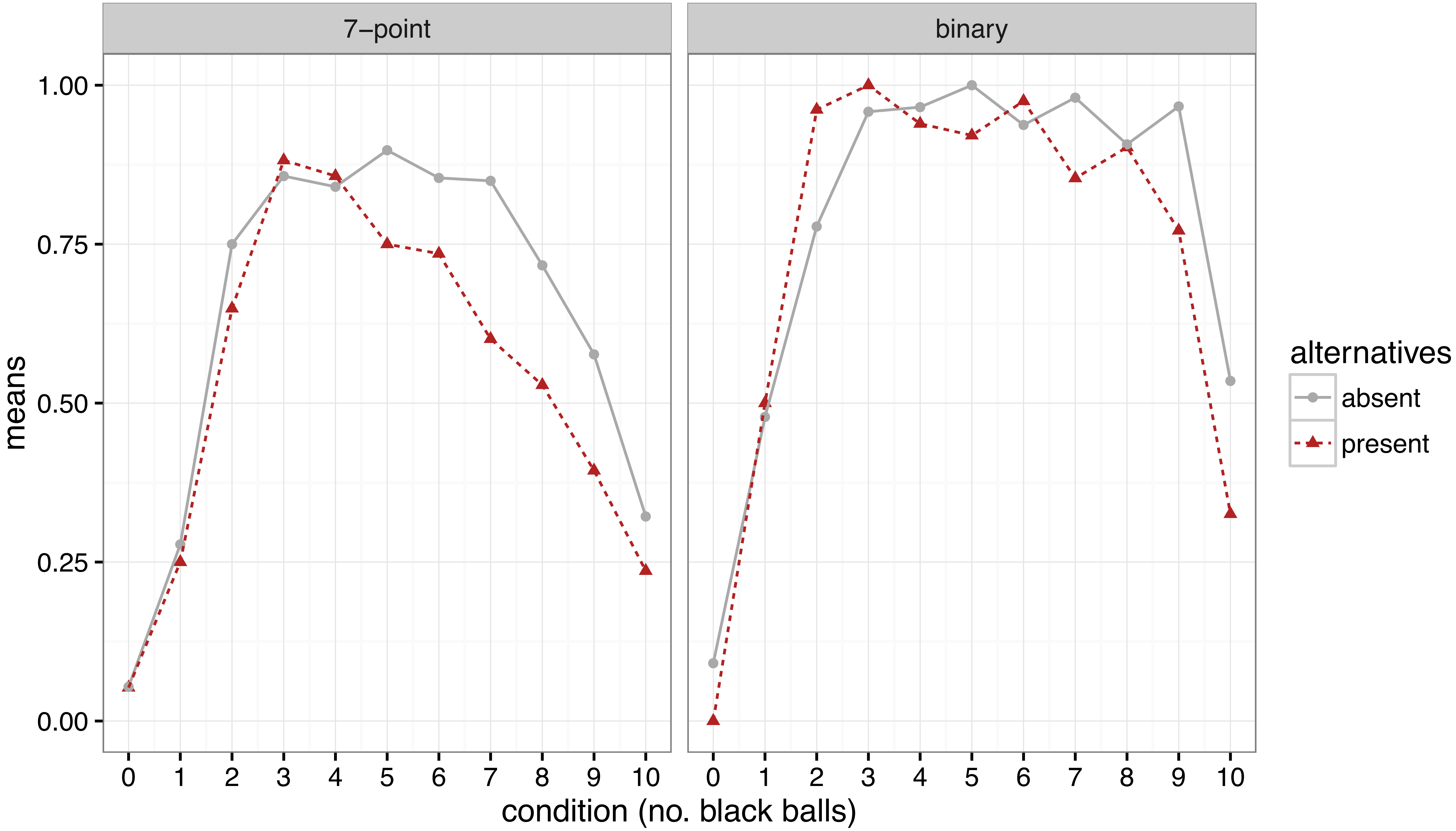
Experimental data

- two task types
  - 7-point rating scale
  - binary truth-value judgement task
- filler sentences with most and many

	A	B	C	D
task type	7-point	7-point	binary	binary
fillers?	yes	no	yes	no
N	119	114	109	107

(c.f., van Tiel & Geurts (2014), van Tiel (2014), Degen & Tanenhaus (2015))

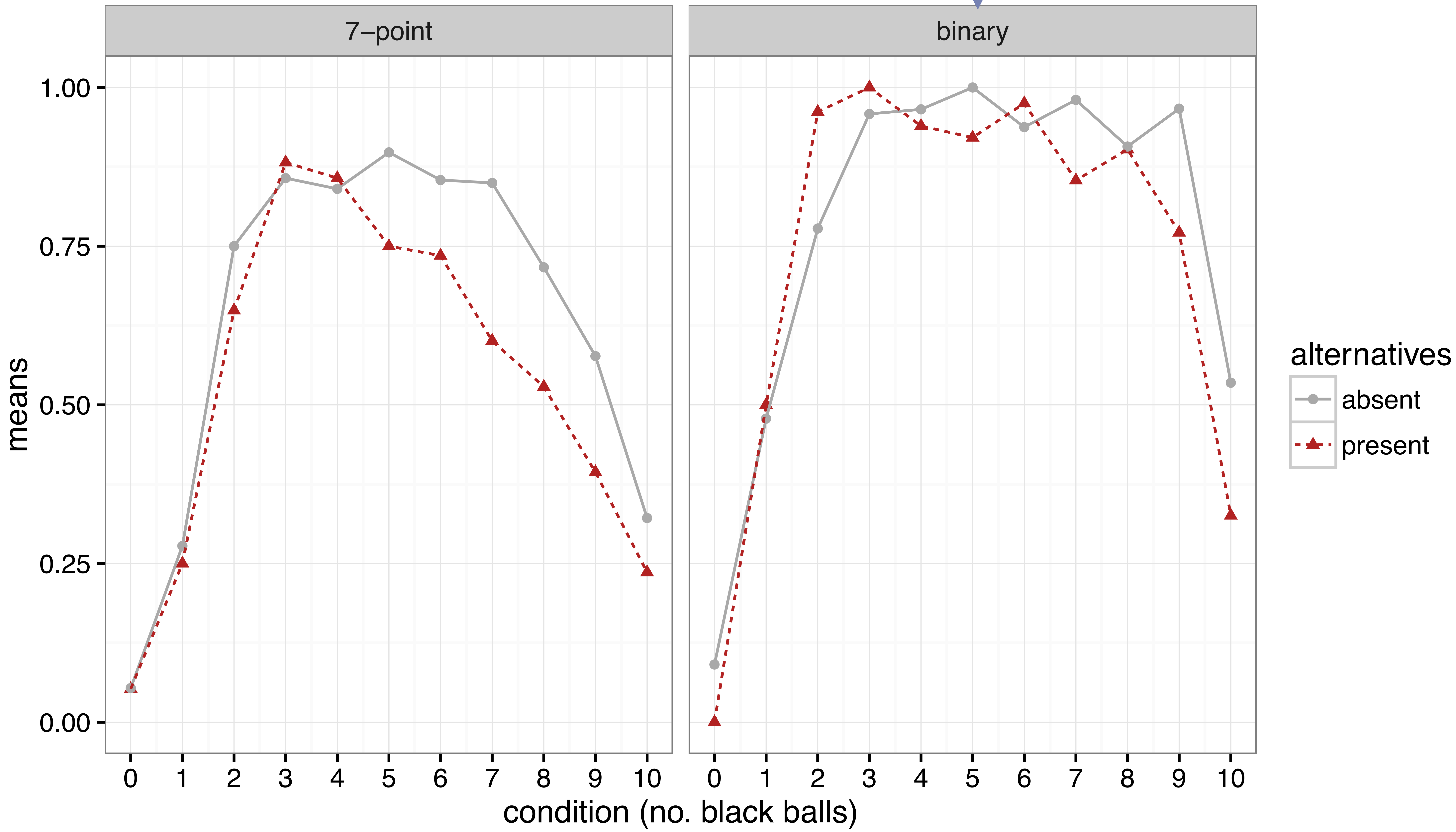
Results



7–point
binary
means
1.00
0.75
0.50
0.25
0.00
0 1 2 3 4 5 6 7 8 9 10
condition (no. black balls)
alternatives
absent
present

Results

measures **truth conditions**



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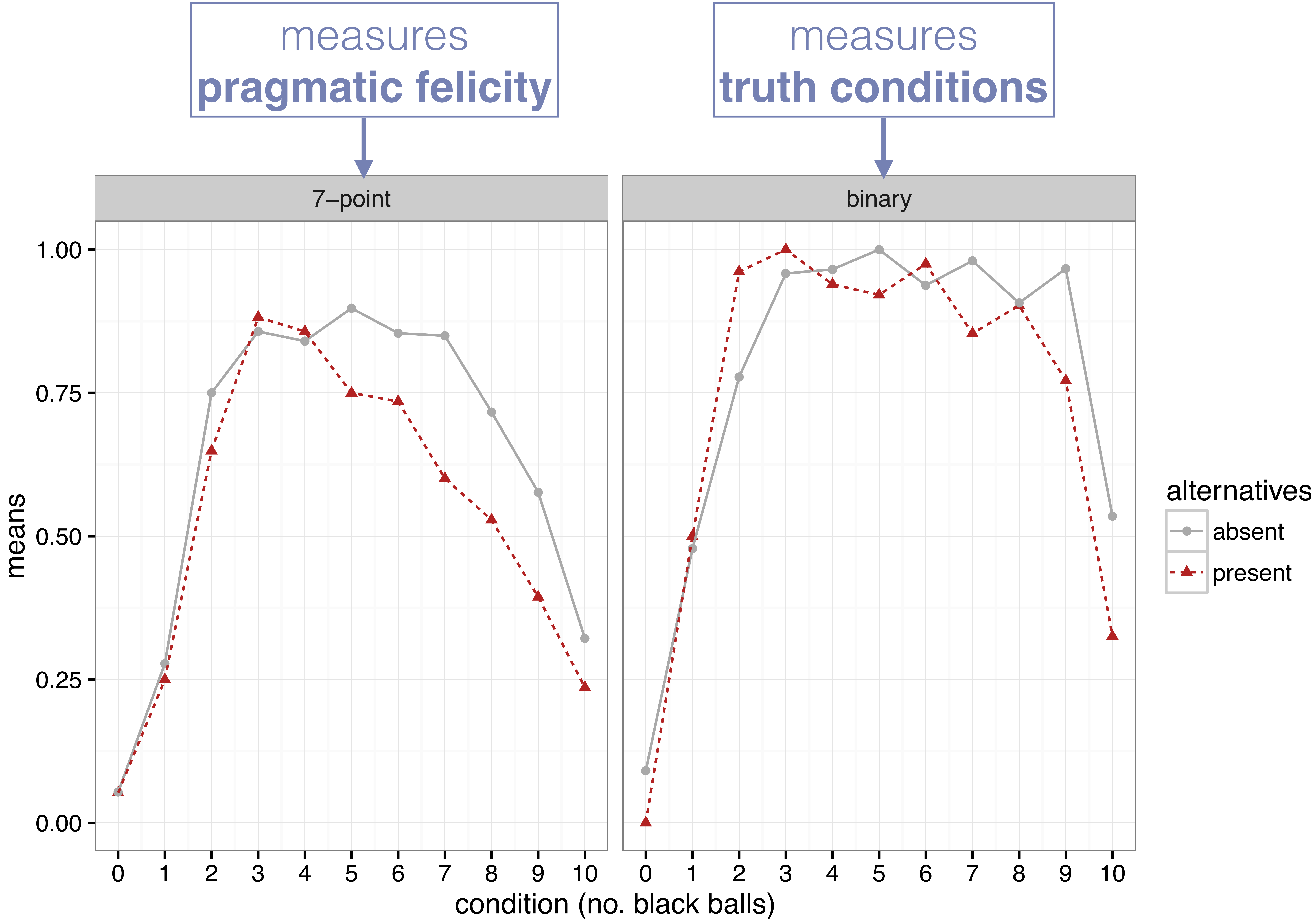

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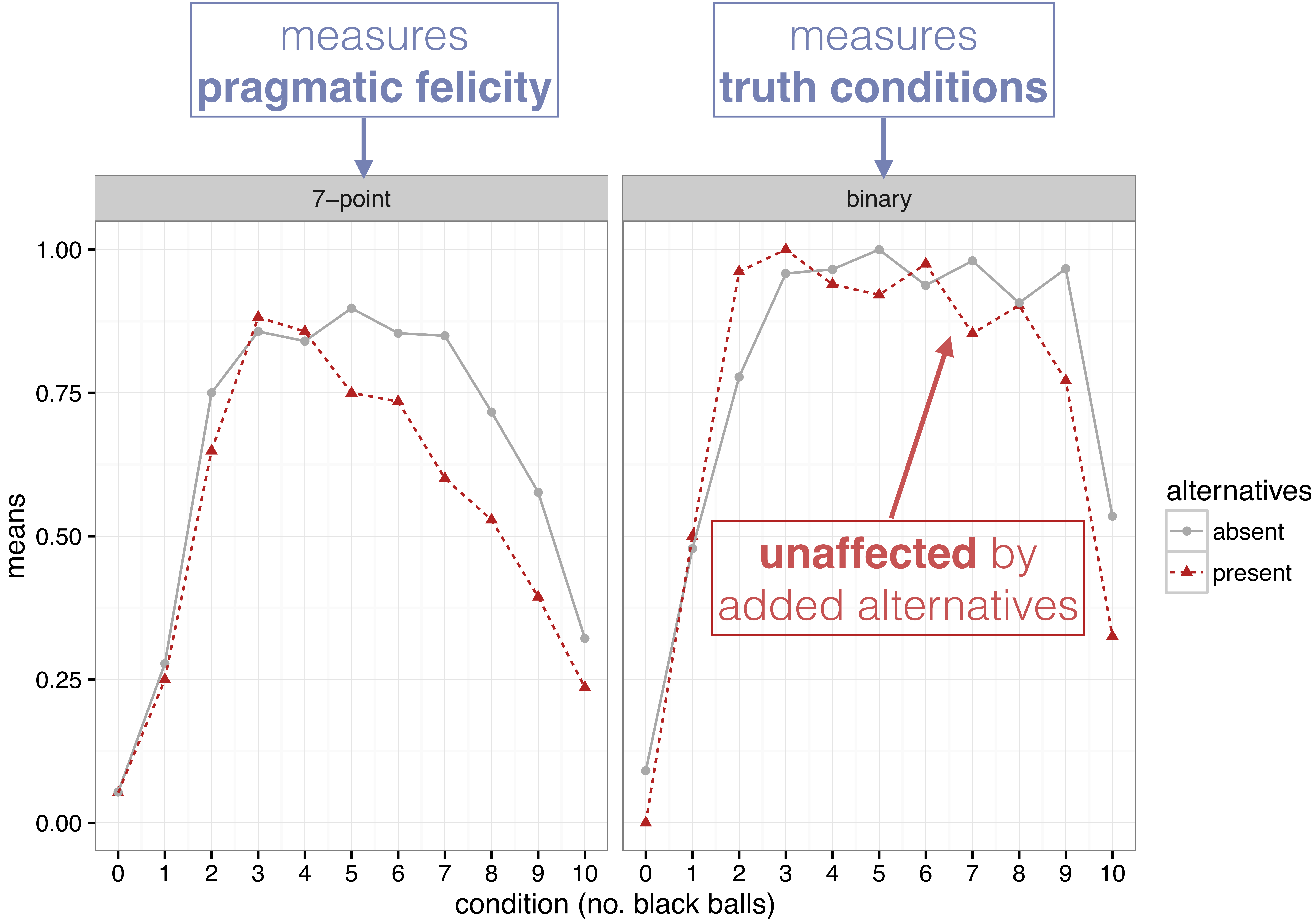

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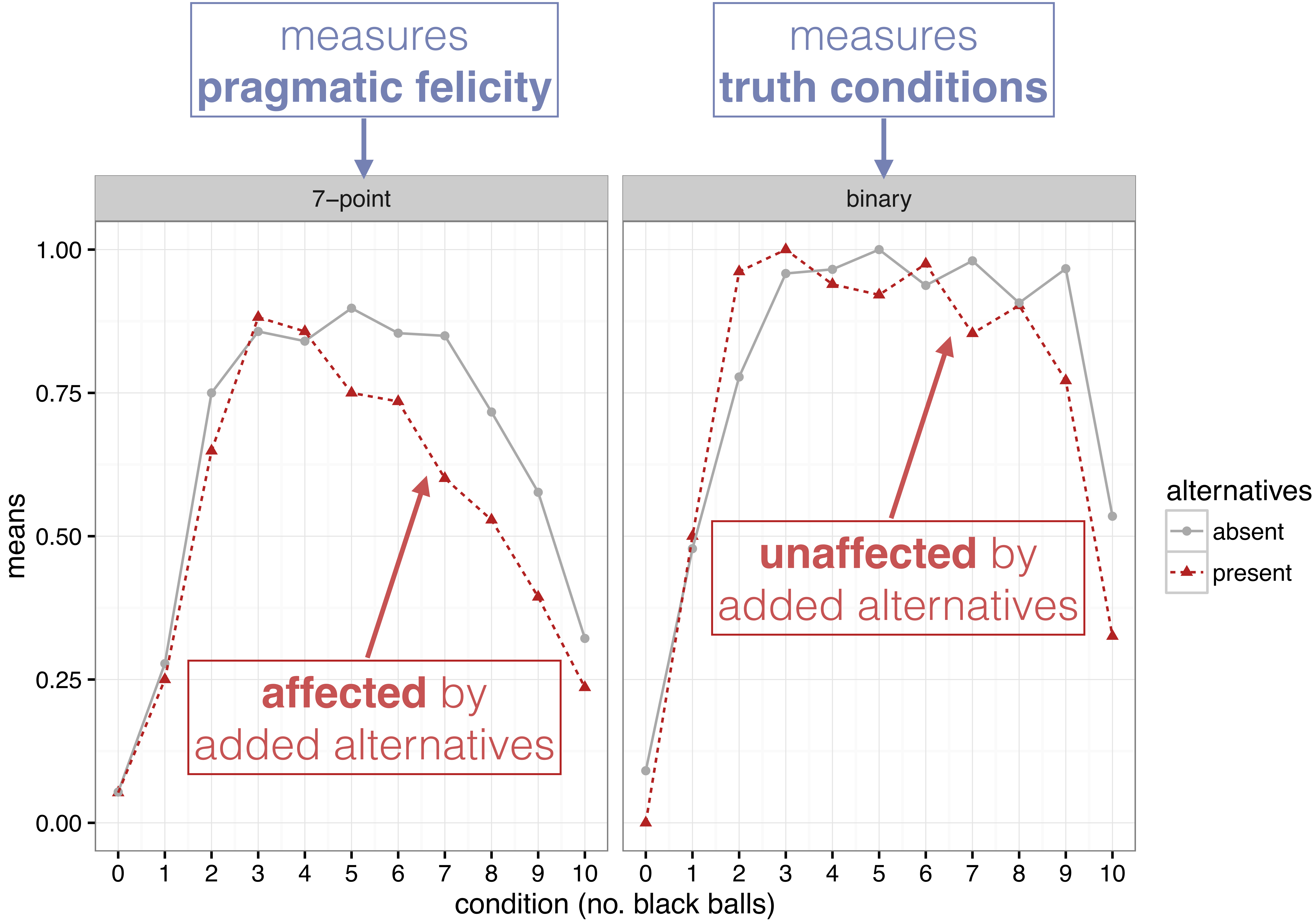

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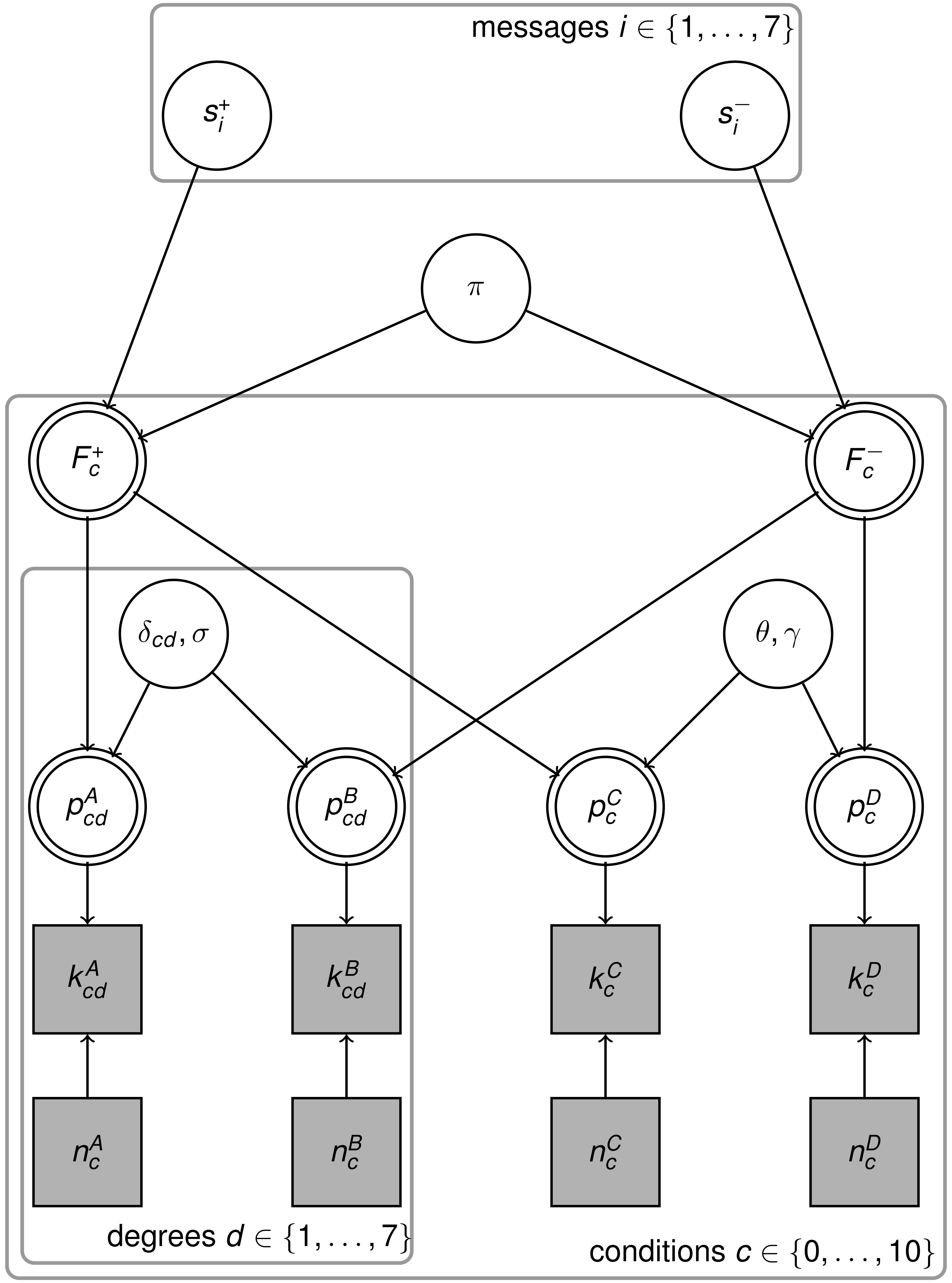
approach

combine:

1. theory-driven probabilistic pragmatics models
2. link function (from regression modeling)

Data-generating model



$$s_i^{+/-} \sim \text{Beta}(1, 1)$$

$$\frac{1}{\pi} \sim \text{Gamma}(0.5, 0.5)$$

$$F_c^{+/-} = \text{F}(c\,;\ \tilde{s}^{\,+/-}, \pi)$$

$$\sigma \sim \text{Uniform}(0, 0.4)$$

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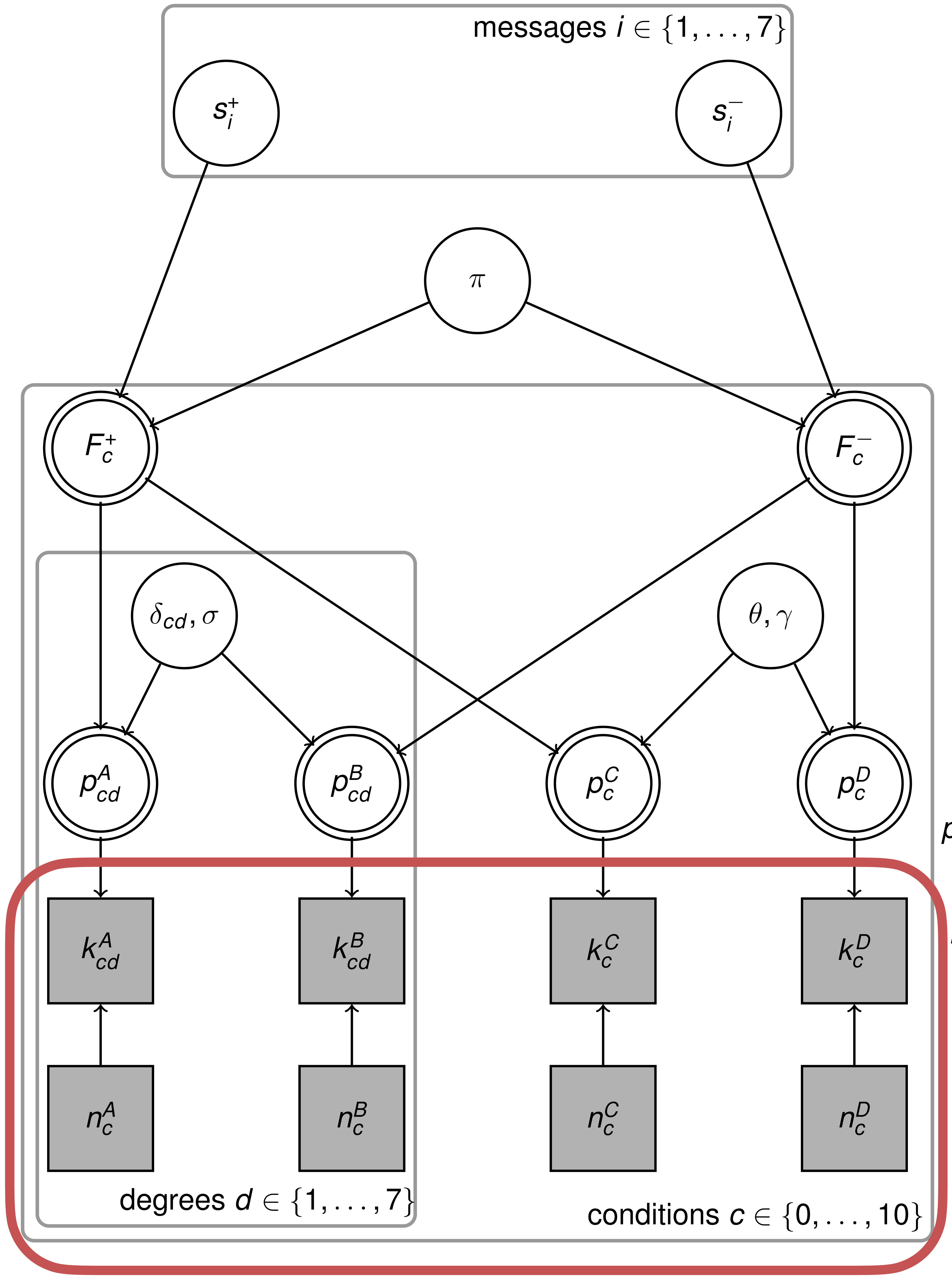
Data-generating model

data



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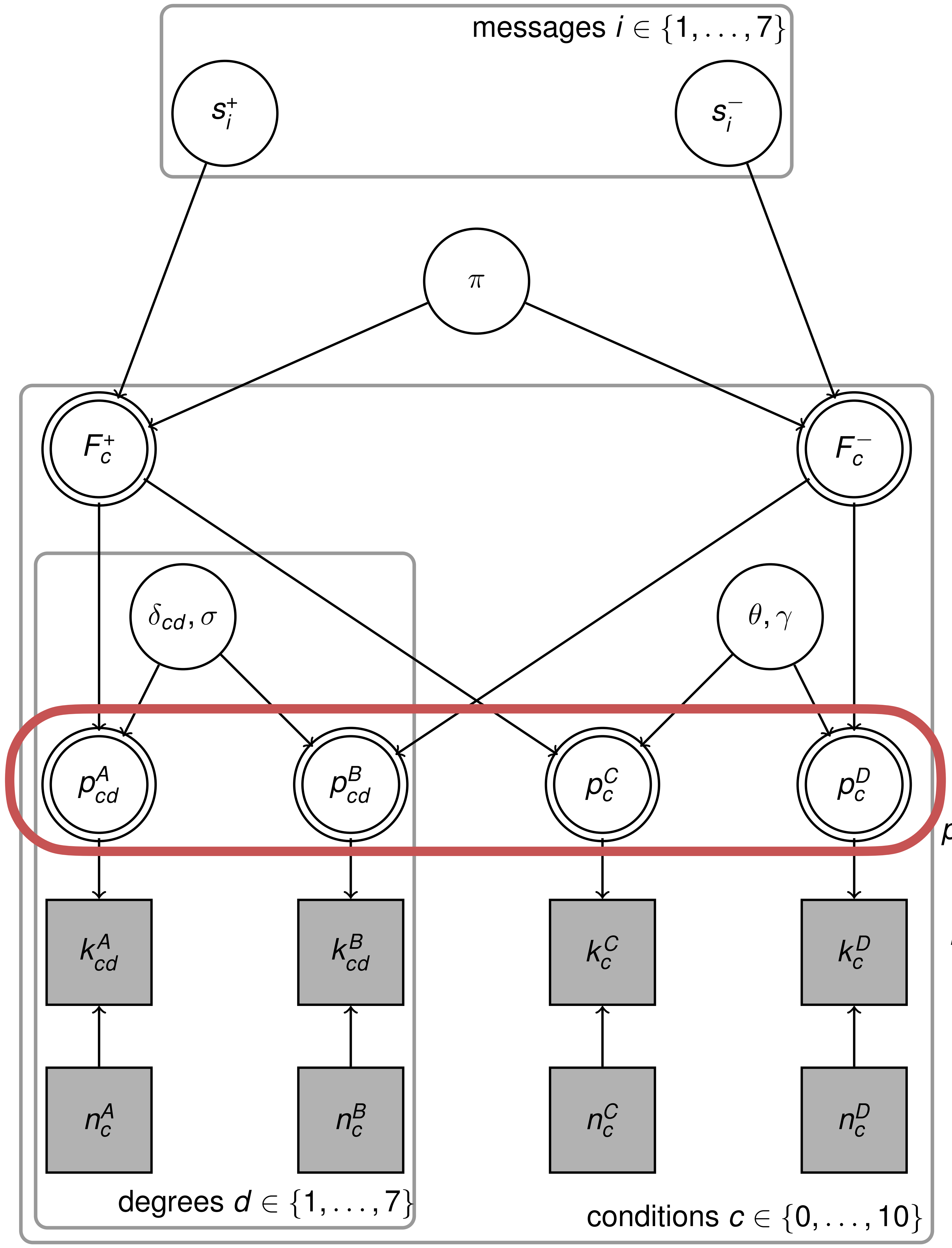

choice probabilities

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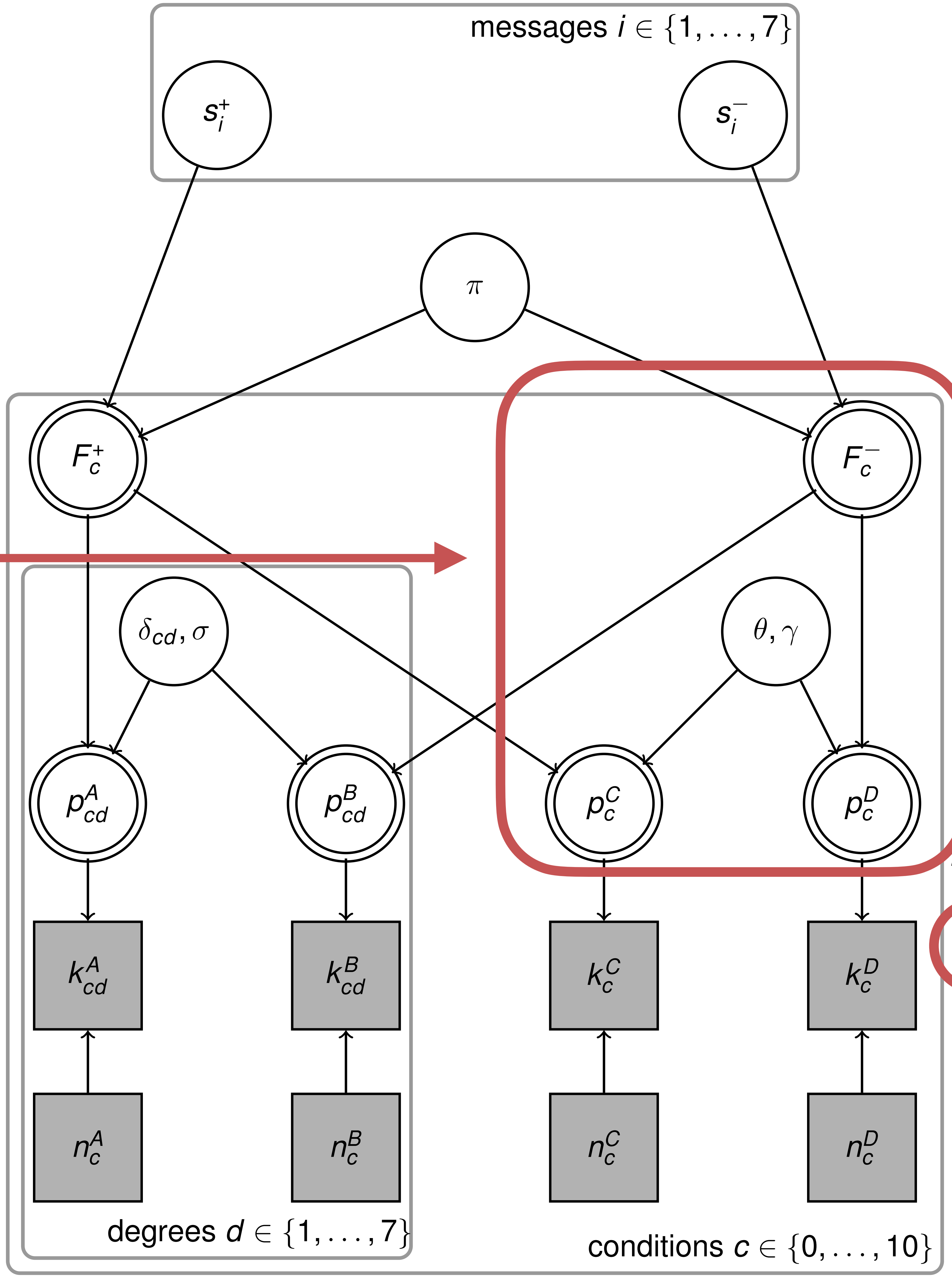
Data-generating model

logistic
linking function



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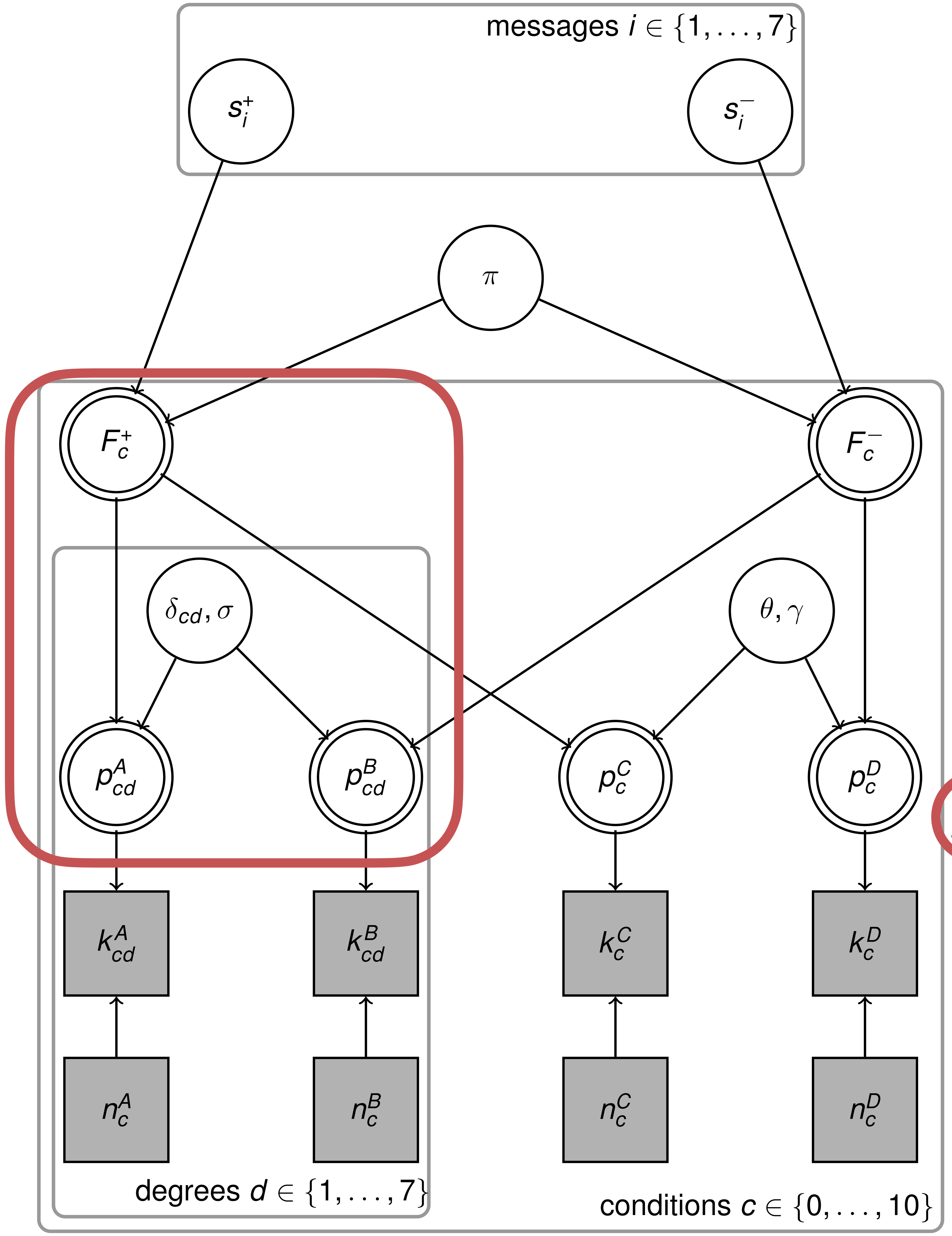
Data-generating model

inverse Gaussian linking function



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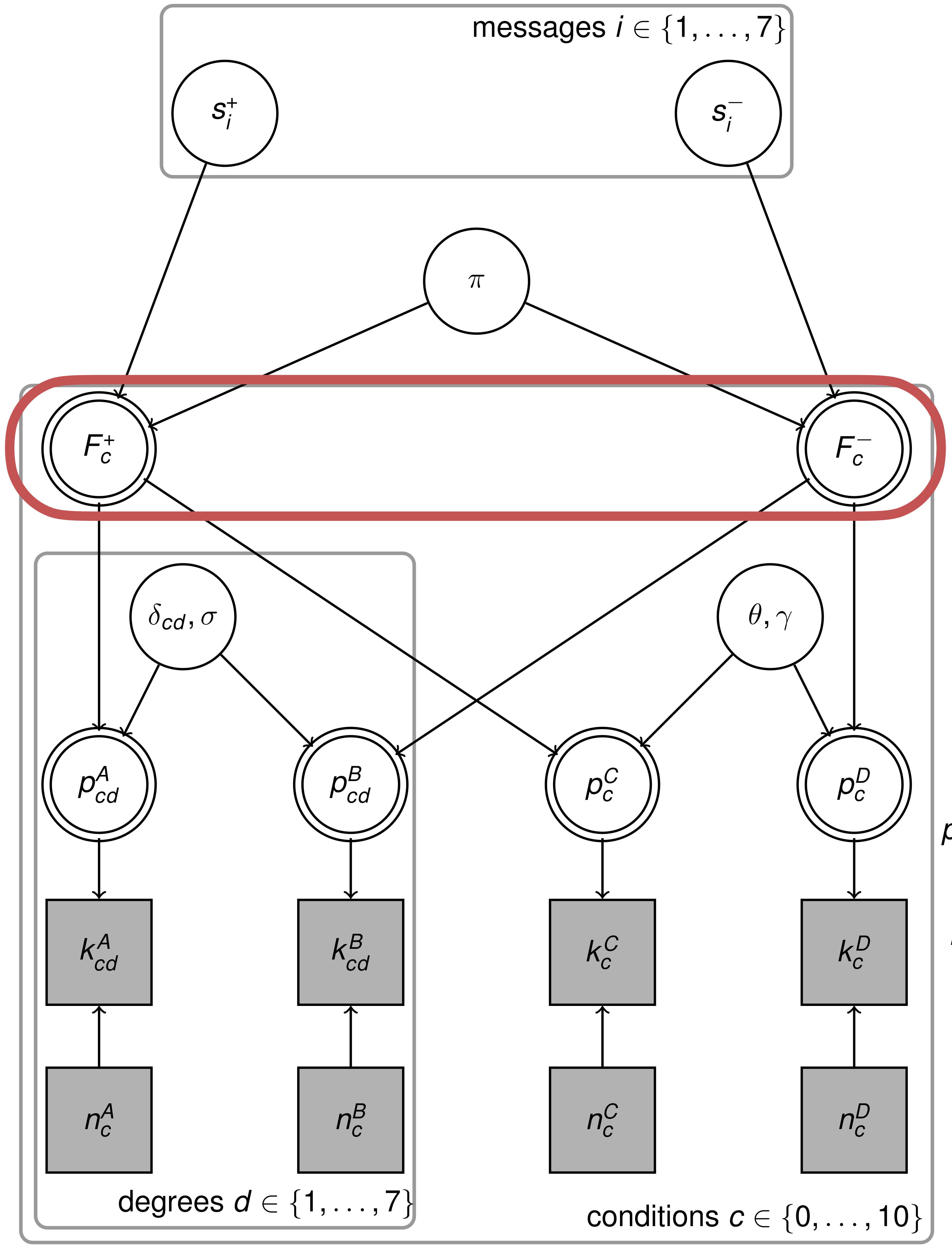
Data-generating model

pragmatic
felicity



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Pragmatic felicity

how good a description is "some" (numerically)

literal listener

$$P_{LL}(t \mid m) \propto P_{prior}(t) \times B(m, t)$$

speaker's expected utility

$$\mathrm{EU}_S(m, t, P_{LL} \,;\, \pi) = \sum_{t'} P_{LL}(t' \mid m) \times \mathrm{U}(t, m, t' \,;\, \pi)$$

EU relative to alternatives X

$$\mathrm{EU}^*(m, t, X \,;\, \pi) = \frac{\mathrm{EU}(m, t, P_{LL} \,;\, \pi) - \min_{m' \in X} \mathrm{EU}(m', t, P_{LL} \,;\, \pi)}{\max_{m' \in X} \mathrm{EU}(m', t P_{LL} \,;\, \pi) - \min_{m' \in X} \mathrm{EU}(m', t, P_{LL} \,;\, \pi)}$$

probability of entertaining X

$$P(X \mid \vec{s}) = \prod_{m \in X} s_m \prod_{m \in M \setminus X} (1 - s_m)$$

pragmatic felicity

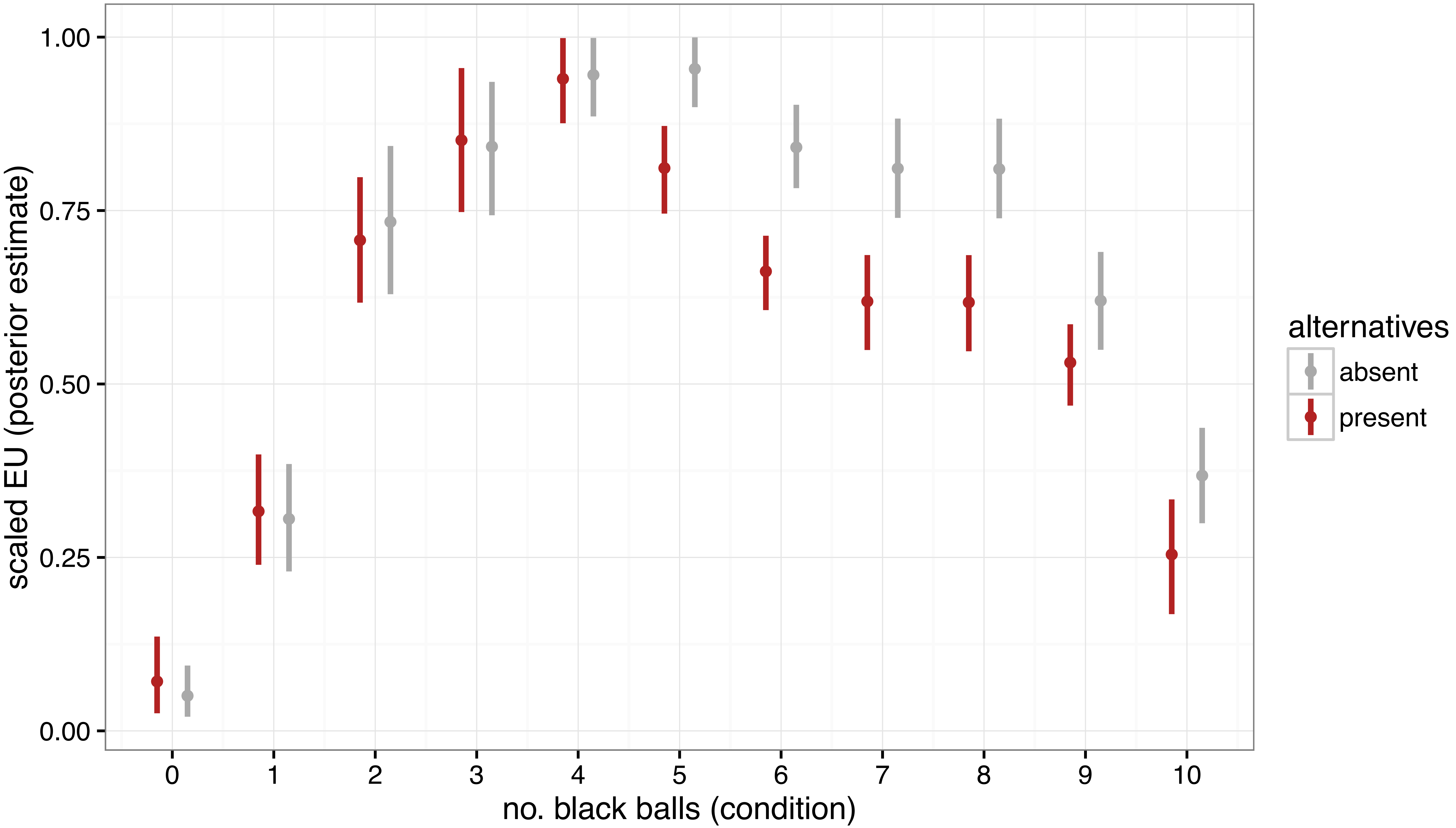
$$\mathrm{F}(m, t \,;\, \vec{s}, \pi) = \sum_X P(X \mid \vec{s})\, \mathrm{EU}^*(m, t, X \,;\, \pi)$$

pragmatic felicity

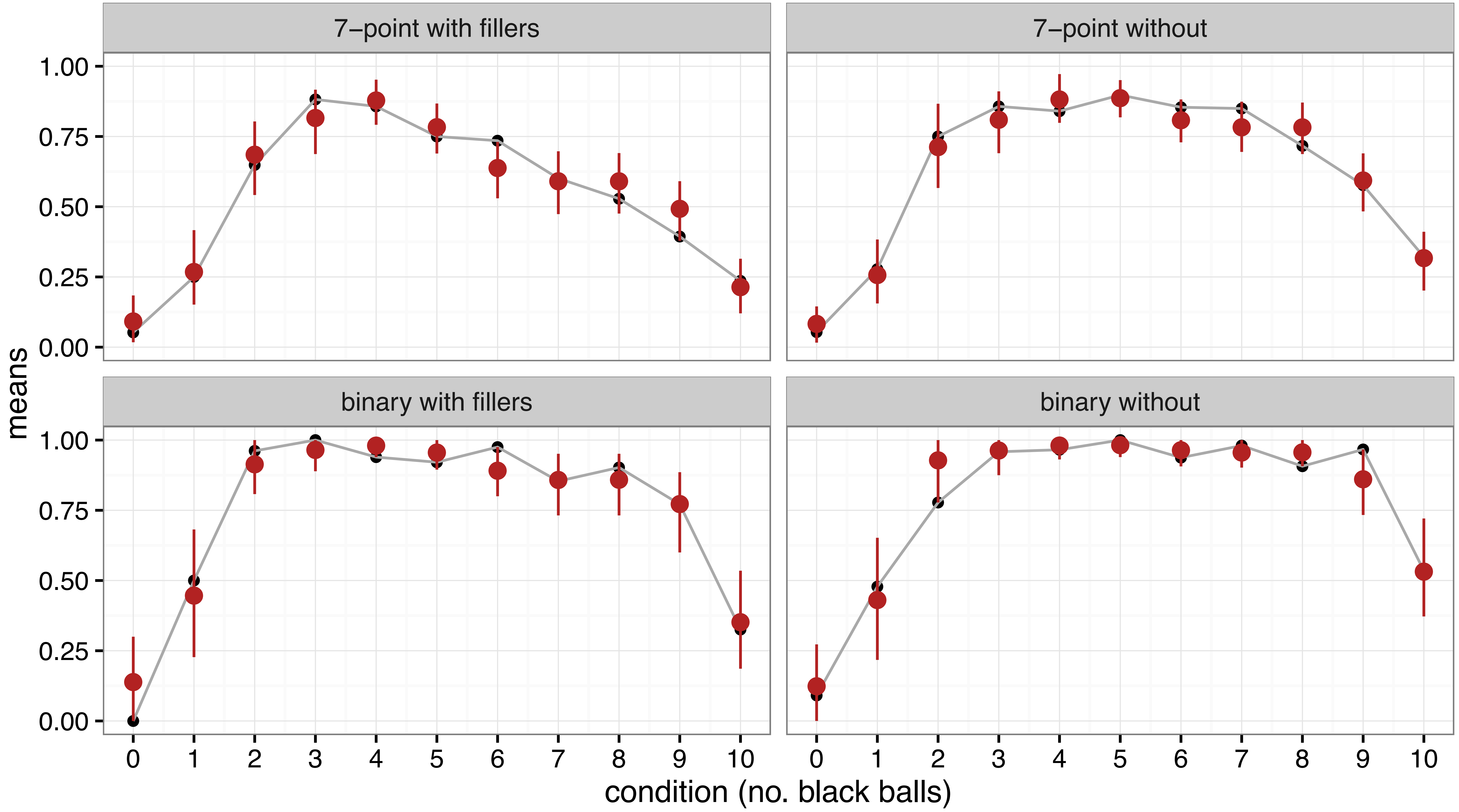
measures how good "*some*" is as a description of *t*, relative to other quantifiers, weighed by how salient these quantifiers are

Posterior pragmatic felicity $F^{+/-}$

CSP-Subheading



Posterior predictive checks



Conclusions of case study

- 7-point and binary tasks could measure the same thing
- full data-generating model identifies what a task measures
- **theory-driven Bayesian data analysis is powerful and insightful**