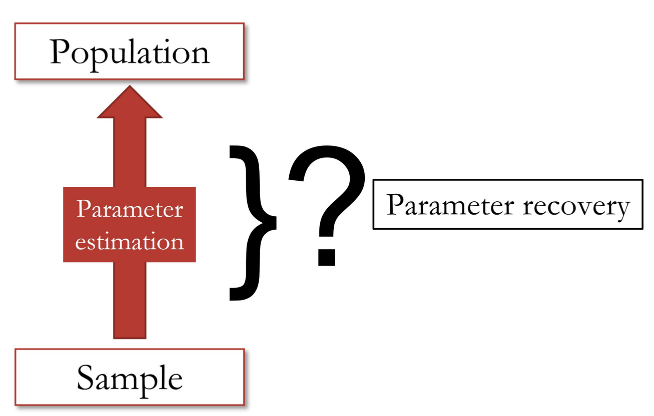
Classical Maximum Likelihood Estimation in Shifted-Wald Models of Response Times

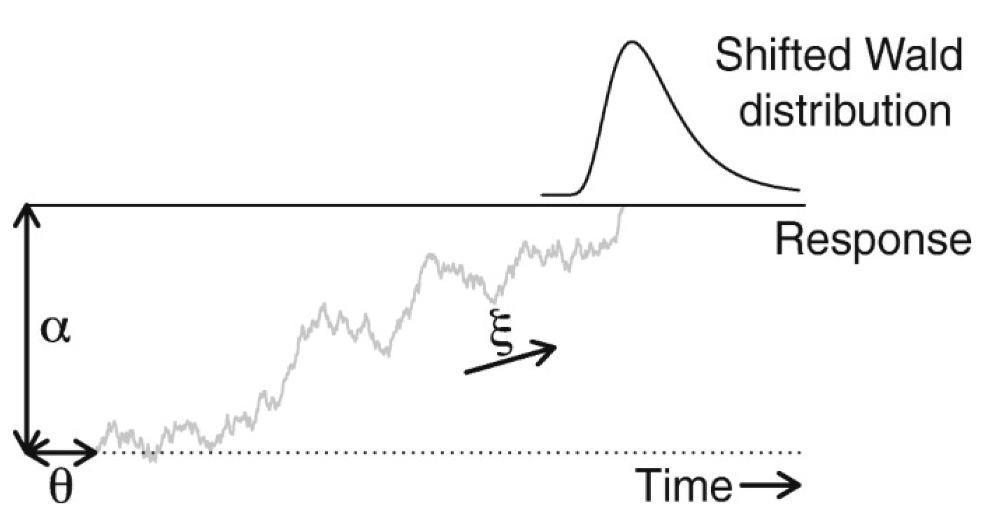
Bryanna L. Scheuler & Thomas J. Faulkenberry
Department of Psychological Sciences, Tarleton State University

Background

Response times are a crucial component of understanding and measuring cognitive processes. There are many probability distributions that can be used to describe distributions of response times. Thus, given some observed data, we use various techniques to estimate the population parameters of the distribution that could have potentially generated the observed sample. But, how do we know that the parameters we are estimating from a response time distribution are representative of the true population parameters? To assess the validity of our estimation technique, we must conduct a parameter recovery study.



This study focused on classical maximum likelihood estimation (CMLE), a method that works by maximizing the likelihood function that results when we observe some data (Myung, 2003). While one can use calculus to maximize some likelihood functions, the more common technique is to use computer methods to minimize the negative log-likelihood function. These techniques are conceptually easy to describe, but there are instances where the algorithm fails (e.g., stuck in a local minimum, etc.). Thus, it is essential to perform a parameter recovery study to gauge whether our CMLE algorithm can actually return the correct parameter values to us.
We tested CMLE on a specific model of response times known as the shifted-Wald model. The shifted-Wald model is composed of three parameters: shift, drift rate, and response threshold.



Method

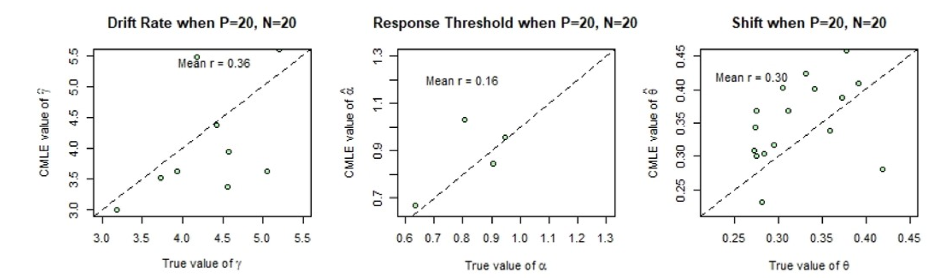
For this study, we simulated data using shifted-Wald parameter targets reported by Faulkenberry et al. (2018). This was composed of four main steps:

- Generate 'artificial' people from parent population
- Generate response times for artifical people
- Fit shifted-Wald model to RT distribution with CMLE
- Compare estimates to original target parameters

This process was applied to 5 sub-experiments in a design used by Farrell and Ludwig (2008). These sub-experiments had 5, 20, or 80 participants, with 20, 80, or 500 trials per particpant.

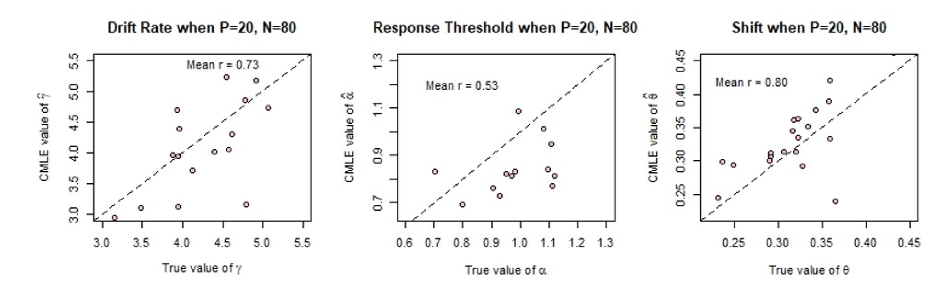
Experiments

Experiment 1: Participants = 20, Trials = 20



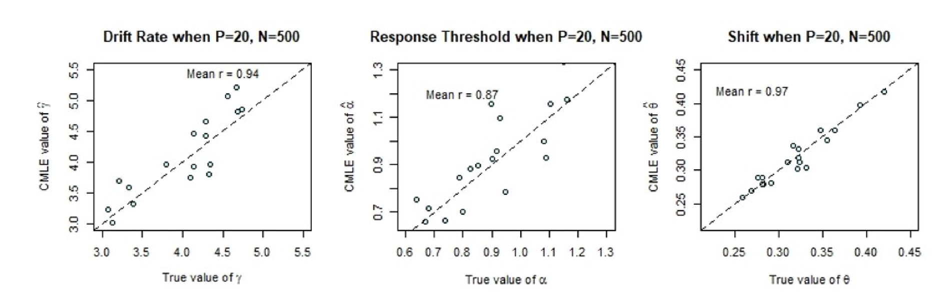
Comparison Method	Drift Rate (γ)	Response Threshold (α)	Shift (θ)
Correlation	0.36	0.16	0.30
RMSD	2.42	3.47	0.24
Mean Bias	0.58(0.58)	0.96(0.98)	-0.05(0.06)

Experiment 2: Participants = 20, Trials = 80



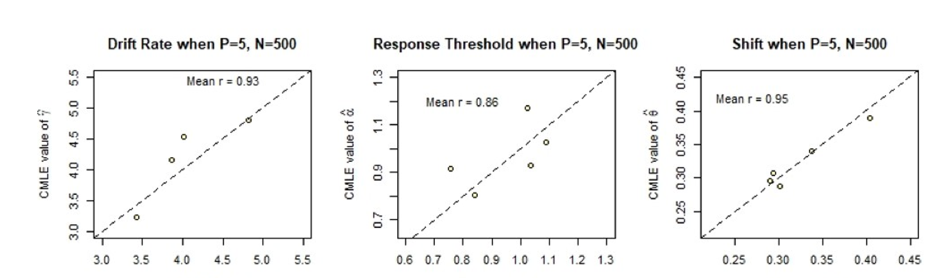
Comparison Method	Drift Rate (γ)	Response Threshold (α)	Shift (θ)
Correlation	0.73	0.53	0.80
RMSD	0.66	0.30	0.04
Mean Bias	0.06(0.15)	0.03(0.07)	0.00(0.01)

Experiment 3: Participants = 20, Trials = 500



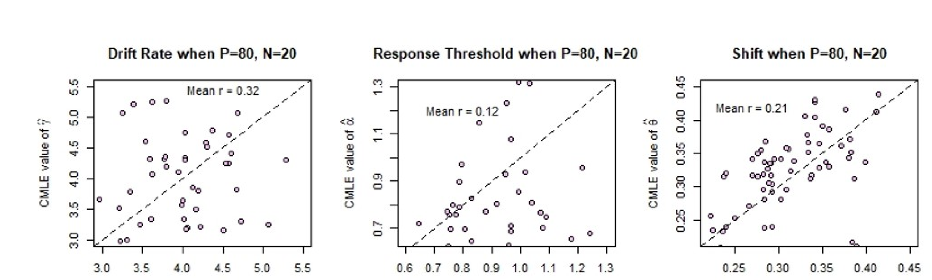
Comparison Method	Drift Rate (γ)	Response Threshold (α)	Shift (θ)
Correlation	0.94	0.87	0.97
RMSD	0.24	0.09	0.01
Mean Bias	0.01(0.05)	0.00(0.02)	0.00(0.00)

Experiment 4: Participants = 5, Trials = 500



Comparison Method	Drift Rate (γ)	Response Threshold (α)	Shift (θ)
Correlation	0.93	0.86	0.95
RMSD	0.23	0.09	0.01
Mean Bias	0.02(0.10)	0.01(0.04)	0.00(0.01)

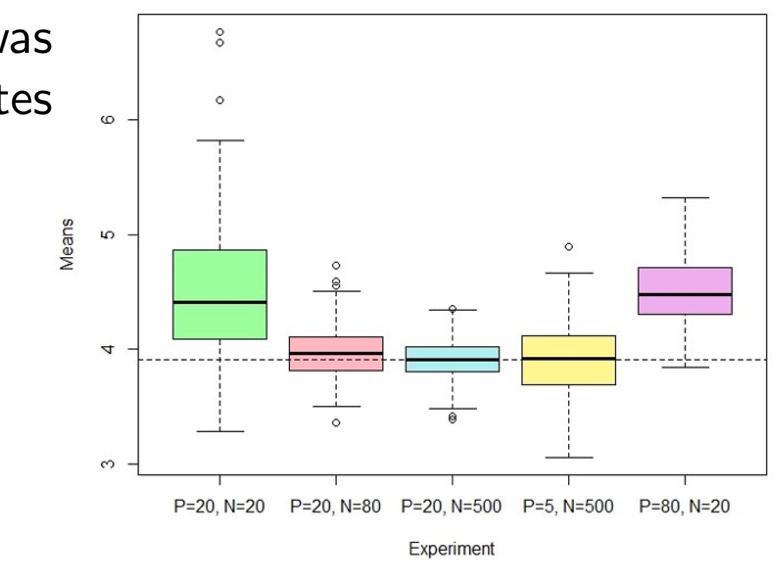
Experiment 5: Participants = 80, Trials = 20



Comparison Method	Drift Rate (γ)	Response Threshold (α)	Shift (θ)
Correlation	0.32	0.12	0.21
RMSD	2.65	4.17	0.27
Mean Bias	0.61(0.31)	0.98(0.48)	-0.05(0.04)

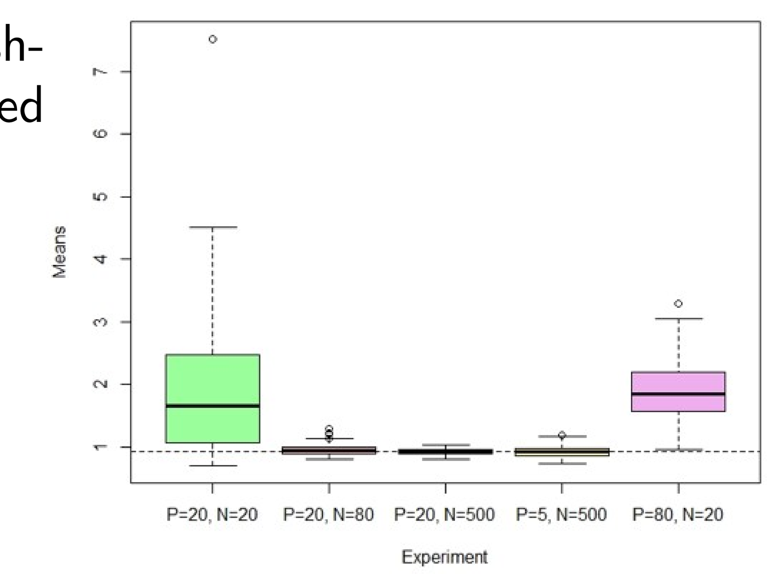
Recovering drift rate?

The target mean for drift rate was 3.91. The mean estimated drift rates were:
Experiment 1 = 4.49 (0.60)
Experiment 2 = 3.97 (0.23)
Experiment 3 = 3.91 (0.17)
Experiment 4 = 3.92 (0.32)
Experiment 5 = 4.52 (0.31)



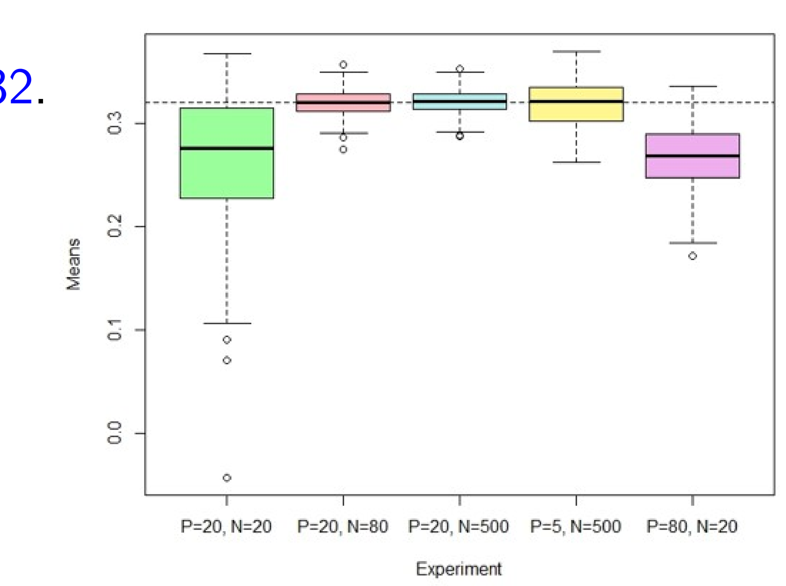
Recovering response threshold?

The target mean for response threshold was 0.92. The mean estimated response thresholds were:
Experiment 1 = 1.88 (0.98)
Experiment 2 = 0.95 (0.08)
Experiment 3 = 0.92 (0.04)
Experiment 4 = 0.92 (0.09)
Experiment 5 = 1.89 (0.45)



Recovering shift?

The target mean for shift was 0.32. The mean estimated shifts were:
Experiment 1 = 0.27 (0.06)
Experiment 2 = 0.32 (0.01)
Experiment 3 = 0.32 (0.01)
Experiment 4 = 0.32 (0.02)
Experiment 5 = 0.27 (0.03)



Discussion

There are still many questions left to be answered:

- How do other methods compare to CMLE when estimating shifted-Wald parameters?
- Is there an optimum trial size when using CMLE for these models?
- How can we use this information about participant versus trial size to make informed decisions about sampling plans?