



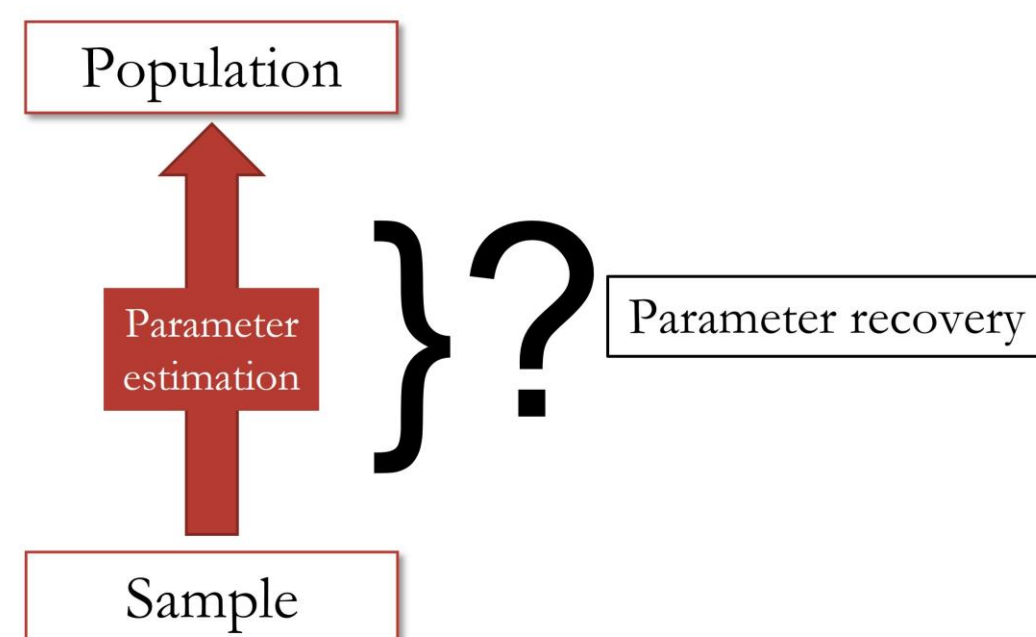
Evaluating Single-Level and Hierarchical Maximum Likelihood Estimation in Shifted-Wald Models



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Parameter Recovery

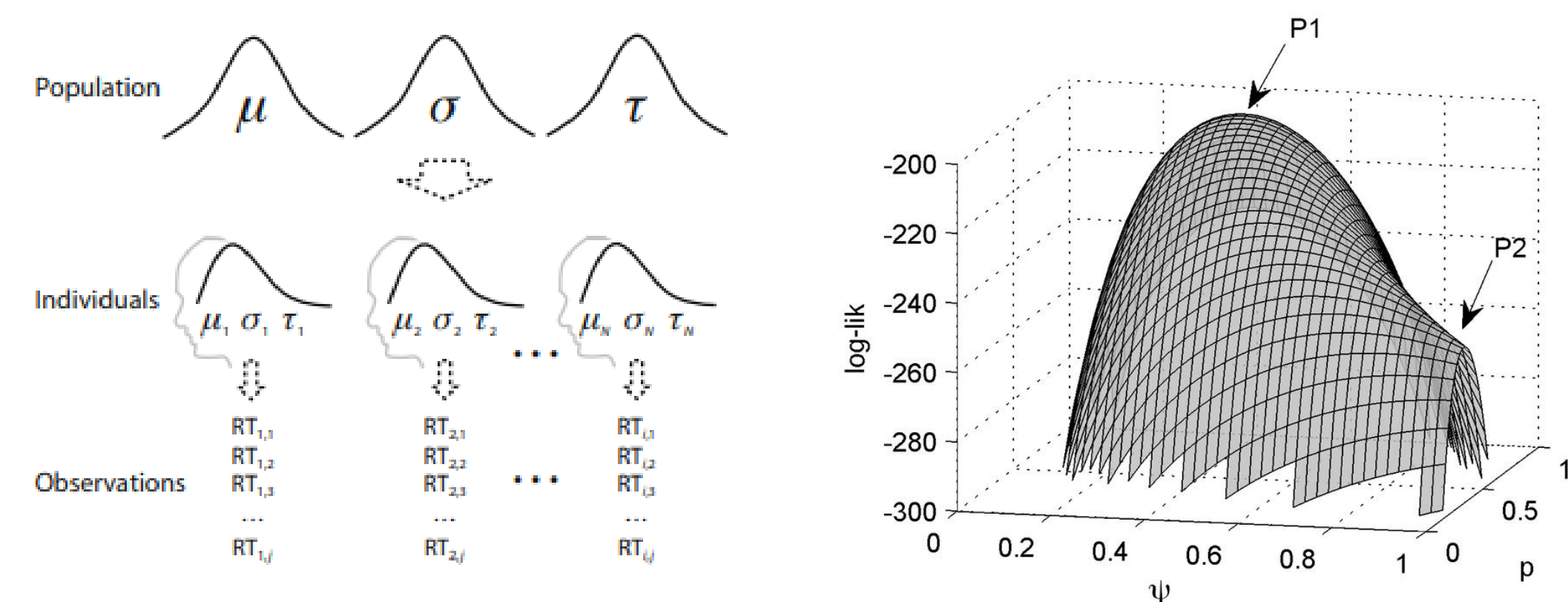
Response times are a crucial component of understanding and measuring cognitive processes. By fitting a model to a distribution of observed response times, we can estimate population parameters that could have potentially generated the observed sample. But, [how do we know that the estimated parameters are representative of the true population parameters?](#) To assess the validity of our estimation techniques, we must conduct a [parameter recovery study](#).



Estimation Methods

This study focused on two methods for estimating parameters:

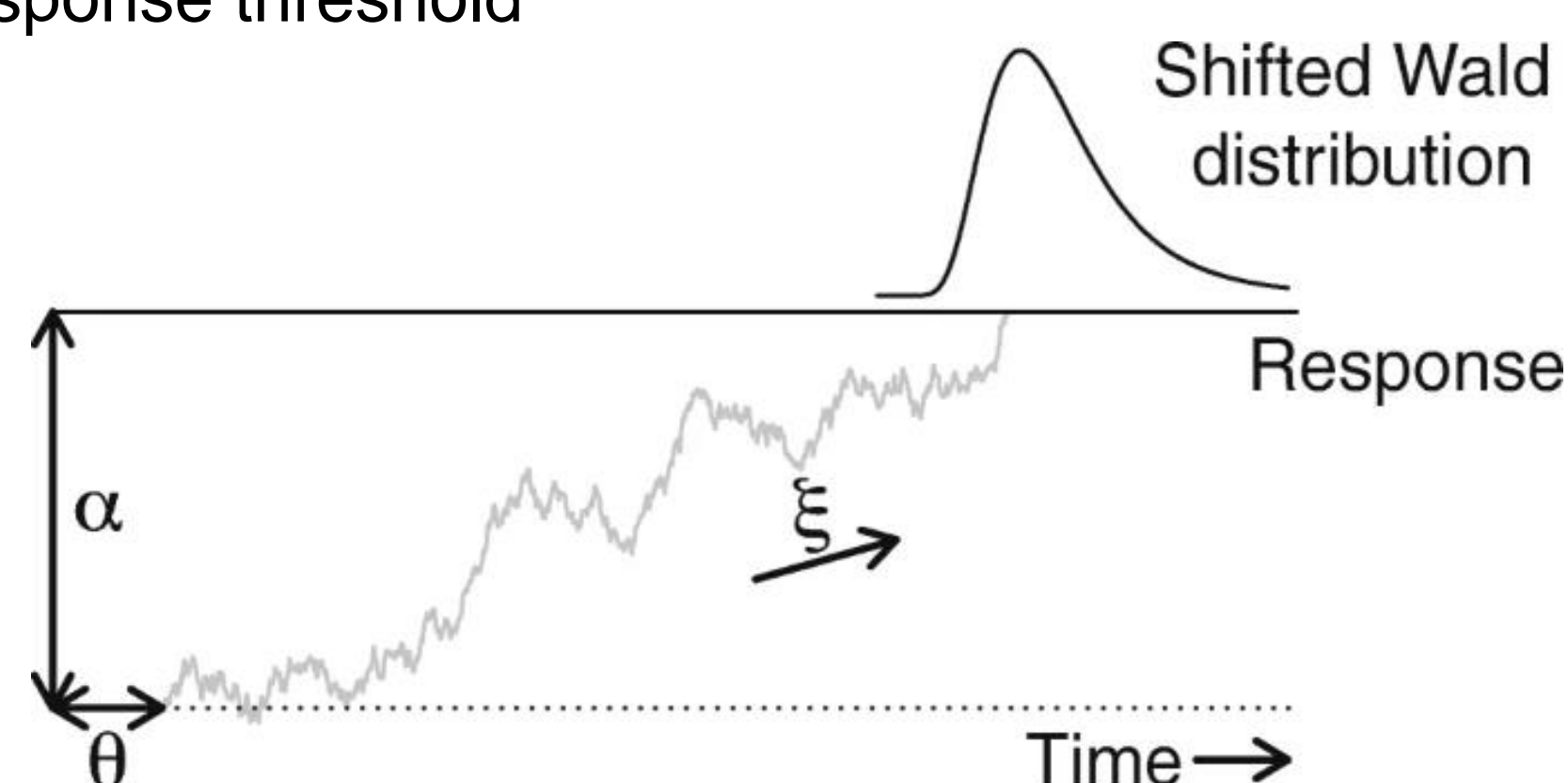
- Classical (single-level) maximum likelihood estimation (CMLE)
- Hierarchical maximum likelihood estimation (HMLE)



Shifted-Wald Models

For this study we used a model of response times known as the [shifted-Wald model](#), which is composed of three parameters:

- Shift
- Drift rate
- Response threshold



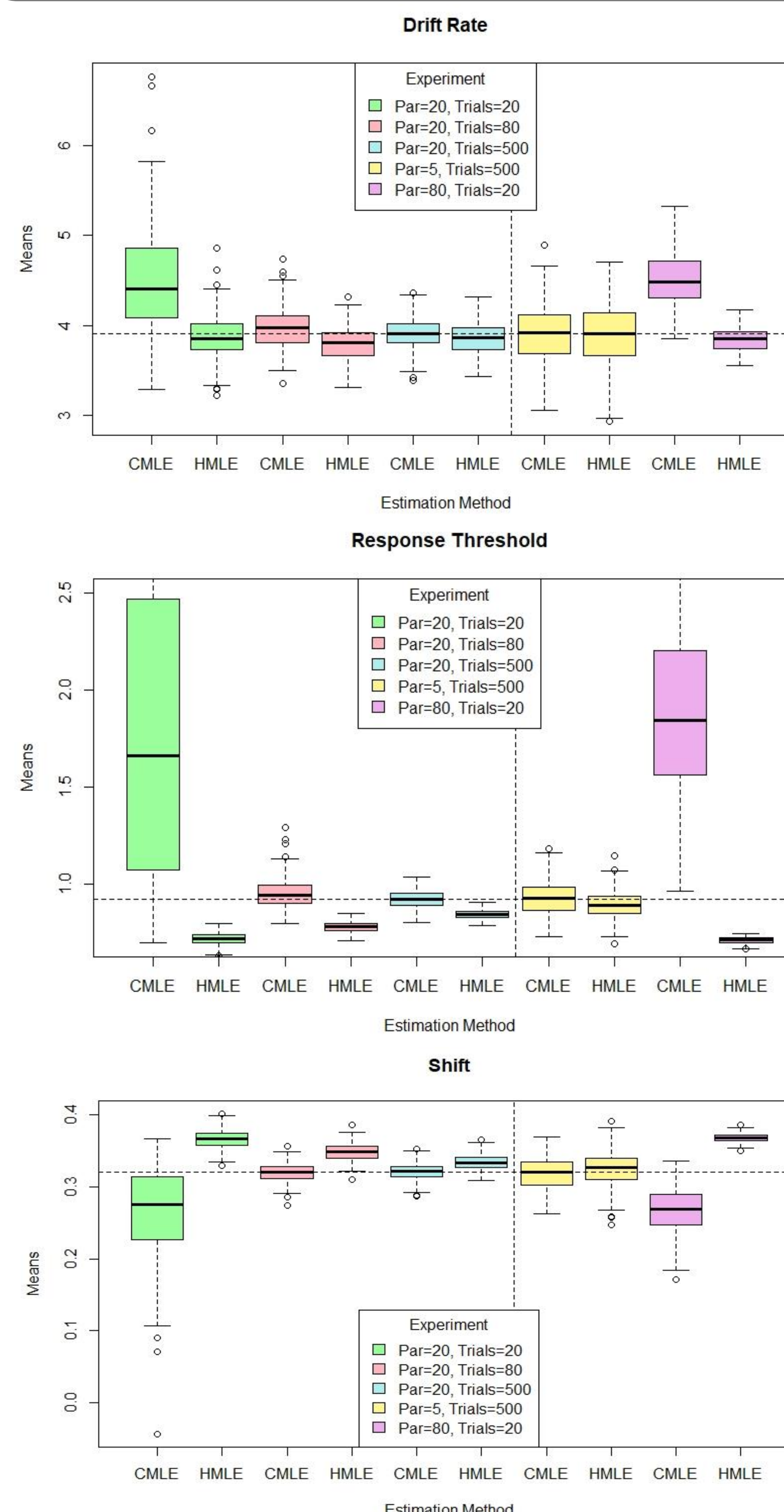
Methods

For this study, we simulated data using shifted-Wald parameter targets reported by Faulkenberry et al. (2018). This was composed of four main steps:

- Generate 'artificial' people from parent population
- Generate response times for artificial people
- Fit shifted-Wald model to RT distribution with CMLE and HMLE
- Compare estimates to original target parameters

This process was applied to 5 sub-experiments in a design used by Farrell and Ludwig (2008). These sub-experiments had 5, 20, or 80 participants, with 20, 80, or 500 trials per participant.

Results: Parameters



Results: Error and Bias

	Root Mean Square Error		
	Root Mean Square Error in Seconds		
	ν	α	ϑ
<i>Par = 20, Trials = 20</i>			
HMLE Group Level	0.198	0.202	0.046
HMLE Individual Level	0.673	0.226	0.046
CMLE Individual Level	2.419	3.474	0.236
<i>Par = 20, Trials = 80</i>			
HMLE Group Level	0.175	0.139	0.028
HMLE Individual Level	0.401	0.157	0.029
CMLE Individual Level	0.657	0.296	0.036
<i>Par = 20, Trials = 500</i>			
HMLE Group Level	0.144	0.074	0.014
HMLE Individual Level	0.187	0.101	0.017
CMLE Individual Level	0.236	0.095	0.012
<i>Par = 5, Trials = 500</i>			
HMLE Group Level	0.275	0.060	0.019
HMLE Individual Level	0.364	0.115	0.016
CMLE Individual Level	0.231	0.092	0.012
<i>Par = 80, Trials = 20</i>			
HMLE Group Level	0.107	0.209	0.048
HMLE Individual Level	0.678	0.232	0.048
CMLE Individual Level	2.647	4.171	0.268

Note. CMLE = classical maximum likelihood estimation; HMLE = hierarchical maximum likelihood estimation

	Mean Bias (B) and Standard Deviation(SD) in Seconds					
	ν		α		ϑ	
	B	SD	B	SD	B	SD
<i>Par = 20, Trials = 20</i>						
HMLE Group Level	-0.045	0.251	-0.202	0.032	0.046	0.013
HMLE Individual Level	-0.050	0.203	-0.200	0.042	0.046	0.007
CMLE Individual Level	0.578	0.581	0.961	0.975	-0.053	0.062
<i>Par = 20, Trials = 80</i>						
HMLE Group Level	-0.111	0.188	-0.139	0.026	0.028	0.012
HMLE Individual Level	-0.100	0.114	-0.137	0.032	0.028	0.005
CMLE Individual Level	0.064	0.148	0.034	0.073	-0.001	0.008
<i>Par = 20, Trials = 500</i>						
HMLE Group Level	-0.061	0.171	-0.074	0.026	0.013	0.010
HMLE Individual Level	-0.062	0.053	-0.078	0.020	0.014	0.004
CMLE Individual Level	0.008	0.052	0.003	0.021	0.000	0.003
<i>Par = 5, Trials = 500</i>						
HMLE Group Level	-0.014	0.342	-0.023	0.070	0.005	0.023
HMLE Individual Level	-0.008	0.149	-0.033	0.054	0.004	0.009
CMLE Individual Level	0.017	0.102	0.006	0.042	0.000	0.006
<i>Par = 80, Trials = 20</i>						
HMLE Group Level	-0.066	0.117	-0.209	0.017	0.048	0.006
HMLE Individual Level	-0.075	0.100	-0.211	0.021	0.048	0.003
CMLE Individual Level	0.615	0.308	0.980	0.476	-0.051	0.036

Note. CMLE = classical maximum likelihood estimation; HMLE = hierarchical maximum likelihood estimation

- There is a trade-off in estimation parameters
- If a study is constrained to a smaller trial size, utilizing HMLE could ensure more accurate estimation

Discussion

There are still many questions left to be answered:

- How would an alternative approach, (i.e., Bayesian) change our results?
- Is there an optimum number of participants or trials?
- How can we utilize these results to inform participant/ trial size decisions?



For references, R script, and more, please see QR code.