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FIT FUNCTIONS R SCRIPT
# various utility functions for parameter recovery routines
# density function for Wald (unshifted)
dwald = function(x, gamma, alpha) {
 return((alpha/(sqrt(2*pi*x^3)))*exp(-(alpha-gamma*x)^2/(2*x)))
# function to generate random shifted Wald data
# adapted from pp. 79-80, Dagpunar, J. (1988). Principles of Random Variate
Generation. Clarendon Press, Oxford.
# code modified from Heathcote (2004)
rwald = function(n, gamma, alpha, theta) {
 y2 = rchisq(n, 1)
 v2onm = v2/gamma
 u = runif(n)
 r1 = (2*a1pha + y2onm - sqrt(y2onm*(4*a1pha+y2onm)))/(2*gamma)
 r2 = (alpha/gamma)^2/r1
 ifelse(u < alpha/(alpha+gamma*r1), theta+r1, theta+r2)
# basic function that returns Root Mean Squared Error
rmse = function(error) {
 sgrt (mean (error 2))
# negative log likelihood for shifted Wald
nll. wald = function(par, dat) {
 return(-sum(log(dwald(dat-par[3], gamma=par[1], alpha=par[2]))))
# Hierach. negative log likelihood for shifted Wald
nll.wald.h = function(par, dat) {
 n subjects <- dim(dat)[1]
 n_trials <- dim(dat)[2]
par <- exp(par)
gamma group mean <- par[1]
gamma group sd <- par[2]
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alpha group mean <- par[3]

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alpha group sd <- par[4]
theta group mean <- par[5]
theta_group_sd <- par[6]
gamma ind <- par[6 + 1:(n subjects-1)]
alpha ind \langle -par[6 + (n subjects-1) + 1: (n subjects-1)]
theta ind \langle -par[6 + 2*(n subjects-1) + 1:(n subjects-1)]
gamma ind <- c(gamma ind, n subjects*gamma group mean - sum(gamma ind))
alpha_ind <- c(alpha_ind, n_subjects*alpha_group_mean - sum(alpha_ind))
theta ind <- c(theta ind, n subjects*theta group mean - sum(theta ind))
theta mat<- matrix(rep(theta ind, each=n trials), n subjects, n trials, byrow=TRUE)
if (any (dat-theta mat <0)) {return(Inf)}
n11 \leftarrow 0
  for (s in 1:n subjects) {
    nll <- nll - sum(log(dwald(dat[s,]-theta ind[s], gamma=gamma ind[s],
alpha=alpha ind[s])))
nll <- nll -sum(dnorm(gamma ind, mean=gamma group mean, sd=gamma group sd), log=TRUE)
nll <- nll -sum(dnorm(alpha ind, mean=alpha group mean, sd=alpha group sd), log=TRUE)
nll <- nll -sum(dnorm(theta ind, mean=theta group mean, sd=theta group sd), log=TRUE)
  return (n11)
# Start point estimate for SW, based on first two moments
# assumes s = p \cdot min(x), where x is a data vector
# from Heathcote (2004)
waldinit = function (x, p = 0.9) {
  theta = p*min(x)
  x = x - theta
  gamma = sqrt(mean(x)/var(x))
  alpha = gamma*mean(x)
  return (c (gamma, alpha, theta))
waldinit. h = function(x, p = 0.9) {
  #x: matrix of rt w rows indicating subject and columns indicating trial
 n subjects \langle -\dim(x)[1] \rangle
 n trials \langle -\dim(x)[2] \rangle
   theta ind \langle -p*apply(x, 1, min) \# vector of shift parameters
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theta_group_mean <- mean(theta_ind) #group mean shift parameter
  theta group sd <- sd(theta ind)
                                 #SD of shift across group
  theta_mat<- matrix(rep(theta_ind, each=n_trials), n_subjects, n_trials, byrow=TRUE)
#diff shift per person > create matrix
 x = x - theta mat
 gamma ind = sqrt(apply(x, 1, mean) / apply(x, 1, var)) #gamma vector
  gamma group mean <- mean(gamma ind)
  gamma group sd <- sd(gamma ind)
 alpha_ind <- gamma_ind *apply(x, 1, mean) #alpha vector
 alpha group mean <- mean(alpha ind) #group mean alpha
 alpha group sd <- sd(alpha ind)
 #to make sure you pull the right parameter, we have to be careful; pull group first
 par <- c (gamma group mean, gamma group sd,
          alpha group mean, alpha group sd,
          theta_group_mean, theta_group_sd,
          gamma ind[-n subjects], alpha ind[-n subjects], theta ind[-n subjects])
 return(log(par))
CMLE R SCRIPT
#source("fitFunctions.R")
# define simulation parameters
nSub = 20
nTrials = 20
# parent distribution means and sds (from F, Vick, Bowman, 2018)
G = 3.91 \# gamma
G. sd = 0.70
A = 0.92 \# alpha
A. sd = 0.17
H = 0.32 \# theta
H. sd = 0.05
```

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# each simulation run starts HERE
# build simulated RT matrix
# rows = subjects, columns = trials
rts = matrix(0, nrow=nSub, ncol=nTrials)
# build matrix to store "target" parameters
# rows = subjects, columns = gamma, alpha, theta
targets = matrix(0, nrow=nSub, ncol=3)
# randomly draw target shifted Wald parameters from parent distribution
# (one unique parameter value for each subject)
for (i in 1:nSub) {
  targets[i, 1] = rnorm(1, mean=G, sd=G.sd) # draw random gamma
  targets[i, 2] = rnorm(1, mean=A, sd=A.sd) # draw random alpha
  targets[i, 3] = rnorm(1, mean=H, sd=H.sd) # draw random theta
# from these target SW values, generate the distribution of observed RTs for each
subject
for (i in 1:nSub) {
  rts[i,] = rwald(nTrials,
                  gamma = targets[i, 1],
                  alpha = targets[i, 2],
                  theta = targets[i, 3])
}
# now let's use CMLE to fit shifted Wald distribution to each row of observed RTs
# first, we'll store these estimates in a matrix
predictions = matrix(0, nrow=nSub, ncol=3)
# now do the fits (one per subject)
for (i in 1:nSub) {
  fit = optim(waldinit(rts[i,]), nll.wald, dat=rts[i,])
  predictions[i,] = fit$par
# now lets compare the predictions (fits) to the targets
# mean of parameter estimates
apply(predictions, 2, mean)
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# RMSD
apply (predictions-targets, 2, rmse)
# mean bias
apply(predictions-targets, 2, mean)
# correlation
diag(cor(predictions, targets))
HMLE R SCRIPT
#source("fitFunctions.R")
nSub = 20
nTrials = 20
# parent distribution means and sds (from F, Vick, Bowman, 2018)
G = 3.91 \# gamma
G. sd = 0.70
A = 0.92 \# alpha
A. sd = 0.17
H = 0.32 # theta
H. sd = 0.05
#matrices
rmse mat = matrix(NA, 200, 3+(3*nSub))
mb mat = matrix(NA, 200, 3+(3*nSub))
ind_means_mat = matrix(NA, 200, (3*nSub))
for (x in 1:200) {
  # each simulation run starts HERE
  # build simulated RT matrix
  # rows = subjects, columns = trials
  rts = matrix(0, nrow=nSub, ncol=nTrials)
  # build matrix to store "target" parameters
  # rows = subjects, columns = gamma, alpha, theta
  targets = matrix(0, nrow=nSub, ncol=3)
  # randomly draw target shifted Wald parameters from parent distribution
  # (one unique parameter value for each subject)
```

```
for (i in 1:nSub) {
    targets[i,1] = rnorm(1, mean=G, sd=G.sd) # draw random gamma
    targets[i,2] = rnorm(1, mean=A, sd=A.sd) # draw random alpha
    targets[i, 3] = rnorm(1, mean=H, sd=H.sd) # draw random theta
 # from these target SW values, generate the distribution of observed RTs for each
subject
 for (i in 1:nSub) {
    rts[i,] = rwald(nTrials,
                    gamma = targets[i, 1],
                    alpha = targets[i, 2],
                    theta = targets[i, 3])
 }
 # now let's use CMLE to fit shifted Wald distribution to each row of observed RTs
 # first, we'll store these estimates in a matrix
 #predictions = matrix(0, nrow=nSub, ncol=3)
 # now do the fits (one per subject)
 # mean of parameter estimates
 #apply(predictions, 2, mean)
 # RMSD
 #apply(predictions-targets, 2, rmse)
 # mean bias
 #apply(predictions-targets, 2, mean)
 # correlation
 #diag(cor(predictions, targets))
 #HIERARCHICAL MODEL FIT
 fit. h = optim(waldinit. h(rts), nll. wald. h, dat=rts)
 # now lets compare the predictions (fits) to the targets
 #will need to do
 par <- exp(fit.h$par)
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gamma_group_mean <- par[1]
gamma group sd <- par[2]
alpha_group_mean <- par[3]
alpha group sd <- par[4]
theta group mean <- par[5]
theta group sd <- par[6]
gamma ind <- par[6 + 1:(nSub-1)]
alpha ind \langle -par[6 + (nSub-1) + 1:(nSub-1)]
theta ind \langle -par[6 + 2*(nSub-1) + 1:(nSub-1)]
gamma_ind <- c(gamma_ind, nSub*gamma_group_mean - sum(gamma_ind))
alpha ind <- c(alpha ind, nSub*alpha group mean - sum(alpha ind))
theta ind <- c(theta ind, nSub*theta group mean - sum(theta ind))
##RMSE
#g level error
gg rmse = sqrt((G- gamma group mean)^2)
ga_rmse = sqrt((A- alpha_group_mean)^2)
gh_rmse = sqrt((H- theta_group_mean)^2)
#i level error
ig rmse = sqrt((targets[,1]-gamma ind)^2)
ia rmse = sqrt((targets[, 2]-alpha ind)^2)
ih_rmse = sqrt((targets[, 3]-theta_ind)^2)
rmse mat[x,] = c(gg rmse, ga rmse, gh rmse, ig rmse, ia rmse, ih rmse)
##MEAN BIAS
#g level mb
gg_mb = -1*(G-gamma_group_mean)
ga mb = -1*(A- alpha group mean)
gh mb = -1*(H- theta group mean)
#i level mb
ig mb = -1*(targets[, 1]-gamma ind)
ia mb = -1*(targets[, 2]-alpha ind)
ih mb = -1*(targets[, 3]-theta ind)
mb_mat[x,] = c(gg_mb, ga_mb, gh_mb, ig_mb, ia_mb, ih_mb)
#MEANS (individual level)
ind means mat[x,] = c(gamma ind, alpha ind, theta ind)
```

```
write.csv(rmse_mat, file= "hrmse.csv")
write.csv(mb_mat, file= "hmb.csv")
write.csv(ind_means_mat, file = "MeansMat.csv")
```