Cognitive processes in mental arithmetic: A confirmatory Bayesian analysis

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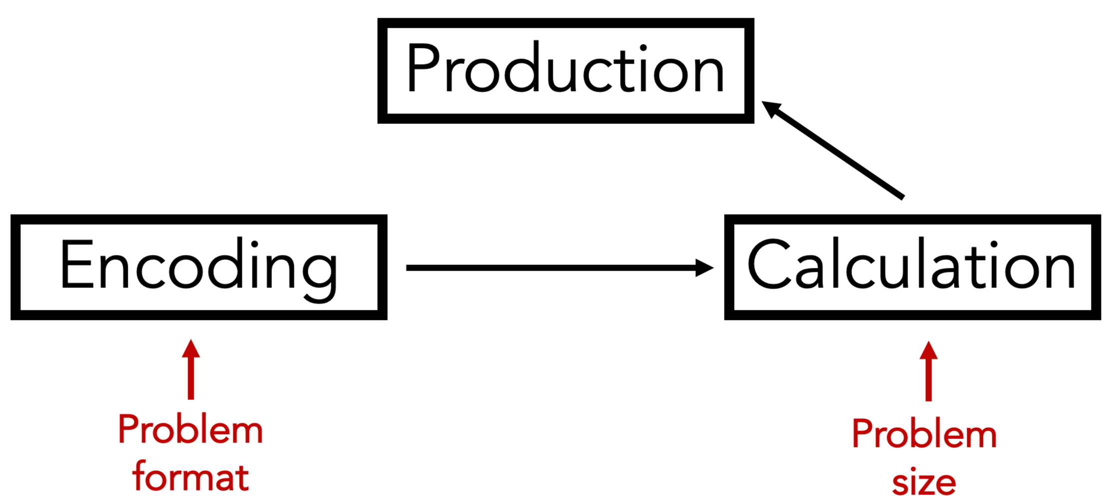
Problem

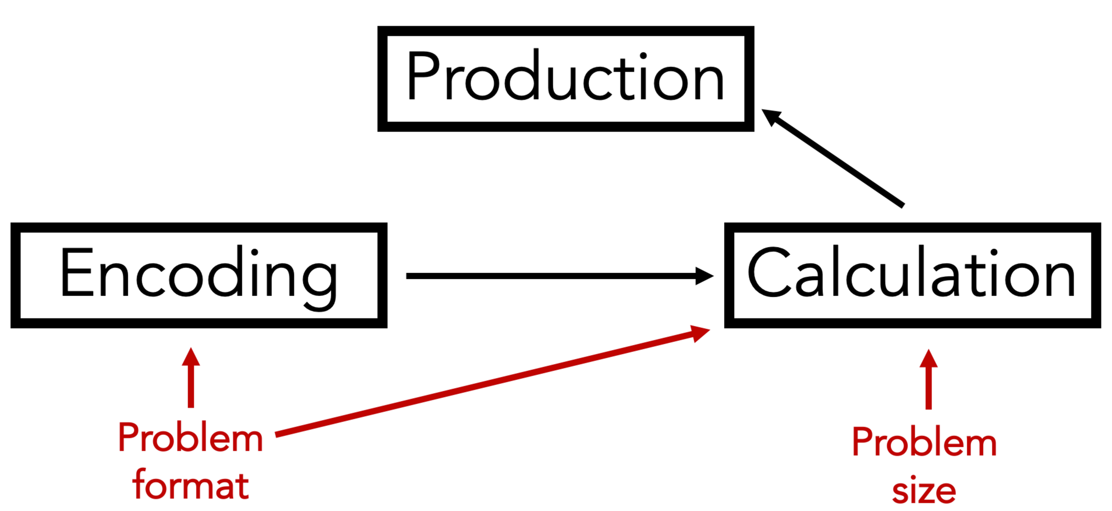
Mental arithmetic occurs in three stages (Ashcraft, 1992):

1. Encoding
2. Calculation
3. Production

Researchers commonly use response times (RTs) to study these mental arithmetic stages and the situations that can impact the underlying cognitive processes. For example, an increase in RTs due to manipulation of format illustrates functioning of the encoding stage. Similarly, an increase in RTs due to manipulation of problem difficulty (e.g., the problem size effect) illustrates functioning of the calculation stage.



While researchers agree to the division of mental arithmetic into these stages, there is significant debate as to whether encoding and calculation are independent. For example, while a manipulation of problem format affects encoding, does it also directly affect calculation processes?



Competing models

Model 1 (e.g., Dehaene & Cohen, 1995):

- Stages are independent, so a manipulation of encoding will not cause changes to the calculation stage.
- Statistical model is additive:

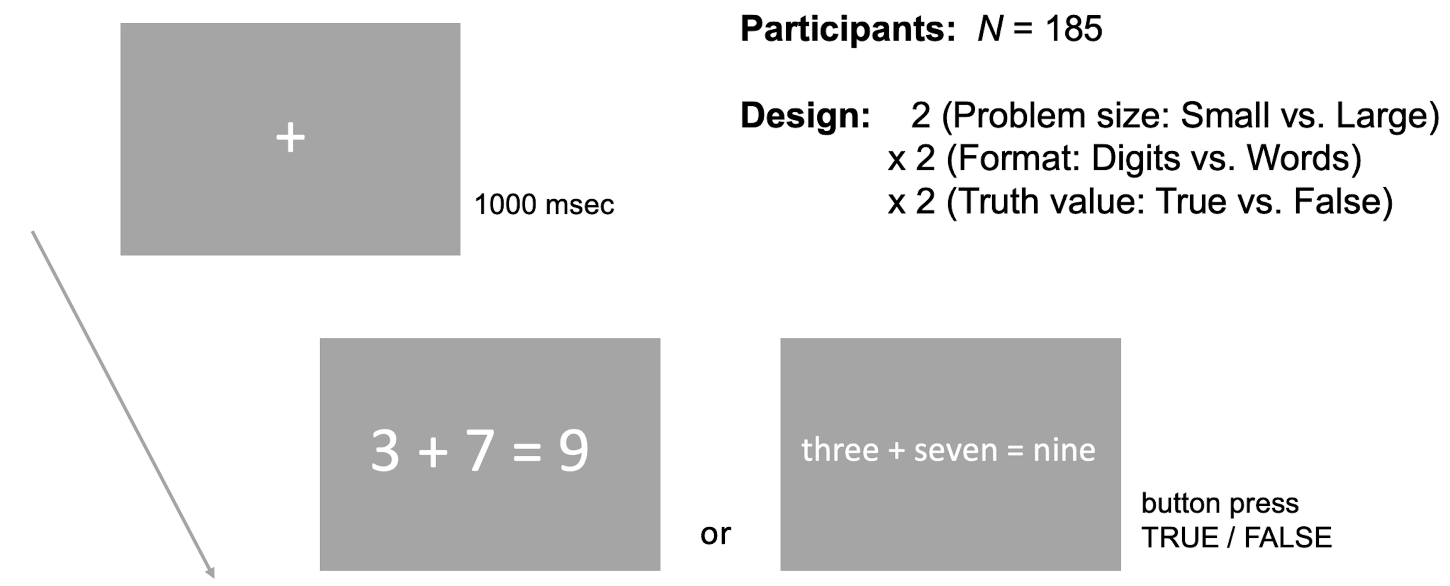
$$\mathcal{M}_1 : \text{RT} \sim \text{format} + \text{size}$$

Model 2 (e.g., Campbell & Fugelsang, 2001):

- Stages are not independent, so a manipulation of encoding will directly impact calculation processes.
- Statistical model is interactive:

$$\mathcal{M}_2 : \text{RT} \sim \text{format} + \text{size} + \text{format} \cdot \text{size}$$

Experiment design



Bayesian inference

For inference, we use Bayesian statistics (Faulkenberry, Ly, & Wagenmakers, 2020), which allows us to mathematically quantify the extent to which the observed data updates the relative evidence in favor of one model over another.

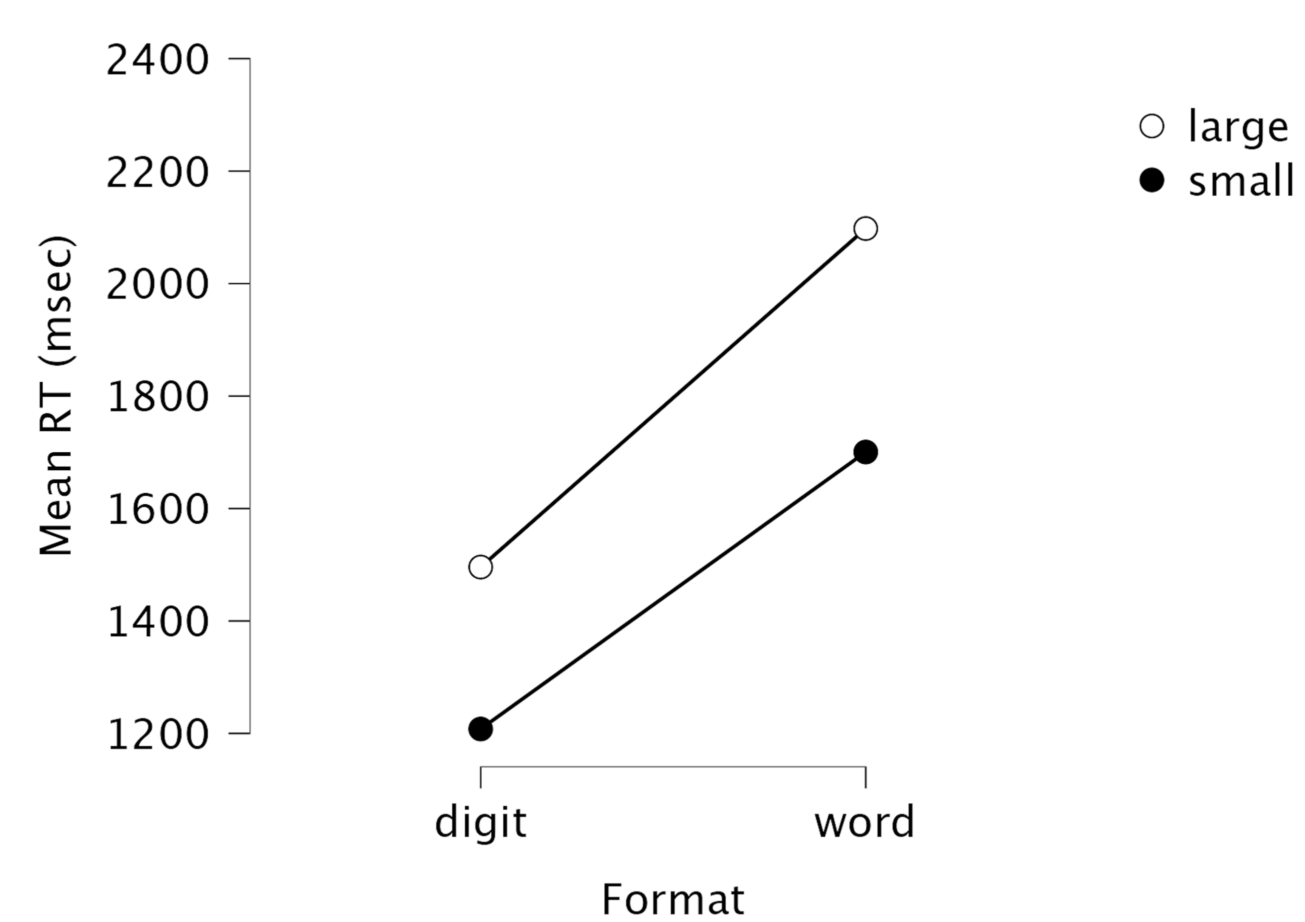
1. Specifically, we can can evaluate which model is best predicted by our observed data using the Bayes factor:

$$\text{BF}_{12} = \frac{P(\text{data} \mid \mathcal{M}_1)}{P(\text{data} \mid \mathcal{M}_2)}$$

2. Example: suppose $\text{BF}_{12} = 10$. This means that the observed data are 10 times more likely under $\mathcal{M}_1$ than under $\mathcal{M}_2$.
3. To compute Bayes factors, we use methods of Rouder et al. (2012) implemented in the software package JASP (https://jasp-stats.org)

Results - true problems

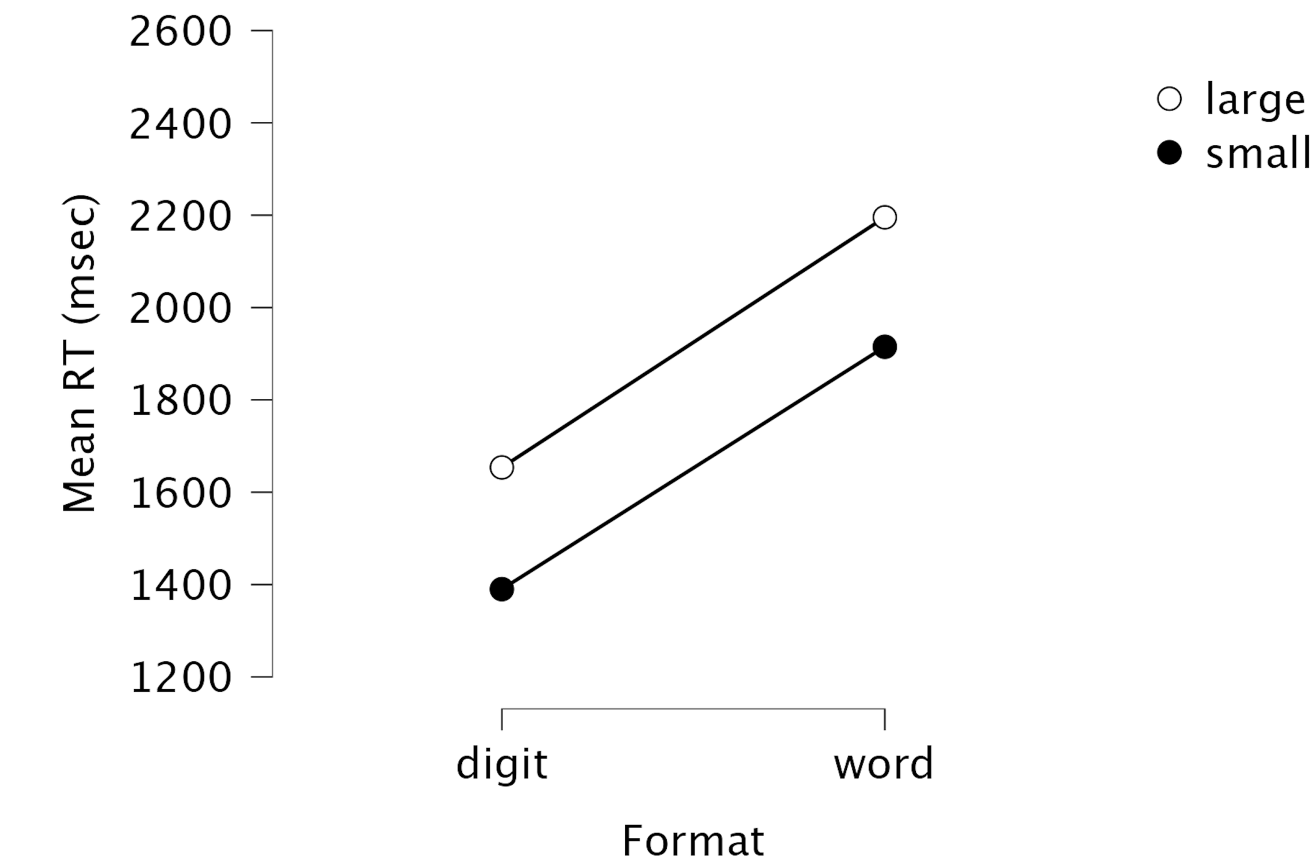
For true problems, data more likely under Model 2 (interactive model)



- $\text{BF}_{21} = 18.77$ – observed data 18.77 times more likely under $\mathcal{M}_2$ (interactive model) than under $\mathcal{M}_1$ (additive model)
- posterior probability of winning model: $P(\mathcal{M}_2 \mid \text{data}) = 0.949$

Results - false problems

For false problems, data more likely under Model 1 (additive model)



- $\text{BF}_{12} = 7.56$ – observed data 7.56 times more likely under $\mathcal{M}_1$ (additive model) than under $\mathcal{M}_2$ (interactive model)
- posterior probability of winning model: $P(\mathcal{M}_1 \mid \text{data}) = 0.883$

Discussion

- For true problems, the interactive model was preferred.
- For false problems, the additive model was preferred
- Why is there a difference in preferred models between true and false problems?
  - Extra verification processes? Need to replicate with production task.
  - Frampton & Faulkenberry (2018) (see also Campbell & Fugelsang, 2001) found that even when the additive model was the best fitting model, there were shifts in problem-solving strategies that occurred with problems in word format – these may imply that format does affect calculation processes.

References

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