Using MATLAB to Develop Macroeconomic Models

Bob Taylor
The MathWorks, Inc.

Introduction

- Construct Model
- Prepare Data
- Identify Model
- Calibrate Model
- Analysis

CONSTRUCT MODEL

Smets-Wouters Model

Linearized Smets-Wouters Model

From Smets-Wouters Structural VARMA to Reduced-Form VAR

Differences with Smets-Wouters Model

Smets-Wouters Model

- Smets and Wouters (2002)
  - Dynamic Stochastic General Equilibrium (DSGE) model
  - Focus is on European economies
  - Nonlinear system of equations to model macro and household economies
  - Notable feature is that the money supply is not observed directly
  - Linearized model is a structural Vector Auto-Regressive Moving Average (VARMA) model

Linearized Smets-Wouters Model

- Smets and Wouters (2004, 2007)
  - Linearized model for the United States economy
  - System of 14 equations with a companion system to model potential output
    - 14 endogenous variables
    - 7 exogenous disturbances
  - Equivalent to a structural VARMA(3,2) model with 7 equations and numerous parameter restrictions
  - Ignoring restrictions, you can invert the MA lag operator to obtain an unrestricted reduced-form VAR(3) model

From Smets-Wouters Structural VARMA to Reduced-Form VAR

- $\mathbf{A}_0 \mathbf{Y}_t = \mathbf{a}'' + \sum_i \mathbf{A}_i'' \mathbf{Y}_{t-i} + \sum_i \mathbf{B}_i'' \mathbf{U}_{t-i} + \mathbf{B}_0 \mathbf{U}_t$
- $\mathbf{B}_0 \mathbf{U}_t = \mathbf{A}_0 \mathbf{W}_t$
- $\mathbf{U}_t = \mathbf{B}_0^{-1} \mathbf{A}_0 \mathbf{W}_t$
- $\mathbf{A}_0 \mathbf{Y}_t = \mathbf{a}'' + \sum_i \mathbf{A}_i'' \mathbf{Y}_{t-i} + \sum_i \mathbf{B}_i'' \mathbf{B}_0^{-1} \mathbf{A}_0 \mathbf{W}_{t-i} + \mathbf{A}_0 \mathbf{W}_t$
- $\mathbf{Y}_t = \mathbf{A}_0^{-1} \mathbf{a}'' + \sum_i \mathbf{A}_0^{-1} \mathbf{A}_i'' \mathbf{Y}_{t-i} + \sum_i \mathbf{A}_0^{-1} \mathbf{B}_i'' \mathbf{B}_0^{-1} \mathbf{A}_0 \mathbf{W}_{t-i} + \mathbf{W}_t$
- $\mathbf{Y}_t = \mathbf{a}' + \sum_i \mathbf{A}_i' \mathbf{Y}_{t-i} + \sum_i \mathbf{B}_i' \mathbf{W}_{t-i} + \mathbf{W}_t$
- $\mathbf{Y}_t - \sum_i \mathbf{A}_i' \mathbf{Y}_{t-i} = \mathbf{a}' + \sum_i \mathbf{B}_i' \mathbf{W}_{t-i} + \mathbf{W}_t$
- $A'(L) \mathbf{Y}_t = \mathbf{a}' + B'(L) \mathbf{W}_t$
- $B'(L)^{-1} A'(L) \mathbf{Y}_t = B'(L)^{-1} \mathbf{a}' + \mathbf{W}_t$
- $A(L) \mathbf{Y}_t = \mathbf{a} + \mathbf{W}_t$
- $\mathbf{Y}_t - \sum_i \mathbf{A}_i \mathbf{Y}_{t-i} = \mathbf{a} + \mathbf{W}_t$
- $\mathbf{Y}_t = \mathbf{a} + \sum_i \mathbf{A}_i \mathbf{Y}_{t-i} + \mathbf{W}_t$

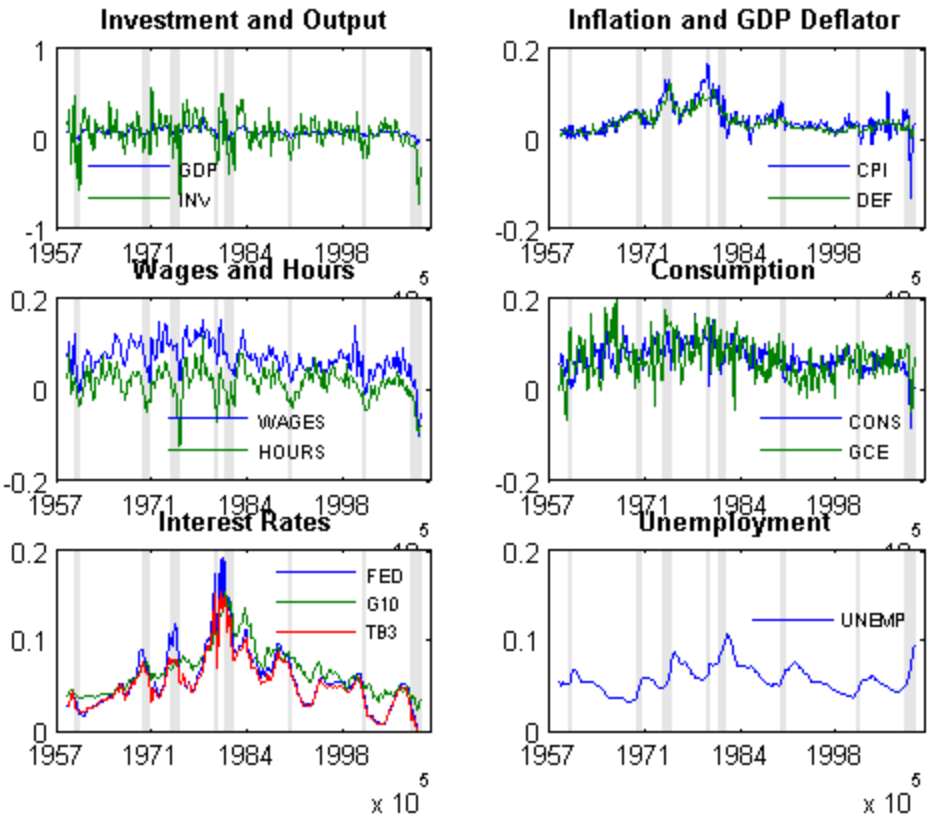
Differences with Smets-Wouters Model

- Time series choices
  - Replace Fed Funds rate with 3-month Treasury Bills
  - Replace hourly wages paid with total wages paid
  - Added unemployment since a psychological indicator of economic health
- Time series formats
  - Smets-Wouters mixes integrated and differenced data
  - We will use differenced (or rate) data exclusively
- Nominal versus real
  - Use nominal data since GDP deflator is in the model
- Detrending
  - Smets-Wouters detrends some series with a common GDP growth rate trend
  - Rate data induces stationarity by differencing instead of detrending
- Order of auto-regression
  - Theory says order of model is 3 although lag 3 matrix is extremely sparse
  - Let data determine the optimal lag order for the model

PREPARE DATA

Data from St. Louis Federal Reserve
Time Series for Model
Stylized Model

Data from St. Louis Federal Reserve



Time Series for Model

Series	Description	FRED Series
$rGDP_t$	Change in log nominal GDP	GDP
$rDEF_t$	Change in log GDP deflator	GDPDEF
$rWAGES_t$	Change in log total paid compensation	COE
$rHOURS_t$	Change in log non-farm business sector hours worked	HOANBS
$rTB3_t$	Interest rate for 3-month Treasury bill	TB3MS
$rCONS_t$	Change in log personal consumption expenditures	PCEC
$rINV_t$	Change in log gross private domestic investment	GPDI
$rUNEMP_t$	Rate of unemployment	UNRATE

Stylized Model

- Reduced-form unrestricted VAR model

$$\mathbf{Y}_t = \mathbf{a} + \sum_i \mathbf{A}_i \mathbf{Y}_{t-i} + \mathbf{W}_t$$

$$\mathbf{W}_t \sim N(\mathbf{0}, \mathbf{Q})$$

$$\mathbf{Y}_t = \begin{bmatrix} rGDP_t \\ rDEF_t \\ rWAGES_t \\ rHOURS_t \\ rTB3_t \\ rCONS_t \\ rINV_t \\ rUNEMP_t \end{bmatrix}$$

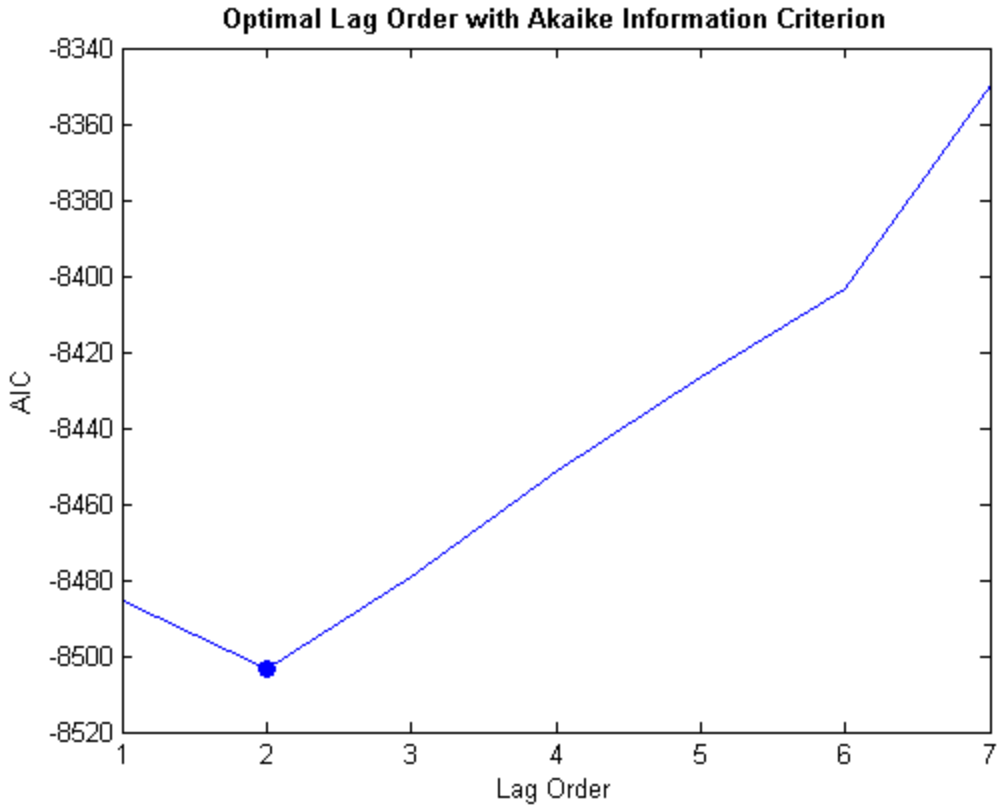
IDENTIFY MODEL

Optimal Lag Order

Time Series Selection

Backtest to Assess Forecast Accuracy

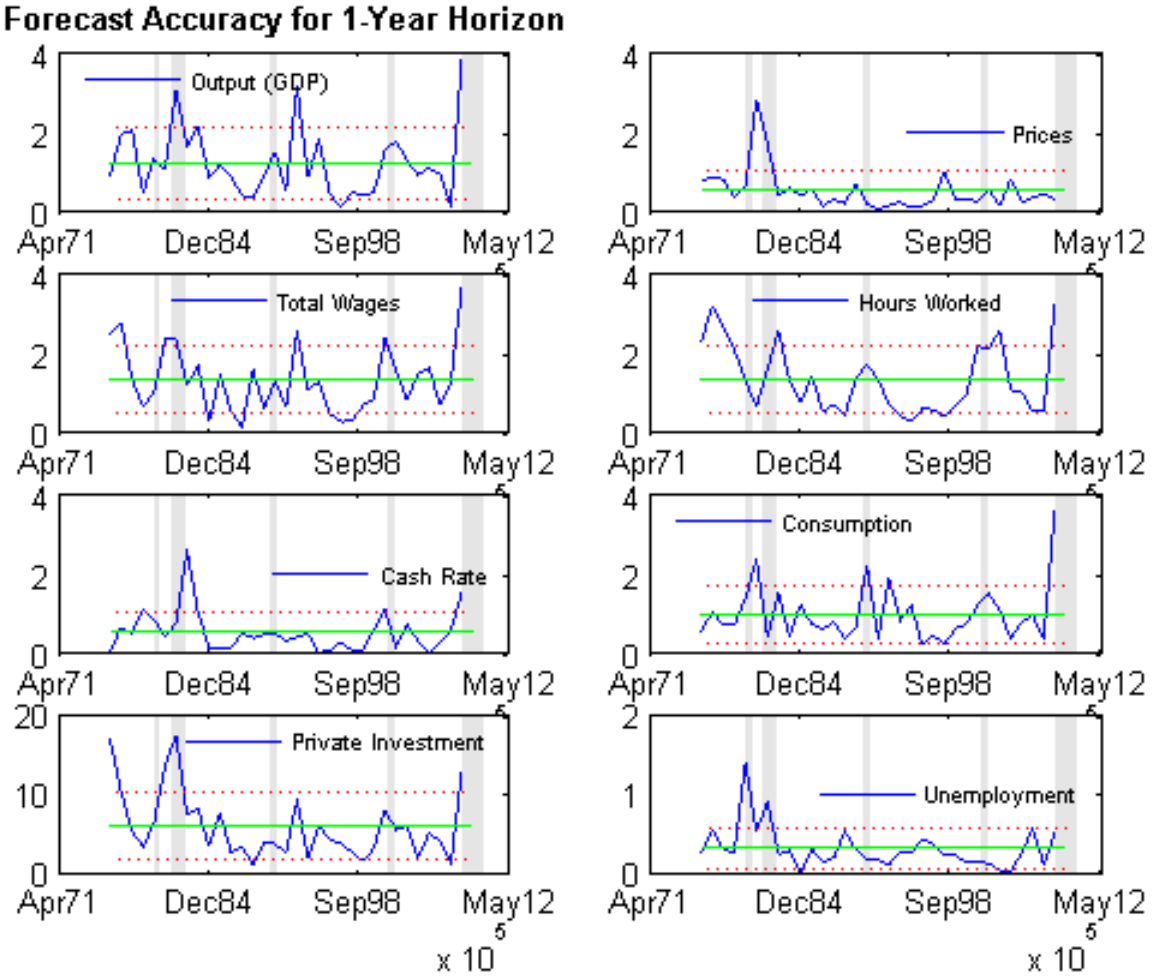
Optimal Lag Order



Time Series Selection

- Which series are relevant?
- Which restrictions are important?
- Depends upon what you are trying to do
- Two viewpoints
  - Ex-ante
  - Ex-post
- Smets-Wouters offers a good collection of choices

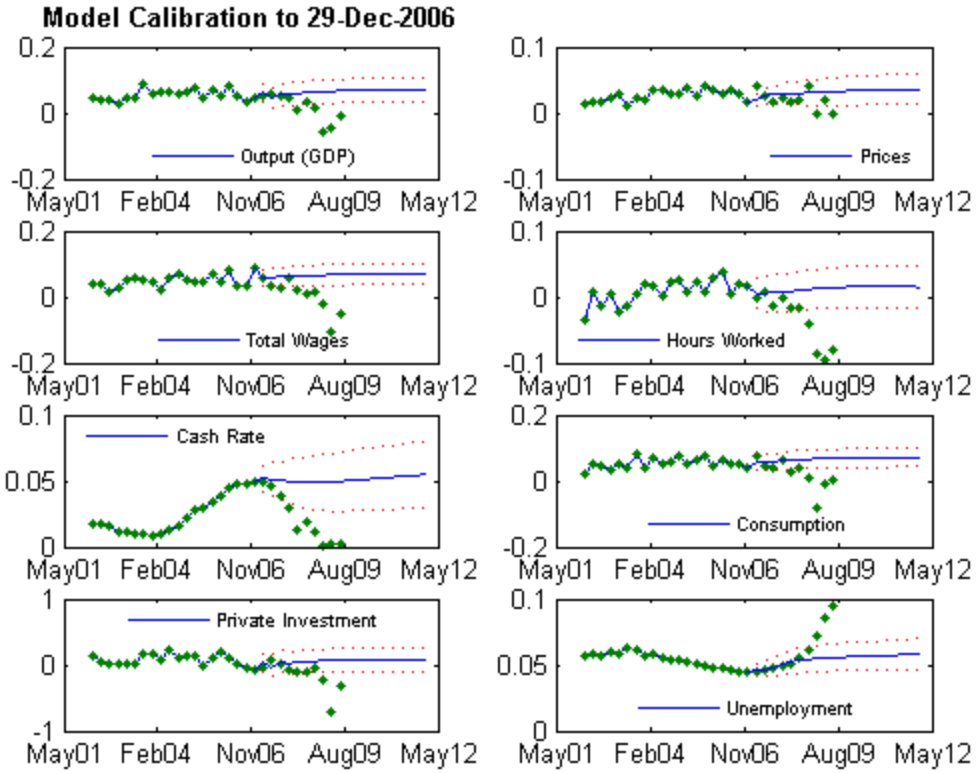
Backtest to Assess Forecast Accuracy



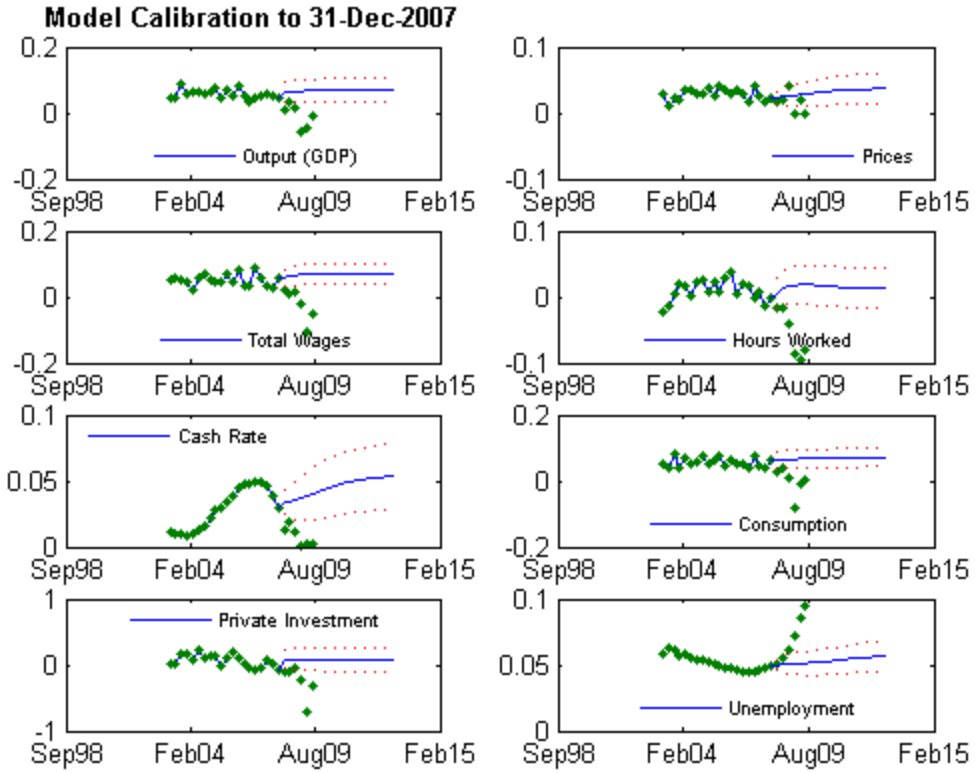
CALIBRATE MODEL

Calibration to 31-Dec-2006
Calibration to 31-Dec-2007
Calibration to 31-Dec-2008
Calibration to 30-Sep-2009

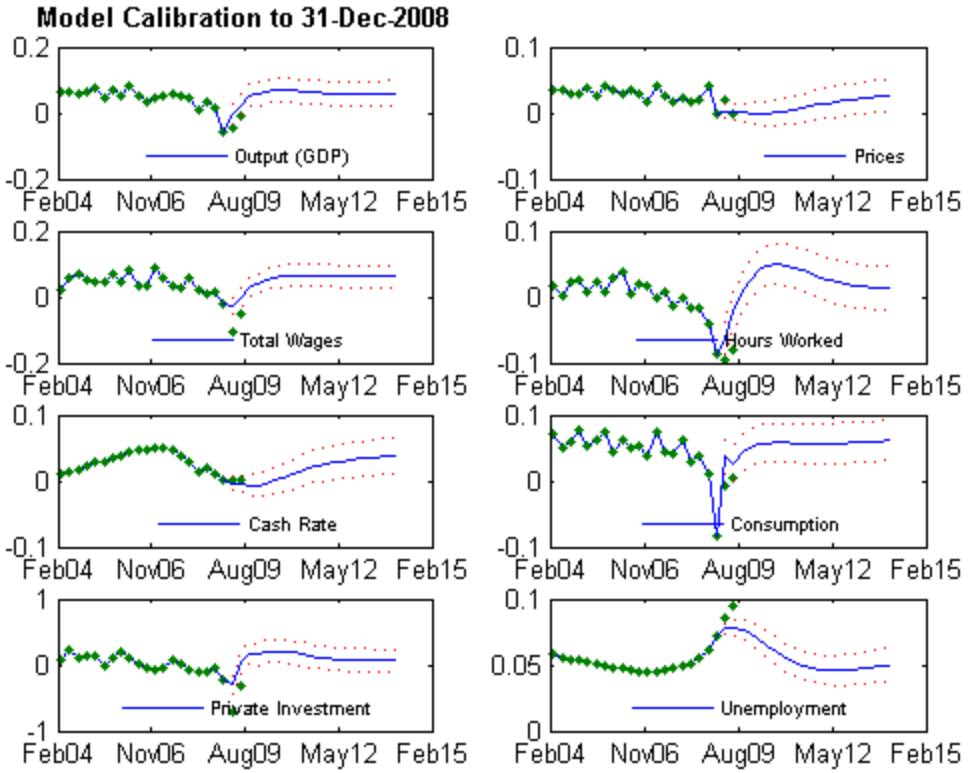
Calibration to 31-Dec-2006



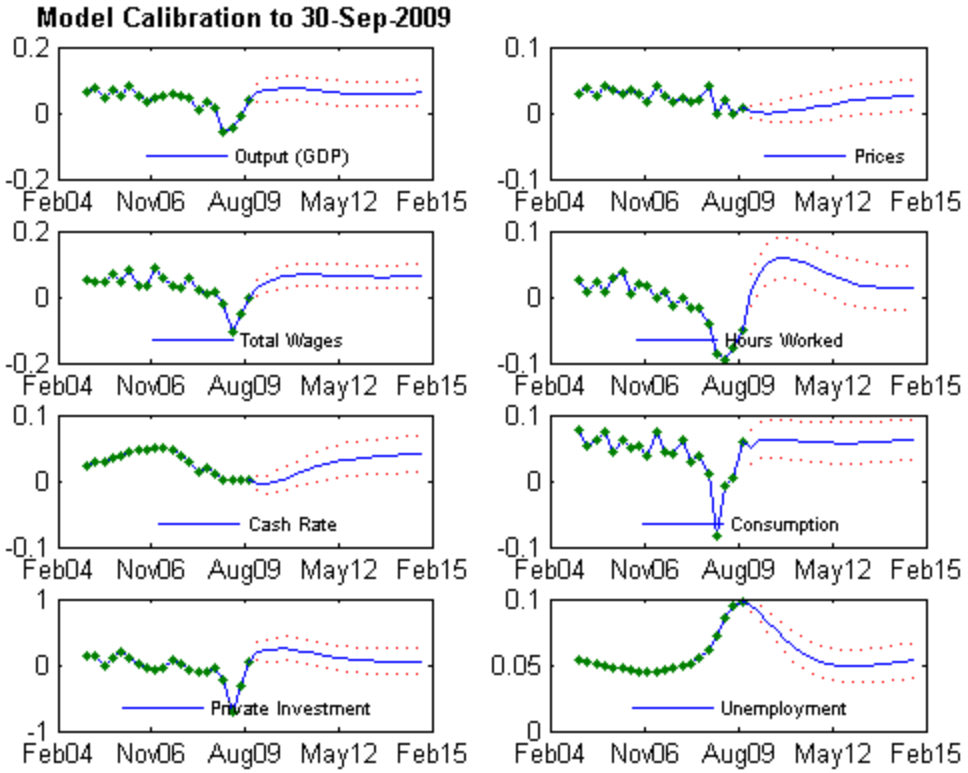
Calibration to 31-Dec-2007



Calibration to 31-Dec-2008



Calibration to 30-Sep-2009



ANALYSIS

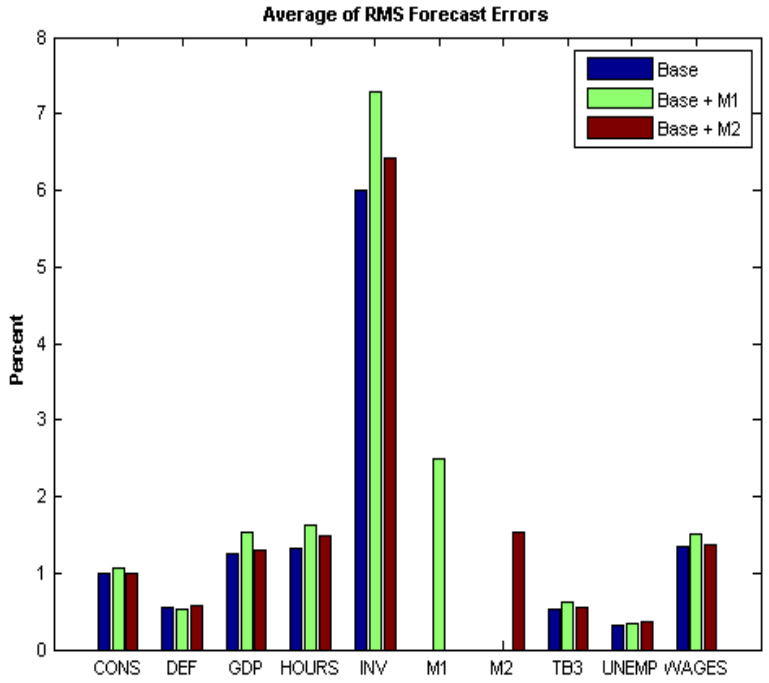
Model Augmentation
Money Supply and RMS Forecast Errors
Impulse Response Analysis
Impulse Response Analysis Detail
Real GDP Forecast

Model Augmentation

- Smets-Wouters does not observe money supply
- What happens if you add the money supply?
  - Try both M1 and M2
  - No forecast improvements with the addition of observed money supply
  - M2 is better than M1

Money Supply and RMS Forecast Errors

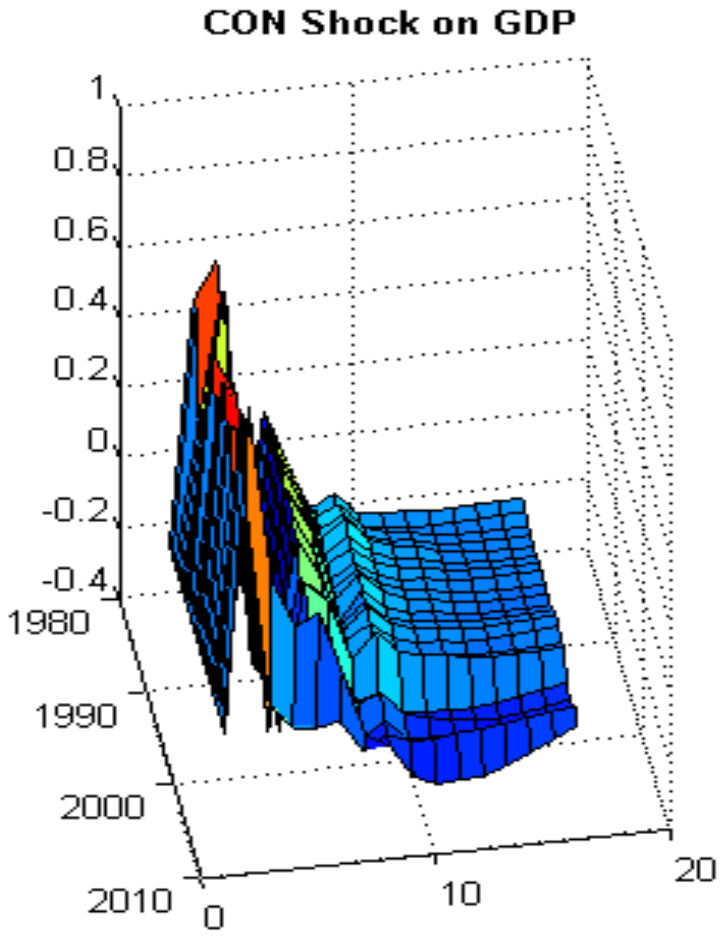
	Base	Base + M1	Base + M2
GDP	1.25	1.53	1.30
DEF	0.54	0.54	0.57
WAGES	1.33	1.51	1.36
HOURS	1.32	1.63	1.48
TB3	0.53	0.63	0.55
CONS	1.00	1.06	1.00
INV	6.00	7.28	6.42
UNEMP	0.33	0.33	0.35

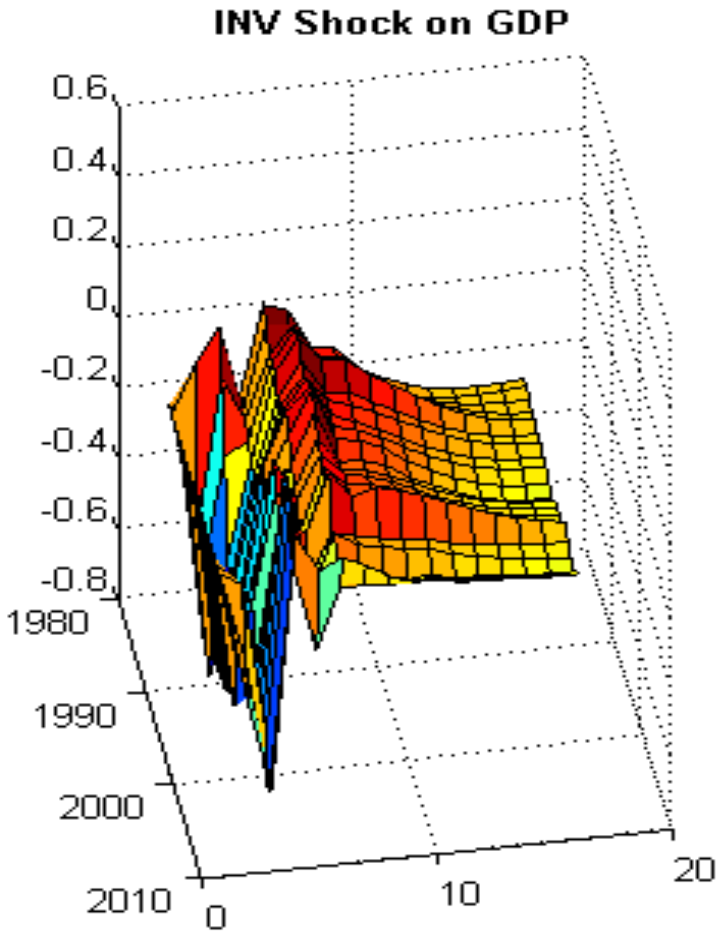


Impulse Response Analysis

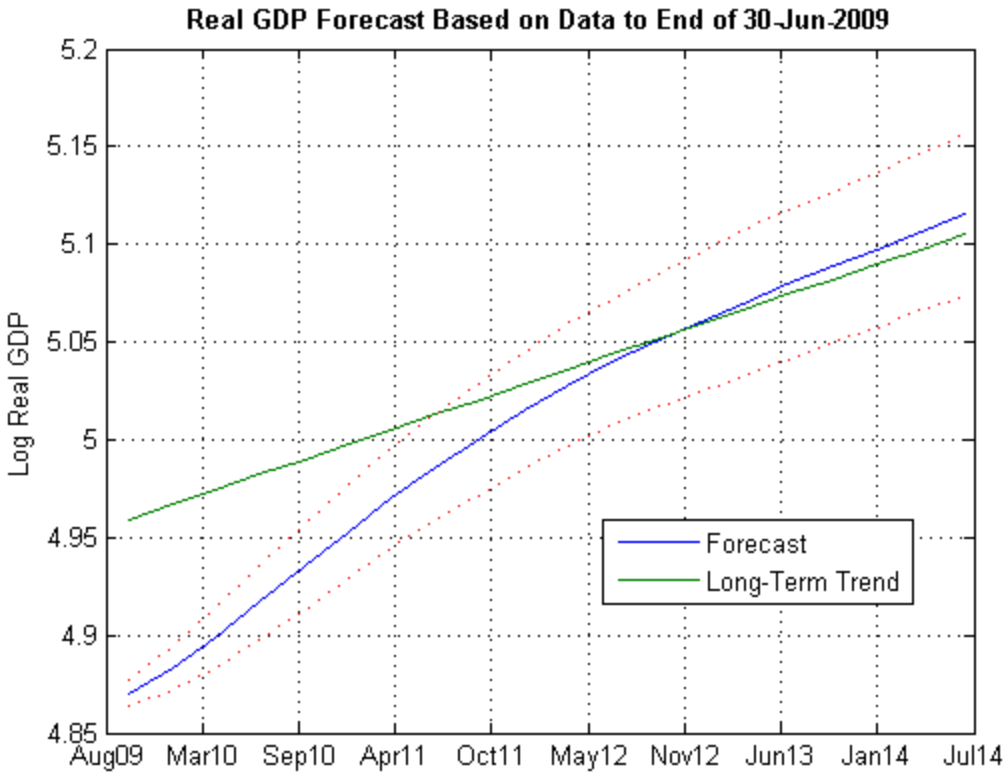
- Time series models in VARMA form are systems of difference equations
- Such systems respond to shocks at different rates according to the lag structure
- Impulse response analysis examines the manner in which the system absorbs shocks as it returns to equilibrium

Impulse Response Analysis Detail





Real GDP Forecast



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