

# CS151 Intro to Data Structures

Implementing Stacks

Queues

# Announcements

HW2, Lab3, Lab4 due Friday

Lab5 today - due Oct 11th

Manual checkoff by me or TAs

# Agenda

- Stack Review
- Linked List based Stack implementation
- Amortized Analysis
- Queues

# Stacks - FILO

- First In Last Out
- *stack* of plates in the dining hall
- Operations:
  - push
  - pop
  - peek
  - isEmpty

# Stack Review - what will this code print?

```
public static void main(String[] args) {  
    Stack<Integer> stack = new Stack<Integer>();  
  
    stack.push(10);  
    stack.push(20);  
    stack.push(30);  
  
    int popped = stack.pop();  
    System.out.println("Popped: " + popped);  
  
    int top = stack.peek();  
    System.out.println("Top: " + top);  
  
    stack.push(40);  
    System.out.println("New Top after push: " + stack.peek());  
  
    while (!stack.isEmpty()) {  
        System.out.println("Popped: " + stack.pop());  
    }  
}
```

# Implementing a Stack with an Array

Goal:  $O(1)$  operations

Our class implementation:

- fixed size array (no expansions!)
- How did we implement push?
- How did we implement pop?
- How did we implement peek?

# Now let's implement stack with a linked list!

Goal:  $O(1)$  operations.

What to consider:

- When we PUSH where should we insert to?
  - Front, back, middle?
- When we POP where should we remove from?
  - Reminder: Stack should be FIFO

# Linked List Stack Performance

Space complexity is

- $O(n)$

Runtime Complexity:

- push:
  - $O(1)$
- Pop:
  - $O(1)$
- Peek:
  - $O(1)$



# Stack Summary

- FIFO wrapper around Array / Linked List
- Allows for limited data structures operations all with  $O(1)$  cost
- Real world applications: call stack, browser history, postfix calculator

# Amortized Analysis

<https://courses.cs.washington.edu/courses/cse373/17wi/summaries/amortized-runtime.pdf>

# Amortized Analysis

*average* run time complexity of an operation.

Compares the total cost of a series of operations with how many of those operations happened.

# Amortized Runtime Analysis

$$\frac{\textit{total cost of operations}}{\textit{total number of operations}}$$

Where an “operation” is the operation a client is doing through your public interface, like `insert(5)` or `pop()` or `add(3)`.

# Exercise: Amortized Analysis of Array Insert

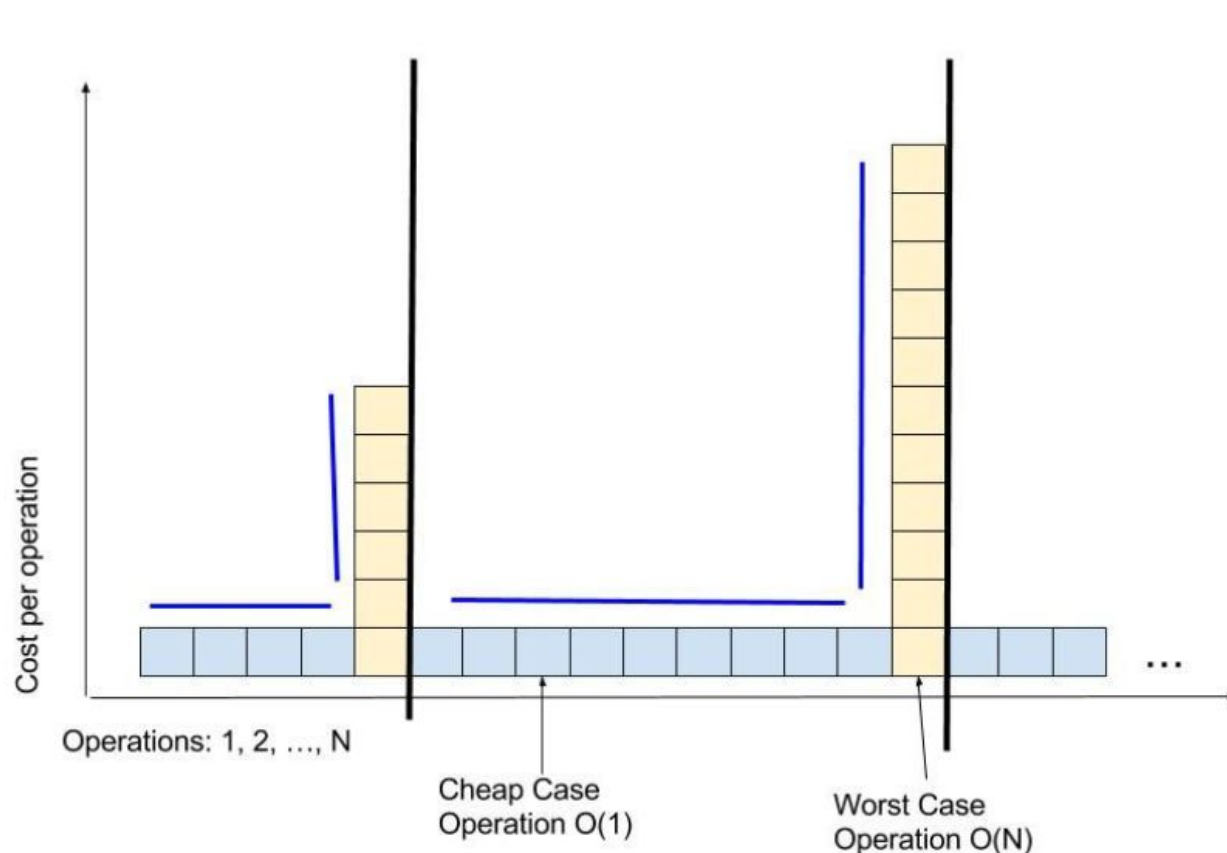
Let's say you have an expandable array with N elements: [1, 2, 3... N]

When you try to add the N+1 th element, a resize occurs.

$$\frac{\textit{total cost of operations}}{\textit{total number of operations}}$$

# Amortized Analysis

Intuition: you want to “build up enough credit” with a series of cheap operations, so that when you have one (or more) expensive operations, you can average out the cost of the expensive one



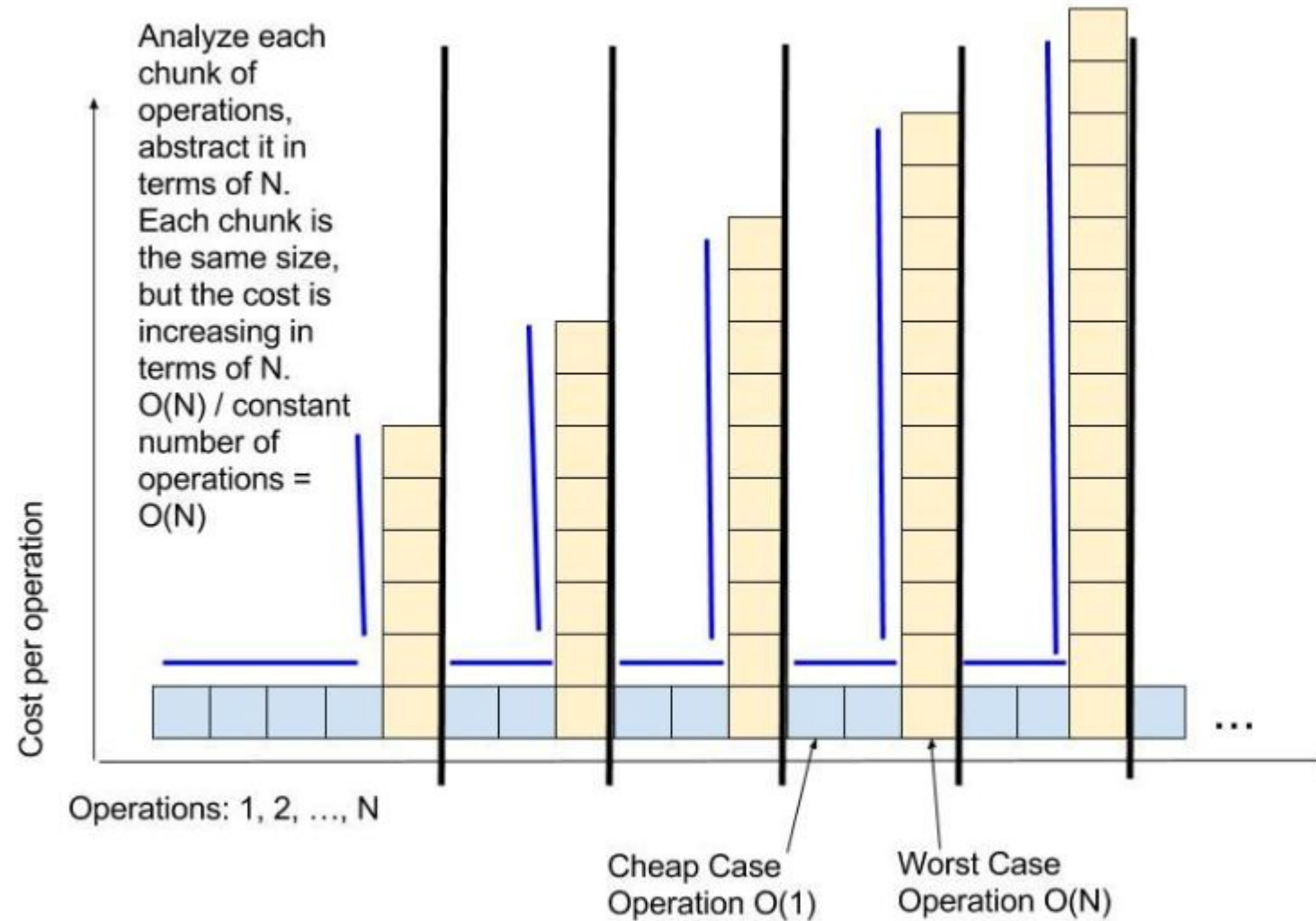
# Exercise: Amortized Analysis of Array Insert

Let's say you have an expandable array **which adds two extra slots each time it is full**

Amortized cost of insert? Calculate for 9 inserts.

$$\frac{\text{total cost of operations}}{\text{total number of operations}}$$

# Amortized Analysis





# Amortized Analysis

Average case runtime analysis

Intuition: you want to “build up enough credit” with a series of cheap operations, so that when you have one (or more) expensive operations, you can average out the cost of the expensive one

Comparing the total cost of a series of operations with how many operations happened

# Queues

FIFO Stacks

# Stack Property

First-in Last-out (FILO)

Where might a FILO stack not make sense?

Line for the cash register

Printer Queue

# FIFO: First-in First-out

The first item in, is the first item out

Add-to the back, remove from the front

This is a **Queue**

Inserting – “enqueue”

Removing - “dequeue”

# Queue Interface

```
public interface Queue<E> {  
    int size();  
    boolean isEmpty();  
    E first();  
    void enqueue(E e);  
    E dequeue();  
}
```

- **null** is returned from `dequeue()` and `first()` when queue is empty

# Queue Example

Cash register code

# Example

<i>Operation</i>	<i>Output</i>	<i>Q</i>

# Example

*Operation*  
enqueue(5)



*Output*    *Q*





# Example

*Operation*  
enqueue(5)



*Output*    *Q*  
—            (5)

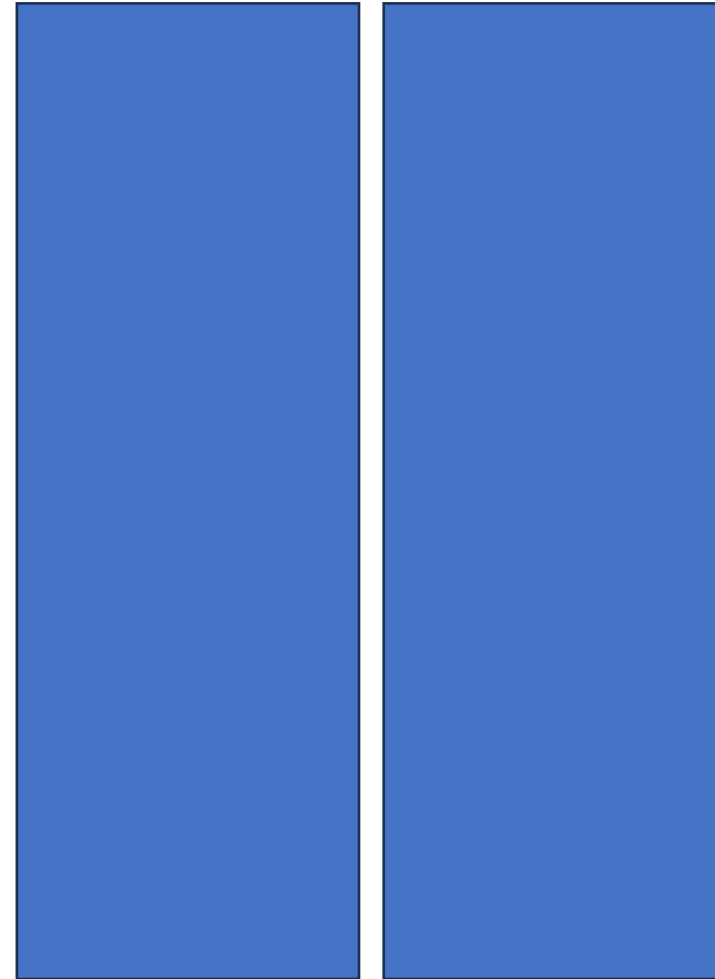


# Example

<i>Operation</i>	<i>Output</i>	<i>Q</i>
enqueue(5)	—	(5)
enqueue(3)	—	(5, 3)
dequeue()	5	(3)
enqueue(7)	—	(3, 7)
dequeue()	3	(7)
first()	7	(7)
dequeue()	7	()
dequeue()	<i>null</i>	()
isEmpty()	<i>true</i>	()
enqueue(9)	—	(9)
enqueue(7)	—	(9, 7)
size()	2	(9, 7)
enqueue(3)	—	(9, 7, 3)
enqueue(5)	—	(9, 7, 3, 5)
dequeue()	9	(7, 3, 5)

# Example

<i>Operation</i>	<i>Output</i>	<i>Q</i>
enqueue(5)	—	(5)
enqueue(3)	—	(5, 3)
dequeue()		
enqueue(7)		
dequeue()		
first()		
dequeue()		
dequeue()		
isEmpty()		
enqueue(9)		
enqueue(7)		
size()		
enqueue(3)		
enqueue(5)		
dequeue()		



# Example

<i>Operation</i>	<i>Output</i>	<i>Q</i>
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first()	7	(7)
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enqueue(9)	—	(9)
enqueue(7)	—	(9, 7)
size()	2	(9, 7)
enqueue(3)	—	(9, 7, 3)
enqueue(5)	—	(9, 7, 3, 5)
dequeue()	9	(7, 3, 5)

# Implementing a Queue with an Array

Goal:  $O(1)$  operations

How can we achieve this?

1. Fixed size underlying array (no expansions)
2. Where should we insert?
  - a. front, back, middle?
3. Where should we remove from?
  - a. we should preserve first in first out property!