CS151 Intro to Data Structures

Hashmaps

Announcements & Outline

Next homework and Lab due Monday December 1st

Lab today is manual grading. Have me or TA check you off.

Today:

- HashMap Review
- ProbeHashMaps
- HW7 discussion

Hash Map Reivew

Hash Map:

- Efficient data structure with constant time* access, insertion, and removal
- * assuming no collisions or expansions

Hash Function Review

Book's `AbstractHashMap` hash method uses:

h_1`(k) = k.hashCode()` // java memory address

h_2`(x) = ((ax + b) % p) % N`

```
h = h2(h1(k))
```

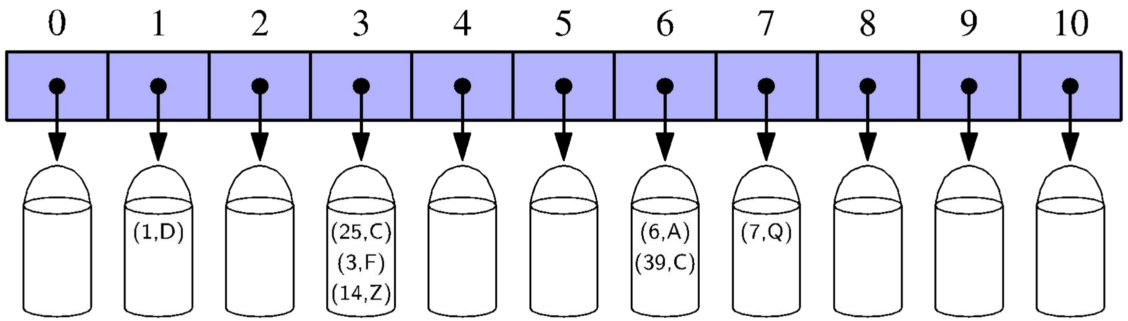
Performance Analysis

	ArrayMap	Collision Resistant Hash Map
get		
put		
remove		

Review: Handling Collisions

ChainHashMap:

- When more than one key hash to the same index, we have a bucket
- Each index holds a collection of entries

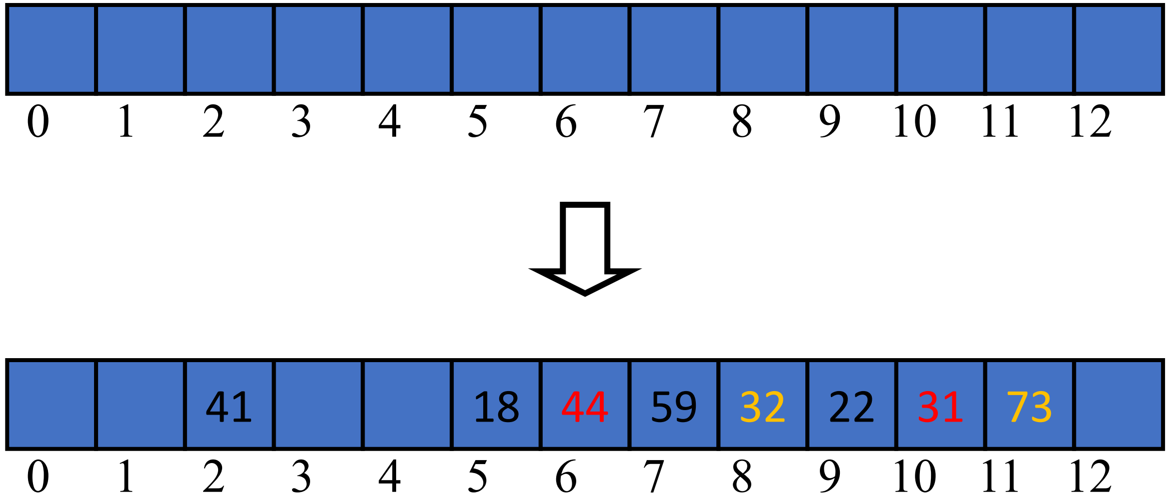


- Worst case:
  - all elements collide into the same bucket
  - O(n) operations

Open Addressing and Probing

- Example: $h(x) = x\%13$
- insert 18(5), 41(2), 22(9), 44(5), 59(7), 32(6), 31(5), 73(8)

Keep "*probing*"
(h(k)+1)%n
(h(k)+2)%n
....
(h(k)+i)%n
until you find an empty slot!



ProbeHashMap

Let’s look at an implementation of ProbeHashMap

Open Addressing and Probing

Linear Probing (what we just saw):

- Keep "*probing*" until you find an empty slot

  (h(k)+1) % n
  (h(k)+2) % n
  ....
  (h(k)+i) % n

- Colliding items cluster together – future collisions to cause a longer sequence of probes

Open Addressing and Probing

Quadratic Probing:

- Keep "*probing*" until you find an empty slot

  (h(k)+f(1)) % n

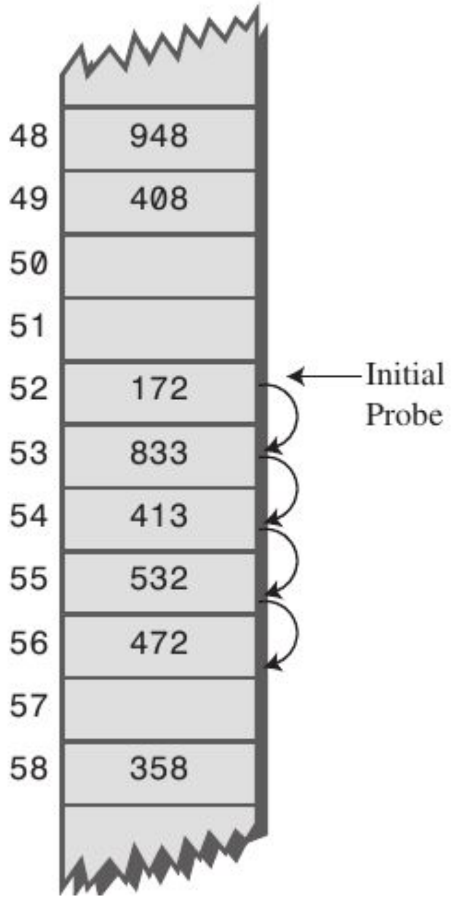
  (h(k)+f(2)) % n

  ….

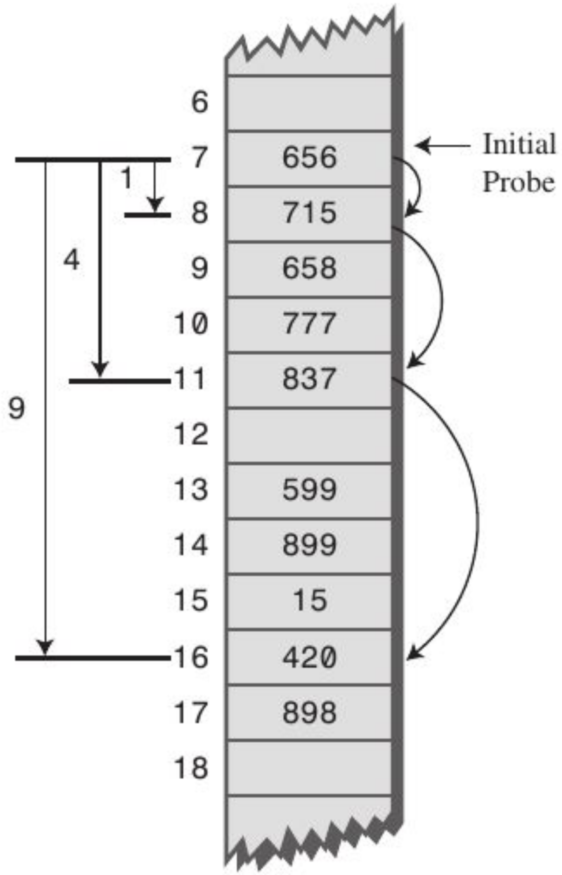
  (h(k)+f(i)) % n

  where f(i) = i^2

Linear Probing vs Quadratic Probing



Linear Probing



Quadratic Probing

- Quadratic probing still creates large clusters!
- Unlike linear probing, they are clustered away from the initial hash position
- If the primary hash index is x, probes go to x+1, x+4, x+9, x+16, x+25 and so on, this results in ***Secondary Clustering***

Approach #3: **Double Hashing**

Let's try to avoid clustering.

To probe, let's use a **second hash function**

- Keep "*probing*" until you find an empty slot

  (h(k)+f(1)) % n

  (h(k)+f(2)) % n

  ….

  (h(k)+f(i)) % n

Where f(i) = i * **h'(k)**

Approach #3: **Double Hashing**

Keep "*probing*" until you find an empty slot

(h(k)+f(1)) % n

(h(k)+f(2)) % n

....

(h(k)+f(i)) % n

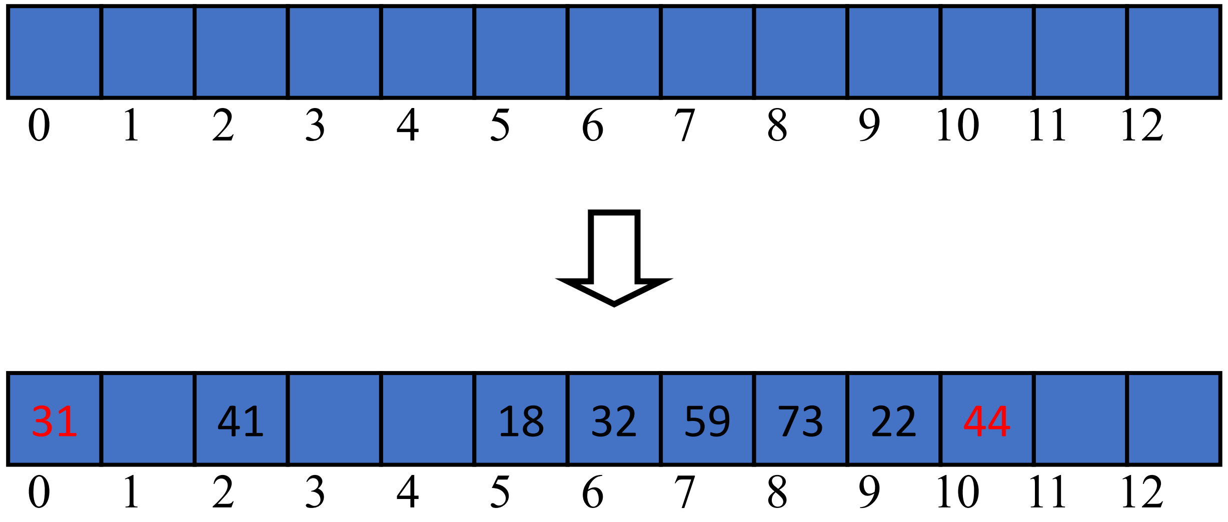
Where f(i) = i * **h'(k)**

A common choice for **h'(k)** = q - (k % q)
where q is prime and < n

Example

k	$h(k)$	h'(k)	Probes		
18	5	3	5		
41	2	1	2		
22	9	6	9		
44	5	5	5	10	
59	7	4	7		
32	6	3	6		
31	5	4	5	9	0
73	8	4	8		

- Insert 18, 41, 22, 44, 59, 32, 31, 73



probe:

(h(k) + f(k)) % n

h(k) = k % 13

f (k) = i * **h'(k)**

h'(k) = 7 - k % 7

Performance Analysis

	ChainHashMap Best Case	ChainHashMap Worst Case	ProbeHashMap Best Case	ProbeHashMap Worst Case
get				
put				
remove				

Which is better in practice?

Open Addressing vs Chaining

- Probing is significantly faster in practice
- locality of references – much faster to access a series of elements in an array than to follow the same number of pointers in a linked list

Performance Analysis

	ArrayMap	HashMap with good hashing and good probing
get		
put		
remove		

Performance of Hashtable

	array	linked list	BST (balanced)	HashTable
search	O(n)	O(n)	O(logn)	O(1)
insert	O(1) *	O(1) / O(n)	O(logn)	O(1)
remove	O(n)	O(1) / O(n)	O(logn)	O(1)

Load Factor

- HashMaps have an underlying array... what if it gets full?
  - For ChainHashMap collisions increase
  - For ProbeHashMap we need to resize!

- Load Factor = # of elements stored / capacity

- A common strategy is to resize the hash map when the load factor exceeds a predefined threshold (often 0.75)
  - tradeoff between memory and runtime

HW7 Discussion

Homework 7

- NYPD “Stop Question and Frisk” dataset
- How to work with large data

From Wikipedia, the free encyclopedia

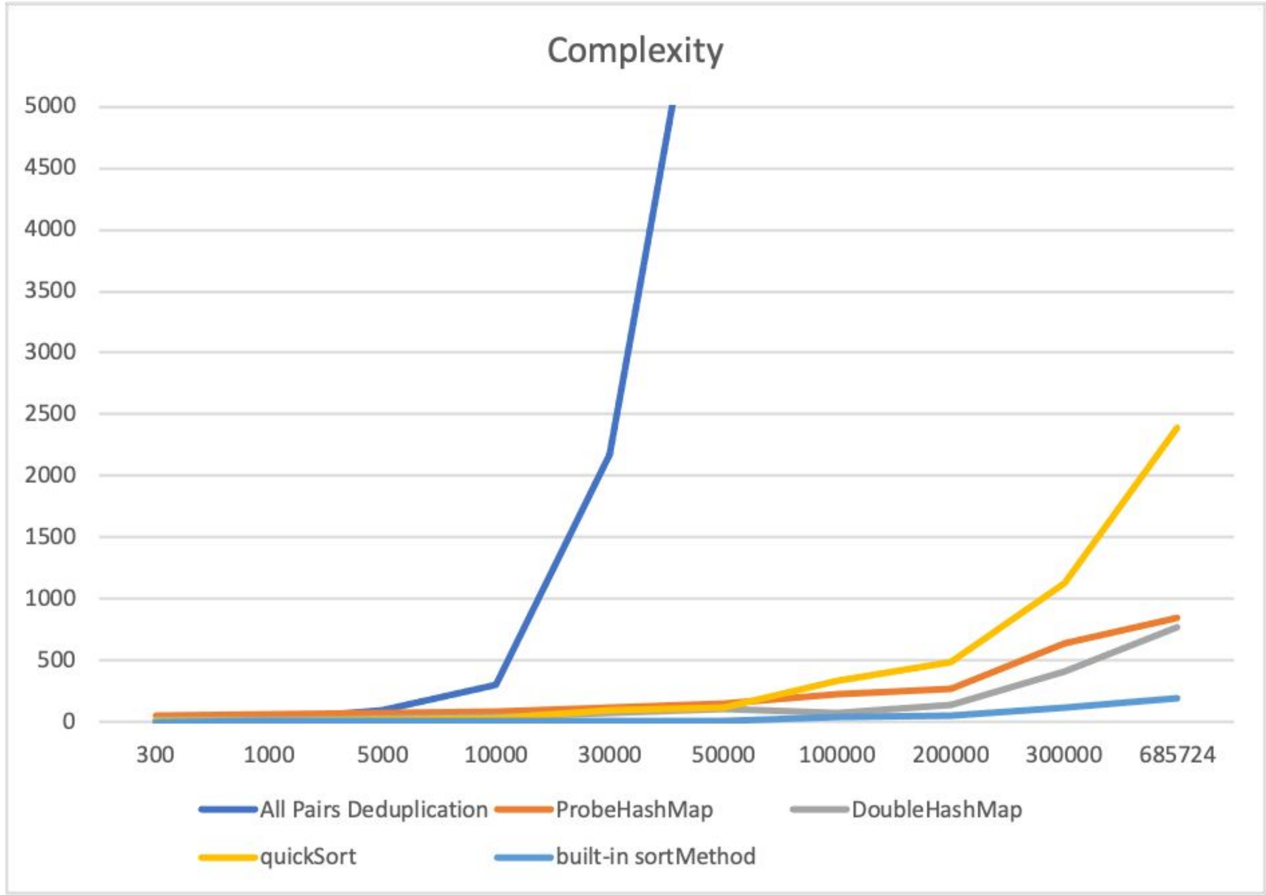
A ***Terry* stop** in the United States allows the police to briefly detain a person based on reasonable suspicion of involvement in criminal activity.[1][2] Reasonable suspicion is a lower standard than probable cause which is needed for arrest. When police stop and search a pedestrian, this is commonly known as a **stop and frisk**. When police stop an automobile, this is known as a traffic stop. If the police stop a motor vehicle on minor infringements in order to investigate other suspected criminal activity, this is known as a **pretextual stop**. Additional rules apply to stops that occur on a bus.[3]

Homework 7

- How many times was the same person stopped for questioning?

Homework 7 Part 2: Complexity Analysis

- Line graph
- x axis: number of entries
- y axis: time in seconds



MergeSort

What sorting algorithms have we seen thus far?

1. Selection sort
   a. How does it work?
   b. Runtime complexity
2. Heap sort
   a. How does it work?
   b. Runtime complexity?

Divide and Conquer algorithm

1. **Divide**: recursively break down the problem into sub-problems
2. **Conquer:** recursively solve the sub-problems
3. **Combine:** combine the solutions to the sub-problems until they are a solution to the entire problem

Binary search is a divide and conquer algorithm

Usually involves recursion

Merge Sort

1. **Divide**: Divide the unsorted list into lists with only one element

2. **Conquer**: merge them back together in a sorted manner

3. **Combine:** merge the sorted sequences

Merge Sort

https://youtu.be/4VqmGXwpLqc?si=WpYuXYLtJOuhvd77&t=24

Merge Sort

Sort a sequence of numbers A, $|A| = n$

Base: $|A| = 1$, then it's already sorted

General

- divide: split A into two halves, each of size $\frac{n}{2}$ ($\left\lfloor\frac{n}{2}\right\rfloor$ and $\left\lceil\frac{n}{2}\right\rceil$)
- conquer: sort each half (by calling mergeSort recursively)
- combine: merge the two sorted halves into a single sorted list

Example

6	8	4	1	7	2	5	3

Example

6	8	4	1	7	2	5	3

6	8	4	1

7	2	5	3

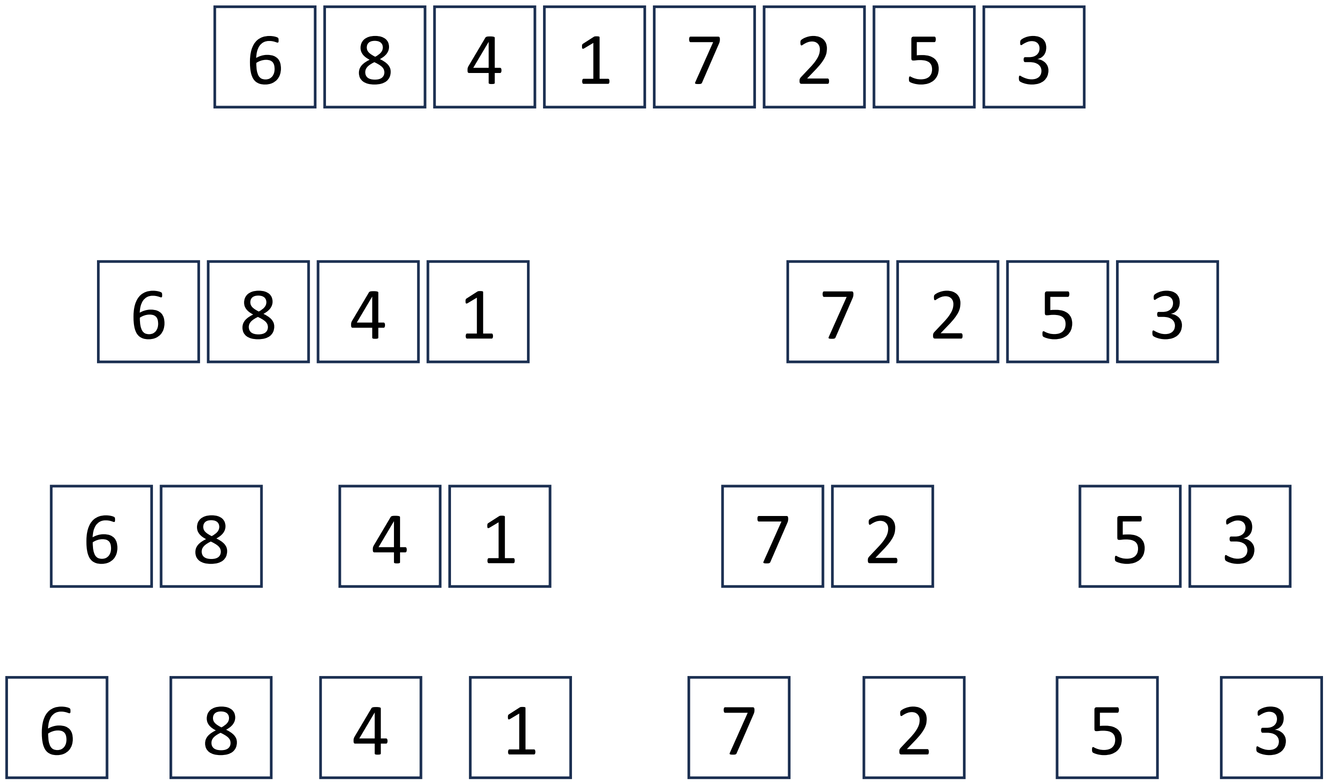
Example

6 8 4 1 7 2 5 3

6 8 4 1   7 2 5 3

6 8   4 1   7 2   5 3

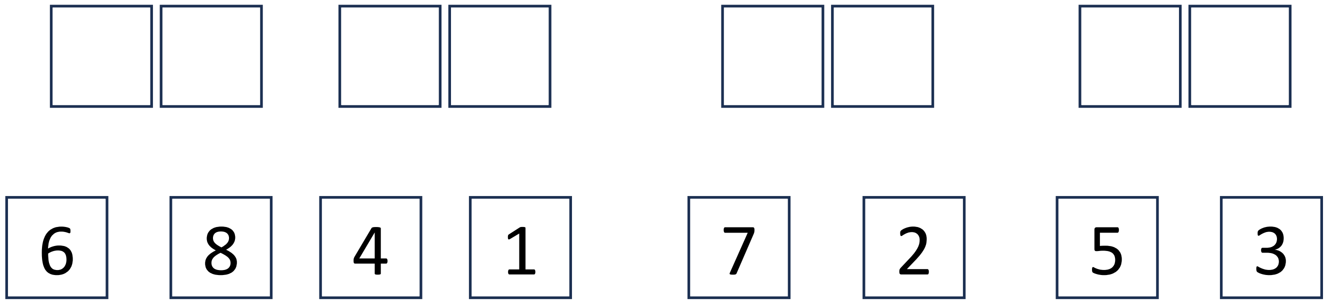
Example



Example

6	8	4	1	7	2	5	3

Example

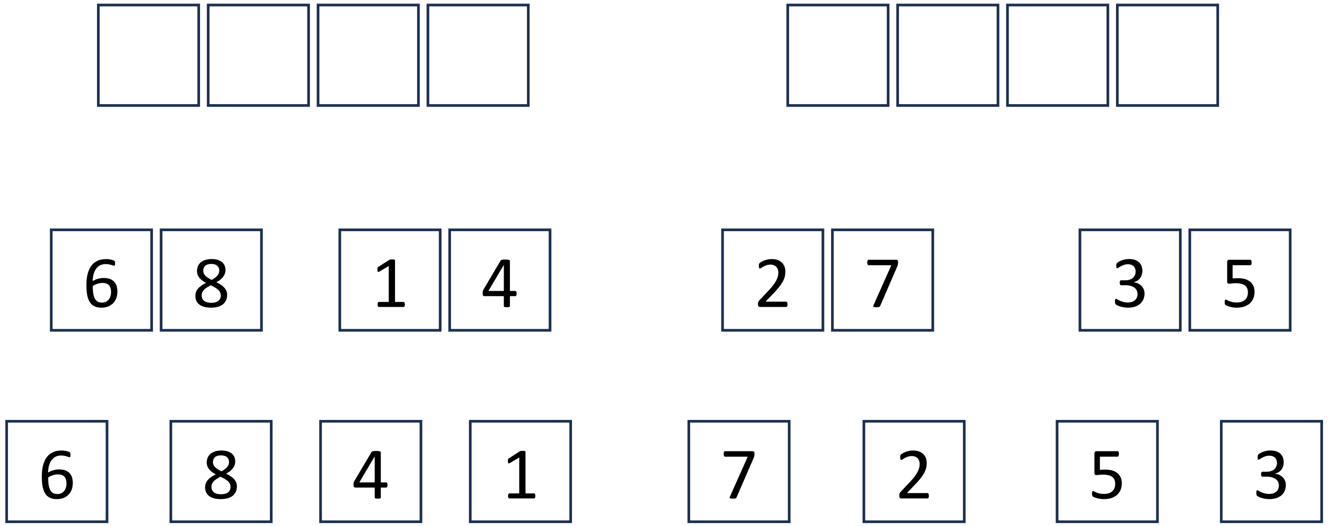


Example

6 8 1 4 2 7 3 5

6 8 4 1 7 2 5 3

Example



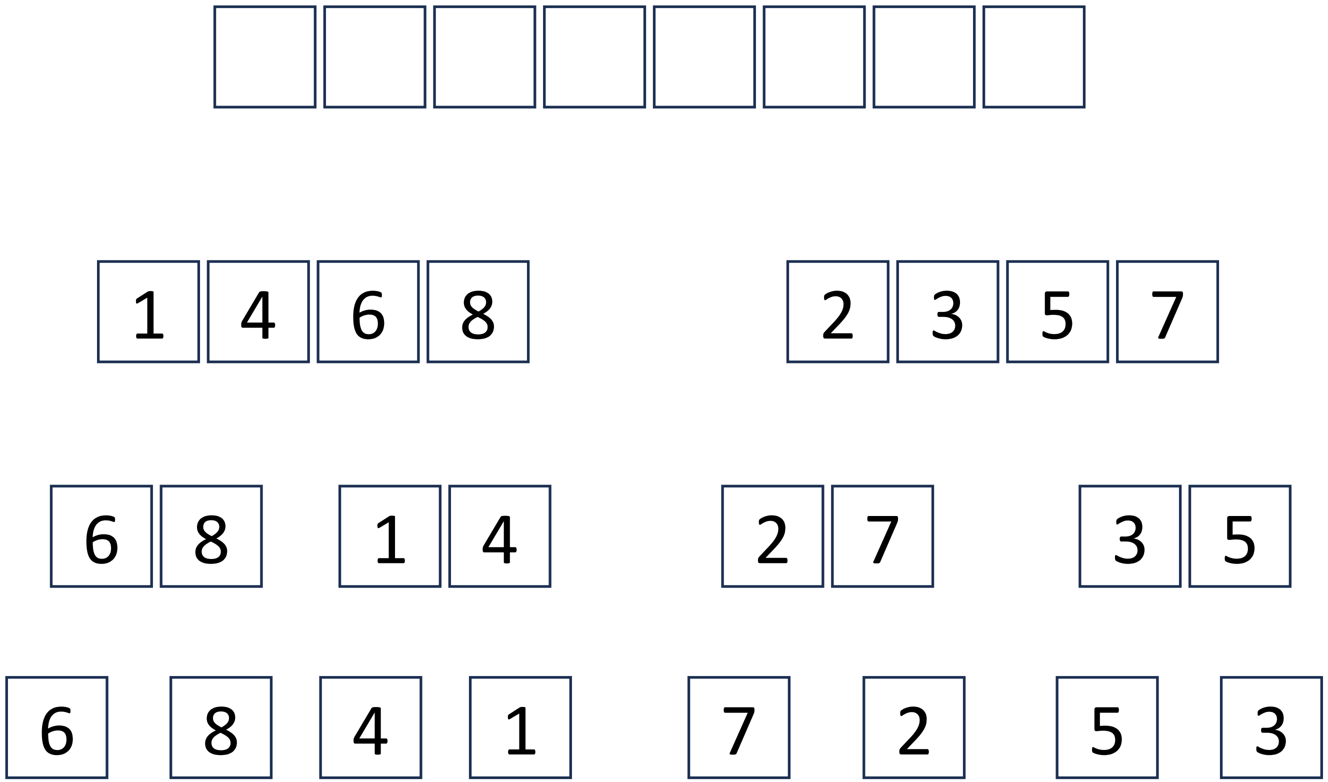
Example

1 4 6 8   2 3 5 7

6 8   1 4   2 7   3 5

6   8   4   1   7   2   5   3

Example



Example

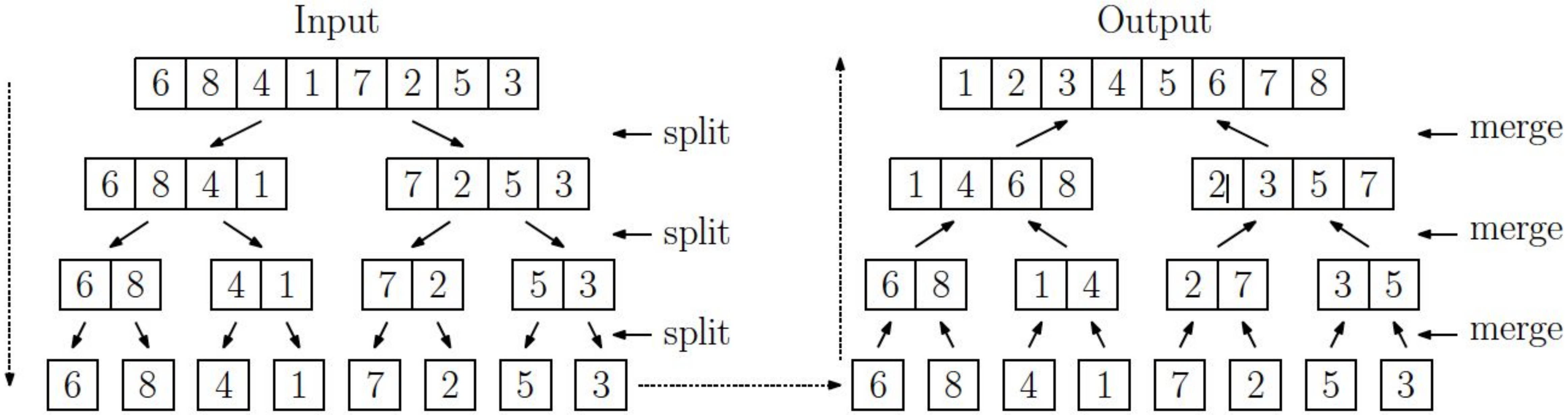
1 2 3 4 5 6 7 8

1 4 6 8 2 3 5 7

6 8 1 4 2 7 3 5

6 8 4 1 7 2 5 3

Example - summary



Merge - how do we sort two sorted lists?

```
Algorithm merge(A, B)
  S = []

  while(!A.isEmpty() and !B.isEmpty())
   if A[0] < B[0]
      S.add(A.removeFirst())
   else
      S.add(B.removeFirst())

   while (!A.isEmpty())
      S.add(A.removeFirst())
   while (!B.isEmpty())
      S.add(B.removeFirst())
   return S
```

runtime complexity?
O(n)

where n is A.length + B.length

Merge Sort Implementation

Summary

ChainHashMap - handles collisions by bucketing collisions in a Linked List

ProbeHashMap - handles collisions by finding the “next” open slot

1. Linear probe
2. Quadratic probe
3. Double Hash

Chain and Probe Hash Maps have equivalent runtime complexity (Big-O notation), but Probe is faster in practice