CS151 Intro to Data Structures

Implementing Stacks

Queues

Important Dates

HW2, Lab3, Lab4 due Friday 10/3

Lab5 today - due 10/10

Manual checkoff by me or TAs

Midterm after break

If you have testing accommodations let me know immediately

Agenda

- Stack Review
- Stack implementations
- Amortized Analysis
- Queues
  - Last DS before your exam

Stacks - FILO

- First In Last Out

- *stack* of plates in the dining hall

- Operations:
  - push
  - pop
  - peek
  - isEmpty

Stack Review - what will this code print?

```
public static void main(String[] args) {
    Stack<Integer> stack = new Stack<Integer>();

    stack.push(10);
    stack.push(20);
    stack.push(30);

    int popped = stack.pop();
    System.out.println("Popped: " + popped);

    int top = stack.peek();
    System.out.println("Top: " + top);

    stack.push(40);
    System.out.println("New Top after push: " + stack.peek());

    while (!stack.isEmpty()) {
        System.out.println("Popped: " + stack.pop());
    }
}
```

Implementing a Stack with an Array

Goal: O(1) operations

Let’s try to code it

Our class implementation:

- fixed size array (no expansions!)
- How did we implement push?
- How did we implement pop?
- How did we implement peek?

Now let’s implement stack with a linked list!

Goal: O(1) operations.

What to consider:

- When we PUSH where should we insert to?
  - Front, back, middle?

- When we POP where should we remove from?
  - Reminder: Stack should be FILO

Linked List Stack Performance

Space complexity is

- O(n)

Runtime Complexity:

- push:
  - O(1)
- Pop:
  - O(1)
- Peek:
  - O(1)

Stack Summary

- FILO wrapper around Array / Linked List
- Allows for limited data structures operations all with O(1) cost
- Real world applications: call stack, browser history, postfix calculator

Amortized Analysis

https://courses.cs.washington.edu/courses/cse373/17wi/summaries/amortized-runtime.pdf

Amortized Analysis

average run time complexity of an operation.

Compares the total cost of a series of operations with how many of those operations happened.

Amortized Runtime Analysis

$$\frac{total\ cost\ of\ operations}{total\ number\ of\ operations}$$

Where an “operation” is the operation a client is doing through your public interface, like insert(5) or pop() or add(3).

Exercise: Amortized Analysis of Array Insert

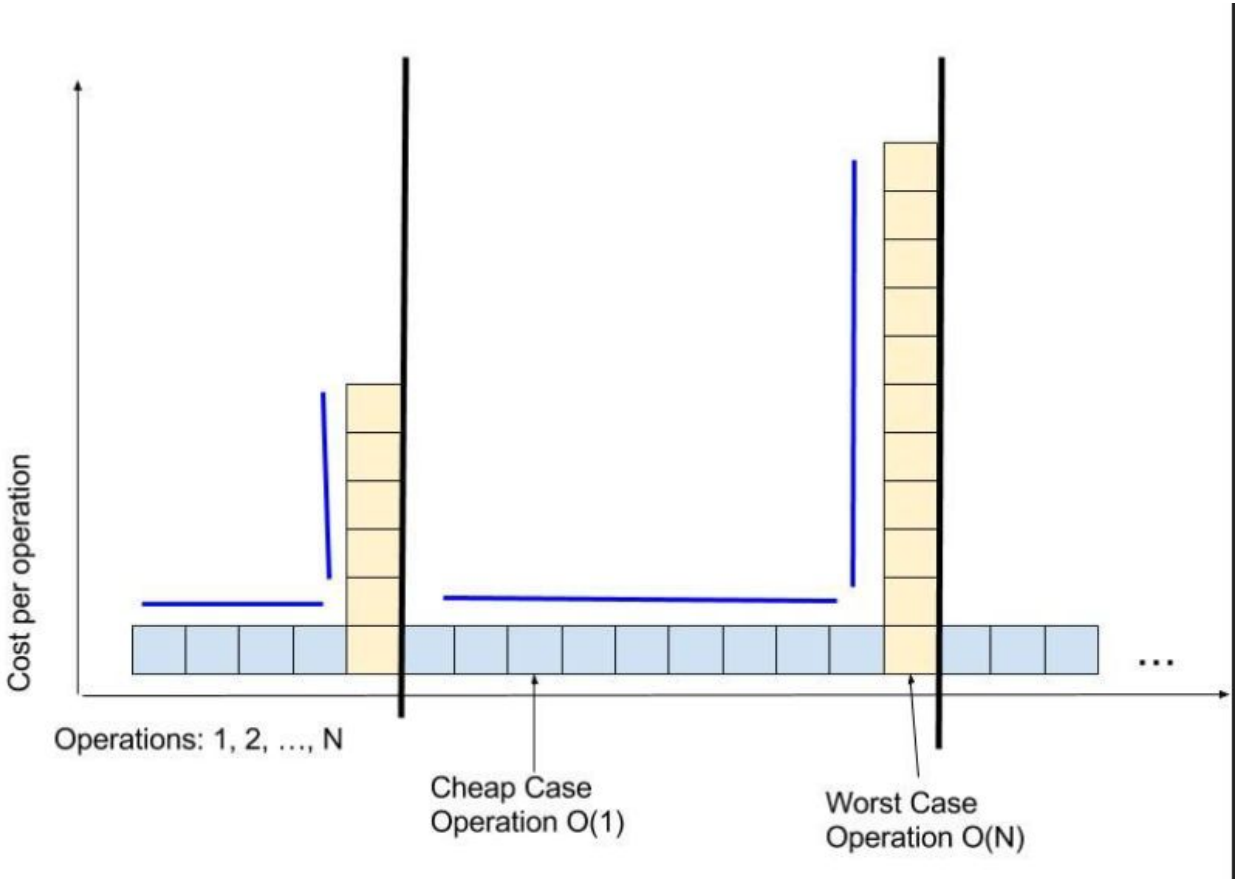
Let’s say you have an expandable array with N elements: [1, 2, 3... N]

When you try to add the N+1 th element, a resize occurs.

$$\frac{total\ cost\ of\ operations}{total\ number\ of\ operations}$$

Amortized Analysis

Intuition: you want to “build up enough credit” with a series of cheap operations, so that when you have one (or more) expensive operations, you can average out the cost of the expensive one



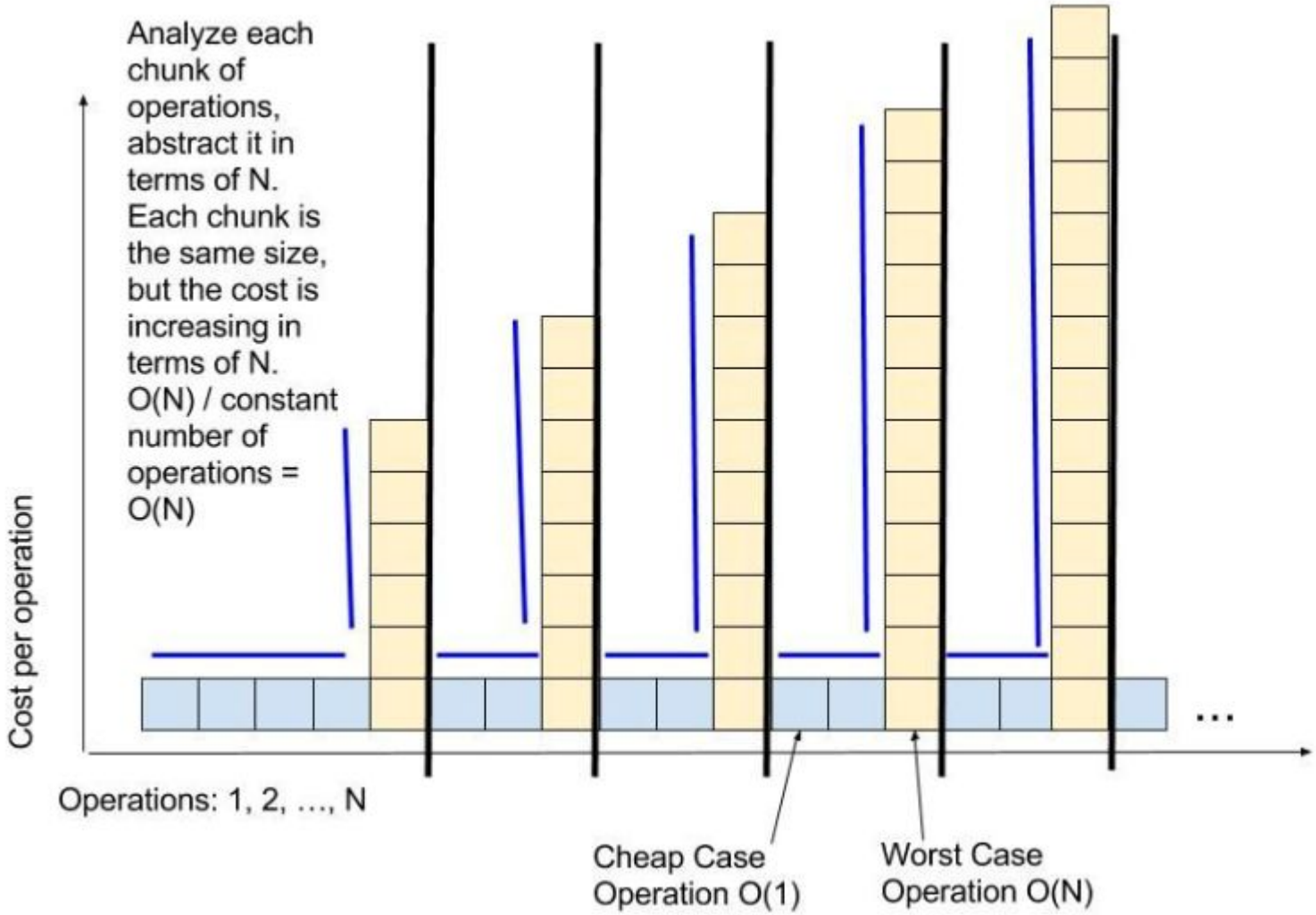
Exercise: Amortized Analysis of Array Insert

Let's say you have an expandable array **which adds two extra slots each time it is full**

Amortized cost of insert? Calculate for 9 inserts with an expandable array that begins with capacity 2

$$\frac{total\ cost\ of\ operations}{total\ number\ of\ operations}$$

Amortized Analysis



Amortized Analysis

Average case runtime analysis

Intuition: you want to “build up enough credit” with a series of cheap operations, so that when you have one (or more) expensive operations, you can average out the cost of the expensive one

Comparing the total cost of a series of operations with how many operations happened

Queues

FIFO Stacks

Stack Property

First-in Last-out (FILO)

Where might a FILO stack not make sense?

- Line for the cash register
- Printer Queue

FIFO: First-in First-out

The first item in, is the first item out

Add-to the back, remove from the front

This is a **Queue**

Inserting – “`enqueue`”

Removing - “`dequeue`”

`Queue` Interface

```
public interface Queue<E> {
  int size();
  boolean isEmpty();
  E first();
  void enqueue(E e);
  E dequeue();
}
```

- `null` is returned from `dequeue()` and `first()` when queue is empty

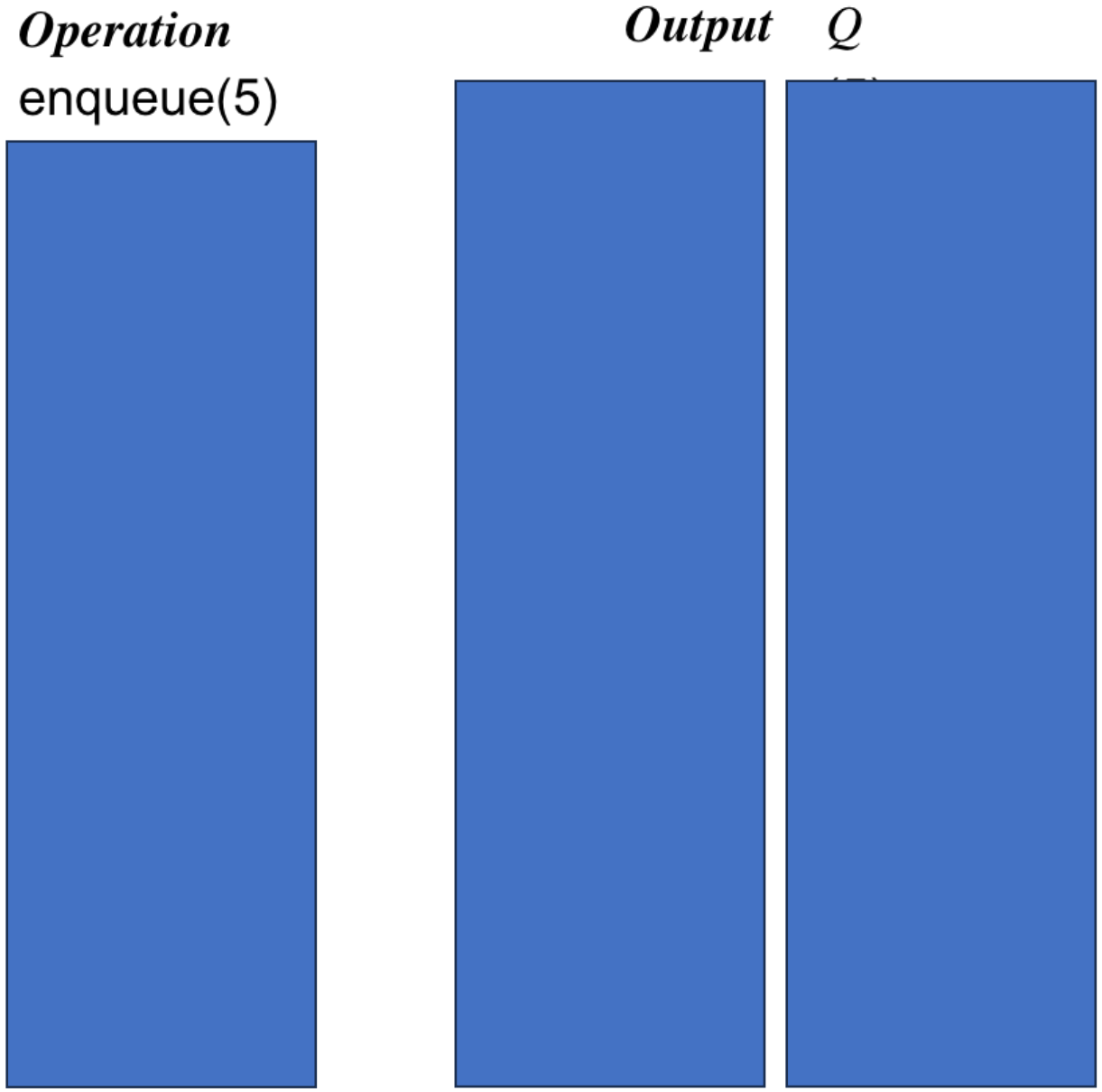
Queue Example

Cash register code

Example

Operation	*Output*	*Q*
enqueue(5)	–	(5)
enqueue(3)	–	(5, 3)
dequeue()	*5*	(3)
enqueue(7)	–	(3, 7)
dequeue()	*3*	(7)
first()	*7*	(7)
dequeue()	*7*	()
dequeue()	*null*	()
isEmpty()	*true*	()
enqueue(9)	–	(9)
enqueue(7)	–	(9, 7)
size()	*2*	(9, 7)
enqueue(3)	–	(9, 7, 3)
enqueue(5)	–	(9, 7, 3, 5)
dequeue()	*9*	

Example



Example

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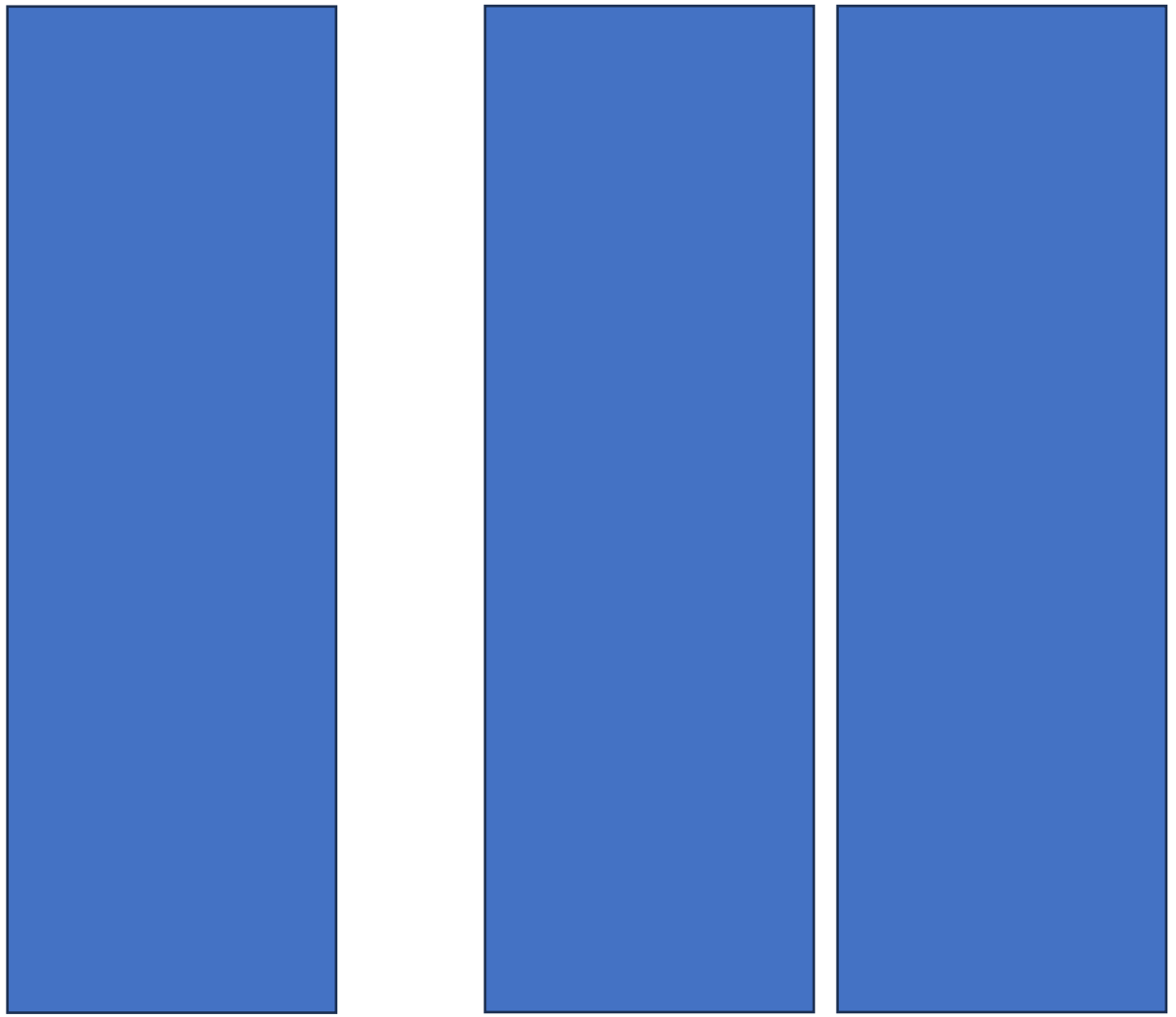

Example

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dequeue()	*7*	()
dequeue()	*null*	()
isEmpty()	*true*	()
enqueue(9)	–	(9)
enqueue(7)	–	(9, 7)
size()	*2*	(9, 7)
enqueue(3)	–	(9, 7, 3)
enqueue(5)	–	(9, 7, 3, 5)
dequeue()	*9*	(7, 3, 5)

Example

Operation	*Output*	*Q*
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enqueue(3)	–	(5, 3)
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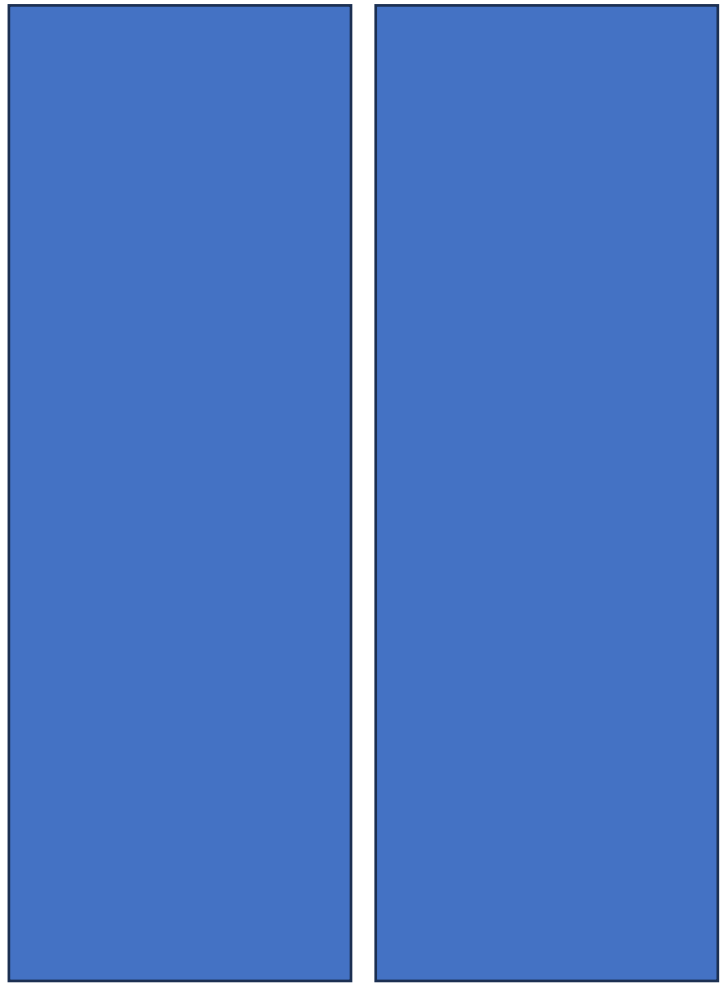
dequeue()		



Example

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enqueue(3)	–	(5, 3)
dequeue()	*5*	(3)
enqueue(7)	–	(3, 7)
dequeue()	*3*	(7)
first()	*7*	(7)
dequeue()	*7*	()
dequeue()	*null*	()
isEmpty()	*true*	()
enqueue(9)	–	(9)
enqueue(7)	–	(9, 7)
size()	*2*	(9, 7)
enqueue(3)	–	(9, 7, 3)
enqueue(5)	–	(9, 7, 3, 5)
dequeue()	*9*	(7, 3, 5)

Implementing a Queue with an Array

Goal: O(1) operations

How can we achieve this?

1. Fixed size underlying array (no expansions)

1. Where should we insert?
   a. front, back, middle?

1. Where should we remove from?
   b. we should preserve first in first out property!

Summary

- Stacks are FILO data structures with O(1) operations
  - can be implemented with an array or LL
- Queues are FIFO data structures with O(1) operations
  - can be implemented with an array or LL
  - saw array today will see LL next week
- Amortized runtime analysis
  - average