CS151 Intro to Data Structures

Graphs

Announcements

- Lab11: extra credit on final exam
- Complete your course evaluations
- Practice questions posted
- Final Exam
  - Dec 15 9:30-12:30 Park 159
  - two cheat sheets allowed

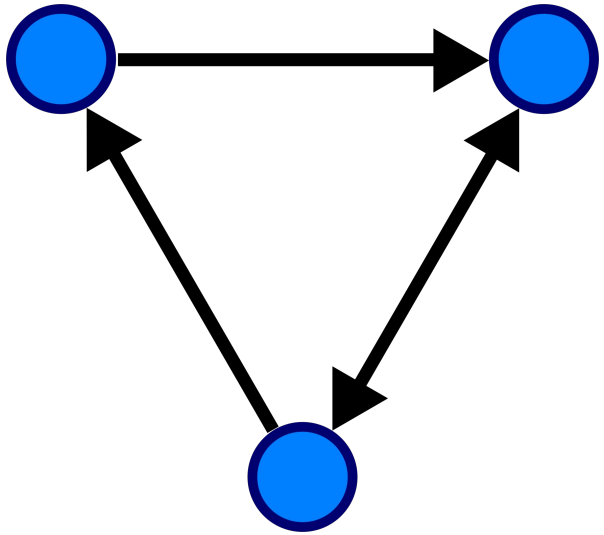
- Office hour change:
  - cancelled friday
  - Thursday 2-5pm with priority for 3:30-5

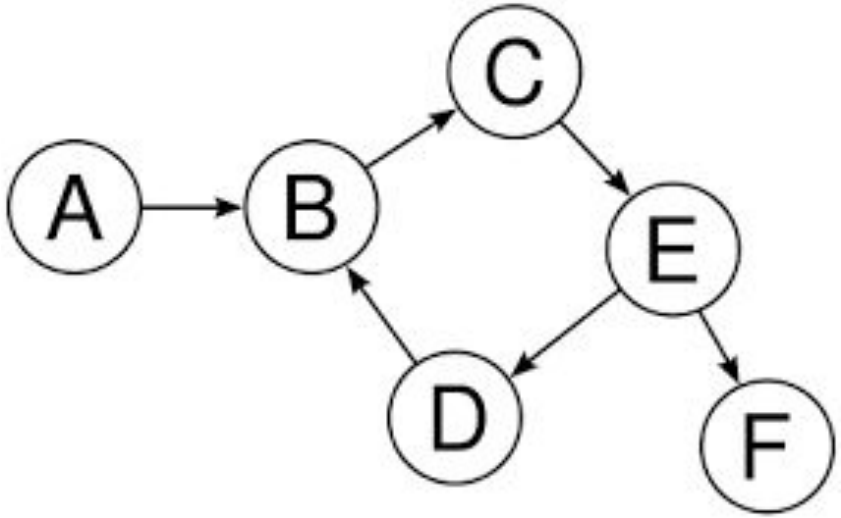
Graphs

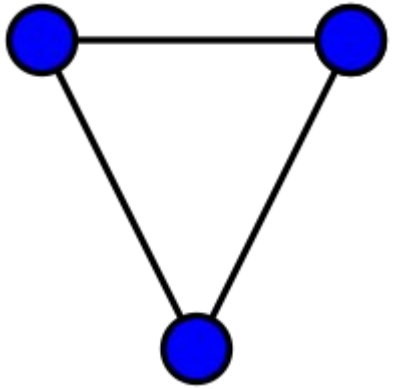
- Terminology
- Data Structures for Graphs
  - Adjacency Lists
  - Adjacency Matrix
- Shortest Paths
  - Djikstra’s Algorithm

Graphs

- A way of representing relationships between pairs of objects
- Consist of ***Vertices (V)*** with pairwise connections between them ***Edges (E)***
- A ***Graph G*** is a set of vertices and edges **(V, E)**

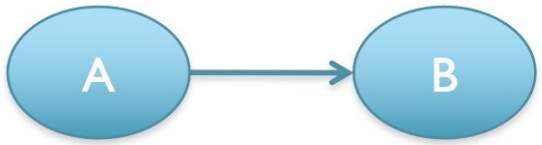






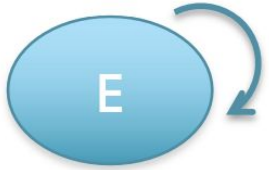
Edges

- An edge (u, v) connects vertices u and v
- Edges can be ***directed*** or ***undirected***
- An edge is said to be ***incident*** to a vertex if the vertex is one of the endpoints



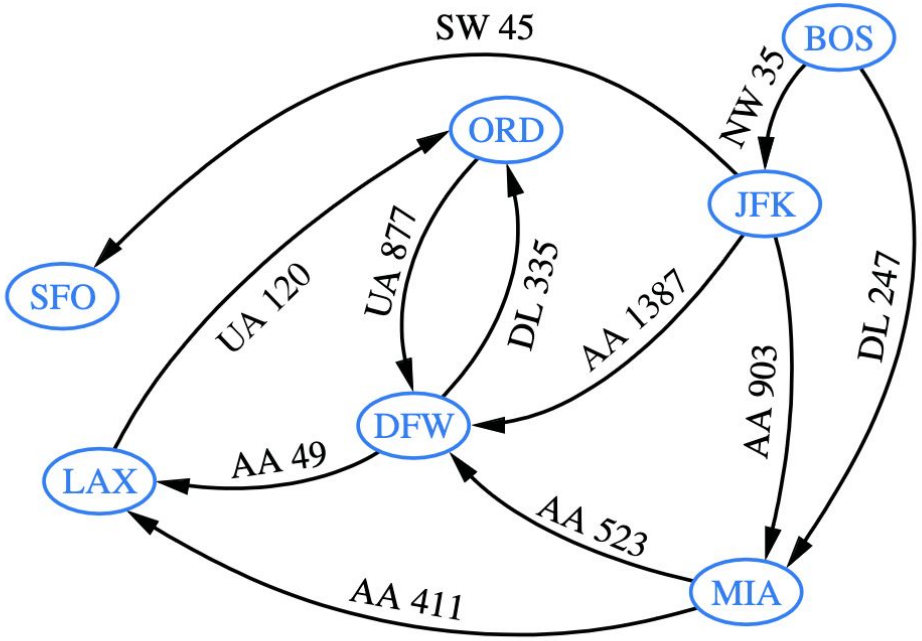
Directed Edge

Undirected Edge



Self Edge
(Unusual but usually allowed)

Directed vs Undirected Graphs



Example of a directed graph representing a flight network.

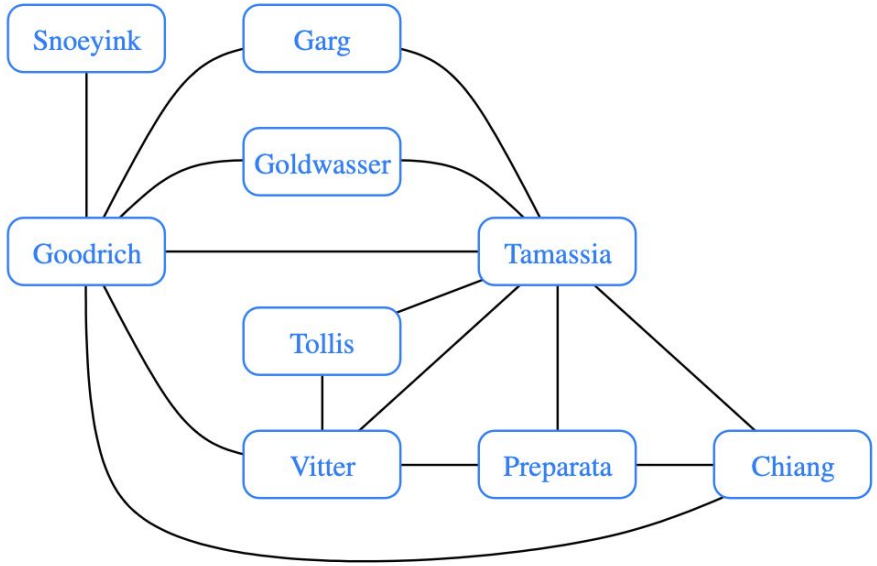


Figure 14.1: Graph of coauthorship among some authors.

Graphs

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- **Data Structures for Graphs**
  - Adjacency Lists
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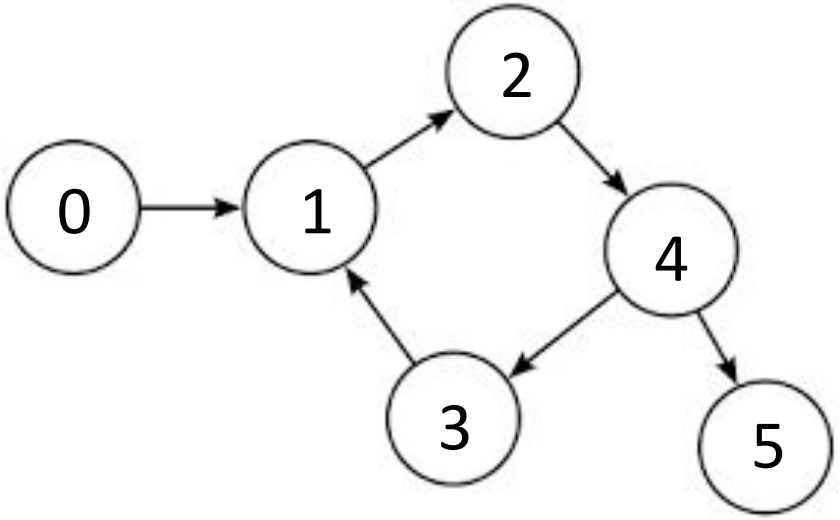
Representing a graph

Adjacency List -

For each vertex v, we maintain a separate list containing the edges that are outgoing from v



Graph ADT

numVertices( ):	Returns the number of vertices of the graph.
vertices( ):	Returns an iterable of all the vertices of the graph.
numEdges( ):	Returns the number of edges of the graph.
edges( ):	Returns an iterable of all the edges of the graph.
getEdge(u, v):	Returns the edge from vertex u to vertex v, if one exists; otherwise return null. For an undirected graph, there is no difference between getEdge(u, v) and getEdge(v, u).
endVertices(e):	Returns an array containing the two endpoint vertices of edge e. If the graph is directed, the first vertex is the origin and the second is the destination.
opposite(v, e):	For edge e incident to vertex v, returns the other vertex of the edge; an error occurs if e is not incident to v.
outDegree(v):	Returns the number of outgoing edges from vertex v.
inDegree(v):	Returns the number of incoming edges to vertex v. For an undirected graph, this returns the same value as does outDegree(v).

Graph ADT

outgoingEdges(v):	Returns an iterable of all outgoing edges from vertex v.
incomingEdges(v):	Returns an iterable of all incoming edges to vertex v. For an undirected graph, this returns the same collection as does outgoingEdges(v).
insertVertex(x):	Creates and returns a new Vertex storing element x.
insertEdge(u, v, x):	Creates and returns a new Edge from vertex u to vertex v, storing element x; an error occurs if there already exists an edge from u to v.
removeVertex(v):	Removes vertex v and all its incident edges from the graph.
removeEdge(e):	Removes edge e from the graph.

Representing a graph as an **AdjacencyList**

For each vertex v, we maintain a separate list containing the edges that are outgoing from v

Given that we will be inserting and removing vertices and edges, which data structure should we use?

Representing a graph as an **AdjacencyList**

How might we implement the following methods with an AdjacencyList representation?

1. addVertex

2. addEdge

3. removeVertex

4. removeEdge

Representing a graph - Adjacency List

Runtime Complexity: (In terms of V and E rather than n)

- addVertex:
  - O(1)
- addEdge:
  - O(E)
- removeVertex:
  - O(V*E)
- removeEdge:
  - O(E)

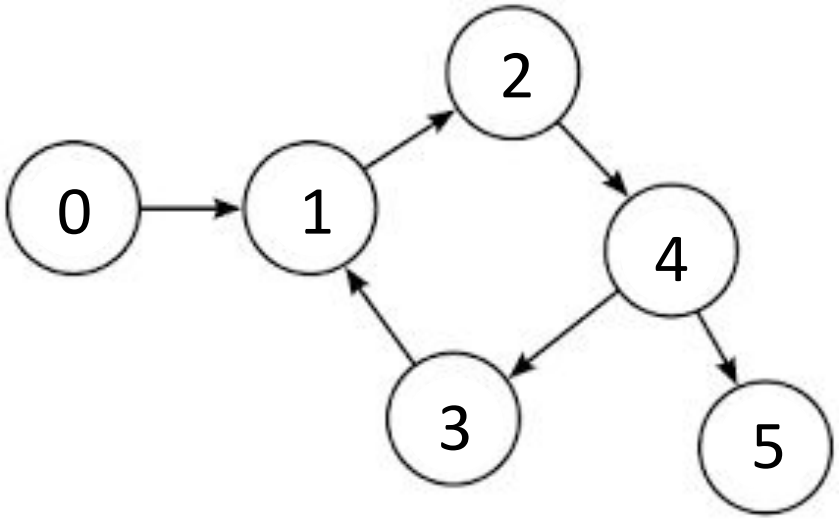
Representing a graph

Adjacency Matrix -

each index in the array is another array

Maintains an VxV matrix

where each slot (i,j) represents an outgoing edge from i to j



	1				
		1			
				1	
	1				
			1		1

Representing a graph

Let’s implement a graph as an Adjacency Matrix

Representing a graph - Adjacency Matrix

Runtime Complexity: (In terms of V and E rather than n)

- addVertex:
  - O(V)
  - O(V^2) if we need to expand
- addEdge:
  - O(1)
- removeVertex:
  - O(V)
- removeEdge:
  - O(1)

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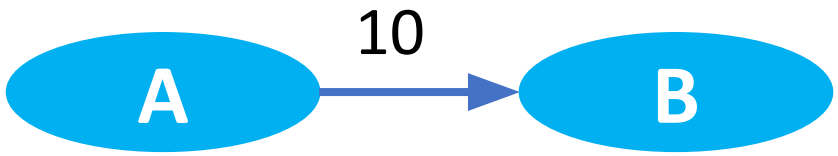
Weighted Graphs

Edges have weights/costs



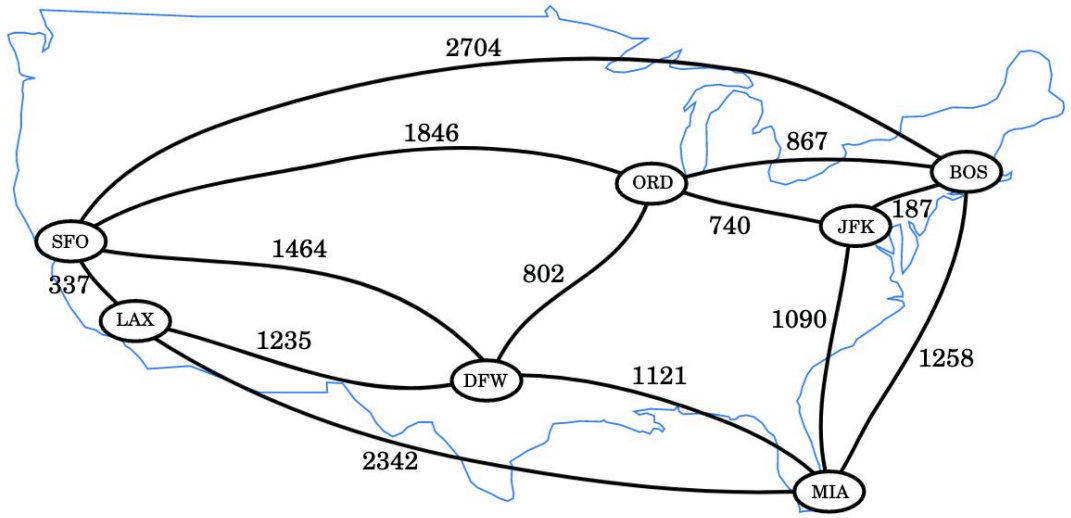


Figure 14.14: A weighted graph whose vertices represent major U.S. airports and whose edge weights represent distances in miles. This graph has a path from JFK to LAX of total weight 2,777 (going through ORD and DFW). This is the minimum-weight path in the graph from JFK to LAX.

Shortest Paths

A **path** is defined as a set of edges

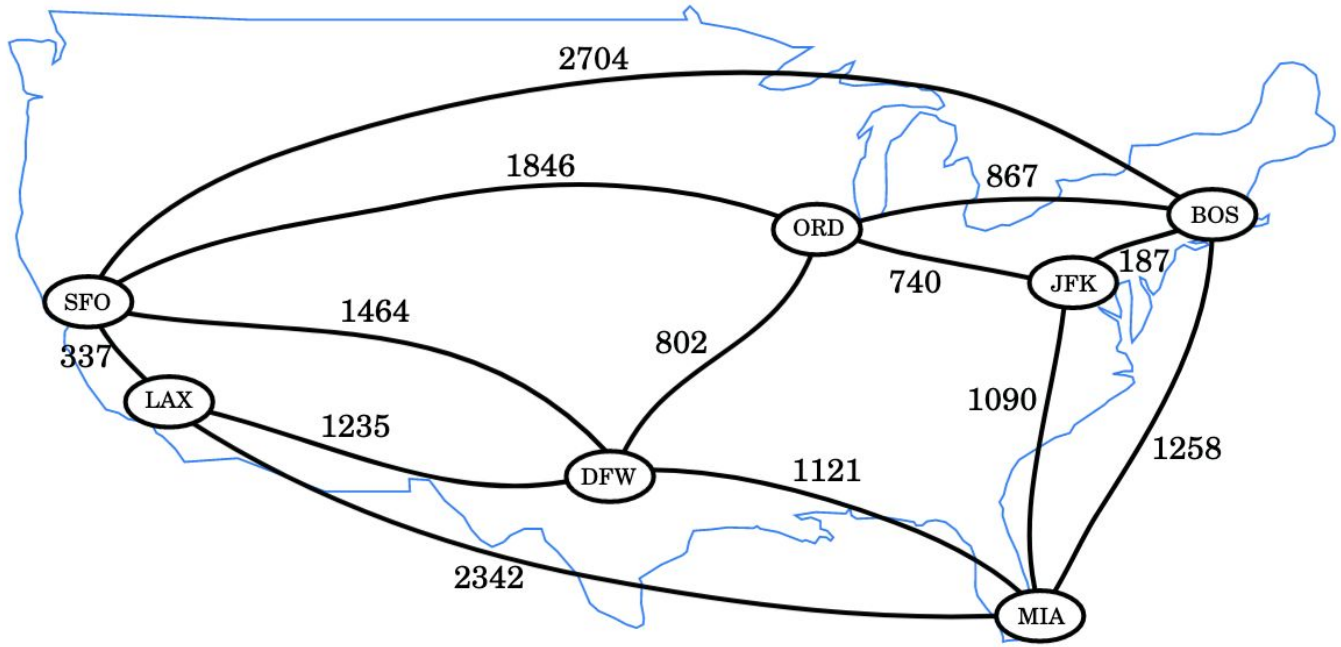
$$P = ((v_0, v_1), (v_1, v_2), \ldots, (v_{k-1}, v_k))$$

The *length* of a path is the sum of the weights of the edges

$$w(P) = \sum_{i=0}^{k-1} w(v_i, v_{i+1}).$$

Shortest Paths

What is the length of the path P = ((SFO, DFW), (DFW, MIA), (MIA, JFK))

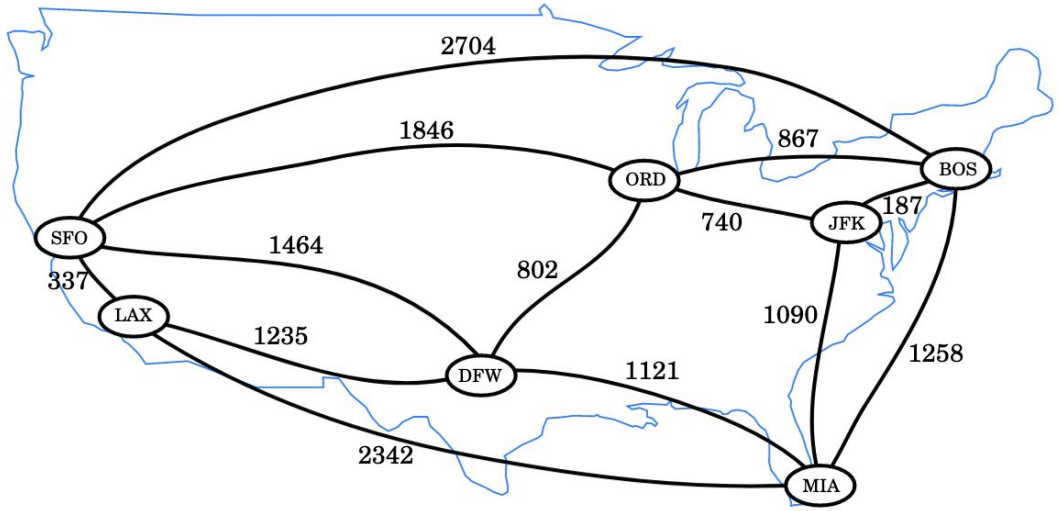


Shortest Paths

What is the shortest path from SFO to JFK?

There are many possible paths...

((SFO, ORD), (ORD, JFK))
((SFO, LAX), (LAX, MIA), (MIA, JFK))
((SFO, BOS), (BOS, JFK))
....
((SFO, DFW), (DFW, ORD), (ORD, JFK))

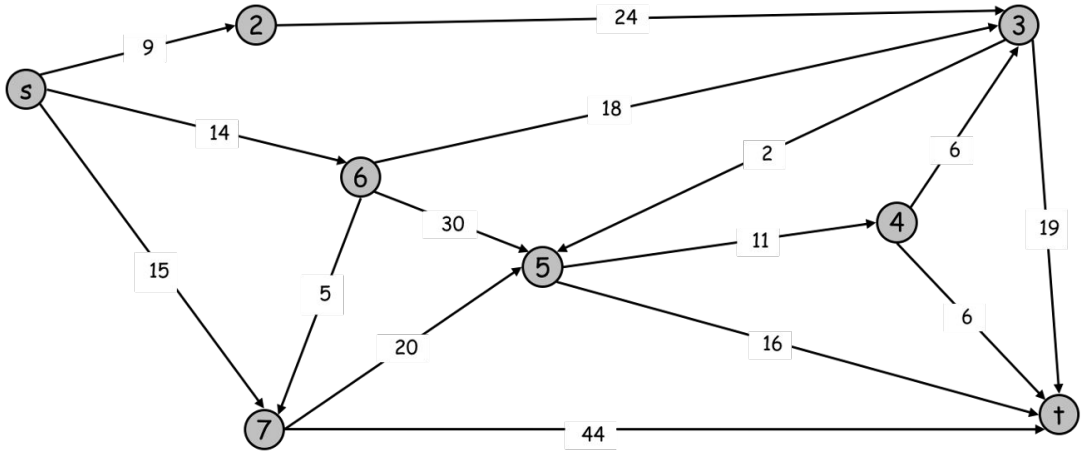


Dijkstra's algorithm

- graph search algorithm that finds the shortest path between nodes in a weighted graph

- maintains a set of vertices whose shortest distance from the *source* has already been determined, which it gradually refines
  - uses a *min heap* to select the vertex with the smallest distance

Dijkstra's algorithm



1. init:
   a. assign a init distance for each node
      i. 0 for src, INF for all other nodes
   b. create a min-heap and add the source

2. while heap is non-empty:
   c. poll node p and mark as visited
   d. For each neighboring node not yet visited:
      i. distance of neighbor = dist(p) + weight of edge (p, neighbor)
      ii. If this distance is less than the current dist, update it.
      iii. insert in heap if distance changed