CS340 Analysis of Algorithms
Fall 2025

Lab:	**1**	**Professor:**	**Elizabeth Dinella**
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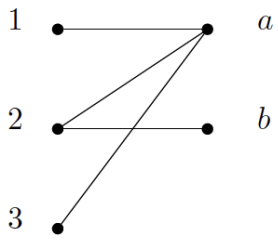
Title:	**Max Bipartite Matching**	**URL:**	**https://bmc-cs-340.github.io**

For the following problems, there are n residents and m hospital positions. Residents and hospitals still give a list of suitable candidates, however, they no longer rank them. In other words, for each pair of (resident, position) we can check whether this placement is mutually agreeable. Our goal is to find the largest number of placements (matchings).

1 Pseudo Code Attempt 1 - Brute Force

```
1  Function BFmaxmatching()
2      create a set P of all pairs (resident, position)
3      create a set S of all subsets of P
4      maxM = 0
5      for every elemement M∈S do
6          check if every resident appears at most once in M
7          check if every hospital appears at most once in M
8          check if every pair (i,j)∈M, resident i and hospital j are agreeable
9          if the answer to all above checks are yes, and if |M|>maxM then
10             maxM=|M|
11     return maxM
```

Answer the following questions for the graph G on the right.



1. Highlight a maximum matching in this graph.
2. List all pairs in P
3. What is the size of P for this graph?
4. List at least four different elements in S (use set notations for each element), and for each element state its size, hightlight the pairs it corresponds to in G, and state whether, on line 9 of the pseudo code, the answer is yes. Include elements of at leasts three diffferent sizes.
5. What is the size S for this graph?

Answer these questions for any bipartite graph:

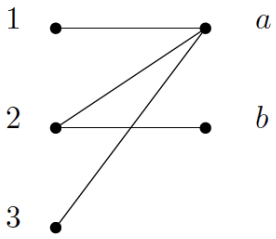
1. What is $|P|$ as a function of n and m?
2. What is $|S|$ as a function of n and m?
3. How many iterations will the for loop execute?

2 Pseudo Code Attempt 2 - Greedy

```
1  Function Gmaxmatching()
2      M = ∅
3      while there is at least one agreeable pair (resident, position) do
4          for each resident r do
5              let N(r) be the set of hospital positions that are acceptable for r
6          for each hospital position h do
7              let N(h) be the set of residents that are acceptable for h
8          let x be any entity (a resident or a position) with the smallest N(x). If there are multiple
             choices, choose any.
9          match x to an element y∈N(x) with the smallest N(y) and add (x,y) to M
10         remove x and y from the input sets
11     return |M|
```

Answer the following questions for the graph on the right.

1. Trace the algorithm on this graph and highlight the pairs in M
2. Is M a maximum matching for this input?



Answer these questions for any bipartite graph:

1. Given an upper-bound on the number of iterations of the while-loop, as a function of n and m
2. Is it always true that the max matching size is $minn,m$?
3. If you answered no above, give a bipartite graph with max matching size smaller than $min(n,m)$.