

# Lab 10

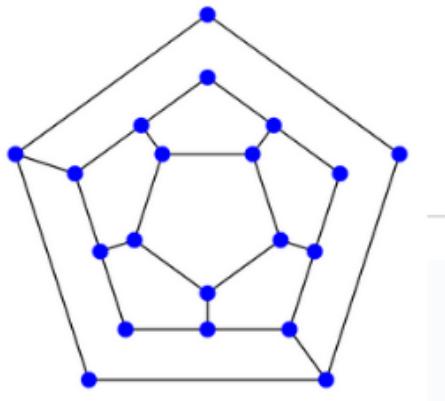
Deadline: 12/11/25

**Problem 1.** Show that Clique, Independent Set, and Vertex Cover are polynomial-time equivalent by giving reductions between them. (Hint: You don't need six reductions—a cycle of three suffices.)

For a graph  $G$  and integer  $k$ :

- **Clique:** Does  $G$  have a clique of size  $k$ ?
- **Independent Set:** Does  $G$  have an independent set of size  $k$ ?
- **Vertex Cover:** Does  $G$  have a vertex cover of size  $k$ ?

**Problem 2.** Does the following graph have a Hamiltonian cycle? If so, give one. If not, explain why.



**Problem 3.** You have 3 machines and jobs with processing times [2, 3, 4, 6, 2, 2].

- Run the following greedy algorithm: assign each job (in the given order) to the currently least-loaded machine. What is the makespan?
- Find an optimal assignment. What is the optimal makespan?
- Construct an instance where greedy does worse relative to optimal.
- For parts (a) and (c), compute the ratio (greedy makespan)/(optimal makespan).

**Submit on Gradescope.**