CS340 Analysis of Algorithms

Handout:	**7**	**Professor:**	**Dianna Xu**
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Sample proof of correctness on the reduction from 3SAT to Independent Set. Please refer to lecture notes for details of the reduction construction.
Correctness: We will show that F is satisfiable if and only if G has an independent set of size k

($\Longrightarrow$): If F is satisfiable, then each of the k clauses of F must have at least one true literal. Select such a literal from each clause. Let V' denote the corresponding set of vertices picked from each of the clause clusters (one from each cluster) in G. We claim that V' is an indenpendent set of size k, for the follow reasons:

1. Since there are k clauses, and we pick one vertex from each clause cluster, clearly $|V'| = k$
2. We only take one vertex from each clause cluster, and
3. Two conflicting literals can not be both in the solution set of F, which means that if $x \in V'$ then $\overline{x} \notin V'$

By construction, there are only two kinds of edges in G. The edges that connect all pairs of the clause cluster vertices and the conflict edges between a literal x and its negation $\overline{x}$. 2 and 3 above make sure that for each edge of G, at most one end point is in V'. Therefore V' is an independent set.

($\Longleftarrow$): Suppose G has an independent set V' of size k. By construction (because we put an edge between all pairs of vertices in any clause cluster in G), two vertices from the same clause cluster can not both be in V' (or V' would not be an independent set). Since $|V'| = k$, and there are k clauses, V' has exactly one vertex from each clause cluster. Also note that if a vertex $x \in V'$, then $\overline{x}$ is necessarily adjacent to x (via the conflict edges that we inserted into G), and therefore $\overline{x}$ can not be in V'. Therefore, there exists an assignment in which every literal corresponding to a vertex appearing in V' is set to 1. Such an assignment satisfies one literal in each clause in F, and therefore the entire formula is satisfied.