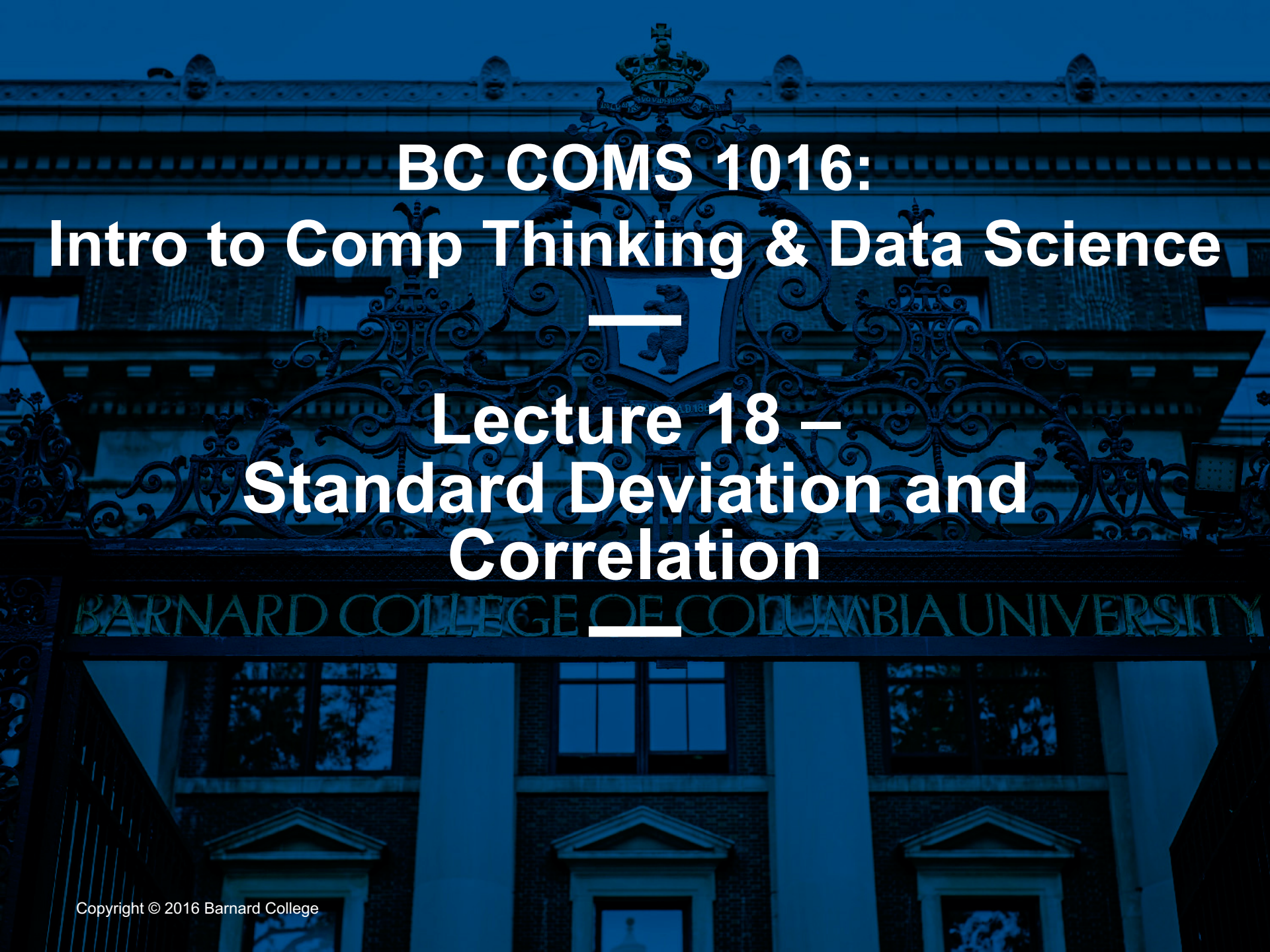


BC COMS 1016: Intro to Comp Thinking & Data Science

—

Lecture 18 – Standard Deviation and Correlation

Copyright © 2016 Barnard College

Announcements

- Lab07 – Normal Distribution and Variance of Sample Means (short)
  - Due Wednesday 11/23
- Homework 7 - Confidence Intervals, Resampling, the Bootstrap, and the Central Limit Theorem
  - Due Thursday 11/24
  - Not the shortest
- Homeworks:
  - Run all cells before submitting
- Dropping 2 homeworks and labs

Copyright © 2016 Barnard College

Confidence Intervals

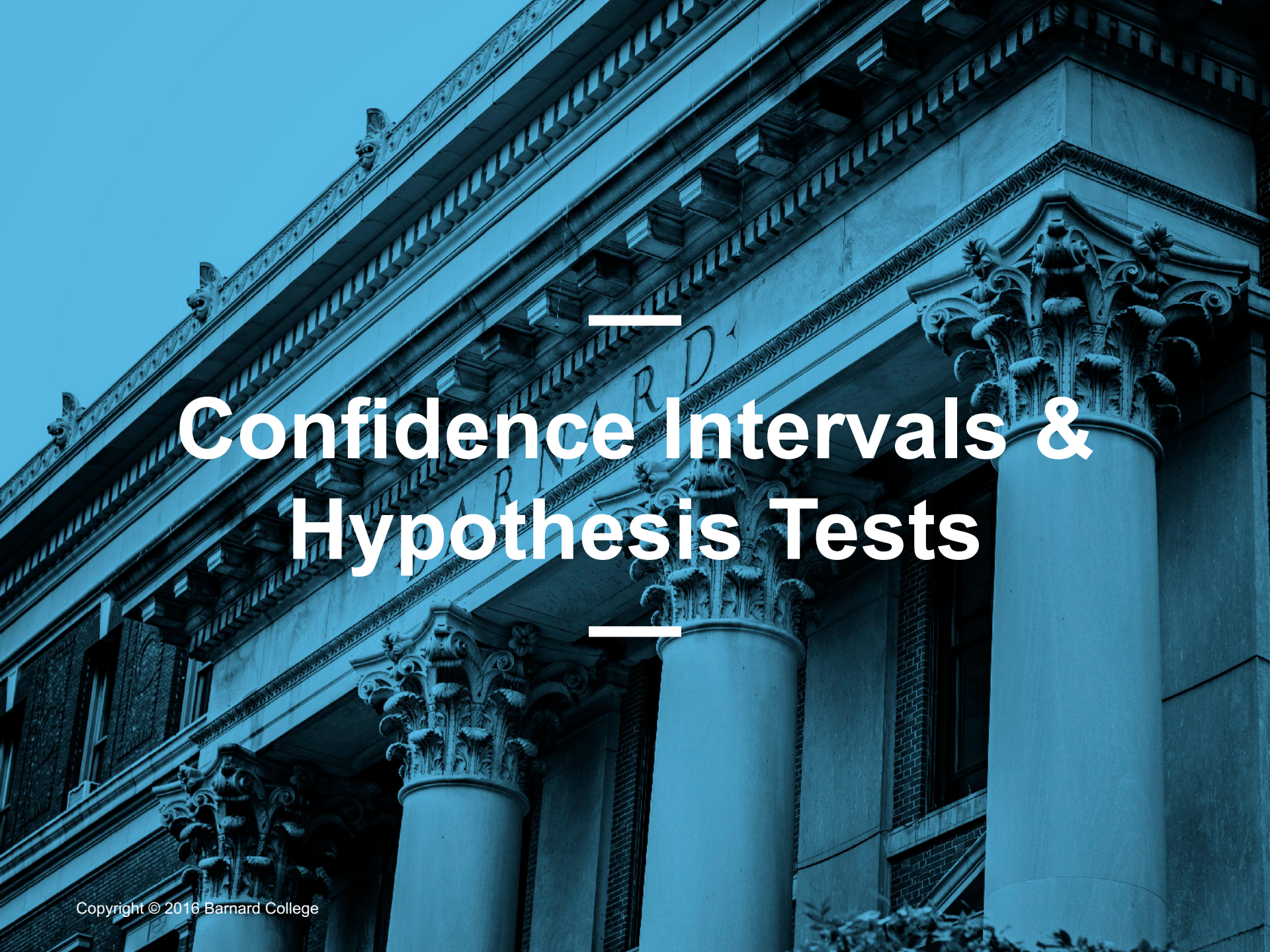
Copyright © 2016 Barnard College

95% Confidence Interval

- Interval of **estimates of a parameter**
- Based on random sampling
- 95% is called the confidence level
  - Could be any percent between 0 and 100
  - Higher level means wider intervals

- The **confidence is in the process** that gives the interval:
  - It generates a “good” interval about 95% of the time

Copyright © 2016 Barnard College



Confidence Intervals & Hypothesis Tests

Copyright © 2016 Barnard College

Using a CI for Testing

- Null hypothesis: **Population average = *x***
- Alternative hypothesis: **Population average ≠ *x***
- Cutoff for P-value: *p*%
- Method:
  - Construct a (100-*p*)% confidence interval for the population average
  - If *x* is not in the interval, reject the null
  - If *x* is in the interval, can't reject the null

Copyright © 2016 Barnard College

Data Science in this course

- Exploration
  - Discover patterns in data
  - Articulate insights (visualizations)
- Inference
  - Make reliable conclusions about the world
  - Statistics is useful
- **Prediction**
  - **Informed guesses about unseen data**

Copyright © 2016 Barnard College

Center & Spread

Copyright © 2016 Barnard College

Questions/Goals

- How can we quantify natural concepts like “center” and “variability”?
- Why do many of the empirical distributions that we generate come out bell shaped?
- How is sample size related to the accuracy of an estimate?

Copyright © 2016 Barnard College

Average and the Histogram

Copyright © 2016 Barnard College

The average (mean)

Data: 2, 3, 3, 9

Average = (2+3+3+9)/4 = 4.25

- Need not be a value in the collection
- Need not be an integer even if the data are integers
- Somewhere between min and max, but not necessarily halfway in between
- Same units as the data
- Smoothing operator: collect all the contributions in one big pot, then split evenly

Copyright © 2016 Barnard College

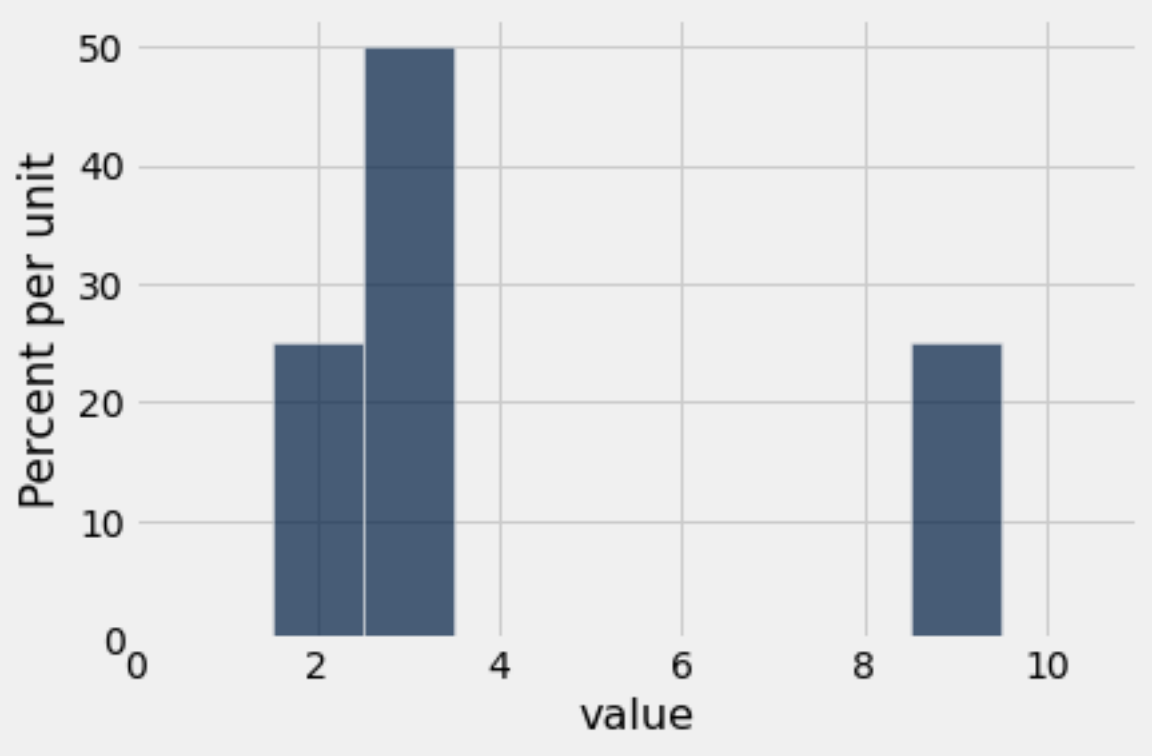
Relation to the histogram

- The average depends only on the **proportions** in which the distinct values appears
- The average is the **center of gravity** of the histogram
- It is the point on the horizontal axis where the histogram balances

Copyright © 2016 Barnard College

Average as balance point

- Average is 4.25



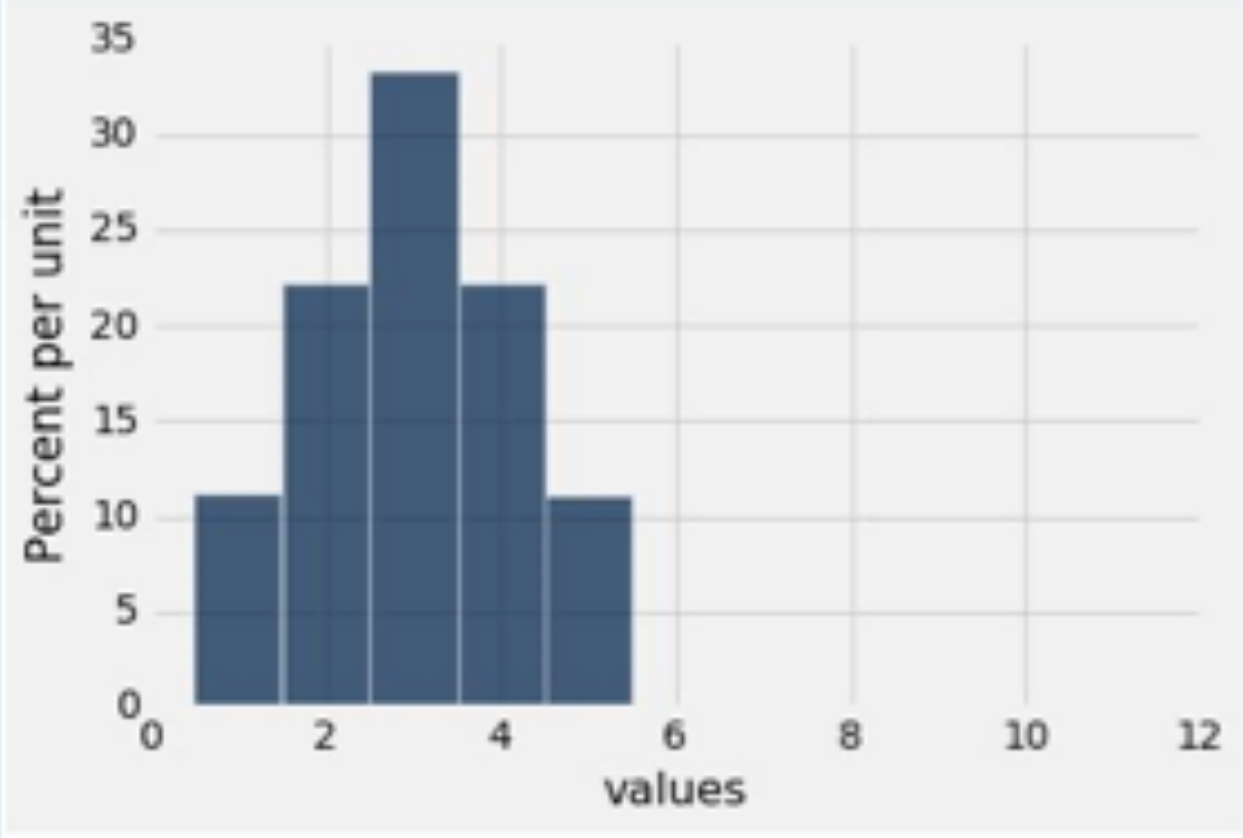
Copyright © 2016 Barnard College

Average and Median

Copyright © 2016 Barnard College

Question

- What list produces this histogram?

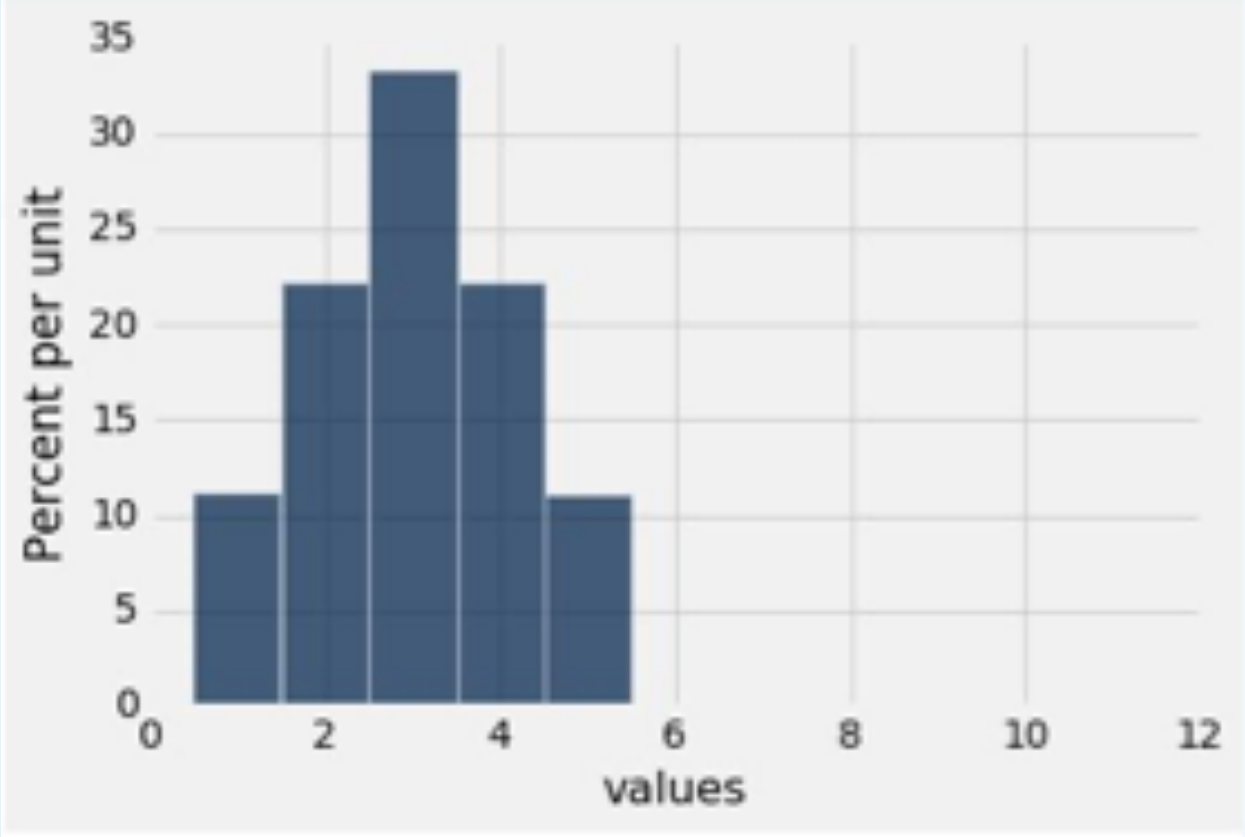


Copyright © 2016 Barnard College

Question

- What list produces this histogram?

1, 2, 2, 3, 3
3, 4, 4, 5



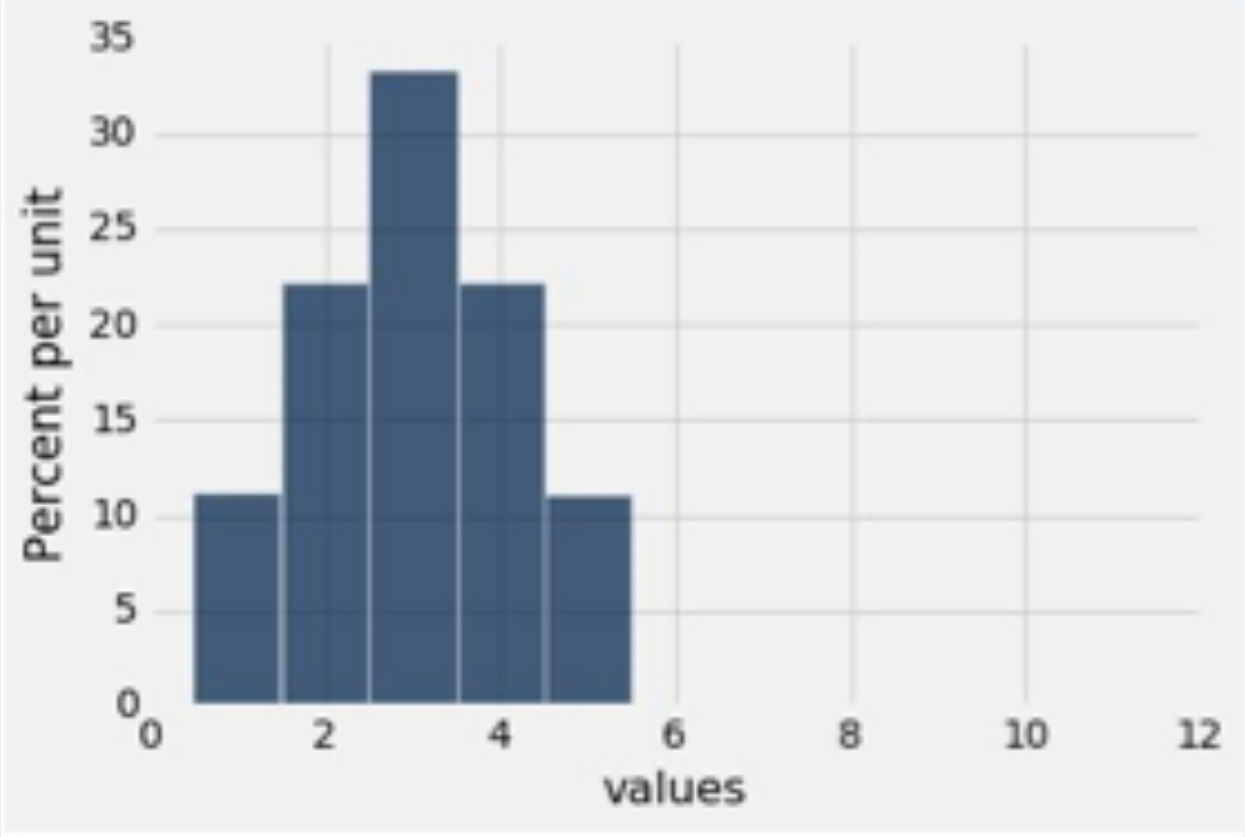
Copyright © 2016 Barnard College

Question

- What list produces this histogram?

1, 2, 2, 3, 3
3, 4, 4, 5

- Average?



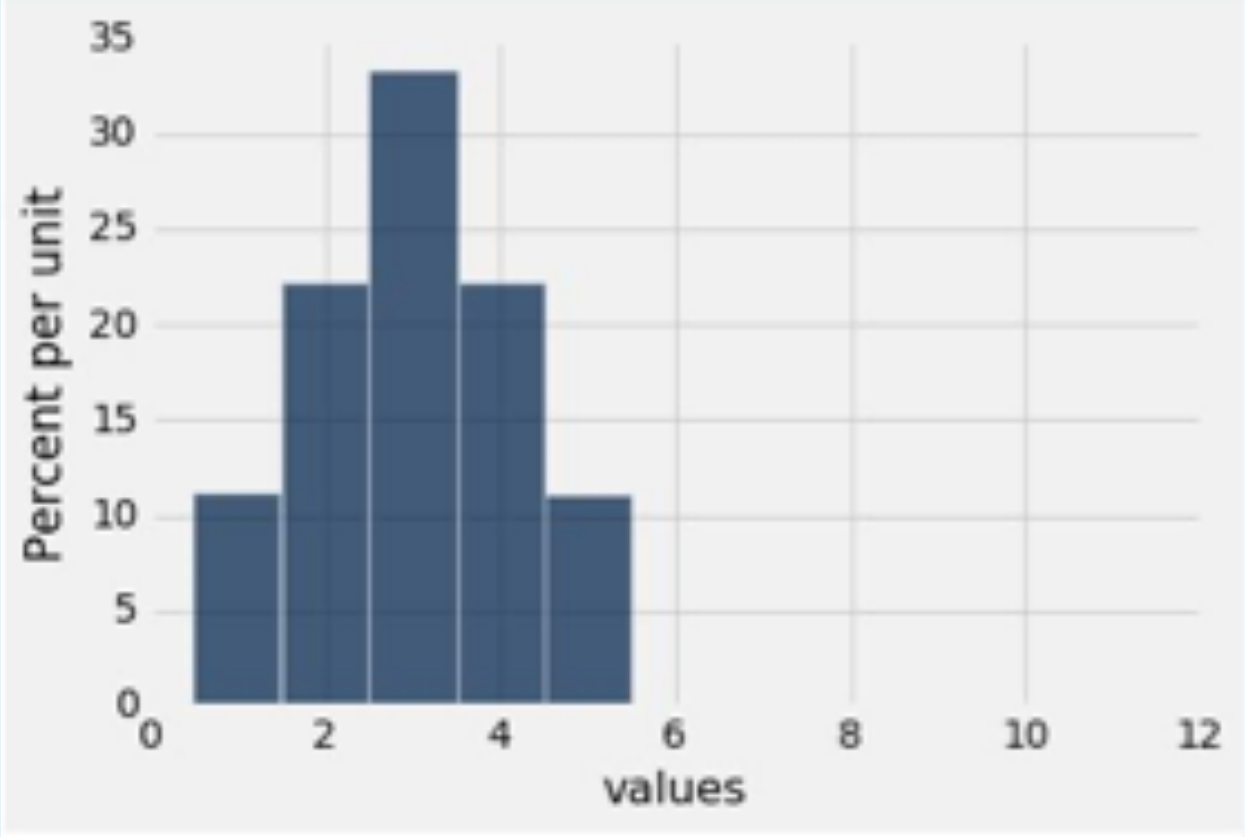
Copyright © 2016 Barnard College

Question

- What list produces this histogram?

1, 2, 2, 3, 3
3, 4, 4, 5

- Average?
  - 3

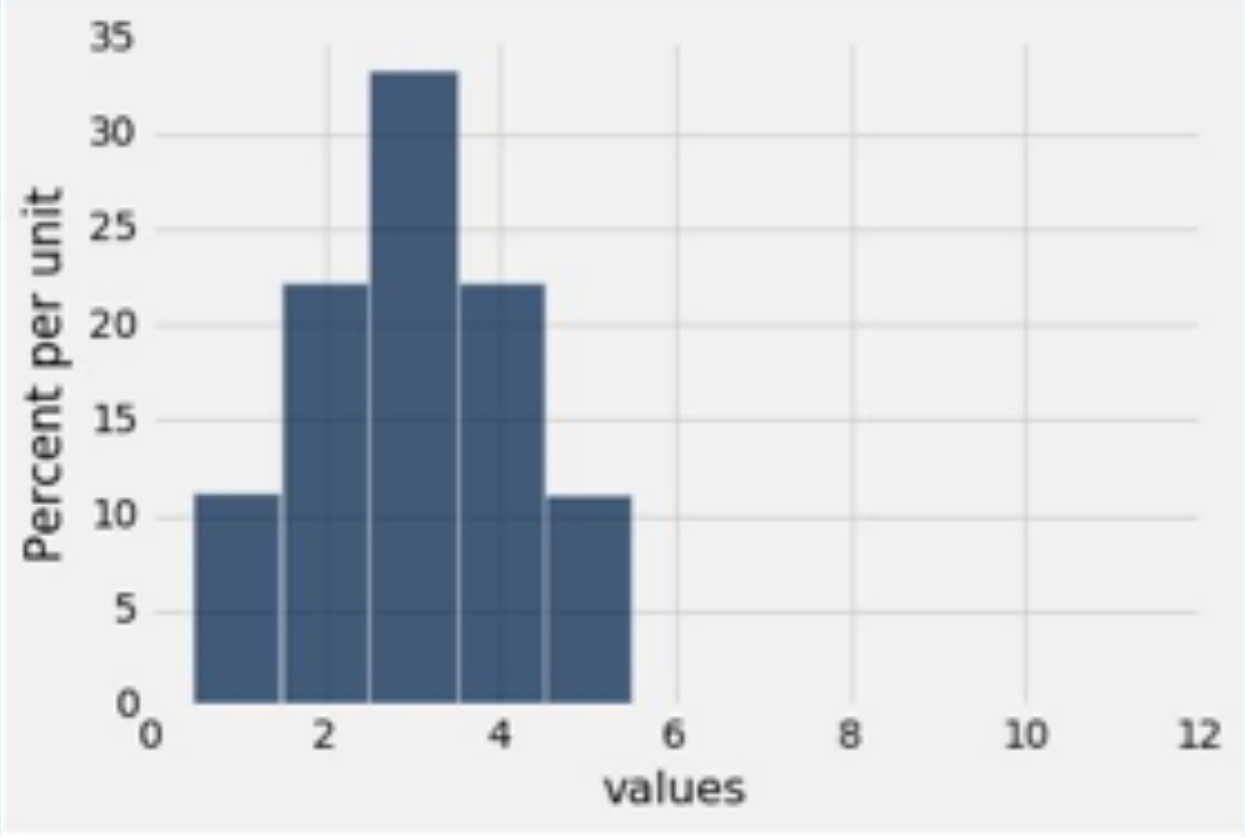


Copyright © 2016 Barnard College

Question

- What list produces this histogram?

1, 2, 2, 3, 3
3, 4, 4, 5



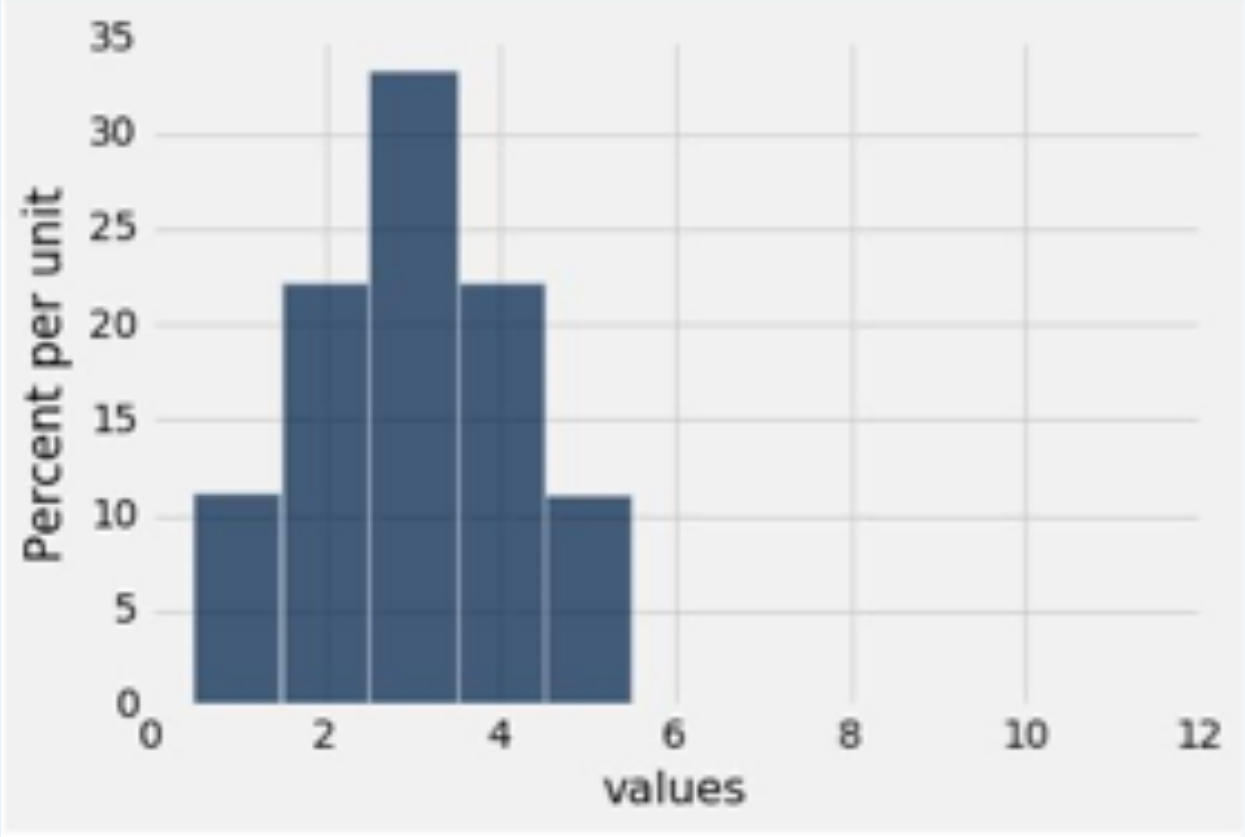
- Average?
  - 3
- Median?

Copyright © 2016 Barnard College

Question

- What list produces this histogram?

1, 2, 2, 3, 3
3, 4, 4, 5

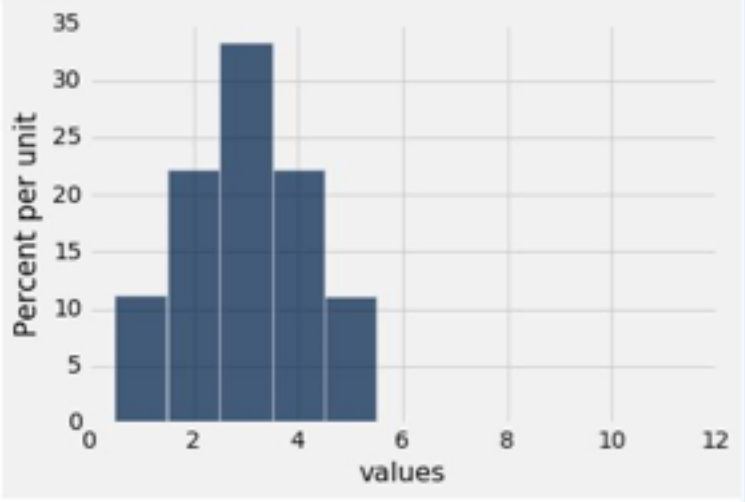


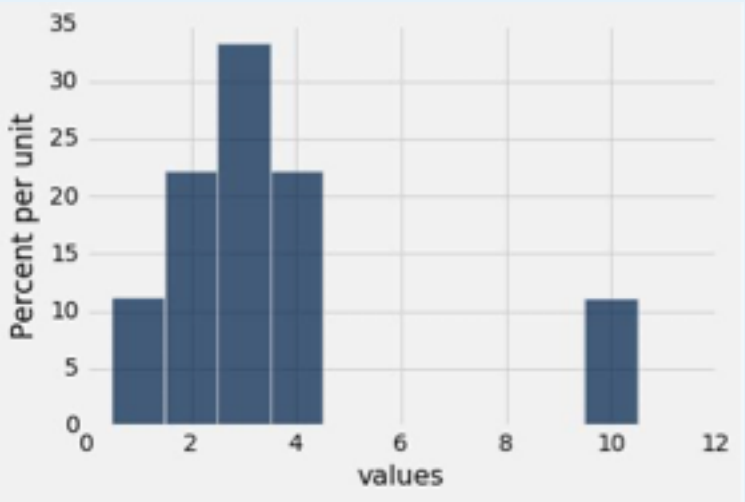
- Average?
  - 3
- Median?
  - 3

Copyright © 2016 Barnard College

Question 2

- Are the medians of these two distributions the same or different? Are the means the same or different? If you say “different,” then say which one is bigger





Copyright © 2016 Barnard College

Answer 2

- List 1
  - 1, 2, 2, 3, 3, 3, 4, 4, 5
- List 2
  - 1, 2, 2, 3, 3, 3, 4, 4, 10
- Medians = 3
- Mean(List1) = 3
- Mean (List 2) = 3.55556

Copyright © 2016 Barnard College

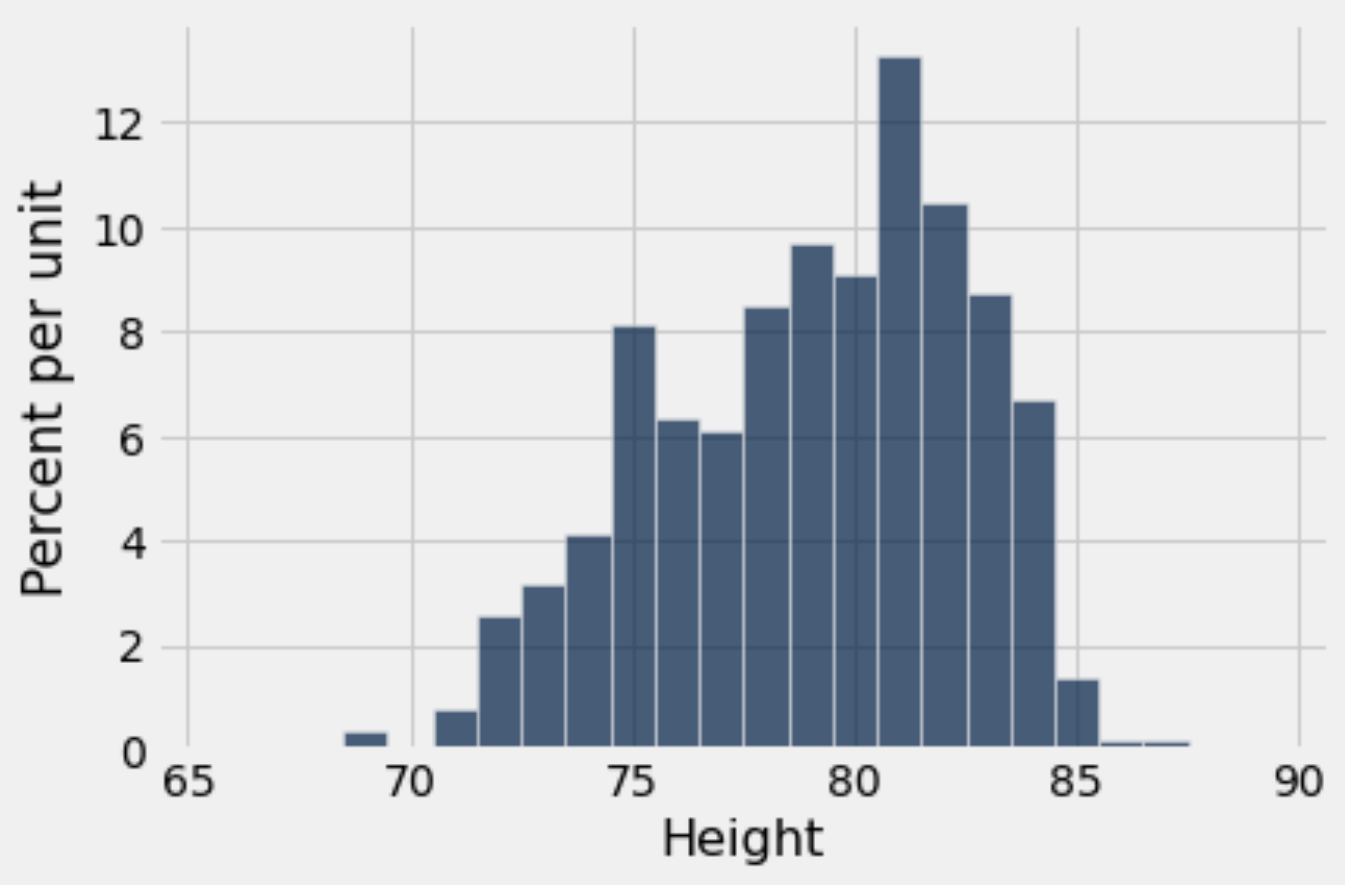
Comparing Mean and Median

- **Mean:** Balance point of the histogram
- **Median:** Half-way point of data; half the area of histogram is on either side of median
- If the distribution is symmetric about a value, then that value is both the average and the median.
- If the histogram is skewed, then the mean is pulled away from the median in the direction of the tail.

Copyright © 2016 Barnard College

Question

- Which is bigger, median or mean?



Copyright © 2016 Barnard College

Standard Deviation

Copyright © 2016 Barnard College

Defining Variability

- **Plan A:** “biggest value - smallest value”
  - Doesn’t tell us much about the shape of the distribution
- **Plan B**:
  - Measure variability around the mean
  - Need to figure out a way to quantify this

Copyright © 2016 Barnard College

How far from the average?

- Standard deviation (SD) measures roughly how far the data are from their average
- SD = root mean square of deviations from average

Steps: 5 4 3 2 1

- SD has the same units as the data

Copyright © 2016 Barnard College

Why use Standard Deviation

- There are two main reasons.
- **The first reason:**
  - No matter what the shape of the distribution, the bulk of the data are in the range “average plus or minus a few SDs”
- **The second reason:**
  - Relation with the bellshaped curve
  - Discuss this later in the lecture

Copyright © 2016 Barnard College

Chebyshev's Inequality

Copyright © 2016 Barnard College

How big are most values?

No matter what the shape of the distribution, the bulk of the data are in the range "average ± a few SDs"

Chebyshev's Inequality

No matter what the shape of the distribution, the proportion of values in the range "average ± *z* SDs" is

at least $1 - 1/z^2$

Copyright © 2016 Barnard College

Chebyshev's Bounds

the proportion of values in the range "average ± *z* SDs" is at least 1 - 1/*z*2

Range	Proportion

Copyright © 2016 Barnard College

Chebyshev's Bounds

the proportion of values in the range "average ± *z* SDs" is at least 1 - 1/*z*2

Range	Proportion
average ± 2 SDs	at least 1 - 1/4 (75%)

Copyright © 2016 Barnard College

Chebyshev's Bounds

the proportion of values in the range "average ± *z* SDs" is at least 1 - 1/*z*2

Range	Proportion
average ± 2 SDs	at least 1 - 1/4 (75%)
average ± 3 SDs	at least 1 - 1/9 (88.888...%)

Copyright © 2016 Barnard College

Chebyshev's Bounds

the proportion of values in the range "average ± *z* SDs" is at least 1 - 1/*z*2

Range	Proportion
average ± 2 SDs	at least 1 - 1/4 (75%)
average ± 3 SDs	at least 1 - 1/9 (88.888...%)
average ± 4 SDs	at least 1 - 1/16 (93.75%)

Copyright © 2016 Barnard College

Chebyshev's Bounds

the proportion of values in the range "average ± *z* SDs" is at least 1 - 1/*z*2

Range	Proportion
average ± 2 SDs	at least 1 - 1/4 (75%)
average ± 3 SDs	at least 1 - 1/9 (88.888...%)
average ± 4 SDs	at least 1 - 1/16 (93.75%)
average ± 5 SDs	at least 1 - 1/25 (96%)

True no matter what the distribution looks like

Copyright © 2016 Barnard College

Understanding HW05 Results

Statistics:
Minimum: 7.5
Maximum: 29.0
Mean: 24.55
Median: 25.0
Standard Deviation: 3.96

- At least 50% of the class had scores between 20.59 and 28.51

- At least 75% of the class had scores between 16.62 and 32.47

Copyright © 2016 Barnard College

Standard Units

Copyright © 2016 Barnard College

Standard Units

- How many SDs above average?
- ***z* = (value - average)/SD**
  - Negative z: value below average
  - Positive z: value above average
  - z = 0: value equal to average
- When values are in standard units: average = 0, SD = 1
- Chebyshev: At least 96% of the values of *z* are between -5 and 5

Copyright © 2016 Barnard College

Question

What whole numbers are closest to

(1) Average age

(2) The SD of ages

Age in Years	Age in Standard Units
27	-0.0392546
33	0.992496
28	0.132704
23	-0.727088
25	-0.383171
33	0.992496
23	-0.727088
25	-0.383171
30	0.476621
27	-0.0392546

Copyright © 2016 Barnard College

Answers

(1) Average age is close to 27 (standard unit here is close to 0)

(2) The SD is about 6 years (standard unit at 33 is close to 1. 33 – 27 = 6)

Age in Years	Age in Standard Units
27	-0.0392546
33	0.992496
28	0.132704
23	-0.727088
25	-0.383171
33	0.992496
23	-0.727088
25	-0.383171
30	0.476621
27	-0.0392546

Copyright © 2016 Barnard College

The SD and the Histogram

- Usually, it's not easy to estimate the SD by looking at a histogram.

- But if the histogram has a bell shape, then you can

Copyright © 2016 Barnard College

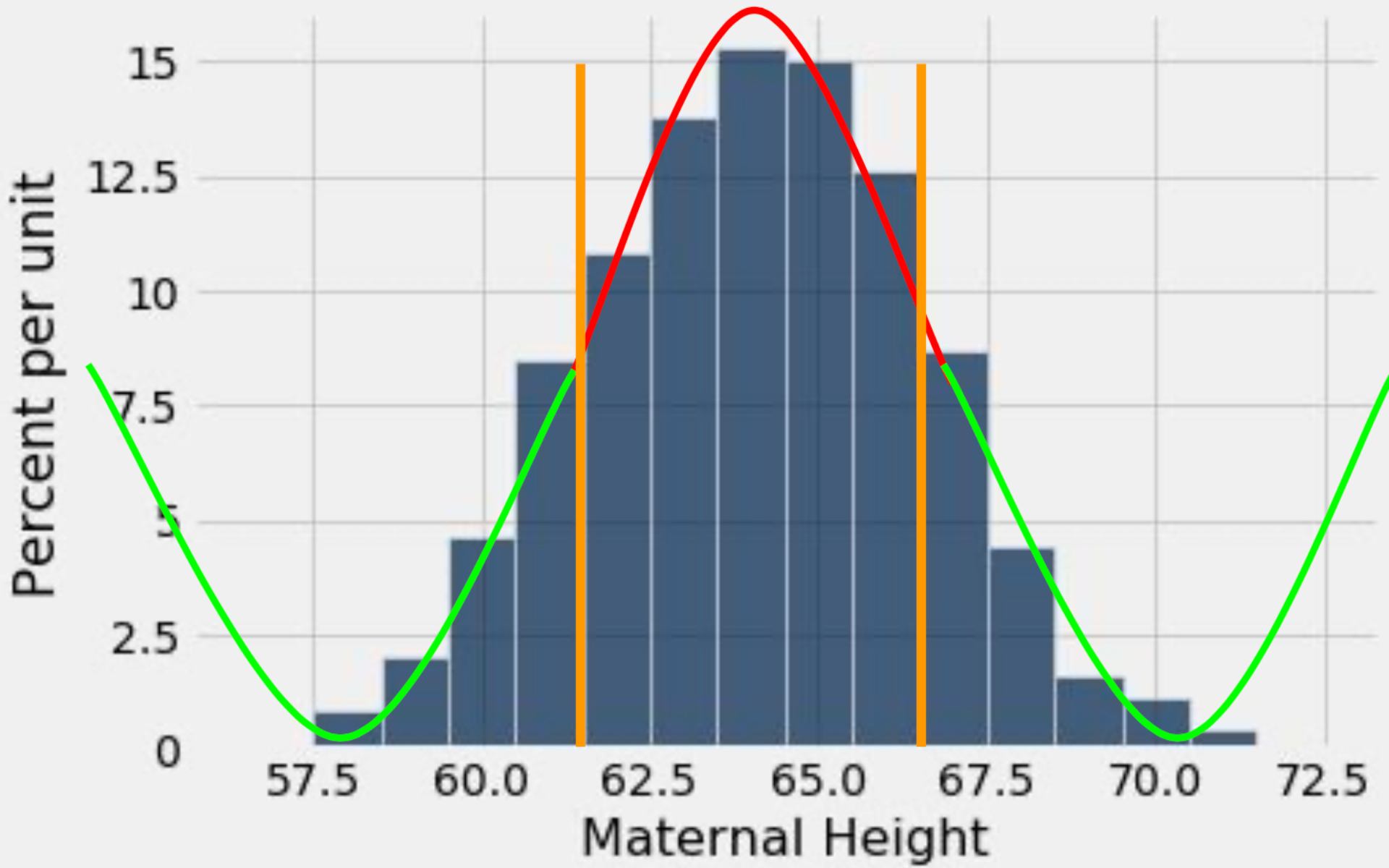
The SD and Bell Shaped Curves

If a histogram is bell-shaped, then

- the average is at the center
- the SD is the distance between the average and the points of inflection on either side

Copyright © 2016 Barnard College

Points of Inflection



Normal Distribution

Copyright © 2016 Barnard College

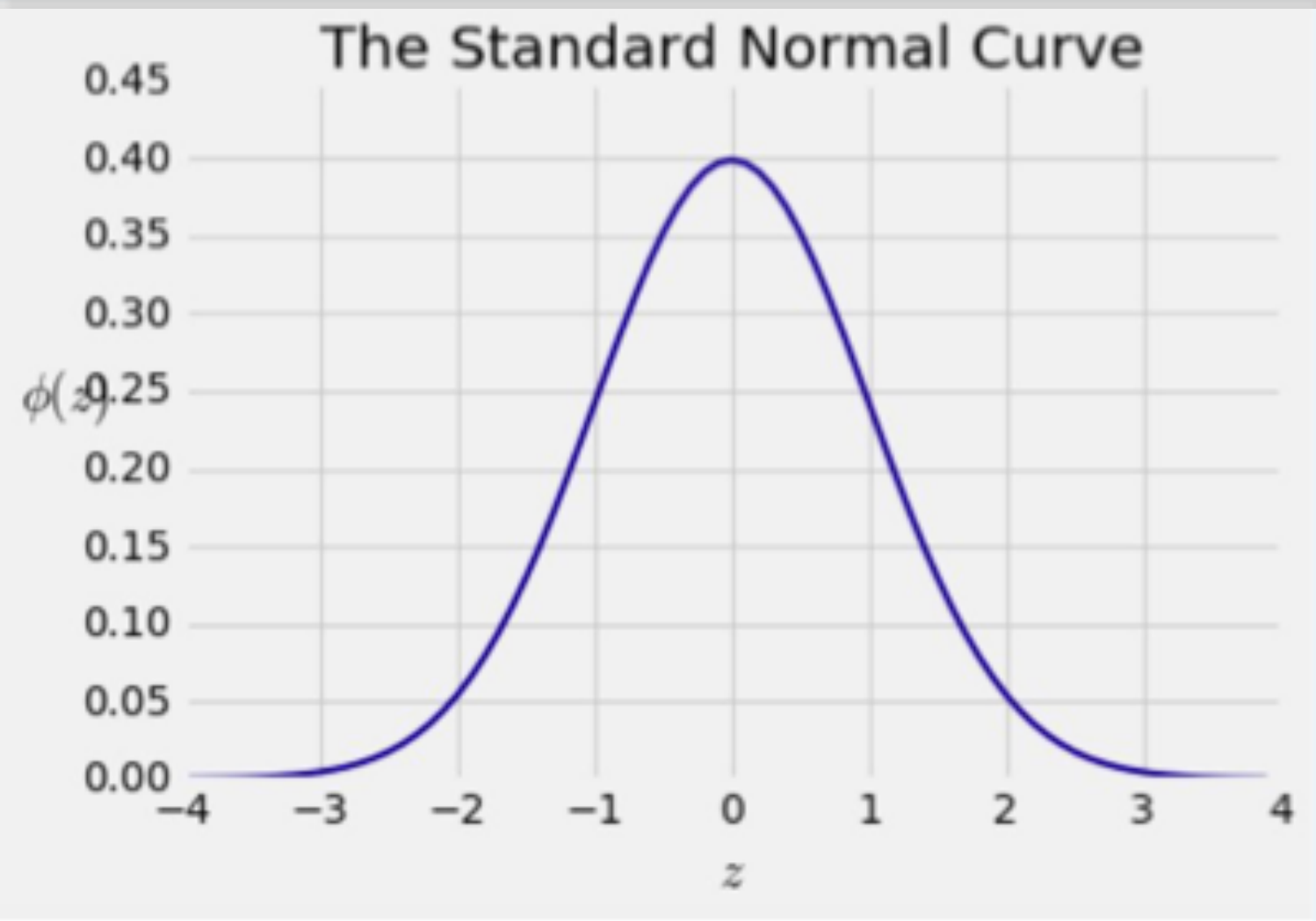
Standard Normal Curve

Equation for the normal curve

$$\phi(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}, \qquad -\infty < z < \infty$$

Copyright © 2016 Barnard College

Bell Curve



Copyright © 2016 Barnard College

How Big are Most of the Values

No matter what the shape of the distribution,

the bulk of the data are in the range "average ± a few SDs"

If a histogram is bell-shaped, then

- Almost all of the data are in the range "average ± 3 SDs

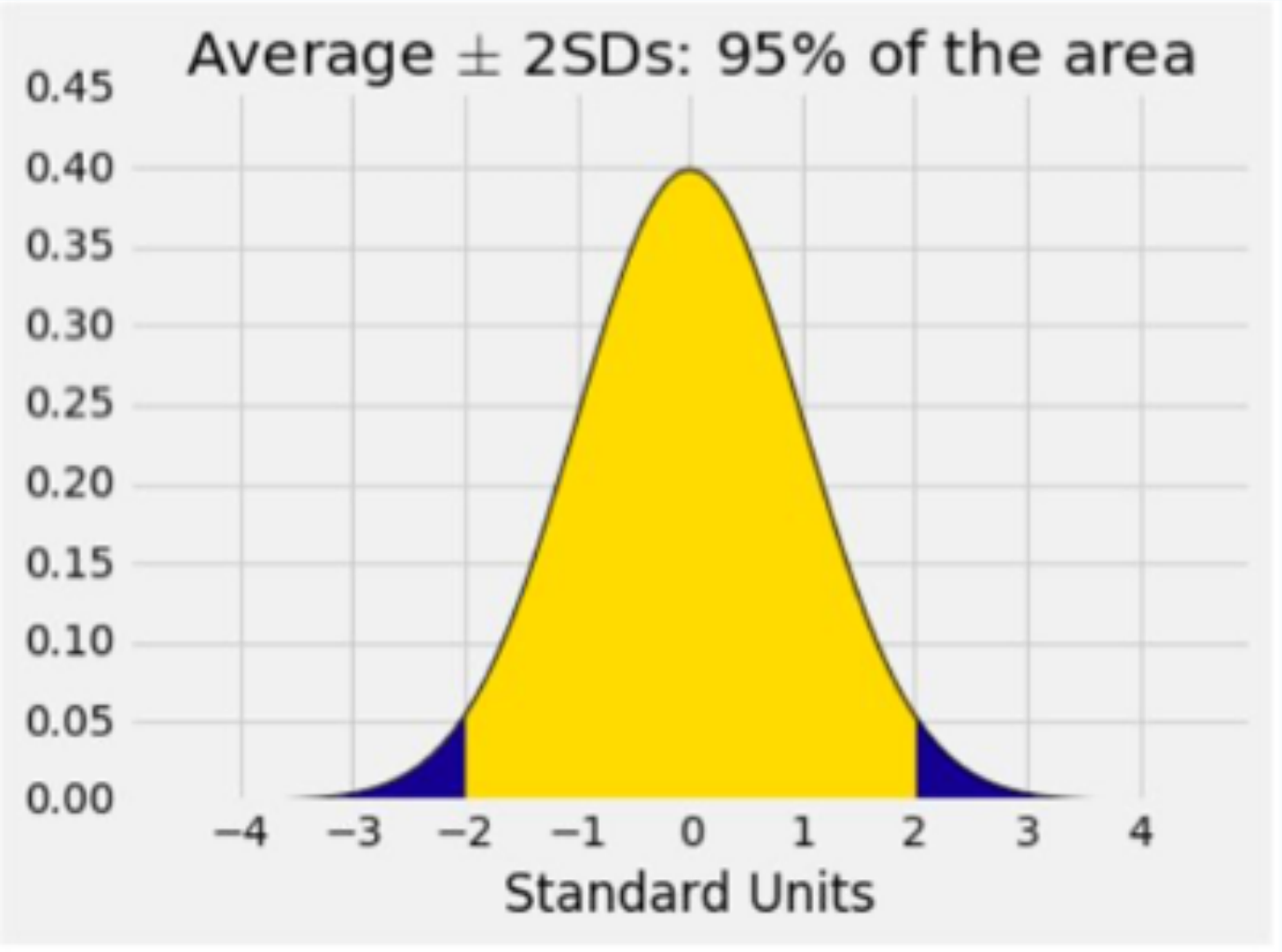
Copyright © 2016 Barnard College

Bounds and Approximations

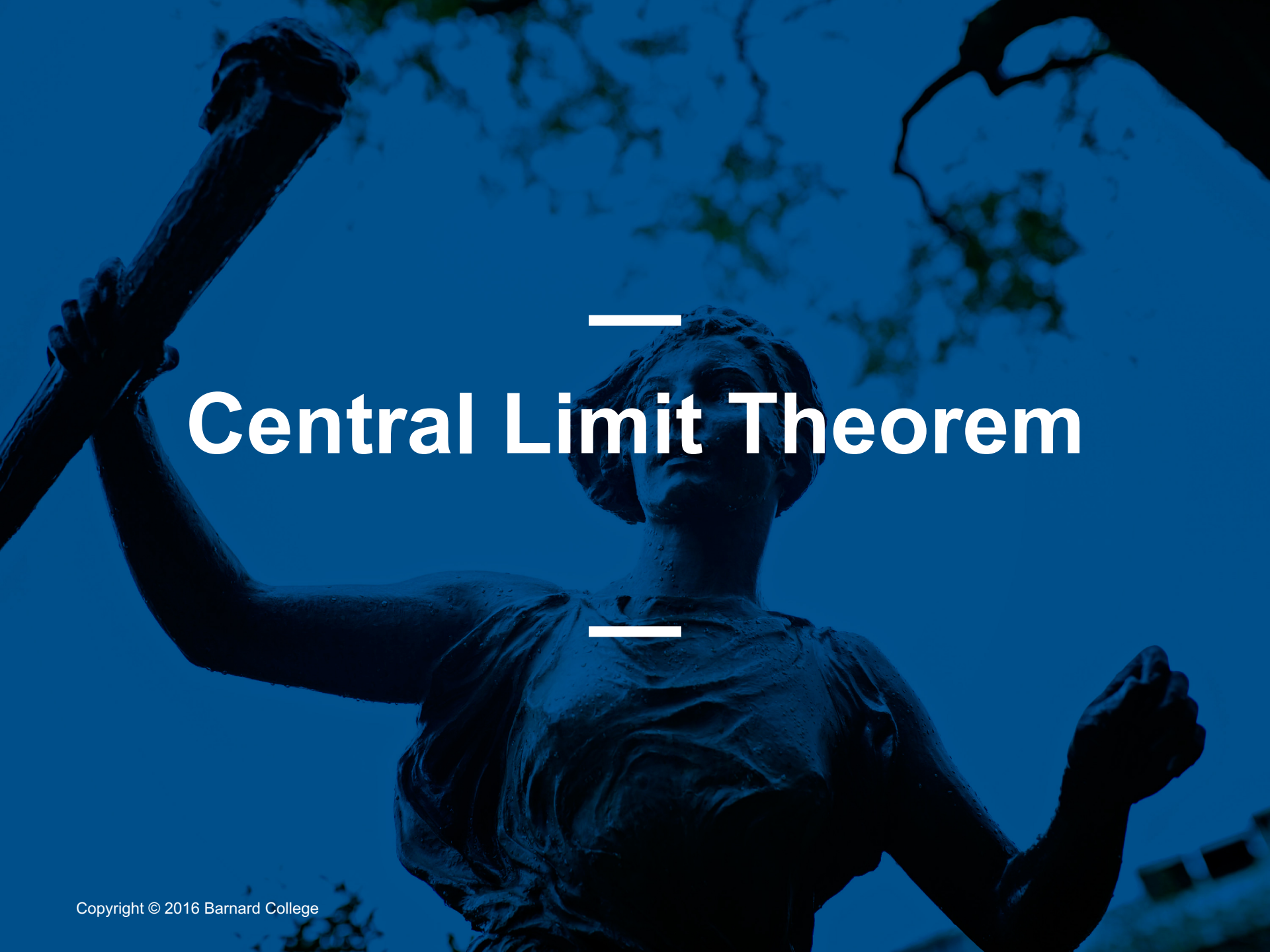
Percent in Range	All Distributions	Normal Distributions
Average +- 1 SD	At least 0%	About 68%
Average +- 2 SDs	At least 75%	About 95%
Average +- 3 SDs	At least 88.888…%	About 99.73%

Copyright © 2016 Barnard College

A “Central” Area



Copyright © 2016 Barnard College



Central Limit Theorem

Copyright © 2016 Barnard College

Central Limit Theorem

If the sample is

- large, and
- drawn at random with replacement,

Then, *regardless of the distribution of the population,*

the probability distribution of the sample sum (or the sample average) is roughly normal

Copyright © 2016 Barnard College

Sample Average

- We often only have a sample
- We care about sample averages because they estimate population averages.
- The Central Limit Theorem describes how the normal distribution (a bell-shaped curve) is connected to random sample averages.
- CLT allows us to make inferences based on averages of random samples

Copyright © 2016 Barnard College

Correlation

Copyright © 2016 Barnard College

Prediction

- To predict the value of a variable:
  - Identify (measurable) attributes that are associated with that variable
  - Describe the relation between the attributes and the variable you want to predict
  - Then, use the relation to predict the value of a variable

Copyright © 2016 Barnard College

Visualizing Two Numerical Variables

- Trend
  - Positive association
  - Negative association
- Pattern
  - Any discernible “shape” in the scatter
  - Linear
  - Non-linear

Visualize, then quantify

Copyright © 2016 Barnard College

The Correlation Coefficient *r*

- Measures **linear** association
- Based on standard units
- $-1 \leq r \leq 1$
  - $r = 1$: scatter is perfect straight line sloping up
  - $r = -1$: scatter is perfect straight line sloping down
- $r = 0$: No linear association; *uncorrelated*

Copyright © 2016 Barnard College