Package 'BioGSP'

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Type Package

Title Biological Graph Signal Processing for Spatial Data Analysis

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Description Implementation of Graph Signal Processing (GSP) methods including Spectral
Graph Wavelet Transform (SGWT) for analyzing spatial patterns in biological data.
Based on Hammond, Vandergheynst, and Gribonval (2011) ``Wavelets on Graphs via
Spectral Graph Theory''. Provides tools for multi-scale analysis of spatial signals,
including forward and inverse transforms, energy analysis, and visualization functions
tailored for biological applications.

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Encoding UTF-8

RoxygenNote 7.2.3

Imports Matrix, igraph, RANN, RSpectra, ggplot2, patchwork, ggpubr,
viridis, methods

Suggests spdep, spatialEco, sp, gasper, egg, testthat (>= 2.1.0),
remotes, dplyr, tidyr, knitr, rmarkdown

Depends R (>= 3.5.0)

NeedsCompilation no

Contents

BioGSP-package   *BioGSP: Biological Graph Signal Processing for Spatial Data Analysis*

Description

The BioGSP package provides a comprehensive implementation of Graph Signal Processing (GSP) methods including Spectral Graph Wavelet Transform (SGWT) for analyzing spatial patterns in biological data. This implementation is based on Hammond, Vandergheynst, and Gribonval (2011) "Wavelets on Graphs via Spectral Graph Theory".

Details

The package enables multi-scale analysis of spatial signals by:

- Building graphs from spatial coordinates using k-nearest neighbors
- Computing graph Laplacian eigendecomposition for spectral analysis
- Designing wavelets in the spectral domain using various kernel functions
- Decomposing signals into scaling and wavelet components at multiple scales
- Providing reconstruction capabilities with error analysis
- Offering comprehensive visualization and analysis tools

Main Functions

SGWT Main function for SGWT analysis

sgwt_forward Forward SGWT transform

sgwt_inverse Inverse SGWT transform

sgwt_energy_analysis Energy distribution analysis

plot_sgwt_decomposition Visualization of SGWT components

Cal_GCC Graph Cross-Correlation analysis

demo_sgwt Demonstration with synthetic data

Applications

The BioGSP package is particularly useful for:

- Spatial biology: Analyzing cell distribution patterns in tissue imaging (CODEX, Visium, etc.)

- Single-cell genomics: Spatial transcriptomics and proteomics analysis

- Neuroscience: Brain connectivity and signal analysis

- Pathology: Tumor microenvironment and tissue architecture analysis

- Developmental biology: Spatial pattern formation and cell fate mapping

- Immunology: Immune cell spatial organization and interactions

Author(s)

BioGSP Development Team

References

Hammond, D. K., Vandergheynst, P., & Gribonval, R. (2011). Wavelets on graphs via spectral graph theory. Applied and Computational Harmonic Analysis, 30(2), 129-150.

Examples

```
## Not run:
# Load the package
library(BioGSP)

# Run a quick demo
demo_result <- demo_sgwt()

# Generate synthetic data
set.seed(123)
n <- 100
data <- data.frame(
  x = runif(n, 0, 10),
  y = runif(n, 0, 10),
  signal = sin(runif(n, 0, 2*pi))
)

# Apply SGWT
result <- SGWT(data, signal = "signal", k = 8, J = 4)

# Analyze results
energy_analysis <- sgwt_energy_analysis(result)
print(energy_analysis)

## End(Not run)
```

Cal_Eigen   *Calculate eigenvalues and eigenvectors with knee detection*

Description

Calculate eigenvalues and eigenvectors of a spatial graph with automatic detection of the low-frequency cutoff using knee detection

Usage

```
Cal_Eigen(
  data.in = NULL,
  x_col = "x",
  y_col = "y",
  k = 25,
  k_fold = 15,
  sensitivity = 2
)
```

Arguments

data.in	Data frame with spatial coordinates
x_col	Character string specifying the column name for X coordinates (default: "x")
y_col	Character string specifying the column name for Y coordinates (default: "y")
k	Number of nearest neighbors (default: 25)
k_fold	Eigendecomposition parameter (default: 15)
sensitivity	Sensitivity parameter for knee detection (default: 2)

Value

List containing knee point, eigenvectors, and eigenvalues

Examples

```
## Not run:
# Create spatial data
data <- data.frame(x = runif(100), y = runif(100))
result <- Cal_Eigen(data, k = 10)

# With custom column names
data2 <- data.frame(X = runif(100), Y = runif(100))
result2 <- Cal_Eigen(data2, x_col = "X", y_col = "Y", k = 10)

## End(Not run)
```

Cal_GCC *Calculate Graph Cross-Correlation (GCC) - DEPRECATED*

Description

DEPRECATED: This function is deprecated. Use sgwt_similarity instead for more comprehensive signal similarity analysis with energy normalization and advanced features.

Calculate Graph Cross-Correlation between two signals using Graph Fourier Transform. This is a simplified approach that only considers low-frequency GFT components.

Usage

```
Cal_GCC(
  data.in = NULL,
  knee = NULL,
  signal1 = NULL,
  signal2 = NULL,
  eigenvector = NULL
)
```

Arguments

data.in	Data frame containing the signals
knee	Knee point for frequency cutoff
signal1	Name of first signal column
signal2	Name of second signal column
eigenvector	Matrix of eigenvectors

Value

Cosine similarity value

Examples

```
## Not run:
# DEPRECATED - use sgwt_similarity instead
# gcc_value <- Cal_GCC(data, knee = 10, signal1 = "sig1", signal2 = "sig2", eigenvector =

# NEW RECOMMENDED APPROACH:
# sgwt1 <- SGWT(data, signal = "signal1", k = 25, J = 4)
# sgwt2 <- SGWT(data, signal = "signal2", k = 25, J = 4)
# similarity <- sgwt_similarity(sgwt1, sgwt2)

## End(Not run)
```

---

cal_laplacian   *Calculate Graph Laplacian Matrix*

Description

Compute unnormalized, normalized, or random-walk Laplacian from an adjacency matrix.

Usage

```
cal_laplacian(W, type = c("unnormalized", "normalized", "randomwalk"))
```

Arguments

W	A square adjacency matrix (can be dense or sparse).
type	Type of Laplacian to compute: "unnormalized", "normalized", or "randomwalk".

Value

Laplacian matrix of the same class as input.

Examples

```
## Not run:
W <- matrix(c(0, 1, 1, 1, 0, 1, 1, 1, 0), nrow = 3)
cal_laplacian(W, type = "normalized")

## End(Not run)
```

---

codex_toy_data   *Toy CODEX Spatial Cell Type Data*

Description

A synthetic dataset mimicking CODEX multiplexed imaging data for demonstrating Spectral Graph Wavelet Transform (SGWT) analysis on spatial cell type distributions. The dataset contains spatial coordinates and cell type annotations for multiple immune cell populations arranged in realistic spatial clusters.

Usage

```
data(codex_toy_data)
```

Format

A data frame with 18604 rows and 5 columns:

cellLabel Character. Unique identifier for each cell

Y_cent Numeric. Y coordinate of cell centroid (0-115 range)

X_cent Numeric. X coordinate of cell centroid (0-116 range)

Annotation5 Character. Full descriptive cell type name

ROI_num Character. Region of interest identifier ("ROI_0" through "ROI_15")

Details

The dataset contains 16 regions of interest (ROI_0 through ROI_15) with different spatial patterns and varying cell counts (945-1497 cells per ROI). Each ROI represents a distinct tissue region with unique spatial arrangements of the same cell types.

ROI Distribution:

- **ROI_0**: 952 cells

- **ROI_1**: 945 cells

- **ROI_2**: 1155 cells

- **ROI_3**: 1421 cells

- **ROI_4**: 1096 cells

- **ROI_5**: 1420 cells

- **ROI_6-ROI_15**: 958-1497 cells each

Cell types across all ROIs include:

- **BCL6- B Cell** (~3719 cells): Primary B cell population

- **CD4 T** (~4092 cells): Helper T cells - largest population

- **CD8 T** (~3346 cells): Cytotoxic T cells

- **DC** (~2233 cells): Dendritic cells

- **M1** (~1490 cells): M1 macrophages

- **CD4 Treg** (~1490 cells): Regulatory T cells

- **BCL6+ B Cell** (~931 cells): Activated B cells

- **Endothelial** (~746 cells): Vascular cells

- **M2** (~370 cells): M2 macrophages

- **Myeloid** (~186 cells): Other myeloid cells

- **Other** (~1 cells): Miscellaneous cell types

This synthetic data is designed to demonstrate:

- Spatial clustering patterns of different cell types

- Multi-scale spatial analysis using SGWT

- Cross-cell type correlation analysis

- Graph construction and eigenvalue analysis

- Wavelet decomposition of spatial signals

Source

Generated synthetically using clustered normal distributions with realistic parameters based on real CODEX data characteristics.

Examples

```
# Load the toy dataset
data(codex_toy_data)

# Examine the structure
str(codex_toy_data)
head(codex_toy_data)

# Summary of cell types
table(codex_toy_data$Annotation5)

# Summary by ROI
table(codex_toy_data$ROI_num)
table(codex_toy_data$ROI_num, codex_toy_data$Annotation5)

# Quick visualization of spatial distribution
if (requireNamespace("ggplot2", quietly = TRUE)) {
  library(ggplot2)
  ggplot(codex_toy_data, aes(x = X_cent, y = Y_cent, color = Annotation5)) +
    geom_point(size = 0.8, alpha = 0.7) +
    facet_wrap(~ROI_num, scales = "free") +
    labs(title = "Toy CODEX Spatial Cell Distribution by ROI",
         x = "X Coordinate", y = "Y Coordinate") +
    theme_minimal() +
    scale_y_reverse()
}

# Basic SGWT analysis example
## Not run:
# Focus on BCL6- B Cell cells in ROI_1 for SGWT analysis
bcl6nb_data <- codex_toy_data[codex_toy_data$Annotation5 == "BCL6- B Cell" &
                              codex_toy_data$ROI_num == "ROI_1", ]

# Create binned representation
library(dplyr)
binned_data <- codex_toy_data %>%
  filter(Annotation5 == "BCL6- B Cell", ROI_num == "ROI_1") %>%
  mutate(
    x_bin = cut(X_cent, breaks = 20, labels = FALSE),
    y_bin = cut(Y_cent, breaks = 20, labels = FALSE)
  ) %>%
  group_by(x_bin, y_bin) %>%
  summarise(cell_count = n(), .groups = 'drop')

# Prepare for SGWT
complete_grid <- expand.grid(x_bin = 1:20, y_bin = 1:20)
sgwt_data <- complete_grid %>%
  left_join(binned_data, by = c("x_bin", "y_bin")) %>%
  mutate(
    cell_count = ifelse(is.na(cell_count), 0, cell_count),
    x = x_bin,
    y = y_bin,
    signal = cell_count / max(cell_count, na.rm = TRUE)
  ) %>%
  select(x, y, signal)
```

```
# Apply SGWT
sgwt_result <- SGWT(data.in = sgwt_data,
                    signal = "signal",
                    k = 8,
                    J = 3,
                    kernel_type = "mexican_hat")

## End(Not run)
```

```
compare_kernel_families
```
Compare different kernel families

Description

Visualize and compare different kernel families (both scaling and wavelet filters)

Usage

```
compare_kernel_families(
  x_range = c(0, 3),
  scale_param = 1,
  plot_results = TRUE
)
```

Arguments

x_range   Range of x values to evaluate (default: c(0, 3))

scale_param   Scale parameter for all functions (default: 1)

plot_results   Whether to plot the comparison (default: TRUE)

Value

Data frame with x values and kernel values for each family

Examples

```
comparison <- compare_kernel_families()
comparison <- compare_kernel_families(x_range = c(0, 5), scale_param = 1.5)
```

```
compute_sgwt_filters
```
Compute SGWT filters

Description

Compute wavelet and scaling function coefficients in the spectral domain

Usage

```
compute_sgwt_filters(
  eigenvalues,
  scales,
  lmax = NULL,
  kernel_type = "mexican_hat"
)
```

Arguments

`eigenvalues`	Eigenvalues of the graph Laplacian
`scales`	Vector of scales for the wavelets
`lmax`	Maximum eigenvalue (optional)
`kernel_type`	Kernel family that defines both scaling and wavelet filters (default: "mexican_hat", options: "mexican_hat", "meyer", "heat")

Value

List of filters (scaling function + wavelets)

Examples

```
eigenvals <- c(0, 0.1, 0.5, 1.0, 1.5)
scales <- c(2, 1, 0.5)
filters <- compute_sgwt_filters(eigenvals, scales)
filters_meyer <- compute_sgwt_filters(eigenvals, scales, kernel_type = "meyer")
filters_heat <- compute_sgwt_filters(eigenvals, scales, kernel_type = "heat")
```

```
cosine_similarity
```
Calculate cosine similarity between two vectors

Description

Calculate cosine similarity between two numeric vectors with numerical stability

Usage

```
cosine_similarity(x, y, eps = 1e-12)
```

Arguments

x	First vector
y	Second vector
eps	Small numeric for numerical stability when norms are near zero (default 1e-12)

Value

Cosine similarity value (between -1 and 1)

Examples

```
x <- c(1, 2, 3)
y <- c(2, 3, 4)
similarity <- cosine_similarity(x, y)
# With custom eps for numerical stability
similarity2 <- cosine_similarity(x, y, eps = 1e-10)
```

---

demo_sgwt	*Demo function for SGWT*

Description

Demonstration function showing basic SGWT usage with synthetic data

Usage

```
demo_sgwt()
```

Value

List containing demo data, SGWT results, and energy analysis

Examples

```
## Not run:
demo_result <- demo_sgwt()
print(demo_result$energy)

## End(Not run)
```

```
FastDecompositionLap
```
Fast eigendecomposition of Laplacian matrix

Description

Perform fast eigendecomposition using RSpectra for large matrices

Usage

```
FastDecompositionLap(
  laplacianMat = NULL,
  k_fold = 1.5,
  which = "LM",
  sigma = NULL,
  opts = list(),
  lower = TRUE,
  ...
)
```

Arguments

laplacianMat	Laplacian matrix
k_fold	Multiplier for number of eigenvalues to compute (default: 1.5)
which	Which eigenvalues to compute ("LM", "SM", etc.)
sigma	Shift parameter for eigenvalue computation
opts	Additional options for eigenvalue computation
lower	Whether to compute from lower end of spectrum
...	Additional arguments

Value

List with eigenvalues (evalues) and eigenvectors (evectors)

Examples

```
## Not run:
# Create a Laplacian matrix and decompose
L <- matrix(c(2, -1, -1, -1, 2, -1, -1, -1, 2), nrow = 3)
decomp <- FastDecompositionLap(L, k_fold = 2)

## End(Not run)
```

---

find_knee_point *Find knee point in a curve*

Description

Simple knee point detection using the maximum curvature method

Usage

```
find_knee_point(y, sensitivity = 1)
```

Arguments

y   Numeric vector of y values

sensitivity   Sensitivity parameter (not used in this simple implementation)

Value

Index of the knee point

Examples

```
y <- c(1, 2, 3, 10, 11, 12)  # curve with a knee
knee_idx <- find_knee_point(y)
```

---

gft *Graph Fourier Transform*

Description

Compute the Graph Fourier Transform (GFT) of a signal using Laplacian eigenvectors.

Usage

```
gft(signal, U)
```

Arguments

signal   Input signal (vector or matrix)

U   Matrix of eigenvectors (dense matrix preferred)

Value

Transformed signal in the spectral domain (vector or matrix)

---

hello_sgwt   *Hello function for SGWT package demonstration*

Description

Simple hello function to demonstrate package loading

Usage

```
hello_sgwt()
```

Value

Character string with greeting

Examples

```
hello_sgwt()
```

---

install_and_load   *Install and load packages*

Description

Utility function to install and load packages from CRAN or GitHub

Usage

```
install_and_load(packages)
```

Arguments

packages   Named vector where names are package names and values are source URLs

Value

NULL (side effect: installs and loads packages)

Examples

```
## Not run:
packages <- c("ggplot2" = "ggplot2", "devtools" = "r-lib/devtools")
install_and_load(packages)

## End(Not run)
```

```
plot_FM                          Plot frequency modes
```

Description

Plot frequency modes from graph Fourier analysis

Usage

```
plot_FM(input = NULL, FM_idx = c(1:20), ncol = 5)
```

Arguments

input	Input data (currently not used, for future compatibility)
FM_idx	Indices of frequency modes to plot (default: 1:20)
ncol	Number of columns in plot layout (default: 5)

Value

Combined plot of frequency modes

Examples

```
## Not run:
# This function requires specific data structure (df_hex_combine)
# plot_FM(FM_idx = 1:10, ncol = 5)

## End(Not run)
```

```
plot_sgwt_decomposition
                    Plot SGWT decomposition results
```

Description

Visualize SGWT decomposition components including original signal, scaling function, wavelet coefficients, and reconstructed signal

Usage

```
plot_sgwt_decomposition(
  sgwt_result,
  data.in,
  x_col = "x",
  y_col = "y",
  plot_scales = NULL,
  ncol = 3
)
```

Arguments

`sgwt_result`	SGWT result object from SGWT() function
`data.in`	Original data frame with spatial coordinates
`x_col`	Character string specifying the column name for X coordinates (default: "x")
`y_col`	Character string specifying the column name for Y coordinates (default: "y")
`plot_scales`	Which wavelet scales to plot (default: first 4)
`ncol`	Number of columns in the plot layout (default: 3)

Value

ggplot object with combined plots

Examples

```
## Not run:
# Assuming you have SGWT results
plots <- plot_sgwt_decomposition(sgwt_result, data.in)
print(plots)

# With custom column names
plots2 <- plot_sgwt_decomposition(sgwt_result, data.in, x_col = "X", y_col = "Y")
print(plots2)

## End(Not run)
```

SGWT   *Spectral Graph Wavelet Transform (SGWT)*

Description

Main function for performing Spectral Graph Wavelet Transform analysis on spatial data. Provides a comprehensive interface for multi-scale analysis of spatial signals using graph wavelets.

Usage

```
SGWT(
  data.in = NULL,
  x_col = "x",
  y_col = "y",
  signal = NULL,
  k = 25,
  scales = NULL,
  J = 5,
  scaling_factor = 2,
  kernel_type = "mexican_hat",
  laplacian_type = "normalized",
  k_fold = 15,
  return_all = TRUE
)
```

Arguments

`data.in`	Data frame containing spatial coordinates and signal data.
`x_col`	Character string specifying the column name for X coordinates (default: "x")
`y_col`	Character string specifying the column name for Y coordinates (default: "y")
`signal`	Character string specifying the column name of the signal to analyze, or a numeric vector of signal values.
`k`	Number of nearest neighbors for graph construction (default: 25)
`scales`	Vector of scales for the wavelets. If NULL, scales are auto-generated.
`J`	Number of scales to generate if scales is NULL (default: 5)
`scaling_factor`	Scaling factor between consecutive scales (default: 2)
`kernel_type`	Kernel family ("mexican_hat", "meyer", or "heat") that defines both scaling and wavelet filters
`laplacian_type`	Type of graph Laplacian ("unnormalized", "normalized", or "randomwalk", default: "normalized")
`k_fold`	Parameter for eigendecomposition (default: 15)
`return_all`	Whether to return all analysis results (default: TRUE)

Value

If return_all = TRUE, returns a list containing:

decomposition SGWT decomposition results

reconstructed_signal Reconstructed signal for validation

reconstruction_error RMSE between original and reconstructed signal

original_signal Original input signal

graph_info Graph construction information (adjacency matrix, Laplacian, eigenvalues, eigenvectors)

data Original input data

parameters Analysis parameters used

If return_all = FALSE, returns only the SGWT decomposition.

References

Hammond, D. K., Vandergheynst, P., & Gribonval, R. (2011). Wavelets on graphs via spectral graph theory. Applied and Computational Harmonic Analysis, 30(2), 129-150.

Examples

```
## Not run:
# Generate synthetic spatial data
set.seed(123)
n_points <- 100
x_coords <- rep(1:10, each = 10) + rnorm(n_points, 0, 0.1)
y_coords <- rep(1:10, times = 10) + rnorm(n_points, 0, 0.1)
signal_data <- sin(0.5 * x_coords) * cos(0.3 * y_coords) + rnorm(n_points, 0, 0.1)
```

```
demo_data <- data.frame(x = x_coords, y = y_coords, signal = signal_data)

# Apply SGWT
result <- SGWT(data.in = demo_data, signal = "signal", k = 8, J = 4)

# With custom column names
demo_data2 <- data.frame(X = x_coords, Y = y_coords, signal_1 = signal_data)
result2 <- SGWT(data.in = demo_data2, x_col = "X", y_col = "Y", signal = "signal_1", k =

# View reconstruction error
print(result$reconstruction_error)

## End(Not run)
```

---

sgwt-globals   *Global variables used in ggplot2 aesthetics*

Description

This file declares global variables used in ggplot2 aesthetics to avoid R CMD check NOTEs about undefined global functions or variables.

---

sgwt_auto_scales   *Generate automatic scales for SGWT*

Description

Generate logarithmically spaced scales for SGWT

Usage

```
sgwt_auto_scales(lmax, J = 5, scaling_factor = 2)
```

Arguments

lmax	Maximum eigenvalue
J	Number of scales
scaling_factor	
	Scaling factor between consecutive scales

Value

Vector of scales

Examples

```
scales <- sgwt_auto_scales(lmax = 2.0, J = 5, scaling_factor = 2)
```

```
sgwt_energy_analysis
```
Analyze SGWT energy distribution across scales

Description

Calculate and analyze energy distribution across different scales in the SGWT decomposition

Usage

```
sgwt_energy_analysis(sgwt_result)
```

Arguments

```
sgwt_result    SGWT result object from SGWT() function
```

Value

Data frame with energy analysis results

Examples

```
## Not run:
# Assuming you have SGWT results
energy_analysis <- sgwt_energy_analysis(sgwt_result)
print(energy_analysis)

## End(Not run)
```

```
sgwt_forward
```
Forward SGWT transform

Description

Decompose signal into wavelet coefficients using SGWT

Usage

```
sgwt_forward(
  signal,
  eigenvectors,
  eigenvalues,
  scales,
  lmax = NULL,
  kernel_type = "mexican_hat"
)
```

Arguments

`signal`	Input signal vector
`eigenvectors`	Eigenvectors of the graph Laplacian
`eigenvalues`	Eigenvalues of the graph Laplacian
`scales`	Vector of scales for the wavelets
`lmax`	Maximum eigenvalue (optional)
`kernel_type`	Kernel family that defines both scaling and wavelet filters (default: "mexican_hat", options: "mexican_hat", "meyer", "heat")

Value

List containing coefficients, filters, scales, eigenvalues, and eigenvectors

Examples

```
## Not run:
# Assuming you have eigenvalues, eigenvectors, and a signal
result <- sgwt_forward(signal, eigenvectors, eigenvalues, scales)
result_meyer <- sgwt_forward(signal, eigenvectors, eigenvalues, scales, kernel_type = "me
result_heat <- sgwt_forward(signal, eigenvectors, eigenvalues, scales, kernel_type = "hea

## End(Not run)
```

---

sgwt_get_kernels   *Get a unified kernel family (low-pass and band-pass) by kernel_type*

Description

Returns a pair of functions implementing the scaling (low-pass) and wavelet (band-pass) kernels for a given kernel family. This enforces consistency: a single kernel_type defines both filters.

Usage

```
sgwt_get_kernels(kernel_type = "mexican_hat")
```

Arguments

`kernel_type`	Kernel family name ("mexican_hat", "meyer", or "heat")

Value

A list with two functions: list(scaling = function(x, scale_param), wavelet = function(x, scale_param))

---

sgwt_inverse   *Inverse SGWT transform*

Description

Reconstruct signal from wavelet coefficients

Usage

```
sgwt_inverse(sgwt_decomp)
```

Arguments

sgwt_decomp   SGWT decomposition object from sgwt_forward

Value

Reconstructed signal vector

Examples

```
## Not run:
# Assuming you have an SGWT decomposition
reconstructed <- sgwt_inverse(sgwt_decomp)

## End(Not run)
```

---

sgwt_similarity   *Comprehensive Signal Similarity Analysis*

Description

Compute similarity between two signals using either raw signals (via SGWT) or pre-computed SGWT decompositions. This function provides a unified interface for signal similarity analysis with energy normalization and advanced features.

Usage

```
sgwt_similarity(
  signal1,
  signal2,
  data.in = NULL,
  x_col = "x",
  y_col = "y",
  k = 25,
  J = 4,
  kernel_type = "mexican_hat",
  eps = 1e-12,
  low_only = FALSE,
  return_parts = FALSE,
  ...
)
```

Arguments

`signal1`	Either a character string (column name in data.in), numeric vector, or SGWT result object
`signal2`	Either a character string (column name in data.in), numeric vector, or SGWT result object
`data.in`	Data frame containing signals (required if signal1/signal2 are column names)
`x_col`	Column name for X coordinates (default: "x")
`y_col`	Column name for Y coordinates (default: "y")
`k`	Number of nearest neighbors for graph construction (default: 25)
`J`	Number of wavelet scales (default: 4)
`kernel_type`	Wavelet kernel type (default: "mexican_hat")
`eps`	Numerical stability parameter (default: 1e-12)
`low_only`	If TRUE, use only low-frequency similarity (default: FALSE)
`return_parts`	If TRUE, return detailed components; if FALSE, return scalar similarity (default: FALSE)
`...`	Additional arguments passed to SGWT()

Value

Similarity score (scalar) or detailed similarity analysis (list) depending on return_parts

Examples

```
## Not run:
# Method 1: Direct signals in data frame
data <- data.frame(x = runif(100), y = runif(100),
                 signal1 = rnorm(100), signal2 = rnorm(100))
sim1 <- sgwt_similarity("signal1", "signal2", data.in = data)

# Method 2: Pre-computed SGWT results
sgwt1 <- SGWT(data, signal = "signal1", k = 25, J = 4)
sgwt2 <- SGWT(data, signal = "signal2", k = 25, J = 4)
sim2 <- sgwt_similarity(sgwt1, sgwt2)

# Method 3: Mixed - one SGWT result, one raw signal
sim3 <- sgwt_similarity(sgwt1, "signal2", data.in = data)

## End(Not run)
```

---

sgwt_weighted_similarity
Energy-normalized weighted similarity between two SGWT results

---

Description

Compute low-frequency cosine similarity (scaling), non-low cosine similarity (flattened wavelet coefficients), and an overall energy-normalized weighted score. If 'low_only = TRUE', compute only the low-frequency cosine and set 'S = c_low'.

Usage

```
sgwt_weighted_similarity(
  sgwt_a,
  sgwt_b,
  eps = 1e-12,
  validate = TRUE,
  return_parts = TRUE,
  low_only = FALSE
)
```

Arguments

`sgwt_a`	SGWT output for signal A. Either the full list returned by SGWT (with '$de-composition') or a decomposition list as returned by 'sgwt_forward()'.
`sgwt_b`	SGWT output for signal B. Same format as 'sgwt_a'.
`eps`	Small numeric for numerical stability when norms are near zero (default 1e-12).
`validate`	Logical; if TRUE, check consistency of dimensions, scale count/order, and kernel family.
`return_parts`	Logical; if TRUE (default), return a list with components; if FALSE, return only the scalar S.
`low_only`	Logical; if TRUE, compute **low-frequency-only** similarity (skip non-low and set 'S = c_low').

Value

If 'return_parts=TRUE', a list with: * 'c_low' — cosine on scaling coefficients * 'c_nonlow' — cosine on flattened wavelet coefficients (**NA if 'low_only = TRUE'**) * 'w_low' — macro weight for the low-frequency part (in [0,1]) * 'w_NL' — 1 - w_low (non-low weight) * 'S' — final weighted similarity in [-1,1] * 'E_low_a', 'E_NL_a', 'E_low_b', 'E_NL_b' — energy diagnostics per signal (**E_NL_* = NA if 'low_only'**) * 'n', 'J' — length of signal and number of wavelet scales (**J = NA if 'low_only'**) If 'return_parts=FALSE', returns the scalar 'S'.

Examples

```
## Not run:
# Assume two SGWT results sgwt_res1 and sgwt_res2 from SGWT(..., return_all=TRUE)
sim <- sgwt_weighted_similarity(sgwt_res1, sgwt_res2)
sim_low <- sgwt_weighted_similarity(sgwt_res1, sgwt_res2, low_only = TRUE)
str(sim)

## End(Not run)
```

```
simulate_multiscale
```
Simulate Multiple Center Patterns

Description

Generate spatial patterns with multiple circular centers at different scales. Creates concentric circle patterns with inner circle A and outer ring B at various radius combinations.

Usage

```
simulate_multiscale(
  grid_size = 60,
  n_centers = 3,
  Ra_seq = c(10, 5, 1),
  Rb_seq = c(10, 5, 1),
  seed = 123
)
```

Arguments

grid_size	Size of the spatial grid (default: 60)
n_centers	Number of pattern centers to generate (default: 3)
Ra_seq	Vector of inner circle radii (default: c(10, 5, 1))
Rb_seq	Vector of outer ring radii (default: c(10, 5, 1))
seed	Random seed for reproducible center placement (default: 123)

Value

List of data frames, each containing X, Y coordinates and circleA, circleB binary signals

Examples

```
## Not run:
# Generate multi-center patterns with default parameters
patterns <- simulate_multiscale()

# Custom parameters
Ra_seq <- seq(from = 10, to = 3, length.out = 6)
Rb_seq <- seq(from = 20, to = 3, length.out = 6)
patterns <- simulate_multiscale(Ra_seq = Ra_seq, Rb_seq = Rb_seq, n_centers = 3)

## End(Not run)
```

```
simulate_ringpattern
```
                              Simulate Concentric Ring Patterns

---

Description

Generate concentric ring patterns with dynamic outer ring movement. Creates a solid inner circle with a moving outer ring that closes in over time.

Usage

```
simulate_ringpattern(
  grid_size = 60,
  radius_seq = seq(2.5, 20, by = 2.5),
  n_movements = 10,
  center_x = NULL,
  center_y = NULL
)
```

Arguments

`grid_size`	Size of the spatial grid (default: 60)
`radius_seq`	Vector of inner circle radii to simulate (default: seq(2.5, 20, by = 2.5))
`n_movements`	Number of movement steps for the outer ring (default: 10)
`center_x`	X coordinate of pattern center (default: grid_size/2)
`center_y`	Y coordinate of pattern center (default: grid_size/2)

Value

List of data frames, each containing X, Y coordinates, movement indicators, and signal_1 (solid circle), signal_2 (concentric ring) binary signals

Examples

```
## Not run:
# Generate concentric ring patterns with default parameters
patterns <- simulate_ringpattern()

# Custom parameters
radius_seq <- seq(2.5, 20, by = 2.5)
patterns <- simulate_ringpattern(radius_seq = radius_seq, n_movements = 5)

## End(Not run)
```

`visualize_multiscale`

Visualize Multiple Center Simulation Results

Description

Create visualization plots for multiple center simulation patterns

Usage

```
visualize_multiscale(
  sim_data,
  Ra_seq,
  Rb_seq,
  bg_color = "grey",
  signal1_color = "red",
  signal2_color = "blue"
)
```

Arguments

`sim_data`	Output from simulate_multiscale function
`Ra_seq`	Vector of Ra values used in simulation
`Rb_seq`	Vector of Rb values used in simulation
`bg_color`	Background color for visualization (default: "grey")

```
signal1_color
                Color for signal 1 visualization (default: "red")
signal2_color
                Color for signal 2 visualization (default: "blue")
```

Value

Combined ggplot object with all pattern visualizations

Examples

```
## Not run:
# Generate and visualize patterns
Ra_seq <- seq(from = 10, to = 3, length.out = 6)
Rb_seq <- seq(from = 20, to = 3, length.out = 6)
sim_data <- simulate_multiscale(Ra_seq = Ra_seq, Rb_seq = Rb_seq, n_centers = 3)
plot_grid <- visualize_multiscale(sim_data, Ra_seq, Rb_seq)
print(plot_grid)

## End(Not run)
```

```
visualize_ringpattern
```
                    Visualize Concentric Ring Simulation Results

---

Description

Create visualization plots for concentric ring simulation patterns

Usage

```
visualize_ringpattern(
  sim_data,
  radius_seq,
  bg_color = "grey",
  signal1_color = "#16964a",
  signal2_color = "#2958a8"
)
```

Arguments

sim_data	Output from simulate_ringpattern function
radius_seq	Vector of radius values used in simulation
bg_color	Background color for visualization (default: "grey")
signal1_color	
	Color for signal 1 visualization (default: "#16964a")
signal2_color	
	Color for signal 2 visualization (default: "#2958a8")

Value

Combined ggplot object with all pattern visualizations

Examples

```
## Not run:
# Generate and visualize patterns
radius_seq <- seq(2.5, 20, by = 2.5)
sim_data <- simulate_ringpattern(radius_seq = radius_seq)
plot_grid <- visualize_ringpattern(sim_data, radius_seq)
print(plot_grid)

## End(Not run)
```

```
visualize_sgwt_kernels
```
Visualize SGWT kernels and scaling functions

Description

Visualize the scaling function and wavelet kernels used in SGWT based on the eigenvalue spectrum and selected parameters

Usage

```
visualize_sgwt_kernels(
  eigenvalues,
  scales = NULL,
  J = 4,
  scaling_factor = 2,
  kernel_type = "mexican_hat",
  lmax = NULL,
  eigenvalue_range = NULL,
  resolution = 1000
)
```

Arguments

eigenvalues	Vector of eigenvalues from graph Laplacian
scales	Vector of scales for the wavelets (if NULL, auto-generated)
J	Number of scales to generate if scales is NULL (default: 4)
scaling_factor	
	Scaling factor between consecutive scales (default: 2)
kernel_type	Type of wavelet kernel ("mexican_hat" or "meyer", default: "mexican_hat")
lmax	Maximum eigenvalue (optional, computed if NULL)
eigenvalue_range	
	Range of eigenvalues to plot (default: full range)
resolution	Number of points for smooth curve plotting (default: 1000)

Value

List containing the filter visualization plot and filter values

Examples

```
## Not run:
# Generate some example eigenvalues
eigenvals <- seq(0, 2, length.out = 100)

# Visualize kernels with specific parameters
viz_result <- visualize_sgwt_kernels(
  eigenvalues = eigenvals,
  J = 4,
  scaling_factor = 2,
  kernel_type = "mexican_hat"
)
print(viz_result$plot)

## End(Not run)
```

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