

**Домашнее задание по теме «Кратные интегралы и числовые ряды»
(модуль 1)**

**по дисциплине «Теория вероятностей и математическая статистика»,
ИУ1Б, 3-й семестр**

Задача 1 (2 балла). Поменять порядок интегрирования в повторном интеграле.
Выполнить рисунок области интегрирования.

| № варианта | Условие задачи |
|---------------|---|
| 1 | $\int_0^1 dx \int_{x/2}^{2x} f(x, y) dy + \int_1^2 dx \int_{x/2}^{2/x} f(x, y) dy.$ |
| 2 | $\int_0^4 dx \int_{\sqrt{4x-x^2}}^{\sqrt{16-x^2}} f(x, y) dy.$ |
| 3 | $\int_{-1}^1 dy \int_{y^2-1}^{1-y^2} f(x, y) dx.$ |
| 4 | $\int_0^2 dx \int_{-\sqrt{4-x^2}}^{\sqrt{2x-x^2}} f(x, y) dy.$ |
| 5 | $\int_2^4 dy \int_{y/2}^y f(x, y) dx.$ |

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| 6 | $\int_0^3 dx \int_{x^2}^{3+2x} f(x, y) dy.$ |
| 7 | $\int_{-\sqrt{2}}^{\sqrt{2}} dy \int_{y^2-1}^{y^2/2} f(x, y) dx.$ |
| 8 | $\int_1^2 dx \int_{2/x}^{2x} f(x, y) dy.$ |
| 9 | $\int_0^4 dx \int_{-\sqrt{4x-x^2}}^{\sqrt{16-x^2}} f(x, y) dy.$ |
| 10 | $\int_0^1 dx \int_{-1+\sqrt{2x-x^2}}^{1-\sqrt{2x-x^2}} f(x, y) dy.$ |
| 11 | $\int_{-8/3}^0 dy \int_{-2(y+1)}^{\sqrt{4+y^2}} f(x, y) dx.$ |

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| 12 | $\int_0^2 dx \int_{-\sqrt{4x-x^2}}^0 f(x, y) dy.$ |
| 13 | $\int_{-1}^0 dy \int_{-\sqrt{-y}}^{\sqrt{y+1}} dx.$ |
| 14 | $\int_{-\sqrt{2}}^{\sqrt{2}} dy \int_{-y^2/2}^{1-y^2} f(x, y) dx.$ |
| 15 | $\int_{-\sqrt{3}}^{\sqrt{3}} dy \int_{-\sqrt{1+y^2}}^{\sqrt{1+y^2}} f(x, y) dx.$ |
| 16 | $\int_0^4 dx \int_{2-\sqrt{8-(x-2)^2}}^{\sqrt{4x-x^2}} f(x, y) dy.$ |
| 17 | $\int_{-1}^1 dx \int_{-\sqrt{2-x^2}}^x f(x, y) dy + \int_1^{\sqrt{2}} dx \int_{-\sqrt{2-x^2}}^{\sqrt{2-x^2}} f(x, y) dy.$ |

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| 18 | $\int_0^2 dx \int_{-\sqrt{4-x^2}}^{\sqrt{4x-x^2}-2} f(x, y) dy.$ |
| 19 | $\int_{-4}^{-2} dx \int_{-\sqrt{-x^2-4x}}^{\sqrt{-x^2-4x}} f(x, y) dy + \int_{-2}^{\sqrt{8}} dx \int_{-\sqrt{8-x^2}}^{\sqrt{8-x^2}} f(x, y) dy.$ |
| 20 | $\int_0^1 dx \int_{2x-1}^{(x+1)/2} f(x, y) dy.$ |
| 21 | $\int_{-2}^0 dx \int_{-x-2}^{\sqrt{-x}} f(x, y) dy + \int_0^2 dx \int_{x-2}^{\sqrt{x}} f(x, y) dy.$ |
| 22 | $\int_{-2}^2 dx \int_{-2+\sqrt{4-x^2}}^{2+\sqrt{4-x^2}} f(x, y) dy.$ |
| 23 | $\int_0^1 dy \int_{2y-1}^{(y+1)/2} f(x, y) dx.$ |

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| 24 | $\int_{-4}^0 dx \int_{-\sqrt{-x}}^{2-x} f(x, y) dy.$ |
| 25 | $\int_{-\sqrt{3}}^0 dx \int_{-\sqrt{1+x^2}}^{\sqrt{1+x^2}} f(x, y) dy.$ |
| 26 | $\int_{-3}^0 dx \int_0^{3+x} f(x, y) dy + \int_0^3 dx \int_{2x}^{3+x} f(x, y) dy.$ |
| 27 | $\int_0^2 dx \int_{-\sqrt{4x-x^2}}^0 f(x, y) dy.$ |
| 28 | $\int_{-1}^0 dy \int_{-\sqrt{-y}}^{\sqrt{y+1}} dx.$ |

Задача 2 (2 балла). Вычислить объем тела, ограниченного поверхностями. Выполнить рисунок.

| № варианта | Условие задачи |
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| 1 | $x^2 + y^2 + z^2 = 5, z = x^2 + y^2 + 1$ (внутри параболоида). |
| 2 | $x^2 + y^2 - z^2 = 9, z = 0, z = 4.$ |
| 3 | $z = x^2 + y^2; (x - 1)^2 + y^2 = 1; z = 0.$ |
| 4 | $x^2 + y^2 = 1, x^2 + y^2 = 4, z = 0; x + y + z = 4.$ |
| 5 | $(x - 1)^2 + y^2 = 1; z = 0; x + y + z = 4.$ |
| 6 | $x^2 + y^2 + z^2 = 16, z = \sqrt{7}, z = 2\sqrt{3}.$ |
| 7 | $z = 9 - x^2 - y^2, z = 0, x^2 + y^2 = 4$ (вне цилиндра). |
| 8 | $x^2 + y^2 - z^2 = 4; x^2 + y^2 = 9.$ |
| 9 | $x^2 + (y - 2)^2 = 4; z = 0; z = 6 - x.$ |

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| 10 | $x^2 + y^2 + z^2 = 6; z = x^2 + y^2$ (внутри параболоида). |
| 11 | $z = \sqrt{x^2 + y^2 + 1}; z = \sqrt{3 - x^2 - y^2}.$ |
| 12 | $x^2 + y^2 = 1; x^2 + y^2 + z^2 = 4$ (вне цилиндра). |
| 13 | $z = 6; z = 10 - x^2 - y^2.$ |
| 14 | $z = 5 - x^2 - y^2; z = 1.$ |
| 15 | $z = x^2 + y^2; z = 1; z = 4.$ |
| 16 | $x^2 + y^2 - z^2 = 4; x^2 + y^2 = 9.$ |
| 17 | $z = x^2 + y^2; (x - 1)^2 + y^2 = 1; z = 0.$ |
| 18 | $z = 5 - x^2 - y^2; z = 1.$ |
| 19 | $x^2 + y^2 + z^2 = 16, z = \sqrt{7}, z = 2\sqrt{3}.$ |
| 20 | $z = 6; z = 10 - x^2 - y^2.$ |

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| 21 | $x^2 + y^2 + z^2 = 6; z = x^2 + y^2$ <p>(внутри параболоида).</p> |
| 22 | $(x - 1)^2 + y^2 = 1; z = 0; x + y + z = 4.$ |
| 23 | $x^2 + y^2 - z^2 = 9, z = 0, z = 4.$ |
| 24 | $x^2 + y^2 = 1; x^2 + y^2 + z^2 = 4$ <p>(вне цилиндра).</p> |
| 25 | $x^2 + y^2 + z^2 = 5, z = x^2 + y^2 + 1$ <p>(внутри параболоида).</p> |
| 26 | $z = x^2 + y^2; z = 1; z = 4.$ |
| 27 | $z = \sqrt{x^2 + y^2 + 1}; z = \sqrt{3 - x^2 - y^2}.$ |
| 28 | $z = 9 - x^2 - y^2, z = 0, x^2 + y^2 = 4$ <p>(вне цилиндра).</p> |

Задача 3 (3 балла). Исследовать сходимость рядов. В случае знакопеременного ряда исследовать на абсолютную и условную сходимость.

| № вар. | Условие задачи | | |
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| 1 | $\sum_{n=1}^{\infty} \frac{(n-1)^3}{n^4 + 3n^2 + 2}$ | $\sum_{n=1}^{\infty} \frac{2^n n!}{n^n}$ | $\sum_{n=1}^{\infty} (-1)^n \left(\frac{5n}{5n+2} \right)$ |
| 2 | $\sum_{n=1}^{\infty} \frac{n^3}{n^{17} + n^2}$ | $\sum_{n=1}^{\infty} n \operatorname{arctg} \frac{1}{n}$ | $\sum_{n=1}^{\infty} (-1)^n \frac{1000 \cdot 1002 \cdot 1004 \dots (998 + 2n)}{1 \cdot 4 \cdot 7 \dots (3n - 2)}$ |
| 3 | $\sum_{n=1}^{\infty} (-1)^n \frac{n+1}{\sqrt{n^3}}$ | $\sum_{n=1}^{\infty} \left(\frac{7n-1}{7n+2} \right)^n$ | $\sum_{n=1}^{\infty} \frac{1 \cdot 4 \cdot 7 \dots (3n-2)}{(2n+5)!}$ |

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| 4 | $\sum_{n=1}^{\infty} \frac{3n^2 - 5}{2n^2 + 1}$ | $\sum_{n=1}^{\infty} \frac{2^n}{n}$ | $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{\sqrt{n}} \sin \frac{1}{n}$ |
| 5 | $\sum_{n=1}^{\infty} \frac{1}{n} \operatorname{tg} \frac{1}{\sqrt{n}}$ | $\sum_{n=1}^{\infty} \frac{n^3}{(n+1)!}$ | $\sum_{n=1}^{\infty} (-1)^{n-1} \left(\frac{3n}{3n+1} \right)^{n^2}$ |
| 6 | $\sum_{n=1}^{\infty} \left(\frac{n}{n+3} \right)^{n^2}$ | $\sum_{n=2}^{\infty} \frac{\ln n}{n}$ | $\sum_{n=1}^{\infty} (-1)^{n-1} \sin \frac{\pi}{n+2}$ |
| 7 | $\sum_{n=1}^{\infty} \cos \frac{1}{n}$ | $\sum_{n=2}^{\infty} \frac{2n+19}{\sqrt{n^2-1}}$ | $\sum_{n=1}^{\infty} (-1)^n \frac{(2n)!}{(n!)^2}$ |

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| 8 | $\sum_{n=1}^{\infty} n \ln \frac{n^3 + 3}{n^3}$ | $\sum_{n=1}^{\infty} \frac{(2n)! 2^n}{(n!)^2 \cdot 3^n}$ | $\sum_{n=1}^{\infty} (-1)^n (2n-1)^2 \operatorname{tg} \frac{\pi}{n^2}$ |
| 9 | $\sum_{n=1}^{\infty} (-1)^{n-1} \left(1 - \cos \frac{1}{\sqrt{n}}\right)$ | $\sum_{n=1}^{\infty} \left(\frac{4n+1}{5n+2}\right)^{n/2}$ | $\sum_{n=1}^{\infty} \frac{1000 \cdot 1002 \cdot 1004 \dots (998 + 2n)}{1 \cdot 4 \cdot 7 \dots (3n-2)}$ |
| 10 | $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{2n^2 + 15}{7n^2}$ | $\sum_{n=1}^{\infty} \left(\frac{n-1}{n+1}\right)^{n^2}$ | $\sum_{n=1}^{\infty} \frac{1 \cdot 3 \dots (2n-1)}{5^n \cdot n!}$ |
| 11 | $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \operatorname{arctg} \frac{5}{\sqrt{n}}$ | $\sum_{n=1}^{\infty} \frac{e^{-\sqrt{n}}}{n}$ | $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1 \cdot 11 \cdot 21 \dots (10n-9)}{(2n-1)!}$ |

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| 12 | $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n}} \operatorname{arctg} \frac{1}{\sqrt[5]{n^4 + 1}}$ | $\sum_{n=1}^{\infty} \frac{n^5}{3^n}$ | $\sum_{n=1}^{\infty} (-1)^{n-1} \left(\frac{3n}{3n-3} \right)^{2n^2}$ |
| 13 | $\sum_{n=0}^{\infty} \frac{1}{\sqrt{3n+1}}$ | $\sum_{n=1}^{\infty} \frac{(n!)^3}{(3n)!}$ | $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} e^{-\sqrt{n}}}{\sqrt{n}}$ |
| 14 | $\sum_{n=1}^{\infty} (-1)^n n \operatorname{arctg} \frac{1}{n}$ | $\sum_{n=1}^{\infty} \left(\frac{5n}{n+3} \right)^n$ | $\sum_{n=1}^{\infty} \frac{(2n)!}{(3n+4)3^n}$ |
| 15 | $\sum_{n=0}^{\infty} \frac{2 \cdot 5 \cdot \dots (3n+2)}{(n+2)!}$ | $\sum_{n=2}^{\infty} \frac{\ln n}{n}$ | $\sum_{n=1}^{\infty} (-1)^n \left(\frac{5n-1}{5n+1} \right)^n$ |

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| 16 | $\sum_{n=1}^{\infty} \sqrt[3]{n^2} \arcsin^2 \frac{1}{\sqrt[3]{3n}}$ | $\sum_{n=1}^{\infty} \frac{5^{n-1}}{(n-1)!}$ | $\sum_{n=1}^{\infty} (-1)^{n-1} \left(\frac{n+3}{n+4} \right)^{n^2};$ |
| 17 | $\sum_{n=1}^{\infty} \left(1 - \cos \frac{1}{\sqrt{n}} \right)$ | $\sum_{n=1}^{\infty} \frac{3n^2 - 5}{2n^2 + 1}$ | $\sum_{n=1}^{\infty} (-1)^{n-1} \left(\frac{1}{3n+1} \right)^n$ |
| 18 | $\sum_{n=1}^{\infty} \left(\frac{5n}{5n+2} \right)^n$ | $\sum_{n=2}^{\infty} \frac{(-1)^{n-1}}{(\ln n)^n}$ | $\sum_{n=1}^{\infty} \frac{1 \cdot 11 \cdot 21 \dots (10n-9)}{(2n-1)!}$ |
| 19 | $\sum_{n=1}^{\infty} (-1)^{n-1} n \sin \frac{1}{n}$ | $\sum_{n=1}^{\infty} \frac{2^n n!}{n^n}$ | $\sum_{n=1}^{\infty} \frac{1}{2^n} \cdot \left(\frac{n+3}{n} \right)^n$ |

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| 20 | $\sum_{n=1}^{\infty} \left(\frac{3n}{3n+1} \right)^{n^2}$ | $\sum_{n=2}^{\infty} \frac{1}{n \sqrt{\ln^3 n}}$ | $\sum_{n=2}^{\infty} (-1)^{n-1} \operatorname{arctg} \frac{1}{\sqrt{n+2}}$ |
| 21 | $\sum_{n=1}^{\infty} \frac{2^n}{7^n} \left(\frac{n}{n+1} \right)^{n^2}$ | $\sum_{n=1}^{\infty} \sqrt[3]{n} \operatorname{tg} \frac{10}{n^2}$ | $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{(3n+1)(3n+2)}$ |
| 22 | $\sum_{n=1}^{\infty} \left(\frac{n}{n+3} \right)^{n^2}$ | $\sum_{n=1}^{\infty} \ln \left(1 + \frac{1}{n} \right)$ | $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{3^n n!}{n^n}$ |
| 23 | $\sum_{n=1}^{\infty} n^2 \cdot \operatorname{tg}^5 \frac{\pi}{\sqrt{n^3}}$ | $\sum_{n=1}^{\infty} \frac{1}{5^n} \left(\frac{n+1}{n} \right)^{n^2}$ | $\sum_{n=1}^{\infty} (-1)^n \frac{3n^2 + 1}{2n^2 - 1}$ |

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| 24 | $\sum_{n=1}^{\infty} \left(\frac{4n+1}{5n+2} \right)^{n/2}$ | $\sum_{n=1}^{\infty} \frac{n^n}{n!}$ | $\sum_{n=1}^{\infty} (-1)^n \cdot n \cdot \operatorname{arctg} \frac{1}{n}$ |
| 25 | $\sum_{n=2}^{\infty} \frac{\ln n}{n(\ln^4 n + 1)}$ | $\sum_{n=1}^{\infty} \frac{n^5}{3^n}$ | $\sum_{n=1}^{\infty} (-1)^{n-1} \cdot \frac{n^2}{2n^2 + 1}$ |
| 26 | $\sum_{n=1}^{\infty} \frac{(n-1)^3}{n^4 + 3n^2 + 2}$ | $\sum_{n=1}^{\infty} \left(\frac{n+2}{n+4} \right)^{n^2}$ | $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} \cdot 3 \cdot 5 \cdot 7 \dots (2n+1)}{2 \cdot 5 \cdot 8 \dots (3n-1)}$ |
| 27 | $\sum_{n=1}^{\infty} \frac{\operatorname{arctg} n}{n^2 + 1}$ | $\sum_{n=1}^{\infty} \frac{4^n n!}{n^n}$ | $\sum_{n=1}^{\infty} (-1)^{n-1} \cdot \frac{(n-1)^3}{n^4 + 3n^2 + 2}$ |

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| 28 | $\sum_{n=1}^{\infty} 2^n \operatorname{tg} \frac{1}{3^n}$ | $\sum_{n=1}^{\infty} \left(\frac{n-1}{n+1} \right)^{n^2}$ | $\sum_{n=1}^{\infty} (-1)^{n-1} \cdot \frac{n^n}{n!}$ |
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