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2025/dms- trigonometry

Compound angle formulae

A displacement of a body from a fixed point can be expressed as  $10 \sin 2t + 0.67$  meters or  $10 \sin \omega t = 0.33$  amperes.

The angle  $2t+0.67$  and  $\omega t -0.33$  are called compound angles because they are a sum or a difference of two angles.

The compound angle formula of sine and cosine are sines of A and B are

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A-B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A+B) = \cos A \sin B - \sin A \cos B$$

$$\cos(A-B) = \cos A \sin B + \sin A \cos B$$

The above can be used to derive the compound angle formulae below

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

The compound angle formulae are true for all values of A and B into the formula they may be shown to be true

Examples :-

1. Expand and supply the following expressions

- a)  $\sin(\pi+\alpha)$
- b)  $-\cos(90^\circ+B)$
- c)  $\sin(A-B) - \sin(A+B)$

Answers:

- a)  $\sin(\pi+\alpha) = \sin \pi \cos \alpha + \cos \pi \sin \alpha$  from  $\sin(A+B)$
- b)  $-\cos(90^\circ+B) = -\cos 90^\circ \cos B - \sin 90^\circ \sin B$
- c)  $\sin(A-B) - \sin(A+B) = \sin A \cos B - \cos A \sin B - \sin A \cos B + \cos A \sin B$   
 $= -2 \cos A \sin B$

Assignment:

1. Prove that

$$\cos(y - \pi) + \sin\left(y + \frac{\pi}{2}\right) = 0$$

2. Show that

$$\tan\left(x + \frac{\pi}{4}\right)\tan\left(x - \frac{\pi}{4}\right) = -1$$

3. If  $\sin P = 0.8142$  and  $\cos Q = 0.4432$  evaluate correct to 3 decimal places

- a)  $\sin(P-Q)$
- b)  $\cos(P+Q)$
- c)  $\tan(P+Q)$

4. solve the equation  $4\sin(x-20^\circ) = 5\cos x$  for values of  $x$  between  $0^\circ$  and  $90^\circ$

5. If  $\sin P = 0.8142$  and  $\cos Q = 0.4432$  evaluate correct to 3 decimal places

- a)  $\sin(P-Q)$
  - b)  $\cos(P+Q)$
  - c)  $\tan(P+Q)$
- using compound angle formulae.

6. Solve the equation

$$4\sin(x-20^\circ) = 5\cos x$$

For values between  $0^\circ$  and  $90^\circ$