The First Meet Of Cosmology Club Of BNU

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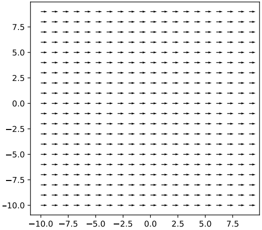
Cosmological Principle

The Cosmological Principle is the assertion that, on sufficiently large scales (beyond those traced by the large-scale structure of the distribution of galaxies), the Universe is both homogeneous and isotropic.

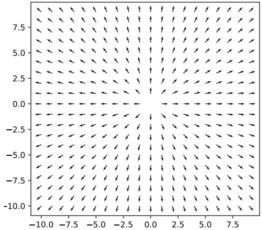
1. homogeneous the property of being identical everywhere in space
2. isotropic the property of looking the same in every direction

Cosmological Principle

1. homogeneous, but not isotropic(ex:Bianchi models)



2. isotropic, but not homogeneous(ex:LTB models)



General Relativity

1. Geometry of space-time depend on the distribution of matter

$$G_{ab} = \kappa^2 T_{ab}$$

2. isotropic, but not homogeneous(ex:LTB models)

$$\nabla_a T^{ab} = 0$$

and for free particles

$$\frac{d^2x^i}{ds^2} + T^i_{kl}\frac{dx^k}{ds}\frac{dx^l}{ds} = 0$$

1. The Robertson–Walker Metric

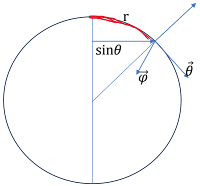
$$ds^2 = -dt^2 + a^2(\frac{dr^2}{1+Kr^2} + r^2 d\Omega^2)$$

where $K = 0$ for flat three-dimensional flat space, $K = 1$ for positively curved, $K = -1$ for negatively curved spaces.

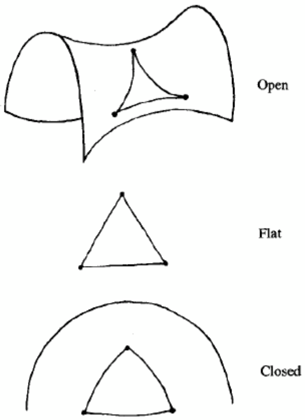
2. positively curved space, ex: 2-dimensional sphere

$$ds^2 = a^2(d\theta^2 + \sin^2\theta d\psi^2) = a^2(\frac{dr^2}{1-r^2} + r^2 d\psi^2)$$

where $r = \sin\theta$, $dr^2 = (1 - \sin^2\theta)d\theta = (1 - r^2)d\theta^2$



General Relativity



1. Christoffel symbol for flat FRW metric

$$T^0_{00} = 0 \;\; T^0_{i0} = T^0_{0i} = 0 \;\; T^0_{ij} = \delta_{ij} a\dot{a} \;\; T^i_{0j} = \delta_{ij}\frac{\dot{a}}{a} \;\; others = 0$$

   where i, j, k represent $1, 2$ or 3, and α, β represent $1, 2, 3$ or 0

2. for free particle (τ represent proper time)

$$\frac{d^2x^0}{d\tau^2} + \dot{a}a\delta_{ij}\frac{dx^i}{d\tau}\frac{dx^j}{d\tau} = 0$$

$$\frac{d^2x^i}{d\tau^2} + 2H\frac{dx^i}{d\tau}\frac{dx^0}{d\tau} = 0$$

free particle

Define Lorentz factor $\gamma = 1/\sqrt{1-u^2} = \frac{dx_0}{d\tau} = \frac{dt}{d\tau}$(natural unit system), $u^i = \frac{dx^i}{dx_0} = \frac{dx^i}{dt}, u^2 = u^i u_i$

$$
\begin{aligned}
\frac{dx^i}{d\tau} &= \gamma \frac{dx^i}{dt} = \gamma u^i \\
\frac{d^2x^i}{d\tau^2} &= \frac{d}{d\tau}(\gamma u^i) = \gamma \frac{d}{dt}(\gamma u^i) = \gamma\dot{\gamma}u^i + \gamma^2 \dot{u}^i \\
\frac{d^2x^0}{d\tau^2} &= \frac{dx^0}{d\tau}\gamma = \frac{d}{d\tau}\gamma = \frac{dt}{d\tau}\frac{d}{dt}\gamma = \gamma\frac{d}{dt} = \gamma\dot{\gamma}
\end{aligned}
$$

so

$$\dot{\gamma} = -H(\gamma - \frac{1}{\gamma})$$

$$\dot{\gamma}u^i + \gamma\dot{u}^i = -2H\gamma u^i \Rightarrow \dot{(\gamma u)} + H(\gamma u) = 0$$

For light, τ always equal to 0, so we need another parameter λ, $\dfrac{dt}{d\lambda} = E = \nu$. As a result[1]

$$\frac{d\nu}{dt} + \frac{\dot{a}}{a}\nu = 0$$

$$E \propto \frac{1}{a}$$

redshift z define as $z = \dfrac{\nu_s}{\nu_o b} - 1 = \dfrac{a_o b}{a_s} - 1$, if we define today's scale factor $a_o b = 1$, this lead

$$z = \frac{1}{a} - 1$$

[1] Modern Cosmology P31

Ideal fluid

1. For ideal fluid. $T_{ab} = (\rho + p)U_a U_b + p g_{ab}$ With $\nabla_a t^{ab} = 0$

$$\dot{\rho} + 3H(p + \rho) = 0$$

2. on the other hand, with the first law of thermodynamics

$$d(a^3\rho) + pd(a^3) = 0 \Rightarrow \dot{\rho} + 3H(p + \rho) = 0$$

3. define $p = w\rho$ for dust matter $w = 0$, for radiation $w = 1/3$, for cosmology constant $w = -1(T_\lambda \propto_{ab})$, so

$$\rho_m = \rho_{mo}(1+z)^3$$
$$\rho_r = \rho_{r0}(1+z)^4$$
$$\rho_\lambda = \rho_{\lambda 0}$$

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Friedmann equations

1. Friedmann model=GR+FRW(metric)+ideal fluid
2. Friedmann equations

$$H^2 = \frac{\dot{a}}{a} = \frac{8\pi G}{3}\rho - \frac{k}{a^2}$$
$$\dot{H} + H^2 = \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3}$$

3. consider a universe above dust matter, radiation, cosmology constant and curvature, and define $\Omega_i = \frac{8}{3H^2}\rho_i$ $,\Omega_k = -\frac{k}{H^2a^2}$, those lead

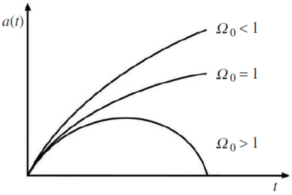
$$\Omega_m + \Omega_r + \Omega_\Lambda + \Omega_k = 1$$
$$H^2 = H_0^2(\Omega_{mo}(1+z)^3 + \Omega_{ro}(1+z)^4 + \Omega_\Lambda + \Omega_{mo}(1+z)^{-2})$$
$$q = \frac{a\ddot{a}}{\dot{a}^2} = \frac{1}{2}(\Omega_m + 2\Omega_r - 2\Omega_\Lambda)$$

Friedmann equations

1. Ignore radiation $\Omega_{r0} \sim 0$

$$\Omega_m + \Omega_\Lambda + \Omega_k = 1$$
$$H^2 = H_0^2(\Omega_{mo}(1+z)^3 + \Omega_\Lambda + \Omega_{mo}(1+z)^{-2})$$
$$q = \frac{a\ddot{a}}{\dot{a}^2} = \frac{1}{2}(\Omega_m - 2\Omega_\Lambda)$$

2. Open or close depend on $\Omega_m + \Omega_\Lambda$($> 1$ close, $= 1$ flat, < 1 open)



1. The Bianchi models
   1.1 Metric

$$
\begin{aligned}
ds^2 = &- dt^2 \\
&+ R(t)^2[e^{2a(t)}dx^2 + e^{2b(t)}dy^2 + e^{2c(t)}dz^2],
\end{aligned}
$$

   1.2 Evolution equation

$$
\begin{aligned}
&3\frac{\dot{R}^2}{R^2} + \dot{a}\dot{b} + \dot{a}\dot{c} + \dot{b}\dot{c} = 8\pi G\rho_B, \\
&2\frac{\ddot{R}}{R} + (\frac{\dot{R}}{R})^2 + 3\frac{\dot{R}}{R}(\dot{b} + \dot{c}) + \ddot{b} + \ddot{c} + \dot{b}^2 + \dot{c}^2 + \dot{b}\dot{c} \\
&= -8\pi G {T^1}_1, \\
&2\frac{\ddot{R}}{R} + (\frac{\dot{R}}{R})^2 + 3\frac{\dot{R}}{R}(\dot{a} + \dot{c}) + \ddot{a} + \ddot{c} + \dot{a}^2 + \dot{c}^2 + \dot{a}\dot{c} \\
&= -8\pi G {T^2}_2, \\
&2\frac{\ddot{R}}{R} + (\frac{\dot{R}}{R})^2 + 3\frac{\dot{R}}{R}(\dot{a} + \dot{b}) + \ddot{a} + \ddot{b} + \dot{a}^2 + \dot{b}^2 + \dot{a}\dot{b}
\end{aligned}
$$

Beyond FRW

1. The LTB
   1.1 Metric

$$ds^2 = -dt^2 + X^2(t,r)dr^2 + R^2(t,r)d\Omega^2$$

   1.2 Read "Dark energy Theory and Observations" sec 10.1

Thank you!

Backup slide

Some additional content