

Solutions

Math 321-01 Spring 2015

Quiz 3 04.02.15

Name: _____

Show all work clearly and in order, and circle your final answers. Justify your answers algebraically whenever possible; You have 20 minutes to take this 10 point quiz.

1. (5 points) Players A, B , and C toss a fair coin in order. The first to throw a head wins. What is the probability that the player C wins the game.

$$C \text{ wins} = \{ TTH, TTTTTH, TTTTTTTH, \dots \}$$

By independence we get

$$\begin{aligned} P(C \text{ wins}) &= P(TTH) + P(TTTTTH) + \dots + \\ &= \frac{1}{2^3} + \frac{1}{2^6} + \frac{1}{2^9} + \dots + \\ &= \frac{1}{8} \cdot \sum_{i=0}^{\infty} \frac{1}{8^i} = \frac{1}{8} \cdot \left(\frac{1}{1 - \frac{1}{8}} \right) = \frac{1}{7} \end{aligned}$$

2. (5 points) We toss a fair coin twice. If the outcomes of the two coin tosses are the same, we win; otherwise, we lose. Let A be the event that the first coin comes up heads, B be the event that the second coin comes up heads, and C be the event that we win. For each of the following statements decide whether it is True or False and **justify your answers!!!**.

False (i) Events A and B are not independent. $S = \{TT, HT, TH, HH\}$
 $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{2}$
 $A \cap B = \{HH\}$ thus $P(A \cap B) = \frac{1}{4} = \frac{1}{2} \cdot \frac{1}{2} = P(A) \cdot P(B)$
 Thus A & B are independent.

True (ii) Events A and C are independent.
 we need to check if $P(A \cap C) = P(A) \cdot P(C)$.
 $A \cap C = \{HH\}$ so that $P(A \cap C) = \frac{1}{4}$,
 $C = \{HH, TT\}$ thus $P(C) = \frac{2}{4}$, so $P(A \cap C) = \frac{1}{4} = P(A) \cdot P(C)$

False (iii) The probability of winning is $\frac{3}{4}$.
 $C = \{HH, TT\}$ so $P(C) = \frac{2}{4} = \frac{1}{2}$.