Chapter 26

Capacitance and Dielectrics. Solutions of Home Work Problems

26.1 Problem 26.10 *(In the text book)*

A variable air capacitor used in a radio tuning circuit is made of N semicircular plates each of radius R and positioned a distance d from its neighbors, to which it is electrically connected. As shown in Figure (26.10), a second identical set of plates is enmeshed with its plates halfway between those of the first set. The second set can rotate as a unit. Determine the capacitance as a function of the angle of rotation θ, where $\theta = 0$ corresponds to the maximum capacitance.

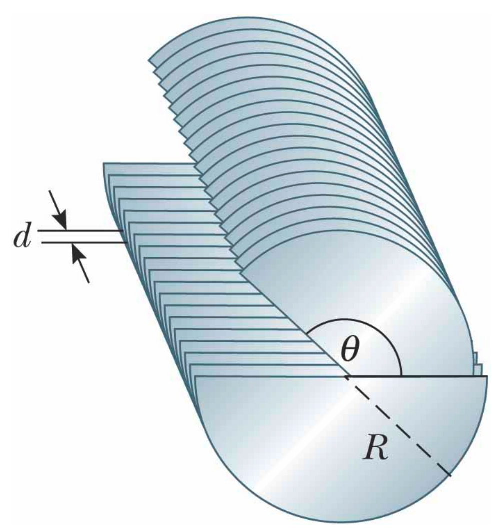

Figure 26.10:

Solution

With $\theta = \pi$, the plates are out of mesh and the overlap area is zero. With $\theta = 0$, the overlap area is that of a semi-circle, $\frac{1}{2}\pi R^2$. By proportion, the effective area of a single sheet of charge is $A_{effective} = \frac{1}{2}(\pi - \theta)R^2$

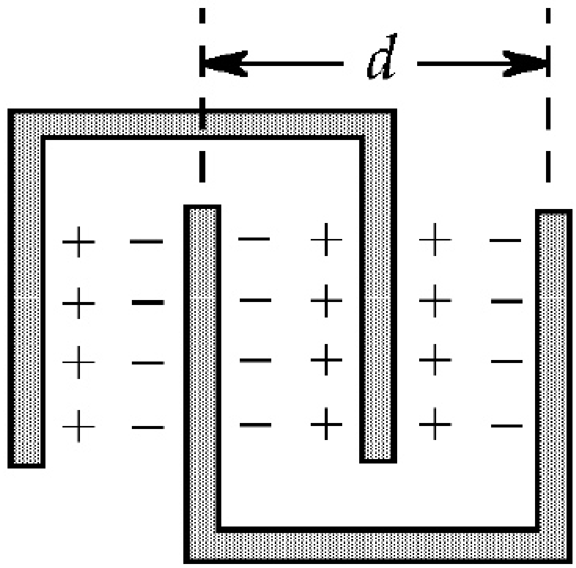

Figure 26.11:

When there are two plates in each comb, the number of adjoining sheets of positive and negative charge is 3, as shown in the sketch. When there are N plates on each comb, the number of parallel capacitors is $2N - 1$ and the total capacitance is

$$C = (2N-1)\frac{\epsilon_\circ A_{effective}}{\text{distance}} = \frac{(2N-1)(\pi-\theta)R^2/2}{d/2} = \frac{(2N-1)(\pi-\theta)R^2}{d}$$

26.2 Problem 26.27 *(In the text book)*

Find the equivalent capacitance between points a and b for the group of capacitors connected as shown in Figure (26.27). Take $C_1 = 5.00\ \mu F$, $C_2 = 10.0\ \mu F$, and $C_3 = 2.00\ \mu F$.

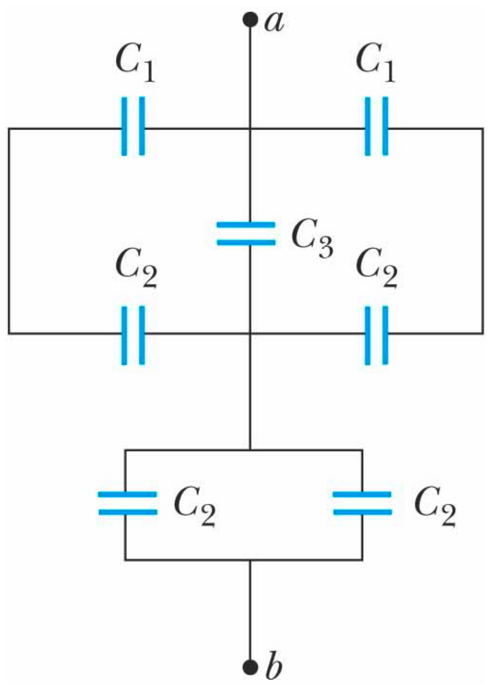

Figure 26.27:

Solution

You can think of C_3 as a source of potential difference, then C_1 and C_2 are connected in series with equivalent capacitance C_{p1}:

$$C_{p1} = \frac{C_1 C_2}{C_1 + C_2}$$

The two C_2 capacitors are connected in parallel with equivalent capacitance C_{p2} of:

$$C_{p2} = C_2 + C_2 = 2C_2$$

The circuit then reduces to C_{p1}, C_3 and C_{p1} connected in parallel with equivalent capacitance C_{p3} of

$$C_{p3} = C_3 + 2C_{p1}$$

Finally we end up with two capacitors C_{p2} and C_{p3} in series, and the total equivalent capacitance is:

$$C = \frac{C_{p2}C_{p3}}{C_{p2} + C_{p3}} = \frac{2C_2 \times (C_3 + 2C_{p1})}{2C_2 + C_3 + 2C_{p1}} = \frac{2C_2 \times \left(C_3 + 2\frac{C_1C_2}{C_1+C_2}\right)}{2C_2 + C_3 + 2\frac{C_1C_2}{C_1+C_2}}$$

Using numerical values we get:

$$C = 6.04\ \mu F$$

26.3 Problem 26.31 *(In the text book)*

(a) A 3.00-μF capacitor is connected to a 12.0-V battery. How much energy is stored in the capacitor?

(b) If the capacitor had been connected to a 6.00-V battery, how much energy would have been stored?

Solution

(a) The energy stored in the capacitor from the 12-V battery is:

$$U = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2} \times 3.00 \times 10^{-6} \times (12)^2 = 2.16 \times 10^{-4}\ J$$

(b) and from the 6-V battery:

$$U = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2} \times 3.00 \times 10^{-6} \times (6)^2 = 5.4 \times 10^{-5}\ J$$

26.4 Problem 26.47 *(In the text book)*

A parallel-plate capacitor in air has a plate separation of 1.50 cm and a plate area of 25.0 cm^2. The plates are charged to a potential difference of 250 V and disconnected from the source. The capacitor is then immersed in distilled water. Determine

(a) the charge on the plates before and after immersion,

(b) the capacitance and potential difference after immersion, and

(c) the change in energy of the capacitor. Assume the liquid is an insulator.

Solution

Originally:

$$C_i = \frac{\epsilon_\circ A}{d} = \frac{Q}{(\Delta V)_i} = \frac{8.85 \times 10^{-12} \times 25.0 \times 10^{-4}}{1.50 \times 10^{-2}} = 1.475 \times 10^{-12}\ F = 1.475\ pF$$

(a) The charge is the same before and after immersion, with value

$$Q = C_i(\Delta V)_i = 1.475 \times 10^{-12} \times 250 = 3.69 \times 10^{-10}\ C = 369\ pC$$

(b) After immersion:

$$C_f = \kappa_w C_i = 80.0 \times 1.475 \times 10^{-12} = 1.18 \times 10^{-10}\ C = 118\ pC$$

and

$$(\Delta V)_f = \frac{(\Delta V)_i}{\kappa_w} = \frac{250}{80.0} = 3.12\ V$$

(c) The original energy:

$$U_i = \frac{1}{2}C_i(\Delta V)_i^2$$

and the final energy:

$$U_f = \frac{1}{2}C_f(\Delta V)_f^2 = \frac{1}{2}\kappa_w C_i\left(\frac{(\Delta V)_i}{\kappa_w}\right)^2 = \frac{1}{2}C_i\frac{(\Delta V)_i^2}{\kappa_w}$$

and

$$\Delta U = U_f - U_i = \frac{1}{2}C_i(\Delta V)_i^2 \times \left(\frac{1}{\kappa_w} - 1\right) = \frac{1}{2} \times 1.475 \times 10^{-12} \times (250)^2 \times \left(\frac{1}{80} - 1\right) = 4.55 \times 10^{-8}\ J$$

26.5 Problem 26.61 *(In the text book)*

A parallel-plate capacitor is constructed by filling the space between two square plates with blocks of three dielectric materials, as in Figure (26.61). You may assume that $\ell \gg d$.

(a) Find an expression for the capacitance of the device in terms of the plate area A and d, κ_1, κ_2, and κ_3.

(b) Calculate the capacitance using the values $A = 1.00\ cm^2$, $d = 2.00\ mm$, $\kappa_1 = 4.90$, $\kappa_2 = 5.60$, and $\kappa_3 = 2.10$.

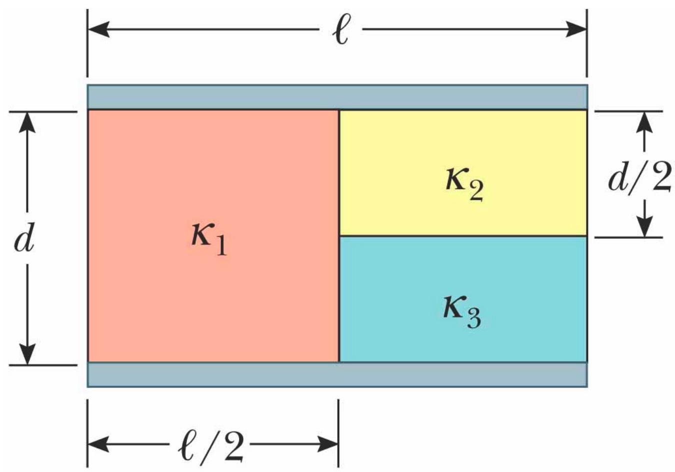

Figure 26.61:

Solution

The capacitor can be thought of as three capacitors. C_1 with κ_2 in series with C_2 the one with κ_3 and both are connected in parallel with capacitor C_3 with κ_3. Note that:

$$C_1 = \frac{\kappa_1 \epsilon_\circ A/2}{d} = \frac{\kappa_1 \epsilon_\circ A}{2d}, \qquad C_2 = \frac{\kappa_2 \epsilon_\circ A/2}{d/2} = \frac{\kappa_2 \epsilon_\circ A}{d} \quad \text{and} \quad C_3 = \frac{\kappa_3 \epsilon_\circ A}{d}$$

so the total capacitance is:

$$C = C_1 + \frac{C_2 C_3}{C_2 + C_3} = \frac{\kappa_1 \epsilon_\circ A}{2d} + \frac{\epsilon_\circ A}{d}\left(\frac{\kappa_2 \kappa_3}{\kappa_2 + \kappa_3}\right) = \frac{\epsilon_\circ A}{d}\left(\frac{1}{2}\kappa_1 + \frac{\kappa_2 \kappa_3}{\kappa_2 + \kappa_3}\right)$$

Using the numerical values we get:

$$C = 1.76 \times 10^{-12}\ F = 1.76\ pF$$