Probability Midterm I

Math 321
13 February 2015

Name: _____
by writing my name i swear by the honor code

Course Instructor: Shirali Kadyrov

1. (*20 points*) An instructor gives his class a set of 20 problems with the information that the midterm will consist of a random selection of 5 of them. If a student has figured out how to do 12 of the problems, what is the probability that he or she will answer at least 4 problems correctly?

$\#S = \binom{20}{5}$, A = student answers 4

B = student answers 5

$\#A = \binom{12}{4}\cdot\binom{8}{1}$

$\#B = \binom{12}{5}$

$P(A \cup B) = P(A) + P(B) = \dfrac{\binom{12}{4}\binom{8}{1} + \binom{12}{5}}{\binom{20}{5}}$

2. (*20 points*) Among 35 students in the class, 12 earned "A" on the midterm test and 10 earned "A" on the Final exam, and 20 did not earn "A" on either exam.

a. (*10 pts*) What is the probability that a randomly selected student earned "A" on both exams?

A = student gets A on midterm

B = student gets A on final.

$P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$= \frac{12}{35} + \frac{10}{35} - (1 - P((A \cup B)^c)) = \frac{22}{35} - 1 + \frac{20}{35} = \frac{7}{35}$

b. (*10 pts*) Are the events "Getting A on the midterm" and "Getting A on the Final" independent? Justify!

No, as $P(A \cap B) = \frac{7}{35} \neq \frac{12}{35} \cdot \frac{10}{35} = P(A) \cdot P(B)$

3. (*20 points*) A fair die is rolled twice.

a. (*7 pts*) Describe the sample space S. What is the number of elements of S?

$S = \{(m,n) : m = 1,2,3,4,5,6;\ n = 1,2,3,4,5,6\}$

$\#S = 6 \cdot 6 = 36$

b. (*7 pts*) Write down the elements of the event

$B =$ "The sum of faces showing is 9",

and find $P(B)$.

$B = \{(3,6), (6,3), (4,5), (5,4)\}$

$P(B) = \frac{4}{36} = \frac{1}{9}$

c. (*6 pts*) Given that the event B occurred, what is the probability that the face of the first toss is less than that of the second toss?

$A =$ 1st face is less than 2nd face, $A \cap B = \{(3,6), (4,5)\}$

$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{2/36}{4/36} = \frac{1}{2}$

4. (*20 points*) English and American spellings are *rigour* and *rigor*, respectively. An English speaking man staying at Almaty hotel writes this word, and a letter taken at random from this spelling is found to be a vowel. If 40% of the English speaking men at the hotel are Englishmen, and 60% are Americans, what is the probability that the writer is an Englishman?

A_1 = English man wrote

A_2 = American wrote

B = vowel selected

rigour has 3 vowels out of 6 so $P(B|A_1) = \frac{3}{6}$

rigor has 2 vowels out of 5 so, $P(B|A_2) = \frac{2}{5}$

$P(A_1) = \frac{40}{100}$, $P(A_2) = \frac{60}{100}$

Bayes thm:

$$P(A_1|B) = \frac{P(B|A_1) \cdot P(A_1)}{P(B|A_1)P(A_1) + P(B|A_2) \cdot P(A_2)}$$

$$= \frac{\frac{1}{2} \cdot \frac{40}{100}}{\frac{1}{2} \cdot \frac{40}{100} + \frac{2}{5} \cdot \frac{60}{100}}$$

5. (*20 points*) An urn contains 15 chips, numbered 1 through 15. Two are drawn simultaneously (i.e. without replacement).

a. (*10 pts*) What is the probability that the numbers on the chips are at most 10?

$\#S = \binom{15}{2}$, A = numbers are at most 10.

we need to select 2 from $\{1, 2, \ldots, 10\}$

so $\#A = \binom{10}{2}$

Thus $P(A) = \dfrac{\binom{10}{2}}{\binom{15}{2}}$

b. (*10 pts*) What are the chances that the numbers on the chips will differ by more than 2?

A = numbers differ by more than 2

$A^c = B \cup C$ where

B = numbers differ by 1

C = numbers differ by 2

$B = \{\{1,2\}, \{2,3\}, \{3,4\}, \ldots, \{14,15\}\}$

$C = \{\{1,3\}, \{2,4\}, \{3,5\}, \ldots, \{13,15\}\}$

$\#B = 14$, $\#C = 13$, $P(A) = 1 - P(A^c)$

thus, $P(A) = 1 - \dfrac{\#B + \#C}{\#S} = 1 - \dfrac{27}{\binom{15}{2}}$