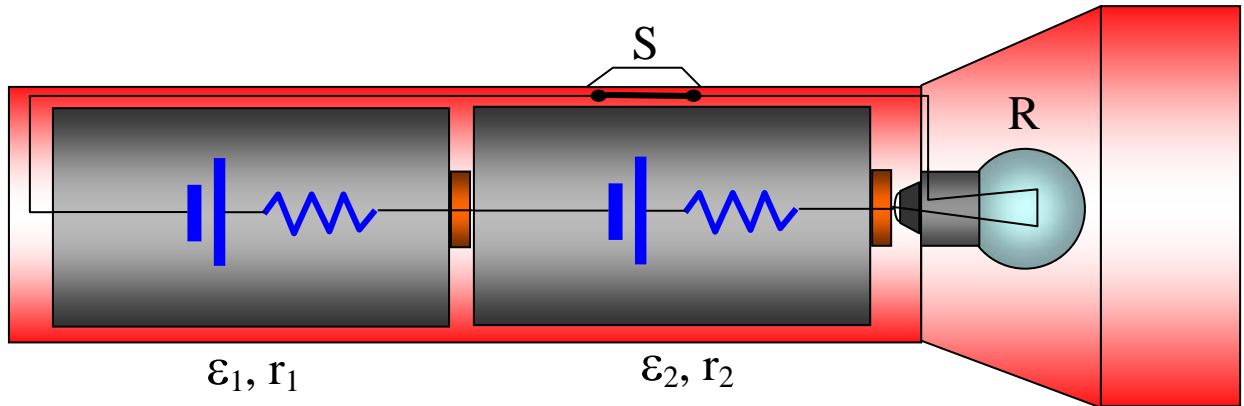


Homework 5

chapter 28: 2, 7, 31, 43

Problem 28.2

Two 1.50-V batteries (with their positive terminals in the same direction) are inserted in series into the barrel of a flashlight. One battery has an internal resistance of $0.255\ \Omega$, the other an internal resistance of $0.153\ \Omega$. When the switch is closed, a current of 600 mA occurs in the lamp. (a) What is the lamp's resistance? (b) What fraction of the chemical energy transformed appears as internal energy in the batteries?



a) Solution 1

Using Ohm's law, we can relate the potential difference across the light bulb (lamp) with the current in the circuit and the resistance of the lamp.

$$(1) \quad IR = V$$

The two batteries connected in series produce that potential difference. The potential differences each battery creates, depend on their electromotive forces and internal resistances. Since the batteries are connected in series the voltage across both batteries is equal to the sum of voltages produced by each battery.

$$(2) \quad V = (\epsilon_1 - Ir_1) + (\epsilon_2 - Ir_2)$$

We obtained two equations with two unknowns (R, V). The rest is math. Solving by substitution the resistance of the battery is:

$$\begin{aligned} R &= \frac{(\varepsilon_1 - Ir_1) + (\varepsilon_2 - Ir_2)}{I} = \frac{\varepsilon_1 + \varepsilon_2}{I} - r_1 - r_2 = \\ &= \frac{1.5V + 1.5V}{0.6A} - 0.255\Omega - 0.153\Omega = 4.59\Omega \end{aligned}$$

Solution 2.

We could use Kirchhoff's rule for the circuit. Only one equation, with unknown resistance R of the lamp, can be obtained for one loop.

$$\varepsilon_1 - Ir_1 + \varepsilon_2 - Ir_2 - IR = 0$$

Hence

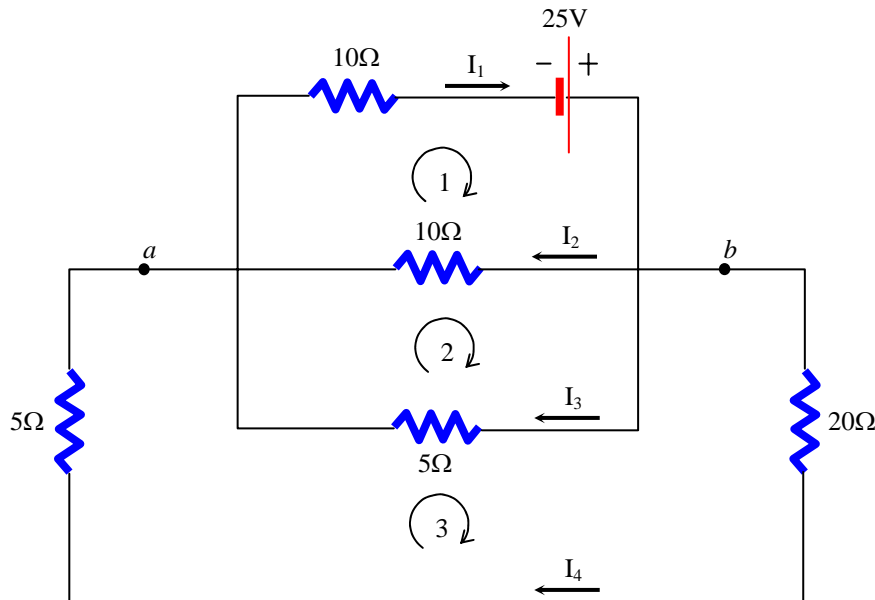
$$R = \frac{\varepsilon_1 + \varepsilon_2}{I} - r_1 - r_2$$

b) Electric power is also dissipated in the batteries due to their internal resistance. (Recall that we can treat the battery like a resistor in series with the ideal source of the electromotive force.) From the scalar properties of power we can find the power dissipated in any subset of the elements in the circuit by adding the power dissipated in the elements. Therefore the fraction of the power dissipated in the batteries is:

$$\% = \frac{P_1 + P_2}{P_1 + P_2 + P_R} = \frac{I^2 r_1 + I^2 r_2}{I^2 r_1 + I^2 r_2 + I^2 R} = \frac{r_1 + r_2}{r_1 + r_2 + R} = \frac{0.255\Omega + 0.153\Omega}{0.255\Omega + 0.153\Omega + 4.6\Omega} = 8.16\%$$

Problem 28.7

Consider the circuit shown in Figure P28.9. Find (a) the current in the $20\text{-}\Omega$ resistor and (b) the potential difference between point a and b .



Solution 1

We could solve this problem "step by step" finding the equivalent resistance of various subsets of the system of resistors leading to the answer from Ohm's law. Let's find the current I_1 flowing through the battery by finding the equivalent resistance of all the resistors. The currents in the $20\text{ }\Omega$ resistor and the left hand side $5\text{ }\Omega$ resistor are equal. These resistors are connected in series. Therefore

$$R_{20\&5} = 20\Omega + 5\Omega = 25\Omega$$

These two resistors, the other $5\text{ }\Omega$ resistor, and the $10\text{ }\Omega$ resistor connected to the high potential terminal of the battery have the

same potentials on their sides, respectively. They are connected in parallel. This system has an equivalent resistance of

$$R_{(20\&5)\&5\&10} = \left(\frac{1}{25\Omega} + \frac{1}{5\Omega} + \frac{1}{10\Omega} \right)^{-1} = 2.94\Omega$$

The current flowing through the $10\ \Omega$ resistor connected to the lower potential terminal of the battery is equal to the current flowing through the remaining resistors ($I_1 = I_2 + I_3 + I_4$). The resistor is in series with the rest of the system. The equivalent resistance of the entire circuit is therefore

$$R = 2.94\Omega + 10\Omega = 12.94\Omega$$

Using Ohm's law, we find the current flowing through the battery and the $10\ \Omega$ resistor connected to the lower potential terminal of the battery.

$$I_1 = \frac{\varepsilon}{R} = \frac{25V}{12.94\Omega} = 1.93A$$

b) It is easier to first find the potential difference between the two points indicated in the problem. Using Ohm's law, we can find the voltage across the $10\ \Omega$ resistor and add the voltage across the battery.

$$V_b - V_a = -I_1 \cdot R_{10} + \varepsilon = -1.93A \cdot 10\Omega + 25V = 5.68V$$

a) The current flowing through the 20Ω resistor has the same value as the current flowing through the 20Ω & 5Ω system. These two resistors are connected in series. Therefore

$$I_4 = \frac{V_b - V_a}{R_{20\&5}} = \frac{5.68V}{20\Omega + 5\Omega} = 0.227A$$

Solution 2

a) It is easier to use Kirchhoff's rules to write the equations necessary to find any unknown in the circuit. For this circuit we can find three independent equations from the loop rule, and one equation from the junction rule. The four values of the currents can be found from this set of equations. We can assume that their directions are arbitrary but let's make our assignment following our intuition.

Using the loop rule for the marked loops and the junction rule for junction A we obtain four equations.

$$\begin{aligned}-10\Omega \cdot I_1 - 10\Omega \cdot I_2 &= -25\text{V} \\ +10\Omega \cdot I_2 - 5\Omega \cdot I_3 &= 0 \\ +5\Omega \cdot I_3 - 25\Omega \cdot I_4 &= 0 \\ -I_1 + I_2 + I_3 + I_4 &= 0\end{aligned}$$

Since we are interested in current I_4 , in our solution we will eliminate the other currents. I will eliminate them from the fourth equation using the expressions found from the first three equations

$$\begin{aligned}I_3 &= 5I_4 \\ I_2 &= 0.5I_3 = 0.5 \cdot 5I_4 = 2.5I_4 \\ I_1 &= -I_2 + 2.5\text{A} = -2.5I_4 + 2.5\text{A}\end{aligned}$$

and obtain an equation for the current in the 20Ω resistor

$$(2.5\Omega + 2.5\Omega + 5\Omega + 1\Omega) \cdot I_4 = 2.5\text{V}$$

Therefore

$$I_4 = \frac{2.5V}{2.5\Omega + 2.5\Omega + 5\Omega + 1\Omega} = 0.227A$$

Comment.

Solving linear equations with a large number of unknowns is rather a tedious process. Scientific calculators are equipped with appropriate programs to help with these tasks. Learn how to do it on your calculator.

In case your calculator does not have this program, learn a more efficient method than by substitution. Below I included a solution using Cramer's method

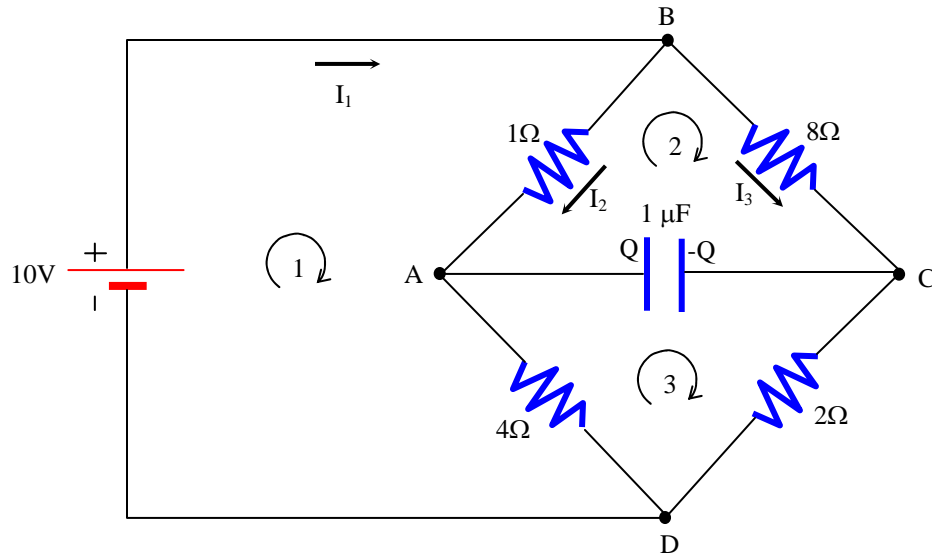
$$\begin{aligned} D &= \begin{vmatrix} -10 & -10 & 0 & 0 \\ 0 & 10 & -5 & 0 \\ 0 & 0 & 5 & -25 \\ -1 & 1 & 1 & 1 \end{vmatrix} = -10 \cdot (-1)^{1+1} \begin{vmatrix} 10 & -5 & 0 \\ 0 & 5 & -25 \\ 1 & 1 & 1 \end{vmatrix} - 10 \cdot (-1)^{1+2} \begin{vmatrix} 0 & -5 & 0 \\ 0 & 5 & -25 \\ -1 & 1 & 1 \end{vmatrix} = \\ &= -10 \cdot (10 \cdot 5 \cdot 1 + (-5) \cdot (-25) \cdot 1 + 0 \cdot 0 \cdot 1 - 1 \cdot 5 \cdot 0 - 1 \cdot (-25) \cdot 10 - 1 \cdot 0 \cdot (-5)) + \\ &+ 10 \cdot (0 \cdot 5 \cdot 1 + (-5) \cdot (-25) \cdot (-1) + 0 \cdot 0 \cdot (-1) - (-1) \cdot 5 \cdot 0 - 1 \cdot (-25) \cdot 0 - 1 \cdot 0 \cdot (-5)) = -5500 \end{aligned}$$

$$\begin{aligned} D_4 &= \begin{vmatrix} -10 & -10 & 0 & -25 \\ 0 & 10 & -5 & 0 \\ 0 & 0 & 5 & 0 \\ -1 & 1 & 1 & 0 \end{vmatrix} = -25 \cdot (-1)^{1+4} \begin{vmatrix} 0 & 10 & -5 \\ 0 & 0 & 5 \\ -1 & 1 & 1 \end{vmatrix} = 25 \cdot \left(5 \cdot (-1)^{2+3} \begin{vmatrix} 0 & 10 \\ -1 & 1 \end{vmatrix} \right) = \\ &= 25 \cdot (-5) \cdot 10 = -1250 \end{aligned}$$

$$I_4 = \frac{D_4}{D} = \frac{-1250}{-5500} = 0.227A$$

Problem 28.31

The circuit in Figure P28.37 has been connected for a long time. (a) What is the voltage across the capacitor? (b) If the battery is disconnected, how long does it take the capacitor to discharge to one tenth of its initial voltage?



Solution 1

If the battery was connected for a long time, the current "flowing" through the capacitor has a zero value. This information simplifies the equations we can obtain from Kirchhoff's rules. We can immediately assume that the current flowing through the 1Ω resistor has the same value as the current flowing through the 4Ω resistor. We can reduce the number of junctions we have to consider. (Notice that junctions A and C generate trivial equations.)

$$10\text{V} - (1 + 4)\Omega \cdot I_2 = 0 \quad (\text{loop 1})$$

$$1\Omega \cdot I_2 - 8\Omega \cdot I_3 + V_C = 0 \quad (\text{loop 2})$$

$$-V_C - 2\Omega \cdot I_3 + 4\Omega \cdot I_2 = 0 \quad (\text{loop 3})$$

$$I_1 - I_2 - I_3 = 0 \quad (\text{junction B})$$

We obtained four equations with four unknowns. In fact, the first three equations do not include current I_1 through the battery. Therefore the first three equations are necessary to find the answer. From the first equation we can find one unknown

$$I_2 = \frac{10\text{V}}{1\Omega + 4\Omega} = 2\text{A}$$

From the second equation

$$I_3 = \frac{1\Omega \cdot I_2 + V_C}{8\Omega}$$

Substituting this expression we find the voltage across the capacitor

$$V_C = 3\Omega \cdot I_2 = 6\text{V}$$

Solution 2.

In this particular case there is a simpler solution. Notice that when the capacitor is "fully" charged (only then), the 1Ω and 4Ω are connected in series. We can relate the potential at junction B with the potential at junction A.

$$V_A - V_D = I_2 \cdot 4\Omega = \frac{10\text{V}}{4\Omega + 1\Omega} \cdot 4\Omega = 8\text{V}$$

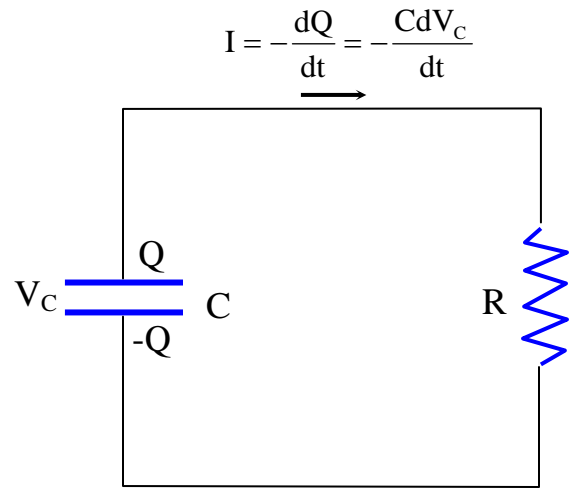
Similarly

$$V_C - V_D = I_3 \cdot 2\Omega = \frac{10\text{V}}{2\Omega + 8\Omega} \cdot 2\Omega = 2\text{V}$$

Therefore

$$V_A - V_C = (V_A - V_D) - (V_C - V_D) = 6\text{V}$$

b) When the battery is disconnected, the capacitor is discharged through the system of resistors. A current of the same value will be flowing through the 1Ω and the 8Ω resistors. They are connected in series. Similarly, we can see that the 4Ω and 2Ω resistors are connected in series. Both pairs are connected in parallel at points A and C. The equivalent resistance of the system is therefore:



$$(1) \quad R = \left(\frac{1}{1\Omega + 8\Omega} + \frac{1}{4\Omega + 2\Omega} \right)^{-1} = 3.6\Omega$$

The loop rule applied to the equivalent circuit presented in the figure relates the rate of change in the voltage across the capacitor with the value of that voltage at any instant.

$$(2) \quad V_C - R \cdot \left(-\frac{CdV_C}{dt} \right) = 0$$

We obtained a differential equation for function $V_C(t)$. Choosing the instant when the battery is disconnected as the reference instant $t_0=0s$, we can express explicitly the time dependence of the voltage across the capacitor.

$$\frac{dV_C}{V_C} = -\frac{1}{RC}dt$$

$$\ln V_C \Big|_{V_C(0)}^{V_C(t)} = - \frac{t}{RC} \Big|_0^t$$

$$V_C(t) = V_C(0) \cdot e^{-\frac{t}{RC}}$$

From this function we can write an equation for the time t_{10} at which the voltage discharges to a given value.

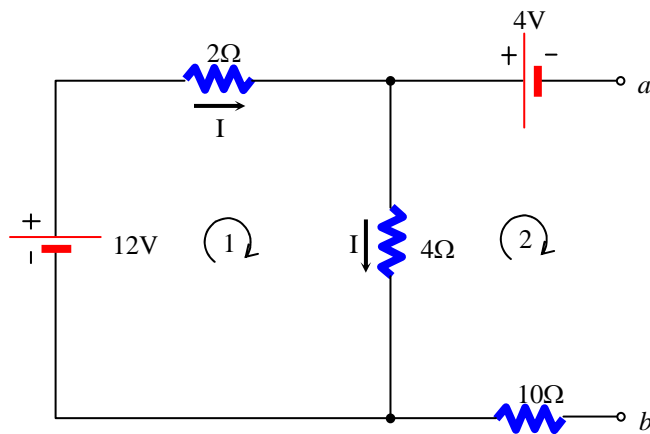
$$\frac{1}{10} \cdot V_C(0) = V_C(0) \cdot e^{-\frac{t_{10}}{RC}}$$

The solution to this equation is:

$$t_{10} = -RC \ln \frac{1}{10} = 3.6\Omega \cdot 1\mu F \cdot \ln 10 = 8.29\mu s$$

Problem 28.43

Calculate the potential difference between points a and b in Figure P28.54, and identify which point is at the higher potential.



Using Kirchhoff's rules we can write three independent equations for this circuit. Although since points a and b are not connected, we can immediately predict that the current does not flow in the 4V battery and the 10 Ω

resistor. The junction rule, therefore, implicates the same value for the current in the other two resistors. For simplicity's sake I will skip the units but first I made sure all the quantities are expressed in the SI unit system. Consistent with the markings in the figure my equations are as follows. :

$$\begin{aligned} 12\text{V} - 2\Omega \cdot I - 4\Omega \cdot I &= 0 && \text{(loop 1)} \\ 4\Omega \cdot I - 4\text{V} - (V_a - V_b) + 10\Omega \cdot 0 &= 0 && \text{(loop 2)} \\ (I - I + 0) &= 0 && \text{(top junction)} \end{aligned}$$

Substituting for the current in the second equation, the potential difference between point a and b is

$$(V_a - V_b) = 4\Omega \cdot I - 4\text{V} = 4\Omega \cdot \frac{12\text{V}}{6\Omega} - 4\text{V} = 4\text{V}$$

The result indicates that the potential at point a is higher.