Solutions

Math 321-01 Spring 2015
Quiz 7 22.04.15

Name: _____

Show all work clearly and in order, and circle your final answers. Justify your answers algebraically whenever possible; You have 20 minutes to take this 10 point quiz.

1. (*6 points*) Suppose that in Kabanbay Batyr avenue there were 5 car accidents occurred within 25 days.

a. (*5 pts*) What is the probability that 2 accidents will occur during the next 5 days?

X = # of accidents in 5 days

$5/25 = \frac{1}{5}$ ← number of expected accidents per day

So, $E(X) = 5 \cdot \frac{1}{5} = 1 = \lambda$.

$X \sim Pois(1)$

$P(X=2) = e^{-1} \cdot \frac{1^2}{2!} = \frac{1}{2e}$

b. (*1 pt*) Is the probability that 4 accidents will occur over the 10 days the square of your answer to part (a)? Explain?

Y = # of accidents in 10 days

Then $E(Y) = 2$, $Y \sim Pois(2)$

So, $P(Y=4) = e^{-2} \cdot \frac{2^4}{4!} = \frac{2}{3e^2} \neq \left(\frac{1}{2e}\right)^2$

So, Ans is NO.

Table 4.3.1

z	0	1	2	3	4	5	6	7	8	9
−3.	0.0013	0.0010	0.0007	0.0005	0.0003	0.0002	0.0002	0.0001	0.0001	0.0000
⋮						⋮				
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
−0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7703	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9278	0.9292	0.9306	0.9319
⋮						⋮				
3.	0.9987	0.9990	0.9993	0.9995	0.9997	0.9998	0.9998	0.9999	0.9999	1.0000

2. (*6 points*) Suppose that 100 fair dice tossed.

a. (*1 pt*) What is the expected value of the sum of the faces showing?

$X = X_1 + \ldots + X_{100}$, X_i = face of i-th die

$E(X_i) = 3.5 \Rightarrow E(X) = 350$

b. (*1 pt*) Find the standard deviation of the sum of the faces showing? ($\sqrt{91/6 - 3.5^2} \approx 1.7$)

$Var(X_i) = \frac{91}{6} - 3.5^2$

$Var(X) = 100\left(\frac{91}{6} - 3.5^2\right) \Rightarrow \sigma(X) = 17$

c. (*4 pts*) Estimate the probability that the sum of the faces showing exceeds 370. Include a continuity correction in your analysis.

By CLT

$P(X > 370) = P(X \geq 371) = P\left(\frac{X-350}{17} \geq \frac{371-350}{17}\right)$

$\approx P\left(Z \geq \frac{371-0.5-350}{17}\right) \approx P(Z \geq 1.21)$

$= 1 - \Phi(1.21) = 1 - 0.8869 = 0.1131$