

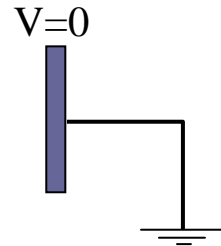
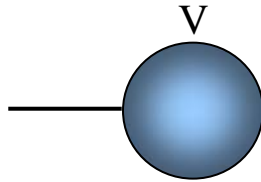
<http://iml.umkc.edu/physics/wrobel/phy250/homework.htm>

# Homework 3

chapter 26: 3, 17, 41, 56

### Problem 26.3

An isolated charged conducting sphere of radius 12.0 cm creates an electric field of  $4.90 \cdot 10^4$  N/C at a distance 21.0 cm from its center. (a) What is its surface charge density? (b) What is its capacitance?



*a) Using Gauss' law for a spherical closed surface concentric with the sphere, we can express the magnitude of the electric field around the sphere in terms of the charge on the sphere.*

$$(1) \quad E = \frac{kQ}{r^2}$$

*where  $Q$  represents the total charge on the sphere and  $r$  is the distance from the center of the sphere to the point at which we consider the electric field. (We cannot use the formula for a point charge here!)*

*For a conducting sphere all excess charge is accumulated on the surface. We can use the definition of area charge density to answer the question.*

$$(2) \quad \sigma = \frac{dQ}{dA}$$

*Because of the symmetry in this problem, the sphere is charged uniformly. Therefore the charge density is a simple function of the charge and the area of the sphere:*

$$\sigma = \frac{Q}{A} = \frac{Er^2}{k \cdot 4\pi R^2} = \frac{\epsilon_0 Er^2}{R^2} =$$

$$= \frac{8.85 \cdot 10^{-12} \frac{C^2}{Nm^2} \cdot 4.9 \cdot 10^4 \frac{N}{m} \cdot (0.21m)^2}{(0.12m)^2} = 1.33 \frac{\mu C}{m^2}$$

where  $R$  represents the radius of the charged sphere.

b) First we have to imagine where the second plate of the capacitor is. When the sphere is charged the charge must be transferred from somewhere. For all practical purposes we can assume that it is transferred from far away (infinity). That place constitutes the second plate.

In order to find the capacitance of this capacitor we must relate the potential difference  $V$  between the "plates" with the charge transferred  $Q$ . When the charge of the sphere is  $Q$ , the potential at the sphere (with the reference point at infinity) is

$$(3) \quad V_s = \frac{kQ}{R}$$

The "other plate" is chosen for the potential reference therefore its potential is  $V_\infty = 0V$ . Therefore the relation between the charge transferred and the potential difference between the plates is:

$$V = V_s - V_\infty = \frac{k}{R} \cdot Q$$

We can see that the potential difference is proportional to the charge transferred as is required for a capacitor. From the definition,

$$(4) \quad Q = CV$$

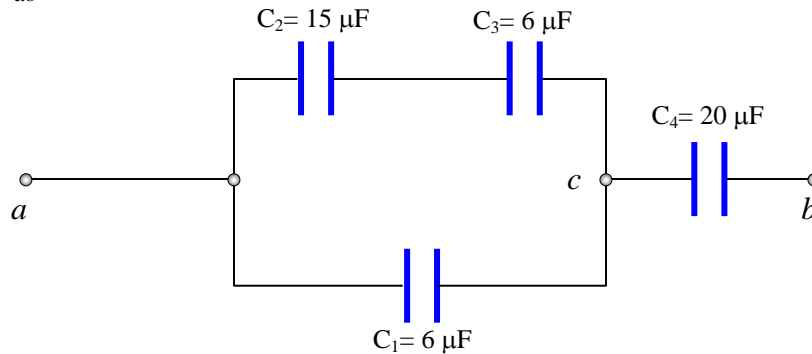
the capacitance of the sphere is:

$$C = \frac{R}{k} = \frac{0.12\text{m}}{9 \cdot 10^9 \frac{\text{Nm}^2}{\text{C}^2}} = 13.3\text{pF}$$

*Note. The capacitance does not depend on the charge stored on the sphere. We should not use the charge we found in part a to solve part b.*

### Problem 26.17

Four capacitors are connected as shown in Figure P26.17. (a) Find the equivalent capacitance between points  $a$  and  $b$ . (b) Calculate the charge of each capacitor, taking  $\Delta V_{ab}$ .



a) Capacitors “ $C_2$ ”, and “ $C_3$ ” will always be charged with equal charges. They are connected in series. Therefore the equivalent capacitance  $C_{eq,(23)}$  of the system “ $C_2$ - $C_3$ ” is:

$$(1) \quad C_{eq,(23)} = \left( \frac{1}{C_2} + \frac{1}{C_3} \right)^{-1} = 2.5 \mu\text{F}$$

The plates of capacitors “ $C_2$ ”, are connected to the same potentials as the system “ $C_2$ - $C_3$ ”. The equivalent capacitance  $C_{eq,(123)}$  of all the three capacitors is:

$$(2) \quad C_{eq,(123)} = C_1 + C_{eq,(23)} = 6 \mu\text{F} + 2.5 \mu\text{F} = 8.5 \mu\text{F}$$

This system of capacitors will always be charged with equal charge and capacitor “ $C_4$ ”. They are connected in series. Hence the equivalent capacitance of the entire system is

$$(3) \quad C = \left( \frac{1}{C_4} + \frac{1}{C_{eq,(123)}} \right)^{-1} = \left( \frac{1}{20 \mu\text{F}} + \frac{1}{8.5 \mu\text{F}} \right)^{-1} = 5.96 \mu\text{F}$$

*b) With 15V potential difference across the entire system, the charge on the entire system is*

$$(4) \quad Q = C \cdot \Delta V_{ab} = 5.96 \mu\text{F} \cdot 15\text{V} = 89.5 \mu\text{C}$$

*and because the entire system results from a connection in series of capacitor “ $C_4$ ” and system “ $C_{123}$ ”, each of the latter are charged with the same charge*

$$Q_4 = 89.5 \mu\text{C}$$

$$Q_{123} = 89.5 \mu\text{C}$$

*Charge  $Q_{123}$  is spread between the two parallel branched of this system. Using the equivalent capacitance  $C_{eq,123}$  of the system of capacitors “ $C_1$ ”, “ $C_2$ ” and “ $C_3$ ”, we can find that the potential difference between point “a” and “c” is*

$$\Delta V_{ac} = \frac{Q_{123}}{C_{eq,(123)}} = \frac{89.5 \mu\text{C}}{8.5 \mu\text{F}} = 10.5\text{V}$$

*It is potential difference across “ $C_1$ ” capacitor, hence the charge on this capacitor is*

$$Q_1 = C_1 \cdot \Delta V_{ab} = 6 \mu\text{F} \cdot 10.5\text{V} = 63.2 \mu\text{C}$$

*Capacitors “ $C_2$ ” and “ $C_3$ ” are connected in series meaning that each of them is charged with the charge on the two together.*

*This charge can be found from the definition of capacitance*

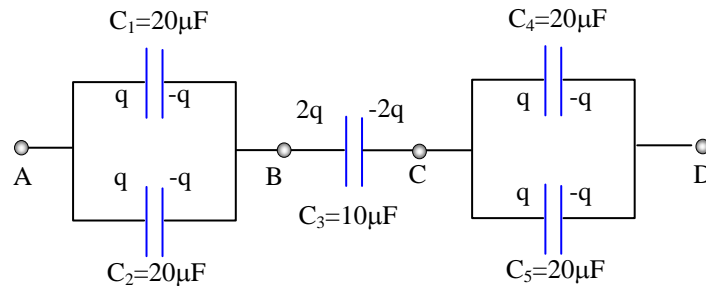
$$Q_2 = Q_3 = Q_{23} = C_{eq,(23)} \cdot \Delta V_{ac} = 2.5 \mu\text{F} \cdot 10.5\text{V} = 26.3 \mu\text{C}$$

*or from the charge on the system of capacitors “ $C_1$ ”, “ $C_2$ ” and “ $C_3$ ”*

$$Q_2 = Q_3 = Q_{123} - Q_{23} = 89.5 \mu\text{C} - 63.2 \mu\text{C} = 26.3 \mu\text{C}$$

### Problem 26.41

Each capacitor in the combination shown in Fig. 26.41 has a breakdown voltage of 15V. What is the breakdown voltage for the combination?



*The breakdown voltage for the combination has such a value that potential difference across at least one capacitor reaches 15 V. To solve the problem one has to determine to which capacitor the highest voltage is applied.*

*Capacitors  $C_1$  and  $C_2$  are connected to the same points A and B. The voltage across each has the same value. From their capacitances one should conclude that they always are equally charged.*

*The right-hand side plates of capacitors  $C_1$  and  $C_2$  and the left-hand side plate of capacitor  $C_3$  are isolated – the charge has no route to escape. It requires that charge on the plate of capacitor  $C_3$  is equal to the sum of charge on the other two plates yielding the charge on capacitor  $C_3$  twice as high as the charge on  $C_1$  or  $C_2$ . Since the capacitance of capacitor  $C_3$  is only half of the capacitance of  $C_1$ , the voltage across this capacitor is four times greater.*

*The distribution of charge on the last two capacitors results in the voltage across the remaining two capacitors equal to that of capacitor  $C_1$ .*

*The system breaks down when the (highest) voltage across capacitor  $C_3$  reaches 15V. At this time voltage between points A and B as well as voltage between point C and D is 3.75V. Hence the voltage across entire system is*

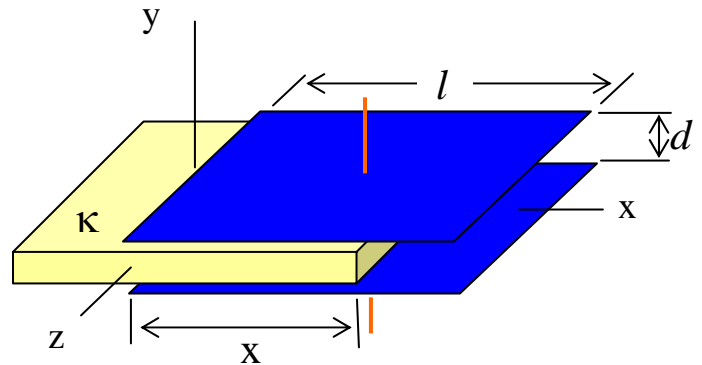
$$\begin{aligned} V_A - V_D &= (V_A - V_B) + (V_B - V_C) + (V_C - V_D) = \\ &= 3.75\text{V} + 15\text{V} + 3.75\text{V} = 22.5\text{V} \end{aligned}$$



### Problem 26.56

A capacitor is constructed from two square plates of sides  $l$  and separation  $d$ . A material of dielectric constant  $\kappa$  is inserted a distance  $x$  into the capacitor, as shown in Figure P26.56. Assume that  $d$  is much smaller than  $x$ . (a) Find the equivalent capacitance of the device. (b) Calculate the energy stored in the capacitor, letting  $\Delta V$  represent the potential difference. (c) Find the direction and magnitude of the force exerted on the dielectric, assuming a constant potential difference  $\Delta V$ . Ignore friction. (d) Obtain a numerical value for the force assuming the  $l = 5$  cm,  $\Delta V = 2000$  V,  $d = 2$  mm, and the dielectric is glass ( $\kappa = 4.5$ ). (Suggestion: The system can be considered as two capacitors connected in parallel.)

a) The system can be considered as two capacitors, one with a dielectric and one without a dielectric. The top plates of the two capacitors always have the same potential, and the bottom plates always have the same potential. The two capacitors are connected in parallel.



If the separation between the plates is much smaller than the dimensions of the plates of both capacitors, both capacitors can be considered to be parallel-plate capacitors. Therefore their capacitance may be expressed in terms of their dimensions and the properties of the medium between the plates

$$C_1 = \kappa \epsilon_0 \frac{lx}{d} \quad \text{and} \quad C_2 = \epsilon_0 \frac{l(l-x)}{d}$$

The equivalent capacitance is therefore

$$C = C_1 + C_2 = \frac{\kappa \epsilon_0 lx}{d} + \frac{\epsilon_0 l(l-x)}{d} = \frac{\epsilon_0}{d} [l(l-x) + \kappa lx]$$

*b) The energy stored in a capacitor depends on the capacitance of the capacitor and the potential difference between the plates*

$$U = \frac{C \cdot (\Delta V)^2}{2} = \frac{\epsilon_0 [l(l-x) + \kappa lx] (\Delta V)^2}{2d}$$

*c,d) The symmetry of interaction of each plate with the dielectric slab (about the xz-plane) requires that the y-component of the resulting electrostatic force exerted on the dielectric by both plates have a zero value. Symmetry about a vertical plane (xy-plane) also requires that the z-component of the force be zero. (Only the x-component of the force is not zero.)*

*As the dielectric slab is inserted (or removed) the potential energy of the system is affected. The x-component of the conservative (electrostatic) force associated with this interaction therefore has a value of*

$$\begin{aligned} F_x &= -\frac{\partial U}{\partial x} = -\frac{\partial}{\partial x} \frac{\epsilon_0 [l(l-x) + \kappa lx] (\Delta V)^2}{2d} = -\frac{\epsilon_0 (\Delta V)^2}{2d} (\kappa - 1)l = \\ &= -\frac{8.9 \cdot 10^{-12} \frac{C^2}{Nm^2} \cdot (2000V)^2}{2 \cdot 0.002m} (4.5 - 1) \cdot 0.05m = -1.55 \cdot 10^{-3} N \end{aligned}$$

*The negative value indicates that the dielectric is pushed out from the capacitor.*