Solutions

Math 321-01 Spring 2015
Quiz 6 18.03.15 Name: ______________________

Show all work clearly and in order, and circle your final answers. Justify your answers algebraically whenever possible; You have 20 minutes to take this 10 point quiz.

**1.** (*5 points*) Let $Y$ be an exponential random variable so that $f_Y(y) = \lambda e^{-\lambda y}$ for $y \geq 0$. Show that $\mathrm{Var}(Y) = 1/\lambda^2$.

$E(Y^2) - E(Y)^2 = Var(Y)$

Integration by parts:

$$E(Y) = \int_0^\infty y \cdot \lambda e^{-\lambda y}\,dy = \left| \begin{matrix} u = \lambda y & du = \lambda\,dy \\ dv = e^{-\lambda y}dy & v = -\frac{e^{-\lambda y}}{\lambda} \end{matrix} \right| = uv - \int v\,du$$

$$= -ye^{-\lambda y}\Big|_0^\infty + \int_0^\infty e^{-\lambda y}\,dy = 0 - \frac{e^{-\lambda y}}{\lambda}\Big|_0^\infty = \frac{1}{\lambda}$$

$$E(Y^2) = \int_0^\infty y^2 \lambda e^{-\lambda y}\,dy = \left| \begin{matrix} u = \lambda y^2 & du = 2\lambda y\,dy \\ dv = e^{-\lambda y}dy & v = -\frac{e^{-\lambda y}}{\lambda} \end{matrix} \right| = uv - \int v\,du$$

$$= -y^2 e^{-\lambda y}\Big|_0^\infty + 2\int_0^\infty y e^{-\lambda y}\,dy = \frac{2}{\lambda}\int_0^\infty y \lambda e^{-\lambda y}\,dy = \frac{2}{\lambda}E(Y)$$

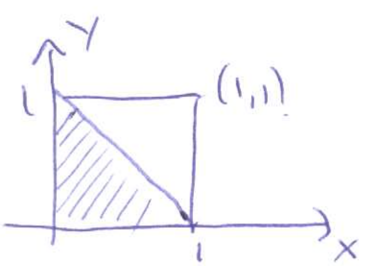
$$= \frac{2}{\lambda^2}$$

So, $\mathrm{Var}\,Y = \frac{2}{\lambda^2} - \left(\frac{1}{\lambda}\right)^2 = \frac{1}{\lambda}$

**2.** (*5 points*) Two cards are drawn from a standard pocker deck. Let $X$ be the number of kings drawn and $Y$ the number of queens. Find the joint pdf $p_{X,Y}(x,y)$.

$$P_{X,Y}(x,y) = \frac{\binom{4}{x}\binom{4}{y}\binom{44}{2-x-y}}{\binom{52}{2}}, \qquad \begin{array}{l} 0 \le x \le 2 \\ 0 \le y \le 2 \\ 0 \le x+y \le 2 \end{array}$$

**3.** (*2 points*) Suppose that $f_{X,Y}(x,y) = 6(1-x-y)$ for $x$ and $y$ defined over the unit square, subject to the restriction that $0 \le x+y \le 1$. Find the marginal pdf for $X$.




$$\left.\begin{array}{l} 0 \le x+y \le 1 \\ 0 \le x \le 1 \\ 0 \le y \le 1 \end{array}\right\} \Rightarrow \quad 0 \le y \le 1-x$$

So, $$f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dy = \int_0^{1-x} 6(1-x-y)\,dy$$

$$= 6y - 6xy - 3y^2 \Big|_0^{1-x} = 6(1-x) - 6x(1-x) - 3(1-x)^2$$

$$= 3(1-x)(2-2x-1+x) = 3(1-x)^2$$