# Homework   4

chapter 27: 3, 19, 29, 57

Problem 27.3

Suppose that the current through a conductor decreases exponentially with time according to the equation $I(t) = I_0 \, e^{-t/\tau}$ where I_0 is the initial current (at $t = 0$), and τ is a constant having dimensions of time. Consider a fixed observation "point" within the conductor. (a) How much charge passes this "point" between $t = 0$ and $t = \tau$? (b) How much charge passes this "point" between $t = 0$ and $t = 10\tau$? (c) How much charge passes this "point" between $t = 0$ and $t = \infty$?

$$I(t) = I_0 e^{-\frac{t}{\tau}}$$
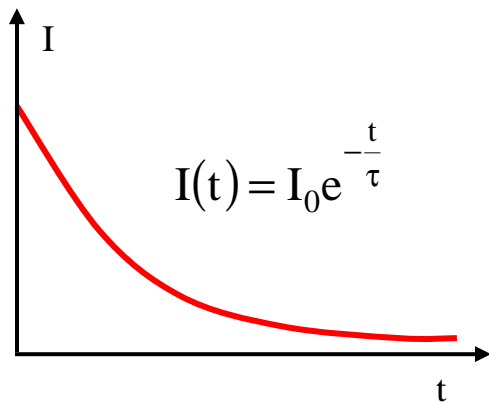
The word "point" in the text really refers to the cross sectional surface containing that point. We often idealize a wire assuming that it is a one-dimensional body.

From the definition of electric current, the differential charge dq passing through the surface is proportional to the corresponding differential time dt'. The proportionality coefficient (the current), for any instant t', is given to us in the text.

$$dq = I(t')dt' = I_0 e^{-\frac{t'}{\tau}}dt'$$

If we want to find the charge q(t) passing through the surface between the two instances 0s and t, we must "add" (integrate) the charges transferred in all the differential time intervals in that time interval [0,t].

$$q(t) = \int_0^t I_0 e^{-\frac{t'}{\tau}}dt' = -I_0\tau e^{-\frac{t'}{\tau}}\Big|_0^t = I_0\tau\left(1 - e^{-\frac{t}{\tau}}\right)$$

Now we have to put the value of the time asked in the questions.

a) $\quad q(\tau) = I_0\tau\left(1 - e^{-1}\right) = 0.632 I_0\tau$

b) $\quad q(10\tau) = I_0\tau\left(1 - e^{-10}\right) = 0.99995 I_0\tau$

c) $\quad q(\infty) = I_0\tau\left(1 - e^{-\infty}\right) = I_0\tau$

Problem 27.19

An aluminum wire with a diameter of 0.1 mm has a uniform electric field of 0.2 V/m imposed along its entire length. The temperature is the wire is 50 °C. Assume one free electron per atom. (a) Use the information in Table 27.2 and determine the resistivity. (b) What is the current density in the wire? (c) What is the total current in the wire? (d) What is the drift speed of the conduction electrons? (e) What potential difference must exist between the ends of a 2 m length of wire to produce the stated electric field?

a) Based on the table, resistivity of aluminum at $T_0 = 20\,°C$ is $\rho_0 = 2.82 \cdot 10^{-8}\ \Omega m$ and the temperature coefficient of resistivity $\alpha = 3.9 \cdot 10^{-3}\ °C^{-1}$. From the temperature dependency of resistivity, at $T = 50\,°C$ aluminum has resistivity of

$$\rho = \rho_0 \cdot [1 + \alpha(T - T_0)] = 2.82 \cdot 10^{-8}\,\Omega m \cdot [1 + 3.9 \cdot 10^{-3}\,°C^{-1} \cdot (50°C - 20°C)] =$$
$$= 3.15 \cdot 10^{-8}\,\Omega m = 31.5\ n\Omega m$$

b) From Ohm's law, current density is proportional to the electric field vector. Hence, current density has the same direction as the electric field vector and its magnitude is

$$J = \frac{1}{\rho} \cdot E = \frac{1}{3.15 \cdot 10^{-8}\,\Omega m} \cdot 0.2\,\frac{V}{m} = 6.35 \cdot 10^{6}\,\frac{A}{m^2} = 6.35\,\frac{MA}{m^2}$$

c) Current in the wire is equal to the flux of current density over a cross-sectional surface of the wire

$$I = \int_S \mathbf{J} \cdot d\mathbf{A} = JA = \left(6.35 \cdot 10^{6}\,\frac{A}{m^2}\right) \cdot \pi \cdot (0.05 \cdot 10^{-3}\,m)^2 = 5 \cdot 10^{-2}\,A = 50mA$$

d) The magnitude of current density is proportional to the concentration of charge carriers (electrons), the absolute value of the charge of the carriers and the drift speed of the carriers. Since each atom of aluminum contributes one charge carrier, there concentration is equal to the number of atoms per unit volume. It can be determined from the density and the atomic mass of aluminum. The drift speed is therefore

$$v_d = \frac{J}{ne} = \frac{J}{\frac{\rho N_A}{M_{Al}} \cdot e} = \frac{6.35 \cdot 10^6 \, A/m^2}{\frac{2.7 \cdot 10^3 \, kg/m^3 \cdot 6.02 \cdot 10^{23} \, mol^{-1}}{27 \cdot 10^{-3} \, kg/mol} \cdot 1.6 \cdot 10^{-19} C} =$$

$$= 6.59 \cdot 10^{-4} \frac{m}{s} = 659 \frac{\mu m}{s}$$

e) From the relation between electric field vector an potential

$$V_+ - V_- = -\int_0^L E \cdot ds \cdot \cos 180° = E \cdot L = 0.2 \frac{V}{m} \cdot 2m = 0.4V$$

Problem 27.29

A 500-W heating coil designed to operate at 110 V is made of nichrome wire 0.50 mm in diameter. (a) Assuming that the resistivity of the nichrome remains constant at its 20 °C value, find the length of wire used. (b) Now consider the variation of resistivity with temperature. What power will the coil of part (a) actually deliver when it is heated to 1200 °C?

a) The resistance R of the heating coil depends on the resistivity ρ of the material, the cross sectional area A (dependent on the diameter d)and the length L of the wire.

1)
$$R = \rho \frac{L}{A} = \rho \frac{4L}{\pi d^2}$$

We can relate the power P of the heater at potential difference V, with the resistance R of the heating coil.

2)
$$P = \frac{V^2}{R}$$

The rest is math. The above two equations contain only two unknowns (R,L). Solving by substitution we can find the length of the wire.

$$L = \frac{\pi d^2 R}{4\rho} = \frac{\pi d^2 V^2}{4\rho P} = \frac{\pi \cdot \left(5 \cdot 10^{-4} \, m\right)^2 \cdot \left(110V\right)^2}{4 \cdot \left(1.5 \cdot 10^{-6} \, \Omega m\right) \cdot 500W} = 3.17m$$

b) In equation (2) we have to use a different value for the resistance in order to find the power at T=1200°C.

The temperature increase causes an increase in the resistivity of the heating coil. We would need to know the temperature coefficient of resistivity α for nichrome, in order to find the resistance of the coil at 1200°C.

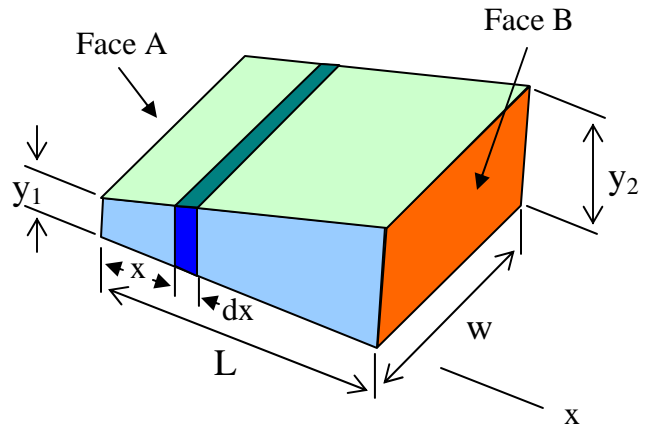*3)* $\qquad R_T = R \cdot [1 + \alpha(T - T_0)]$

Again solving by substitution

$$P_T = \frac{V^2}{R \cdot [1 + \alpha(T - T_0)]} = \frac{P}{1 + \alpha(T - T_0)} =$$

$$= \frac{500W}{1 + \left(0.4 \cdot 10^{-3} \, \frac{1}{°C}\right)(1200°C - 20°C)} = 340W$$

Problem 27.57

Material with uniform resistivity ρ is formed into a wedge is shown in Figure P27.71. Show that the resistance between face A and face B of this wedge is

$$R = \rho \frac{L}{w(y_2 - y_1)} \ln\left(\frac{y_2}{y_1}\right)$$

a) Both face A and face B form the contacts connected to the resistor (conductive wedge). Each face is therefore an equipotential surface and all equipotential surfaces are parallel to the two faces.

If the wedge is divided into differential slices parallel to the surfaces, each slice is a resistor with the length dx and cross-sectional area determined by the with and height of the slice

$$dR = \rho \frac{dx}{A} = \rho \frac{dx}{w \cdot \left(\dfrac{y_2 - y_1}{L} x + y_1\right)}$$

In the wedge, the slices are connected in series (back o back). Equivalent resistance of the system requires adding (integrating) the differential resistance of all slices. Therefore

$$R = \int_0^L \rho \frac{dx}{w \cdot \left(\dfrac{y_2 - y_1}{L} x + y_1\right)} = \frac{\rho}{w} \cdot \frac{L}{y_2 - y_1} \cdot \ln\left(\frac{y_2 - y_1}{L} x + y_1\right)\Big|_0^L =$$

$$= \frac{\rho}{w} \cdot \frac{L}{y_2 - y_1} \cdot \left(\ln\left(\frac{y_2 - y_1}{L} L + y_1\right) - \ln y_1\right) = \frac{\rho L}{w(y_2 - y_1)} \cdot \ln\left(\frac{y_2}{y_1}\right)$$