Chapter 27

Current and Resistance. Solutions of Home Work Problems

27.1 Problem 27.14 *(In the text book)*

A resistor is constructed of a carbon rod that has a uniform cross-sectional area of 5.00 mm^2. When a potential difference of 15.0 V is applied across the ends of the rod, the rod carries a current of 4.00 $\times 10^{-3} A$. Find

(a) the resistance of the rod and

(b) the rod's length.

Solution

(a) The resistance of the rod is:

$$R = \frac{\rho \ell}{A} = \frac{\Delta V}{I} = \frac{15}{4.00 \times 10^{-3}} = 3.75 \times 10^3 \ \Omega = 3.75 \ k\Omega$$

(b) The length of the rod is then:

$$\ell = \frac{RA}{\rho} = \frac{3.75 \times 10^3 \times 5.00 \times 10^{-6}}{3.5 \times 10^{-5}} = 536 \ m$$

27.2 Problem 27.30 *(In the text book)*

A carbon wire and a nichrome wire are connected in series, so that the same current exists in both wires. If the combination has a resistance of 10.0 $k\Omega$ at $0°C$, what is the resistance of each wire at $0°C$ so that the resistance of the combination does not change with temperature? The total or equivalent resistance of resistors in series is the sum of their individual resistances.

Solution

Let the resistance of the carbon wire at $0°C$ be R_c and that of the nichrome wire be R_n. Let also α_c and α_n be the temperature coefficients of the two wires respectively.We would like to find R_c and R_n that would make the total resistance of the whole wire independent of the temperature. The total resistance of the combination is $R = 10^4\ \Omega$. So, at $0°C$ we get:

$$R = R_n + R_c \tag{27.1}$$

At any other temperature R remains the same, so

$$R = R_c\left[1 + \alpha_c \Delta T\right] + R_n\left[1 + \alpha_n \Delta T\right] \tag{27.2}$$

Now subtracting Equation (27.1) from Equation (27.2)

$$\begin{aligned} 0 &= R_c\alpha_c\Delta T + R_n\alpha_n\Delta T \\ &= R_c\alpha_c + R_n\alpha_n \qquad \text{or} \\ R_c &= -R_n\frac{\alpha_n}{\alpha_c} \end{aligned} \tag{27.3}$$

Using Equation (27.3) in Equation (27.1) we get:

$$R = R_n - R_n\frac{\alpha_n}{\alpha_c} = R_n\left(1 - \frac{\alpha_n}{\alpha_c}\right) = R_n\left(\frac{\alpha_c - \alpha_n}{\alpha_c}\right) \qquad \text{and} \qquad R_n = R\left(\frac{\alpha_c}{\alpha_c - \alpha_n}\right)$$

Similarly:

$$R_c = R\left(\frac{\alpha_n}{\alpha_n - \alpha_c}\right)$$

Using the numerical values: $R = 10^4\ \Omega$, $\alpha_n = 4.00 \times 10^{-4}\ °C^{-1}$ and $\alpha_c = -5.00 \times 10^{-4}\ °C^{-1}$ we get:

$$R_n = 10^4 \times \left(\frac{-5.00 \times 10^{-4}}{-5.00 \times 10^{-4} - 4.00 \times 10^{-4}}\right) = 5.56\ k\Omega$$

and

$$R_c = 10^4 \times \left(\frac{4.00 \times 10^{-4}}{4.00 \times 10^{-4} + 5.00 \times 10^{-4}}\right) = 4.44\ k\Omega$$

27.3 Problem 27.46 *(In the text book)*

Residential building codes typically require the use of 12-gauge copper wire (diameter 0.2053 cm) for wiring receptacles. Such circuits carry currents as large as 20 A. A wire of smaller diameter (with a higher gauge number) could carry this much current, but the wire could rise to a high temperature and cause a fire.

(a) Calculate the rate at which internal energy is produced in 1.00 m of 12-gauge copper wire carrying a current of 20.0 A.

(b) What If? Repeat the calculation for an aluminum wire. Would a 12-gauge aluminum wire be as safe as a copper wire?

Solution

(a) The resistance of a 12-gauge one-meter copper wire is:

$$R = \frac{\rho\ell}{A} = \frac{\rho\ell}{\pi(d/2)^2} = \frac{4\rho\ell}{\pi d^2} = \frac{4 \times 1.7 \times 10^{-8} \times 1}{\pi(0.2053)^2} = 5.14 \times 10^{-3}\ \Omega$$

The rate of internal energy production is

$$P = I^2 R = (20)^2 5.14 \times 10^{-3} = 2.06\ W$$

(b) The internal power production in a 12-gauge aluminum wire is:

$$P_{Al} = I^2 R = \frac{I^2 4\rho_{Al}\ell}{d^2}$$

while that in an identical copper wire is:

$$P_{Cu} = I^2 R = \frac{I^2 4\rho_{Cu}\ell}{d^2}$$

Dividing the last two equations we get;

$$\frac{P_{Al}}{P_{Cu}} = \frac{\rho_{Al}}{\rho_{Cu}}, \qquad \text{and} \qquad P_{Al} = \frac{2.82 \times 10^{-8}}{1.70 \times 10^{-8}} \times 2.06 = 3.41\ W$$

Aluminum wire of the same length and diameter will get hotter than copper.

27.4 Problem 27.52 *(In the text book)*

The cost of electricity varies widely through the United States; \$0.120/kWh is one typical value. At this unit price, calculate the cost of

(a) leaving a 40.0-W porch light on for two weeks while you are on vacation,

(b) making a piece of dark toast in 3.00 min with a 970-W toaster, and

(c) drying a load of clothes in 40.0 min in a 5 200-W dryer.

Solution

You pay the electric company for the energy used, i.e. for:

$$E = P\Delta t$$

(a) For a 40-W lamp left on for two weeks, you consume:

$$P\Delta t = 40\ W \times (2\text{ week}) \times \left(\frac{7\ d}{1\text{ week}}\right) \times \left(\frac{86400\ s}{1\ d}\right)\left(\frac{1\ J}{1\ W\cdot s}\right) = 48.4\ MJ$$

The electric company charges per kWh, so

$$P\Delta t = 40\ W \times (2\text{ week}) \times \left(\frac{7\ d}{1\text{ week}}\right) \times \left(\frac{24\ h}{1\ d}\right)\left(\frac{1\ K}{1000}\right) = 13.4\ kWh$$

and the cost is:

$$\text{Cost} = 13.4 \times 0.12 = \$1.61$$

item The cost of making a toast is:

$$\text{Cost} = 970 \times 2 \times 7 \times 24 \times \frac{1}{1000} \times 0.12 = \$0.00582 = 0.582\ ¢$$

(b) The cost of drying cloths:

$$Cost = 5200 \times \frac{40}{60} \times \frac{1}{1000} \times 0.12 = \$0.416$$

27.5 Problem 27.68 *(In the text book)*

An oceanographer is studying how the ion concentration in sea water depends on depth. She does this by lowering into the water a pair of concentric metallic cylinders (Figure (27.68)) at the end of a cable and taking data to determine the resistance between these electrodes as a function of depth. The water between the two cylinders forms a cylindrical shell of inner radius r_a, outer radius r_b, and length L much larger than r_b. The scientist applies a potential difference ΔV between the inner and outer surfaces, producing an outward radial current I. Let ρ represent the resistivity of the water.

(a) Find the resistance of the water between the cylinders in terms of L, ρ, r_a, and r_b.

(b) Express the resistivity of the water in terms of the measured quantities L, r_a, r_b, ΔV, and I.

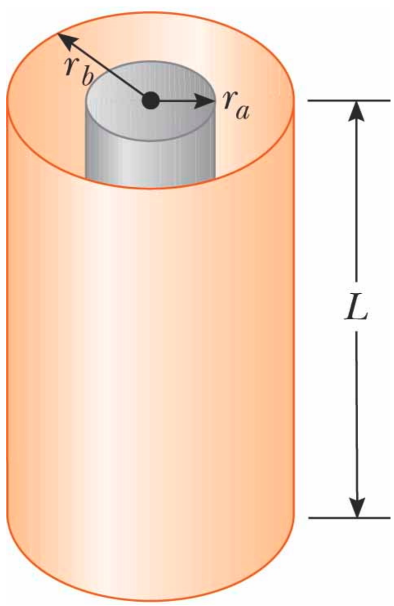

Figure 27.68:

Solution

(a) Take a thin cylindrical shell of water of radius r, thickness dr, and length L which contributes a resistance of dR:

$$dR = \frac{\rho dr}{A} = \frac{\rho dr}{2\pi r L} = \left(\frac{\rho}{2\pi L}\right)\frac{dr}{r}$$

The resistance of the whole water is the sum of the contributions of all the thin shells between r_a and r_b,

$$R = \left(\frac{\rho}{2\pi L}\right)\int_{r_a}^{r_b}\frac{dr}{r} = \left(\frac{\rho}{2\pi L}\right)\ln\left(\frac{r_b}{r_a}\right)$$

(b) The resistance $R = \Delta V/I$, so the resistivity ρ of the water is:

$$\rho = \frac{2\pi L \Delta V}{I \ln(r_b/r_a)}$$

x