Solutions

Math 321-01 Spring 2015
Quiz 8 29.04.15

Name: _____

Show all work clearly and in order, and circle your final answers. Justify your answers algebraically whenever possible; You have 15 minutes to take this 10 point quiz.

**1.** (*6 points*) About 20% of Kazakhstan population has type $B$ blood.

**a.** (*5 pts*) Among 3 selected people, what is the probability that only the 3rd one has type $B$?

success = type B

$X$ = 1st success $\sim$ Geom (0.2)

$P(X=3) = (1-p)^2 \cdot p = 0.8^2 \cdot 0.2 = 0.128$

**b.** (*1 pt*) (Bonus) What is the expected number of people to be tested until 3 type $B$ will occur?

$r = 3$
$p = 0.2$

$E(\text{Negative Binomial}) = \frac{r}{p} = \frac{3}{0.2} = 15$

**2.** (*6 points*) Suppose that a set of measurements $Y_1, Y_2, \cdots, Y_{100}$ is taken from a gamma pdf for which $E(Y) = 1.5$ and $Var(Y) = 0.75$.

**a.** (*3 pts*) What are $E(Y)$ and $Var(Y)$ for the general gamma pdf with parameters $r$ and $\lambda$?

$$E(Y) = \frac{r}{\lambda}$$

$$Var\,Y = \frac{r}{\lambda^2}$$

**b.** (*2 pts*) Determine the parameters $r$ and $\lambda$ for the gamma pdf in the problem.

$$\frac{r}{\lambda} = \frac{3}{2}, \quad \frac{r}{\lambda^2} = \frac{3}{4} \Rightarrow \quad r = 3, \quad \lambda = 2$$

**c.** (*1 pt*) (Bonus) How many $Y_i$'s would you expect to find in the interval $[1, 2]$?

$$p = \int_1^2 f_Y(y)\,dy, \qquad f_Y(y) = \frac{\lambda^r}{(r-1)!} \cdot y^{r-1} e^{-\lambda y} = 4y^2 e^{-2y}$$

$$= \int_1^2 4y^2 e^{-2y}\,dy = -\frac{1}{2} \cdot 4y^2 e^{-2y} \Big|_1^2 + \int_1^2 \frac{1}{2} 8y e^{-2y}\,dy$$

$$= -8e^{-4} + 2e^{-2} + \left(-\frac{1}{2}\right) \cdot 4y e^{-2y} \Big|_1^2 + \int_1^2 2e^{-2y}\,dy$$

$$= -8e^{-4} + 2e^{-2} - 4e^{-4} + 2e^{-2} - e^{-2y} \Big|_1^2$$

$$= -13e^{-4} + 5e^{-2}$$

$X = \#\, Y_i$ in $[1, 2]$

$X \sim Bin(100, p)$, $E(X) = 100 \cdot p$

$= 100(5e^{-2} - 13e^{-4})$