Math 321-01 Spring 2015
Quiz 4 25.02.15 Name: _____

Show all work clearly and in order, and circle your final answers. Justify your answers algebraically whenever possible; You have 20 minutes to take this 10 point quiz.

1. (*3 points*) Give an example of a random variable other than ones considered in this quiz.

X = number of heads appearing in 100 coin toss.

2. (*3 points*) A display case contains 30 gems, of which 8 are real diamonds and the rest 22 are fake diamonds. A burglar removes 4 gems at random, one at a time and without replacement. What is the probability that the last gem he steels is the second real diamond in the set of 4?

Let B = last gem is the 2nd real diamond in 4.

A = last gem is a diamond

C = exactly one of the first three is a diamond.

Then $P(B) = P(A \cap C) = P(C) \cdot P(A|C)$

$$P(C) = \frac{\binom{8}{1}\cdot\binom{22}{2}}{\binom{30}{3}}, \quad P(A|C) = \frac{\binom{7}{1}}{\binom{27}{1}} = \frac{7}{27}$$

Thus, $$P(B) = \frac{\binom{8}{1}\cdot\binom{22}{2}}{\binom{30}{3}} \cdot \frac{7}{27}$$

3. (*4 points*) Suppose a fair die is tossed 3 times. Let X be the largest of the three faces that appear.

(i) Find the pdf $p_X(k)$.

$$p_X(k) = P(X=k) = P(X \le k) - P(X \le k-1)$$

$$= \frac{k}{6} \cdot \frac{k}{6} \cdot \frac{k}{6} - \frac{(k-1)}{6} \cdot \frac{(k-1)}{6} \cdot \frac{(k-1)}{6}$$

$$p_X(k) = \frac{k^3 - (k-1)^3}{6^3} \quad \text{for } k = 1, 2, \ldots, 6$$

(ii) Find the cdf for the random variable X.

$$F_X(k) = P(X \le k) = \frac{k^3}{6^3}$$