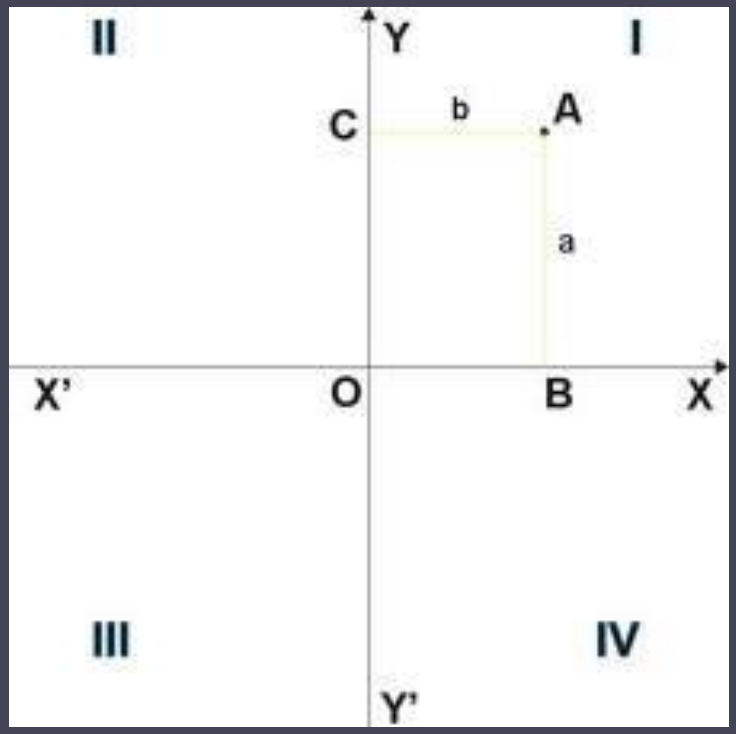
Introduction to Algorithms Lecture 5

By: Darmen Kariboz

Outline - Geometry

- Point in Plane
- Line equation
- Distance between Point and line
- Intersection of two lines
- Point in a Triangle
- Polygon area

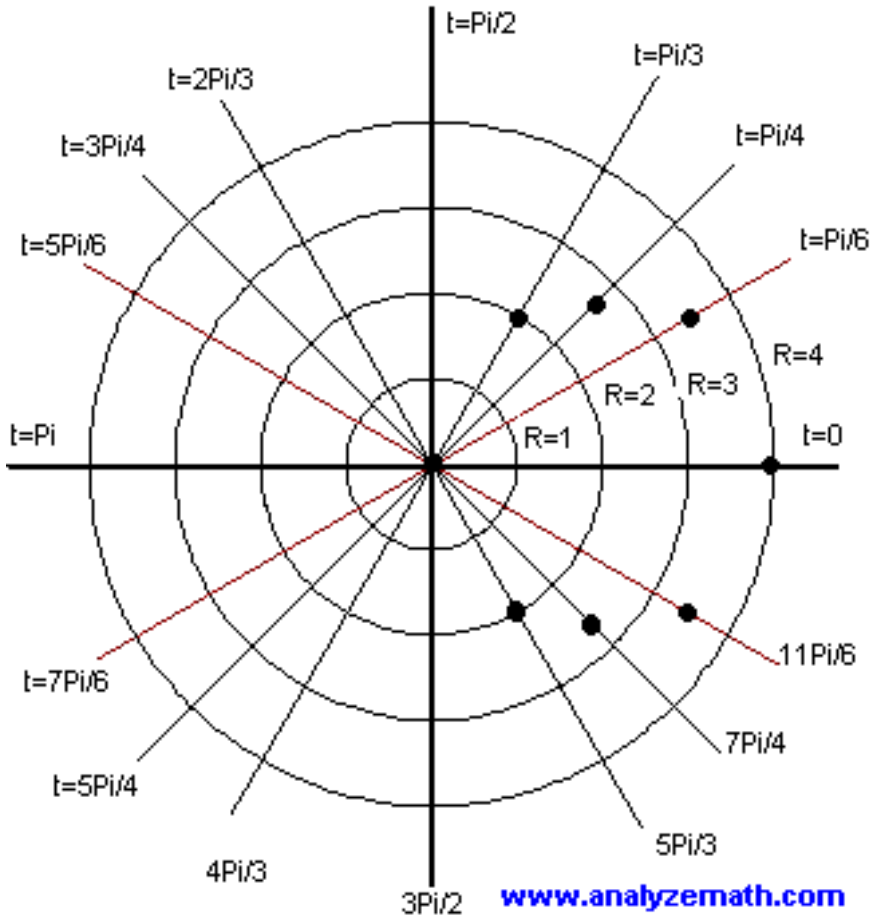
Point in Plane



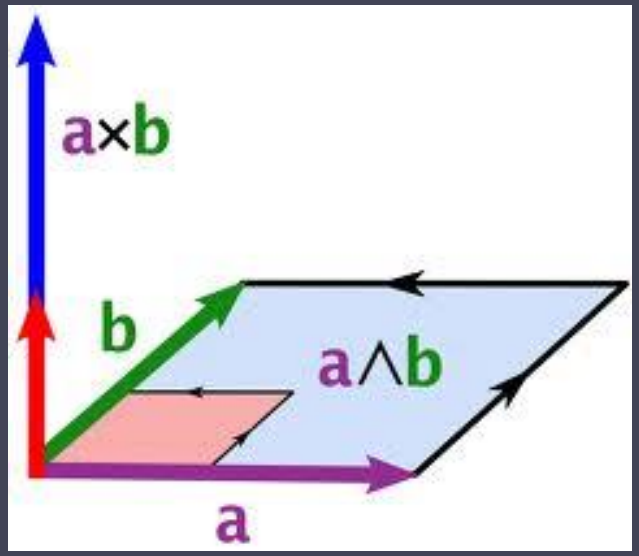
Every point in Descartes system is represented as two coordinates X and Y.

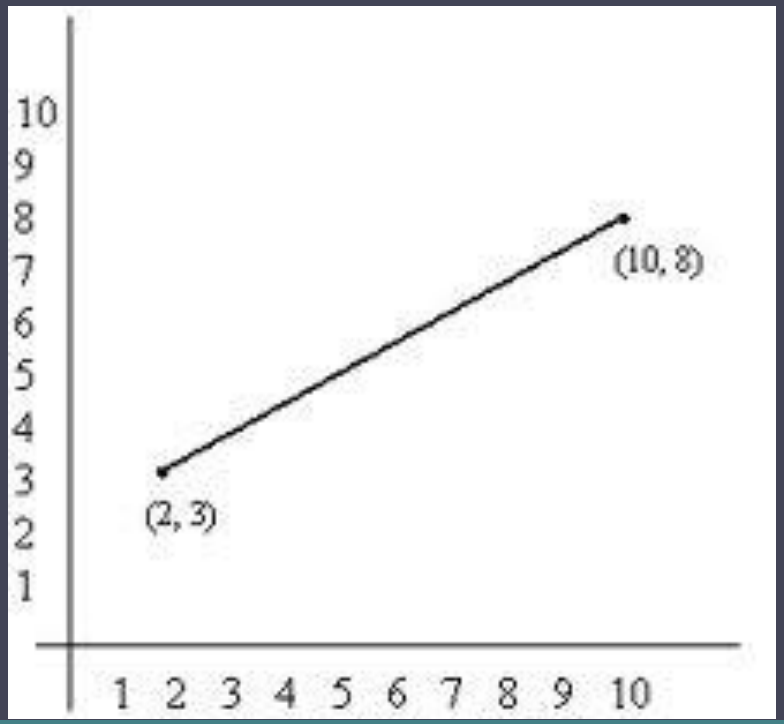
Polar system

- In mathematics, the **polar coordinate system** is a two-dimensional coordinate system in which each point on a plane is determined by a distance from a fixed point and an angle from a fixed direction.



- **X = r*cos(α)**
- **Y = r*sin(α)**





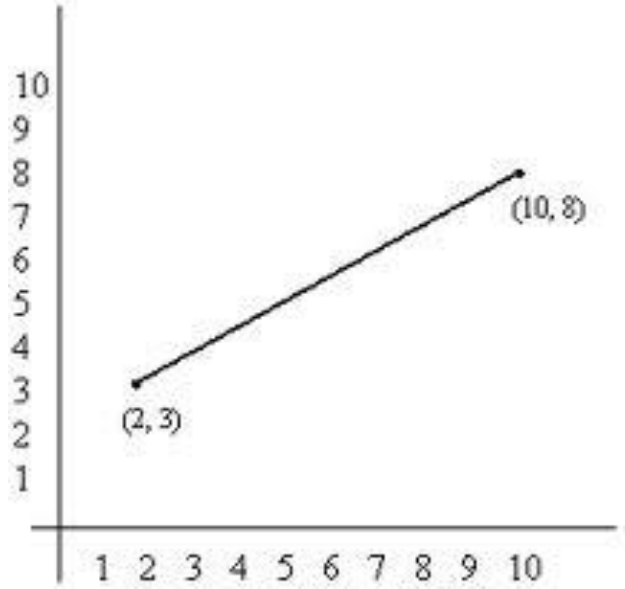
Line Equation

Line equation is an equation that describes line in a plane.

$Ax+By+C = 0$

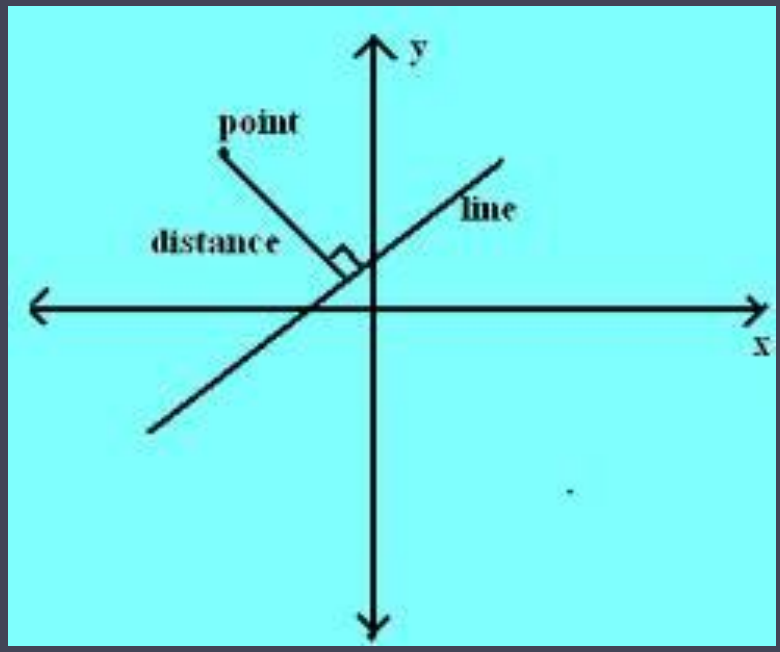
Line equation

- If two coordinates are given, line equation can be found.



- $A = y_1 - y_2$
- $B = x_2 - x_1$
- $C = - (x_1 * A + y_1 * B)$
-

Distance between point and Line



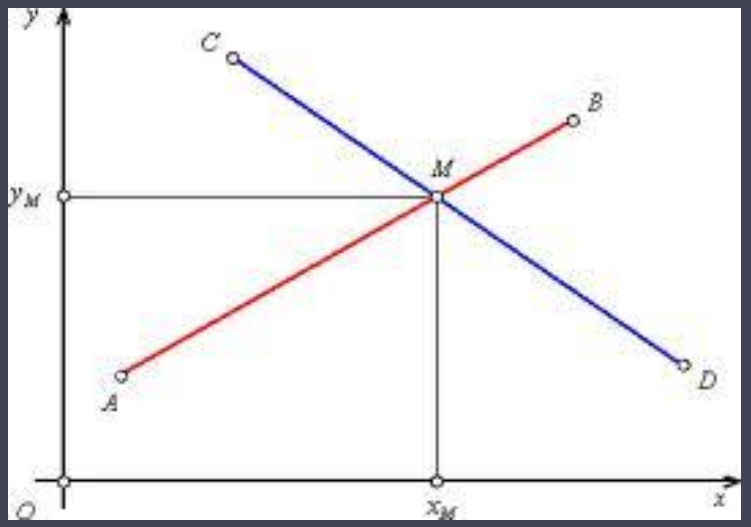
To find distance between line and point. You need to find **normal vector** to point.

Find distance between point and line

- Distance from (Xo ,Yo), to line can be found by given formula.

$$\frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}.$$

Intersection of two lines



Problem is to find coordinates of point, where two lines intersects

Intersection

- If lines are given in terms of line equations:
  A1*x+B1*y+C1=0 and
  A2*x+B2*y+C2=0
- if (A1*B2 - A2*B1 ≠ 0) then
- *x=(C1*B2-C2*B1)/(A1*B2-A2*B1)*
- *y=(A1*C2-A2*C1)/(A1*B2-A2*B1)*

Point in a triangle or not?

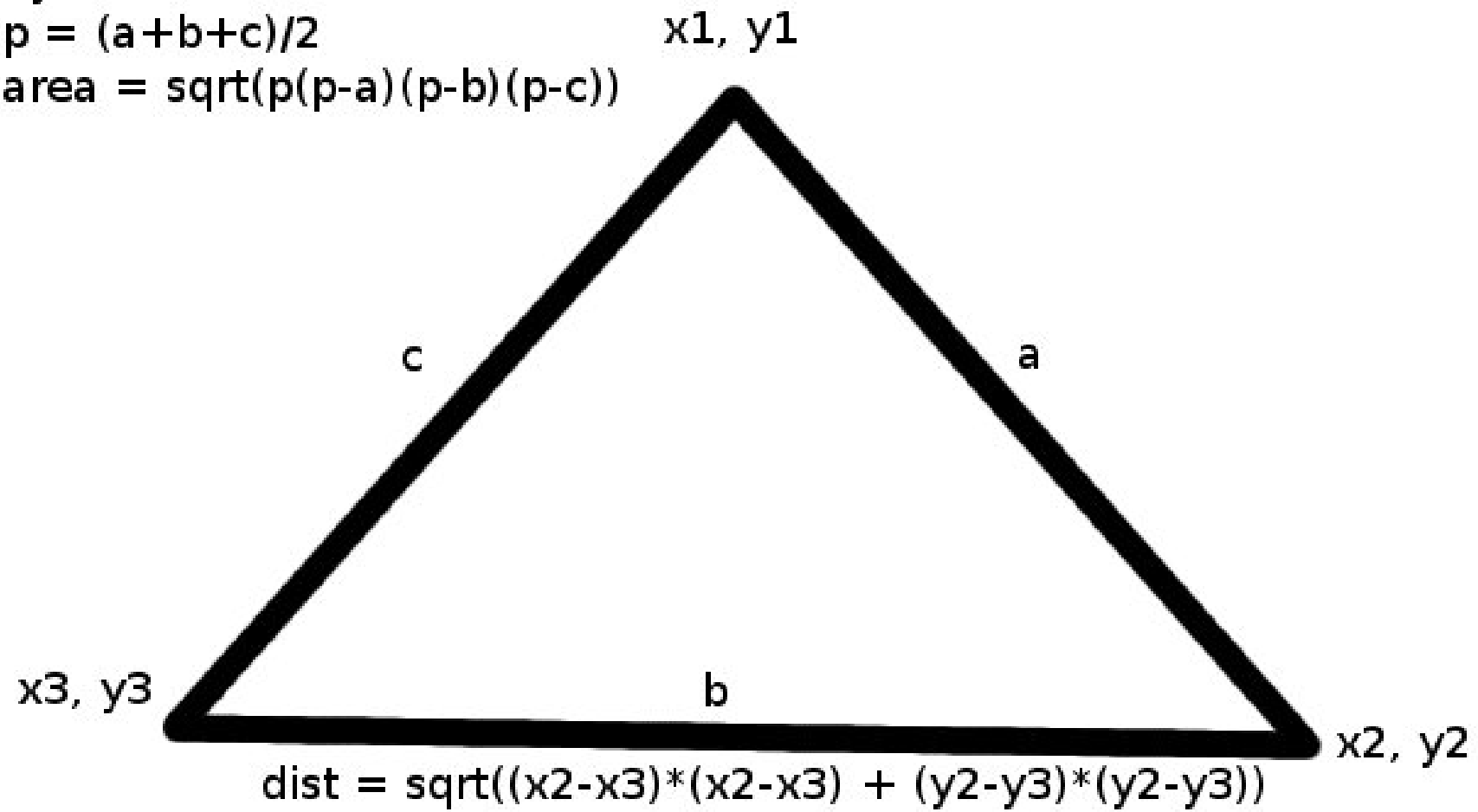
Problem is to determine if the given point inside the triangle or outside

By Heron's Formula

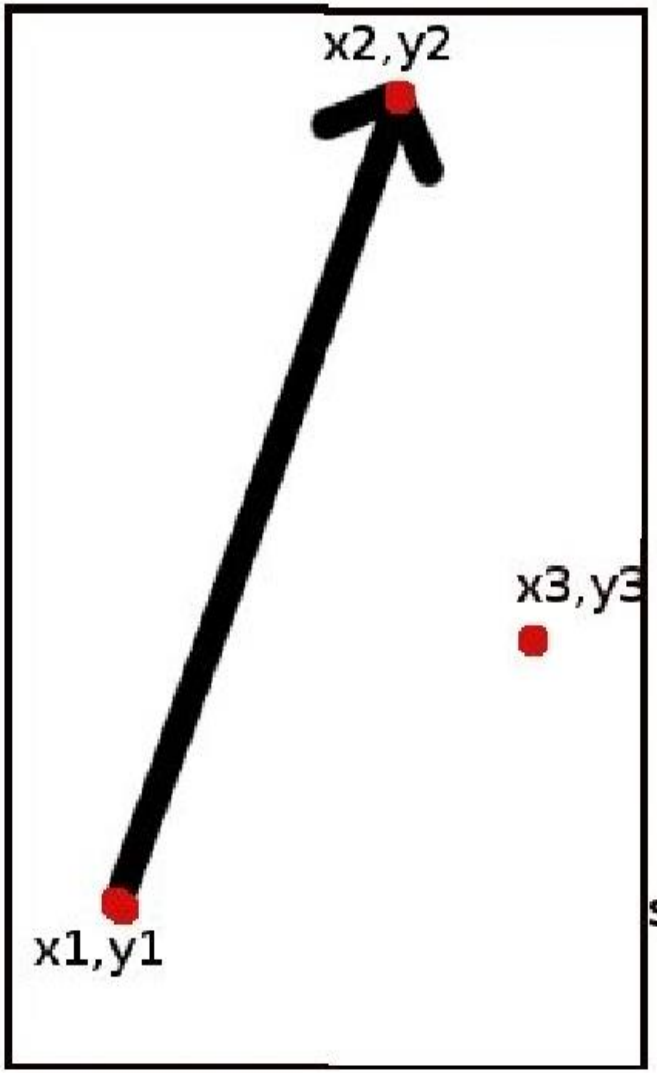
by Herone:

p = (a+b+c)/2

area = sqrt(p(p-a)(p-b)(p-c))



By Line Equation



$$\frac{x - x1}{x2 - x1} = \frac{y - y1}{y2 - y1}$$

$Ax+By+C=0$ formula of a line

$$(x-x1)(y2-y1)=(x2-x1)(y-y1)$$

$$(x-x1)(y2-y1)-(y-y1)(x2-x1)=0$$

$$x(y2-y1)-x1(y2-y1)-y(x2-x1)+y1(x2-x1)=0$$

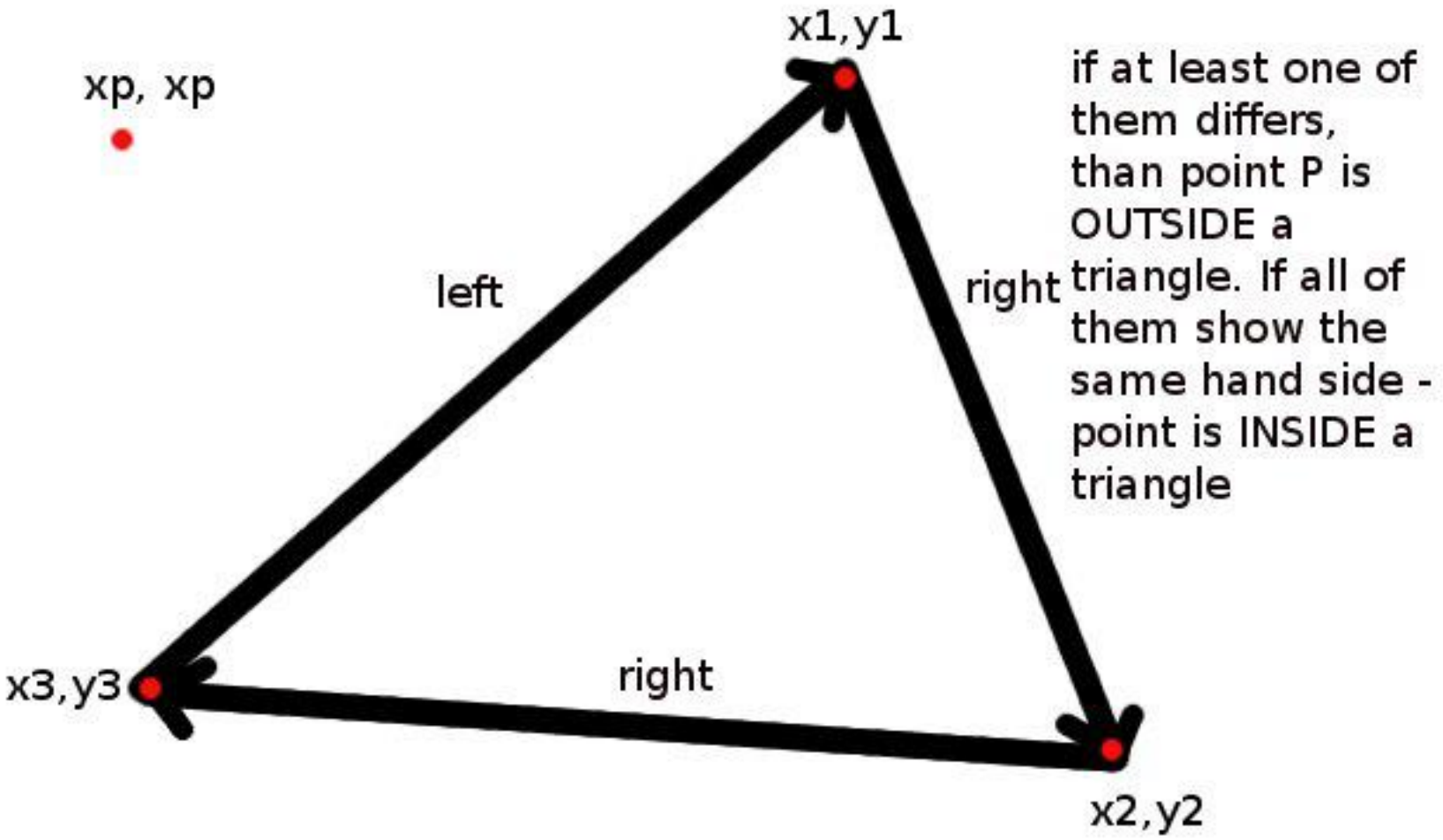
$$x(y2-y1)+y(x1-x2)+y1(x2-x1)-x1(y2-y1)=0$$

$A = y2-y1$
$B = x1-x2$
$C = -B*y1-A*x1$

$sign = A*x3+B*y3+C$

sign>0 left side
sign<0 right side
sign=0 on a line

By Line Equation



Polygon Area

Polygon(let's say convex polygon) is given by list of all corners coordinates in clockwise order.

Area of polygon(by Heron's Formula)

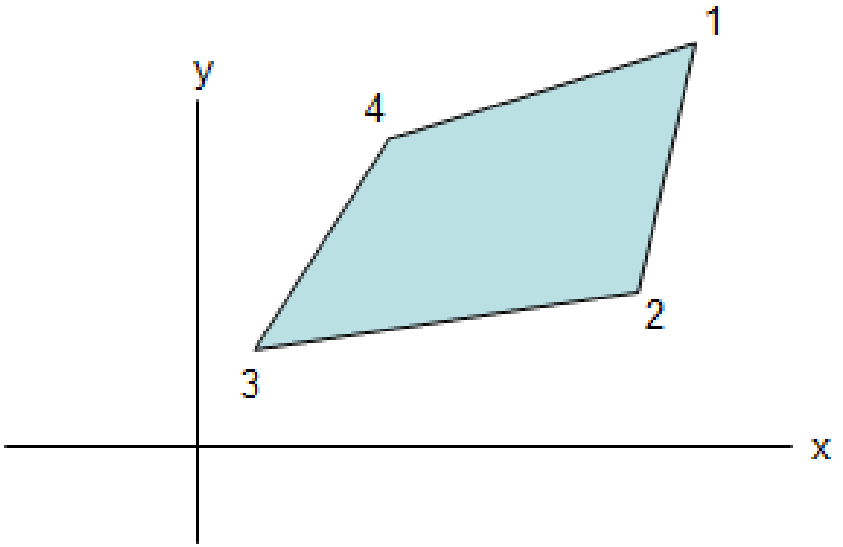
- Find central point of polygon(x,y)
- Find sum of all triangles formed by central point and each edge of polygon.
- Heron's formula:

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\text{where } s = \frac{1}{2}(a+b+c)$$

Area of Polygon

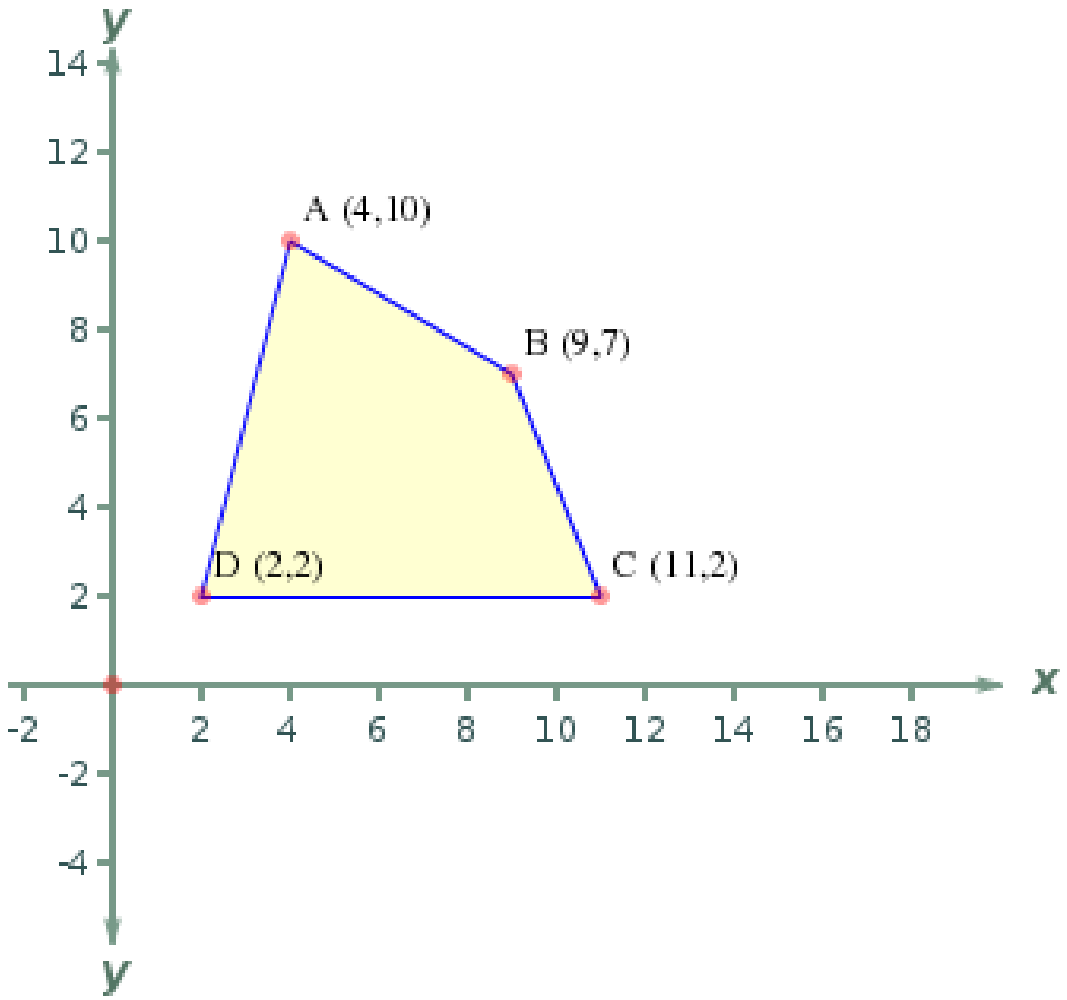
A method for finding the area of any polygon when the coordinates of its vertices are known.



General formula:

$$\left| \frac{(x_1 y_2 - y_1 x_2) + (x_2 y_3 - y_2 x_3) \ \ldots\ldots + (x_n y_1 - y_n x_1)}{2} \right|$$

Example



	x	y		
A	4	10		
B	9	7	28 - 90	=-62
C	11	2	18 - 77	=-59
D	2	2	22 - 4	=18
A	4	10	20 - 8	=12

Total -91

Area = |-91/2| =

45.5

Home Work

- **Polygon area**
- Compare two formulas of finding polygon areas. Output both values for given polygon.

- **Find intersection of two lines in 3D.**
- Given two lines. Each having 6 integer values. Describing lines in 3d.

Midterm next week

- During Lecture
- Lectures 1-5
- Paper based
- Theoretical Questions as well as Practical
- 5 Students with best results will go to OzYurt