Homework 6

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MAT 251: Probability and Mathematical Statistics

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Sections covered 4.2, 4.3, 4.4

Exercise 1. The average number of flaws in a fibre optic cable in 50m of cable is 1.2.
  (i) What is the probability of exactly three flaws in 150m of cable? (Answer: 0.212)
  (ii) What is the probability of at least two flaws in 100m of cable? (Answer: 0.691)
  (iii) What is the probability of exactly one flaw in the first 50m of cable and exactly one flaw in the second 50m of cable? (Answer: 0.13)

Exercise 2. The average number of visitors to a webserver per minute is 4, what is the probability that:
  (i) There are two or fewer visitors in one minute?; (Ans: 0.238)
  (ii) There are exactly two visitors in 30 seconds?. (Ans: 0.271)

Exercise 3. A coin is tossed until it comes up heads for the 20th time. Estimate the probability that more than 50 tosses are needed.

Exercise 4. Let $X_1, X_2, \ldots, X_{100}$ be a random sample from an exponential distribution with mean $1/2$. Use CLT to estimate $P(\sum_{i=1}^{100} X_i > 57)$
  (A) 0.08   (B) 0.016   (C) 0.31   (D) 0.38   (E) 0.46

Exercise 5. Let X represent the profit made on a random day in a certain store. Assume that X is approximately normal with mean \$360 and standard deviation \$50. What is the probability that the profit exceeds \$400?
  (A) 0.2119   (B) 0.2881   (C) 0.5319   (D) 0.7881   (E) 0.8450

Exercise 6. Suppose that in Abylaykhan avenue there were 5 car accidents occurred within 25 days.
  (a) What is the probability that 2 accidents will occur during the next 5 days? Answer: $1/(2e)$
  (b) Is the probability that 4 accidents will occur over the 10 days the square of your answer to part (a)?

Exercise 7. Products produced by a machine has a 3% defective rate.
  (a) What is the probability that the first defective occurs in the fifth item inspected?
  (b) What is the probability that the first defective occurs in the first five inspections?

Exercise 8. Humidity in Kaskelen is modelled as a normal distribution with mean 40% and standard deviation 3%. What is the probability that the humidity will exceed 44%?

Exercise 9. Suppose a radioactive source is metered for two hours, during which time the total number of alpha particles counted is 482. What is the probability that exactly three particles will be counted in the next two minutes?

Exercise 10. A fair coin is tossed two hundred times. Let $X_i = 1$ if the i-th toss comes up heads and $X_i = 0$ otherwise, $i = 1, 2, ..., 200$. Set $X = \sum_{i=1}^{200} X_i$. Approximate the probability $P(|X - E(X)| \leq 5)$.

Exercise 11. Assume that in a certain game you win 3$ with probability 0.2 and loose 1$ with probability 0.8. Assume that you played 100 times. Approximate the probability that you have won a positive amount.