

# Review

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**Exercise 1.** Suppose that three events -  $A$ ,  $B$ , and  $C$  - are defined on a sample space  $S$ . Use union, intersection, and complement operations to represent each of the following:

- (a) none of the three events occurs
- (b) all three of the events occur
- (c) only event  $A$  occurs
- (d) exactly one event occurs
- (e) exactly two event occurs

**Exercise 2.** Suppose that an urn contains 5 white balls and 3 black balls. A ball is drawn from the urn randomly (with all possibilities equally likely), and then a second ball is drawn randomly.

- (a) Describe the sample space  $S$  of the experiment.
- (b) What is the probability of getting two balls with the same color? Justify!
- (c) What is the probability of getting one ball of each color? Justify!

**Exercise 3.** A fair coin is tossed. If a head turns up, a fair die is tossed; if a tail turns up, two fair dice are tossed. What is the probability that the face (or the sum of the faces) showing on the die (or the dice) is equal to 6?.

**Exercise 4.** Suppose  $A$  and  $B$  are two events with  $P(A) = 0.4$  and  $P(A \cup B) = 0.7$ .

- (i) For what value of  $P(B)$  would  $A$  and  $B$  be mutually exclusive? Justify!
- (ii) For what value of  $P(B)$  would  $A$  and  $B$  be independent? Justify!

**Exercise 5.** Players  $A$ ,  $B$ , and  $C$  toss a fair coin in order. The first to throw a head wins. What is the probability that the player  $B$  wins the game.

**Exercise 6.** A continuous random variable  $Y$  has cdf

$$F_Y(y) = \begin{cases} 0 & \text{if } y < 1 \\ \ln y & \text{if } 1 \leq y \leq e \\ 1 & \text{if } e < y \end{cases}$$

- a) Find  $P(Y < 2)$
- b) Find  $P(2 < Y < 2.5)$
- c) Find pdf  $f_Y(y)$ .

**Exercise 7.** A manufacturer has 100 memory chips in stock, 4% of which are likely to be defective. A random sample of 30 chips is selected and shipped to a factory that assembles laptops. Let  $X$  denote the number of computers that receive faulty memory chips. Find  $E(X)$ .

**Exercise 8.** Suppose that 100 fair dice tossed.

- a) What is the expected value of the sum of the faces showing?
- b) Find the standard deviation of the sum of the faces showing? ( $\sqrt{91/6 - 3.5^2} \approx 1.7$ )
- c) Estimate the probability that the sum of the faces showing exceeds 370.

**Exercise 9.** A sample of 37 observations is taken. The sample mean is 80.96 and the sample standard deviation is 12. Then, for the hypothesis  $H_0 : \mu = 85$  v.s.  $H_1 : \mu \neq 85$

- a) use  $\alpha = 5\%$  to test the hypothesis
- b) use  $\alpha = 0.15$  to test the hypothesis
- c) Construct 99% Confidence interval for the population mean

**Exercise 10.** A sample of 6 observations, 18, 20, 16, 19, 17, 18, is taken. Suppose the population is normally distributed. Then,

- a) using 5% significance level, test the hypothesis  $H_0 : \mu = 18$  v.s.  $H_1 : \mu > 18$ .
- b) Compute  $p$ -value and interpret it.
- c) Find 90% confidence interval for the true mean.

**Exercise 11.** A nutritionist found that in a sample of 80 families, 22% said they eat apples at least once a week. Construct the 90% CI for the population proportion.