

# Problem

1. Write a pure Python function that computes  $V(n)$ :

For  $n \geq 1$ :

$$V(n) = \sum_{k=0}^{n-1} U(k)$$

$$U(n) = [\log(\sin(n) + V(n))]^4$$

For  $n = 0$ :

$$V(0) = 42$$

You can use: `from math import sin, log`

2. Time this function for  $n = 20$ .

3. Profile the running time.

4. Use a memoization method to speed up the calculation. Is it faster?

5. Write a `numpy` version of this function. You can use:

```
from numpy import sin as sin_, log as log_, sum as sum_
```

Is it faster?

# Solution

1.

```
In [1]: from math import sin, log

def U(n):
    return log(sin(n) + V(n)) ** 4

def V(n):
    if n == 0:
        return 42
    else:
        return sum([U(k) for k in range(n)])
```

2.

In [2]: `%timeit V(20)`

194 ms  $\pm$  1.22 ms per loop (mean  $\pm$  std. dev. of 7 runs, 10 loops each)

3.

In [3]: `%prun V(20)`

4719347 function calls (2622201 primitive calls) in 0.734 seconds

Ordered by: internal time

ncalls	totttime	percall	cumtime	percall	filename:lineno(function)
1048576/20	0.299	0.000	0.786	0.039	943180024.py:3(U)
1048577/1	0.248	0.000	0.640	0.640	943180024.py:6(V)
1048575	0.076	0.000	0.076	0.000	{built-in method math.sin}
1048575	0.056	0.000	0.056	0.000	{built-in method math.log}
524288	0.052	0.000	0.052	0.000	{built-in method builtins.sum}
14	0.000	0.000	0.000	0.000	socket.py:626(send)
1	0.000	0.000	0.000	0.000	{method 'execute' of 'sqlite3.
Connection' objects}					
6/4	0.000	0.000	0.027	0.007	{method 'run' of '_contextvar
s.Context' objects}					
1	0.000	0.000	0.000	0.000	{method 'send' of '_socket.soc
ket' objects}					
1	0.000	0.000	0.000	0.000	{method 'disable' of '_lsprof.
Profiler' objects}					
2	0.000	0.000	0.000	0.000	{method 'recv' of '_socket.soc
ket' objects}					
2/1	0.000	0.000	0.730	0.730	{built-in method builtins.exe
c}					
72	0.000	0.000	0.000	0.000	enum.py:1544(_get_value)
4	0.000	0.000	0.034	0.008	base_events.py:1910(_run_once)
152/148	0.000	0.000	0.000	0.000	{built-in method builtins.isin
stance}					
16	0.000	0.000	0.000	0.000	enum.py:1551(__or__)
5	0.000	0.000	0.000	0.000	attrsettr.py:43(__getattr__)
5	0.000	0.000	0.000	0.000	attrsettr.py:66(_get_attr_opt)
2	0.000	0.000	0.000	0.000	{method '__exit__' of 'sqlite
3.Connection' objects}					
3/1	0.000	0.000	0.000	0.000	{method 'poll' of 'select.epol
l' objects}					
1	0.000	0.000	0.000	0.000	inspect.py:3102(_bind)
31	0.000	0.000	0.000	0.000	enum.py:726(__call__)
2	0.000	0.000	0.000	0.000	socket.py:774(recv_multipart)

_queue)	1	0.000	0.000	0.006	0.006	kernelbase.py:302(poll_control
	8	0.000	0.000	0.000	0.000	enum.py:1562(__and__)
	2	0.000	0.000	0.000	0.000	iostream.py:216(_check_mp_mod
e)	1	0.000	0.000	0.000	0.000	poll.py:80(poll)
	1	0.000	0.000	0.000	0.000	traitlets.py:1527(_notify_obse
	1	0.000	0.000	0.000	0.000	history.py:845(writeout_cache)
rvers)	1	0.000	0.000	0.000	0.000	queues.py:256(get_nowait)
	6/4	0.000	0.000	0.027	0.007	events.py:86(_run)
	1	0.000	0.000	0.000	0.000	iostream.py:718(_rotate_buffer
s)	31	0.000	0.000	0.000	0.000	enum.py:1129(__new__)
	3/1	0.000	0.000	0.000	0.000	selectors.py:451(select)
	8	0.000	0.000	0.000	0.000	traitlets.py:676(__get__)
	1	0.000	0.000	0.000	0.000	queues.py:225(get)
	3	0.000	0.000	0.000	0.000	ioloop.py:742(_run_callback)
	1	0.000	0.000	0.000	0.000	iostream.py:616(_flush)
	6	0.000	0.000	0.000	0.000	typing.py:392(inner)
	1	0.000	0.000	0.000	0.000	history.py:839(_writeout_outpu
	1	0.000	0.000	0.000	0.000	kernelbase.py:324(_flush)
t_cache)	1	0.000	0.000	0.000	0.000	zmqstream.py:427(flush)
	2	0.000	0.000	0.000	0.000	traitlets.py:3631(set)
	2	0.000	0.000	0.000	0.000	iostream.py:278(_really_send)
rom_self)	1	0.000	0.000	0.000	0.000	selector_events.py:129(_read_f
	10	0.000	0.000	0.000	0.000	{built-in method builtins.hasa
	ttr}	2	0.000	0.000	0.000	traitlets.py:718(_validate)
s)	1	0.000	0.000	0.000	0.000	iostream.py:710(_flush_buffer
	25	0.000	0.000	0.000	0.000	{built-in method builtins.len}
	2	0.000	0.000	0.000	0.000	traitlets.py:689(set)
	1	0.000	0.000	0.000	0.000	decorator.py:199(fix)
	3	0.000	0.000	0.000	0.000	base_events.py:814(_call_soon)

tate)	8	0.000	0.000	0.000	0.000	traitlets.py:629(get)
	3	0.000	0.000	0.000	0.000	zmqstream.py:663(_rebuild_io_s
	2	0.000	0.000	0.000	0.000	traitlets.py:3474(validate)
	4	0.000	0.000	0.000	0.000	queue.py:97(empty)
er)	7	0.000	0.000	0.000	0.000	base_events.py:734(time)
	3	0.000	0.000	0.000	0.000	zmqstream.py:686(_update_handl
	1	0.000	0.000	0.000	0.000	queues.py:186(put)
	7	0.000	0.000	0.000	0.000	{built-in method builtins.max}
s_events)	5	0.000	0.000	0.000	0.000	selector_events.py:750(_proces
	2	0.000	0.000	0.000	0.000	traitlets.py:727(_cross_valida
	2	0.000	0.000	0.000	0.000	traitlets.py:708(__set__)
	2	0.000	0.000	0.000	0.000	traitlets.py:3624(validate_ele
ments)	5	0.000	0.000	0.000	0.000	<frozen importlib._bootstrap>:
t)	1390(_handle_fromlist)					
	1	0.000	0.000	0.000	0.000	queues.py:209(put_nowait)
	1	0.000	0.000	0.000	0.000	inspect.py:3237(bind)
	2	0.000	0.000	0.007	0.003	iostream.py:276(<lambda>)
s)	1	0.000	0.000	0.000	0.000	traitlets.py:1512(_notify_trai
	2	0.000	0.000	0.024	0.012	zmqstream.py:583(_handle_event
	3	0.000	0.000	0.000	0.000	threading.py:299(__enter__)
	3	0.000	0.000	0.000	0.000	events.py:36(__init__)
e)	2	0.000	0.000	0.000	0.000	socket.py:703(send_multipart)
	4	0.000	0.000	0.000	0.000	typing.py:1258(__hash__)
	1	0.000	0.000	0.000	0.000	asyncio.py:225(add_callback)
	1	0.000	0.000	0.000	0.000	traitlets.py:1523(notify_chang
s)	1	0.000	0.000	0.000	0.000	inspect.py:2918(apply_default
	1	0.000	0.000	0.000	0.000	zmqstream.py:468(update_flag)
	1	0.000	0.000	0.000	0.000	threading.py:311(_acquire_rest



ore)	10	0.000	0.000	0.000	0.000	{method 'popleft' of 'collecti
ons.deque' objects}	5	0.000	0.000	0.000	0.000	{built-in method builtins.geta
ttr}	2	0.000	0.000	0.000	0.000	queues.py:322(_consume_expire
d)	2	0.000	0.000	0.000	0.000	traitlets.py:2304(validate)
	2	0.000	0.000	0.012	0.006	zmqstream.py:556(_run_callbac
k)	9	0.000	0.000	0.000	0.000	{method 'append' of 'collectio
ns.deque' objects}	7	0.000	0.000	0.000	0.000	{built-in method time.monotoni
c}	2	0.000	0.000	0.000	0.000	iostream.py:213(_is_master_pro
cess)	3	0.000	0.000	0.000	0.000	{method 'acquire' of '_thread.
lock' objects}	1	0.000	0.000	0.000	0.000	threading.py:424(notify_all)
	1	0.000	0.000	0.000	0.000	inspect.py:2865(args)
	8	0.000	0.000	0.000	0.000	{method '__exit__' of '_threa
d.lock' objects}	2	0.000	0.000	0.000	0.000	base_events.py:1895(_add_calllb
ack)	28	0.000	0.000	0.000	0.000	typing.py:2154(cast)
	2	0.000	0.000	0.000	0.000	typing.py:1465(__subclasscheck
__)	1	0.000	0.000	0.000	0.000	concurrent.py:182(future_set_r
esult_unless_cancelled)	2	0.000	0.000	0.000	0.000	base_events.py:785(call_soon)
	3	0.000	0.000	0.000	0.000	threading.py:302(__exit__)
	2	0.000	0.000	0.000	0.000	{method 'set_result' of '_asyn
cio.Future' objects}	2	0.000	0.000	0.000	0.000	typing.py:1194(__instancecheck
__)	6	0.000	0.000	0.000	0.000	{built-in method builtins.nex
t}						

	2/1	0.000	0.000	0.729	0.729	<string>:1(<module>)
	6	0.000	0.000	0.000	0.000	{method 'append' of 'list' obj
ects}						
	2	0.000	0.000	0.012	0.006	iostream.py:157(_handle_event)
	1	0.000	0.000	0.000	0.000	history.py:833(_writeout_input
_cache)						
	1	0.000	0.000	0.000	0.000	threading.py:394(notify)
	1	0.000	0.000	0.000	0.000	threading.py:308(_release_sav
e)						
	4	0.000	0.000	0.000	0.000	zmqstream.py:542(sending)
	2	0.000	0.000	0.000	0.000	{built-in method builtins.issu
bclass}						
	1	0.000	0.000	0.000	0.000	threading.py:627(clear)
	2	0.000	0.000	0.022	0.011	zmqstream.py:624(_handle_recv)
	2	0.000	0.000	0.000	0.000	selectors.py:275(_key_from_fd)
	1	0.000	0.000	0.000	0.000	queues.py:173(qsize)
	1	0.000	0.000	0.000	0.000	inspect.py:2888(kwargs)
	1	0.000	0.000	0.000	0.000	queues.py:317(__put_internal)
	1	0.000	0.000	0.000	0.000	{built-in method posix.getppi
d}						
	2	0.000	0.000	0.000	0.000	traitlets.py:3486(validate_ele
ments)						
	1	0.000	0.000	0.000	0.000	base_events.py:838(call_soon_t
hreadsafe)						
	4	0.000	0.000	0.000	0.000	{method 'get' of 'dict' object
s}						
	5	0.000	0.000	0.000	0.000	{method 'upper' of 'str' objec
ts}						
	4	0.000	0.000	0.000	0.000	queue.py:209(_qsize)
	2	0.000	0.000	0.000	0.000	<frozen abc>:121(__subclasssche
ck__)						
	1	0.000	0.000	0.000	0.000	{built-in method _heapq.heappo
p}						
	2	0.000	0.000	0.000	0.000	{built-in method math.ceil}
	1	0.000	0.000	0.000	0.000	{method 'values' of 'mappingpr
oxy' objects}						
	1	0.000	0.000	0.000	0.000	_base.py:337(_invoke_callback

s)	2	0.000	0.000	0.000	0.000	{built-in method posix.getpid}
	1	0.000	0.000	0.000	0.000	selector_events.py:141(_write_
to_self)	2	0.000	0.000	0.000	0.000	{built-in method _abc._abc_sub
classcheck}	1	0.000	0.000	0.000	0.000	{built-in method _thread.alloc
ate_lock}	2	0.000	0.000	0.000	0.000	{built-in method _contextvars.
copy_context}	10	0.000	0.000	0.000	0.000	inspect.py:2777(kind)
	1	0.000	0.000	0.000	0.000	history.py:55(only_when_enable
d)	2	0.000	0.000	0.000	0.000	{method '__enter__' of '_threa
d.Lock' objects}	4	0.000	0.000	0.000	0.000	{built-in method builtins.has
h}	1	0.000	0.000	0.000	0.000	poll.py:31(register)
	2	0.000	0.000	0.000	0.000	{built-in method builtins.min}
	1	0.000	0.000	0.000	0.000	decorator.py:229(fun)
	1	0.000	0.000	0.000	0.000	unix_events.py:81(_process_sel
f_data)	2	0.000	0.000	0.000	0.000	{method '__exit__' of '_threa
d.RLock' objects}	5	0.000	0.000	0.000	0.000	zmqstream.py:538(receiving)
	3	0.000	0.000	0.000	0.000	{method 'items' of 'mappingpro
xy' objects}	1	0.000	0.000	0.000	0.000	futures.py:394(_call_set_stat
e)	1	0.000	0.000	0.000	0.000	_base.py:537(set_result)
	1	0.000	0.000	0.000	0.000	{method '__enter__' of '_threa
d.RLock' objects}	1	0.000	0.000	0.000	0.000	{method 'items' of 'dict' obje
cts}	5	0.000	0.000	0.000	0.000	base_events.py:2005(get_debug)
	1	0.000	0.000	0.000	0.000	queues.py:177(empty)
	1	0.000	0.000	0.000	0.000	queues.py:309(_get)

```

1      0.000      0.000      0.000      0.000 threading.py:314(_is_owned)
1      0.000      0.000      0.000      0.000 zmqstream.py:659(_check_close
d)
2      0.000      0.000      0.000      0.000 {method 'extend' of 'list' obj
ects}
2      0.000      0.000      0.000      0.000 iostream.py:255(closed)
3      0.000      0.000      0.000      0.000 base_events.py:539(_check_clos
ed)
4      0.000      0.000      0.000      0.000 inspect.py:3058(parameters)
1      0.000      0.000      0.000      0.000 queues.py:312(_put)
4.    2      0.000      0.000      0.000      0.000 {built-in method builtins.ite
r}
2      0.000      0.000      0.000      0.000 {method 'cancelled' of 'async

```

In [2]: `from functools import lru_cache`

```

@lru_cache(maxsize=None)
def cached_U(n):
    return log(sin(n) + V(n)) ** 4

def cached_V(n):
    if n == 0:
        return 42
    else:
        return sum([cached_U(k) for k in range(n)])

```

In [5]: `%timeit cached_V(20)`

```

1      0.000      0.000      0.000      0.000 locks.py:224(clear)
886 ns ± 5.35 ns per loop (mean ± std. dev. of 7 runs, 1,000,000 loops each)

```

5.

```
In [3]: from numpy import sin as sin_, log as log_, sum as sum_, arange

def numpy_U(n):
    return log_(sin_(n) + numpy_V(n)) ** 4

def numpy_V(n):
    if n == 0:
        return 42
    else:
        return sum_([numpy_U(k) for k in arange(n)])
```

```
In [7]: %timeit numpy_V(20)
```

2.7 s ± 43.7 ms per loop (mean ± std. dev. of 7 runs, 1 loop each)

