Symmetry Adapted Approach towards Efficient Trotter Decomposition

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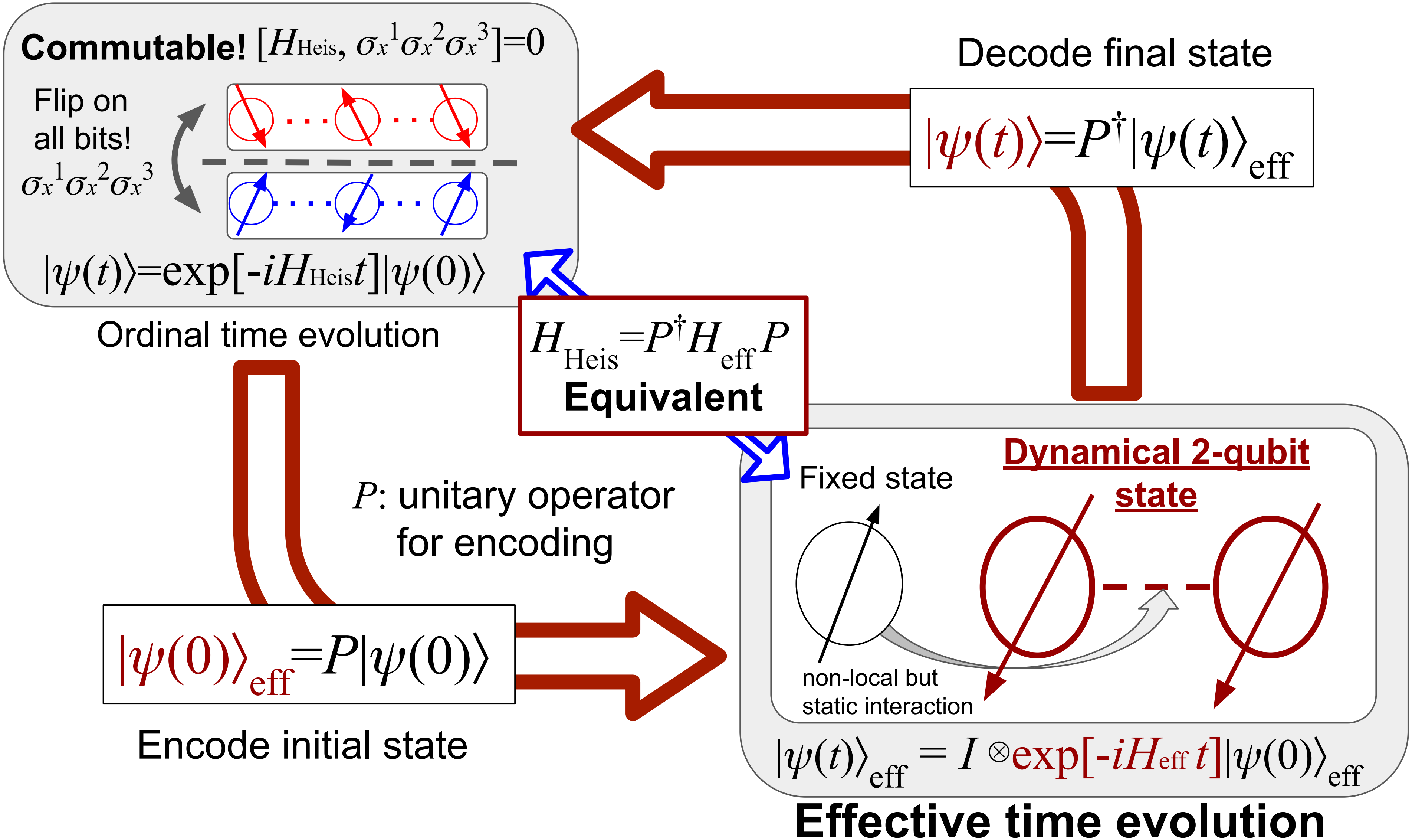
The latest version of the slides: https://docs.google.com/presentation/d/1aMk9SCzIXwKfbbBZ6ho_1levsaFfuWQ5FDAKZfLde0Y/edit?usp=sharing
Source Code (=submitted files) on GitHub: https://github.com/BOBO1997/osp_solutions



Commutable! $[H_{\text{Heis}}, \sigma_x^1\sigma_x^2\sigma_x^3]=0$
Flip on all bits! $\sigma_x^1\sigma_x^2\sigma_x^3$
$|\psi(t)\rangle=\exp[-iH_{\text{Heis}}t]|\psi(0)\rangle$
Ordinal time evolution
Decode final state
$|\psi(t)\rangle=P^\dagger|\psi(t)\rangle_{\text{eff}}$
$H_{\text{Heis}}=P^\dagger H_{\text{eff}}P$
Equivalent
P: unitary operator for encoding
$|\psi(0)\rangle_{\text{eff}}=P|\psi(0)\rangle$
Encode initial state
Fixed state
Dynamical 2-qubit state
non-local but static interaction
$|\psi(t)\rangle_{\text{eff}}=I\otimes\exp[-iH_{\text{eff}}t]|\psi(0)\rangle_{\text{eff}}$
Effective time evolution

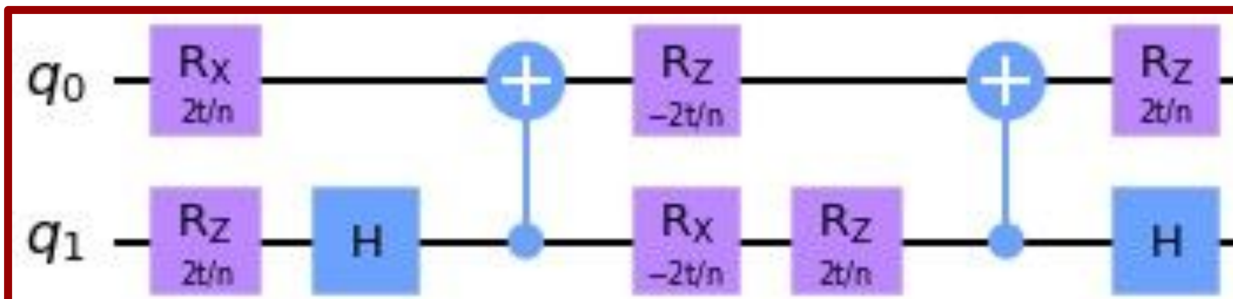
The effective Hamiltonian

$$H_{\text{eff}} = \underbrace{\sigma x^2 + \sigma z^3}_{(=A)} + \underbrace{\sigma z^2 + \sigma x^3}_{(=B)} - (\underbrace{\sigma z^2 \sigma x^3 + \sigma x^2 \sigma z^3}_{(=C)})$$

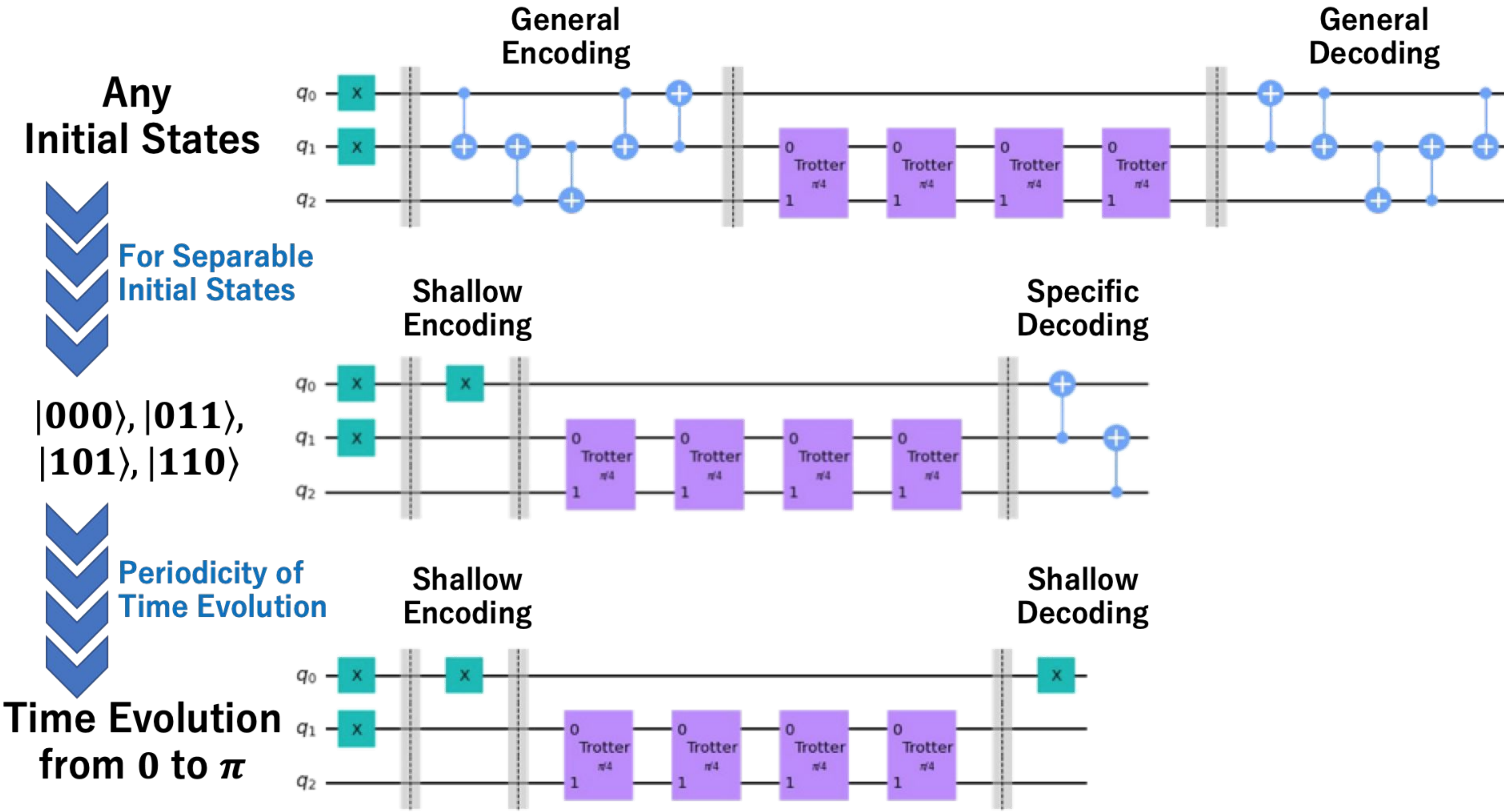
Trotter decomposition

$$e^{-iH_{\text{eff}}\, t} \sim \left(e^{-iAt/n} e^{-iB\, t/n} e^{iC\, t/n}\right)^n$$

The Trotter Block

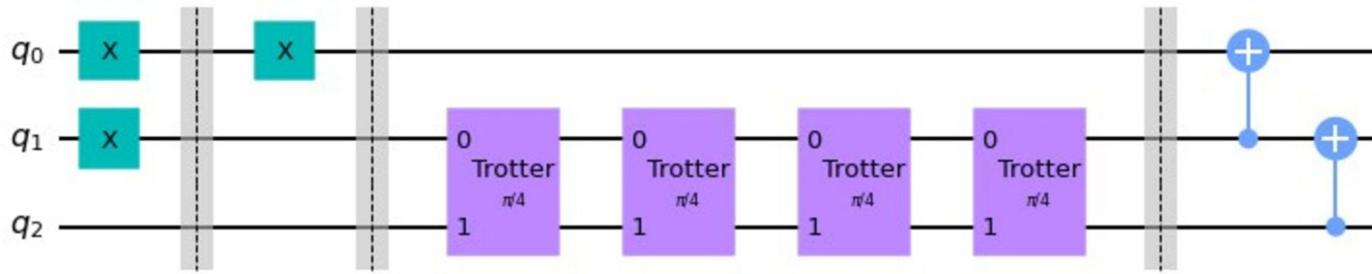


After optimization,
the depth is independent of the Trotter steps!!!



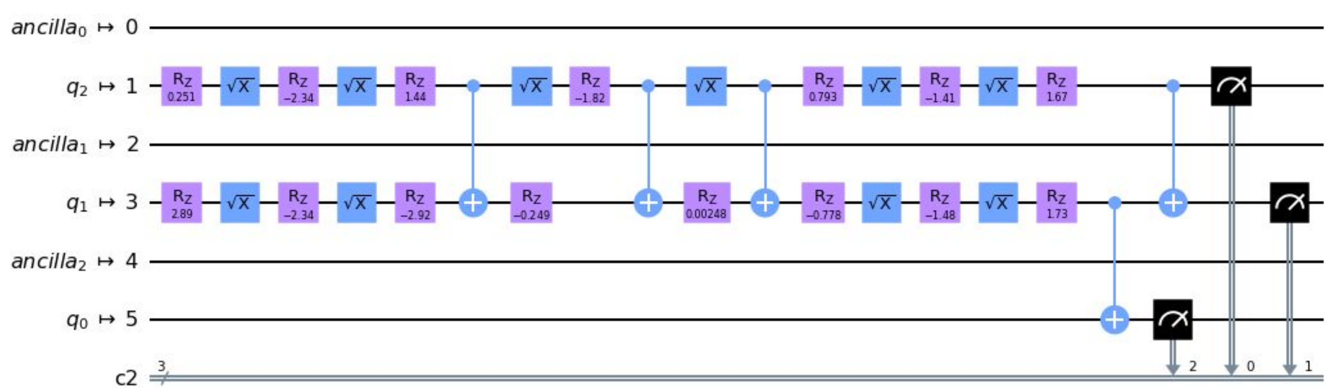
Any Initial States
For Separable Initial States
|000⟩, |011⟩, |101⟩, |110⟩
Periodicity of Time Evolution
Time Evolution from 0 to π
General Encoding
General Decoding
Shallow Encoding
Specific Decoding
Shallow Encoding
Shallow Decoding
q0
q1
q2
X
Trotter
π/4

CNOT depth: 2n+2
for n trotter steps



qiskit.compiler.transpile
(optimization_level=3)

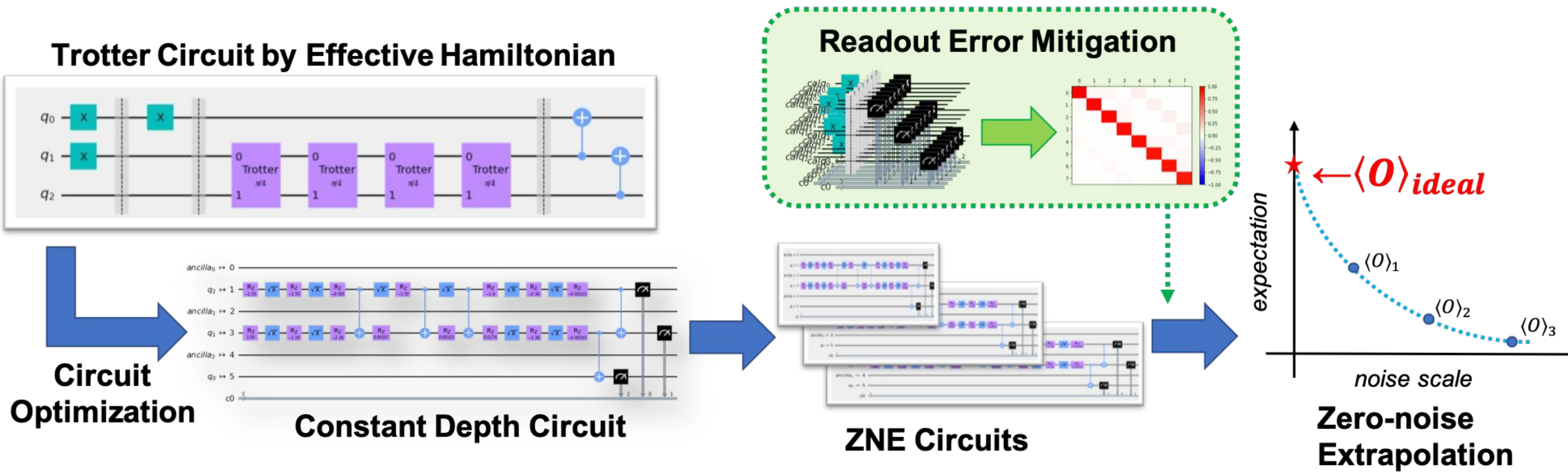
CNOT depth: 3+2
constant depth!!



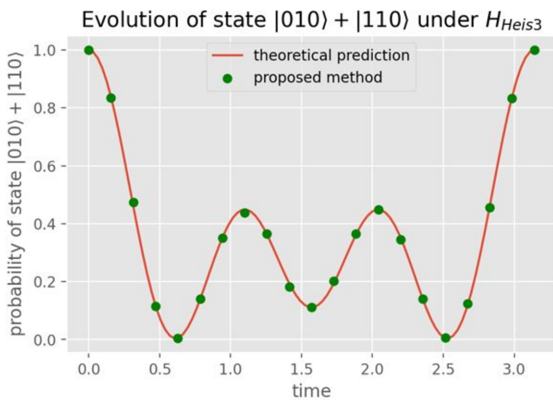
<u>Remark</u>

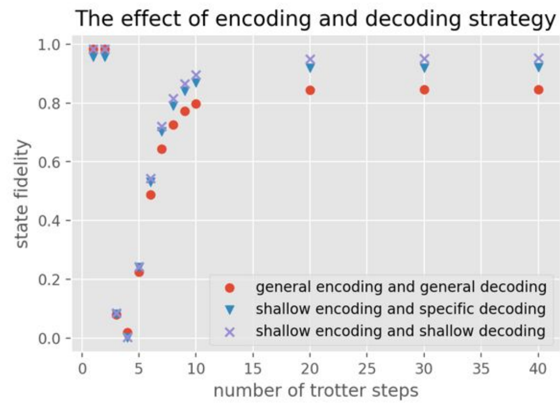
- qiskit.compiler.transpile provides a strong evidence of this depth reducibility
- Rigorous theoretical proof → Future work

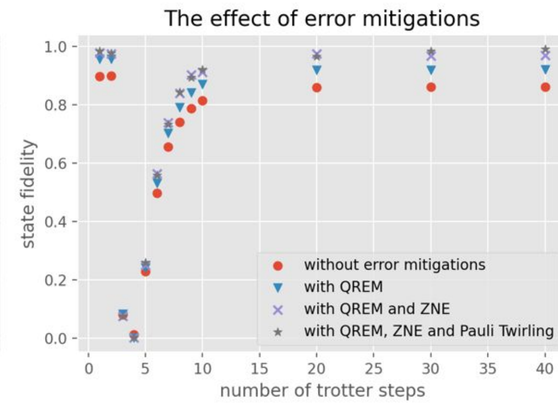
Workflow for each Pauli measurement in tomography circuits



Numerical Simulations







Noise-free simulation of the proposed method

The proposed method computes the time evolution correctly

Noisy simulation of the proposed method

- Even general encoding achieves fidelity > 0.8
- Error mitigation is effective → fidelity > 0.99
- 30 Trotter steps are enough (fidelity converges)

Results

Fidelity

Settings	`fake_jakarta`	`ibmq_jakarta`
General encoding and general decoding		
without any QEM	0.7856 ± 0.0015	0.8039 ± 0.0048
with QREM	0.8448 ± 0.0015	0.9032 ± 0.0054
with QREM and ZNE	0.9393 ± 0.0053	$\boxed{0.9866 \pm 0.0017}$
with QREM, ZNE and Pauli Twirling	0.9801 ± 0.0031	-
Shallow encoding and specific decoding		
without any QEM	0.8631 ± 0.0017	0.8637 ± 0.0041
with QREM	0.9234 ± 0.0016	0.9728 ± 0.0040
with QREM and ZNE	0.9840 ± 0.0024	$\boxed{0.9857 \pm 0.0043}$
with QREM, ZNE and Pauli Twirling	0.9714 ± 0.0048	0.9624 ± 0.0167
Shallow encoding and shallow decoding		
without any QEM	0.8863 ± 0.0012	0.8803 ± 0.0044
with QREM	0.9533 ± 0.0017	0.9852 ± 0.0061
with QREM and ZNE	0.9855 ± 0.0036	$\boxed{0.9929 \pm 0.0015}$
with QREM, ZNE and Pauli Twirling	0.9801 ± 0.0031	0.9768 ± 0.0034