Using DFS Search and Enumerate Method to Find All Solutions in 13 Convex Figures in Tangram Game

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***Abstract*—Tangram, a well-known traditional puzzle, is a tool in elementary education which is often associated with creative thinking. It consists of 7 boards (five triangles, one square and one parallelogram). Using tangram, we could build over 1600 kinds of shapes, such as houses, towers, and animals. However, it is very difficult to supplement a complete graphic with 7 boards. Therefore, this program was developed. In this program, the author used the DFS search algorithm and enumeration method to find all the solutions in the 13 convex figures in the Tangram. Then the author improved this search method to make the search algorithm run faster. The program is built using the Qt application development framework and C++ computer language and uses a user interface to display all the different results about the 13 convex graphics. The result of the research is that every convex polygon made of seven boards is not unique. Some graphics have only 16 transformations, but some will produce 224 different combinations of positions and colors.**

Keywords: DFS Algorithm; Tangram; convex figure; enumerate method

I. INTRODUCTION

Tangram can be used in many fields. In the Beijing Olympic Games, the traditional Tangram elements can be used in some posters [1]. In the field of fashion design, the arrangement of 7 boards can give people some fashionable ideas to create new clothes [2]. Therefore, there are many studies on Tangram. In 1942, the "A Theorem on the Tangram" improved the use of 7 boards can splice only and only if 13 convex polygon figures [3]. However, for each graphic, how many ways they can be combined using 7 boards is not mentioned. Therefore, the program is developed. This program may help in in the field of fashion design, designers sometimes find that the graphics or colors that appear on the clothes do not meet their aesthetic standards, but if graphics are changed, they will cause other shapes to change. Then arrange all the graphics and let the designer get from it is a good choice to solve the problem. In the graphic cryptography area, the program may provide a method in unlock the password [5]. Because through the enumeration method of this program, you can search for all the solution sets of a certain pattern. In education, especially in preschool education and basic education, Tangram is also widely cited. Through various solutions of splicing graphics, students can think creatively and understand geometric concepts in the game [6, 7].

This paper uses DFS search and enumeration methods to find all solutions in 13 convex graphs. Through the DFS search algorithm, all the points in the convex polygon are found and saved in the array. Using the enumeration method, the jigsaw puzzle (five triangles, one square, and one parallelogram) is rotated and matched at all points. Find the appropriate position of each, and finally use the backtracking algorithm to find all the solutions in the convex polygon.

II. PROGRAM DESIGN RULES AND STANDARDS

Before developing the program, the author needs to make some rules and standards. Tangram (five triangles, one square and one parallelogram) and 13 convex polygon figures are composed of many half squares, each half square (triangle) is of this shape (Figure 1). Definition: The length of a leg of an isosceles right triangle is 1 unit.

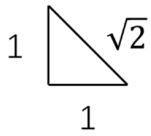

Figure 1. Basic element of the graphic

A. The stander of seven pieces of tangram

Tangram consists of 7 boards, and there are shapes and colors (Figure 2). Two big triangles (red and magenta), one middle tangram (green), two small triangles (blue and cyan), one square (silver) and one parallelogram (yellow). The function of color is to display the result in the user interface to make it beautiful and intuitive.

In the program, the author will default their tangram ID (tangramID) from left to right (Figure 2). For example, the large red triangle will be called tangramID_1, the green middle triangle will be called tangramID_3, and so on.

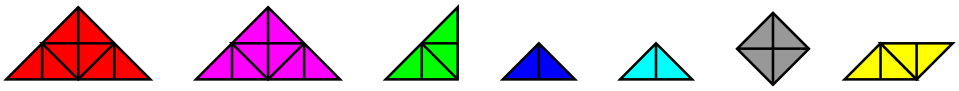

Figure 2. The shape of the 7 boards and their corresponding colors

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B. *The stander for 13 convex polygonal pieces*

There are only and only if 13 types of convex polygons that can be assembled in tangram [3]. 13 convex polygon figures are composed of many triangles (Figure 1), each of them is composed of 32 triangles. These are all of them, as shown below (Figure 3).

In the program, the author according to their numbers as tangram convex figures ID (Tangram_Convex_Figures). For example, the first one will be called Tangram_Convex_Figures_1, the last one will be called Tangram_Convex_Figures_13.

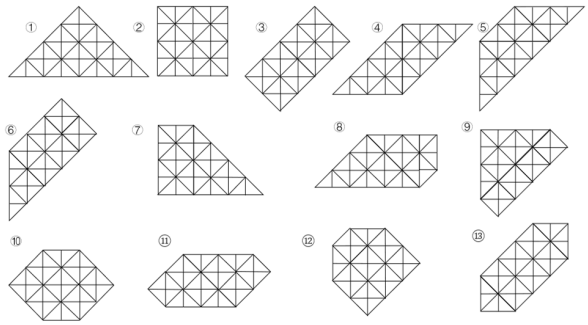

Figure 3. 13 convex polygonal pieces

III. WHAT ARE THE STEPS OF TANGRAM SOFTWARE DEVELOPMENT?

This part introduces the modeling process of figures and how to achieve the conditions to achieve the method to find all the solutions. This will be discussed in two separate steps: *A.* The process of modeling, and *B.* Achieve requirements of the program.

A. *The process of modeling*

The modeling process is divided into 13 convex polygon modeling and tangram modeling. This will be discussed in two separate steps, *1)* 13 convex polygon modeling and *2)* Tangram modeling.

1) 13 convex polygon modeling

Among the 13 convex polygonal figures, this is the standard for establishing a rectangular coordinate system (Figure 4). We can calculate the coordinates of the vertices of the convex polygon, according to these points, we can find the functional analytic formula ($f(x) = kx + b,\ k \neq 0$) of the outermost edge of the polygon, then use the for loop to enumerate all the integer coordinates in the convex polygon, and then match each coordinate to the inner side of the edge enclosed by the linear equation of two variables, and finally all the points in the graph can be found.

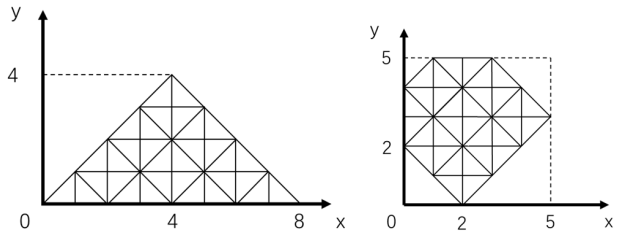

Figure 4. Coincidence and non-coincidence of edge and coordinate system

For example, the figure Tangram_Convex_Figures_1 at the bottom left (Figure 4, left) can be calculated by the three equations $f_1(x) = x$, $f_2(x) = 0$ and $f_3(x) = -x + 8$. Then for each of each pair (x, y), this will be constrained by the inequalities $f_1 < 0$, $f_2 > 0$ and $f_3 < 0$, finally we can get all the pairs in Tangram_Convex_Figures_1.

In the C language, a very important concept is the pointer. It can handle some problems efficiently. In this program, we have already obtained all the collections of a convex polygon, and there must be only 8 directions for a point (Figure 5), so when there is a line segment between two points. At this time, according to the positional relationship in Figure 5, point this point to the target point.

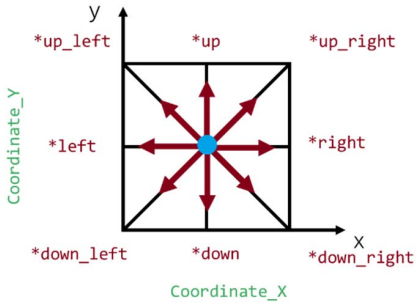

Figure 5. A point has 8 directions, and each direction name

So far, we have recorded the model of 13 convex polygons in the program, and run it when the program starts

2) Tangram modeling

In this part of tangram modeling, the idea of building a model is similar to that of convex polygon modeling. Both pointers are used to find points that are related to them (a line segment between two points) based on the coordinates of the points. However, it is worth mentioning that, first of all, in the tangram, we should realize that each board is not only one case, but the graphics need to be rotated. Secondly, the parallelogram is special because it needs to consider the generation of mirrored graphics. Therefore, the author divides it into three categories: triangle rotation, square rotation, and parallelogram rotation. Then, the modeling methods will be discussed one by one.

Theorem 1: only 4 rotations need to be considered

In Tangram, because each board is composed of several isosceles right-angled triangles (Figure 1), the internal angle of each board must be a multiple of 45 degrees, so we only need to take a certain state of the board as the basic state. Yes, then 45

degrees, 90 degrees, 135 degrees, 180 degrees, 225 degrees, 270 degrees, 315 degrees, 360 degrees (0 degrees), these 8 discussions are enough, but in this program, we don't need this a lot of. We only need to rotate 4 times, because we also need to be restricted by the edge of the figure that needs to coincide with the convex polygon after the rotation.

Take tangramID_4 in Tangram_Convex_Figures_1 as an example (Figure 6), with a point as the center, 8 situations (Figure 6, left and right) will appear when the board rotates, but the edge of the right graphics board does not coincide with the convex polygon. Therefore, only 4 rotations need to be considered.

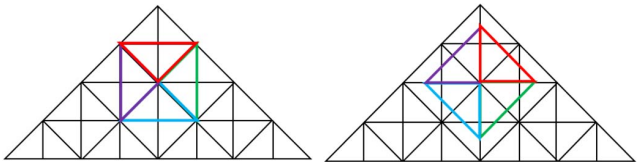

Figure 6. only 4 situations need to be discussed for one point

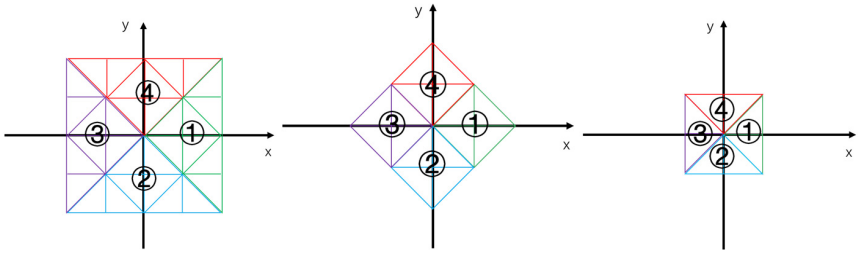

Figure 7. All possible rotations of all triangles in the Tangram

b) Square rotation

The rotation of a square is similar to the rotation of a triangle, but the difference is that there will be repetitions when the square is rotated, so we need to perform a deduplication operation. In this procedure, the author puts the deduplication process at the end. When a collection is found, it will be compared with all previously found collections. If it is found to indicate the same location, the situation will be discarded. Details will appear in 3.2 Achieve requirements of the program.

There is only one square in the Tangram. According to Theorem 1, each category will produce 4 kinds of rotations (Figure 8). The number in the figure is the square rotation ID (rotationID).

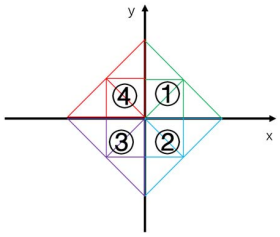

Figure 8. All possible rotations of square in the Tangram

c) Parallelogram rotation

In the Tangram, the parallelogram is the most complicated figure. Not only need to consider the repetitive shape when rotating, but the parallelogram also has the problem of mirror symmetry. When developing the program, the author spent a lot of time thinking about why some answers were not calculated by the computer, finally found that the problem appeared in the mirror symmetry of the parallelogram. The parallel quadrilaterals appearing in Figure 2 is named tangramID_7, and the parallelogram generated by mirroring is named tangramID_8.

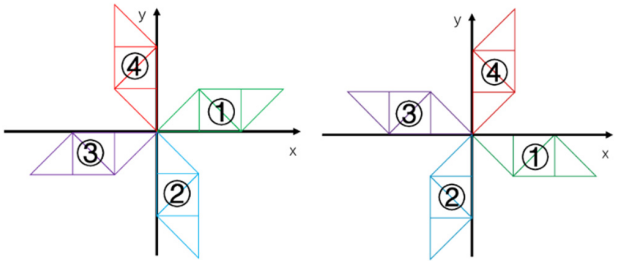

Figure 9. All possible rotations of Parallelogram and its mirror image symmetry in the Tangram

Tangram has only one parallelogram. However, we need to be aware that it has a mirror-symmetrical figure, and then according to Theorem 1, we can draw 8 different ways of rotating a parallelogram (Figure 9).

B. Achieve requirements of the program

In this paper, the author has mainly described the establishment of the model. Including seven figures in the

tangram and 13 convex polygonal shapes. In 3.2, the author will mainly focus on how to complete the goal map. The important steps have 4 steps, matching, judgment, deletion and de-duplication.

For the points in a set, we perform a matching operation, then a judgment operation, and finally a deletion operation.

a) What does the match operation mean

The matching operation is a function. The input parameters are point coordinates, tangramID and rotationID. The function return value is a Boolean (true or false).

Then, we expand the graph according to the three parameters passed in (according to the given point coordinates, tangramID and rotationID, we can build a triangle, square or parallelogram model at the current point coordinates), when the vertices of the board can be When it coincides with the vertices of the convex polygon, it means that the board is inside the convex polygon, and then we perform the judgment operation. Return true if the match is successful, otherwise false.

Take case Tangram_Convex_Figures_1, point: (2,0), tangramID_5, rotationID_3 and rotationID_4 as an example (Figure 10).

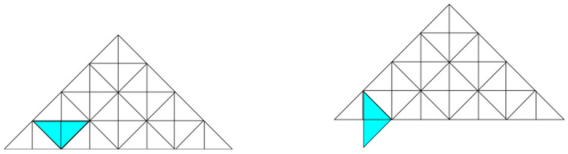

Figure 10. The graph on the left satisfies the matching condition, while the graph on the right does not meet the condition

b) What does the judge operation mean

The judge operation is a function. The input parameters are point coordinates, tangramID and rotationID. The function return value is a Boolean (true or false).

When we complete the matching operation, the judgment operation will be performed. The function of the judgment operation is to confirm that the graphics can cover the area found by the matching operation. By judging whether all the points of the convex polygon's point coordinate expansion map have the same direction vector (according to the current tangramID and rotationID, all the fixed direction vectors of the current map can be known), and whether the area is occupied.

Take case Tangram_Convex_Figures_1, tangramID_1, tangramID_3 as an example. When tangramID_1 uses rotation ID_1 to occupy (2,2) points (Figure, left), tangram_3 cannot occupy the position with rotation_3 at (5,3) points, because there will be overlapping parts (Figure 11, right).

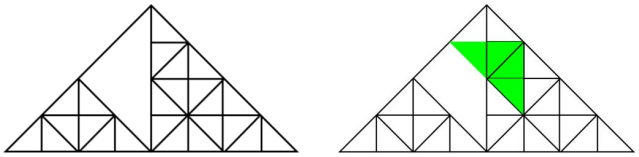

Figure 11. Due to overlap, tangramID_3 cannot occupy this area

c) What does delete operation mean

The delete operation is a function. The input parameters are point coordinates, tangramID and rotationID. The function return value is a Boolean (true or false).

When the judgment operation is satisfied, the delete operation will be executed in the next step. The meaning of the delete operation is to prevent other boards from overlapping when looking for positions. By changing the pointer relationship between the center point and the associated point (8 direction pointers related to the center point, Figure 5) to a null pointer, the delete operation function is realized.

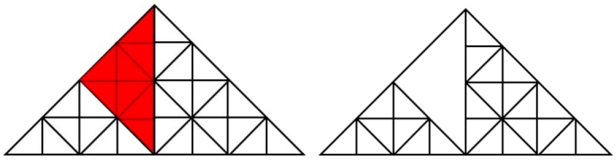

Figure 12. When the judgment operation is completed, the pointer between the internal point and the point is changed to a null pointer

d) What does de-duplication operation mean

When we have completed a complete search (solutions for all 7 boards in the Tangram have been found), we need to perform a deduplication process. Because squares and parallelograms reappear during the rotation. The way to remove duplication is to convert tangramID, rotationID, and point coordinates into vertex coordinates, and then compare the position of each chessboard in this solution with other solutions. If it does not exist, add it to the solution.

e) Use enumeration ideas to find all the solutions

The method of finding all solutions is similar to the method of enumeration. The author uses the backtracking algorithm to traverse all the points sorted by the DFS algorithm. For each point, we will match each plate of the Tangram in turn, and rotate each plate 4 times, that is, each point will perform 28 matching operations (7 each plate rotates 4 times). If the match is successful, the judgment operation and the deletion operation are performed. When all seven boards have found solutions as the stopping condition, then we perform deduplication processing, and finally we will find a set of solutions, that is, all solutions of a convex polygon

IV. OPTIMIZATION OF THE PROGRAM

A. Use the idea of greedy algorithm to make the program run faster

There are seven boards in Tangram. For different search types, the time to finally find the solution set is different. In “How do adults solve the number tangram problem?”, It mentioned that experimenters tend to match larger, discernible objects before smaller ones [4]. Therefore, the author matches the order of the tangrams according to the area from largest to smallest and finds that the search time is greatly reduced. Through the optimized search method, the search time will be

greatly reduced, and the time will be reduced about 81.07%.

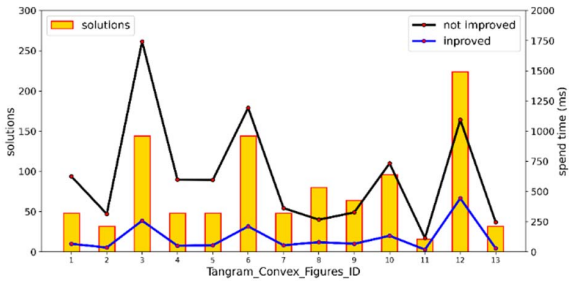

Figure 13. Comparison of the time spent on the two algorithms

V. CONCLUSION

According to the enumeration process of the computer, the data of all the solutions in 13 convex polygons can be finally obtained, which can be obtained (Table 1, The order of FigureID corresponds to Figure 3).

TABLE I. THE NUMBER OF TOTAL SOLUTIONS FOR 13 CONVEX POLYGONS

FigureID	1	2	3	4	5	6	7	8	9	10	11	12	13
number	48	32	144	48	48	144	48	80	64	96	16	224	32

In the development of cryptography, graphic cryptography was born. The graphic password has different meanings according to the location of the graphic. When the graphic position is correct, the password is correct. Tangram graphics can be encrypted as a graphic password [5]. Therefore, using this program can simulate a brute force cracking method. In pedagogy, Tangram is often used in basic education. Since the tangram can be combined with a variety of patterns, each pattern has a variety of solutions, which can stimulate students' creative thinking and imagination [6]. Tangram is a board with geometric patterns. By arranging the position of the wooden planks, more geometric figures can be spliced, so preschool children can also be guided to understand geometric patterns [7]. This program can be used as a software for teachers to instruct teachers before class and provide graphic solutions for teachers in the following courses.

The limitations and deficiencies of this subject, and what are the remaining issues to be solved. First of all, compared with the method of mathematical proof, this topic is to let the computer enumerate and calculate all the results after the conditions are restricted. I think it is not perfect. Secondly, the modeling process is complicated, because the enumeration method is used, so a lot of repetitive work is required. The author makes the assumption that through the matrix research method, the matching algorithm of the tangram can also be completed.

The author believes that the next step of the research work: First, we can break through 13 convex polygons and introduce the concept of concave polygons to make more patterns. Second, we can introduce computer vision modeling methods to recognize objects through the camera, and then the computer completes the patchwork of graphics and the calculation of all solutions.

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