

Assignment 10

B.Anusha

Download all python codes from

<https://github.com/BOJJAVOYINAANUSHA/ASSIGNMENT10/tree/main/ASSIGNMENT10/assignment10.py>

and latex-tikz codes from

<https://github.com/BOJJAVOYINAANUSHA/ASSIGNMENT10/blob/main/ASSIGNMENT10/ASSIGNMENT10.tex>

∴ Our problem is

$$\min_{\mathbf{x}} Z = (60 \ 80) \mathbf{x} \quad (2.0.5)$$

$$s.t. \quad \begin{pmatrix} 3 & 4 \\ 5 & 2 \end{pmatrix} \mathbf{x} \leq \begin{pmatrix} 8 \\ 11 \end{pmatrix} \quad (2.0.6)$$

$$\mathbf{x} \geq \mathbf{0} \quad (2.0.7)$$

Lagrangian function is given by

$$\begin{aligned} L(\mathbf{x}, \lambda) &= (60 \ 80) \mathbf{x} + \left\{ \left[(3 \ 4) \mathbf{x} - 8 \right] \right. \\ &\quad \left. + \left[(5 \ 2) \mathbf{x} - 11 \right] \right. \\ &\quad \left. + \left[(-1 \ 0) \mathbf{x} \right] + \left[(0 \ -1) \mathbf{x} \right] \right\} \lambda \end{aligned} \quad (2.0.8)$$

where,

$$\lambda = \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \end{pmatrix} \quad (2.0.9)$$

Now,

$$\nabla L(\mathbf{x}, \lambda) = \begin{pmatrix} 60 + (3 \ 5 \ -1 \ 0) \lambda \\ 80 + (4 \ 2 \ 0 \ -1) \lambda \\ (3 \ 4) \mathbf{x} - 8 \\ (5 \ 2) \mathbf{x} - 11 \\ (-1 \ 0) \mathbf{x} \\ (0 \ -1) \mathbf{x} \end{pmatrix} \quad (2.0.10)$$

∴ Lagrangian matrix is given by

$$\begin{pmatrix} 0 & 0 & 3 & 5 & -1 & 0 \\ 0 & 0 & 4 & 2 & 0 & -1 \\ 3 & 4 & 0 & 0 & 0 & 0 \\ 5 & 2 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \mathbf{x} \\ \lambda \end{pmatrix} = \begin{pmatrix} -60 \\ -80 \\ 8 \\ 11 \\ 0 \\ 0 \end{pmatrix} \quad (2.0.11)$$

1 QUESTION No. 2.12(OPTIMIZATION)

Reshma wishes to mix two types of food P and Q in such a way that the vitamin contents of the mixture contain at least 8 units of vitamin A and 11 units of vitamin B. Food P costs Rs. 60/kg and Food Q costs Rs. 80/kg. Food P contains 3 units/kg of vitamin A and 5 units/kg of vitamin B while food Q contains 4 units/kg of vitamin A and 2 units/kg of vitamin B. Determine the minimum cost of the mixture.

2 SOLUTION

Food	Vitamin A	Vitamin B	Cost
P	3 units/kg	5 units/kg	60 Rs/kg
Q	4 units/kg	2 units/kg	80 Rs/kg
Requirement	8 units/kg	11 units/kg	

TABLE 2.1: Food Requirements

Let the mixture contain x kg of food P and y kg of food Q be y such that

$$x \geq 0 \quad (2.0.1)$$

$$y \geq 0 \quad (2.0.2)$$

According to the question,

$$3x + 4y \geq 8 \quad (2.0.3)$$

$$5x + 2y \geq 11 \quad (2.0.4)$$

Considering λ_1, λ_2 as only active multiplier,

$$\begin{pmatrix} 0 & 0 & 3 & 5 \\ 0 & 0 & 4 & 2 \\ 3 & 4 & 0 & 0 \\ 5 & 2 & 0 & 0 \end{pmatrix} \begin{pmatrix} \mathbf{x} \\ \lambda \end{pmatrix} = \begin{pmatrix} -60 \\ -80 \\ 8 \\ 11 \end{pmatrix} \quad (2.0.12)$$

resulting in,

$$\begin{pmatrix} \mathbf{x} \\ \lambda \end{pmatrix} = \begin{pmatrix} 0 & 0 & 3 & 5 \\ 0 & 0 & 4 & 2 \\ 3 & 4 & 0 & 0 \\ 5 & 2 & 0 & 0 \end{pmatrix}^{-1} \begin{pmatrix} -60 \\ -80 \\ 8 \\ 11 \end{pmatrix} \quad (2.0.13)$$

$$\Rightarrow \begin{pmatrix} \mathbf{x} \\ \lambda \end{pmatrix} = \begin{pmatrix} 0 & 0 & \frac{-28}{196} & \frac{56}{196} \\ 0 & 0 & \frac{70}{196} & \frac{-42}{196} \\ \frac{196}{196} & \frac{70}{196} & 0 & 0 \\ \frac{56}{196} & \frac{-42}{196} & 0 & 0 \end{pmatrix} \begin{pmatrix} -60 \\ -80 \\ 8 \\ 11 \end{pmatrix} \quad (2.0.14)$$

$$\Rightarrow \begin{pmatrix} \mathbf{x} \\ \lambda \end{pmatrix} = \begin{pmatrix} 2 \\ \frac{1}{2} \\ -20 \\ 0 \end{pmatrix} \quad (2.0.15)$$

$$\therefore \lambda = \begin{pmatrix} -20 \\ 0 \end{pmatrix} \leq \mathbf{0}$$

\therefore Optimal solution is given by

$$\mathbf{x} = \begin{pmatrix} 2 \\ \frac{1}{2} \end{pmatrix} \quad (2.0.16)$$

$$Z = (60 \ 80) \mathbf{x} \quad (2.0.17)$$

$$= (60 \ 80) \begin{pmatrix} 2 \\ \frac{1}{2} \end{pmatrix} \quad (2.0.18)$$

$$= 160 \quad (2.0.19)$$

By using cvxpy in python ,

$$\mathbf{x} = \begin{pmatrix} 2.11436237 \\ 0.41422822 \end{pmatrix} \quad (2.0.20)$$

$$Z = 159.99999999 \quad (2.0.21)$$

Hence , $\boxed{x = 2}$ of food P and $\boxed{y = \frac{1}{2}}$ of food Q should contained in the mixture and the obtained minimum cost of the mixture is Rs. $\boxed{Z = 160}$.

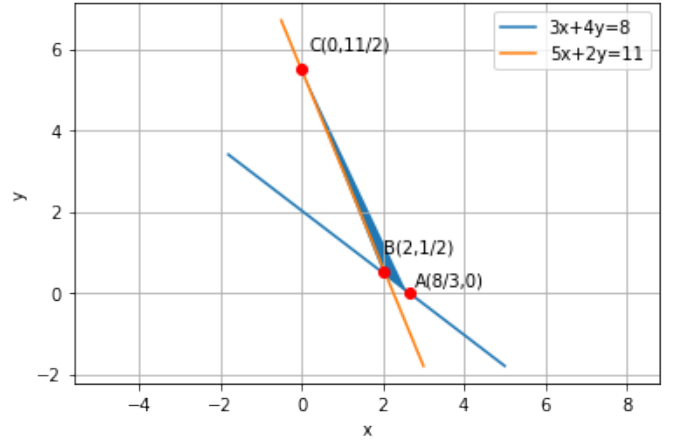


Fig. 2.1: Graphical Solution