Assignment 10

B.Anusha

Download all python codes from

```
https://github.com/BOJJAVOYINAANUSHA/
    ASSIGNMENT10/blob/main/
    ASSIGNMENT10/assignment10.py
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and latex-tikz codes from

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https://github.com/BOJJAVOYINAANUSHA/
    ASSIGNMENT10/blob/main/
    ASSIGNMENT10/ASSIGNMENT10.tex
```

1 Question No. 2.12(Optimization)

Reshma wishes to mix two types of food P and Q in such a way that the vitamin contents of the mixture contain at least 8 units of vitamin A and 11 units of vitamin B. Food P costs Rs. 60/kg and Food Q costs Rs. 80/kg. Food P contains 3 units/kg of vitamin A and 5 units/kg of vitamin B while food Q contains 4 units/kg of vitamin A and 2 units/kg of vitamin B. Determine the minimum cost of the mixture.

2 Solution

Food	Vitamin A	Vitamin B	Cost
P	3 units/kg	5 units/kg	60 Rs/kg
Q	4 units/kg	2 units/kg	80 Rs/kg
Requirement	8 units/kg	11 units/kg	

TABLE 2.1: Food Requirements

Let the mixture contain x kg of food P and y kg of food Q be y such that

$$x \geq 0 \tag{2.0.1}$$

$$y \geq 0 \tag{2.0.2}$$

According to the question,

$$3x + 4y \geq 8 \tag{2.0.3}$$

$$5x + 2y \geq 11 \tag{2.0.4}$$

$\therefore$ Our problem is

$$\min_{\mathbf{x}} Z = \begin{pmatrix} 60 & 80 \end{pmatrix} \mathbf{x} \tag{2.0.5}$$

$$s.t. \quad \begin{pmatrix} 3 & 4 \\ 5 & 2 \end{pmatrix} \mathbf{x} \leq \begin{pmatrix} 8 \\ 11 \end{pmatrix} \tag{2.0.6}$$

$$\mathbf{x} \leq \mathbf{0} \tag{2.0.7}$$

Lagrangian function is given by

$$\begin{aligned} &L(\mathbf{x}, \boldsymbol{\lambda}) \\ &= \begin{pmatrix} 60 & 80 \end{pmatrix} \mathbf{x} + \left\{ \left[\begin{pmatrix} 3 & 4 \end{pmatrix} \mathbf{x} - 8 \right] \right. \\ &+ \left[\begin{pmatrix} 5 & 2 \end{pmatrix} \mathbf{x} - 11 \right] \\ &+ \left. \left[\begin{pmatrix} -1 & 0 \end{pmatrix} \mathbf{x} \right] + \left[\begin{pmatrix} 0 & -1 \end{pmatrix} \mathbf{x} \right] \right\} \boldsymbol{\lambda} \end{aligned} \tag{2.0.8}$$

where,

$$\boldsymbol{\lambda} = \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \end{pmatrix} \tag{2.0.9}$$

Now,

$$\nabla L(\mathbf{x}, \boldsymbol{\lambda}) = \begin{pmatrix} 60 + \begin{pmatrix} 3 & 5 & -1 & 0 \end{pmatrix} \boldsymbol{\lambda} \\ 80 + \begin{pmatrix} 4 & 2 & 0 & -1 \end{pmatrix} \boldsymbol{\lambda} \\ \begin{pmatrix} 3 & 4 \end{pmatrix} \mathbf{x} - 8 \\ \begin{pmatrix} 5 & 2 \end{pmatrix} \mathbf{x} - 11 \\ \begin{pmatrix} -1 & 0 \end{pmatrix} \mathbf{x} \\ \begin{pmatrix} 0 & -1 \end{pmatrix} \mathbf{x} \end{pmatrix} \tag{2.0.10}$$

$\therefore$ Lagrangian matrix is given by

$$\begin{pmatrix} 0 & 0 & 3 & 5 & -1 & 0 \\ 0 & 0 & 4 & 2 & 0 & -1 \\ 3 & 4 & 0 & 0 & 0 & 0 \\ 5 & 2 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \mathbf{x} \\ \boldsymbol{\lambda} \end{pmatrix} = \begin{pmatrix} -60 \\ -80 \\ 8 \\ 11 \\ 0 \\ 0 \end{pmatrix} \tag{2.0.11}$$

Considering λ_1, λ_2 as only active multiplier,

$$\begin{pmatrix} 0 & 0 & 3 & 5 \\ 0 & 0 & 4 & 2 \\ 3 & 4 & 0 & 0 \\ 5 & 2 & 0 & 0 \end{pmatrix} \begin{pmatrix} \mathbf{x} \\ \boldsymbol{\lambda} \end{pmatrix} = \begin{pmatrix} -60 \\ -80 \\ 8 \\ 11 \end{pmatrix} \tag{2.0.12}$$

resulting in,

$$\begin{pmatrix} \mathbf{x} \\ \boldsymbol{\lambda} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 3 & 5 \\ 0 & 0 & 4 & 2 \\ 3 & 4 & 0 & 0 \\ 5 & 2 & 0 & 0 \end{pmatrix}^{-1} \begin{pmatrix} -60 \\ -80 \\ 8 \\ 11 \end{pmatrix} \tag{2.0.13}$$

$$\implies \begin{pmatrix} \mathbf{x} \\ \boldsymbol{\lambda} \end{pmatrix} = \begin{pmatrix} 0 & 0 & \frac{-28}{196} & \frac{56}{196} \\ 0 & 0 & \frac{70}{196} & \frac{-42}{196} \\ \frac{-28}{196} & \frac{70}{196} & 0 & 0 \\ \frac{56}{196} & \frac{-42}{196} & 0 & 0 \end{pmatrix} \begin{pmatrix} -60 \\ -80 \\ 8 \\ 11 \end{pmatrix} \tag{2.0.14}$$

$$\implies \begin{pmatrix} \mathbf{x} \\ \boldsymbol{\lambda} \end{pmatrix} = \begin{pmatrix} 2 \\ \frac{1}{2} \\ -20 \\ 0 \end{pmatrix} \tag{2.0.15}$$

$\because \boldsymbol{\lambda} = \begin{pmatrix} -20 \\ 0 \end{pmatrix} \preceq \mathbf{0}$

$\therefore$ Optimal solution is given by

$$\mathbf{x} = \begin{pmatrix} 2 \\ \frac{1}{2} \end{pmatrix} \tag{2.0.16}$$

$$Z = \begin{pmatrix} 60 & 80 \end{pmatrix} \mathbf{x} \tag{2.0.17}$$

$$= \begin{pmatrix} 60 & 80 \end{pmatrix} \begin{pmatrix} 2 \\ \frac{1}{2} \end{pmatrix} \tag{2.0.18}$$

$$= 160 \tag{2.0.19}$$

By using cvxpy in python ,

$$\mathbf{x} = \begin{pmatrix} 2.11436237 \\ 0.41422822 \end{pmatrix} \tag{2.0.20}$$

$$Z = 159.99999999 \tag{2.0.21}$$

As the feasible region is unbounded, therefore, 160 may or maynot be the minimum value of Z. For this, we graph the inequality, $3x + 4y < 8$ and check whether the resulting half plane has points in common with the feasible region or not.
It can be seen that the feasible region has no common point with $3x + 4y < 8$. Therefore, the minimum cost of the mixture will be Rs.160 at the line segment joining the points $\begin{pmatrix} \frac{8}{3} & 0 \end{pmatrix}$ and $\begin{pmatrix} 2 & \frac{1}{2} \end{pmatrix}$.

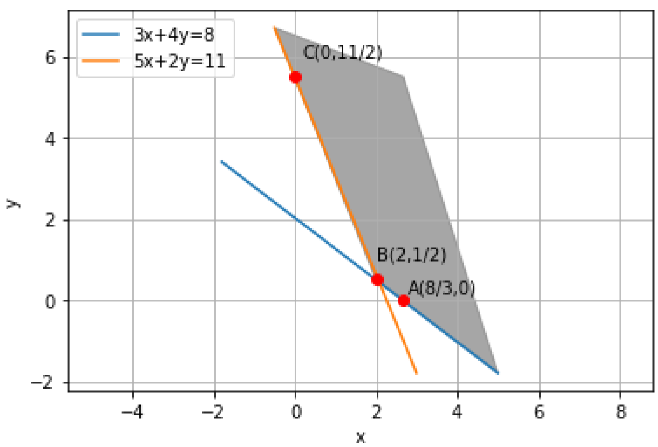

Fig. 2.1: Graphical Solution