

ASSIGNMENT 4

B.ANUSHA

Download all python codes from

<https://github.com/BOJJAVOYINAANUSHA/ASSIGNMENT4/tree/main/ASSIGNMENT4/assignment.py>

Latex-tikz codes from

<https://github.com/BOJJAVOYINAANUSHA/ASSIGNMENT4/tree/main/ASSIGNMENT4/ASSIGNMENT.tex>

Calculating the eigen vectors corresponding to $\lambda_1 = 0, \lambda_2 = 1$ respectively.

$$\mathbf{V}\mathbf{x} = \lambda\mathbf{x} \quad (2.0.9)$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} = 0 \implies \mathbf{p}_1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (2.0.10)$$

$$\begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix} \mathbf{x} = 0 \implies \mathbf{p}_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (2.0.11)$$

By eigen decomposition on \mathbf{V} ,

$$\mathbf{V} = \mathbf{P}\mathbf{D}\mathbf{P}^T \quad (2.0.12)$$

Where,

$$\mathbf{P} = (\mathbf{p}_1 \quad \mathbf{p}_2) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad (2.0.13)$$

$$\mathbf{D} = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \quad (2.0.14)$$

To find the vertex of the parabola ,

$$\begin{pmatrix} \mathbf{u}^T + \eta \mathbf{p}_1^T \\ \mathbf{V} \end{pmatrix} \mathbf{c} = \begin{pmatrix} -\mathbf{f} \\ \eta \mathbf{p}_1 - \mathbf{u} \end{pmatrix} \quad (2.0.15)$$

$$\text{where, } \eta = \mathbf{u}^T \mathbf{p}_1 = -2 \quad (2.0.16)$$

$$\implies \begin{pmatrix} 0 & -4 \\ 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{c} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad (2.0.17)$$

from the above it can be observed that,

$$\mathbf{c} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad (2.0.18)$$

Now to evaluate the direction vector \mathbf{m} ,

$$\implies \mathbf{m}^T (\mathbf{V}\mathbf{q} + \mathbf{u}) = 0 \quad (2.0.19)$$

$$\implies \mathbf{m}^T \left(\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \begin{pmatrix} 0 \\ -2 \end{pmatrix} \right) = 0 \quad (2.0.20)$$

$$\implies \mathbf{m}^T \begin{pmatrix} 1 \\ -2 \end{pmatrix} = 0 \quad (2.0.21)$$

$$\implies \mathbf{m} = \begin{pmatrix} -2 \\ -1 \end{pmatrix} \quad (2.0.22)$$

1 QUESTION No 2.45

Find the equation of the normal to the curve $x^2 = 4y$ which passes through the point $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$.

2 SOLUTION

Given curve,

$$x^2 = 4y \quad (2.0.1)$$

$$\implies x^2 - 4y = 0 \quad (2.0.2)$$

Comparing with the standard equation :

$$\mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \quad (2.0.3)$$

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} 0 \\ -2 \end{pmatrix}, \mathbf{f} = 0 \quad (2.0.4)$$

\therefore

$$|\mathbf{V}| = 0 \quad (2.0.5)$$

\therefore the given curve (2.0.2) represents a parabola . we can find the eigen values corresponding to the \mathbf{V} ,

$$|\mathbf{V} - \lambda \mathbf{I}| = \begin{vmatrix} 1 - \lambda & 0 \\ 0 & -\lambda \end{vmatrix} = 0 \quad (2.0.6)$$

$$\implies (1 - \lambda)(-\lambda) = 0 \quad (2.0.7)$$

\therefore Eigen values are

$$\lambda_1 = 0, \lambda_2 = 1 \quad (2.0.8)$$

Now to obtain the equation of normal using,

$$\Rightarrow \mathbf{m}^T(\mathbf{x} - \mathbf{q}) = 0 \quad (2.0.23)$$

$$\Rightarrow \begin{pmatrix} -2 & -1 \end{pmatrix} \left(\mathbf{x} - \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right) = 0 \quad (2.0.24)$$

$$\Rightarrow \begin{pmatrix} -2 & -1 \end{pmatrix} \mathbf{x} + 4 = 0 \quad (2.0.25)$$

- Plot of Normal to the given curve -

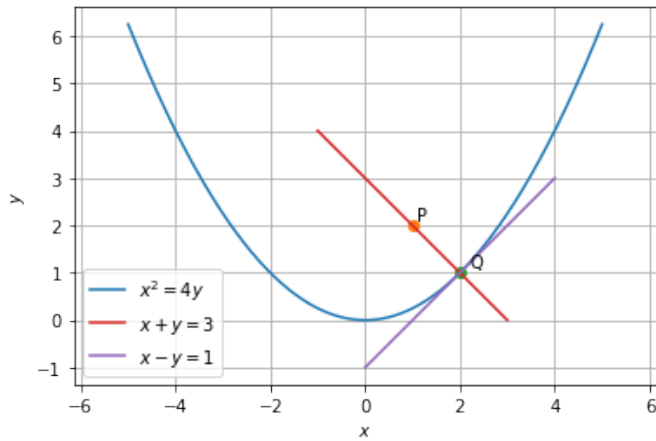


Fig. 2.1: Normal to Parabola.