ASSIGNMENT 4

B.ANUSHA

Download all python codes from

https://github.com/BOJJAVOYINAANUSHA/ASSIGNMENT4/tree/main/ASSIGNMENT4/CODES

Latex-tikz codes from

https://github.com/BOJJAVOYINAANUSHA/ASSIGNMENT4/tree/main/ASSIGNMENT4

1 Question No 2.45

Find the equation of the normal to the curve $x^2 = 4y$ which passes through the point $\begin{pmatrix}1\\2\end{pmatrix}$.

2 SOLUTION

Given curve,

$$x^2 = 4y \tag{2.0.1}$$

$$\implies x^2 - 4y = 0 \tag{2.0.2}$$

Comparing with the standard equation :

$$\mathbf{x}^T\mathbf{V}\mathbf{x} + 2\mathbf{u}^T\mathbf{x} + f = 0 \tag{2.0.3}$$

$$\mathbf{V} = \begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}, \mathbf{u} = \begin{pmatrix}0\\-2\end{pmatrix}, \mathbf{f} = 0 \tag{2.0.4}$$

$\because$

$$|\mathbf{V}| = 0 \tag{2.0.5}$$

$\therefore$ the given curve (2.0.2) represents a parabola . we can find the eigen values corresponding to the $\mathbf{V}$,

$$|\mathbf{V} - \lambda\mathbf{I}| = \begin{vmatrix}1-\lambda & 0\\0 & -\lambda\end{vmatrix} = 0 \tag{2.0.6}$$

$$\implies (1-\lambda)(-\lambda) = 0 \tag{2.0.7}$$

$\therefore$ Eigen values are

$$\lambda_1 = 0, \lambda_2 = 1 \tag{2.0.8}$$

Calculating the eigen vectors corresponding to $\lambda_1 = 0, \lambda_2 = 1$ respectively.

$$\mathbf{V}\mathbf{x} = \lambda\mathbf{x} \tag{2.0.9}$$

$$\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}\mathbf{x} = 0 \implies \mathbf{p_1} = \begin{pmatrix}0\\1\end{pmatrix} \tag{2.0.10}$$

$$\begin{pmatrix}0 & 0\\0 & -1\end{pmatrix}\mathbf{x} = 0 \implies \mathbf{p_2} = \begin{pmatrix}1\\0\end{pmatrix} \tag{2.0.11}$$

By eigen decomposition on $\mathbf{V}$,

$$\mathbf{V} = \mathbf{P}\mathbf{D}\mathbf{P}^T \tag{2.0.12}$$

Where,

$$\mathbf{P} = \begin{pmatrix}\mathbf{p_1} & \mathbf{p_2}\end{pmatrix} = \begin{pmatrix}0 & 1\\1 & 0\end{pmatrix} \tag{2.0.13}$$

$$\mathbf{D} = \begin{pmatrix}\lambda_1 & 0\\0 & \lambda_2\end{pmatrix} = \begin{pmatrix}0 & 0\\0 & 1\end{pmatrix} \tag{2.0.14}$$

To find the vertex of the parabola ,

$$\begin{pmatrix}\mathbf{u} + \kappa\mathbf{p_1^T}\\\mathbf{V}\end{pmatrix}\mathbf{C} = \begin{pmatrix}-\mathbf{f}\\\kappa\mathbf{p_1} - \mathbf{u}\end{pmatrix} \tag{2.0.15}$$

$$Where, \kappa = \mathbf{u^T}\mathbf{p_1} = -2 \tag{2.0.16}$$

$$\implies \begin{pmatrix}0 & -4\\1 & 0\\0 & 0\end{pmatrix}\mathbf{C} = \begin{pmatrix}0\\0\\0\end{pmatrix} \tag{2.0.17}$$

Removing last row and representing (2.0.17) as agumented matrix and then converting the matrix to echelon form.

$$\begin{pmatrix}0 & 4 & 0\\1 & 0 & 0\end{pmatrix} \xleftrightarrow{R_2 \leftrightarrow R_1} \begin{pmatrix}1 & 0 & 0\\0 & 4 & 0\end{pmatrix} \tag{2.0.18}$$

$$\begin{pmatrix}1 & 0 & 0\\0 & 4 & 0\end{pmatrix} \xleftrightarrow{R_2 \leftarrow \frac{R_2}{4}} \begin{pmatrix}1 & 0 & 0\\0 & 1 & 0\end{pmatrix} \tag{2.0.19}$$

from the above it can be observed that,

$$\mathbf{C} = \begin{pmatrix}0\\0\end{pmatrix} \tag{2.0.20}$$

Now to evalute the direction vector m,

$$\mathbf{m}^T(\mathbf{V}\mathbf{q}+\mathbf{u}) = 0 \tag{2.0.21}$$

$$\mathbf{m}^T\left(\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}\begin{pmatrix}1\\2\end{pmatrix}+\begin{pmatrix}0\\-2\end{pmatrix}\right) = 0 \tag{2.0.22}$$

$$\mathbf{m}^T\begin{pmatrix}1\\-2\end{pmatrix} = 0 \tag{2.0.23}$$

$$\mathbf{m} = \begin{pmatrix}1\\-2\end{pmatrix} \tag{2.0.24}$$

Now to obtain the equation of normal using,

$$\mathbf{m}^T(\mathbf{x}-\mathbf{q}) = 0 \tag{2.0.25}$$

$$\begin{pmatrix}1 & -2\end{pmatrix}\left(\mathbf{x}-\begin{pmatrix}1 & 2\end{pmatrix}\right) = 0 \tag{2.0.26}$$

$$\begin{pmatrix}1 & -2\end{pmatrix}\mathbf{x}+3 = 0 \tag{2.0.27}$$

- Plot of Normal to the given curve -

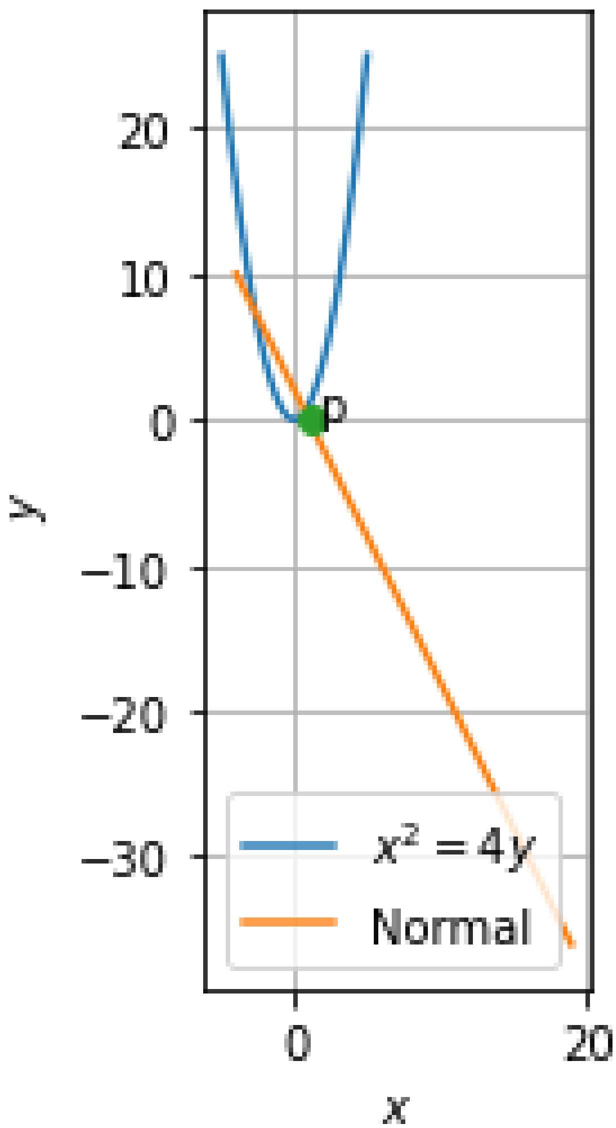

Fig. 2.1: Normal to Parabola.