

ASSIGNMENT-8

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1 QUESTION No-2.26(MATRICES)

Using elementary transformations, find the inverse of each of the matrices:

1) $\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$

2 SOLUTION

1) Given that

$$\mathbf{A} = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} \quad (2.0.1)$$

The augmented matrix $[A|I]$ is as given below:-

$$\left(\begin{array}{cc|cc} 2 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{array} \right) \quad (2.0.2)$$

We apply the elementary row operations on $[A|I]$ as follows :-

$$[A|I] = \left(\begin{array}{cc|cc} 2 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{array} \right) \quad (2.0.3)$$

$$\xleftrightarrow{R_1 \leftarrow R_1 - R_2} \left(\begin{array}{cc|cc} 1 & 0 & 1 & -1 \\ 1 & 1 & 0 & 1 \end{array} \right) \quad (2.0.4)$$

$$\xleftrightarrow{R_2 \leftarrow R_2 - R_1} \left(\begin{array}{cc|cc} 1 & 0 & 1 & -1 \\ 0 & 1 & -1 & 2 \end{array} \right) \quad (2.0.5)$$

By performing elementary transformations on augmented matrix $[A|I]$, we obtained the augmented matrix in the form $[I|A]$. Hence we can conclude that the matrix A is invertible and inverse of the matrix is:-

$$\therefore \mathbf{A}^{-1} = \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix} \quad (2.0.6)$$

2) QR decomposition of $\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$

Let α and β be the column vectors of given matrix \mathbf{A} ,

$$\alpha = \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \beta = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad (2.0.7)$$

QR decomposition of matrix form is:

$$(\alpha \ \beta) = (\mathbf{u}_1 \ \mathbf{u}_2) \begin{pmatrix} k_1 & r_1 \\ 0 & k_2 \end{pmatrix} \quad (2.0.8)$$

Finding values of the above equation, we get:

$$\Rightarrow k_1 = \|\alpha\| = \left\| \begin{pmatrix} 2 \\ 1 \end{pmatrix} \right\| \quad (2.0.9)$$

$$\therefore k_1 = \sqrt{5} \quad (2.0.10)$$

$$\Rightarrow \mathbf{u}_1 = \frac{\alpha}{k_1} = \frac{1}{\sqrt{5}} \begin{pmatrix} 2 \\ 1 \end{pmatrix} \quad (2.0.11)$$

$$\therefore \mathbf{u}_1 = \begin{pmatrix} \frac{2}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} \end{pmatrix} \quad (2.0.12)$$

$$\Rightarrow r_1 = \frac{\mathbf{u}_1^T \beta}{\|\mathbf{u}_1\|^2} = \frac{\frac{1}{\sqrt{5}} \begin{pmatrix} 2 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix}}{1} \quad (2.0.13)$$

$$\therefore r_1 = \frac{3}{\sqrt{5}} \quad (2.0.14)$$

$$\Rightarrow \mathbf{u}_2 = \frac{\beta - r_1 \mathbf{u}_1}{\|\beta - r_1 \mathbf{u}_1\|} = \frac{\begin{pmatrix} 1 \\ 1 \end{pmatrix} - \frac{3}{\sqrt{5}} \begin{pmatrix} \frac{2}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} \end{pmatrix}}{\left\| \begin{pmatrix} 1 \\ 1 \end{pmatrix} - \frac{3}{\sqrt{5}} \begin{pmatrix} \frac{2}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} \end{pmatrix} \right\|} \quad (2.0.15)$$

$$\therefore \mathbf{u}_2 = \begin{pmatrix} \frac{-1}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} \end{pmatrix} \quad (2.0.16)$$

$$\Rightarrow k_2 = \mathbf{u}_2^T \beta = \begin{pmatrix} \frac{-1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad (2.0.17)$$

$$\therefore k_2 = \frac{1}{\sqrt{5}} \quad (2.0.18)$$

From equations (2.0.10), (2.0.12), (2.0.14), (2.0.16), (2.0.18) and using (2.0.7) the obtained **QR** decomposition is

$$\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} \frac{2}{\sqrt{5}} & \frac{-1}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \end{pmatrix} \begin{pmatrix} \sqrt{5} & \frac{3}{\sqrt{5}} \\ 0 & \frac{1}{\sqrt{5}} \end{pmatrix} \quad (2.0.19)$$