ASSIGNMENT 3

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1 QUESTION No 2.57

Draw a circle of radius 3 units.Take two points P and Q on one its extended diameter each at a distance of 7 units from its centre. Draw tangents to the circle from these two points P and Q.

2 SOLUTION

The center and radius of the circle without any loss of generality is given in table 2.1

	Circle
Centre	$\mathbf{O}=\begin{pmatrix}0\\0\end{pmatrix}$
Radius	$r=3$

TABLE 2.1: Input values

- Let P and Q are the points on one of its extended diameter each at a distance of 7cm. from its centre.

$$\therefore \mathbf{P} = \begin{pmatrix}-7\\0\end{pmatrix}, \mathbf{Q} = \begin{pmatrix}7\\0\end{pmatrix} \quad (2.0.1)$$

Lemma 2.1. *Let PA, PB, QC and QD are the tangents from points P and Q with the distance of 7cm. to the points A, B, C and D on circle with radius 3. We know a tangent is always perpendicular to the radius .*

$$\therefore OA \perp AP \quad (2.0.2)$$
$$\therefore OC \perp CQ \quad (2.0.3)$$

Now,

$$(\mathbf{O} - \mathbf{A})^T(\mathbf{A} - \mathbf{P}) = 0 \quad (\because OA \perp AP) \quad (2.0.4)$$

$$\mathbf{A}^T(\mathbf{A} - \mathbf{P}) = 0 \quad \left(\because \mathbf{O} = \begin{pmatrix}0\\0\end{pmatrix}\right) \quad (2.0.5)$$

$$\mathbf{A}^T\mathbf{A} - \mathbf{A}^T\mathbf{P} = 0 \quad (2.0.6)$$

$$\|\mathbf{A}\|^2 = \mathbf{A}^T\mathbf{P} \quad (2.0.7)$$

$$\|\mathbf{A}\|^2 = \mathbf{P}^T\mathbf{A} \quad \left(\because \mathbf{A}^T\mathbf{P} = \mathbf{P}^T\mathbf{A}\right) \quad (2.0.8)$$

$$\mathbf{P}^T\mathbf{A} = 9 \quad \left(\because \|\mathbf{A}\|^2 = 9\right) \quad (2.0.9)$$

$$\begin{pmatrix}-7 & 0\end{pmatrix}\mathbf{A} = 9 \quad \left(\because \mathbf{P} = \begin{pmatrix}-7\\0\end{pmatrix}\right) \quad (2.0.10)$$

$$\begin{pmatrix}1 & 0\end{pmatrix}\mathbf{A} = \frac{-9}{7} \quad (2.0.11)$$

$$\mathbf{A} = \begin{pmatrix}\frac{-9}{7}\\0\end{pmatrix} + \lambda\begin{pmatrix}0\\1\end{pmatrix} \quad (2.0.12)$$

$$\mathbf{A} = \mathbf{a} + \lambda\mathbf{m} \quad (2.0.13)$$

$$\mathbf{a} = \begin{pmatrix}\frac{-9}{7}\\0\end{pmatrix}, \mathbf{m} = \begin{pmatrix}0\\1\end{pmatrix} \quad (2.0.14)$$

We know,

$$\|\mathbf{a} + \lambda\mathbf{m}\|^2 = 9 \quad (2.0.15)$$

$$(\mathbf{a} + \lambda\mathbf{m})^T(\mathbf{a} + \lambda\mathbf{m}) = r^2 \quad (2.0.16)$$

$$\lambda^2 = \frac{r^2 - \|\mathbf{a}\|^2}{\|\mathbf{m}\|^2} \quad (2.0.17)$$

$$\lambda = \pm 2.71 \quad (2.0.18)$$

Substitute λ value in (2.0.12) we get,

$$\mathbf{A} = \begin{pmatrix}\frac{-9}{7}\\2.71\end{pmatrix}, \mathbf{B} = \begin{pmatrix}\frac{-9}{7}\\-2.71\end{pmatrix} \quad (2.0.19)$$

Similarly,

$$(\mathbf{O} - \mathbf{C})^T(\mathbf{C} - \mathbf{Q}) = 0 \quad (\because OC \perp CQ) \quad (2.0.20)$$

$$\mathbf{C}^T(\mathbf{C} - \mathbf{Q}) = 0 \quad \left(\because \mathbf{O} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}\right) \quad (2.0.21)$$

$$\mathbf{C}^T\mathbf{C} - \mathbf{C}^T\mathbf{Q} = 0 \quad (2.0.22)$$

$$\|\mathbf{C}\|^2 = \mathbf{C}^T\mathbf{Q} \quad (2.0.23)$$

$$\|\mathbf{C}\|^2 = \mathbf{Q}^T\mathbf{C} \quad \left(\because \mathbf{C}^T\mathbf{Q} = \mathbf{Q}^T\mathbf{C}\right) \quad (2.0.24)$$

$$\mathbf{Q}^T\mathbf{C} = 9 \quad \left(\because \|\mathbf{C}\|^2 = 9\right) \quad (2.0.25)$$

$$\begin{pmatrix} 7 & 0 \end{pmatrix}\mathbf{C} = 9 \quad \left(\because \mathbf{Q} = \begin{pmatrix} 7 \\ 0 \end{pmatrix}\right) \quad (2.0.26)$$

$$\begin{pmatrix} 1 & 0 \end{pmatrix}\mathbf{C} = \frac{9}{7} \quad (2.0.27)$$

$$\mathbf{C} = \begin{pmatrix} \frac{9}{7} \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (2.0.28)$$

$$\mathbf{C} = \mathbf{c} + \mu\mathbf{m} \quad (2.0.29)$$

$$\mathbf{c} = \begin{pmatrix} \frac{9}{7} \\ 0 \end{pmatrix}, \mathbf{m} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (2.0.30)$$

We know,

$$\|\mathbf{c} + \mu\mathbf{m}\|^2 = 9 \quad (2.0.31)$$

$$(\mathbf{c} + \mu\mathbf{m})^T(\mathbf{c} + \mu\mathbf{m}) = r^2 \quad (2.0.32)$$

$$\mu^2 = \frac{r^2 - \|\mathbf{c}\|^2}{\|\mathbf{m}\|^2} \quad (2.0.33)$$

$$\mu = \pm 2.71 \quad (2.0.34)$$

Substitute μ value in (2.0.28) we get

$$\mathbf{Q} = \begin{pmatrix} \frac{9}{7} \\ 2.71 \end{pmatrix}, \mathbf{R} = \begin{pmatrix} \frac{9}{7} \\ -2.71 \end{pmatrix} \quad (2.0.35)$$
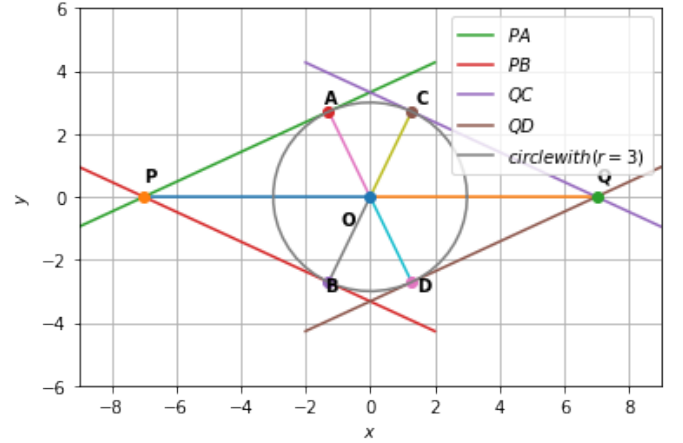
- Plot of Tangents PA, PB, QC and QD :

Fig. 2.1: Tangent lines to circle of radius 3 units.