ASSIGNMENT 3

ANUSHA

Download all python codes from

```
https://github.com/BOJJAVOYINAANUSHA/
    ASSIGNMENT_3/blob/main/ASSIGNMENT3
    /assignment3.py
```

and latex-tikz codes from

```
https://github.com/BOJJAVOYINAANUSHA/
    ASSIGNMENT_3/blob/main/ASSIGNMENT3
    /ASSIGNMENT3.tex
```

1 Question No 2.57

Draw a circle of radius 3 units.Take two points P and Q on one its extended diameter each at a distance of 7 units from its centre. Draw tangents to the circle from these two points P and Q.

2 Solution

The data from the question is in the table 2.1

	circle
Centre	$\mathbf{O}=\begin{pmatrix}0\\0\end{pmatrix}$
Radius	r=3
Radius	d=7

TABLE 2.1: Input values

Lemma 2.1. *The points of contact for the tangent drawn from a point*

$$\mathbf{Q} = d\mathbf{e}_1, \textit{where } \mathbf{e}_1 = \begin{pmatrix}1\\0\end{pmatrix} \tag{2.0.1}$$

to the circle are given by

$$\mathbf{x} = \frac{r^2}{d}\mathbf{e}_1 \pm r\sqrt{1-\frac{r^2}{d^2}}\mathbf{e}_2, \textit{where } \mathbf{e}_2 = \begin{pmatrix}0\\1\end{pmatrix} \tag{2.0.2}$$

Let the point at distance d from **O** *be*

$$\mathbf{Q} = d\mathbf{e}_1, \textit{where } \mathbf{e}_1 = \begin{pmatrix}1\\0\end{pmatrix} \tag{2.0.3}$$

If $\mathbf{x}$ *be a point of contact for the tangent from* **Q**,

$$QR \perp RO \tag{2.0.4}$$

$$\implies (\mathbf{O}-\mathbf{x})^T(\mathbf{x}-\mathbf{Q}) = 0 \tag{2.0.5}$$

$$\textit{or, } \mathbf{Q}^T\mathbf{x} = \left\|\mathbf{x^2}\right\| = r^2 \tag{2.0.6}$$

$$\implies \mathbf{e}_1{}^{\mathbf{T}}\mathbf{x} = \frac{r^2}{d} \tag{2.0.7}$$

$$\because \mathbf{O} = 0. \tag{2.0.8}$$

The above equation can be expressed in parametric form as

$$\mathbf{x} = \frac{r^2}{d}\mathbf{e}_1 + \lambda\mathbf{e}_2 \tag{2.0.9}$$

Substituting the above in

$$\|\mathbf{x}\|^2 = r^2, \tag{2.0.10}$$

yields

$$\left\|\frac{r^2}{d}\mathbf{e}_1 + \lambda\mathbf{e}_2\right\|^2 = r^2 \tag{2.0.11}$$

$$\implies \lambda^2 = r^2\left[1-\frac{r^2}{d^2}\right] \tag{2.0.12}$$

$$\textit{or, } \lambda = \pm r\sqrt{1-\frac{r^2}{d^2}} \tag{2.0.13}$$

Substitute r and d values in the above equation, we get

$$\therefore \lambda = \pm 2.71 \tag{2.0.14}$$

Now we can substitute r, d and λ *in* (2.0.9).

$$\mathbf{x} = \frac{3^2}{7}\begin{pmatrix}1\\0\end{pmatrix} + \pm 2.71\begin{pmatrix}0\\1\end{pmatrix} \tag{2.0.15}$$

$$\mathbf{x} = \begin{pmatrix}1.285\\0\end{pmatrix} + \begin{pmatrix}0\\\pm 2.71\end{pmatrix} \tag{2.0.16}$$

$$\implies \mathbf{C} = \begin{pmatrix}1.285\\2.71\end{pmatrix}, \mathbf{D} = \begin{pmatrix}1.285\\-2.71\end{pmatrix} \tag{2.0.17}$$

From (2.0.1)

$$\mathbf{Q} = 7\begin{pmatrix}1\\0\end{pmatrix} \tag{2.0.18}$$

$$\mathbf{Q} = \begin{pmatrix}7\\0\end{pmatrix} \tag{2.0.19}$$

Similarly, The points of contact for the tangent drawn from a point

$$\mathbf{P} = d\mathbf{e}_{\mathit{1}}, \textit{where } \mathbf{e}_{\mathit{1}} = \begin{pmatrix}-1\\0\end{pmatrix} \tag{2.0.20}$$

Referencing (2.0.9)*, we have*

$$\mathbf{y} = \frac{r^2}{d}\mathbf{e}_{\mathit{1}} + \lambda\mathbf{e}_2 \tag{2.0.21}$$

Now we can substitute the r, d and λ values in the above equation, we get:

$$\mathbf{y} = \frac{3^2}{7}\begin{pmatrix}-1\\0\end{pmatrix} + \pm 2.71\begin{pmatrix}0\\1\end{pmatrix} \tag{2.0.22}$$

$$\mathbf{y} = \begin{pmatrix}-1.285\\0\end{pmatrix} + \begin{pmatrix}0\\\pm 2.71\end{pmatrix} \tag{2.0.23}$$

$$\implies \mathbf{A} = \begin{pmatrix}-1.285\\2.71\end{pmatrix}, \mathbf{B} = \begin{pmatrix}-1.285\\-2.71\end{pmatrix} \tag{2.0.24}$$

From (2.0.20)

$$\mathbf{P} = 7\begin{pmatrix}-1\\0\end{pmatrix} \tag{2.0.25}$$

$$\mathbf{P} = \begin{pmatrix}-7\\0\end{pmatrix} \tag{2.0.26}$$

- Plot of Tangents PA, PB, QC and QD:

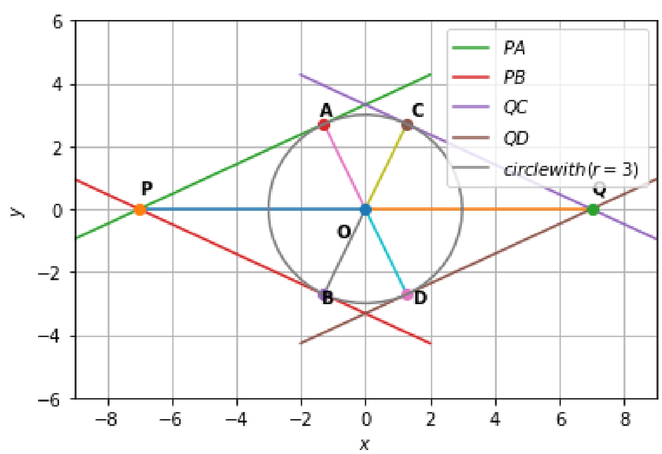

Fig. 2.1: Tangent lines to circle of radius 3 units.