ASSIGNMENT 3

B.ANUSHA

Download all python codes from

```
https://github.com/BOJJAVOYINAANUSHA/
    ASSIGNMENT_3/blob/main/ASSIGNMENT3
    /assignment3.py
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and latex-tikz codes from

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https://github.com/BOJJAVOYINAANUSHA/
    ASSIGNMENT_3/blob/main/ASSIGNMENT3
    /ASSIGNMENT3.tex
```

1 Question No 2.57

Draw a circle of radius 3 units.Take two points P and Q on one its extended diameter each at a distance of 7 units from its centre. Draw tangents to the circle from these two points P and Q.

2 Solution

The center and radius of the circle without any loss of generality is given in table 2.1

	Circle
Centre	$\mathbf{O}=\begin{pmatrix}0\\0\end{pmatrix}$
Radius	r=3

TABLE 2.1: Input values

- Let P and Q are the points on one of its extended diameter each at a distance of 7cm. from its centre.

$$\therefore \mathbf{P} = \begin{pmatrix}-7\\0\end{pmatrix}, \mathbf{Q} = \begin{pmatrix}7\\0\end{pmatrix} \tag{2.0.1}$$

1) To find coordinates of points where tangent touches the circle.
   - Let $\mathbf{M}$ be any point on x-axis whose coordinates are:

$$\mathbf{M} = \begin{pmatrix}M_1\\0\end{pmatrix} \tag{2.0.2}$$

   - Tangents are drawn from $\mathbf{M}$ to any circle with centre $\mathbf{O}$ at origin and radius $\mathbf{r}$.

Lemma 2.1. *The coordinates of points* $\mathbf{N}_1$ *and* $\mathbf{N}_2$ *where tangent touches the circle are given by:*

$$\mathbf{N} = \mathbf{n} + \lambda\mathbf{m} \tag{2.0.3}$$

$$where,\ \mathbf{m} = \begin{pmatrix}0\\1\end{pmatrix} \tag{2.0.4}$$

$$\mathbf{n} = \begin{pmatrix}\frac{r^2}{M_1}\\0\end{pmatrix} \tag{2.0.5}$$

$$\lambda = \pm\sqrt{\frac{r^2 - \|\mathbf{n}\|^2}{\|\mathbf{m}\|^2}} \tag{2.0.6}$$

Proof. We know a tangent is always perpendicular to the radius .

$$\therefore N_1O \perp N_1M, \tag{2.0.7}$$

$$N_2O \perp N_2M \tag{2.0.8}$$

- Now,

$$\implies (\mathbf{O} - \mathbf{N})^T(\mathbf{N} - \mathbf{M}) = 0 \tag{2.0.9}$$

$$\mathbf{N}^T(\mathbf{N} - \mathbf{M}) = 0 \quad (\because \mathbf{O} = 0) \tag{2.0.10}$$

$$\mathbf{N}^T\mathbf{N} - \mathbf{N}^T\mathbf{M} = 0 \tag{2.0.11}$$

$$\mathbf{N}^T\mathbf{M} = \|\mathbf{N}\|^2 \tag{2.0.12}$$

$$\implies \mathbf{M}^T\mathbf{N} = \|\mathbf{N}\|^2\, (\because \mathbf{N}^T\mathbf{M} = \mathbf{M}^T\mathbf{N}) \tag{2.0.13}$$

- Now,using (2.0.2) in above equation we get:

$$\implies \begin{pmatrix}M_1 & 0\end{pmatrix}\mathbf{N} = r^2 \tag{2.0.14}$$

$$\implies \begin{pmatrix}1 & 0\end{pmatrix}\mathbf{N} = \begin{pmatrix}\frac{r^2}{M_1}\\0\end{pmatrix} \tag{2.0.15}$$

$$\implies \mathbf{N} = \begin{pmatrix}\frac{r^2}{M_1}\\0\end{pmatrix} + \lambda\begin{pmatrix}0\\1\end{pmatrix} \tag{2.0.16}$$

$$\implies \mathbf{N} = \mathbf{n} + \lambda\mathbf{m} \tag{2.0.17}$$

$$\text{where, } \mathbf{n} = \begin{pmatrix} \frac{r^2}{M_1} \\ 0 \end{pmatrix} \text{and } \mathbf{m} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{2.0.18}$$

- Also we know,

$$\|\mathbf{n} + \lambda\mathbf{m}\|^2 = r^2 \tag{2.0.19}$$

$$(\mathbf{n} + \lambda\mathbf{m})^T(\mathbf{n} + \lambda\mathbf{m}) = r^2 \tag{2.0.20}$$

$$\lambda^2 = \frac{r^2 - \|\mathbf{n}\|^2}{\|\mathbf{m}\|^2} \tag{2.0.21}$$

$$\lambda = \pm\sqrt{\frac{r^2 - \|\mathbf{n}\|^2}{\|\mathbf{m}\|^2}} \tag{2.0.22}$$

□

2) For tangents to the **Circle** from a point **P**:-
   - Here, we have r=3
   - Now,referencing (2.0.3),we have

$$\mathbf{A} = \mathbf{a} + \lambda_1\mathbf{m} \tag{2.0.23}$$

   where,

$$\mathbf{m} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \mathbf{a} = \begin{pmatrix} \frac{r}{M_1} \\ 0 \end{pmatrix} \tag{2.0.24}$$

   - Using $\mathbf{P} = \begin{pmatrix} M_1 \\ 0 \end{pmatrix} = \begin{pmatrix} -7 \\ 0 \end{pmatrix}$, we get:

$$\mathbf{a} = \begin{pmatrix} \frac{-9}{7} \\ 0 \end{pmatrix} = \begin{pmatrix} -1.285 \\ 0 \end{pmatrix} \tag{2.0.25}$$

   And

$$\lambda_1 = \pm\sqrt{\frac{r^2 - \|\mathbf{a}\|^2}{\|\mathbf{m}\|^2}} \tag{2.0.26}$$

$$\lambda_1 = \pm 2.71 \tag{2.0.27}$$

3) Substituting λ_1, **a** and **m** in (2.0.23) we get the coordinates of **A** and **B** as :

$$\mathbf{A} = \begin{pmatrix} -1.285 \\ 0 \end{pmatrix} + 2.71\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -1.285 \\ 2.71 \end{pmatrix} \tag{2.0.28}$$

$$\mathbf{B} = \begin{pmatrix} -1.285 \\ 0 \end{pmatrix} - 2.71\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -1.285 \\ -2.71 \end{pmatrix} \tag{2.0.29}$$

4) For tangents to the **Circle** from a point **Q**:-
   - Here, we have r=3
   - Referencing (2.0.3),we have

$$\mathbf{C} = \mathbf{c} + \lambda_2\mathbf{m} \tag{2.0.30}$$

   where,

$$\mathbf{m} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \mathbf{c} = \begin{pmatrix} \frac{r}{M_1} \\ 0 \end{pmatrix} \tag{2.0.31}$$

   - Using $\mathbf{Q} = \begin{pmatrix} M_1 \\ 0 \end{pmatrix} = \begin{pmatrix} 7 \\ 0 \end{pmatrix}$ we get :

$$\mathbf{c} = \begin{pmatrix} \frac{9}{7} \\ 0 \end{pmatrix} = \begin{pmatrix} 1.285 \\ 0 \end{pmatrix} \tag{2.0.32}$$

   And

$$\lambda_2 = \pm\sqrt{\frac{r^2 - \|\mathbf{c}\|^2}{\|\mathbf{m}\|^2}} \tag{2.0.33}$$

$$\lambda_2 = \pm 2.71 \tag{2.0.34}$$

5) Substituting λ_2, **c** and **m** in (2.0.30) we get the coordinates of **C** and **D** as :

$$\mathbf{C} = \begin{pmatrix} 1.285 \\ 0 \end{pmatrix} + 2.71\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1.285 \\ 2.71 \end{pmatrix} \tag{2.0.35}$$

$$\mathbf{D} = \begin{pmatrix} 1.285 \\ 0 \end{pmatrix} - 2.71\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1.285 \\ -2.71 \end{pmatrix} \tag{2.0.36}$$

6) Now, the coordinates of A, B, C, D are,

$$\mathbf{A} = \begin{pmatrix} -1.285 \\ 2.71 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -1.285 \\ -2.71 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 1.285 \\ 2.71 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} 1.285 \\ -2.71 \end{pmatrix} \tag{2.0.37}$$

7) Plot of Tangents PA, PB, QC and QD :

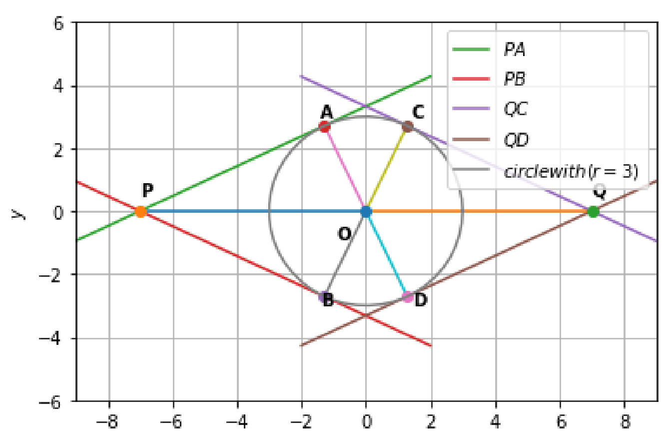

Fig. 2.1: Tangent lines to circle of radius 3 units.