ASSIGNMENT 3

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Download all python codes from

https://github.com/BOJJAVOYINAANUSHA/ASSIGNMENT_3/blob/main/ASSIGNMENT3/assignment3.py

and latex-tikz codes from

https://github.com/BOJJAVOYINAANUSHA/ASSIGNMENT_3/blob/main/ASSIGNMENT3/ASSIGNMENT3.tex

1 Question No 2.57

Draw a circle of radius 3 units.Take two points P and Q on one its extended diameter each at a distance of 7 units from its centre. Draw tangents to the circle from these two points P and Q.

2 Solution

The center and radius of the circle without any loss of generality is given in table 2.1

	Circle
Centre	$\mathbf{O}=\begin{pmatrix}0\\0\end{pmatrix}$
Radius	r=3

TABLE 2.1: Input values

- Let P and Q are the points on one of its extended diameter each at a distance of 7cm. from its centre.

$$\therefore \mathbf{P}=\begin{pmatrix}-7\\0\end{pmatrix}, \mathbf{Q}=\begin{pmatrix}7\\0\end{pmatrix} \tag{2.0.1}$$

- Let PA and PB be tangents from point P with the distance of 7cm. to the points A and B on circle with radius 3 .

We know a tangent is always perpendicular to the radius .

$$\therefore OA \perp AP \tag{2.0.2}$$

Now,

$$(\mathbf{O}-\mathbf{A})^T(\mathbf{A}-\mathbf{P})=0 \quad (\because OA \perp AP) \tag{2.0.3}$$

$$\mathbf{A}^T(\mathbf{A}-\mathbf{P})=0 \quad \left(\because \mathbf{O}=\begin{pmatrix}0\\0\end{pmatrix}\right) \tag{2.0.4}$$

$$\mathbf{A}^T\mathbf{A}-\mathbf{A}^T\mathbf{P}=0 \tag{2.0.5}$$

$$\|\mathbf{A}\|^2=\mathbf{A}^T\mathbf{P} \tag{2.0.6}$$

$$\|\mathbf{A}\|^2=\mathbf{P}^T\mathbf{A} \quad \left(\because \mathbf{A}^T\mathbf{P}=\mathbf{P}^T\mathbf{A}\right) \tag{2.0.7}$$

$$\mathbf{P}^T\mathbf{A}=9 \quad \left(\because \|\mathbf{A}\|^2=9\right) \tag{2.0.8}$$

$$\begin{pmatrix}-7 & 0\end{pmatrix}\mathbf{A}=9 \quad \left(\because \mathbf{P}=\begin{pmatrix}-7\\0\end{pmatrix}\right) \tag{2.0.9}$$

$$\begin{pmatrix}1 & 0\end{pmatrix}\mathbf{A}=\frac{-9}{7} \tag{2.0.10}$$

$$\mathbf{A}=\begin{pmatrix}\frac{-9}{7}\\0\end{pmatrix}+\lambda\begin{pmatrix}0\\1\end{pmatrix} \tag{2.0.11}$$

$$\mathbf{A}=\mathbf{a}+\lambda\mathbf{m} \tag{2.0.12}$$

$$\mathbf{a}=\begin{pmatrix}\frac{-9}{7}\\0\end{pmatrix}, \mathbf{m}=\begin{pmatrix}0\\1\end{pmatrix} \tag{2.0.13}$$

We know,

$$\|\mathbf{a}+\lambda\mathbf{m}\|^2=9 \tag{2.0.14}$$

$$(\mathbf{a}+\lambda\mathbf{m})^T(\mathbf{a}+\lambda\mathbf{m})=r^2 \tag{2.0.15}$$

$$\lambda^2=\frac{r^2-\|\mathbf{a}\|^2}{\|\mathbf{m}\|^2} \tag{2.0.16}$$

$$\lambda=\pm 2.71 \tag{2.0.17}$$

Substitute λ value in (2.0.28) we get,

$$\mathbf{A}=\begin{pmatrix}\frac{-9}{7}\\2.71\end{pmatrix}, \mathbf{B}=\begin{pmatrix}\frac{-9}{7}\\-2.71\end{pmatrix} \tag{2.0.18}$$

- Similarly, Let QC and QD be tangents from point Q with the distance of 7cm. to the points C and D on circle with radius 3.

We know a tangent is always perpendicular to the radius .

$$\therefore OC \perp CQ \tag{2.0.19}$$

Now,

$$(\mathbf{O}-\mathbf{C})^T(\mathbf{C}-\mathbf{Q})=0 \quad (\because OC \perp CQ) \tag{2.0.20}$$

$$\mathbf{C}^T(\mathbf{C}-\mathbf{Q})=0 \quad \left(\because \mathbf{O}=\begin{pmatrix}0\\0\end{pmatrix}\right) \tag{2.0.21}$$

$$\mathbf{C}^T\mathbf{C}-\mathbf{C}^T\mathbf{Q}=0 \tag{2.0.22}$$

$$\|\mathbf{C}\|^2=\mathbf{C}^T\mathbf{Q} \tag{2.0.23}$$

$$\|\mathbf{C}\|^2=\mathbf{Q}^T\mathbf{C} \quad \left(\because \mathbf{C}^T\mathbf{Q}=\mathbf{Q}^T\mathbf{C}\right) \tag{2.0.24}$$

$$\mathbf{Q}^T\mathbf{C}=9 \quad \left(\because \|\mathbf{C}\|^2=9\right) \tag{2.0.25}$$

$$\begin{pmatrix}7 & 0\end{pmatrix}\mathbf{C}=9 \quad \left(\because \mathbf{Q}=\begin{pmatrix}7\\0\end{pmatrix}\right) \tag{2.0.26}$$

$$\begin{pmatrix}1 & 0\end{pmatrix}\mathbf{C}=\frac{9}{7} \tag{2.0.27}$$

$$\mathbf{C}=\begin{pmatrix}\frac{9}{7}\\0\end{pmatrix}+\lambda\begin{pmatrix}0\\1\end{pmatrix} \tag{2.0.28}$$

$$\mathbf{C}=\mathbf{c}+\lambda\mathbf{m} \tag{2.0.29}$$

$$\mathbf{c}=\begin{pmatrix}\frac{9}{7}\\0\end{pmatrix}, \mathbf{m}=\begin{pmatrix}0\\1\end{pmatrix} \tag{2.0.30}$$

We know,

$$\|\mathbf{c}+\mu\mathbf{m}\|^2=9 \tag{2.0.31}$$

$$(\mathbf{c}+\mu\mathbf{m})^T(\mathbf{c}+\mu\mathbf{m})=r^2 \tag{2.0.32}$$

$$\mu^2=\frac{r^2-\|\mathbf{c}\|^2}{\|\mathbf{m}\|^2} \tag{2.0.33}$$

$$\lambda=\pm 2.71 \tag{2.0.34}$$

Substitute μ value in (2.0.28) we get

$$\mathbf{Q}=\begin{pmatrix}\frac{9}{7}\\2.71\end{pmatrix}, \mathbf{R}=\begin{pmatrix}\frac{9}{7}\\-2.71\end{pmatrix} \tag{2.0.35}$$

- Plot of Tangents PA,PB,QC and QD :

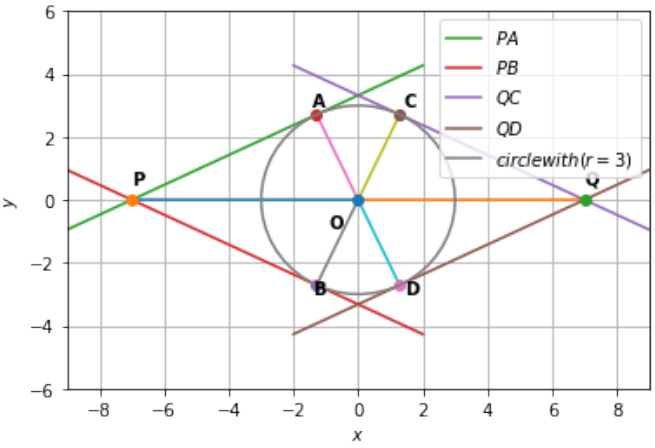

Fig. 2.1: Tangent lines to circle of radius 3 units.