

ASSIGNMENT-6

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1 QUESTION No-2.33 (QUADRATIC FORMS)

Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $\mathbf{x}^T \begin{pmatrix} \frac{1}{25} & 0 \\ 0 & \frac{1}{9} \end{pmatrix} \mathbf{x} = 1$.

2 SOLUTION

Given equation of the ellipse,

$$\mathbf{x}^T \begin{pmatrix} \frac{1}{25} & 0 \\ 0 & \frac{1}{9} \end{pmatrix} \mathbf{x} = 1 \quad (2.0.1)$$

we have,

$$\mathbf{V} = \begin{pmatrix} \frac{1}{25} & 0 \\ 0 & \frac{1}{9} \end{pmatrix} \quad (2.0.2)$$

$$\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f = 1 \quad (2.0.3)$$

$$\mathbf{c} = -\mathbf{V}^{-1} \mathbf{u} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (2.0.4)$$

$$\lambda_1 = \frac{1}{25}, \lambda_2 = \frac{1}{9} \quad (2.0.5)$$

Axes of ellipse is given by: Length of semi major axis, a is

$$a = \sqrt{\frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_1}} \quad (2.0.6)$$

substituting the values in (2.0.6), we get

$$a = 5 \quad (2.0.7)$$

Length of major axis is $2a = 10$ and the length of semi minor axis, b is

$$b = \sqrt{\frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_2}} \quad (2.0.8)$$

substituting the values in (2.0.8), we get

$$b = 3 \quad (2.0.9)$$

Length of the minor axis is $2b = 6$

The vertices are given as

$$\pm \begin{pmatrix} 5 \\ 0 \end{pmatrix} \quad (2.0.10)$$

Coordinates of foci are given by,

$$\mathbf{F} = \pm \left(\sqrt{\frac{(\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f)(\lambda_2 - \lambda_1)}{\lambda_1 \lambda_2}} \right) \mathbf{p}_1 \quad (2.0.11)$$

where, $\mathbf{p}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ since the equation of ellipse is in standard form. Substituting the values in (2.0.11) we have,

$$\mathbf{F} = \pm \begin{pmatrix} 4 \\ 0 \end{pmatrix}. \quad (2.0.12)$$

Eccentricity of the ellipse is given by,

$$e = \frac{\sqrt{\frac{(\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f)(\lambda_2 - \lambda_1)}{\lambda_1 \lambda_2}}}{\sqrt{\frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_1}}} \quad (2.0.13)$$

substituting the values in (2.0.13), we have

$$e = \frac{4}{3}. \quad (2.0.14)$$

Length of the latus rectum is given by,

$$l = \frac{2 \left(\sqrt{\frac{f - \mathbf{u}^T \mathbf{V}^{-1} \mathbf{u}}{\lambda_2}} \right)^2}{\sqrt{\frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_1}}} \quad (2.0.15)$$

substituting the values in (2.0.15), we have

$$l = \frac{18}{5} \quad (2.0.16)$$

The plot of the ellipse is given below

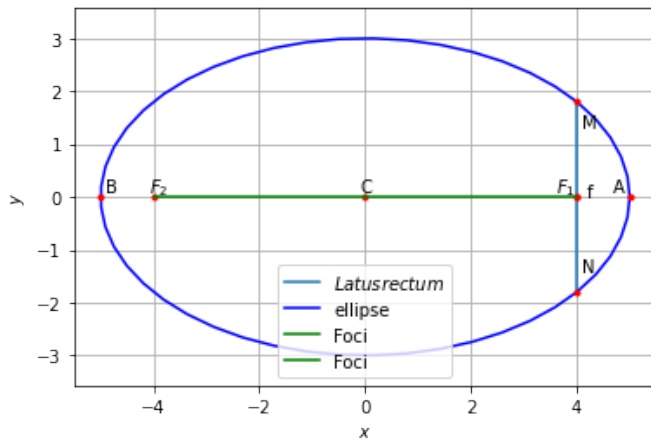


Fig. 2.1: Plot of standard ellipse