ASSIGNMENT-6

B. Anusha

1 QUESTION No-2.33 (Quadratic forms)

Find the coordinates of the foci, the vertices,the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $\mathbf{x}^\top \begin{pmatrix} \frac{1}{25} & 0 \\ 0 & \frac{1}{9} \end{pmatrix} \mathbf{x} = 1$.

2 Solution

Given equation of the ellipse,

$$\mathbf{x}^\top \begin{pmatrix} \frac{1}{25} & 0 \\ 0 & \frac{1}{9} \end{pmatrix} \mathbf{x} = 1 \tag{2.0.1}$$

we have,

$$\mathbf{V} = \begin{pmatrix} \frac{1}{25} & 0 \\ 0 & \frac{1}{9} \end{pmatrix} \tag{2.0.2}$$

$$\mathbf{u}^\top \mathbf{V}^{-1} \mathbf{u} - f = 1 \tag{2.0.3}$$

$$\mathbf{c} = -\mathbf{V}^{-1}\mathbf{u} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{2.0.4}$$

$$\lambda_1 = \frac{1}{25}, \lambda_2 = \frac{1}{9} \tag{2.0.5}$$

Axes of ellipse is given by: Length of semi major axis, a is

$$a = \sqrt{\frac{\mathbf{u}^\top \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_1}} \tag{2.0.6}$$

substituting the values in (2.0.6),we get

$$a = 5 \tag{2.0.7}$$

Length of major axis is 2a = 10
and the length of semi minor axis, b is

$$b = \sqrt{\frac{\mathbf{u}^\top \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_2}} \tag{2.0.8}$$

substituting the values in (2.0.8),we get

$$b = 3 \tag{2.0.9}$$

Length of the minor axis is 2b = 6
The vertices are given as

$$\pm \begin{pmatrix} 5 \\ 0 \end{pmatrix} \tag{2.0.10}$$

Coordinates of foci are given by,

$$\mathbf{F} = \pm \left(\sqrt{\frac{(\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f)(\lambda_2 - \lambda_1)}{\lambda_1 \lambda_2}} \right) \mathbf{p_1} \tag{2.0.11}$$

where, $\mathbf{p_1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ since the equation of ellipse is in standard form. Substituting the values in (2.0.11) we have,

$$\mathbf{F} = \pm \begin{pmatrix} 4 \\ 0 \end{pmatrix}. \tag{2.0.12}$$

Eccentricity of the ellipse is given by,

$$e = \frac{\sqrt{\frac{(\mathbf{u}^\top \mathbf{V}^{-1} \mathbf{u})(\lambda_2 - \lambda_1)}{\lambda_1 \lambda_2}}}{\sqrt{\frac{\mathbf{u}^\top \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_1}}} \tag{2.0.13}$$

substituting the values in (2.0.13),we have

$$e = \frac{4}{3}. \tag{2.0.14}$$

Length of the latus rectum is given by,

$$l = \frac{2\left(\sqrt{\frac{f - \mathbf{u}^\top \mathbf{V}^{-1} \mathbf{u}}{\lambda_2}} \right)^2}{\sqrt{\frac{\mathbf{u}^\top \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_1}}} \tag{2.0.15}$$

substituting the values in (2.0.15),we have

$$l = \frac{18}{5} \tag{2.0.16}$$

The plot of the ellipse is given below

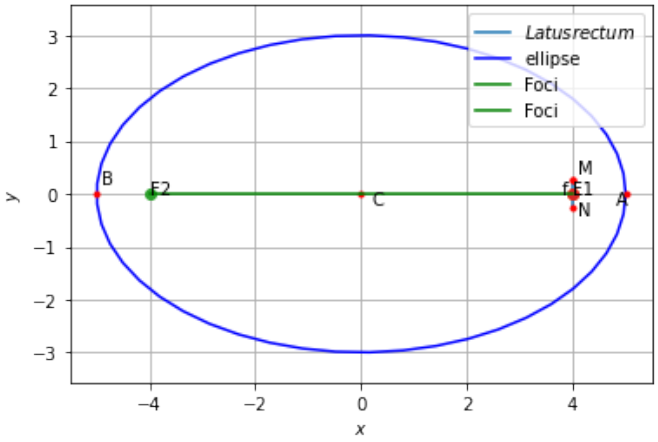

Fig. 2.1: Plot of standard ellipse