

Fax  
DVD  
 Touchscreen  
 Pendrive

Brain or -

- Mainframe
- Micro
- Super
- PC
- Minic

### DIGITAL LOGIC

⇒ Is the study of digital pulses.

In computer, reasoning is done by logic gates.

Logic gates - high speed switches which uses transistors and diodes.

In logic gates a transistor which is used is a one acting as a switch

To switching

ON - high (1)

OFF - low (0)

0 and 1 are known as binary digits (bits)

Description of logic gates is done by truth table.

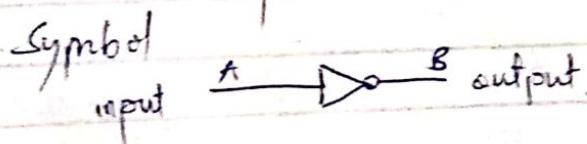
Truth table - is a table consisting of truth values.

### Types of logic gates

- i) AND
- ii) OR
- iii) NOR
- iv) NAND
- v) Exclusive OR
- vi) Exclusive NOR
- vii) NOT

### NOT GATE

- Inverts inputs



A	$\bar{A} = B$
1	0
0	1

## AND GATE



$$A \cdot B = AB = A \wedge B$$

A	B	Q
0	0	0
0	1	0
1	0	0
1	1	1

## OR GATE (A)



$$A + B = AB = A \vee B$$

$$A + B$$

A	B	Q
0	0	0
0	1	1
1	0	1
1	1	1

## NAND GATE (AND-NOT)



$$Y = \overline{A \cdot B}$$

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

### NOR GATE (OR-NOT)

OR  
OR  
OR



$$Y = \overline{A+B}$$

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

### EXCLUSIVE OR GATE

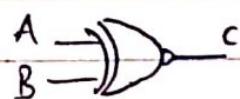


$$Y = \bar{A}B + A\bar{B} = A \oplus B$$

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

EXCLUSIVE NOR GATE

⊖ Exnor

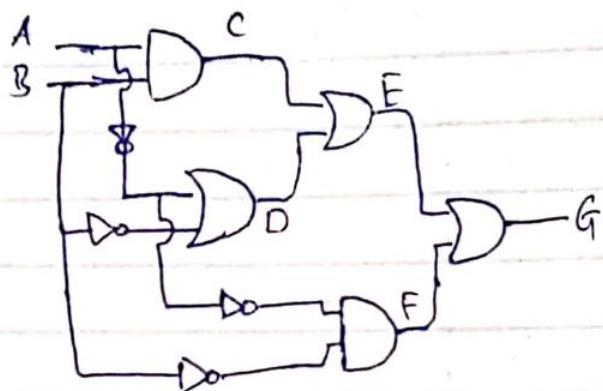


$$C = A \oplus B$$

~~$C = \overline{AB}$~~

$$C = (A + \bar{B}) \cdot (\bar{A} + B)$$

A	B	C
0	0	1
0	1	0
1	0	0
1	1	1

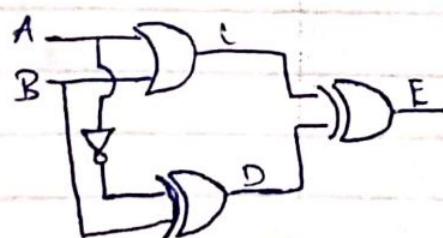


A	B	$\bar{A}$	$\bar{B}$	C	D	E	F	G
0	0	1	1	0	1	1	0	1
0	1	1	0	0	1	1	0	1
1	0	0	1	0	1	1	0	1
1	1	0	0	1	0	1	0	1

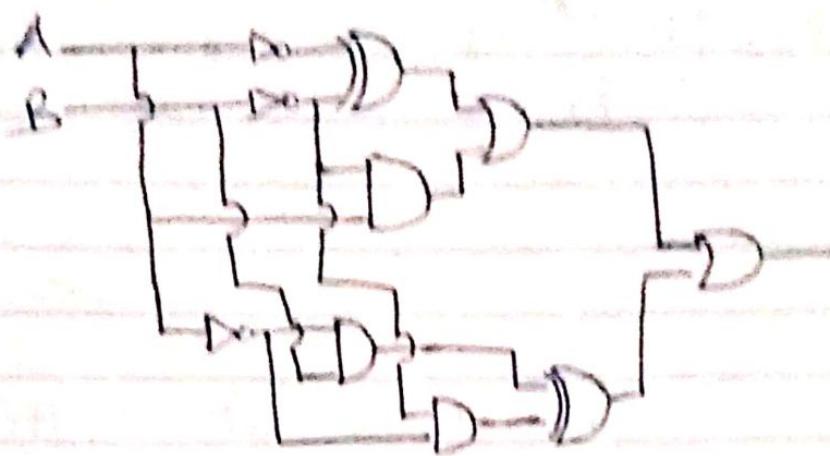
Given i)  $(A \vee B) \oplus (\sim A \oplus B)$

$$\text{ii) } [(\sim A \oplus B) \vee (A \wedge \sim B)] \vee [(\sim A \wedge B) \oplus (\sim A \wedge \sim B)]$$

iv)



v)



Given

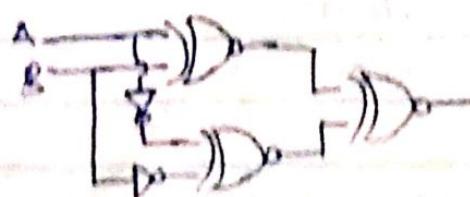
i)  $(A \oplus B) \oplus (\neg A \oplus \neg B)$

ii)  $\neg [\neg (\neg A \vee B) \wedge (A \wedge \neg B)]$

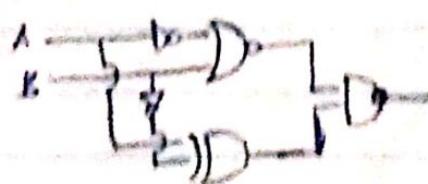
iii)  $\neg [\neg (A \wedge \neg B) \oplus (\neg A \oplus B)]$

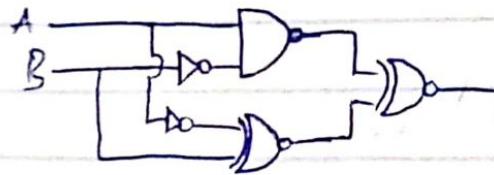
Soln:

i)



ii)





## LAWS OF BOOLEAN ALGEBRA.

Idempotent

$$\begin{aligned} 1: \quad A \wedge A &\equiv A \\ A \vee A &\equiv A \end{aligned}$$

Double negation

$$7: \quad \neg\neg A = A$$

Identity

$$\begin{aligned} 2: \quad A \vee 1 &\equiv 1 \\ A \wedge 1 &\equiv A \\ A \vee 0 &\equiv A \\ A \wedge 0 &\equiv 0 \end{aligned}$$

$$8: \quad \neg(A \wedge B) \equiv \neg A \vee \neg B$$

$$\neg(A \vee B) \equiv \neg A \wedge \neg B$$

$$9: \quad A \vee (A \wedge B) \equiv A$$

Commutative

$$\begin{aligned} 3: \quad A \vee B &\equiv B \vee A \\ A \wedge B &\equiv B \wedge A \end{aligned}$$

$$A \wedge (A \vee B) \equiv A$$

Distributive

$$\begin{aligned} 10: \quad A \vee (B \wedge C) &\equiv (A \vee B) \wedge (A \vee C) \\ A \wedge (B \vee C) &\equiv (A \wedge B) \vee (A \wedge C) \end{aligned}$$

Association

$$\begin{aligned} 11: \quad A \vee (B \vee C) &\equiv (A \vee B) \vee C \\ A \wedge (B \wedge C) &\equiv (A \wedge B) \wedge C \end{aligned}$$

Complement

$$\begin{aligned} 12: \quad A \vee \neg A &\equiv 1 \\ A \wedge \neg A &\equiv 0 \end{aligned}$$

Take  $O = F$   
 $\Delta = T$   
SOP & POS

SOP (Sum Of Products) - for high output  
 POS (Product Of Sum) - for low output

A	B	C	X
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

Using SOP

$$(\neg A \cdot B \cdot C) + (A \cdot \neg B \cdot C) + (A \cdot B \cdot \neg C)$$

$$[(\neg A \cdot B) \cdot C] + [(A \cdot \neg B) \cdot C] + [(A \cdot B) \cdot \neg C]$$

$$[C \cdot ((\neg A \cdot B) + (A \cdot B))] + [(A \cdot B) \cdot \neg C]$$

$$\text{e. } [(\neg A \cdot B) + (A \cdot B)]$$

## DATA PRESENTATION

Binary - are two digits which are represented in terms of HIGH (1) and LOW (0) which are in terms of arithmetic

Binary operations are centred in addition and multiplication

### ADDITION OF BINARY DIGITS (BITS)

High + High = High & Low

High + Low = Low

e.g.:

$$\begin{array}{r} 110 \\ + 11 \\ \hline 1001 \end{array}$$

$$\begin{array}{r} 11100 \\ 1110 \\ \hline 101010 \end{array}$$

$$\begin{array}{r} 1111110 \\ + 1111 \\ \hline 10001101 \end{array}$$

$$\begin{array}{r} 111101101 \\ 1110011 \\ \hline 1001100000 \end{array}$$

### BINARY MULTIPLICATION

$$0 \times 1 = 0$$

$$1 \times 1 = 1$$

$$\begin{array}{r}
 0 + 0 = 0 \\
 1 + 0 = 1 \\
 1 + 1 = 2 \\
 10 + 1 = 3
 \end{array}$$

$$\begin{array}{r}
 11101 \\
 1111 \\
 1110
 \end{array}$$

eg.

$$\begin{array}{r}
 1110 \\
 \times 111 \\
 \hline
 1110 \\
 1110 \\
 + 1110 \\
 \hline
 1110010
 \end{array}$$

### BINARY CONVERSION

1. Binary to decimal (reverse)
2. Binary to hexadecimal
3. Binary to octal
4. BCD (Binary Coded decimal)
5. Gray code
6. Complement (1's, 2's)

L  
 1100  
 0110  
 0011  
 0100  
 1001  
 0000  
 R  
 1100  
 0110  
 0011  
 0100  
 1001  
 0000  
 Dc. Cax 10  
 at

### BINARY TO DECIMAL

Convert to decimal

$$\text{Q) } 11101.11011 = 1 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 + 1 \times 2^{-1} + 0 \times 2^{-2} + 1 \times 2^{-3} + 1 \times 2^{-4}$$

=

$$\text{b) } 11.00111$$

### DECIMAL TO BINARY

-Repeating divide by two until the last digit remainder is zero

least significant binary number is obtained from first remainder

Most significant binary number is obtained for the last remainder.

Example:

$$\text{a) } 5_{10}$$

$$\begin{array}{r} 15 \\ \hline 2 \\ 7 \\ \hline 2 \\ 1 \end{array} = 2 \text{ rem } 1 \quad (\text{LSB})$$

$$\begin{array}{r} 7 \\ \hline 2 \\ 3 \\ \hline 1 \\ 0 \end{array} = 1 \text{ rem } 0$$

$$\frac{1}{2} = 0 \cos 1 \quad (\text{MSB})$$

$$\therefore \Sigma_b = 101_2$$

b)  $F_0$

$$\frac{7}{2} = 3 \cos 1 \quad (\text{LSB})$$

$$\frac{13}{2} = 1 \cos 1$$

$$\frac{1}{2} = 0 \cos 1 \quad (\text{MSB})$$

$$F_0 = 111_2$$

c)  $15_{10}$

$$\frac{15}{2} = 7 \cos 1 \quad (\text{LSB})$$

$$\frac{7}{2} = 3 \cos 1$$

$$\frac{13}{2} = 1 \cos 1$$

$$\frac{1}{2} = 0 \cos 1 \quad (\text{MSB})$$

$$15_{10} = 1111_2$$

d)  $45_{10}$ .

$$\begin{array}{r} 45 \\ \hline 2 \end{array} = 22 \text{ rem } 1 \quad (\text{LSB})$$

$$\begin{array}{r} 22 \\ \hline 2 \end{array} = 11 \text{ rem } 0$$

$$\begin{array}{r} 11 \\ \hline 2 \end{array} = 5 \text{ rem } 1$$

$$\begin{array}{r} 5 \\ \hline 2 \end{array} = 2 \text{ rem } 1$$

$$\begin{array}{r} 2 \\ \hline 2 \end{array} = 1 \text{ rem } 0$$

$$\begin{array}{r} 1 \\ \hline 2 \end{array} = 0 \text{ rem } 1 \quad (\text{MSB})$$

$$45_{10} = 101101_2$$

e)  $165_{10}$ 

$$\begin{array}{r} 165 \\ \hline 2 \end{array} = 82 \text{ rem } 1$$

$$\begin{array}{r} 82 \\ \hline 2 \end{array} = 41 \text{ rem } 0$$

$$\begin{array}{r} 41 \\ \hline 2 \end{array} = 20 \text{ rem } 1$$

$$\frac{20}{2} = 10 \text{ rem } 0$$

$$\frac{10}{2} = 5 \text{ rem } 0$$

$$\frac{5}{2} = 2 \text{ rem } 1$$

$$\frac{2}{2} = 1 \text{ rem } 0$$

$$\frac{1}{2} = 0 \text{ rem } 1$$

$$165_{10} = 10100101_2$$

f)  $0.375_{10}$

$$\begin{aligned}
 &= 0.375 \times 2 = 0.75 && \text{MSB} \\
 &= 0.25 \times 2 = 0.5 && \\
 &\cancel{= 0.5 \times 2} = 1.0 && \\
 &= 0.0 \times 2 = 0.0 && \text{LSB}
 \end{aligned}$$

$$0.375_{10} = 011_2$$

g)  $512.375_{10}$

$$\frac{512}{2} = 256 \text{ rem } 0$$

$$\frac{256}{2} = 128 \text{ rem } 0$$

$$\frac{128}{2} = 64 \text{ rem } 0$$

$$\frac{64}{2} = 32 \text{ rem } 0$$

$$\frac{32}{2} = 16 \text{ rem } 0$$

$$\frac{16}{2} = 8 \text{ rem } 0$$

$$\frac{8}{2} = 4 \text{ rem } 0$$

$$\frac{4}{2} = 2 \text{ rem } 0$$

$$\frac{2}{2} = 1 \text{ rem } 0$$

$$\frac{1}{2} = 0 \text{ rem } 1$$

$$0.375_{10} = 011$$

$$512_{10} = 1000000000$$

$$512 \cdot 375_{10} = 1000000000.011$$

0.83

$$0.83 \times 2 = 0.66 \quad \downarrow$$

$$0.66 \times 2 = 1.32$$

$$0.32 \times 2 = 0.64$$

$$0.64 \times 2 = 1.28$$

$$0.28 \times 2 = 0.56 \quad \underline{1}$$

$$0.56 \times 2 = 1.12$$

### HEXADECIMAL TO BINARY

Hexadecimal, a group of four bits to 0.0000  
 . Hexadecimal is always in base 16

Hex	binary	Hex	binary
0	0000	C	1100
1	0001	D	1101
2	0010	E	1110
3	0011	F	1111
4	0100		
5	0101		
6	0110		
7	0111		
8	1000		
9	1001		
A	1010		
B	1011		

## Convert Hex into B

- a)  $28AF_{16}$
- b)  $1ABE_{16}$
- c)  $F018_{16}$

Soln

a)  $2 \quad 8 \quad A \quad F$   
 $0010 \quad 1000 \quad 1010 \quad 1111$

$$28AF_{16} = 0010100010101111_2$$

b)  $1 \quad A \quad B \quad E$   
 $0001 \quad 1010 \quad 1011 \quad 1110$

$$1ABE_{16} = 0001101010111110_2$$

c)  $F018_{16}$

$F \quad 0 \quad 1 \quad 8$   
 $1111 \quad 0000 \quad 0001 \quad 1000$

$$F018_{16} = 1111000000011000$$

## BINARY TO HEXADECIMAL.

eg

$$\text{a) } 110001110_2$$

$$= \begin{array}{r} 0001110001110 \\ \underline{1} \quad \underline{8} \quad \underline{\text{E}} \end{array}$$

$$= 110001110_2 = 18\text{E}_{16}$$

$$\text{b) } 1111001100111001_2$$

$$= \begin{array}{r} 1111001100111001 \\ \underline{\text{F}} \quad \underline{3} \quad \underline{3} \quad \underline{9} \end{array}$$

$$\Rightarrow 1111001100111001_2 = \text{F339}$$

HEXADECIMAL - DECIMAL

$$\text{eg } 18\text{E}_{16}$$

soln,

$$\begin{array}{r} 18\text{E}_{16} \\ \underline{2} \quad \underline{1} \quad \underline{0} \end{array} = 1 \times 16^2 + 8 \times 16^1 + 14 \times 16^0$$

$$18\text{E}_{16} = 398_{10}$$

b)  $3EA2F_{16}$

$$= 3 \times 16^5 + 14 \times 16^4 + 10 \times 16^3 + 2 \times 16^2 + 15 \times 16^1$$

=

### OCTAL TO BINARY

Octal numbers contains only 3 bits

Octal	bits
0	000
1	001
2	010
3	011
4	100
5	101
6	110
7	111

Convert to binary

a)  $376_8$

$$= 0111110_2$$

b)  $409_8$

$$= 100000111_2$$

Convert binary to octal

a)  $11100110011_8 = 17146_{10}$

### OCTAL TO DECIMAL

a)  $146_s$

$$= 1 \times 8^2 + 4 \times 8^1 + 6 \times 8^0$$
$$146_8 = 102_{10}$$

Convert  $102_{10}$  to octal then to hexa

$102_{10}$

$$\frac{102}{2} = 51 \text{ rem } 0$$

$$\frac{51}{2} = 25 \text{ rem } 1$$

$$\frac{25}{2} = 12 \text{ rem } 1$$

$$\frac{12}{2} = 6 \text{ rem } 0$$

$$\frac{6}{2} = 3 \text{ rem } 0$$

$$\frac{3}{2} = 1 \text{ rem } 1$$

$$\frac{1}{2} = 0 \text{ rem } 1$$

$$102_{10} = 1100110_2$$

$$1100110_2 = 146_s$$

$$1100110_2 = 66_{16}$$

1s COMPLEMENT

- Inversion of binary digits

eg

Find ones complement

$$a) 11100 \rightarrow 00011$$

$$b) 101011 \rightarrow 010100$$

2s COMPLEMENT

Addition of high to ones complement

eg

$$a) 11101 \longrightarrow 00010 \longrightarrow 00011$$

$$b) 00111\ 00110 \longrightarrow 11000\ 11001 \longrightarrow 110001101$$

\*

BINARY CODED DECIMAL (BCD)

Decimal	Binary
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	
9	1000 1001

Convert the following to Binary

a)  $89_{10} = 1000100100100100_2$

b)  $124 = 000100100100_2$

c)  $2567 =$

Convert binary to BCD

a)  $1001100100100100_2 = 9924$

### GRAY CODE

- Help in discarding
- Are unweighted ie has no base
- Unweighted code which help in discarding the part in an ALU
- This help in ALU if the bits are overweighted

Conversion from binary to Gray Code

$$XY\bar{Z} + XWY + \bar{W}YZ$$

$$XY\cdot\bar{Z}\cdot 1 + XWY\cdot 1 + \bar{W}YZ\cdot 1$$

$$X\cdot Y\cdot\bar{Z}\cdot(W+\bar{W}) + XWY\cdot(Z+\bar{Z}) + \bar{W}YZ\cdot(X+\bar{X})$$

$$XY\bar{Z}W + XY\bar{Z}\bar{W} + XWYZ + YWY\bar{Z} + \bar{W}YZX + \bar{W}YZ\bar{X}$$

$$AB + A\bar{B}C + \bar{A}B\bar{C}D$$

## NIBBLES

- Is a group of four bits

e.g.

$$1 \text{ nibble} = 0101$$

$$\text{Nibble} = \text{Nybble}$$

- Byte - is a group of two consecutive nibbles OR
- is a group of eight bits

$$1 \text{ kilobyte} = 2^{10} \text{ Bytes} = 1024 \text{ bytes}$$

## Abbreviations

Nibble	Nibbles
Byte	B
Kilobyte	KB
Megabyte	MB
Gigabyte	GB
Serabyte	TB

Convert 36 GB to B

$$= 36 \times 2^{10} \times 2^{10} \times 10^{10}$$

$$= 3.865 \times 10^{10} B$$

Convert 256 TB to i) KB ii) MB iii) B

i)  $256 \text{ TB} = 256 \times 2^{30} \text{ KB}$

ii)  $256 \text{ TB} = 256 \times 2^{20} \text{ MB}$

iii)  $256 \text{ TB} = 256 \times 2^{40} \text{ B}$

## MEMORY SYSTEM

Date \_\_\_\_\_

Byte is unit of storage of memory computer

- i) Primary storage is nearby processor
- are always volatile
  - are very fast

Performance of the processor depends on the location of a memory

### Types of memory

- Main memory (primary)
- Secondary memory

Main memory - are volatile memory in which the data lost when power is switched off

e.g. Registers, Cache, RAM

### Components of memory system

- i) Main memory
- ii) Secondary memory

### Problems with the memory system

Access time | Execution time

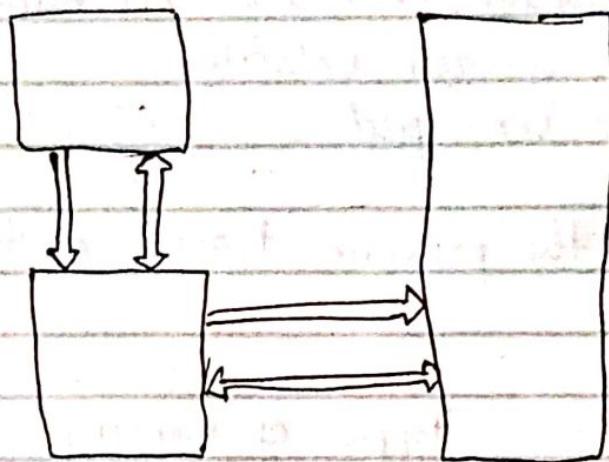
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Unified cache

Instruction

Data

Split cache (Harvard cache)



$$\text{Miss rate} = \frac{\text{No of misses}}{\text{total no of accesses}}$$

$$\text{total no of access} = \text{No of hits} + \text{No misses}$$

$$\text{Hit rate} = \frac{\text{No of hits}}{\text{total no of access}}$$

One line in a cache is equal to 4B

$$8KB = 8 \times 2^{10} \text{ Bytes}$$

$$1 \text{ line} = 4B$$

$$x = 8 \times 2^{10} B$$

$$x = 2^n \text{ lines}$$

### Replacement Algorithm

to accommodate as many info as possible

- Random replacement
- Least recently used (LRU)
- First in - First Out (FIFO)
- Least frequently used (LFU)