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COURSE TITLE: LINEAR ALGEBRA.
COURSE CODE: RMT 122.
COURSE INSTRUCTOR: BENEDICT LOGATHO
SEMISTER TWO:

Subtopic I: Linear equation and matrice
Introduction:
An equation is a mathematical statement which has an equation equal sign (=) btn the algebraic expression

Linear equations are the equation of degree 1 OR it is the equation for the straight line.
The soln of linear equations will generate values when substituted for unknown values make the eqn true.

Standard form of linear equations
@ In one variable $Ax+B=0$
(b) In two variable $Ax+By=C$

where a, b

Generally a linear equation is given by $a_1x_1 + a_2x_2 + \dots a_nx_n = b$ where $a_1, a_2 \dots a_n \neq 0$ and are either real or complex constant/coefficient. b is a constant number which can be real or complex.

A system of linear equation in n unknowns/variables $x_1, x_2 \dots x_n$ or a linear system is a set of M linear equations each in n unknown variables. A linear system can be written as

$$a_{11}x_1 + a_{12} + \dots a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22} + \dots a_{2n}x_n = b_2$$

$$\vdots$$

$$a_{m1}x_1 + a_{mn}x_n = b_m$$

Note: if $b_1 = b_2 = \dots b_m = 0$ then the equation above is called a homogeneous system

→ If $x_1 = x_2 = \dots x_n = 0$ and is a solution to a homogeneous system then it is called trivial solutions otherwise is called non-trivial solution.

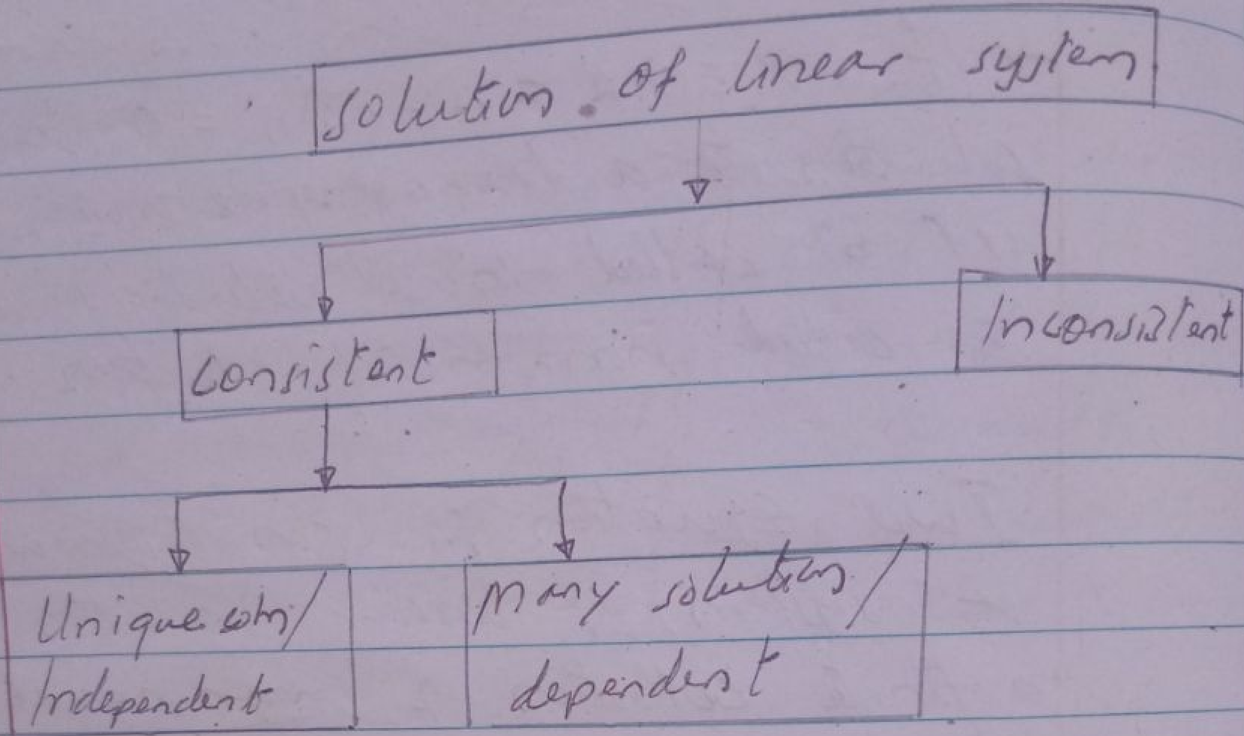
Two equation in two unknown:

A system of linear equation is simply a finite collection of linear equations involving certain variables. In two variable is given by:

$$a_1 x + b_1 y = c_1$$
$$a_2 x + b_2 y = c_2$$

A soln is a pair of (x, y) that satisfy both equation simulteneously.

→ linear system is said to be consistent if it has at least one solution and inconsistent if it has none.



In case of a linear system

$$a_1x + b_1y = c_1 \quad \text{---} \quad L_1$$
$$a_2x + b_2y = c_2 \quad \text{---} \quad L_2$$

then exactly one of the following soln must hold

- The lines L_1 and L_2 interest at a single point
- The lines L_1 and L_2 are parrallel to each other
- The lines L_1 and L_2 coincide they actually are the same lines

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Solving system of linear equation

The method of Elimination Involve transforming a given linear system by means of sequences of succesive steps. Each of the steps consists of performing one of the following three elementary operation which produce equivalent system of equation (which is much simpler to solve and have exactly the same solution

1: Multiply an equation by non-zero constant.

2. Interchange two equation

3. Add a constant multiple of (the

term of one equation to (corresponding term of) another equation

Example: Solve the system of equations by elimination method:

$$x+y=7$$
$$2x+3y=18$$

<u>Solution</u>.

$2|\ x+y=7$
$1|\ 2x+3y=18$

$$\begin{aligned} &\cancel{2x}+2y=14 \\ -&\cancel{2x}+3y=18 \end{aligned}$$

$+y=+4$
$y=4$.

Take eqn (i) substitute value of y.

$x+4=7$.
$x=7-4$
$x=3$.

Example 2: Solve the linear system by Elimination method.

$$\begin{cases} x + 2y + z = 4 \\ 3x + 8y + 7z = 20 \\ 2x + 7y + 9z = 23 \end{cases}$$

Soln.

Add -3 times the first equation to the second equation.

$$x + 2y + z = 4$$
$$2y + 4z = 8$$
$$2x + 7y + 9z = 23$$

Add -2 times the first equation to the third equation.

$$x + 2y + z = 4$$
$$2y + 4z = 8$$
$$3y + 7z = 15$$

multiply the second eqn by 1/2

$$x + 2y + z = 4$$
$$y + 2z = 4$$
$$3y + 7z = 15$$

Add -3 times the second eqn th
third equation.

$$x + 2y + z = 4.$$

$$y + 2z = 4$$

$$z = 3$$

The system has a triangular form that makes its solution easy by back substitution $z = 3$

$$2z + y = 4$$

$$y + 2(3) = 4$$

$$y = -2$$

also $x + 2y + z = 4.$

$$x + 2(-2) + 3 = 4.$$

$$x = 5$$

The solution is unique (x, y, z)
$(5, -2, 3)$

Try

solve the system of linear equation by elimination method.

① $3x - 8y + 10z = 22$

$x - 3y + 2z = 5$

$2x - 9y - 8z = -11$

② $x + 2y + 3z = 2$

$3x - 5y - 4z = 15$

$-2x - 3y + 2z = 2$

③ $3x + 2y = 9$

$x - y = 8$

④ $3x - 2y = 8$

$2x + 5y = -1$

Solution

2. $x + 2y + 3z = 2$ ---- (i)

$3x - 5y - 4z = 15$ ---- (ii)

$-2x - 3y + 2z = 2$ ---- (iii)

Take equation (i) and (ii)

$$3\left\{x + 2y + 3z = 2\right.$$
$$1\left\{3x - 5y - 4z = 15\right.$$

$$-\begin{cases} 3x + 6y + 9z = 6 \\ 3x - 5y - 4z = 15 \end{cases}$$

$$+11y + 13z = -9 \quad \text{---- (A)}$$

Take equation (ii) and (iii)

$$-2\left\{3x - 5y - 4z = 15\right.$$
$$3\left\{-2x - 3y + 2z = 2\right.$$

$$-\begin{cases} -6x + 10y + 8z = -30 \\ -6x - 9y + 6z = 6 \end{cases}$$

$$19y + 2z = -26 \quad \text{---- (B)}$$

Combine equation A and B

$$\begin{cases} 19) & 11y + 13z = -9 \\ 11) & 19y + 2z = -36 \end{cases}$$

$$\begin{cases} 209y + 247z = -171 \\ 209y + 22z = -396 \end{cases}$$

$$225z = 225$$

$$z = 1.$$

Take equation A substitute value of z

$$11y + 13z = -19.$$

$$11y + 13 = -19$$

$$\frac{11y}{11} = \frac{-22}{11}$$

$$y = -2.$$

Take any equation from equation (i), (ii) and (iii) to obtain value of x.

$$x + 2y + 3z = 2.$$

$$x + -4 + 3 = 2$$

$$x = 3.$$

$\therefore$ The value of x, y, z are
3, -2 and 1.

3. $3x + 2y = 9$ ---- (i)
$x - y = 8$ ---- (ii)

$$\begin{cases} 3x + 2y = 9 & \times 1 \\ x - y = 8 & \times 3 \end{cases}$$

$$\begin{cases} \cancel{3x} + 2y = 9 \\ \cancel{3x} - 3y = 24 \end{cases}$$

$$\frac{5y}{5} = \frac{-15}{5}$$

$$y = -3.$$

Take equation (i) to substitute value of y to obtain value of x

$3x + 2y = 9$
~~$3x - 6y = 9$~~
~~$9x$~~

4.

$$3x - 6 = 9$$

$$\frac{3x}{3} = \frac{15}{3}$$

$$x = 5.$$

$\therefore$ The value of x and y are 5 and -3.

4. $3x - 2y = 8$ — ①

$2x + 5y = -1$ — ⑪

$$\begin{aligned} 2\{\ & 3x - 2y = 8 \\ 3\}\ & 2x + 5y = -1 \end{aligned}$$

$$-\left\{\begin{aligned} 6x - 4y &= 16 \\ 6x + 15y &= -3 \end{aligned}\right.$$

$$\frac{-19y}{-19} = \frac{19}{-19}$$

$$y = -1.$$

Take equat ⑪. substitute value of y

$$2x + 5y = -1.$$
$$2x - 5 = -1.$$
$$2x = -1 + 5$$
$$\frac{2x}{2} = \frac{4}{2}$$
$$\underline{x = 2}$$

$\therefore$ The value of x and y are 2 and -1.

1. $3x - 8y + 10z = 22$ ------ (i)
$x - 3y + 2z = 5$ ------ (ii)
$2x - 9y - 8z = -11$ ------ (iii)

Take equation (i) and (ii)

$$\begin{cases} 3x - 8y + 10z = 22 & 1 \\ x - 3y + 2z = 5 & 3 \end{cases}$$

$$-\begin{cases} 3x - 8y + 10z = 22 \\ 3x - 9y + 6z = 15 \end{cases}$$
$$y + 4z = 7 \text{ ------ } (A)$$

Take equation (ii) and (iii)

$$2\left\{\begin{array}{l} x - 3y + 2z = 5 \\ 2x - 9y - 8z = -11 \end{array}\right.$$
1

$$-\left\{\begin{array}{l} \cancel{2x} - 6y + 4z = 10 \\ \cancel{2x} - 9y - 8z = -11 \end{array}\right.$$

$$3y + 12z = 21 \quad \text{--- (B)}$$

Take equation A and B combine

$$y + 4z = 7$$

$$3y + 12z = 21$$

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NO SOLUTION AND INFINITELY MANY SOLUTION.

Example: Solve the following system of linear equation by Elimination method

(a) $x+y+z=3$

$x-y-5z=1$

$2x+3y+5z=6$

(b) $x+y+z=1$

$2x+y-z=3$

$3x+y-3z=5$

Solution.

(a) $R_2 \rightarrow 5R_1+R_2$

$x+y+z=3$

$6x+4y=16$

$2x+3y+5z=6$

$R_3 \rightarrow -5R_1+R_3$

$x+y+z=3$

$6x+4y=16$

$-3x-2y=-9$

$R_2 \rightarrow +2R_3+R_2$

$x+y+z=3$

$0+0+0=-2$

$-3x - 2y = -2$

Because the second equation $0 = 2$ is false we conclude that this system is in consistent. Because the final system is equivalent to the original system hence the original system is ~~is~~ inconsistent or no solution.

(b) solution.

$R_2 \to -1R_1 + R_2$

To eliminate y in 2^{nd} eqn.

$x + y + z = 1.$

$x \quad - 2z = 2$

$R_3 = -1R_1 + R_3$

$x + y + z = 1$

$x \quad - 2z = 2$

$2x \quad - 4z = 4$

$R_3 = -2R_2 + R_3$

$x + y + z = 1$

$x - 2z = 2$

$0 + 0 = 0$ Indicate many solution hence discard eqn

$x + y + z = 1$

$x - 2z = 2$

Introduce parameter to ~~equation~~ equivalent system (i) and (ii) let $z = s$

from (ii) $x = 2s + 2$

from (i) $x + y + z = 1$

$2s + 2 + y + s = 1$

~~$y = -3s + 1$~~ $y = -3s - 1$

$\therefore (x, y, z) = 2s + 2, -3s + 1, s)$ where s is any real number

MATRICES.

A matrix is a rectangular array of number

$$A = \begin{bmatrix} a_{11} & a_{12} \cdots & a_{1n} \\ a_{21} & a_{22} \cdots & a_{2n} \\ a_{m1} & a_{m2} \cdots & a_{mn} \end{bmatrix}$$

If a matrix A has m rows and n column, then we say that A is of order/size/dimension m by n $(m \times n)$

→ The entry or element in the i^{th} row and j^{th} column is a real or complex number and is denoted by a double sub-script notation a_{ij}. We call a_{ij} the $(i, j)^{th}$ entry.

Using matrices to solve linear system.
In order to solve system of linear equation by elimination have transform the given system of linear equation into Augmented matrix.

Example

$x - y - z = 1$

$2x - 3y + z = 10$

$x + y - 2z = 0$

It's augmented matrix is

$$\left[\begin{array}{ccc|c} +1 & -1 & -1 & 1 \\ 2 & -3 & 1 & 10 \\ 1 & 1 & -2 & 0 \end{array}\right]$$

The basic strategy is to create an equivalent system by using elementary row operation as we did in the previous example.

Elementary row operation on matrices.

1. Interchange two Rows ie $R_i \longleftrightarrow R_j$ to make $a_{11} = 1$
2. Multiply a row by a non-zero constant ie CR_j
3. Add a multiple of one row to another row ie $CR_i + R_j \to R_j$

Defn: Row equivalent matrices.

. Two matrices are called row equivalent if one can be obtained from another by a ~~defin~~ finite sequence of elementary row operations.

Example:

$x - y - z = 1$

$2x - 3y + z = 10$

$x + y - 2z = 0$

solution.

Write into augmented matrix.

$$\left[\begin{array}{ccc|c} +1 & -1 & -1 & 1 \\ 2 & -3 & 1 & 10 \\ 1 & 1 & -2 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -1 & -1 & 1 \\ 0 & -1 & 3 & 8 \\ 0 & 2 & -1 & -1 \end{array}\right] \begin{array}{l} -2R_1 + R_2 \to R_2 \\ -1R_1 + R_3 \to R_3. \end{array}$$

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$$\left[\begin{array}{ccc|c} 1 & -1 & -1 & 1 \\ 0 & +1 & -3 & -8 \\ 0 & 2 & -1 & -1 \end{array}\right] \quad -1R_2 \rightarrow R_2$$

$$\left[\begin{array}{ccc|c} 1 & -1 & -1 & 1 \\ 0 & 1 & -3 & -8 \\ 0 & 0 & 5 & 15 \end{array}\right] \quad -2R_2 + R_3 \rightarrow R_3$$

Hence $5z = 15$

$z = 3$

$y - 3z = -8$

$y = -8 + 3 \times 3$

$y = 1$

$x - y - z = 1$

$x = 1 + 1 + 3 = 5$

$x = 5$

(x, y, z) $(5, 1, 3)$

The matrix above is in row-echelon form

NB: Row echelon form and reduced-row-echelon form.

An $M \times n$ matrix is in row echelon form if it has the following three properties.

1. The leading entry at each non-zero row is 1.
2. The leading entry in ~~many~~ any row is to the right of the leading entry in the row above it.
3. All non-zero are above the rows consisting entirely of zero

NB: A leading entry of a row is the leftmost non-zero entry in a non-zero row

- Non-zero row means a row that contains at least one nonzero entry

A matrix in row-echelon form and each leading 1 is the only nonzero entry in its column then is said to be in reduced row echelon form.

Example: The following matrix are in row echelon form where star can be any value including zero.

$$\begin{bmatrix} 1 & * & * \\ 0 & 1 & * \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & * & * & * \\ 0 & 1 & * & * \\ 0 & 0 & 1 & * \end{bmatrix}$$

$$\begin{bmatrix} 1 & * & * & * \\ 0 & 0 & 1 & * \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The following matrices are in reduced row echelon form.

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & * \\ 0 & 1 & 0 & * \\ 0 & 0 & 1 & * \end{bmatrix}$$

$$\begin{bmatrix} 1 & * & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ & \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -3 & 0 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -7 \\ 0 & 1 & 5 \\ 0 & 0 & 0 \end{bmatrix}$$

The following matrices are not in reduced row echelon form give reason.

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

NB: An Augumented matrix can be transformed into several different row-echelon form matrices by using different sequence of elementary row operation but its reduced row echelon form is unique.

NB: The process of transforming a matrix into row-echelon form is known as Gauss-Elimination.

- The process of transforming a matrix into reduced row echelon form is known as Gauss-Jordan Elimination.

TO DO:

solve the following system of linear equation and state whether is consistent or inconsistent.

(a) $x + 5y + 3z = 7$

$2x + 11y - 4z = 6$

solution

To eliminate x multiply by -2 in equation (i) add to eqn (ii)

$-2R_1 + R_2 \rightarrow R_2$

$$+\begin{cases} -2x - 10y - 6z = -14 \\ 2x + 11y - 4z = 6 \end{cases}$$

$+y - 10z = -8$

$y = 10z - 8$.

substitute the value of y in eqn (i)

$x + 5(10z - 8) + 3z = 7$

$x + 50z - 40 + 3z = 7$

$x + 53z = 47$ ——— (A)

$\therefore$

Also, substitute the value of y in eqn (ii)

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$2x + 11(10z - 8) - 4z = 6$

$2x + 110z - 88 - 4z = 6$

$2x + 106z = 96$ — (B)

Combine eqn (A) and (B) simultaneously.

$x + 53z = 47$

$2x + 106z = 96$

multiply by $-\frac{1}{2}$ in eqn 2. add to eqn ① $-\frac{1}{2}R_2 + R_1 \rightarrow R_1$

$x + 53z = 47$

$-x - 53z = 47$

$0 + 0 = 0$ Indicate many solution hence discard equation

$x + 5y + 3z = 7$

$x + 53z = 47$

Introduce parameter to equivalent system (i) let $z = s$

$x = 47 - 53s.$

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$47 - 53s + 5y + 3s = 7$

$40 - 50s + 5y = 0$

$$\frac{50s - 40}{5} = \frac{5y}{5}$$

$y = 10s - 8$

$(x, y, z) = (47 - 53s, 10s - 8, s).$

Hence the system is consistent.

(b) $x + 2y = -2$

$4x + 3y = 7$

solution.

$$\begin{array}{l} 4 \\ 1 \end{array}\begin{cases} x + 2y = -2 \\ 4x + 3y = 7 \end{cases}$$

$$- \begin{cases} 4x + 8y = -8 \\ 4x + 3y = 7 \end{cases}$$

$$\frac{5y}{5} = \frac{-15}{5}$$

$y = -3$

Take eqn (1) substitute value of y

$x + -6 = -2$

$x = 4$

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The system have one solution hence consistent.

(c) $x + 4y - z = 12$

$3x + 8y - 2z = 4.$

solution

To eliminate value of x multiply $+3$ by to the first equation add to R_2

$$+\begin{cases} -3x + -12y - 3z = -36 \\ 3x + 8y - 2z = -4 \end{cases}$$

$-6y - 5z = -40$

$6y + 5z = 40.$

$$-\begin{cases} 3x + 12y - 3z = 36 \\ 3x + 8y - 2z = 4 \end{cases}$$

$4y - z = 32$ —— (A)

$z = 4y - 32.$

Take equation (1) substitute to the value of z

$x + 4y - (4y - 32) = 12$

$x + 4y - 4y + 3z = 32$

$x = 0$

Take equation (ii) substitute the value of x

$8y - 2z = 4$ —— B

Combine eqn (A) and B

$4y - z = 32$

$8y - 2z = 4$

multiply by $-\frac{1}{2}$ in equation B. add to equation A.

$$+\begin{cases} 4y - z = 32 \\ -4y + z = -2 \end{cases}$$

$0 + 0 = 30$

$0 = 30$

No solution, hence the system is inconsistent.

(d) $x - 5y = 6$

$3x + 2y = 1$

$5x + 2y = 1$

solution

Take eqn (i) and (ii)

$$\begin{cases} x - 5y = 6 & 3 \\ 3x + 2y = 1 & 1 \end{cases}$$

$$-\begin{cases} 3x - 15y = 18 \\ 3x + 2y = 1 \end{cases}$$

$$\frac{-17y}{-17} = \frac{17}{-17}$$

$y = -1$.

Take eqn (i) substitute value of y

$x - (-5) = 6$

$x + 6 = 6$

$x = 0$

~~One solution, hence consistent~~

$5 \neq 6$

$-2 \neq 1$ hence inconsistent

because of no solution.

(e) $x + y = 1$

$2x - y = 5$

$3x + 4y = 2.$

solution.

$-2R_1 + R_2 \rightarrow R_2$

$-3y = -3$

$y = 1.$

$3x + 4 = 2$

$\frac{3x}{3} = \frac{-2}{3}$

$x = -2/3.$

$\frac{2}{3} \neq 1$, There is no solution in the system of linear eqn hence inconsistent

(f) $3x + 2y + z = 2$

$4x + 2y + 2z = 8$

$x - y + z = 4.$

solution

$R_1 - R_2 \rightarrow R_2$

$x + z = 6$ ——— (A)

$2R_3 + R_1 \rightarrow R_3$

$$-\begin{cases} 3x + 2y + z = 2 \\ 2x - 2y + 2z = 8 \end{cases}$$

$$5x + 3z = 10 \quad \text{— (B)}$$

$$3\begin{cases} x + z = 6 \\ 5x + 3z = 10 \end{cases}$$

$$-\begin{cases} 3x + \cancel{3z} = 18 \\ 5x + \cancel{3z} = 10 \end{cases}$$

$$-2x = 8$$

$$x = -4.$$

Take eqn (B) substitute the valu of x

$$5x + 3z = 10$$

$$-20 + 3z = 10$$

$$3z = 30$$

$$z = 10$$

Take eqn (iii) substitute value of x and z

$$-4 - y + 10 = 4$$

$$-y + 6 = 4$$

$y = 2$.

$(x, y, z) = (-4, 2, 10)$, One solution hence the system is consistent

(9) $2x + y - 2z = -5$

$3y + z = 7$.

$z = 4$.

Solution:

$3y + 4 = 7$

$3y = 3$

$y = 1$.

$2x + 1 - 8 = -5$

$2x = 2$

$x = 1$.

$(x, y, z) = (1, 1, 4)$

One solution. hence the system is consistent.

(h) $2x - y = 5$

$4x - 2y = t$

(a) Determine t so that the system is consistent

(b) Determine t so that the system is inconsistent

(c) How many different value of t can be selected in b above.

GAUSS ELIMINATION METHOD:

Gauss Elimination method is the method of solving a system of linear equation by transforming the augmented matrix into row-echelon form and then using back-substitution to find the solution set

Example:

$2x+y+z=6$

$-3x-4y+2z=4$

$x+y-z=-2$

its Augmented matrix

$$\left[\begin{array}{ccc|c} 2 & 1 & 1 & 6 \\ -3 & -4 & 2 & 4 \\ 1 & 1 & -1 & -2 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & -2 \\ -3 & -4 & 2 & 4 \\ 2 & 1 & 1 & 6 \end{array}\right] \quad R_1 \leftrightarrow R_3$$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & -2 \\ 0 & -1 & -1 & -2 \\ 0 & -1 & 3 & 10 \end{array}\right] \quad \begin{array}{l} 3R_1+R_2 \rightarrow R_2 \\ -2R_1+R_3 \rightarrow R_3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & -2 \\ 0 & 1 & 1 & 2 \\ 0 & -1 & 3 & 10 \end{array}\right] \quad -1R_2 \rightarrow R_2$$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & -2 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 4 & 12 \end{array}\right] \quad R_2 + R_3 \rightarrow R_3$$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & -2 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right] \quad \tfrac{1}{4}R_3 \rightarrow R_3.$$

$$x + y - z = 2$$
$$y + z = 2$$
$$z = 3$$

by back- substitution

$$y + z = 2$$
$$y = 2 - 3$$
$$= -1.$$

$$x + y \cancel{+y} - z = 2$$
$$x = 2 + y$$
$$3 - 1 = 2$$

$(x, y, z) = (2, -1, 3)$.

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Solve by Gauss elimination method

(a) $x + 2y = -2$
$4x + 3y = 7$

(b) $2x + y - z = 7$
$x - 3y - 3z = 4$
$4x + y + z = 3$

(c) $y + 5z = -4$
$x + 4y + 3z = -2$
$2x + 7y + z = 8$

GAUSS - JORDAN ELIMINATION METHOD

If we continue the Gauss Elimination procedure until we have a reduced row echelon form the procedure is called Gauss - Jordan Elimination

Example:

$x - y - z = 1$
$2x - 3y + z = 10$
$x + y + 2z = 20$

Soln.

from, previous example of matrix method

$$\left[\begin{array}{ccc|c} 1 & -1 & -1 & 1 \\ 0 & 1 & -3 & -8 \\ 0 & 0 & 5 & 15 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -1 & -1 & 1 \\ 0 & 1 & -3 & -8 \\ 0 & 0 & 1 & 3 \end{array}\right] \frac{1}{5}R_3 \to R_3.$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -4 & -7 \\ 0 & 1 & -3 & -8 \\ 0 & 0 & 1 & 3 \end{array}\right] R_2 + R_1 \to R_1$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 3 \end{array}\right] \begin{array}{l} 4R_3 + R_1 \to R_1 \\ 3R_3 + R_2 \to R_2. \end{array}$$

We now have an equivalent matrix in reduced row echelon form

Then $x = 5$
$y = 1$
$z = 3$

The solution set is $(x, y, z) = (5, 1, 3)$

Try:

Solve the Gauss-Jordan Elimination

(a) $2x - 3y - 2z = 0$

$x + y - 2z = 7$

$3x - 5y - 5z = 3$

(b) $x + 2y + 5z = 4$

$y + 4z = 4$

$2x + 4y + 10z = 8$

(c) $x + z = -1$

$3y + 2z = 5$

$3x + z - 3y = -8$

MATRIX OPERATION

Addition of matrix

If $A = [a_{ij}]$ and $B = [b_{ij}]$ are matrix of the same size/dimension then their sum $A+B$ is the matrix obtained by adding corresponding elements of the matrices A and B

i.e. $A+B = [a_{ij} + b_{ij}]$

Example:

$$A = \begin{bmatrix} 3 & 0 & -1 \\ 2 & -7 & 5 \end{bmatrix} \quad B = \begin{bmatrix} 4 & -3 & 6 \\ 9 & 0 & -2 \end{bmatrix}$$

$$C = \begin{bmatrix} 3 & -2 \\ -1 & 6 \end{bmatrix}$$

Then

$$A+B = \begin{bmatrix} 3 & 0 & -1 \\ 2 & -7 & 5 \end{bmatrix} + \begin{bmatrix} 4 & -3 & 6 \\ 9 & 0 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & -3 & 5 \\ 11 & -7 & 3 \end{bmatrix}$$

but $A+C$ is not possible b'se are not of the same size multiplication.

- multiplication of a matrix by a number

If $A = [a_{ij}]$ is a matrix and c is a number then cA is the matrix obtained by multiply each element of A by c

$$cA = [c a_{ij}]$$

Eg: Given that $A = \begin{bmatrix} 3 & 0 & -1 \\ 2 & -1 & 5 \end{bmatrix}$

find 3A.

Soln

$$\begin{bmatrix} 3\times 3 & 3\times 0 & 3\times -1 \\ 3\times 2 & 3\times -1 & 3\times 5 \end{bmatrix}$$

$$\begin{bmatrix} 9 & 0 & -3 \\ 6 & -3 & 15 \end{bmatrix}$$

matrix multiplication.

Suppose that A is an $m \times p$ matrix and B is a $p \times n$ matrix when the product AB is the ~~matrix~~

$m \times n$ defined as the element of AB in its i^{th} row and j^{th} column is the sum of product of corresponding elements in the i^{th} row of A and the j^{th} column of B.

That is if the i^{th} row of A is $[a_{21} \ a_{22} \ a_{23} \ \ldots \ a_{2p}]$ and the j^{th} column of B is

$$\begin{bmatrix} b_{ij} \\ b_{2j} \\ b_{3j} \\ b_{4j} \\ \vdots \\ b_{pj} \end{bmatrix}$$

Then the element in the i^{th} row and j^{th} column at the product of AB is $a_{i1} b_{ij} + a_{i2} b_{2j} + \ldots a_{ip} b_{pj}$

Example:

$A = \begin{bmatrix} 2 & -1 \\ -4 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 5 \\ 3 & 7 \end{bmatrix}$

$$AB = \begin{bmatrix} 2\times 1 + -1\times 3 & 2\times 5 + -1\times 7 \\ -4\times 1 + 3\times 3 & -4\times 5 + 3\times 7 \end{bmatrix}$$

$$AB = \begin{bmatrix} -1 & 3 \\ 5 & 1 \end{bmatrix}$$

Properties of matrix operation.

(i) matrix addition.

(a) $A+B = B+A$ - Commutative

(b) $A+(B+C) = (A+B)+C$ - Associative

(c) $A+O = O+A = A$ - Additive identity.

(d) $A+(-A) = O$ The matrix $-A$ is called Additive inverse or negative of A.

(ii) properties of matrix multiplication.

(a) If A, B and C are of the appropriate size then $A(BC) = (AB)C$

$A\cdot(B+C) = AB + AC$

$(A+B)C = AC + BC$

TRY:

(a) Let $A = \begin{bmatrix} 5 & 2 & 3 \\ 2 & -3 & 4 \end{bmatrix}$

$B = \begin{bmatrix} 2 & -1 & 1 & 0 \\ 0 & 2 & 2 & 2 \\ 3 & 0 & -1 & 3 \end{bmatrix}$

$C = \begin{bmatrix} 1 & 0 & 2 \\ 2 & -3 & 0 \\ 0 & 0 & 3 \\ 2 & 1 & 0 \end{bmatrix}$

Show that $A(BC) = (AB)C$.

(b) Given $A = \begin{bmatrix} 2 & 2 & 3 \\ 3 & -1 & 2 \end{bmatrix}$

$B = \begin{bmatrix} 1 & 0 \\ 2 & 2 \\ 3 & -1 \end{bmatrix}$

$C = \begin{bmatrix} -1 & 2 \\ 1 & 0 \\ 2 & -2 \end{bmatrix}$

Show that $A(B+C) = AB + AC$

(iii) properties of scalar multiplication
if r and s are real numbers and A and B are matrix then

(a) $r(sA) = (rs)A$

(b) $(r+s)A = rA + sA$

(c) $r(A+B) = rA + rB$

(d) $A(rB) = r(AB)$

TRY: Let $r = -2$ $A = \begin{bmatrix} 1 & 2 & 3 \\ -2 & 0 & 1 \end{bmatrix}$

$B = \begin{bmatrix} 2 & -1 \\ 1 & 4 \\ 0 & -2 \end{bmatrix}$

show that $A(rB) = r(AB)$

(iv) properties of Transpose:
if r is a scalar and A and B are matrices then:

(a) $(A^T)^T = A$

(b) $(A+B)^T = A^T + B^T$

(c) $(AB)^T = B^T \cdot A^T$

(d) $(rA)^T = rA^T$

TRY:

Let $A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & -1 & 3 \end{bmatrix}$ and

$B = \begin{bmatrix} 0 & 1 \\ 2 & 2 \\ 3 & -1 \end{bmatrix}$

Show that $(AB)^T = B^T \cdot A^T$

Dfn: A matrix $A = [a_{ij}]$ with real entries is called symmetric if

$$A^T = A$$

Example: The matrices

$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$ $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

are symmetric

TO DO: 31/03/2020

① Solve for x if $3A + 2X = 4B$

Given that $A = \begin{bmatrix} 2 & 0 \\ 4 & 6 \end{bmatrix}$ and

$B = \begin{bmatrix} 1 & 3 \\ 5 & 2 \end{bmatrix}$

TRY:

Let $A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & -1 & 3 \end{bmatrix}$ and

$B = \begin{bmatrix} 0 & 1 \\ 2 & 2 \\ 3 & -1 \end{bmatrix}$

Show that $(AB)^T = B^T \cdot A^T$

Dfn: A matrix $A = [a_{ij}]$ with real entries is called symmetric if

$$A^T = A$$

Example: The matrices

$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$ $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

are symmetric

TO DO: 31/03/2020.

① Solve for x if $3A + 2x = 4B$

Given that $A = \begin{bmatrix} 2 & 0 \\ 4 & 6 \end{bmatrix}$ are

$B = \begin{bmatrix} 1 & 3 \\ 5 & 2 \end{bmatrix}$.

(ii) Let $A = \begin{bmatrix} 1 & -2 & 3 \\ 2 & -4 & 1 \\ 3 & -5 & 2 \end{bmatrix}$

~~(iii)~~ Show that $3A^2 - 2A + I = \begin{bmatrix} 17 & -28 & 15 \\ -13 & 30 & 10 \\ -9 & 22 & 21 \end{bmatrix}$

(iii) Let $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$

(a) Show that $A^2 = 4A - 7I$

(b) Use part (a) to show that

$A^4 = 8A - 63I$

(c) Use part (b) to show that

$A^4 = \begin{bmatrix} -47 & 24 \\ -8 & -47 \end{bmatrix}.$

04/04/2022

THE INVERSE OF A MATRIX.

Dfn:

An ~~and~~ $n \times n$ matrix A is called non-singular (or invertible) if there exists an $n \times n$ matrix B such that

$$AB = BA = I_n$$

The matrix B is called an inverse of A or matrix A is the inverse of B

Example:

$$A = \begin{bmatrix} 2 & 3 \\ 2 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} -1 & 3/2 \\ 1 & -1 \end{bmatrix}$$

$AB = BA = I_2$ then B is the inverse of A.

Example. Find the inverse of

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

Soln. from $AB = I_2 = A \cdot A^{-1}$.

let $A^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$AA^{-1} = I_2$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} a+2c & b+2d \\ 3a+4c & 3b+4d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

If the two matrices are equal then the corresponding entries are equal

$a+2c = 1$ $\qquad b+2d=0$

$3a+4c = 0$ $\qquad 3b+4d=1$

Solving for a, b, c and d above. The solution are

$a = -2$, $c = 3/2$, $b = 1$, $d = -1/2$.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 3/2 & -1/2 \end{bmatrix}$$

$\therefore$

$$A^{-1} = \begin{bmatrix} -2 & 1 \\ 3/2 & -1/2 \end{bmatrix}$$

NB: Not every matrix has an inverse

properties of inverse.

(a) If A is a non-singular matrix then A^{-1} is a non-singular matrix and $(A^{-1})^{-1} = A$.

(b) If A and B are non-singular matrices then AB is non-singular and $(AB)^{-1} = B^{-1}A^{-1}$

(c) If A is a non-singular matrix then $(A^T)^{-1} = (A^{-1})^T$

Finding inverse by Gauss-Jordan reduction method.

step 1: Form the $2 \times 2n$ matrix $[A : I_n]$ obtained by adjoining the identity matrix I_n to the given matrix

step 2: Compute the reduced row echelon form of matrix obtained in step 1 by using elementary row operations

$A_{23} = (-1)^{2+3} (M_{23})$

$(-1)\, 10 \quad = -10$

If we think of the sign $(-1)^{i+j}$ as being located in position (i, j) of an $n \times n$ matrix then we get checkboard as

$$\begin{bmatrix} + & - \\ - & + \end{bmatrix} \quad \begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix} \quad \begin{bmatrix} + & - & + & - \\ - & + & - & + \\ + & - & + & - \\ - & + & - & + \end{bmatrix}$$

$n = 2 \qquad n = 3 \qquad n = 4.$

Try: Find the determinant of.

$$\begin{bmatrix} 1 & 2 & -3 & 4 \\ -4 & 2 & 1 & 3 \\ 3 & 0 & 0 & -3 \\ 2 & 0 & -2 & 3 \end{bmatrix}$$

Application of determinant

i) In solving system of linear equation

ii) In finding the inverse of a square matrix

(iii) Finding the area of triangles

~~Finding~~ solving system of linear equation by determinant method known cramar's rule.

Consider the system of two equation.

$$a_1x + b_1y = c_1$$

$$a_2x + b_2y = c_2.$$

by elimination method.

$$a_1b_2x + b_1b_2y = c_1b_2$$

$$-a_2b_1x + -b_1b_2y = -c_2b_1$$

$$(a_1b_2 - a_2b_1)x = c_1b_2 - c_2b_1$$

$$x = \frac{c_1b_2 - c_2b_1}{a_1b_2 - a_2b_1} = \frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

From $a_1x + b_1y = c_1$

$$a_2x + b_2y = c_2$$

multiply by $-a_2$ in eqn ① and a_1 in eqn 2

$$-a_1a_2x - a_2b_1y = -a_2c_1$$

$$a_1a_2x + a_1b_2y = a_1c_2$$

$$(a_1b_2 - a_2b_1)y = a_1c_2 - a_2c_1$$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & -3 & 1 & 0 & 0 \\ 0 & -4 & 4 & -1 & 1 & 0 \\ 0 & 0 & 0 & -2 & -3 & 1 \end{array}\right]$$

Since C consist of entirely zero the the inverse does not exist.

TO DO: 04/04/2022:

1. Given $x + 2y = 10$

$3x + (6+t)y = 30$

(a) Determine a particular value of t so that the system has infinitely many solution.

(b) Determine a particular value of t so that the system has unique solution

2. Find the matrix A and B such that $A+B = \begin{bmatrix} 3 & 0 & 3 \\ -2 & 1 & -4 \end{bmatrix}$ and

$$2A - B = \begin{bmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{bmatrix}$$

3. Solve for x and y

(a) $\frac{3}{x} - \frac{6}{y} = 2$

$\frac{4}{x} + \frac{7}{y} = -3$

(b) $\frac{1}{x-2} + \frac{3}{y+1} = 13$

$\frac{4}{x-2} - \frac{5}{y+1} = 1$

06/04/2022

DETERMINANT:

Each n-square matrix $A = [a_{ij}]$ is assigned a special scalar called the determinant of A denoted by det A or $|A|$ or
$$\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}$$

Determinant of order 1 and 2

$$|a_{11}| = a_{11} \text{ and } \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

$$a_{11} \cdot a_{22} - a_{21} \cdot a_{12}$$

Example:

$$\begin{vmatrix} 5 & 3 \\ 4 & 6 \end{vmatrix} = 5(6) - 4(3) = 18$$

Determinant of order 3

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$$

$$- a_{12} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$= a_{11}(a_{22}a_{33} - a_{32}a_{23}) - a_{12}(a_{21}a_{33} - a_{31}a_{23}) +$
$a_{13}(a_{21}a_{32} - a_{31}a_{22})$

Example: let $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 5 & -2 \\ 1 & -3 & 4 \end{bmatrix}$ and

$B = \begin{bmatrix} 3 & 2 & 1 \\ -4 & 5 & -1 \\ 2 & -3 & 4 \end{bmatrix}$ find the det(A) and det(B).

Solution

Given

$$B = \begin{vmatrix} 3 & 2 & 1 \\ -4 & 5 & -1 \\ 2 & -3 & 4 \end{vmatrix} = 3\begin{vmatrix} 5 & -1 \\ -3 & 4 \end{vmatrix} - 2\begin{vmatrix} -4 & -1 \\ 2 & 4 \end{vmatrix} + 1\begin{vmatrix} -4 & 5 \\ 2 & -3 \end{vmatrix}$$

$3(20-3) - 2(-16+2) + 1(12-10)$

$3 \times 17 - 2 \times -14 + 2$

$$A = \begin{vmatrix} 2 & 1 & 1 \\ 0 & 5 & -2 \\ 1 & -3 & 4 \end{vmatrix} = 2\begin{vmatrix} 5 & -2 \\ -3 & 4 \end{vmatrix} - 1\begin{vmatrix} 0 & -2 \\ 1 & 4 \end{vmatrix} + 1\begin{vmatrix} 0 & 5 \\ 1 & -3 \end{vmatrix}$$

$$2(20-6) - (0+2) + (0-5)$$

$$2 \times 14 - 2 - 5 = 21.$$

properties of determinant.

i) The determinant of a matrix and its transpose are equal that is $\det(A^T) = \det(A)$

(ii) Let A be a square matrix.

(a) If A has a row ~~or~~ column of zero then $|A| = 0$

(b) If A has two identical row and column then $|A| = 0$

(c) If A is a triangular matrix then $|A| =$ product of diagonal element

(iii) Suppose B is obtained from A by elementary row/column operation

(a) If two rows/columns of A were interchanged then $|B| = -|A|$

(b) If a row/column of A were multiplied by a scalar k then $|B| = k|A|$.

(c) If a multiple of a row/column of A were added to another row/column of A then $|B| = |A|$

(ii) The determinant of a product of the matrices A and B is the product of their determinant

ie $\det(AB) = \det(A) \cdot \det(B)$.

Minor, Cofactor and its application.

Let $A = [a_{ij}]$ be an $n \times n$ matrix

Let M_{ij} be the $(n-1) \times (n-1)$ submatrix of A obtained by deleting the i^{th} row and j^{th} column of A.

The determinant $\det(M_{ij})$ is called the minor of a_{ij}

The cofactor A_{ij} of a_{ij} is defined as $A_{ij} = (-1)^{i+j} \det(M_{ij})$

Thus cofactor is used to find the determinant of any square matrix.

$$\begin{vmatrix} a_{11} & a_{21} & a_{31} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}(\text{cofactor of } a_{11}) +$$

$a_{21}(\text{cofactor of } a_{21}) + a_{31}(\text{cofactor of } a_{31}) +$

$a_{21}(\text{cofactor of } a_{21}) + a_{22}(\text{cofactor of } a_{22}) + a_{23}(\text{cofactor } a_{23})$

Example: Let $A = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 5 & 6 \\ 7 & 1 & 2 \end{bmatrix}$ then

$$M_{12} = \begin{vmatrix} 4 & 6 \\ 7 & 2 \end{vmatrix} = 8 - 42 = -34$$

$$M_{23} = \begin{vmatrix} 3 & -1 \\ 7 & 1 \end{vmatrix} = 3 + 7 = 10$$

Also, $A_{12} = (-1)^{1+2}(M_{12})$

$= (-1)(-34) = 34$

$A_{23} = (-1)^{2+3} (M_{23})$

$(-1)\,10 = -10$

If we think of the sign $(-1)^{i+j}$ as be located in position (i, j) of an $n \times n$ matrix then we get checkboard as

$$\begin{bmatrix} + & - \\ - & + \end{bmatrix} \quad \begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix} \quad \begin{bmatrix} + & - & + & - \\ - & + & - & + \\ + & - & + & - \\ - & + & - & + \end{bmatrix}$$

$n=2$ $\quad$ $n=3$ $\quad$ $n=4$

Try: Find the determinant of.

$$\begin{bmatrix} 1 & 2 & -3 & 4 \\ -4 & 2 & 1 & 3 \\ 3 & 0 & 0 & -3 \\ 2 & 0 & -2 & 3 \end{bmatrix}$$

Application of determinant

i) In solving system of linear equation

ii) In finding the inverse of a square matrix

(iii) Finding the area of triangle

~~finding~~ solving system of linear equation by determinant method known cramer's rule.

Consider the system of two equation

$$a_1x + b_1y = c_1$$

$$a_2x + b_2y = c_2.$$

by elimination method.

$$a_1b_2x + b_1b_2y = c_1b_2$$

$$-a_2b_1x + -b_1b_2y = -c_2b_1$$

$$(a_1b_2 - a_2b_1)x = c_1b_2 - c_2b_1$$

$$x = \frac{c_1b_2 - c_2b_1}{a_1b_2 - a_2b_1} = \frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

From $a_1x + b_1y = c_1$

$$a_2x + b_2y = c_2$$

multiply by $-a_2$ in eqn (1) and a_1 in eqn 2

$$-a_1a_2x - a_2b_1y = -a_2c_1$$

$$a_1a_2x + a_1b_2y = a_1c_2$$

$$(a_1b_2 - a_2b_1)y = a_1c_2 - a_2c_1$$

$$y = \frac{a_1c_2 - a_2c_1}{a_1b_2 - a_2b_1} = \frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}},$$

$$y = \frac{\Delta y}{\Delta}$$

~~From~~ For a system of 3 unknown eqn

$$a_1x + b_1y + c_1z = k_1$$
$$a_2x + b_2y + c_2z = k_2$$
$$a_3x + b_3y + c_3z = k_3.$$

$$x = \frac{\begin{vmatrix} k_1 & b_1 & c_1 \\ k_2 & b_2 & c_2 \\ k_3 & b_3 & c_3 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}} = \frac{\Delta x}{D}$$

$$y = \frac{\begin{vmatrix} a_1 & k_1 & c_1 \\ a_2 & k_2 & c_2 \\ a_3 & k_3 & c_3 \end{vmatrix}}{D}$$

$$z = \frac{\Delta z}{\Delta} = \frac{\begin{vmatrix} a_1 & b_1 & k_1 \\ a_2 & b_2 & k_2 \\ a_3 & b_3 & k_3 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}}$$

NB: $D \neq 0$

Example: solve for x, y, z by Cramer's rule, determinant method.

$$x + y + z = 5$$
$$x - 2y - 3z = -1$$
$$2x + y - z = 3$$

Soln: First compute the determinant D of the coefficient

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & -2 & -3 \\ 2 & 1 & -1 \end{vmatrix} = \begin{vmatrix} -2 & -3 \\ 1 & -1 \end{vmatrix} - \begin{vmatrix} 1 & -3 \\ 2 & -1 \end{vmatrix} + \begin{vmatrix} 1 & -2 \\ 2 & 1 \end{vmatrix}$$

$$= 5$$

Then. $x = \frac{\Delta x}{D} = \frac{\begin{vmatrix} 5 & 1 & 1 \\ -1 & -2 & 3 \\ 3 & 1 & -1 \end{vmatrix}}{5} = \frac{20}{5}$

$x = 4.$

$y = \frac{\Delta y}{D} = \frac{\begin{vmatrix} 1 & 5 & 1 \\ 1 & -1 & -3 \\ 2 & 3 & -1 \end{vmatrix}}{5} = \frac{-10}{5} = -2$

$z = \frac{\Delta z}{D} = \begin{vmatrix} 1 & 1 & 5 \\ 1 & -2 & -1 \\ 2 & 1 & 3 \end{vmatrix} = \frac{15}{5} = 3.$

$(x, y, z) = (4, -2, 3).$

Find the inverse by determinant

step 1: Find the determinant of matrix

step 2: Find the cofactor of each - then element in the given matrix

step 3: Find the Adjoint which is the transposed cofactor step 4: $A^{-1} = \frac{1}{\det|A|} \cdot \text{Adjoint } A$.

The area of triangle which coordinate (x_1, y_1), (x_2, y_2), (x_3, y_3) is given by

$$\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

matrix inverse in solving system of linear equation. given

$$Ax = B$$
$$A^{-1}(Ax) = A^{-1}B$$
$$(A^{-1}A)x = A^{-1}B$$
$$Ix = A^{-1}B$$
$$x = A^{-1}B$$

Try: solve by matrix inverse

$$x + y = 4$$
$$3y + z = 7$$
$$2x + 3y + 3z = 21$$