THE DERIVATIVES

Defn:

If the fxn f is continous at x_1, then the tangent line to the graph of f at the point $P(x_1, f(x_1))$ is

i) Is the line through P having slope $m(x_1)$ given by

$$m(x_1) = \lim_{\Delta x \to 0} \frac{f(x_1 + \Delta x) - f(x_1)}{\Delta x}$$

If this limit exists

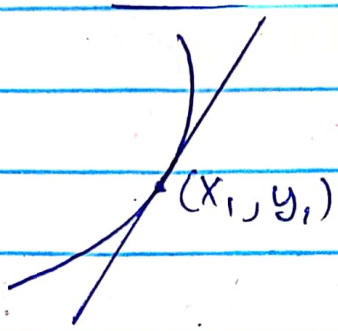
ii) Is the line $x = x_1$ if

$$\lim_{\Delta x \to 0} \frac{f(x_1 + \Delta x) - f(x_1)}{\Delta x} = \infty$$

Example

1 Find the slope of the tangent line to the curve $y = x^2 - 4x + 3$ at the point (x_1, y_1)

Soln



$$f(x_1) = x_1^2 - 4x_1 + 3$$

$$f(x_1 + \Delta x) = (x_1 + \Delta x)^2 - 4(x_1 + \Delta x) + 3$$

$$m(x_1) = \lim_{\Delta x \to 0} \frac{(x_1 + \Delta x)^2 - 4x_1 + 4\Delta x + 3 - x^2}{\Delta x}$$

$$m(x_1) = \lim_{\Delta x \to 0} \frac{x_1^2 + 2x_1\Delta x + \Delta x^2 - 4\Delta x - x^2}{\Delta x}$$

$$m(x_1) = \lim_{\Delta x \to 0} \frac{2x_1\Delta x + \Delta x^2 - 4\Delta x}{\Delta x}$$

$$m(x_1) = \lim_{\Delta x \to 0} 2x_1 + \Delta x - 4$$

m(x₁) Applying $\lim_{\Delta x \to 0}$ $\Delta x = 0$

$$m(x_1) = \lim_{\Delta x \to 0} 2x_1 + \lim_{\Delta x \to 0} \Delta x - \lim_{\Delta x \to 0} 4$$

$$m(x_1) = 2x_1 + 0 - 4$$

$$m(x_1) = 2x_1 - 4$$

∴ The slope of a tangent $= 2x_1 - 4$

2 Find an eqn of the tangent line to the curve of example (1) above at the point (4, 3)

Soln

$$m(x_1) = 2x_1 - 4$$

$$x_1 = 4$$

$$m(4) = 2(4) - 4$$

$$m(4) = 4$$

eqn; point - slope form

$$m(x_1) = \frac{y - y_1}{x - x_1}$$

$$4 = \frac{y - 3}{x - 4}$$

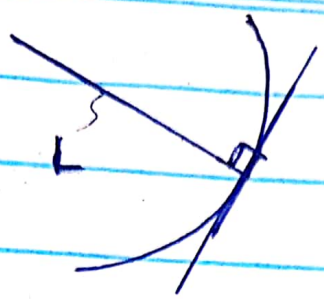
$$y - 3 = 4x - 16$$

$$y = 4x - 13$$

∴ Equation is $y = 4x - 13$

Defn;

The normal line to the curve at the given point is the line perpendicular to the tangent line at that point



Let L be a given line. To find the slope of L, we write its eqn in the slope intercept form which is $y = -2x + 4/3$ (slope-intercept form) then -2 will be our slope.

EXERCISE

1 Find a slope of the tangent to the graph at the point $P(x_1, y_1)$

a) $y = 9 - x^2$

b) $y = x^2 - 6x + 9$

c) $y = x^2/4$

d) $y = 7 - 6x - x^2$

e) $y = x^3 - 3x$

2 Find an eqn of a tangent line and an eqn of the normal ine to the given curve at the indicated point

a) $y = x^2 - 4x - 5$; $(-2, 7)$

b) $y = x^2 + 2x + 1$; $(1, 4)$

c) $y = x^3/8$; $(4, 8)$

d) $y = 2x - x^3$; $(-2, 4)$

e) $y = 6/x$; $(3, 2)$

3 Find an eqn of the tangent line to the curve $y = 2x^2 + 3$ that is parallel to the line $8x - y + 3 = 0$

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4 Find an eqn of the normal line to the curve $y = x^3 - 3x$ that is parallel to the line $2x + 18y - 9 = 0$

5 Find an eqn of the tangent line to the curve $y = \sqrt{4x - 3}$ that is perpendicular to the line $x + 2y - 11 = 0$

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