@ 10-1 on 69

@ PRODUCTION SCHEDULING. A company has plants located in Michigan, New York and Ohio where it manufactures laptop computers, desktop computers and servers. The number of units of each product that can be produced per day at each plant are given in the table below. The company has orders for 2,150 laptop computers, 2300 desktop computers and 2500 servers. How many days should be the company operate each plant in order to exactly fill these orders?

plant	Michigan	New York	Ohio
Laptop	10	70	60
Desktop	20	50	80
server	40	30	90

Solution:

let
x be michigan
y be New York ??
z be ohio

$$10x + 70y + 60z = 2150$$
$$20x + 50y + 80z = 2300$$
$$40x + 30y + 90z = 2500$$

It augmented matrix.

$$\left[\begin{array}{ccc|c} 1 & 7 & 6 & 215 \\ 2 & 5 & 8 & 230 \\ 4 & 3 & 9 & 250 \end{array}\right]$$

(a)
$$\left[\begin{array}{ccc|c} 1 & 7 & 6 & 215 \\ 2 & 5 & 8 & 230 \\ 4 & 3 & 9 & 250 \end{array}\right] \begin{array}{l} -2R_1 + R_2 \to R_2 \\ -4R_1 + R_3 \to R_3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 7 & 6 & 215 \\ 0 & -9 & -4 & -200 \\ 0 & -25 & -15 & -610 \end{array}\right] -\tfrac{1}{9}R_2 \to R_2$$

$$\left[\begin{array}{ccc|c} 1 & 7 & 6 & 215 \\ 0 & 1 & \frac{4}{9} & \frac{200}{9} \\ 0 & -25 & -15 & -610 \end{array}\right] 25R_2 + R_3 \to R_3$$

$$\left[\begin{array}{ccc|c} 1 & 7 & 6 & 215 \\ 0 & 1 & \frac{4}{9} & \frac{200}{9} \\ 0 & 0 & \frac{-35}{9} & -\frac{490}{9} \end{array}\right]$$

By back substitution.

$$x + 7y + 6z = 215$$
$$y + \tfrac{4}{9}z = \frac{200}{9}$$
$$-\frac{35}{9}z = -\frac{490}{9}$$

$$z = 14.$$

$$y + \tfrac{4}{9}(14) = \frac{200}{9}$$

$$y = 16$$

$$x + 7(16) + 6(14) = 215$$

$$x = 19$$

∴ The number of days for the plants to op at michigan, nawyne and ohio are 19, 16 and 14 respectively.

(1) 2

$$x_1 - 2x_2 + x_3 + x_4 + 2x_5 = 2$$
$$-2x_1 + 4x_2 + 2x_4 + 2x_3 - 2x_5 = 0$$
$$3x_1 - 6x_2 + x_3 + x_4 + 5x_5 = 4$$
$$-x_1 + 2x_2 + 3x_3 + x_4 + x_5 = 3$$

Solution.

Its Augmented matrix

$$\left[\begin{array}{ccccc|c} 1 & -2 & 1 & 1 & 2 & 2 \\ -2 & 4 & 2 & 2 & -2 & 0 \\ 3 & -6 & 1 & 1 & 5 & 4 \\ -1 & 2 & 3 & 1 & 1 & 3 \end{array}\right] \begin{array}{l} 2R_1 + R_2 \to R_2 \\ -3R_1 + R_3 \to R_3 \\ R_1 + R_4 \to R_4 \end{array}$$

$$\left[\begin{array}{ccccc|c} 1 & -2 & 1 & 1 & 2 & 2 \\ 0 & 0 & 4 & 4 & 2 & 4 \\ 0 & 0 & -2 & -2 & -1 & -2 \\ 0 & 0 & 4 & 2 & 3 & 5 \end{array}\right] \begin{array}{l} -1R_2 + R_4 \to R_4 \\ -\frac{1}{2}R_3 \to R_3 \end{array}$$

$$\left[\begin{array}{ccccc|c} 1 & -2 & 1 & 1 & 2 & 2 \\ 0 & 0 & 4 & 4 & 2 & 4 \\ 0 & 0 & 1 & 1 & \frac{1}{2} & 1 \\ 0 & 0 & 0 & -2 & 1 & 1 \end{array}\right] \begin{array}{l} -\frac{1}{2}R_4 \to R_4 \\ -4R_3 + R_2 \to R_2 \\ -R_3 + R_1 \to R_1 \end{array}$$

$$\left[\begin{array}{ccccc|c} 1 & -2 & 0 & 0 & \frac{3}{2} & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & \frac{1}{2} & 1 \\ 0 & 0 & 0 & 1 & -\frac{1}{2} & -\frac{1}{2} \end{array}\right] -1R_4 + R_3 \to R_3$$

$$\left[\begin{array}{cccccc} 1 & -2 & 0 & 0 & \frac{3}{2} & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 & 1 & -\frac{1}{2} & -\frac{1}{2} \end{array}\right]$$

(b) $\left[\begin{array}{ccccc|c} 1 & -2 & 0 & 0 & 3/2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 3/2 \\ 0 & 0 & 0 & 1 & -1/2 & -1/2 \end{array}\right]$

$x_1 - 2x_2 + 3/2\, x_5 = 1.$

$x_3 + x_5 = 3/2$

$x_4 - 1/2\, x_5 = -1/2.$

Let

x_2 be n

x_5 be m.

$x_1 = 1 + 2x_2 - 3/2\, x_5$

$x_1 = 1 + 2n - 3/2\, m$

$x_3 = 3/2 - x_5$

$x_3 = 3/2 - m$

$x_4 = 1/2\, x_5 - 1/2$

$x_4 = 1/2\, m - \frac{1}{2}$

$x_4 = \frac{m-1}{2}.$

$\therefore x_1 = 2n - 3/2\, m + 1$

$x_2 = n$

$x_3 = 3/2 - m$

$x_4 = \frac{m-1}{2}$

$x_5 = m.$

1. (C) 10 - Qn 62.

Write each system in problems 61-68 as a matrix equation and solve using inverses. [Note the inverse of each coefficient matrix was found earlier in this exercise set in the indicated problem].

$$3x_1 - 4x_2 = k_1$$
$$-2x_1 + 3x_2 = k_2.$$

(A) $k_1 = 3,\ k_2 = -1.$

(B) $k_1 = 6,\ k_2 = 5$

(C) $k_1 = 0, k_2 = -4.$

Solution.

It's Augmented matrix

$$\left[\begin{array}{cc|c} 3 & -4 & k_1 \\ -2 & 3 & k_2 \end{array}\right]$$

(A) $k_1 = 3,\ k_2 = -1.$

$$\left[\begin{array}{cc|c} 3 & -4 & 3 \\ -2 & 3 & -1 \end{array}\right]$$

Let $A = \begin{bmatrix} 3 & -4 \\ -2 & 3 \end{bmatrix}$

$$\left[\begin{array}{cc:cc} 3 & -4 & 1 & 0 \\ -2 & 3 & 0 & 1 \end{array}\right] \tfrac{1}{3}R_1 \to R_1$$

$$\left[\begin{array}{cc:cc} 1 & -\tfrac{4}{3} & \tfrac{1}{3} & 0 \\ -2 & 3 & 0 & 1 \end{array}\right] 2R_1 + R_2$$

$$\left[\begin{array}{cc:cc} 1 & -\tfrac{4}{3} & \tfrac{1}{3} & 0 \\ 0 & \tfrac{1}{3} & \tfrac{2}{3} & 1 \end{array}\right] 3R_2 \to R_2$$

$$\left[\begin{array}{cc:cc} 1 & -\tfrac{4}{3} & \tfrac{1}{3} & 0 \\ 0 & 1 & 2 & 3 \end{array}\right] \tfrac{4}{3}R_2 + R_1 \to R_1$$

(i)

$$\left[\begin{array}{cc:cc} 1 & 0 & 3 & 4 \\ 0 & 1 & 2 & 3 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix}.$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

8

$$x_1 = 3\times3 + 4\times-1. = 5$$

$$x_2 = 2\times3 + 3\times-1 = 3$$

$$x_1 = 5, \quad x_2 = 3.$$

(B) $K_1 = 6, \quad K_2 = 5$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = A^{-1} \cdot \begin{bmatrix} K_1 \\ K_2 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 6 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3\times6 + 5\times4 \\ 2\times6 + 3\times5 \end{bmatrix}$$

8

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 38 \\ 27 \end{bmatrix}$$

$$x_1 = 38, \quad x_2 = 27.$$

(C)

(c) $K_1 = 0, \quad K_2 = -4$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = A^{-1} \begin{bmatrix} K_1 \\ K_2 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 0 \\ -4 \end{bmatrix}$$

8

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3\times0 + 4\times-4 \\ 2\times0 + 3\times-4 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -16 \\ -12 \end{bmatrix}$$

$$x_1 = -16, \quad x_2 = -12.$$

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(d) 10-5 on 71. it's clear that $x = 0$, y [illegible] is a solution to each of the systems given in problem 71. Use Cramer's rule to determine whether this solution is unique. [Hint: if $D \neq 0$, what can you conclude? If $D = 0$, what can you conclude?].

(a) $x - 4y + 9z = 0$
$4x - y + 6z = 0$
$x - y + 3z = 0$

(b) $3x - y + 3z = 0$
$5x + 5y + 9z = 0$
$-2x + y - 3z = 0$

solution

(a) $x - 4y + 9z = 0$
$4x - y + 6z = 0$
$x - y + 3z = 0$

Its augmented matrix.

$$\left[\begin{array}{ccc|c} 1 & -4 & 9 & 0 \\ 4 & -1 & 6 & 0 \\ 1 & -1 & 3 & 0 \end{array}\right]$$

let

$$A = \begin{bmatrix} 1 & -4 & 9 \\ 4 & -1 & 6 \\ 1 & -1 & 3 \end{bmatrix}$$

d)

$$|A| = \begin{vmatrix} -1 & 6 \\ -1 & 3 \end{vmatrix} - -4\begin{vmatrix} 4 & 6 \\ 1 & 3 \end{vmatrix} + 9\begin{vmatrix} 4 & -1 \\ 1 & -1 \end{vmatrix}.$$

$$|A| = -3 + 6 - (-4(12-6)) + 9(-4+1)$$

$$|A| = 3 + 24 + -27$$

$$|A| = 0$$

Determinant $= 0$, therefore the solution is NOT unique since the value of x, y, z does not exist

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(d) (b) $3x - y + 3z = 0$
$5x + 5y - 9z = 0$
$-2x + y - 3z = 0$

solution.

Its Augmented matrix

$$\left[\begin{array}{ccc|c} 3 & -1 & 3 & 0 \\ 5 & 5 & -9 & 0 \\ -2 & 1 & -3 & 0 \end{array}\right]$$

let

$$D = \begin{bmatrix} 3 & -1 & 3 \\ 5 & 5 & -9 \\ -2 & 1 & -3 \end{bmatrix}$$

$$|D| = 3\begin{vmatrix} 5 & -9 \\ 1 & -3 \end{vmatrix} - -1\begin{vmatrix} 5 & -9 \\ -2 & -3 \end{vmatrix} + 3\begin{vmatrix} 5 & 5 \\ -2 & 1 \end{vmatrix}.$$

$|D| = 3(-15+9) - -1(-15-18) + 3(5+10).$

$|D| = -18 - 33 + 45$

$|D| = -6.$

Hence, the solution is unique since the value of (x, y, z) are obtained through cramer rule.

$|A|$

Qn2 (d) solve the system of equations using the inverse of the coefficient matrix of the equivalent matrix equation.

$$5w - 4x + 3y - 2z = -6$$
$$w + 4x - 2y + 3z = -5$$
$$2w - 3x + 6y - 9z = 14$$
$$3w - 5w + 2y - 4z = -3$$

solution.

It's Augmented matrix.

$$\left[\begin{array}{cccc|c} 5 & -4 & 3 & -2 & -6 \\ 1 & 4 & -2 & 3 & -5 \\ 2 & -3 & 6 & -9 & 14 \\ 3 & -5 & 2 & -4 & -3 \end{array}\right]$$

let $A = \begin{bmatrix} 5 & -4 & 3 & -2 \\ 1 & 4 & -2 & 3 \\ 2 & -3 & 6 & -9 \\ 3 & -5 & 2 & -4 \end{bmatrix}$

Determinant of A.

$$|A| = 5\begin{vmatrix} 4 & -2 & 3 \\ -3 & 6 & -9 \\ -5 & 2 & -4 \end{vmatrix} + 4\begin{vmatrix} 1 & -2 & 3 \\ 2 & 6 & -9 \\ 3 & 2 & -4 \end{vmatrix} + 3\begin{vmatrix} 1 & 4 & 3 \\ 2 & -3 & -9 \\ 3 & -5 & -4 \end{vmatrix}$$

$$+ 2\begin{vmatrix} 1 & 4 & -2 \\ 2 & -3 & 6 \\ 3 & -5 & 2 \end{vmatrix}.$$

$$-90 + -40 - 336 + 164$$

$$|A| = -302$$

2(d) minors of A

$M_{11} = \begin{vmatrix} 4 & -2 & 3 \\ -3 & 6 & -9 \\ -5 & 2 & -4 \end{vmatrix} = -18$

$M_{12} = \begin{vmatrix} 1 & -2 & 3 \\ 2 & 6 & -9 \\ 3 & 2 & -4 \end{vmatrix} = -10$

$M_{13} = \begin{vmatrix} 1 & 4 & 3 \\ 2 & -3 & -9 \\ 3 & -5 & -4 \end{vmatrix} = -112$

$M_{14} = \begin{vmatrix} 1 & 4 & -2 \\ 2 & -3 & 6 \\ 3 & -5 & 2 \end{vmatrix} = 82$

$M_{21} = \begin{vmatrix} -4 & 3 & -2 \\ -3 & 6 & -9 \\ -5 & 2 & -4 \end{vmatrix} = 75$

$M_{22} = \begin{vmatrix} 5 & 3 & -2 \\ 2 & 6 & -9 \\ 3 & 2 & -4 \end{vmatrix} = 59$

$M_{23} = \begin{vmatrix} 5 & -4 & -2 \\ 2 & -3 & -9 \\ 3 & -5 & -4 \end{vmatrix} = 147$

$M_{24} = \begin{vmatrix} 5 & -4 & 3 \\ 2 & -3 & 6 \\ 3 & -5 & 2 \end{vmatrix} = 61$

$M_{31} = \begin{vmatrix} -4 & 3 & -2 \\ 4 & -2 & 3 \\ -5 & 2 & -4 \end{vmatrix} = -1$

$M_{32} = \begin{vmatrix} 5 & 3 & -2 \\ 1 & -2 & 3 \\ 3 & 2 & -4 \end{vmatrix} = 33$

$M_{33} = \begin{vmatrix} 5 & -4 & -2 \\ 1 & 4 & 3 \\ 3 & -5 & -4 \end{vmatrix} = -23$

$M_{34} = \begin{vmatrix} 5 & -4 & 3 \\ 1 & 4 & -2 \\ 3 & -5 & 2 \end{vmatrix} = -29$

$M_{41} = \begin{vmatrix} -4 & 3 & -2 \\ 4 & -2 & 3 \\ -3 & 6 & -9 \end{vmatrix} = 45$

$M_{42} = \begin{vmatrix} 5 & 3 & -2 \\ 1 & -2 & 3 \\ 2 & 6 & -9 \end{vmatrix} = 25$

$M_{43} = \begin{vmatrix} 5 & -4 & -2 \\ 1 & 4 & 3 \\ 2 & -3 & -9 \end{vmatrix} = -173$

$M_{44} = \begin{vmatrix} 5 & -4 & 3 \\ 1 & 4 & -2 \\ 2 & -3 & 6 \end{vmatrix} = 97$

2.(d) Minors of A.

$$= \begin{bmatrix} -18 & -10 & -112 & 82 \\ 75 & -59 & 147 & 61 \\ -1 & 33 & -23 & -29 \\ 45 & 25 & 173 & 97 \end{bmatrix}$$

Cofactor of A $= \begin{bmatrix} + & - & + & - \\ - & + & - & + \\ + & - & + & - \\ - & + & - & + \end{bmatrix}$

$$\begin{bmatrix} -18 & 10 & -112 & -82 \\ -75 & -59 & -147 & 61 \\ -1 & -33 & -23 & 29 \\ -45 & 25 & -173 & 97 \end{bmatrix}$$

Adjoint matrix of A.

$$\begin{bmatrix} -18 & -75 & -1 & -45 \\ 10 & -59 & -33 & 25 \\ -112 & -147 & -23 & -173 \\ -82 & 61 & 29 & 97 \end{bmatrix}$$

(e) $A^{-1} = \dfrac{\text{Adjoint } A}{|A|} = \begin{bmatrix} +\frac{18}{302} & \frac{75}{302} & \frac{1}{302} & \frac{45}{302} \\ -\frac{10}{302} & \frac{59}{302} & \frac{33}{302} & -\frac{25}{302} \\ +\frac{112}{302} & \frac{147}{302} & \frac{23}{302} & \frac{173}{302} \\ \frac{82}{302} & -\frac{61}{302} & -\frac{29}{302} & -\frac{97}{302} \end{bmatrix}$

2. (d)

$$\begin{bmatrix} W \\ X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} +\frac{18}{302} & \frac{25}{302} & \frac{1}{302} & \frac{45}{302} \\ -\frac{10}{302} & \frac{59}{302} & \frac{33}{302} & -\frac{25}{302} \\ \frac{112}{302} & \frac{147}{302} & \frac{23}{302} & \frac{173}{302} \\ \frac{82}{302} & -\frac{61}{302} & -\frac{29}{302} & -\frac{97}{302} \end{bmatrix} \begin{bmatrix} -6 \\ -5 \\ 14 \\ -3 \end{bmatrix}$$

$$\begin{bmatrix} W \\ X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 2 \\ -1 \end{bmatrix}$$

02

2. (e) 6.6 Qn 38 solve using Cramer's rule.

$3X - Y + 2Z = 1$
$X - Y + 2Z = 3$
$-2X + 3Y + Z = 1$

Solution

1st Augmented matrix.

$$\left[\begin{array}{ccc|c} 3 & -1 & 2 & 1 \\ 1 & -1 & 2 & 3 \\ -2 & 3 & 1 & 1 \end{array}\right]$$

let $A = \begin{bmatrix} 3 & -1 & 2 \\ 1 & -1 & 2 \\ -2 & 3 & 1 \end{bmatrix}$

$$|A| = 3\begin{vmatrix} -1 & 2 \\ 3 & 1 \end{vmatrix} + 1\begin{vmatrix} 1 & 2 \\ -2 & 1 \end{vmatrix} + 2\begin{vmatrix} 1 & -1 \\ -2 & 3 \end{vmatrix}$$

2·2 (e) $|A| = -14.$

$A_x = \begin{vmatrix} 1 & -1 & 2 \\ 3 & -1 & 2 \\ 1 & 3 & 1 \end{vmatrix}$ $|A_x| = \begin{vmatrix} -1 & 2 \\ 3 & 1 \end{vmatrix} + \begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix} + 2\begin{vmatrix} 3 & -1 \\ 1 & 3 \end{vmatrix}$

$|A_x| = 14.$

$x = \frac{|A_x|}{|A|} = \frac{14}{-14} = -1.$

$x = -1.$

02

$\frac{|A_y|}{|A|} = \frac{\begin{vmatrix} 3 & 1 & 2 \\ 1 & 3 & 2 \\ -2 & 1 & 1 \end{vmatrix}}{-14} = -6/7$

$y = -6/7$

$z = \frac{|A_z|}{|A|} = \frac{\begin{vmatrix} 3 & -1 & 1 \\ 1 & -1 & 3 \\ -2 & 3 & 1 \end{vmatrix}}{-14} = \frac{-22}{-14} = 1.57$

2(e)

$z = 1.57$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ -6/7 \\ 1.57 \end{bmatrix}$ do not mix fraction and decimal in your sln

2. (b) 6.4 on 31.

Nutrition A 3-oz serving of roasted, skinless chicken breast contains 140 cal, 27g of protein, 3g of fat, 13 mg of calcium, and 64 mg of sodium. One-half cup of potato salad contains 180 cal, 4g of protein, 11g of fat, 24mg of calcium, and 662 mg of sodium. One broccoli spear contains 50 cal, 5g of protein, 1g of fat, 82 mg of calcium, and 20 mg of sodium. (Source: Home and Garden Bulletin NO 72. U.S. Government printing office, washington D.C. 20402).

(a) Write 1x5 matrices C, P, and B that represent the nutritional

(b) Find C+2P+3B and tell what the entries represent.

Solution.

2 (a) $C = [140 \quad 27 \quad 3 \quad 13 \quad 64]$

$P = [180 \quad 4 \quad 11 \quad 24 \quad 662]$

$B = [50 \quad 5 \quad 1 \quad 82 \quad 20]$

(b) Find C+2P+3B and tell what the entries represent.

$2P = [180\times2 \quad 4\times2 \quad 11\times2 \quad 24\times2 \quad 662\times2]$

$2P = [360 \quad 8 \quad 22 \quad 48 \quad 1324]$

$3B = [50\times3 \quad 5\times3 \quad 1\times3$

(b) 6.4 Qn 31

(b)

$2P = [180\times2 \quad 4\times2 \quad 11\times2 \quad 24\times2 \quad 662\times2]$

$[360 \quad 8 \quad 22 \quad 48 \quad 1324]$

$3B = [150 \quad 15 \quad 3 \quad 246 \quad 60]$

$C + 2P + 3B =$

$[150 \quad 27 \quad 3 \quad 13 \quad 64] + [360 \quad 8 \quad 22 \quad 44 \quad 1324] +$

$[150 \quad 15 \quad 3 \quad 246 \quad 60]$

$C + 2P + 3B = [650 \quad 50 \quad 28 \quad 307 \quad 1448]$

02

2. (a) 6.3 Qn 40

Solve the system of linear equation using Gauss-elimination or Gauss-Jordan elimination. Use a graphing calculator to the check your answer

$-w + 2x - 3y + z = -8$
$-w + x + y - z = -4$
$w + x + y + z = 22$
$-w + x - y - z = -14$

solution

it augmented matrix.

$$\left[\begin{array}{cccc|c} -1 & 2 & -3 & 1 & -8 \\ -1 & 1 & 1 & -1 & -4 \\ 1 & 1 & 1 & 1 & 22 \\ -1 & 1 & -1 & -1 & -14 \end{array}\right]$$

2(a) $\left[\begin{array}{cccc|c} -1 & 2 & -3 & 1 & -8 \\ -1 & 1 & 1 & -1 & -4 \\ 1 & 1 & 1 & 1 & 22 \\ -1 & 1 & -1 & -1 & -14 \end{array}\right] R_1 \leftrightarrow R_3.$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 22 \\ -1 & 1 & 1 & -1 & -4 \\ -1 & 2 & -3 & 1 & -8 \\ -1 & 1 & -1 & -1 & -14 \end{array}\right] \begin{array}{l} R_1 + R_2 \to R_2 \\ R_1 + R_3 \to R_3 \\ R_1 + R_4 \to R_4 \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 22 \\ 0 & 2 & 2 & 0 & 18 \\ 0 & 3 & -2 & 2 & 14 \\ 0 & 2 & 0 & 0 & 8 \end{array}\right] \begin{array}{l} \frac{1}{2} R_2 \to R_2 \\ -3R_2 + R_3 \to R_3 \\ -2R_2 + R_4 \to R_4 \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 22 \\ 0 & 1 & 1 & 0 & 9 \\ 0 & 0 & -5 & 2 & -13 \\ 0 & 0 & -2 & 0 & -10 \end{array}\right] R_4 \leftrightarrow R_3$$

2(a) $\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 22 \\ 0 & 1 & 1 & 0 & 9 \\ 0 & 0 & -2 & 0 & -10 \\ 0 & 0 & -5 & 2 & -13 \end{array}\right] -\frac{1}{2} R_3 \to R_3$

5

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 22 \\ 0 & 1 & 1 & 0 & 9 \\ 0 & 0 & 1 & 0 & 5 \\ 0 & 0 & -5 & 2 & -13 \end{array}\right] 5R_3 + R_4 \to R_4$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 22 \\ 0 & 1 & 1 & 0 & 9 \\ 0 & 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 2 & 12 \end{array}\right] \quad \tfrac{1}{2}R_4 \to R_4$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 22 \\ 0 & 1 & 1 & 0 & 9 \\ 0 & 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 & 6 \end{array}\right]$$

$$\begin{aligned} w+x+y+z &= 22 \\ x+y &= 9 \\ y &= 5 \\ z &= 6 \end{aligned}$$

Back substitution

$x+y=9$

$x+5=9$

$\underline{x=4}$

$w+4+5+6=22$

$w=22-15$

$w=7$

$(w,x,y,z)=(7,4,5,6).$

02

Qn 4 (a) Compute the determinant

$$\begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & -7 & -13 \\ 0 & 0 & 0 & 2/7 \end{bmatrix}$$

Where are the steps on reaching this triangular matrix

Soln

choose the 4th row to check the determinant

form properties of determinant for triangular matrix

$$0 + 0 - 0 + \tfrac{2}{7}\begin{vmatrix} 1 & 2 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & -7 \end{vmatrix}$$

02

$$\tfrac{2}{7} \times -7 = -2.$$

Determinant $= -2$

4(b) Use adjoint to compute A^{-1}?

$$A = \begin{bmatrix} 2 & 3 & 1 \\ 0 & 1 & 1 \\ 2 & -1 & -2 \end{bmatrix}$$

Soln

i) check the determinant of $|A|$

$$2\begin{vmatrix} 1 & 1 \\ -1 & -2 \end{vmatrix} - 3\begin{vmatrix} 0 & 1 \\ 2 & -2 \end{vmatrix} + \begin{vmatrix} 0 & 1 \\ 2 & -1 \end{vmatrix} = 2.$$

ii) minors of A

$$M_{11} = \begin{vmatrix} 1 & 1 \\ -1 & -2 \end{vmatrix} = -1 \qquad M_{12} = \begin{vmatrix} 0 & 1 \\ 2 & -2 \end{vmatrix} = -2$$

b) $M_{13} = \begin{vmatrix} 0 & 1 \\ 2 & -1 \end{vmatrix} = -2$

$M_{21} = \begin{vmatrix} 3 & 1 \\ -1 & -2 \end{vmatrix} = -5$

$M_{22} = \begin{vmatrix} 2 & 1 \\ 2 & -1 \end{vmatrix} = -6$

$M_{23} = \begin{vmatrix} 2 & 3 \\ 2 & -1 \end{vmatrix} = -8$

$M_{31} = \begin{vmatrix} 3 & 1 \\ 1 & 1 \end{vmatrix} = 2$

$M_{32} = \begin{vmatrix} 2 & 1 \\ 0 & 1 \end{vmatrix} = 2$

$M_{33} = \begin{vmatrix} 2 & 3 \\ 0 & 1 \end{vmatrix} = 2$

minors of $A = \begin{bmatrix} -1 & -2 & -2 \\ -5 & -6 & -8 \\ 2 & 2 & 2 \end{bmatrix}$

4 (b) (iii) Cofactor of A

$$\begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$$

$$\begin{bmatrix} -1 & 2 & -2 \\ 5 & -6 & 8 \\ 2 & -2 & 2 \end{bmatrix}$$

(ii)

$$\begin{bmatrix} -1 & 2 & 2 \\ 5 & -6 & 8 \\ 2 & -2 & 2 \end{bmatrix}$$

(iii) transpose of cofactors = adjoint

$$\begin{bmatrix} -1 & 2 & -2 \\ 5 & -6 & 8 \\ 2 & -2 & 2 \end{bmatrix}^T = \begin{bmatrix} -1 & 5 & 2 \\ 2 & -6 & -2 \\ -2 & 8 & 2 \end{bmatrix}$$

Now, $A^{-1} = \dfrac{\text{Adjoint }(A)}{|A|} = \dfrac{1}{2} \times \begin{bmatrix} -1 & 5 & 2 \\ 2 & -6 & -2 \\ -2 & 8 & 2 \end{bmatrix}.$

$$A^{-1} = \begin{bmatrix} -1/2 & 5/2 & 1 \\ 1 & -3 & -1 \\ -1 & 4 & 1 \end{bmatrix}$$

(i) 3-5 ques 2

Repeat exercise 1 for the following linear systems:

$$2x_1 + 3x_2 + 7x_3 = 0$$
$$-2x_1 \quad\quad - 4x_3 = 0$$
$$x_1 + 2x_2 + 4x_3 = 0$$

Solution.

In the exercise 1 required to find the value of variable by Jordan elimination.

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1. Using the augmented matrix

$$\left[\begin{array}{ccc|c} 2 & 3 & 7 & 0 \\ -2 & 0 & -4 & 0 \\ 1 & 2 & 4 & 0 \end{array}\right] \quad R_3 \leftrightarrow R_1.$$

$$\begin{bmatrix} 1 & 2 & 4 & 0 \\ -2 & 0 & -4 & 0 \\ 2 & 3 & 7 & 0 \end{bmatrix} \quad \begin{array}{l} 2R_1 + R_2 \rightarrow R_2 \\ -2R_1 + R_3 \rightarrow R_3. \end{array}$$

$$\begin{bmatrix} 1 & 2 & 4 & 0 \\ 0 & 4 & 4 & 0 \\ 0 & -1 & -1 & 0 \end{bmatrix} \quad \begin{array}{l} \cancel{4R_2 + R_3 \rightarrow R_3}. \\ 4R_3 + R_2 \rightarrow R_3 \end{array}$$

$$\begin{bmatrix} 1 & 2 & 4 & 0 \\ 0 & 4 & 4 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$x_1 + 2x_2 + 4x_3 = 0$$

$$4x_2 + 4x_3 = 0$$

$$0 + 0 + 0 = 0$$

Indicate many solution, hence introduce the parameter by let x_2 be t

$$4t + 4x_3 = 0$$

$$4x_3 = -4t$$

$$x_3 = -t$$

$$x_1 + 2t - 4t = 0$$

$$x_1 = 4t - 2t$$

$$x_1 = 2t.$$

$$(x_1, x_2, x_3) = (2t, t, -t).$$