# Unit 2

# Mathematical models

# Introduction

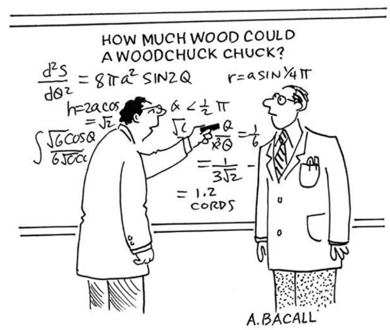
This unit is primarily concerned with a central theme of the module – how you can use mathematics to help investigate and solve practical problems.

The unit starts with an everyday problem – how do you decide when to set off on a certain journey, in order to reach your destination by a particular time? In Section 1, we consider how to make a rough estimate of the journey time. This section uses some simple mathematical ideas and, at the same time, introduces a general strategy for tackling problems by creating a **mathematical model**.

The aims of a mathematical model are to:

- describe the important features of a real situation mathematically – for example, by using numbers, formulas or graphs
- allow you to make predictions about the situation.

For example, such models can be used to predict traffic flows on roads or to investigate the likely impact of changes to speed limits on sections of motorway. However, a model does simplify the real situation, and emphasises certain aspects of it (such as the speed of a vehicle) and ignores others (such as the weather conditions). Even so, the results from models are often useful in practice, since in many cases an approximate answer is perfectly adequate.



**"Assuming, of course, that a woodchuck could chuck wood."**

In Section 2, we look at two models that are used to advise motorists on the gap they should leave on the road between their car and the vehicle ahead, and in particular we look at how these models take account of different features of the situation. This section illustrates the fact that problems can be approached in different ways, such as numerically, graphically or through general relationships such as formulas. It also highlights the importance of communicating mathematical ideas in an appropriate way for a wide audience.

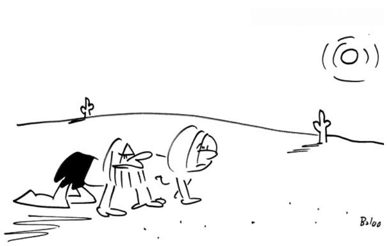
Section 3 concentrates on the use of formulas and shows how these can be written concisely. This is an important section because formulas are frequently used in models and also because the skills and terminology covered here form a foundation for the rest of the module. So you are advised to work through the examples, activities and practice quizzes for this section carefully.

Section 4 introduces the use of inequalities, which can be used to specify some of the restrictions and limits on models concisely. Finally, Section 5 provides some advice on how you can improve your mathematics and on how to use the feedback you will receive on your assignments.

Some of the activities in this unit involve the use of your calculator. For example, Activity 18 on page 110 is in the MU123 Guide. Instructions for using your calculator are provided in the MU123 Guide.

# 1 Planning a journey

## 1.1 Clarifying the question


"YOU AND YOUR SCENIC ROUTES!"

Often, the first step in tackling a practical problem is to decide what questions you need to ask. For example, consider the problem of getting to the right place at the right time, which arises in many everyday activities such as going to work or to an appointment, on holiday or just meeting friends. Here a key question is: when should you set off?

The answer to this question may depend on several factors, such as what kind of transport is available in the area, which route you decide to take and how fast you can travel. You may also want to consider other factors when planning a journey, such as the amount of pollution produced or which is the most scenic route!

This section is based on the following scenario. Suppose that two students want to travel from Great Malvern in Worcestershire to Milton Keynes in Buckinghamshire to attend an Open Day at The Open University (Figure 1). If they decide to travel by car and want to arrive by 10:30 am, at what time should they set off?


**Figure 1** The start and finish of the journey

One way of tackling this problem is to use a route-planning system, known as a *route planner*, on your phone or a satellite navigation device (satnav) to suggest a route and estimate the journey time. Here are some estimates for the time for the car journey between Great Malvern and Milton Keynes, provided by different route planners using various routes:

1 h 36 min, 1 h 58 min, 2 h 3 min, 2 h 5 min, 2 h 8 min, 2 h 19 min,
2 h 21 min, 2 h 33 min, 2 h 48 min, 3 h 19 min.

You can see that the estimated times vary substantially, with the longest time being more than twice the shortest time! However, most of the times are bunched 'in the middle', so it seems likely that these estimates are the most reliable. You'll see in this section how to make your own estimate for the journey time along a particular route, and you'll consider some of the factors that may affect this estimate and which are considered by route planning devices.

First, consider the questions in the following activity.

## Activity 1 Preliminary questions

(a) Why might the route planners estimate different times for the journey?

(b) From Figure 1, the distance in a straight line between Great Malvern and Milton Keynes appears to be just over 100 kilometres. Assuming that the car travels about 50 kilometres in an hour, what is an estimate for the journey time? Do you think this estimate is good enough for planning the time to set off?

How can you obtain an estimate that is more realistic than the one in part (b) of Activity 1?

As with many problems, it helps to consider a simpler version first. There are many possible routes that the students could take, but to see what's involved in calculating the time, let's concentrate here on one particular route.

Suppose that the students decide to take the route from Great Malvern to Milton Keynes indicated by a thick pink line on the map in Figure 2. This route uses the M40 motorway and various principal roads, also known as A-roads. The question is now:

> If the students take the route indicated in Figure 2, at what time must they set off in order to arrive in Milton Keynes by 10:30 am?

To answer this question, you need to estimate the time for the journey along this route.


**Figure 2** The route for the journey

The time for the journey depends on at least two factors: the *road distance* between Great Malvern and Milton Keynes, and the *speeds* at which the car can travel on different types of roads.

These two factors are discussed in the next two subsections.

## 1.2 Estimating distances

This subsection is about using maps to find distances. A **map scale** indicates how to determine distances on the ground from distances on a map. On the map in Figure 2 there is a line drawn at the top right to show the map scale graphically. This line indicates that a distance of 2 centimetres on the map represents 20 kilometres on the ground. Thus, by measuring a distance on the map in centimetres and using the map scale, you can obtain the corresponding distance on the ground. For practice with scale diagrams, see Maths Help Module 5, Subsection 1.1.

Distances on the map can be measured by hand by laying a piece of string approximately along a route and then measuring the string.

The map scale in Figure 2 can be stated in words as ‘2 cm represents 20 km’, which is equivalent to writing ‘1 cm represents 10 km’. This means that to find the ground distance in kilometres, you multiply the map distance in centimetres by 10. Map scales are often written in the form ‘1 cm = 10 km’, but such a statement is incorrect mathematically because a length of 1 centimetre is not equal to a length of 10 kilometres!

## Activity 2 Estimating the distance from a map

(a) On the map, the distance along the route from Great Malvern to Milton Keynes is 13.8 cm, and the distance along the motorway section of the route is 3 cm (to the nearest mm). What are the corresponding ground distances, in kilometres?

(b) Apart from the section on the motorway, the rest of the route is on principal roads. Use your answers to part (a) to find the distance on the principal roads, in kilometres.

(c) What distance on the map represents 25 km on the ground?

On the map in Figure 2, we interpreted the scale as '1 cm represents 10 km'. Map scales can be written in other forms, so you need to be familiar with these alternatives too. Often, the scale of a map is given as, say, 1 : 500 000 or 1/500 000, which is read as 'one to five hundred thousand'. This means that any distance measured on the map represents 500 000 times that distance on the ground. For example, 1 cm on the map represents 500 000 cm on the ground, 1 mm on the map represents 500 000 mm on the ground, and so on. The number 500 000 is called the **scale factor** of the map.

The next example shows how to convert between the two methods of giving a map scale. For details of SI units, see Unit 1, Subsection 2.3 or Maths Help Module 1, Section 2.

## Example 1 Converting map scales

(a) A map scale is given in words as '1 cm represents 20 km'. What is the scale factor?

(b) A map scale is given as 1 : 250 000. Express this map scale in the form '1 cm represents ?? km'.

### Solution

(a) Here, 1 cm on the map represents 20 km on the ground.

Convert 20 km to centimetres, by using 1 km = 1000 m and 1 m = 100 cm.

Now,

$$\begin{aligned}20\,\text{km} &= (20 \times 1000)\,\text{m}\\ &= 20\,000\,\text{m}\\ &= (20\,000 \times 100)\,\text{cm}\\ &= 2\,000\,000\,\text{cm}.\end{aligned}$$

So, 1 cm on the map represents 2 000 000 cm on the ground. That is, the map scale is 1 : 2 000 000, so the scale factor is 2 000 000.

(b) Here, 1 cm on the map represents 250 000 cm on the ground.

Convert 250 000 cm to kilometres. Now,

$$\begin{aligned}250\,000\,\text{cm} &= (250\,000 \div 100)\,\text{m}\\ &= 2500\,\text{m}\\ &= (2500 \div 1000)\,\text{km}\\ &= 2.5\,\text{km}.\end{aligned}$$

So the map scale is '1 cm represents 2.5 km'.

Here are some similar conversions for you to try.

## Activity 3 Converting map scales

(a) A map scale is given as '1 cm represents 10 km'. What is the scale factor?

(b) A map scale is given as 1 : 500 000. Express this map scale in the form '1 cm represents ?? km'.

The next example shows you how to use a scale factor to work out the length of a journey from its distance on a map, and also to work out the map distance if you know the ground distance.

## Example 2 Using a map scale

The scale of a map is 1 : 250 000.

(a) The distance on the map between two places is 7.5 cm. What is the corresponding distance on the ground? Give your answer to two significant figures.

(b) The distance on the ground between two places is 58.4 km. What is the corresponding distance on the map? Give your answer to three significant figures.

### Solution

(a) A measurement of 1 cm on the map represents 250 000 cm on the ground. So a map distance of 7.5 cm represents a ground distance of $(7.5 \times 250\,000)$ cm. Now,

$$\begin{aligned}(7.5 \times 250\,000)\,\text{cm} &= 1\,875\,000\,\text{cm}\\ &= (1\,875\,000 \div 100 \div 1000)\,\text{km}\\ &= 18.75\,\text{km}\\ &= 19\,\text{km (to 2 s.f.)}.\end{aligned}$$

Alternatively, the map scale 1 : 250 000 can be expressed as '1 cm represents 2.5 km', so a map distance of 7.5 cm represents a ground distance of

$$(7.5 \times 2.5)\,\text{km} = 18.75\,\text{km} = 19\,\text{km (to 2 s.f.)}.$$

(b) The scale factor is 250 000, so the ground distance of 58.4 km is represented by a map distance of (58.4 ÷ 250 000) km. (See Example 1(b).) Now,

$$\begin{aligned}(58.4 \div 250\,000)\,\text{km} &= 0.000\,233\,6\,\text{km}\\ &= (0.000\,233\,6 \times 1000 \times 100)\,\text{cm}\\ &= 23.36\,\text{cm}\\ &= 23.4\,\text{cm (to 3 s.f.)}.\end{aligned}$$

Alternatively, the map scale 1 : 250 000 can be expressed as '1 cm represents 2.5 km', so a ground distance of 58.4 km is represented by a map distance of

$$(58.4 \div 2.5)\,\text{cm} = 23.36\,\text{cm} = 23.4\,\text{cm (to 3 s.f.)}.$$

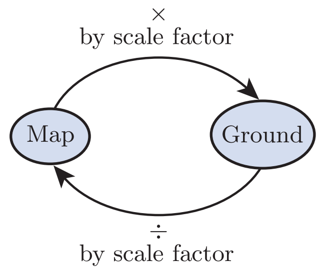



**Figure 3** Using a scale factor to convert between ground and map distances

Figure 3 summarises the process of converting map distances to ground distances, and vice versa. Here are some similar questions for you to try.

## Activity 4 Using map scales

Consider a map with scale 1 : 50 000.

(a) The distance between two towns on the map is 3.4 cm. What is the distance between them on the ground?

(b) The distance along a road is 7.85 km. What is the corresponding distance on the map?

In Activity 2, you made some progress on answering the question about what time the students should set off. You now have an estimate of the distance for both the motorway section (30 km) and the principal road section (108 km) of the route. You have also seen how to calculate ground distances if you are given the scale factor of the map.

To find the time that the journey is likely to take, the next step is to consider the speeds at which you can expect to travel on the different types of roads.

## 1.3 Understanding speed

This subsection is about the concept of **speed**. The speed of an object indicates how far it travels in a particular period of time. For example, if you are travelling at a constant speed and you cover 40 kilometres in 1 hour, then your speed is 40 kilometres per hour. This speed can also be written as 40 km/h ('km/h' is read as 'kilometres per hour').

Therefore

in 1 hour you cover $(40 \times 1)\,\mathrm{km} = 40\,\mathrm{km}$;
in 2 hours you cover $(40 \times 2)\,\mathrm{km} = 80\,\mathrm{km}$;
in 3 hours you cover $(40 \times 3)\,\mathrm{km} = 120\,\mathrm{km}$;
and so on.

Speed can be measured in other units as well: for example, metres per second or miles per hour. These are all examples of **compound units**, since they involve more than one basic unit of measurement.

In practice, you will not be able to travel at a constant speed for a whole journey, as you sometimes have to slow down, speed up or even stop to deal with the traffic and road conditions. Instead, the *average speed* for a journey is used in calculations.

The **average speed** for a journey is calculated using the formula below.

$$\text{average speed} = \frac{\text{distance travelled}}{\text{time taken}}$$

This formula is often written more concisely as

$$\text{average speed} = \frac{\text{distance}}{\text{time}} \qquad \text{or} \qquad \text{speed} = \frac{\text{distance}}{\text{time}}.$$

With formulas such as this one, where the units are not specified, you can work out the unit used to measure the speed from the units for the distance and the time.

For example, suppose that the distance is measured in metres (m) and the time is measured in seconds (s). To work out the unit for the average speed, substitute these units into the formula above.

Since average speed $= \dfrac{\text{distance}}{\text{time}}$, the unit for the speed is $\dfrac{\mathrm{m}}{\mathrm{s}}$.

This unit is usually written as m/s and read as 'metres per second'.

### Example 3 Finding an average speed

Suppose that a car travels a distance of 75 km in 45 minutes on a motorway. What is its average speed? Give your answer in km/h.

#### Solution

First method

Use the formula to find the average speed in km/min and then convert to km/h.

The distance is 75 km and the time is 45 min. So, by the formula,

$$\text{average speed} = \frac{75}{45}\,\text{km/min} = 1.666\ldots\,\text{km/min}.$$

Now convert this answer to km/h by multiplying the full calculator value for the distance travelled in 1 minute by the number of minutes in an hour.

There are 60 minutes in an hour, so

$$\text{average speed} = (1.666\ldots \times 60)\,\text{km/h} = 100\,\text{km/h}.$$

Second method

Convert the time to hours and then find the average speed in km/h.

First, 45 minutes is $\frac{3}{4}$ of an hour, that is, 0.75 h. So, by the formula,

$$\text{average speed} = \frac{75}{0.75}\,\text{km/h} = 100\,\text{km/h}.$$

Here are some similar examples for you to try.

### Activity 5 Finding average speeds

Find the average speeds for the following journeys, rounding your answers to two significant figures. Give your answers to parts (a) and (b) in km/h, and your answer to part (c) in m/s.

(a) A journey of 425 km on the motorway that takes 4 hours

(b) A 30 km journey through a city that takes 1 hour 25 minutes

(c) A 100 m sprint that takes 14 seconds

## 1.4 Finding the time

In Activity 2, you calculated that the route from Great Malvern to Milton Keynes includes 30 km on the M40 motorway and 108 km on principal roads. To find the time required for each part of the journey, you need to use values for the average speeds on each type of road. The average speed of a car on a road depends on many factors, such as how busy the road is, the speed limits, and if there are any delays such as road works.

For the current purpose, you should use the following cautious assumptions about average speeds on these types of roads:

- average speed on principal roads: 50 km/h
- average speed on motorways: 100 km/h.

There are two methods of calculating the time for a journey if you know the distance and the average speed.

**Strategy To find the time, given the distance and the speed**

First method

Find the time to travel 1 km and then find the time to travel the whole distance.

Second method

Use the formula

$$\text{time} = \frac{\text{distance}}{\text{average speed}}.$$

This is a version of the formula

$$\text{average speed} = \frac{\text{distance}}{\text{time}}$$

given in Subsection 1.3.

The unit of speed used in this formula is the one that involves the distance unit and the time unit; for example, if the distance is in kilometres and the time is in hours, then the speed should be measured in km/h.

The next example applies both methods to find the time for the motorway section of the students' journey from Great Malvern to Milton Keynes.

**Example 4 Calculating the time on the motorway**

Find the time for a journey of 30 km at an average speed of 100 km/h. Give your answer in minutes.

### Solution

First method

The average speed is $100\,\text{km/h}$, so $1\,\text{km}$ is travelled in $\frac{1}{100}\,\text{h}$.

Hence, to travel $30\,\text{km}$, it takes $30 \times \frac{1}{100}\,\text{h} = 0.3\,\text{h}$. Since there are 60 minutes in 1 hour,

$$0.3\,\text{h} = (0.3 \times 60)\,\text{min} = 18\,\text{min}.$$

Second method

By the formula, the time is

$$\frac{30}{100}\,\text{h} = 0.3\,\text{h}.$$

Multiplying by 60 to convert the time into minutes gives 18 min, as before.


Another way to solve Example 4 is to use the informal method shown in the cartoon in the margin.

This is possible here because the numbers in the calculation are easy to work with. Even if the numbers are more complicated, you can obtain a rough estimate of the answer by rounding the numbers in the calculation and then using an informal method of this type.

The next activity involves finding the time for the principal roads section of the journey from Great Malvern to Milton Keynes.

### Activity 6 Calculating the time on the principal roads

(a) Use both of the methods above to find the time for a journey of $108\,\text{km}$ at an average speed of $50\,\text{km/h}$. Which method do you prefer?

(b) Use your answer to part (a) and the time found in Example 4 to calculate the total journey time from Great Malvern to Milton Keynes. Round your answer to the nearest 10 minutes.

## 1.5 Checking and interpreting your results

The solution to Activity 6 suggests that the students' total journey time from Great Malvern to Milton Keynes can be estimated to be about $2\frac{1}{2}$ hours. However, it is a good idea to carry out a couple of further checks to ensure that the answer is reasonable and that it answers the question asked.

## Is your answer reasonable?

An important step in solving any problem is to check whether your answer makes sense in the context of the problem and in particular whether it is realistic and roughly what you expected. In this case, you might have compared the answer with other journeys that you had made along similar roads under similar conditions. For example, it usually takes about an hour to travel from Great Malvern to Birmingham – a distance of 70 km, with quite a large section on the M5 motorway. So you might expect a journey of about 140 km to take about 2 hours, and to take longer if it is mostly on principal roads rather than on motorways. This estimate agrees quite well with the journey time obtained earlier.

If you had obtained an answer of say 25 minutes (or 25 hours!) for the whole journey, then the answer *would* have been unreasonable. It might indicate a mistake in the calculations or an unrealistic assumption somewhere – in either case, it would be wise to go back and check both the mathematics and the assumptions made.

In this problem, assumptions were made for average speeds on different types of road. However, traffic conditions vary, and you might expect speeds to be lower during the morning and evening rush hours than during other parts of the day. So, if you need a more accurate estimate of the time, then you may wish to change some of the assumptions for the speed.

To see the effect of modifying the assumed speeds, work through the following activity.

### Activity 7 Changing the assumptions

The speed limits for cars are 112 km/h on the motorway and 96 km/h on principal roads. These are approximate values; the exact speed limits are 70 mph and 60 mph. The table below shows the times, to the nearest minute, for the 30 km motorway section and the 108 km principal roads part, for different speeds.

| **Motorway (30 km)** | | | |
|---|---|---|---|
| Speed in km/h | 80 | 100 | 112 |
| Time taken in minutes | 23 | 18 | 16 |
| **Principal roads (108 km)** | | | |
| Speed in km/h | 40 | 50 | 96 |
| Time taken in minutes | 162 | 130 | 68 |

(a) Based on the values in the table, what is the shortest time for the whole journey if the speed limits are observed? Do you think that this time can be achieved?

(b) If the average speed on the motorway drops from 100 km/h to 80 km/h, how much longer does the journey take? If the average speed on the principal roads drops from 50 km/h to 40 km/h, how much longer does the journey take?

(c) Based on these calculations, would you make any changes to the time allowed for the journey?

The table in Activity 7 shows that, for the range of average speeds considered, the journey time for this route can vary from 1 hour 24 minutes to 3 hours 5 minutes. This range is similar to the times predicted by the route planners in Subsection 1.1! Making realistic assumptions about the speeds at which you can travel on different roads is important in order to predict reasonable journey times.

### Answering the question asked

The question that we are trying to answer is: 'What time should the students set off in order to arrive in Milton Keynes by 10:30 am if they travel by the route in Figure 2?'

If we assume that the average speeds are 100 km/h on the motorway and 50 km/h on the other roads, then the journey time is predicted to be approximately 2 hours 30 minutes. So, to arrive at the destination by 10:30 am, the starting time should be 8:00 am.

However, in practice it would probably be better to allow some extra time in case there are any unexpected delays or the assumptions for the average speed were too high. As you saw in Activity 7, a change in the average speed for the longer, slower part of the journey could affect the time considerably. There may also be other considerations, such as allowing time for parking.

So the final conclusion may be to allow an extra 30 minutes and leave at 7:30 am, perhaps with a further suggestion to take some work to do, in case the journey goes very smoothly!

## 1.6 Route planners and models

The assumptions made in this section about distances on roads and possible average speeds, together with the methods and formulas for calculating the predicted times taken, form the elements of a mathematical model, usually called simply a model. Changing the assumptions made or the formulas used would result in a different model.

You have seen that the assumptions that you make about the average speeds on different types of road can have a large effect on the time predicted for the journey. This may account for some of the discrepancies in the times predicted by the route planners – they have used different models with different assumptions. Some route planners may have taken account of factors such as the time of day, how congested the roads are, whether there are any current roadworks, and so on, and others may have used a simple model like ours, ignoring some or all of these factors.

However, remember that we considered only one particular route that seemed to be reasonably direct and made use of principal roads and the M40 motorway. Many other routes could have been taken, and different routes are likely to take different times.

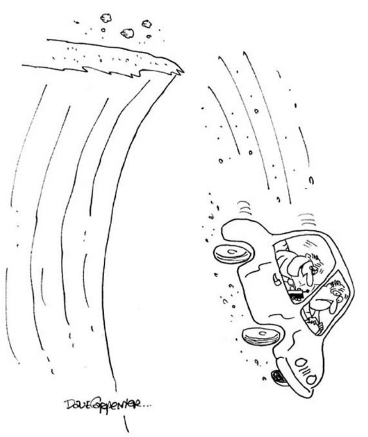
To decide on suitable routes, a route planner computer program stores a network of roads, and a time for travelling along each section of a road is determined from a suitable model. An **algorithm** – a set of instructions to solve a problem step by step – is then used to check the distances and times of all the possible routes systematically and select the shortest or quickest, as appropriate.

### Dijkstra's algorithm

One algorithm used in many route planners is called Dijkstra's algorithm. It was developed in 1959 by a Dutch computer scientist, Edsger Dijkstra. In the algorithm, places are represented by dots, and the lines connecting the dots show the time (or distance) between the two places. The algorithm then systematically searches for the shortest time or shortest distance between two points on the diagram.

So what is the best way to estimate the time for a journey – should you make your own estimate as we have done here, making assumptions about average speeds based on your own experience, or should you rely on a more sophisticated route planner, on a website or on a satnav device?

The answer to that question may depend on how accurate you need the estimate to be and whether a rough estimate obtained from a quick calculation will suffice. Whichever method you choose, you may now be more aware of how different factors can affect the journey time, such as average speeds along different sections of roads, which can change dramatically at different times of the day or on different days. If you decide to use a route planner, perhaps the best advice is to monitor how well the predictions for the journey times match reality in your case, and to choose a route planner that gives the most reliable results. Although route planners give predicted times to the nearest minute, they are unlikely to be this accurate in practice because road conditions are so changeable.



"HERE'S THE PROBLEM. OUR G.P.S. SYSTEM IS IN KILOMETERS, NOT MILES."

## 1.7 The modelling cycle

If you look back at the way we tackled the question of estimating the journey time, then you can see that the problem was broken down into a series of steps.

First, we posed the question: 'What time should the students set off in order to arrive in Milton Keynes by 10:30 am?' We then clarified the problem and saw that it depended on a different question: 'How long will the journey take?'

Next, the problem was simplified by considering just one route.

Then we collected some data (the distances along different parts of the route) and made some assumptions (the average speeds on the motorway and on principal roads). These assumptions made the problem simpler and easier to solve.

We were then able to describe the method of solution mathematically (for example, with a formula) and carry out calculations to find an answer.

Before making final predictions, we considered whether this answer was realistic, and rounded the answer appropriately so that it made sense within the context of the problem. We also investigated changing the assumptions to see how sensitive the journey times were to variations in average speeds.

For some problems, if the conclusions do not seem reasonable, then more extensive changes to the model may be needed. For example, we could have taken the time of day into account and assumed slower average speeds if the journey was during the rush hour.

The types of steps used appear in many problems, and the list of these steps is called the **modelling cycle**. This may seem a rather grand name, but these steps can be applied to more complicated problems, as you saw in the development of the route planner on the video.

This strategy for solving real-world problems is summarised in Figure 4, which indicates why it is called a modelling 'cycle'.

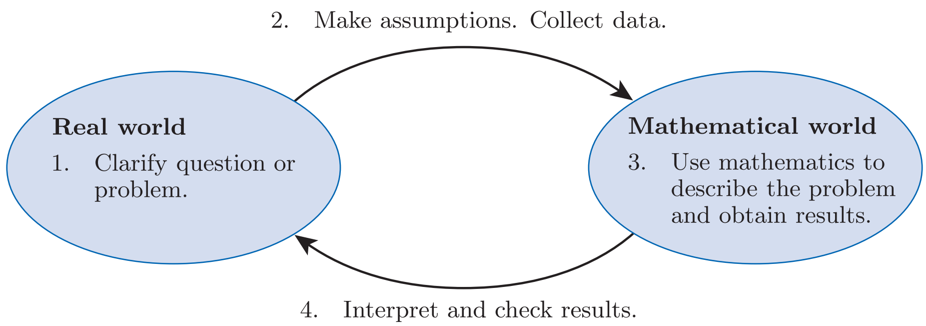



**Figure 4** The modelling cycle

There are four main steps in the modelling cycle.

### The modelling cycle

1. Describe the problem concisely so that you are clear about what you are trying to do. In real life, and particularly if you are working within a team, this may involve discussing the problem with others.

2. Make assumptions to simplify the problem, so that you retain the essential features but will be able to describe it mathematically.At this stage, it is also useful to sort out what you already know about the problem, by collecting data and other information.
3. Describe the problem mathematically using numbers, formulas or graphs, and use these to obtain new results.
4. Consider what these new results mean practically, and check that the predictions seem reasonable. If the predictions do not match reality, then you may need to refine the assumptions, collect further information and go round the cycle again. Your conclusions are only as good as the data you have used and the assumptions you have made!

This modelling cycle can be used as a framework for solving many practical problems involving both basic and advanced mathematical skills.

# 2 Investigating vehicle stopping distances

A common cause of road accidents is drivers failing to leave an adequate gap between their vehicle and the vehicle in front, so the Highway Code includes recommendations for these gaps, at different speeds and in a variety of weather conditions. These recommendations are derived from two different mathematical models. This section compares these two models. It also discusses what each model takes into account, and how the information from the models has been presented so that it can be understood and used by the intended audience.

## 2.1 Two different models

Rule 126 of the Highway Code (2019) contains the following information on stopping distances for cars.

> **Stopping distances**
>
> Drive at a speed that will allow you to stop well within the distance you can see to be clear. You should:
>
> - leave enough space between you and the vehicle in front so that you can pull up safely if it suddenly slows down or stops. The safe rule is never to get closer than the overall stopping distance (see Figure 5)
> - allow at least a two-second gap between you and the vehicle in front on roads carrying faster-moving traffic and in tunnels where visibility is reduced. The gap should be at least doubled on wet roads and increased still further on icy roads.

To apply this two-second rule: when the vehicle in front passes a landmark, count for two seconds; if you pass the landmark before the end of this time, then the gap is too small.

Elsewhere in the Highway Code it states: 'In wet weather stopping distances will be at least double those required for stopping on dry roads.'

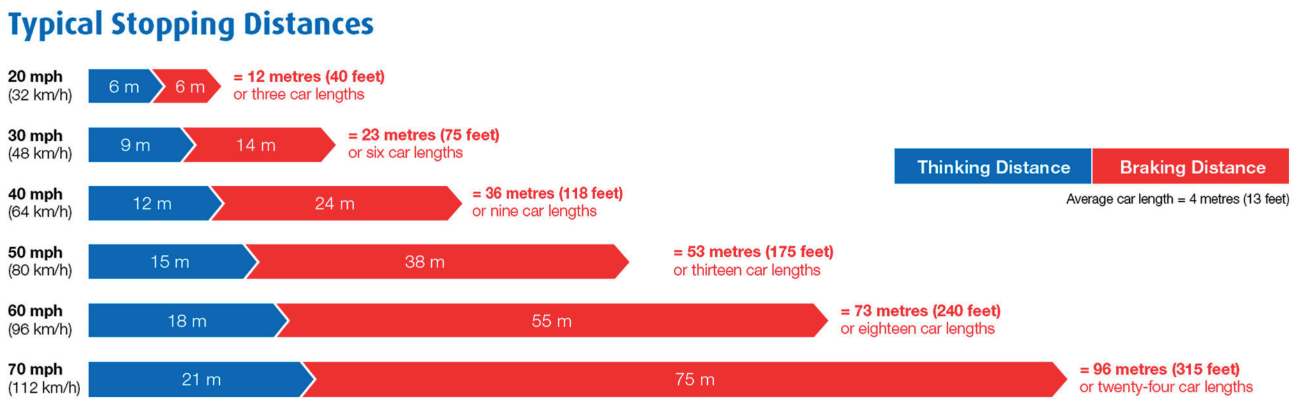



**Figure 5** Typical stopping distances, as stated in Rule 126 of the Highway Code (2019)

Notice that the Highway Code stresses that the driver should ensure that they can stop the car safely, whatever the road conditions. Then it provides some guidance on the distances and times that the driver should allow between their vehicle and the one in front, when travelling at different speeds. Two different recommendations are made, based on different models.

- The *distance model* suggests that a safe distance between your vehicle and the one in front is the distance given by the typical stopping distances chart (Figure 5). The stopping distances are presented both as a chart and as numerical measurements.
- The *time model* suggests that a safe distance between your vehicle and the one in front is the distance given by the two-second rule.

## Activity 8 Looking at the Highway Code

(a) The Highway Code includes both the 'typical stopping distances' chart and the 'two-second rule'. Why do you think both methods have been included?

(b) Which features of the 'typical stopping distances' chart do you think make it easy for people to use? Are there any features that make it difficult to use?

(c) Can you think of any other methods of informing drivers of suitable gaps between cars?

(d) The speeds in the chart are given in both mph (mph means 'miles per hour') and km/h. Use the fact that 1 mile is approximately 1.609 kilometres to check that 40 mph is 64 km/h to the nearest integer.

Activity 8 was about different ways of presenting ideas that involve mathematics, and it showed the importance of considering whether the intended audience will find the information easy to understand. In particular, you saw how useful a chart can be.

## 2.2 The distance model and the modelling cycle

This subsection considers how the distance model used in the Highway Code might have been developed, by following the four stages of the modelling cycle.

### Stage 1: Clarify the question

Both models have been constructed in order to answer the question:

> 'What gap between vehicles should be recommended for drivers travelling at different speeds?'

### Stage 2: Make assumptions and collect data

The distance model is based on typical stopping distances at various speeds. It has been assumed that the stopping distance is determined by two factors:

- the thinking distance (the distance travelled from when the driver first sees a hazard until he or she applies the brakes)
- the braking distance (the distance travelled from when the brakes are first applied to the point when the vehicle stops).

Each of these distances is determined by the speed.

Experiments can be carried out to test the reaction times of drivers, and these experiments would probably produce a range of possible times, depending on the individual and their state of alertness. From the data collected, it is possible to determine a typical reaction time. In this model, a reaction time of $\frac{2}{3}$ of a second has been assumed.

Braking distances can be based on experiments with cars or by relying on data obtained from car manufacturers.

This model ignores other features of the situation such as the road surface, the make and weight of the car, the weather conditions and the tiredness of the driver.

## Stage 3: Use mathematics to obtain results

Having collected the data and made some assumptions, the next stage is to use some mathematics, in this case working out the distances by using formulas.

While the driver is 'thinking', the car is likely to be travelling at a constant speed. If you know the speed of a vehicle and the time during which it travels at that speed, then you can calculate the distance it travels. For example, if a vehicle travels at 30 m/s for 2 seconds, then it travels a distance of 60 m. In general, if an object moves for a certain period of time, then the distance it covers in this time is given by the following formula.

$$\text{distance} = \text{average speed} \times \text{time}$$

This is another version of the formula

$$\text{average speed} = \frac{\text{distance}}{\text{time}}$$

given in Subsection 1.3.

The braking distances can be related to the speed by using the data collected to derive a more complicated formula; you will meet formulas of this type later in the module. Roughly speaking, the effect of the formula is that if the speed doubles, then the braking distance quadruples.

Once a formula has been obtained for both the thinking distance and the braking distance, the total stopping distance can be found by adding these two distances together. Using this formula for different speeds gives the results shown in the chart in Figure 5.

## Stage 4: Interpret and check the results

The distance model could be checked with reality by, for example, observing whether drivers manage to stop their vehicles within the 'typical stopping distance' and also whether collisions occur less frequently when drivers keep this gap between their vehicle and the next.

Now that you have seen how the distance model might have been developed, the next subsection considers how the recommendations of the distance model compare with those of the time model.

## 2.3 Comparing the models

In Subsection 2.1 you saw two methods, given by different mathematical models, for choosing an appropriate gap between your car and the car in front. How do these models compare with each other?

The distance model recommends gaps between vehicles at various speeds, so you can compare this model to the time model by calculating the gaps between vehicles at the same speeds when the two-second rule is observed. So we need to calculate these gaps.

### Gaps for the time model

You can calculate the gap between cars given by the two-second rule by substituting the time of two seconds and the relevant speed into the formula 'distance = speed × time'. Since the units have to match when you substitute into a formula, the speed must be measured in a unit whose 'time part' is seconds – for example, m/s or km/s. To make a comparison with the distance model possible, we want the answer to be in metres. Therefore the speed used in the formula must be expressed in m/s. However, the speeds that we need to consider – those in Figure 5 – are expressed in mph and km/h.

The example below shows how to convert km/h to m/s by breaking the problem down into smaller steps. A similar approach was used in Example 3, and you can use this approach whenever you need to convert between compound units.

#### Example 5 Converting km/h into m/s

Convert 32 km/h to m/s. Give your answer to three significant figures.

**Solution**

A speed of 32 km/h means that in 1 hour the car travels 32 km, that is,

$$32 \times 1000\,\mathrm{m} = 32\,000\,\mathrm{m}.$$

Since there are 60 minutes in an hour and 60 seconds in each minute, there are $60 \times 60 = 3600$ seconds in an hour.

So the car travels 32 000 m in 3600 seconds.

Therefore in 1 second, the car travels

$$\frac{32\,000}{3600}\,\mathrm{m} = 8.888\ldots\,\mathrm{m}.$$

So 32 km/h is 8.89 m/s (to 3 s.f.).

Once the speed is measured in m/s, you can substitute it, and the time 2 seconds, into the formula

$$\text{distance} = \text{speed} \times \text{time}$$

to calculate the gap in metres between vehicles given by the time model. For example, if the speed is 32 km/h, that is, 8.888 ... m/s, then

$$\begin{aligned}\text{distance} &= 8.888\ldots \times 2\,\text{m}\\ &= 17.777\ldots\ \text{m}.\end{aligned}$$

The two-second gap for the speed 32 km/h is therefore 18 m to the nearest whole number.

## Activity 9 Converting the time model

(a) Convert 80 km/h to m/s and write the value in the table below, rounding your answer to two decimal places. The speeds listed in this table are the ones in Figure 5, the 'typical stopping distances' chart.

Vehicle gaps at different speeds

| Speed in km/h | Speed in m/s | Time model gap in m | Distance model gap in m |
|---|---|---|---|
| 32 | 8.89 | 18 | 12 |
| 48 | 13.33 | | 23 |
| 64 | 17.78 | | 36 |
| 80 | | | 53 |
| 96 | 26.67 | | 73 |
| 112 | 31.11 | | 96 |

(b) Fill in the column for the gaps for the time model, rounding your answers to the nearest whole number.

## Drawing a graph

Since the suggested gaps between cars are now measured in metres for both models, we can compare the results for the two models directly. Although it is possible to compare the results by looking at the data in the table in the solution to Activity 9, a **graph** can be helpful. This has the advantage of illustrating overall features, which may not be so clear from the numerical data.

In the next activity you are asked to plot the gaps for the two models on a graph. To help you do that, here are some guidelines for drawing graphs and also an example to remind you how to read values from a graph. For help with graphs, see Maths Help Module 5, Subsection 3.4.

### Tips for drawing a graph or chart based on data

- Include a clear title and the source of the data.
- Label the axes with the names of the quantities and the units.
- Mark the scales clearly, choosing scales that are easy to interpret and that make good use of the space available.

There are different ways of presenting graphs. For example, sometimes the title is included as a figure caption, or as a short description in the text, and the source is quoted with the data table rather than on the graph.

Figure 6 illustrates these points. This is a graph of speed measured in m/s plotted against speed measured in km/h, based on the data given for this conversion in the table in Activity 9. This kind of graph is known as a **conversion graph** because you can use it to convert from one unit to another.

The main features of the graph in Figure 6 have been annotated. The graph has been drawn by choosing the **horizontal axis** to represent the speed in km/h and the **vertical axis** to represent the speed in m/s. The horizontal scale has been marked at intervals of 10 km/h and the vertical scale at intervals of 5 m/s. These scales have been chosen so that it is easy to plot points and read off values.

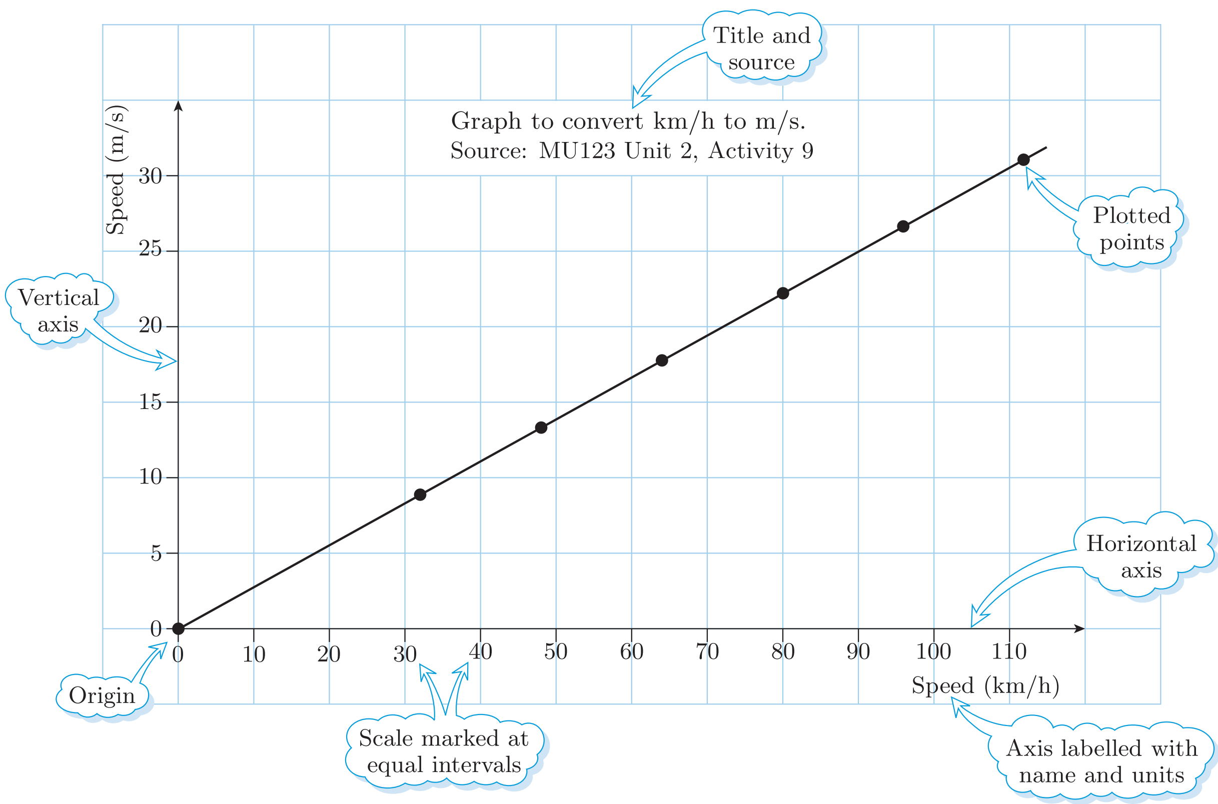



**Figure 6** A conversion graph

Note that it is not always necessary to draw an axis all the way to zero. For example, if all the values on the vertical axis are between 52 and 70, then the points are more spaced out and clearer if part of the vertical axis is omitted. If the axis scale does not start at zero, then this should be indicated either by drawing two angled parallel lines, as shown in Figure 7, or by starting the vertical scale at 50.

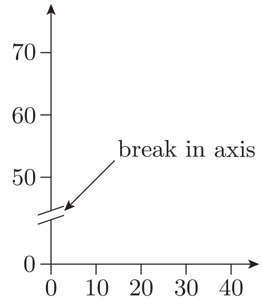



**Figure 7** Showing a break in the vertical axis

Each pair of values from the table has been plotted on the graph in Figure 6. For example, the point representing the pair of values $(32, 8.89)$ has been plotted opposite 32 on the horizontal axis and opposite 8.89 on the vertical axis. The first value of the pair, in this case 32, is known as the **horizontal coordinate** and represents the distance the point is to the right of 0 on the horizontal axis.

The second value in the pair, 8.89, is known as the **vertical coordinate** and represents the distance the point is above 0 on the vertical axis. We say that the coordinates of the point are $(32, 8.89)$. The point with coordinates $(0, 0)$ is called the **origin**.

You can use either dots, as shown in Figure 6, or small crosses to mark points on a graph. Crosses are often easier to use, particularly for hand-drawn graphs, as they mark points precisely and are clearly visible. The points in Figure 6 are joined by a straight line that passes through the origin.

## Interpreting the graph

You can use the graph on the previous page to convert speeds measured in km/h to m/s and vice versa, as illustrated in the next example.

### Example 6 Converting speeds

Use the graph in Figure 6 to make the following conversions.

(a) Convert 75 km/h to m/s.

(b) Convert 5 m/s to km/h.

### Solution

(a) Find 75 on the 'Speed (km/h)' axis, draw a line vertically up to the graph and then draw another line horizontally across to the 'Speed (m/s)' axis, as shown by the short red dashes on the graph in Figure 8. Read off the number on the vertical axis.

A speed of 75 km/h is approximately 21 m/s.

(b) Find 5 on the 'Speed (m/s)' axis, draw a line horizontally across to the graph and then draw another line vertically down to the 'Speed (km/h)' axis, as shown by the long blue dashes on the graph in Figure 8. Read off the number on the horizontal axis.

A speed of 5 m/s is approximately 18 km/h.

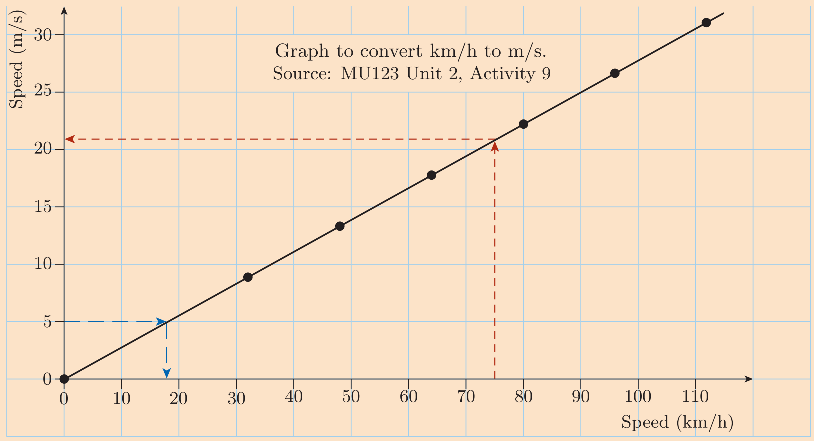



**Figure 8** Converting speeds

Here is a similar activity.

## Activity 10 Converting speeds

Use the graph in Figure 8 to make the following conversions.

(a) Convert 90 km/h to m/s.

(b) Convert 7.5 m/s to km/h.

Now try the next activity, which involves drawing a graph to represent the two models for the gap between vehicles, and then comparing the results.

## Activity 11 Using a graph to compare models

You will need graph paper for this activity.

(a) Draw a graph to show the recommended gaps for the *distance model*. The vertical axis should show the distance (in metres) and the horizontal axis the speed (in m/s). Start by plotting 6 points, each representing the length of the gap in the distance model for a particular speed, as given in the table in the solution to Activity 9. Then join these points with a smooth curve.

(b) On the same axes, plot the points for the *time model* and join the points with a straight line.

(c) What gaps do the two graphs give for a speed of 25 m/s?

(d) Use the two graphs to explain how the gaps given by the two models are different.

In Activity 11 the vertical axis represented the gap in metres and the horizontal axis represented the speed in m/s. So this is a graph of 'gap *against* speed'.

## 2.4 Developing the models further

The actual stopping distances for a particular vehicle and driver are likely to depend on many factors, such as the braking efficiency of the car, the road surface, the slope of the road, the weather conditions and the depth of tread on the tyres. One of the most significant factors could be the time that it takes the driver to react to a hazard. If the driver is not alert or is distracted in some way, then the thinking distance could increase substantially, making the overall stopping distance much greater than that suggested by the models in the Highway Code.

Taking account of these additional factors would involve going around the modelling cycle again, making new assumptions such as increasing the thinking time, developing the mathematical description to include these new assumptions and checking how the new model matches reality.

As noted earlier in the section, it may not be easy to remember or visualise the recommended stopping distances, so a rule that is easier to apply is desirable. A rule based on counting the number of seconds between your car and the one in front is certainly easier to apply, but the question then is: what should the time period be? Recall from Activity 11 that for speeds up to about 18 m/s (or 40 mph), the time model (based on the two-second rule) gives the longer gap, whereas at speeds greater than this, the distance model gives the longer gap. The Highway Code recommends a gap of *at least* two seconds in faster moving traffic.

To make a rule based on time that produced similar gaps to those of the distance model at typical motorway speeds, the time interval would have to be about three seconds. However, a three-second rule would produce much larger gaps than the distance model at lower speeds. So a more realistic time model may be to allow a two-second gap for speeds of up to 18 m/s (or 40 mph) and a three-second gap for speeds greater than 18 m/s (or 40 mph).

This time model is an approximation to the distance model, which is not perfect but can be considered to be sufficiently accurate and more practical.

To summarise this section, you have seen two mathematical models that are used in a practical context, and how the modelling cycle could have been used to develop one of these models. You have also seen the importance of thinking carefully about presenting mathematical information to other people in a form that they can understand easily, whether in a numerical or graphical form or by using formulas.

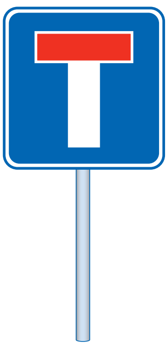
Sometimes a graph or chart can convey information more effectively than text and enable you to extract information that would be difficult to obtain in other ways. Being aware that you can tackle problems in different ways, whether by drawing a graph or diagram, or by looking at some particular numerical cases, as you did in Section 4 of Unit 1, is an important aspect of learning how to solve problems.

# 3 Using formulas

## 3.1 From words to letters

Solving a problem in mathematics often involves using a formula. For example, a formula was used in Section 1 to calculate the time for a journey, given the distance and the average speed. Formulas are used extensively in everyday life; for example, to do temperature conversions, and to calculate utility bills and car-parking charges.

Although some formulas are easy to remember when expressed in words, most formulas are written in a more concise form. This is particularly true when they are used in computer programs or spreadsheets, or when they are more complicated. This section explains how to write formulas concisely. It also introduces some conventions used in formulas.



Road sign indicating a 'no through road'

In everyday life, many things are represented by symbols; for example, a symbol like a **T** on a road sign can warn of 'no through road', and the symbol **P** on a map often indicates a car park. Symbols are concise – they save writing out a whole word or sentence and consequently they make it possible to see key information more clearly. Abbreviations such as h for hours and km for kilometres also make your writing more concise. In mathematics, you are already familiar with some symbols, such as $\sqrt{\ }$ and $\div$. In this subsection, you'll see how letters can be used to represent the different quantities in a formula.

In Section 2, the 'word formula'

$$\text{distance} = \text{average speed} \times \text{time}$$

was used to estimate the distance travelled by a car. If we use the letters

$s$ to represent the average speed,
$t$ to represent the time taken,
$d$ to represent the distance travelled,

then this word formula can be written more concisely as the 'letter formula'

$$d = s \times t.$$

If letters are used instead of words in a formula, then it is essential to say what quantities the letters represent.

## Using a formula

The letters in a formula stand for numbers that are related in the way given by the formula. Thus you can think of a formula as a way of summarising a calculation process. For example, the formula $d = s \times t$ represents the process:

> To find the value of $d$, take the value of $s$ and multiply it by the value of $t$.

When you use a formula, you replace the letters to the right of the equals sign (in the above case, $s$ and $t$) by numbers, and then carry out the calculation to find the value of the letter to the left (in this case, $d$).

This process is known as **substituting** values into the formula.

For example, suppose that a bus travels at an average speed of 50 km/h for 1.2 hours, and we want to find the distance that it has travelled.

Rather than using the word formula to find the distance, we can use the more concise formula $d = s \times t$, where $d$, $s$ and $t$ are defined as before.

Replacing $s$ by 50 and $t$ by 1.2 gives

$$d = 50 \times 1.2 = 60.$$

Hence the distance travelled is 60 km.

We have used particular values of $s$ and $t$ here, but we could easily use the formula again with different values of $s$ and $t$. Since the values of $s$, $t$ and $d$ can vary and represent different numbers in different scenarios, they are known as *variables*. In general, any letter that can represent different numbers is called a **variable**.

So a **formula** is an equation in which one variable, called the **subject** of the formula, appears by itself on the left-hand side of the equation and only the other variables appear on the right-hand side. Thus a formula enables you to calculate the value of the subject when you know the values of the other variables. For example, $d = s \times t$ is a formula whose subject is $d$, because $d$ is the only variable on the left-hand side and you can use this equation to find $d$ if you know the values of $s$ and $t$.

However, note that the word 'formula' is used rather loosely in mathematics; for example, we sometimes say that

$$s \times t$$

is 'a formula for $d$'.

In many formulas the variables represent measurements, and it is important to check that the values you substitute are measured in appropriate units.

## Formulas with set units

Some formulas do not involve units. For example, you saw in Section 4 of Unit 1 that the formula for the $n$th square number is $n^2$. With some formulas the units are already set and cannot be changed. For example, an approximate formula to convert distances in miles to kilometres is $K = 1.6 \times M$, where $M$ is the distance in miles and $K$ is the distance in kilometres, and no other units can be used in this formula. Before you substitute in a formula like this, you must check that the information that you use is expressed in the correct units, and make any conversions. This process is shown in the next example.

### Example 7 Substituting a value into a formula

A European car hire company charges €50 per day for the hire of a small car, plus a booking fee of €20. (€ is the symbol for euros.) So, the total cost of hiring the car is given by the formula

$$T = 50 \times n + 20,$$

where $T$ is the total cost in € and $n$ is the number of days for which the car is hired.

How much does it cost to hire the car for 2 weeks?

### Solution

Check that the given information is in the correct units.

In the formula, the hire time $n$ is measured in *days*, so first convert 2 weeks into days.

There are 7 days in 1 week, so in 2 weeks there are $2 \times 7$ days $= 14$ days. Hence $n = 14$.

Substitute and do the calculation.

Substituting $n = 14$ into the formula gives

$$\begin{aligned} T &= 50 \times 14 + 20 \\ &= 700 + 20 \\ &= 720. \end{aligned}$$

State the conclusion, including the correct units.

Hence the cost of hiring the car for 2 weeks is €720.

Here is a similar type of substitution for you to try.

## Activity 12 Substituting a value into a formula

The length of material needed to make a cushion cover is given by

$$L = 3 \times w + 5,$$

where $L$ is the length of material in cm and $w$ is the width of the cushion in cm. What length of material is needed to make a cover for a cushion of width 0.4 m?

Once you have substituted numbers into a formula, you perform the calculation by using the usual rules of arithmetic. See Unit 1, Subsection 2.1. The mnemonic BIDMAS helps you to remember the order of operations:

> **B**rackets, then **I**ndices (powers and roots), then **D**ivisions and **M**ultiplications, then **A**dditions and **S**ubtractions.

Here is another example of using a formula.

## Example 8 Calculating the time to walk uphill

William Naismith was a Scottish climber who, in 1892, developed a rule for estimating walking times. *Naismith's Rule* estimates that the time taken for a walk up a hill is given by the formula

$$T = \frac{D}{5} + \frac{H}{600},$$

where

| | |
|---|---|
| $T$ | is the time for the walk in hours, |
| $D$ | is the horizontal distance walked in kilometres, |
| $H$ | is the height climbed in metres. |

Naismith's original rule has since been updated for the metric system. The rule is based on the assumptions that someone can walk at a speed of 5 km/h on flat ground and also needs to allow an extra minute to climb a height of 10 metres.

(a) Estimate how long a walk will take if the horizontal distance is 20 km and the height is 1200 m.

(b) Why might you need to allow longer than this estimate?

### Solution

(a) Check that the given information is in the correct units.

In this case, the horizontal distance is 20 km and the height climbed is 1200 m. The units here are those specified for the formula, so no conversion is needed.

Substitute and do the calculation.

Substituting $D = 20$ and $H = 1200$ into the formula gives

$$T = \frac{20}{5} + \frac{1200}{600} = 4 + 2 = 6.$$

State the conclusion, including the correct units.

Hence Naismith's Rule predicts a 6-hour walk.

(b) You may need to allow longer than 6 hours to accommodate rest breaks, or because the terrain is difficult, the walkers are unfit or the weather is bad.

Here is a similar substitution for you to try.

## Activity 13 Using Naismith's Rule

Use Naismith's Rule to estimate the time for a walk in which the horizontal distance is 5000 m and the height is 500 m.

In the next activity, the variable $m$ has no units, so the problem of converting units does not arise.

## Activity 14 Using formulas

(a) The mean is a type of average. You will meet the mean again in Unit 4. The *mean* $m$ of five numbers $a$, $b$, $c$, $d$ and $e$ is given by the formula

$$m = \frac{a+b+c+d+e}{5}.$$

In a hedgerow survey, the numbers of tree and shrub species in five 30-yard sections of a hedge were found to be 4, 5, 6, 4 and 4. What is the mean number of species in a 30-yard section?

(b) In 1974, Dr Max Hooper obtained data on trees and shrubs from 227 English hedges, whose ages he knew from written records. From these data, he derived a formula to estimate the age of a hedge. The age of an English hedge can be estimated by using *Hooper's Rule*:

$$A = 110 \times m + 30,$$

where $A$ is the age in years and $m$ is the mean number of tree and shrub species in a 30-yard section.

Use Hooper's Rule to estimate the age of the hedge in part (a) to the nearest hundred years.

## Formulas for which you can choose the units

The formula for the area of a rectangle is $A = l \times w$, where $A$ is the area, $l$ is the length and $w$ is the width. Here the units for measuring the rectangle have not been specified, so you can choose which units to use, as long as these units are consistent. So if you measure the length of the rectangle in centimetres, then the width should also be measured in centimetres, and the area in square centimetres; if you measure the length in kilometres, then the width should also be measured in kilometres, and the area in square kilometres.

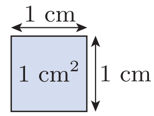



**Figure 9** A square whose sides are 1 cm long has an area of 1 square centimetre, or $1\,\text{cm}^2$

For example, if the length of a rectangle is 3 m and the width is 0.5 m, then the area is

$$3 \times 0.5\,\text{m}^2 = 1.5\,\text{m}^2.$$

If the units given are not consistent with each other, then you should convert the measurements into appropriate units before substituting them into the formula. For example, if the measurement for the width is given as 50 cm instead of 0.5 m, then you should convert this measurement into 0.5 m and proceed as before, or convert the length of 3 m into 300 cm and obtain the area in square centimetres.

Similarly, in the formula $d = s \times t$, mentioned earlier, the units should be consistent; if the units for the speed are km/h, then the units for the time are h and the units for the distance are km.

### Example 9 Substituting values into a formula

A car travels at an average speed of 95 km/h. Use the formula

$$d = s \times t$$

to find the distance the car travels in each of the following times.

(a) 2.5 hours (b) 40 minutes

Give your answers to two significant figures.

### Solution

(a) If the unit for speed is km/h and the unit for time is hours, then the distance is in km, so no conversions are required.

When $s = 95$ and $t = 2.5$,

$$d = 95 \times 2.5 = 237.5.$$

So the distance travelled is 240 km (to 2 s.f.).

(b) First, convert the given time into hours. Since

$$40\,\text{min} = \frac{40}{60}\,\text{h} = 0.666\ldots\,\text{h},$$

the time is 0.666... h.

When $s = 95$ and $t = 0.666\ldots$,

$$d = 95 \times 0.666\ldots = 63.33\ldots\ .$$

So the distance travelled is 63 km (to 2 s.f.).

Here are some problems of this type for you to try.

## Activity 15 Substituting values into a formula

(a) The volume $V$ of a rectangular box is given by

$$V = l \times w \times h,$$

where $l$ is the length, $w$ is the width and $h$ is the height of the box. What is the volume of a box that measures 1.5 m by 2 m by 75 cm?

(b) The average speed $s$ of a vehicle is given by the formula $s = d/t$, where $d$ is the distance travelled and $t$ is the time taken. What is the average speed of a coach that travels 80 km in 1 hour 15 minutes?

One way to help check the consistency of units is to substitute the values for the variables together with their units into the formula. For example, in the area formula $A = l \times w$, the calculation given earlier to find the area of a rectangle with length 3 m and width 50 cm could have been written as

$$\begin{aligned} A &= 3\,\text{m} \times 0.5\,\text{m} \\ &= (3 \times 0.5)\,\text{m}^2 \\ &= 1.5\,\text{m}^2, \end{aligned}$$

which shows that the answer is in square metres. If the width had been substituted as 50 cm instead of 0.5 m, then including the units would have alerted you to the problem, as shown in Figure 10 below.

$A = 3\text{m} \times 50\text{cm}$

$= 150\ \text{m} \times \text{cm}$ ✗

m x cm is <u>NOT</u>
an SI unit of measurement.
To measure this area
$\text{m}^2$ or $\text{cm}^2$ should be used.

**Figure 10** Incorrect use of units in a solution to the area problem

Similarly, the calculation in Example 9(a) could be written as

$$\begin{aligned} d &= 95\,\text{km/h} \times 2.5\,\text{h} \\ &= (95 \times 2.5)\,\text{km} \\ &= 237.5\,\text{km}. \end{aligned}$$

Here, dividing the unit km by the unit h and then multiplying it by h leaves the unit km unchanged.

## 3.2 Writing formulas concisely

Formulas containing a lot of mathematical symbols can look quite complicated. To make them more concise, multiplication signs are usually omitted.

For example,

the formula $d = s \times t$ is usually written as $d = st$;

the formula $A = 110 \times m + 30$ is usually written as $A = 110m + 30$.

However, when you substitute numerical values into a formula, you usually have to put the multiplication signs back in, to make the meaning clear. So $3 \times y$ can be written as $3y$, but $3 \times 2$ cannot be written as 32. You should try to write the multiplication sign $\times$ and the letter $x$ in different ways, so they don't get mixed up!

Another way to write some formulas is to use power notation. For example:

$w^2$ means $w \times w$ and is read as '$w$ squared';
$x^3$ means $x \times x \times x$ and is read as '$x$ cubed';
$y^4$ means $y \times y \times y \times y$ and is read as '$y$ to the power 4' or '$y$ to the 4';
and so on.

One way to check that you understand what a given formula means is to try describing in words how to *use* the formula.

For example, this is how to use Hooper's Rule, $A = 110m + 30$:

To find the value of $A$, multiply the value of $m$ by 110 and then add 30.

### Activity 16 Describing formulas

Describe in words how to use the following formulas, starting each description with: 'To find the value of ...'.

Then in each case work out the value of the subject when $a = 2$ and $b = 5$. (In these formulas, the variables do not represent any particular quantities and there are no units specified.)

(a) $Q = 4a - 5$ (b) $R = \dfrac{a}{3b}$ (c) $P = a^2 + b^2$

## Conventions for writing formulas

There are several conventions that are usually followed when writing formulas concisely.

- In products, numbers are usually written first; for example, the formula $K = 1.6 \times M$, or equivalently $K = M \times 1.6$, is written concisely as

  $$K = 1.6M.$$

  Similarly, $(2a + b) \times 3$ is written concisely as $3(2a + b)$. However, $(2a + b)c$ and $c(2a + b)$ are both acceptable ways of writing $(2a + b) \times c$.
- In products, letters are often written in alphabetical order; for example, $d = s \times t$, or equivalently $d = t \times s$, is usually written as

  $$d = st.$$

  In some formulas, however, there are reasons why the variables are not written in alphabetical order.
- Finally, divisions are usually written in the form of a fraction; for example, $s = d \div t$ is written as $s = \dfrac{d}{t}$ and read as '$s$ equals $d$ over $t$'. Another acceptable form is

  $$s = d/t.$$

Here are some examples for you to try.

### Activity 17 Writing formulas concisely

(a) Write each of the following formulas concisely.

(i) $M = v \times w$ (ii) $A = \frac{1}{2} \times b \times h$ (iii) $V = p \times r \times r \times h$

(b) Write the following formulas with the multiplication signs put back in.

(i) $C = 2pr$ (ii) $V = l^3$ (iii) $s = ut + \frac{1}{2}at^2$

(iv) $A = \frac{1}{2}h(a + b)$

(c) Rewrite the following formulas so that they follow the usual conventions.

(i) $P = (a + b)2$ (ii) $I = TRP \div 100$

Note that in MU123 texts, and other printed materials, units are printed in normal type, whereas variables are printed in italics. This helps to distinguish between, say, the distance 5 metres, which is printed as 5 m, and the expression $5 \times m$ (that is, 5 times the variable $m$), which is printed concisely as $5m$. When handwritten, these look identical and the meaning is obtained from the context. This is one reason why units are usually not included in mathematical calculations that involve variables.

The example below uses a formula written in concise form. There is a tutorial clip of this example on the website that you may like to watch.

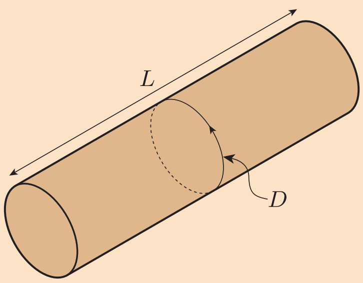
Tutorial clip

### Example 10 Estimating the volume of a log

Foresters can estimate the volume of a log of wood by using the formula

$$V = \frac{LD^2}{4\pi},$$

where $V$ is the volume of the log in cubic metres, $L$ is the length of the log in metres, $D$ is the distance around the middle of the log in metres, and $\pi$ is approximately 3.141 59. Your calculator should have a key for the number $\pi$. The symbol $\pi$ is the Greek letter pi, read as 'pie'.




Estimate the volume of a log that is 1.5 m long and 92 cm around the middle, giving your answer to two significant figures.

#### Solution

Check that the given information is in the correct units.

The length is 1.5 m, so $L = 1.5$. The distance around the middle is 92 cm, but the formula requires the measurement in metres. Since $92\,\text{cm} = (92 \div 100)\,\text{m} = 0.92\,\text{m}$, we have $D = 0.92$.

Substitute and do the calculation.

Substituting $L = 1.5$ and $D = 0.92$ into the formula $V = \dfrac{LD^2}{4\pi}$ gives

$$V = \frac{1.5 \times 0.92^2}{4 \times 3.141\,59\ldots} = \frac{1.2696}{12.566\ldots} = 0.101\ldots\ .$$

State the conclusion, including the correct units.

Hence the volume of the log is $0.10\,\text{m}^3$ (to 2 s.f.).

In the example above, the steps in the calculation were included so that you can see the order in which these steps are carried out. However, as the calculation for $V$ is done using your calculator, it is acceptable to write down the calculation step more briefly like this:

$$V = \frac{1.5 \times 0.92^2}{4\pi} = 0.10 \text{ (to 2 s.f.)}.$$

Several different calculator sequences can be used to calculate the final answer, and some of these sequences involve using the memory and other function keys on your calculator. The next activity explains these key sequences in more detail.

## Activity 18 Doing longer calculations using your calculator

Work through Subsection 3.4 of the MU123 Guide.

Now try substituting values in some more formulas.

## Activity 19 Using letter formulas

(a) The monthly cost of using a phone consists of a fixed monthly charge plus charges for daytime calls. The cost can be calculated using the formula

$$C = 20 + 0.25n,$$

where $C$ is the cost in £, and $n$ is the number of minutes of daytime calls during the month.

What is the phone bill if 94 minutes of daytime calls have been made during the month?

(b) A person's *body mass index*(BMI) is given by the formula

$$I = \frac{M}{H^2},$$

where $I$ is their BMI, $M$ is their mass in kilograms, and $H$ is their height in metres. Strictly, the units for BMI are $\text{kg/m}^2$. However, BMIs are usually quoted without units.

If $I$ is 25 or greater, then the person is classed as overweight.

A woman has a mass of 72 kg and a height of 164 cm. Is she overweight?

(c) If you've been on a long car journey with children, then you've probably heard the question: 'Are we nearly there yet?' The following formula has been suggested to estimate the time it will take before this question is asked:

$$T = \frac{1 + 15A}{0.25C^2},$$

where $T$ is the time in minutes, $A$ is the number of activities the children have, and $C$ is the number of children in the car. A formula like this was suggested in 2006 by Dwight Barkley, a mathematics professor at the University of Warwick, as a fun exercise for families to think about when going on holiday.

If there are 3 children in a car and they have 6 activities, then how long does the formula predict it will be before the question is asked?

## Substituting negative numbers

So far you have substituted only positive numbers into formulas.

When you replace a letter by a negative number, it is usually helpful to include the number in brackets to avoid confusion. Here is an example.

### Example 11 Substituting a negative number

Consider the formula $A = c^2 - 5c + 3$.

Find the value of $A$ when $c = -2$.

### Solution

Putting brackets around $-2$ and substituting it for $c$ gives

$$\begin{aligned} A &= (-2)^2 - 5(-2) + 3 \\ &= (-2) \times (-2) - (-10) + 3 \\ &= 4 + 10 + 3 \\ &= 17. \end{aligned}$$

In the solution to Example 11, the number $-2$ has been enclosed in brackets when it is substituted to ensure that the minus sign is not separated from the 2 by mistake. This makes it clear that $(-2)^2$ has to be evaluated as follows:

$$(-2)^2 = (-2) \times (-2) = 4.$$

If the brackets had been omitted here, then this part of the calculation might have been done *incorrectly* as $-2^2 = -2 \times 2 = -(2 \times 2) = -4$.

In the second occurrence of $-2$ in this solution,

$$-5(-2) \quad \text{was replaced by} \quad -(-10) \quad \text{and then by} \quad +10.$$

To understand these steps, first remember that

$$5(-2) = 5 \times (-2) = -10.$$

(See Unit 1, Subsection 3.1, for the rules for multiplying negative numbers.)

Then remember that in the calculation above $-(-10)$ means 'subtract minus 10', and subtracting the negative number $-10$ is the same as adding the corresponding positive number 10. So in this calculation, $-(-10)$ is the same as $+10$.

However, if the variable being substituted appears first in the calculation on its own, then no brackets are required. For example, if $A = C + 3$ and $C = -2$, then $A = -2 + 3 = 1$.

Sometimes when you are substituting into a formula you have to find the negative of a number. This is the number that is produced by putting a minus sign in front of the number. For example, consider the formula $y = -x$, which means 'to find $y$, take the negative of $x$'. If $x = 3$, for example, then $y = -3$. But what happens when $x$ is a negative number?

For example, substituting $x = -2$ into the formula gives $y = -(-2)$.

So what does $-(-2)$ mean?

A negative number can be thought of as the result of a subtraction from zero. For example, $-3 = 0 - 3$. In the same way, $-(-2)$ can be thought of as the subtraction $0 - (-2)$, which is 2. So replacing $-(-2)$ by 2 gives $y = 2$.

In the same way,

$$-(-3) = 3, \quad -(-20) = 20, \quad -(-10.5) = 10.5, \quad -(-\tfrac{3}{5}) = \tfrac{3}{5}.$$

In general, a negative sign in front of a number changes its sign.

## Activity 20 Substituting a negative number

(a) Consider the formula $G = 6 + a - a^2$. Find the value of $G$ when $a = -4$.

(b) Consider the formula $y = -x + 4$. Find the value of $y$ when $x = -9$.

(c) Consider the formula $v = \dfrac{u-2}{1.2}$. Find the value of $v$ when $u = -1$.

(d) To convert a temperature from Celsius to Fahrenheit, you can use the formula

$$f = 1.8c + 32,$$

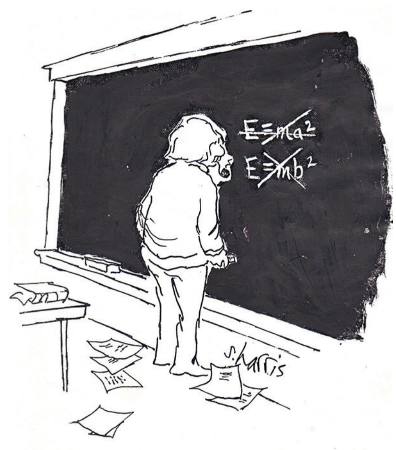
where $f$ is the temperature in degrees Fahrenheit, and $c$ is the temperature in degrees Celsius.

Use this formula to work out the Fahrenheit equivalent of $-10°$ C.

### Gabriel Daniel Fahrenheit

Gabriel Daniel Fahrenheit (1686–1736) was the inventor of the mercury thermometer. For the zero point of his temperature scale, he used the lowest measurable temperature that he could reach in his laboratory. He did this so that no everyday temperature would have a negative value.

## 3.3 Constructing your own formulas




There are many well-known formulas that you can use to solve problems, but sometimes you need to find your own formula. In this subsection you will see how to construct some formulas, and there are further examples throughout the module. You can construct a formula by following the three steps below.

- *First, identify the subject of the formula and the other variables, and their units of measurement.*

  This means that you have to decide the purpose of the formula and what the formula depends on.

  For example, suppose that you want to find a formula for the length of a return journey from one place to another ($A$ to $B$), in terms of the distance between the two places.

  Let's call the length of the return journey $R$, and the distance between the two points $d$, as illustrated in Figure 11. Both $R$ and $d$ are measured in kilometres. You can choose any letters, but formulas are often easier to remember if the letters remind you of the quantities. Also, it is a good idea not to pick letters such as $o$ and $l$ that could be confused with other symbols such as 0 and 1.

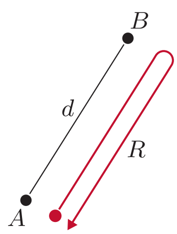



**Figure 11** Identifying the subject and variables of a formula

- *Next, find the relationship between the variables.*

  Here, you need to think about how to work out $R$ from $d$.

  The length of the return journey $R$ is twice the distance between the two places, that is, two lots of $d$, which can be written as $2d$.

  So the formula is $R = 2d$.

- *Finally, write down all the details of the formula.*

  > The formula is $R = 2d$, where $R$ is the length of the return journey in km, and $d$ is the distance between the two points in km.

  Alternatively, and more concisely:

  > The length of the return journey $R$ km is given in terms of the distance $d$ km by the formula $R = 2d$.

Note that you should never include units in a formula. For example, it would be incorrect to write the formula as $R = 2d\,\text{km}$. However, when you use a formula, you need to include the units in your conclusion.

Here is a slightly more complicated example.

### Example 12 Finding a formula – driving to work

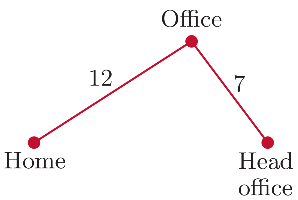
(a) During a working week, Anya drives from home to her office and back five times, and she also makes a number of trips from her office to head office and back. Her office is 12 miles from her home and 7 miles from the head office.

Find a formula for $d$, where $d$ is the total number of miles that Anya drives in a week when she makes $n$ trips to head office.

(b) Use your formula to find the distance driven by Anya in a week when she makes 3 trips to head office.

### Solution

(a) Draw a diagram if it helps you to understand the situation. Then tackle the problem step by step, by considering separately the return journeys from her home to the office and from the office to the head office.




The distance in miles that Anya drives from her home to her office and back is $12 \times 2 = 24$. So in 5 days, the distance in miles that she drives from her home to her office and back is $5 \times 24 = 120$.

The length of the return journey from the office to the head office is $2 \times 7$ miles $= 14$ miles. So the distance she drives to the head office and back in $n$ trips is $n$ lots of 14, that is, $14n$ miles.

The total distance in miles that she drives is the sum of the total distance she travels between home and the office, and the total distance between head office and the office.

Hence a formula for $d$ is

$$d = 120 + 14n,$$

where $d$ is the distance in miles, and $n$ is the number of trips from the office to the head office.

Remember that the unit 'miles' is *not* included in the formula.

(b) Substituting $n = 3$ into the formula in part (a) gives

$$d = 120 + 14 \times 3 = 120 + 42 = 162.$$

So the distance is 162 miles.

Include the unit 'miles' in the conclusion.

You can try finding some formulas yourself in the next activity. If you find the relationship between the variables hard to spot, then try some particular numbers first as that might help you to identify the operations involved. For example, in Example 12 you could have asked yourself how to calculate the total distance if the number of trips to head office is 1, 2, 5, and so on. Looking at some particular numbers often helps you to get a feel for a problem, as you saw in Section 4 of Unit 1.

Then think about which parts of these calculations stay the same and which change. That might help you to discover that the length of the return journey from home to the office is always 120 miles and that this distance always needs to be added to the distance for the trips to the head office and back. Drawing a diagram might help too.

## Activity 21 Finding formulas

(a) Write down a formula for the total distance, $d$ km, travelled on a journey if $m$ km are travelled on the motorway and $p$ km are travelled on principal roads.

(b) A car can travel 15 km on 1 litre of fuel. Write down a formula for the distance $D$ km the car can travel on $f$ litres of fuel.

(c) A car-hire business has 60 cars, and $r$ cars have been rented out. Write down a formula for the number of cars, $N$, that are still available to hire.

(d) To estimate the time of a journey through a town, a mathematical model is modified by adding an allowance for the time spent at junctions. From a survey, it is found that allowing 2 minutes extra for each junction is a realistic adjustment. Write down a formula for $T$, the *extra* time in minutes needed for a journey that goes through $J$ junctions in the town.

The next example shows how checking some particular numbers can help you to spot a pattern that leads to a formula.

Tutorial clip

### Example 13 Finding a formula – the car ferry

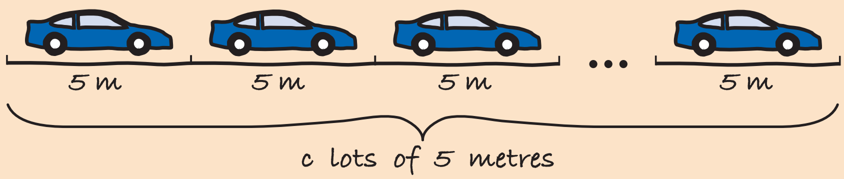
A car ferry can transport both cars and vans. A van requires a space of 9 m, and a car requires a space of 5 m. Find a formula for the length $L$ required for $c$ cars and $v$ vans.

### Solution

Consider the space needed for the cars first, and try some particular numbers to start with.

1 car needs a space of 5 m, so 2 cars need $2 \times 5\,\mathrm{m} = 10\,\mathrm{m}$, 3 cars need $3 \times 5\,\mathrm{m} = 15\,\mathrm{m}$, and so on.

So to find the space needed for $c$ cars, $c$ lots of 5 metres are needed, that is, a distance of $5c$ metres.




Similarly for the vans.

1 van needs a space of 9 m, so 2 vans need $2 \times 9\,\mathrm{m}$, 3 vans need $3 \times 9\,\mathrm{m}$, and so on.

So the space needed for $v$ vans is $v \times 9$ metres, that is, $9v$ metres.

To find the total length, add the distance for the cars and the distance for the vans together.

So a formula for $L$ is $L = 5c + 9v$, where $L$ is the total length in metres, $c$ is the number of cars, and $v$ is the number of vans.

(As a check, work out the length for, say, 2 cars and 3 vans without using the formula:

$$\text{the length is } 2 \times 5\,\mathrm{m} + 3 \times 9\,\mathrm{m} = 10\,\mathrm{m} + 27\,\mathrm{m} = 37\,\mathrm{m}.$$

If you substitute $c = 2$ and $v = 3$ into the formula, then you obtain the same answer. If you get a different answer at this stage, then you should go back and check the steps that you used to develop the formula.)

The following strategy box summarises the key points for writing down formulas.

### Strategy Finding formulas

(a) Identify the subject of the formula and the other variables, and their units of measurement. If possible, choose letters for the variables that remind you of the context.

(b) Find the relationship between the variables.

(c) Write down all the details of the formula, defining the variables and stating their units (if appropriate).

(It is a good idea to try particular numbers first to suggest what the formula is like, and then to check that your formula works.)

Here are some formulas for you to find.

### Activity 22 Finding more formulas

(a) A theatre charges £25 for each adult ticket and £15 for each child ticket, plus an additional booking fee of £5 for each order.

  (i) Write down a formula for the total cost $C$ in £, if $a$ adult tickets are bought in one order.

  (ii) Write down a formula for the total cost $T$ in £, if $a$ adult tickets and $c$ child tickets are bought in one order.

(b) A children's after-school club charges a registration fee of £20 and £8 per session. Find a formula for $P$, where £$P$ is the cost of a child attending $n$ sessions at the club.

(c) A furniture manufacturer requires 3 metres of fabric to cover each chair and uses a roll of fabric containing $r$ metres to cover $n$ chairs. Find a formula for $s$, where $s$ metres is the length of fabric left on the roll.

The next activity asks you to spend a few moments thinking about what you have learned in this section and planning what you should do next.

### Activity 23 Ready to move on?

This section has introduced you to the ideas in the table on the next page. For each idea, spend a few moments reviewing how you got on by asking yourself the following four questions.

- Did you understand the explanation in the text and any worked examples?
- Did you manage to complete the activities successfully?
- Do you feel confident with this idea?
- Have you been able to complete the associated assignment questions?

If your answer to any of these questions is 'No', then plan what you are going to do in order to sort out your difficulties, bearing in mind your time commitments. For example, the quickest way of sorting out a negative answer to the first question is probably to contact your tutor, if you have not already done so. Make a note of the examples, sections of the text or activities that you don't understand so that you can mention these to your tutor. If you have understood the text and managed to do the activities, but still don't feel confident, then you may like to plan some time to work through the practice quizzes on the website, or use some other resources. Or you may decide to discuss the ideas with other people (perhaps in a tutorial or forum) or watch the tutorial clips again.

| Topic | Understood topic | Completed activities | Confident with idea | Completed assignment | What to do next |
|---|---|---|---|---|---|
| Substituting in formulas | | | | | |
| Writing formulas concisely | | | | | |
| Using a letter formula | | | | | |
| Units in formulas | | | | | |
| Constructing formulas | | | | | |

There are no comments on this activity.

# 4 Inequalities

In Section 3, you considered variables and formulas. This section considers some notation that can be used for describing the range of possible values that a variable can take. This notation is useful both when setting up a model, in order to describe known restrictions on the variables, and when stating your conclusions. For example, the restrictions on a variable that represents the speed of a car might be that it is greater than or equal to zero and less than or equal to the speed limit that applies.

In this situation, when there is a particular number that provides a restriction, or limitation, on the value of a variable, we call the number a **limit**. This section introduces a shorthand way of describing such restrictions.

Stating your conclusions to a problem may also involve comparing your answer with a particular number and seeing whether it is greater than or less than that number. For example, in Activity 11 the time model gave a larger gap than the distance model for speeds *less than* 18 m/s, and in Activity 19 you calculated the body mass index and then checked to see if it was 25 or *greater* in order to determine if the person was overweight.

## 4.1 Notation for working with inequalities

If two numbers are not equal, then there is an *inequality* between them. The nature of this inequality can be expressed by using the phrases 'less than' and 'greater than' or the inequality signs $<$ and $>$.

If a number lies to the left of another number on the number line, then it is said to be **less than** the other number. For example, $-5$ lies to the left of $-2$, as shown in Figure 12, so $-5$ is less than $-2$. This statement can be written more concisely by using the inequality sign $<$ for 'less than':

$$-5 < -2.$$

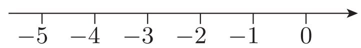



**Figure 12** Part of the number line

It is read as 'minus five is less than minus two'.

If a number lies to the right of another number on the number line, then it is said to be **greater than** the other number. For example, $-1$ lies to the right of $-3$, so $-1$ is greater than $-3$. This statement can be written using the inequality sign $>$ for 'greater than':

$$-1 > -3.$$

It is read as 'minus one is greater than minus three'. The inequality sign always points towards the smaller of the two numbers; for example, $2 < 3$ and $4 > 3$.

Any statement involving inequality signs is called an **inequality**. Each inequality can be written in two different ways. For example, 4 is greater than 2, so you can write

$$4 > 2,$$

but also 2 is less than 4, so you can write

$$2 < 4.$$

Each way of writing an inequality is obtained from the other by swapping the numbers and reversing the inequality sign. This is called reversing the inequality.

As well as the two inequality signs introduced above, there are two other inequality signs, $\leq$ and $\geq$. The four **inequality signs** and their meanings are given in the following box.

### Inequality signs

$<$ is less than

$\leq$ is less than or equal to

$>$ is greater than

$\geq$ is greater than or equal to

The alternative notations

$$\leqslant \quad \text{for} \quad \leq$$

and

$$\geqslant \quad \text{for} \quad \geq$$

are also used.

Inequalities using the signs $<$ and $>$ are often called **strict inequalities** since they do not allow equality.

Here are some examples of correct inequalities:

- $1 < 1.5$, because 1 is less than 1.5.
- $1 \leq 1.5$, because 1 is less than or equal to 1.5 (it is 'less than' 1.5).
- $1 \leq 1$, because 1 is less than or equal to 1 (it is 'equal to' 1).

It may seem strange to write $1 \leq 1.5$ and $1 \leq 1$, when the more precise statements $1 < 1.5$ and $1 = 1$ can be made, and you would not usually write the former statements. The inequality signs $\leq$ and $\geq$ are useful, however, for specifying the range of values that a variable can take, as in the following example.

### Example 14 Specifying the range of a variable

Suppose that the speed of a car on a UK motorway is $s$ km/h. Write down two inequalities that specify the range of possible legal values of $s$.

### Solution

First, decide what you want to say in words.

The speed must be greater than or equal to zero and should be less than or equal to the speed limit on a UK motorway, that is, 112 km/h (70 mph).

Replace the words by the appropriate inequalities.

So the two inequalities are

$$s \geq 0 \quad \text{and} \quad s \leq 112.$$

Most inequalities that you will meet involve variables. A value of the variable for which the inequality is true is said to **satisfy** the inequality. For example, the number 100 satisfies both the inequalities in Example 14.

## 4.2 Illustrating inequalities on a number line

An inequality can be represented on a number line by marking the section of the number line where the inequality is true. A section of the number line without any gaps is known as an **interval**. For example, the straight line above the number line in Figure 13 shows the interval consisting of all the numbers less than or equal to 112, so it illustrates the inequality $s \leq 112$. The small solid circle at the limit 112 indicates that 112 is contained in the interval and is a possible value for $s$. The diagram shows that the possible values for $s$ lie to the left of or exactly on 112.

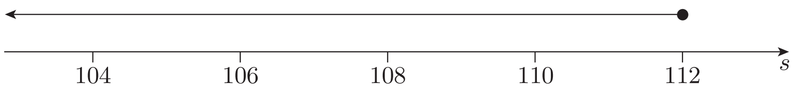



**Figure 13** The interval where $s \leq 112$

A strict inequality can be represented on a number line by using a small empty circle at the limit. For example, Figure 14 illustrates the strict inequality $u > 4$. The possible values for $u$ lie to the right of 4.

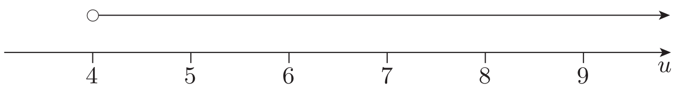



**Figure 14** The inequality $u > 4$

### Activity 24 Using inequality signs

(a) Put the correct sign ($<$ or $>$) in each of the boxes below.

(i) $12 \square 3$ (ii) $-5 \square 3$ (iii) $-2.5 \square -4.5$

(b) Use number lines to represent each of the following inequalities.

(i) $a \geq -3$ (ii) $b < 6$ (iii) $c \leq -2.5$

(c) Reverse each of the inequalities in parts (a) and (b).

Sometimes, two inequalities can be combined to make a **double inequality**, as the next example shows.

Tutorial clip

## Example 15 Using a double inequality

A child who is 5 years or older but not yet 16 is eligible for a child fare on the train. Children under 5 travel free. Suppose that $a$ represents the age of a child in years.

(a) Draw a number line to illustrate the ages eligible for a child fare.

(b) Give a double inequality to describe the age restriction for child fares. Which whole numbers satisfy this inequality?

### Solution

(a) Mark the limits at 5 and 16 on the number line first, and then join the limits with a line.

The ages of children who are eligible for a child fare are shown on the number line in Figure 15.

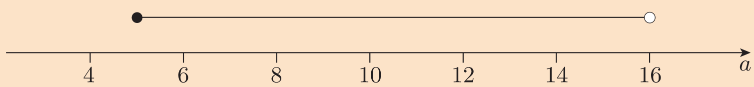



**Figure 15** Child fares

(b) The restrictions on the age for a child fare are

$a$ is greater than or equal to 5, and $a$ is less than 16.

Using inequality signs,

$$a \geq 5 \quad \text{and} \quad a < 16.$$

Now, $a \geq 5$ can be written as $5 \leq a$.

Therefore the inequalities are

$$5 \leq a \quad \text{and} \quad a < 16.$$

These two inequalities can be combined as the double inequality

$$5 \leq a < 16.$$

The whole numbers that satisfy this inequality are

$$5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15.$$

The double inequality $5 \leq a < 16$ is read as

'5 is less than or equal to $a$, which is less than 16',

or as

'$a$ is greater than or equal to 5, and less than 16'.

Here are some similar examples for you to try.

## Activity 25 Restricting variables

A person's body mass index is denoted by the variable $I$. The person is classed as

| | |
|---|---|
| underweight | if $I$ is less than 18.5, |
| healthy weight | if $I$ is 18.5 or more, but less than 25, |
| overweight | if $I$ is 25 or more, but less than 30, |
| obese | if $I$ is 30 or more. |

Express each of these four statements as an inequality (single or double), and illustrate each of them as an interval on a number line.

Inequalities can also be used to illustrate the range of possible numbers that round to a particular value.

The number line in Figure 16 shows the values that round to 7.5 when rounded to one decimal place.

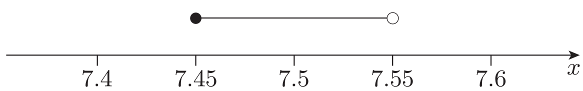



**Figure 16** Numbers that round to 7.5

These are the numbers that lie between 7.45 and 7.55, including 7.45 but not 7.55. So the double inequality that gives these numbers is

$$7.45 \leq x < 7.55.$$

## Activity 26 Describing ranges of numbers

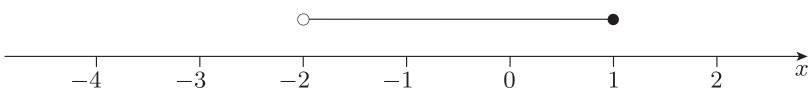
(a) Illustrate each of the following double inequalities on a number line.

(i) $-2 \leq c \leq 2$ (ii) $-1 < b < 6$ (iii) $-4 < x \leq -1$

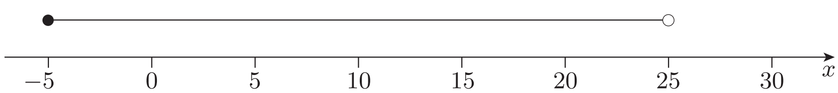
(b) Suppose that a variable $N$ can take any value that is a positive whole number. What values of $N$ satisfy the inequality $2 < N \leq 6$?

(c) Write down inequalities to describe the following intervals.

(i)




(ii)




(d) On a number line, show the numbers $x$ that round to 6 when rounded to the nearest whole number. Write down a double inequality that describes these numbers.

This section should have helped you to develop your skills in using inequalities to describe restrictions on variables. You will meet inequalities again in Unit 7.

# 5 Improving your mathematics

This final section considers several aspects of studying mathematics that you may find helpful. The section is quite short and contains some ideas that you may like to consider throughout your study. Subsection 5.1 looks back at the ways in which you have been solving problems, and suggests a few general strategies that you can use for any mathematical question and that you may find helpful in the future. Subsection 5.2 summarises some advice from students on how to read mathematics, and finally Subsection 5.3 suggests how you can use the feedback you receive on your assignments to improve your work. You may like to come back to this section when you receive your first marked iCMA and TMA.

## 5.1 Some problem-solving strategies

In this unit you have seen how some everyday problems can be investigated by using mathematics. One way of tackling such real-life problems is to use the modelling cycle. In fact, the steps in the modelling cycle are fairly similar to those that you use when you tackle any mathematics problem, whether it is a practical problem or something more abstract, as summarised in the box below.

### Tips for tackling mathematics problems

- Check you understand the problem – and if you are not sure, talk to people (your tutor, fellow students, friends) until you do.
- Collect information that will help you to solve it – this may be data, but it can also include techniques that you have used before that may help in this case too. What do you want to find out, and what do you know already?
- Simplify the problem if you can – this may include trying some numerical examples first or breaking the problem down into smaller achievable steps. You used some numerical examples in Unit 1 when investigating the sums of odd numbers and in Section 3 of this unit when constructing your own formulas.
- Carry out the mathematics. Remember, there are often several different ways of tackling a problem, including numerically and graphically, which may give different insights. Drawing a diagram often helps too.
- Check that your answers are reasonable and rounded appropriately.

Drawing a diagram is a good way of obtaining a different view of a problem. In this unit, you used diagrams to help with formulas and inequalities. Diagrams can also be used as part of your notes, to help you to connect different ideas together and to remember key ideas for later problems.

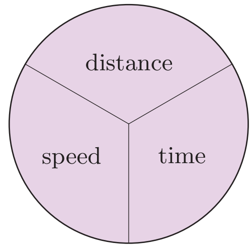



**Figure 17** Remembering formulas

Figure 17 shows an example of a diagram that is very useful for remembering the following formulas:

$$\text{speed} = \frac{\text{distance}}{\text{time}}, \quad \text{time} = \frac{\text{distance}}{\text{speed}}, \quad \text{distance} = \text{speed} \times \text{time}.$$

You may already be able to remember these formulas – if not, using the diagram in Figure 17 might help.

To use this diagram to give the right formula, you should decide which quantity you need on the left-hand side of the formula, and cover up that quantity. Then look at the position of the remaining two quantities. For example, if you cover up 'speed', then you are left with 'distance' over 'time'. The other two formulas work in a similar way.

## 5.2 Reading mathematics

Reading mathematics is a different skill from more general reading because you need to concentrate on each word and symbol, learning and using new vocabulary and notation as you go. This means that it takes longer, and you may sometimes feel stuck if you cannot see how to get from one line to the next. For difficult sections, you may find it helpful to annotate the text by including some extra working to explain the steps or notation.

The cartoons below offer advice on how to cope with some of these difficulties.


## 5.3 Using feedback

Throughout Units 1 and 2 you have been encouraged to assess your progress and to try some of the practice quizzes. So you probably have a fairly good idea of how you are getting on with the module, and you may already be making changes to improve your studying.

Doing the assignments will help you to consolidate the main ideas and provide further evidence of your progress. If you have not already done so, then it's a good idea to try the iCMA and TMA questions for this unit before starting the next unit.

You will receive marks and feedback on the iCMAs and TMAs that you submit. This subsection outlines the kind of feedback you will receive and how to use it. When you receive your feedback, it may be tempting to just look at your overall score and then carry on with the work you are currently studying. However, to gain full benefit from each assignment, you should spend some time, say 20–30 minutes, looking in detail at the feedback you receive. This feedback may suggest how you can improve your solutions, which would help you with later assignments and other parts of the module.

You may find that it is more productive to consider the feedback some time later. Immediately after receiving your score, you may be feeling thrilled with your score or disappointed by it, or even frustrated by any mistakes you made!

## iCMA feedback

The feedback on the iCMAs will contain some notes on correct solutions and references to the parts of units where you can find further information or examples. For each question that you answered incorrectly, you should study the solution provided and check that you understand it, particularly where it differs from your solution. Can you see why your answer is incorrect? You may like to try a similar example from the practice quizzes to make sure that you understand the ideas, or if you are still having difficulties, contact your tutor for help. When you have worked through the feedback, spend a few moments thinking about whether you need to make any changes to the way you tackle iCMA questions in the future.

- Do you need to try more practice quiz questions before attempting each iCMA?
- Do you need to allow yourself more time to complete each iCMA?
- Would it be better to tackle iCMA questions as you work through each section of a unit or to work on them only after you have completed each unit?

## TMA feedback

The feedback that your tutor provides on your TMAs will be tailored to your solutions, and it may include praise on work you have done well or suggestions for alternative techniques that you may find helpful in the future, as well as constructive comments to help you improve your mathematics and the way you present it. The form accompanying your TMA (called the PT3 form) will highlight the main points that you should try to address before you submit your next TMA. It's a good idea to make use of this general advice straight away as you work on the units, so that these ideas will be familiar to you when you tackle the next TMA. For example, your tutor may have commented on how you have explained your solutions, and you can practise improving your skills in this area as you work on the activities in the units.

More detailed comments on your solutions may be provided on your script. You should check through these comments carefully, making sure that you understand them and that you would be able to tackle a similar question successfully in the future, if required to do so. Do you understand why you have lost marks (if any)? If there are any comments or lost marks that you do not understand, contact your tutor.

It is worth making a note of the main points that you need to remember for the next TMA. If you have printed out the assignments, then you may like to write these comments directly on the next TMA. Alternatively, you could highlight the relevant statements on the TMA form or make some more formal notes. It is important to find a helpful way of recording these comments so that you can refer to them easily when you tackle the next TMA.

### Activity 27 Improving your mathematics

This final activity asks you to spend a few minutes considering any changes that you intend to make to the way you study MU123. (You may like to return to part (b) when you have received the feedback on your TMA.)

(a) Subsection 5.2 gave some advice on reading mathematics. Which parts of this advice have you tried already? Which parts do you intend to try?

(b) Based on your work on the assignments and the feedback that you have received, what changes do you intend to make in the preparation of your next assignments? Even relatively simple things such as allowing a bit more time to check through your assignment when you have finished the questions may help to improve your work.

(c) Do you need to make any changes to the amount of time you have for studying or your study sessions?

There are no comments on this activity.

# Learning outcomes

After studying this unit, you should be able to:

- use a map scale to estimate distances
- use formulas relating speed, distance and time
- understand how the modelling cycle can be used to solve problems
- appreciate that mathematical models simplify reality, take account of some features, and ignore others
- appreciate that mathematical ideas can be communicated in different ways (for example, numerically or graphically) and the best way to communicate them may depend on the intended audience
- draw and interpret graphs
- use formulas and find your own formulas to describe simple situations
- understand some conventions for writing formulas
- use inequality signs to describe limits and intervals
- review your studying and make changes to improve it.

# Solutions and comments on Activities

## Solution to Activity 1

**(a)** The route planners may have used different routes or made different assumptions for the speed of the car, the traffic conditions at different times of day or the starting and finishing points of the journey.

**(b)** The car will travel 50 km in the first hour and then another 50 km in the next hour, making 100 km overall. So the journey will take about 2 hours. This estimate is probably not good enough as it takes no account of the actual distances along the roads between Great Malvern and Milton Keynes or the speeds at which cars tend to travel on different types of roads.

## Solution to Activity 2

**(a)** Since 1 cm represents 10 km, the distance along the whole route is

$$(13.8 \times 10)\,\text{km} = 138\,\text{km},$$

and the distance along the motorway is

$$(3 \times 10)\,\text{km} = 30\,\text{km}.$$

**(b)** The total distance along the route is 138 km, and the distance along the motorway is 30 km. So the distance along the principal roads is

$$(138 - 30)\,\text{km} = 108\,\text{km}.$$

**(c)** Since 1 cm represents 10 km, the map distance is

$$(25 \div 10)\,\text{cm} = 2.5\,\text{cm}.$$

## Solution to Activity 3

**(a)** Here, 1 cm on the map represents 10 km on the ground. Now,

$$\begin{aligned}10\,\text{km} &= (10 \times 1000)\,\text{m}\\ &= (10\,000 \times 100)\,\text{cm}\\ &= 1\,000\,000\,\text{cm}.\end{aligned}$$

Thus the map scale is 1 : 1 000 000, so the scale factor is 1 000 000.

**(b)** Here, 1 cm on the map represents 500 000 cm on the ground. Now,

$$\begin{aligned}500\,000\,\text{cm} &= (500\,000 \div 100)\,\text{m}\\ &= (5000 \div 1000)\,\text{km}\\ &= 5\,\text{km}.\end{aligned}$$

So the map scale is '1 cm represents 5 km'.

## Solution to Activity 4

**(a)** The map scale is 1 : 50 000. So a map distance of 3.4 cm represents a ground distance of $3.4 \times 50\,000\,\text{cm} = 170\,000\,\text{cm}$. Now,

$$\begin{aligned}170\,000\,\text{cm} &= (170\,000 \div 100)\,\text{m}\\ &= 1700\,\text{m}\\ &= (1700 \div 1000)\,\text{km}\\ &= 1.7\,\text{km}.\end{aligned}$$

So the distance between the two towns is 1.7 km.

**(b)** The map scale is 1 : 50 000. So a ground distance of 7.85 km is represented by a map distance of $7.85 \div 50\,000$ km. Now,

$$\begin{aligned}(7.85 \div 50\,000)\,\text{km} &= 0.000\,157\,\text{km}\\ &= (0.000\,157 \times 1000 \times 100)\,\text{cm}\\ &= 15.7\,\text{cm}.\end{aligned}$$

So the map distance is 15.7 cm.

## Solution to Activity 5

**(a)** By the formula, the average speed is

$$\frac{425}{4}\,\text{km/h} = 106.25\,\text{km/h} = 110\,\text{km/h (to 2 s.f.)}.$$

**(b)** First method

The journey takes $(60 + 25)$ minutes = 85 minutes.

So, by the formula, the average speed is

$\frac{30}{85}$ km/min.

Now we convert this answer to km/h. The distance covered in 1 minute is $\frac{30}{85}$ km, so the distance covered in 1 hour is $60 \times \frac{30}{85}$ km.

So the average speed is

$$\frac{60 \times 30}{85}\,\mathrm{km/h} = 21.17\ldots\,\mathrm{km/h}$$
$$= 21\,\mathrm{km/h}\ (\text{to 2 s.f.}).$$

(Note that keeping the average speed as a fraction here avoids dealing with an awkward decimal.)

Second method

The time 85 minutes is $85/60$ hours $= 1.416\,66\ldots$ hours. So, by the formula, the average speed is

$$\frac{30}{1.416\,66\ldots}\,\mathrm{km/h} = 21.17\ldots\,\mathrm{km/h}$$
$$= 21\,\mathrm{km/h}\ (\text{to 2 s.f.}).$$

**(c)** By the formula, the average speed is

$$\frac{100}{14}\,\mathrm{m/s} = 7.14\ldots\,\mathrm{m/s} = 7.1\,\mathrm{m/s}\ (\text{to 2 s.f.}).$$

## Solution to Activity 6

**(a)** First method

The average speed is 50 km/h, so 1 km is travelled in 1/50 h.

Hence, to travel 108 km, it takes $108/50\,\mathrm{h} = 2.16\,\mathrm{h}$. Since there are 60 minutes in 1 hour,

$$0.16\,\mathrm{h} = (0.16 \times 60)\,\mathrm{min} = 9.6\,\mathrm{min} \approx 10\,\mathrm{min}.$$

So the journey takes 2 h 10 min to the nearest minute.

Second method

By the formula, the time is

$$\frac{108}{50}\,\mathrm{h} = 2.16\,\mathrm{h},$$

that is, 2 h 10 min to the nearest minute, as before.

Which method you prefer is a personal choice – what is important is finding a method that you can use easily, quickly and without making mistakes.

**(b)** The time for the motorway section was calculated in Example 4 as 0.3 h, so the total journey time is

$$2.16\,\mathrm{h} + 0.3\,\mathrm{h} = 2.46\,\mathrm{h} \approx 2\,\mathrm{h}\ 28\,\mathrm{min},$$

that is, 2 h 30 min to the nearest 10 minutes.

## Solution to Activity 7

| **Motorway (30 km)** | | | |
|---|---|---|---|
| Speed in km/h | 80 | 100 | 112 |
| Time taken in minutes | 23 | 18 | 16 |
| **Principal roads (108 km)** | | | |
| Speed in km/h | 40 | 50 | 96 |
| Time taken in minutes | 162 | 130 | 68 |

**(a)** The shortest journey times for the motorway and principal roads sections are approximately 16 minutes and 68 minutes, respectively, giving a total time of approximately 1 hour 24 minutes. This time cannot be achieved, since it is necessary to slow down or even stop at times during the journey, for example at junctions.

**(b)** If the average speed on the motorway drops from 100 km/h to 80 km/h, then the time increases by 5 minutes. So there is not much change in the overall time. This is because the distance on the motorway is quite short. If the average speed on the principal roads drops from 50 km/h to 40 km/h, then the time increases by 32 minutes.

**(c)** The journey time is quite sensitive to changes in the average speed, so it would probably be wise to allow some extra time for the journey.

## Solution to Activity 8

**(a)** The information given in the 'typical stopping distances' chart shows the effect of the thinking and braking distances, and provides detailed information on these distances, both in metres and as car lengths, to make it easier to visualise. However, it may be difficult to estimate the distances on the road, especially at higher speeds. By comparison, the 'two-second rule' is easier to remember and to use. Including both models gives users a choice of ways to check the gap.

**(b)** Picturing the distance in car lengths should make the distances easier to visualise. Also, splitting the stopping distance into thinking and braking distances indicates that as the speed increases, the braking distance increases rapidly –

roughly, it quadruples each time the speed is doubled. Displaying the total distance as a chart also emphasises the lengths of the gaps that should be left between cars. This is not so apparent from the two-second rule. The chart is clear, but remembering the information it contains may be difficult.

**(c)** An alternative approach is to paint chevrons on the road and advise drivers to keep at least two chevrons apart; this has been tried on sections of some UK motorways. However, this method doesn't take account of different road conditions or types of vehicle.

**(d)** At 40 mph, you travel 40 miles in 1 hour, that is, a distance of

$$40 \times 1.609\,\text{km} = 64.36\,\text{km}.$$

So 40 mph is 64 km/h to the nearest integer.

## Solution to Activity 9

**(a)** As in Example 5, the speed 80 km/h is

$$\frac{80 \times 1000}{60 \times 60}\,\text{m/s} = \frac{80\,000}{3600}\,\text{m/s}$$
$$= 22.22\,\text{m/s (to 2 d.p.)}.$$

**(b)** In each case, the length in metres of the gap in the time model is twice the speed in m/s.

| Speed in km/h | Speed in m/s | Time model gap in m | Distance model gap in m |
|---|---|---|---|
| 32 | 8.89 | 18 | 12 |
| 48 | 13.33 | 27 | 23 |
| 64 | 17.78 | 36 | 36 |
| 80 | 22.22 | 44 | 53 |
| 96 | 26.67 | 53 | 73 |
| 112 | 31.11 | 62 | 96 |

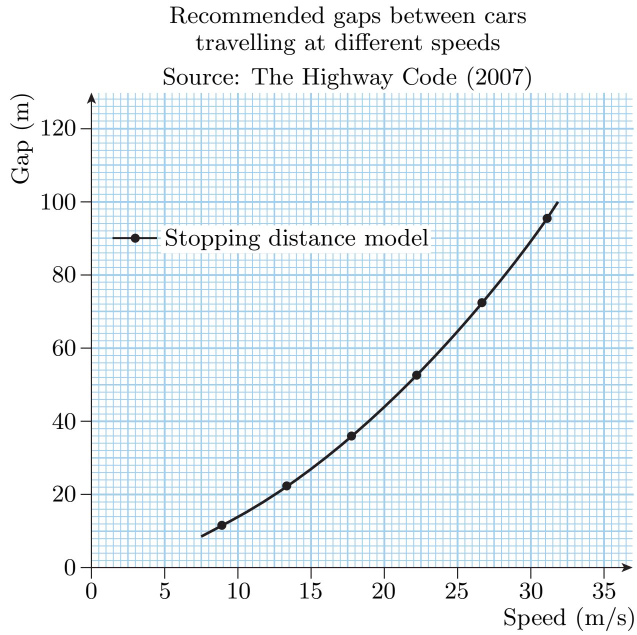
## Solution to Activity 10

**(a)** A speed of 90 km/h is approximately 25 m/s.

**(b)** A speed of 7.5 m/s is approximately 27 km/h.

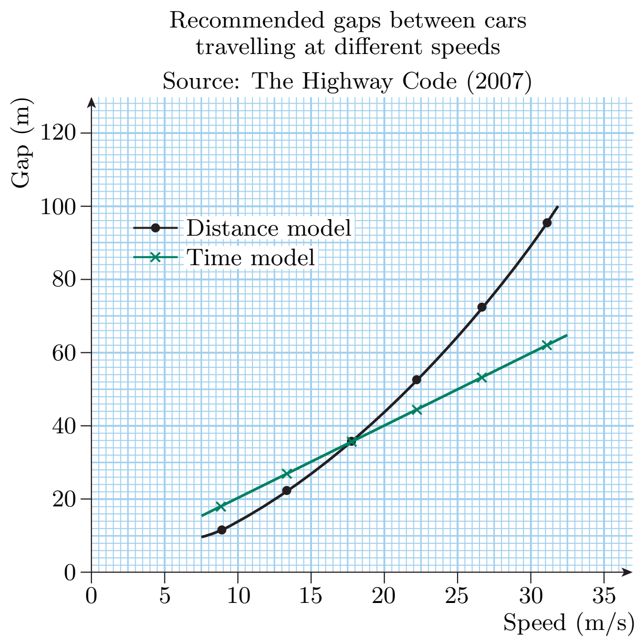
## Solution to Activity 11

**(a)** The graph is shown below.




Check that you have included the title and source of the data and that you have labelled the axes with their titles, units and scales. The points should be joined with a smooth curve.

**(b)** The line is plotted below.




**(c)** At 25 m/s, the time model gives a gap of 50 m and the distance model gives a gap of about 65 m.

**(d)** The distance model gives the smaller gap up to a speed of about 18 m/s. At speeds greater than 18 m/s, the time model gives the smaller gap, and the discrepancy between this model and the distance model increases quite rapidly as the speed increases. Notice that the distance model graph is not a straight line.

## Solution to Activity 12

In the formula, the width $w$ of the material is measured in cm, so first convert 0.4 m to cm, to give $w = (100 \times 0.4)$ cm $= 40$ cm. Substituting $w = 40$ into the formula for $L$ gives

$$L = 3 \times 40 + 5 = 125.$$

So 125 cm, or 1.25 m, of material is needed.

## Solution to Activity 13

In the formula the horizontal distance is measured in km. Now 5000 m $= (5000/1000)$ km $= 5$ km, so $D = 5$ and $H = 500$. Substituting $D$ and $H$ into the formula for $T$ gives

$$T = \frac{5}{5} + \frac{500}{600} = 1\tfrac{5}{6}.$$

Hence the estimate for the time is $1\frac{5}{6}$ hours, or 1 hour 50 minutes.

## Solution to Activity 14

**(a)** When $a = 4$, $b = 5$, $c = 6$, $d = 4$ and $e = 4$,

$$m = \frac{4+5+6+4+4}{5} = \frac{23}{5} = 4.6.$$

So the mean number of species is 4.6.

**(b)** When $m = 4.6$,

$$A = 110 \times 4.6 + 30 = 506 + 30 = 536.$$

So, to the nearest hundred years, the hedge is 500 years old.

## Solution to Activity 15

**(a)** If the length, width and height are measured in metres, then the volume is in cubic metres. First convert the given height into metres. Since

$$75\,\text{cm} = \frac{75}{100}\,\text{m} = 0.75\,\text{m},$$

the height is 0.75 m.

When $l = 1.5$, $w = 2$ and $h = 0.75$,

$$V = 1.5 \times 2 \times 0.75 = 2.25.$$

So the volume is $2.25\,\text{m}^3$.

**(b)** If the unit for distance is km and the unit for time is hours, then the unit for speed is km/h. So first convert the given time into hours. Since

$$75\,\text{min} = \frac{75}{60}\,\text{h} = 1.25\,\text{h},$$

the time is 1.25 h.

When $d = 80$ and $t = 1.25$,

$$s = \frac{80}{1.25} = 64.$$

So the average speed is 64 km/h.

## Solution to Activity 16

**(a)** To find the value of $Q$, multiply $a$ by 4 and then subtract 5.

Substituting $a = 2$ gives

$$Q = 4 \times 2 - 5 = 8 - 5 = 3.$$

**(b)** To find the value of $R$, divide $a$ by the product of 3 and $b$.

Substituting $a = 2$ and $b = 5$ gives

$$R = \frac{2}{3 \times 5} = \frac{2}{15}.$$

**(c)** To find the value of $P$, square $a$ and square $b$, and then add the answers together.

Substituting $a = 2$ and $b = 5$ gives

$$P = 2^2 + 5^2 = 4 + 25 = 29.$$

## Solution to Activity 17

**(a)** **(i)** $M = vw$

**(ii)** $A = \frac{1}{2}bh$ or $A = \dfrac{bh}{2}$

**(iii)** $V = hpr^2$

**(b)** **(i)** $C = 2 \times p \times r$

**(ii)** $V = l \times l \times l$

**(iii)** $s = u \times t + \frac{1}{2} \times a \times t \times t$

**(iv)** $A = \frac{1}{2} \times h \times (a + b)$

**(c)** **(i)** $P = 2(a + b)$

**(ii)** $I = \dfrac{PRT}{100}$

## Solution to Activity 19

**(a)** When $n = 94$,

$$C = 20 + 0.25 \times 94 = 20 + 23.5 = 43.5.$$

Hence the phone bill is £43.50.

**(b)** The formula requires the height in metres, so we calculate

$$164\,\text{cm} = (164 \div 100)\,\text{m} = 1.64\,\text{m}.$$

When $M = 72$ and $H = 1.64$,

$$I = \frac{72}{1.64^2} = \frac{72}{2.6896} = 26.8 \text{ (to 3 s.f.)}.$$

Since $I$ is greater than 25, the woman is classed as being overweight according to her BMI.

**(c)** When $A = 6$ and $C = 3$,

$$T = \frac{1 + 15 \times 6}{0.25 \times 3^2} = \frac{91}{2.25}$$
$$= 40 \text{ (to the nearest minute)}.$$

So the formula predicts that it takes 40 minutes before the children ask if they are nearly there yet. (This prediction would certainly have to be tested to check if it is reasonable!)

## Solution to Activity 20

**(a)** When $a = -4$,

$$G = 6 + (-4) - (-4)^2 = 6 - 4 - 16 = -14.$$

**(b)** When $x = -9$,

$$y = -(-9) + 4 = 9 + 4 = 13.$$

**(c)** When $u = -1$,

$$v = \frac{-1 - 2}{1.2} = \frac{-3}{1.2} = -2.5.$$

**(d)** When $c = -10$,

$$f = 1.8 \times (-10) + 32 = -18 + 32 = 14.$$

Hence $-10°$ C is equivalent to $14°$ F.

## Solution to Activity 21

**(a)** The formula is $d = m + p$, where $d$ is the total distance in km, $m$ is the distance in km on the motorway, and $p$ is the distance in km on principal roads.

**(b)** The formula is $D = 15f$, where $D$ is the distance in km, and $f$ is the number of litres of fuel.

**(c)** The formula is $N = 60 - r$, where $N$ is the number of cars available for hire, and $r$ is the number of cars rented out.

**(d)** The formula is $T = 2J$, where $T$ is the extra time in minutes, and $J$ is the number of junctions.

## Solution to Activity 22

**(a)** **(i)** The total cost is

booking fee + cost of tickets.

If one ticket costs £25, then $a$ tickets cost $a$ lots of £25, that is, £25$a$.

So the formula is $C = 5 + 25a$, where $C$ is the total cost in £, and $a$ is the number of adults.

**(ii)** The total cost is

booking fee + cost of adult tickets
+ cost of child tickets.

The cost of $a$ adult tickets is £25$a$, and the cost of $c$ child tickets is £15$c$.

So the formula is $T = 5 + 25a + 15c$, where $T$ is the total cost in £, $a$ is the number of adult tickets, and $c$ is the number of child tickets.

**(b)** The total cost is

registration fee + cost of sessions.

If one session costs £8, then $n$ sessions cost $n$ lots of £8, that is, £8$n$.

So the formula is $P = 20 + 8n$, where $P$ is the total cost in £, and $n$ is the number of sessions.

**(c)** The amount of fabric left is

length of roll − length needed for chairs.

If one chair requires 3 metres, then $n$ chairs require $3n$ metres.

So the formula is $s = r - 3n$, where $s$ is the amount of fabric left in metres, and $n$ is the number of chairs.

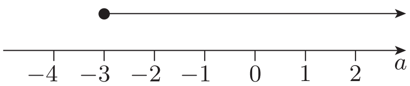
## Solution to Activity 24

**(a) (i)** $12 > 3$

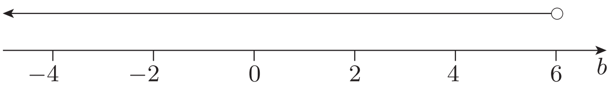
**(ii)** $-5 < 3$

**(iii)** $-2.5 > -4.5$

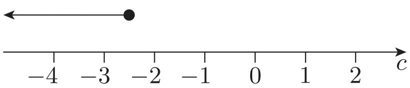
**(b) (i)**




**(ii)**




**(iii)**




**(c)** $3 < 12$, $\quad 3 > -5$, $\quad -4.5 < -2.5$;
$-3 \leq a$, $\quad 6 > b$, $\quad -2.5 \geq c$.

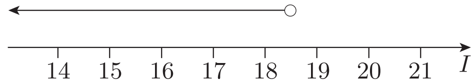
## Solution to Activity 25

The inequalities are as follows:

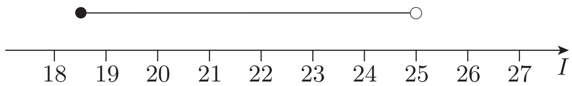
| | |
|---|---|
| underweight | $I < 18.5$, |
| healthy weight | $18.5 \leq I < 25$, |
| overweight | $25 \leq I < 30$, |
| obese | $I \geq 30$. |

These inequalities are each illustrated in the following figure.

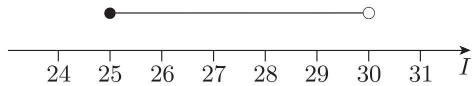
$I < 18.5$



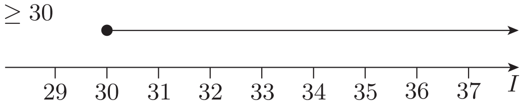

$18.5 \leq I < 25$




$25 \leq I < 30$

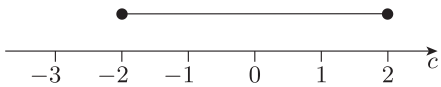



$I \geq 30$

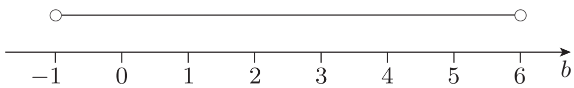



## Solution to Activity 26

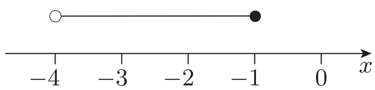
**(a) (i)**



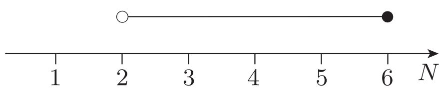

**(ii)**




**(iii)**




**(b)** The interval for $N$ is shown below.

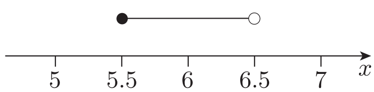



The whole numbers that lie within this interval are 3, 4, 5 and 6.

**(c) (i)** $-2 < x \leq 1$

**(ii)** $-5 \leq x < 25$

**(d)**




The double inequality for these numbers is

$$5.5 \leq x < 6.5.$$