# CS310 Homework5
# Yizhe Qu

```
> restart
```

## Problem1

### a.

```
> eqnCircle1 := (x-2)^2 + (y-5)^2 = 9
```

$$eqnCircle1 := (x-2)^2 + (y-5)^2 = 9 \quad (1)$$

```
> eqnCircle2 := (x-h)^2 + (y-k)^2 = 8
```

$$eqnCircle2 := (x-h)^2 + (y-k)^2 = 8 \quad (2)$$

```
> eqnCenterCircle2 := 7*x - 2*y = -5
```

$$eqnCenterCircle2 := 7x - 2y = -5 \quad (3)$$

```
> solnP1 := solve({eqnCircle1, eqnCircle2}, {x, y})
```

$$solnP1 := \left\{x = -\frac{1}{2}\ \frac{1}{-2+h}\left(2\ RootOf\left(\left(4h^2+4k^2-16h-40k+116\right)\_Z^2+\left(-4h^2k-4k^3-20h^2+16hk+20k^2+80h+80k-560\right)\_Z+h^4+2h^2k^2+k^4-8h^3-8hk^2+40h^2-40k^2-96h+656\right)k-h^2-k^2-10\ RootOf\left(\left(4h^2+4k^2-16h-40k+116\right)\_Z^2+\left(-4h^2k-4k^3-20h^2+16hk+20k^2+80h+80k-560\right)\_Z+h^4+2h^2k^2+k^4-8h^3-8hk^2+40h^2-40k^2-96h+656\right)+28\right), y = RootOf\left(\left(4h^2+4k^2-16h-40k+116\right)\_Z^2+\left(-4h^2k-4k^3-20h^2+16hk+20k^2+80h+80k-560\right)\_Z+h^4+2h^2k^2+k^4-8h^3-8hk^2+40h^2-40k^2-96h+656\right)\right\} \quad (4)$$

```
> solnAllP1 := allvalues(solnP1)
```

$$solnAllP1 := \left\{x = -\frac{1}{2}\ \frac{1}{-2+h}\left(\frac{1}{h^2+k^2-4h-10k+29}\left(\left(h^2k+k^3+5h^2-4hk-5k^2+\left(-h^6-2h^4k^2-h^2k^4+12h^5+20h^4k+16h^3k^2+20h^2k^3+4hk^4-76h^4-160h^3k-164h^2k^2-80hk^3-4k^4+288h^3+640h^2k+528hk^2+80k^3-400h^2-1280hk-496k^2-192h+960k+576\right)^{1/2}-20h-20k+140\right)k\right)-h^2-k^2-\frac{1}{h^2+k^2-4h-10k+29}\left(5\left(h^2k+k^3+5h^2-4hk-5k^2+\left(-h^6-2h^4k^2-h^2k^4+12h^5+20h^4k+16h^3k^2+20h^2k^3+4hk^4-76h^4-160h^3k-164h^2k^2-80hk^3-4k^4+288h^3+640h^2k+528hk^2+80k^3-400h^2\right.\right.\right.\right. \quad (5)$$

$$-1280\,h\,k-496\,k^2-192\,h+960\,k+576)^{1/2}-20\,h-20\,k+140))+28),y$$

$$=\frac{1}{2}\ \frac{1}{h^2+k^2-4\,h-10\,k+29}\left(h^2\,k+k^3+5\,h^2-4\,h\,k-5\,k^2\right.$$

$$+(-h^6-2\,h^4\,k^2-h^2\,k^4+12\,h^5+20\,h^4\,k+16\,h^3\,k^2+20\,h^2\,k^3+4\,h\,k^4-76\,h^4$$

$$-160\,h^3\,k-164\,h^2\,k^2-80\,h\,k^3-4\,k^4+288\,h^3+640\,h^2\,k+528\,h\,k^2+80\,k^3-400\,h^2$$

$$-1280\,h\,k-496\,k^2-192\,h+960\,k+576)^{1/2}-20\,h-20\,k+140))\},\left\{x=\right.$$

$$-\frac{1}{2}\ \frac{1}{-2+h}\left(-\frac{1}{h^2+k^2-4\,h-10\,k+29}((-h^2\,k-k^3-5\,h^2+4\,h\,k+5\,k^2\right.$$

$$+(-h^6-2\,h^4\,k^2-h^2\,k^4+12\,h^5+20\,h^4\,k+16\,h^3\,k^2+20\,h^2\,k^3+4\,h\,k^4-76\,h^4$$

$$-160\,h^3\,k-164\,h^2\,k^2-80\,h\,k^3-4\,k^4+288\,h^3+640\,h^2\,k+528\,h\,k^2+80\,k^3-400\,h^2$$

$$-1280\,h\,k-496\,k^2-192\,h+960\,k+576)^{1/2}+20\,h+20\,k-140)\,k)-h^2-k^2$$

$$+\frac{1}{h^2+k^2-4\,h-10\,k+29}(5\,(-h^2\,k-k^3-5\,h^2+4\,h\,k+5\,k^2$$

$$+(-h^6-2\,h^4\,k^2-h^2\,k^4+12\,h^5+20\,h^4\,k+16\,h^3\,k^2+20\,h^2\,k^3+4\,h\,k^4-76\,h^4$$

$$-160\,h^3\,k-164\,h^2\,k^2-80\,h\,k^3-4\,k^4+288\,h^3+640\,h^2\,k+528\,h\,k^2+80\,k^3-400\,h^2$$

$$-1280\,h\,k-496\,k^2-192\,h+960\,k+576)^{1/2}+20\,h+20\,k-140))+28),y=$$

$$-\frac{1}{2}\ \frac{1}{h^2+k^2-4\,h-10\,k+29}(-h^2\,k-k^3-5\,h^2+4\,h\,k+5\,k^2$$

$$+(-h^6-2\,h^4\,k^2-h^2\,k^4+12\,h^5+20\,h^4\,k+16\,h^3\,k^2+20\,h^2\,k^3+4\,h\,k^4-76\,h^4$$

$$-160\,h^3\,k-164\,h^2\,k^2-80\,h\,k^3-4\,k^4+288\,h^3+640\,h^2\,k+528\,h\,k^2+80\,k^3-400\,h^2$$

$$-1280\,h\,k-496\,k^2-192\,h+960\,k+576)^{1/2}+20\,h+20\,k-140)\}$$

**b.**

```
> P1point1 := simplify(solnAllP1[1])
```

$$P1point1:=\left\{x=-\frac{1}{2}\ \frac{1}{(h^2+k^2-4\,h-10\,k+29)\ (-2+h)}(-h^4-h^2\,k^2+4\,h^3+10\,h^2\,k\right.$$

**(6)**

$$+\left(-\left(h^4+2\,h^2\,k^2+k^4-8\,h^3-20\,h^2\,k-8\,h\,k^2-20\,k^3+40\,h^2+80\,h\,k\right.\right.$$

$$\left.\left.+124\,k^2-96\,h-240\,k-144\right)\,(-2+h)^2\right)^{1/2}k-26\,h^2+4\,k^2$$

$$-5\left(-\left(h^4+2\,h^2\,k^2+k^4-8\,h^3-20\,h^2\,k-8\,h\,k^2-20\,k^3+40\,h^2+80\,h\,k\right.\right.$$

$$\left.\left.+124\,k^2-96\,h-240\,k-144\right)\,(-2+h)^2\right)^{1/2}-12\,h-40\,k+112\Big),y$$

$$=\frac{1}{2}\,\frac{1}{h^2+k^2-4\,h-10\,k+29}\Big(h^2\,k+k^3+5\,h^2-4\,h\,k-5\,k^2$$

$$+\left(-\left(h^4+2\,h^2\,k^2+k^4-8\,h^3-20\,h^2\,k-8\,h\,k^2-20\,k^3+40\,h^2+80\,h\,k\right.\right.$$

$$\left.\left.+124\,k^2-96\,h-240\,k-144\right)\,(-2+h)^2\right)^{1/2}-20\,h-20\,k+140\Big)\Big\}$$

> $P1point2 := simplify(solnAllP1[2])$

$$P1point2 := \left\{x=\frac{1}{2}\,\frac{1}{\left(h^2+k^2-4\,h-10\,k+29\right)\,(-2+h)}\left(h^4+h^2\,k^2-4\,h^3-10\,h^2\,k\right.\right. \tag{7}$$

$$+\left(-\left(h^4+2\,h^2\,k^2+k^4-8\,h^3-20\,h^2\,k-8\,h\,k^2-20\,k^3+40\,h^2+80\,h\,k\right.\right.$$

$$\left.\left.+124\,k^2-96\,h-240\,k-144\right)\,(-2+h)^2\right)^{1/2}k+26\,h^2-4\,k^2$$

$$-5\left(-\left(h^4+2\,h^2\,k^2+k^4-8\,h^3-20\,h^2\,k-8\,h\,k^2-20\,k^3+40\,h^2+80\,h\,k\right.\right.$$

$$\left.\left.+124\,k^2-96\,h-240\,k-144\right)\,(-2+h)^2\right)^{1/2}+12\,h+40\,k-112\Big),y=$$

$$-\frac{1}{2}\,\frac{1}{h^2+k^2-4\,h-10\,k+29}\Big(-h^2\,k-k^3-5\,h^2+4\,h\,k+5\,k^2$$

$$+\left(-\left(h^4+2\,h^2\,k^2+k^4-8\,h^3-20\,h^2\,k-8\,h\,k^2-20\,k^3+40\,h^2+80\,h\,k\right.\right.$$

$$\left.\left.+124\,k^2-96\,h-240\,k-144\right)\,(-2+h)^2\right)^{1/2}+20\,h+20\,k-140\Big)\Big\}$$

> $x1 := rhs(P1point1[1])$

$$x1 := -\frac{1}{2}\,\frac{1}{\left(h^2+k^2-4\,h-10\,k+29\right)\,(-2+h)}\Big(-h^4-h^2\,k^2+4\,h^3+10\,h^2\,k \tag{8}$$

$$+\left(-\left(h^4+2\,h^2\,k^2+k^4-8\,h^3-20\,h^2\,k-8\,h\,k^2-20\,k^3+40\,h^2+80\,h\,k\right.\right.$$

$$\left.\left.+124\,k^2-96\,h-240\,k-144\right)\,(-2+h)^2\right)^{1/2}k-26\,h^2+4\,k^2$$

$$-5\left(-\left(h^4+2\,h^2\,k^2+k^4-8\,h^3-20\,h^2\,k-8\,h\,k^2-20\,k^3+40\,h^2+80\,h\,k\right.\right.$$

$$+124\,k^2-96\,h-240\,k-144)\,(-2+h)^2)^{1/2}-12\,h-40\,k+112)$$

> $y1 := rhs(P1point1[2])$

$$y1 := \frac{1}{2}\,\frac{1}{h^2+k^2-4\,h-10\,k+29}\,(h^2\,k+k^3+5\,h^2-4\,h\,k-5\,k^2 \tag{9}$$
$$+(-(h^4+2\,h^2\,k^2+k^4-8\,h^3-20\,h^2\,k-8\,h\,k^2-20\,k^3+40\,h^2+80\,h\,k$$
$$+124\,k^2-96\,h-240\,k-144)\,(-2+h)^2)^{1/2}-20\,h-20\,k+140)$$

> $x2 := rhs(P1point2[1])$

$$x2 := \frac{1}{2}\,\frac{1}{(h^2+k^2-4\,h-10\,k+29)\,(-2+h)}\,(h^4+h^2\,k^2-4\,h^3-10\,h^2\,k \tag{10}$$
$$+(-(h^4+2\,h^2\,k^2+k^4-8\,h^3-20\,h^2\,k-8\,h\,k^2-20\,k^3+40\,h^2+80\,h\,k$$
$$+124\,k^2-96\,h-240\,k-144)\,(-2+h)^2)^{1/2}\,k+26\,h^2-4\,k^2$$
$$-5\,(-(h^4+2\,h^2\,k^2+k^4-8\,h^3-20\,h^2\,k-8\,h\,k^2-20\,k^3+40\,h^2+80\,h\,k$$
$$+124\,k^2-96\,h-240\,k-144)\,(-2+h)^2)^{1/2}+12\,h+40\,k-112)$$

> $y2 := rhs(P1point2[2])$

$$y2 := -\frac{1}{2}\,\frac{1}{h^2+k^2-4\,h-10\,k+29}\,(-h^2\,k-k^3-5\,h^2+4\,h\,k+5\,k^2 \tag{11}$$
$$+(-(h^4+2\,h^2\,k^2+k^4-8\,h^3-20\,h^2\,k-8\,h\,k^2-20\,k^3+40\,h^2+80\,h\,k$$
$$+124\,k^2-96\,h-240\,k-144)\,(-2+h)^2)^{1/2}+20\,h+20\,k-140)$$

**c.**

> $P1distance := simplify(\mathrm{sqrt}((x2-x1)^2+(y2-y1)^2))$

$$P1distance := \tag{12}$$
$$\left(-\frac{1}{h^2+k^2-4\,h-10\,k+29}\,(h^4+2\,h^2\,k^2+k^4-8\,h^3-20\,h^2\,k-8\,h\,k^2\right.$$
$$\left.-20\,k^3+40\,h^2+80\,h\,k+124\,k^2-96\,h-240\,k-144)\right)^{1/2}$$

**d.**

> $lineHK := 7\cdot h-2\cdot k=-5$

$$lineHK := 7\,h-2\,k=-5 \tag{13}$$

> $kinh := solve(lineHK, k)$

$$kinh := \frac{5}{2}+\frac{7}{2}\,h \tag{14}$$

> $P1distanceInH := subs(k=kinh, P1distance)$

$$P1distanceInH := \tag{15}$$
$$\left(-\frac{1}{h^2+\left(\frac{5}{2}+\frac{7}{2}\,h\right)^2-39\,h+4}\left(h^4+2\,h^2\left(\frac{5}{2}+\frac{7}{2}\,h\right)^2+\left(\frac{5}{2}\right.\right.\right.$$

$$+\frac{7}{2}h\Big)^4-8h^3-20h^2\left(\frac{5}{2}+\frac{7}{2}h\right)-8h\left(\frac{5}{2}+\frac{7}{2}h\right)^2-20\left(\frac{5}{2}+\frac{7}{2}h\right)^3+40h^2$$

$$+80h\left(\frac{5}{2}+\frac{7}{2}h\right)+124\left(\frac{5}{2}+\frac{7}{2}h\right)^2-936h-744\Big)\Big)^{1/2}$$

e.

> $divP1distanceInH := diff(P1distanceInH, h)$

$$divP1distanceInH := \frac{1}{2}\left(-\frac{1}{h^2+\left(\frac{5}{2}+\frac{7}{2}h\right)^2-39h+4}\left(4h^3+4h\left(\frac{5}{2}+\frac{7}{2}h\right)^2\right.\right. \tag{16}$$

$$+14h^2\left(\frac{5}{2}+\frac{7}{2}h\right)+14\left(\frac{5}{2}+\frac{7}{2}h\right)^3-94h^2-96h\left(\frac{5}{2}+\frac{7}{2}h\right)-218\left(\frac{5}{2}\right.$$

$$+\frac{7}{2}h\Big)^2+3678h+1434\Big)+\frac{1}{\left(h^2+\left(\frac{5}{2}+\frac{7}{2}h\right)^2-39h+4\right)^2}\Big(\Big(h^4$$

$$+2h^2\left(\frac{5}{2}+\frac{7}{2}h\right)^2+\left(\frac{5}{2}+\frac{7}{2}h\right)^4-8h^3-20h^2\left(\frac{5}{2}+\frac{7}{2}h\right)-8h\left(\frac{5}{2}\right.$$

$$+\frac{7}{2}h\Big)^2-20\left(\frac{5}{2}+\frac{7}{2}h\right)^3+40h^2+80h\left(\frac{5}{2}+\frac{7}{2}h\right)+124\left(\frac{5}{2}+\frac{7}{2}h\right)^2$$

$$-936h-744\Big)\left(\frac{53}{2}h-\frac{43}{2}\right)\Big)\Big)\Big/$$

$$\left(-\frac{1}{h^2+\left(\frac{5}{2}+\frac{7}{2}h\right)^2-39h+4}\left(h^4+2h^2\left(\frac{5}{2}+\frac{7}{2}h\right)^2+\left(\frac{5}{2}\right.\right.\right.$$

$$+\frac{7}{2}h\Big)^4-8h^3-20h^2\left(\frac{5}{2}+\frac{7}{2}h\right)-8h\left(\frac{5}{2}+\frac{7}{2}h\right)^2-20\left(\frac{5}{2}+\frac{7}{2}h\right)^3+40h^2$$

$$+80h\left(\frac{5}{2}+\frac{7}{2}h\right)+124\left(\frac{5}{2}+\frac{7}{2}h\right)^2-936h-744\Big)\Big)^{1/2}$$

> $hMax := solve(divP1distanceInH = 0, h)$

$$hMax := \frac{43}{53}, \frac{43}{53}-\frac{4}{53}I\sqrt{7}, \frac{43}{53}+\frac{4}{53}I\sqrt{7}, \frac{43}{53}-\frac{2}{53}I\sqrt{134}, \frac{43}{53}+\frac{2}{53}I\sqrt{134} \tag{17}$$

> $distanceMax := subs(h = hMax[1], P1distanceInH)$

$$distanceMax := \frac{1}{4293}\sqrt{136592}\sqrt{4293} \tag{18}$$

**f.**

```
> hValue := solve(P1distanceInH = distanceMax, h)
```

$$hValue := \frac{43}{53} - \frac{4}{477} I\sqrt{938}, \frac{43}{53} + \frac{4}{477} I\sqrt{938}, \frac{43}{53} \quad (19)$$

```
> hSym := hValue[3]
```

$$hSym := \frac{43}{53} \quad (20)$$

```
> kSym := subs(h = hSym, kinh)
```

$$kSym := \frac{283}{53} \quad (21)$$

**g.**

```
> hNum := evalf(hSym)
```

$$hNum := 0.8113207547 \quad (22)$$

```
> kNum := evalf(kSym)
```

$$kNum := 5.339622642 \quad (23)$$

**h.**

```
> drawCircle1 := eqnCircle1
```

$$drawCircle1 := (x-2)^2 + (y-5)^2 = 9 \quad (24)$$

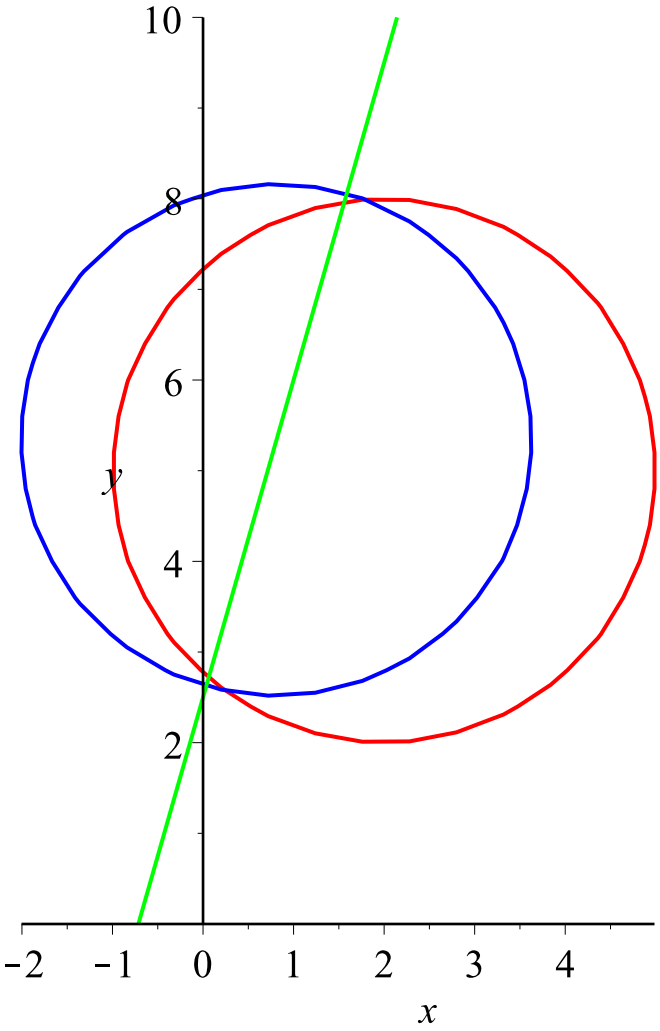
```
> drawCircle2 := subs(h = hNum, k = kNum, eqnCircle2)
```

$$drawCircle2 := (x-0.8113207547)^2 + (y-5.339622642)^2 = 8 \quad (25)$$

```
> drawLine := eqnCenterCircle2
```

$$drawLine := 7x - 2y = -5 \quad (26)$$

```
> with(plots):
> implicitplot([drawCircle1, drawCircle2, drawLine], x = -5..8, y = 0..10, color = [red, blue,
      green], scaling = constrained)
```




**Proplem 2**

**a.**

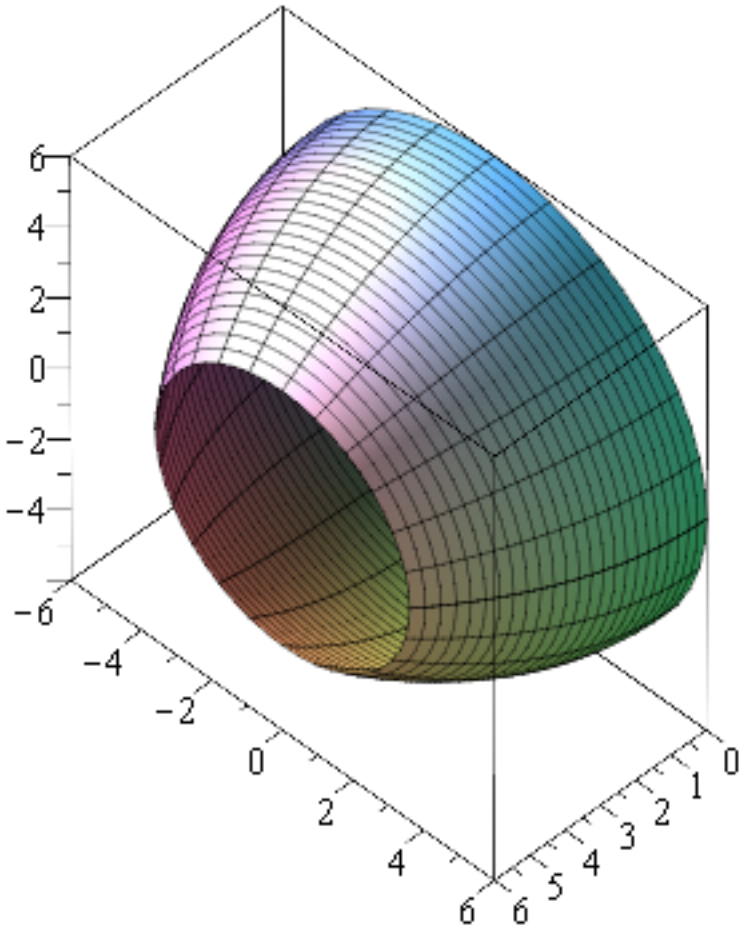
> $bowlShape := B \cdot \left(1 - \frac{2 \cdot x^2}{5 \cdot B^2}\right)$

$$bowlShape := B \left(1 - \frac{2}{5} \frac{x^2}{B^2}\right) \quad (27)$$

> $bowla := subs(B = 6, bowlShape)$

$$bowla := 6 - \frac{1}{15} x^2 \quad (28)$$

> $plot3d([x, bowla \cdot \cos(t), bowla \cdot \sin(t)], x = 0..6, t = 0..2 \cdot \mathrm{Pi}, scaling = constrained)$




**b.**

> $bowlSurface := simplify\left(int\left(2\cdot \mathrm{Pi}\cdot bowlShape\cdot \mathrm{sqrt}\left(1+\left(diff\left(bowlShape,x\right)\right)^2\right),x=0..\mathrm{B}\right)+subs\left(x=B,bowlShape\right)^2\cdot \mathrm{Pi}\right)$

$$bowlSurface := -\frac{1}{6400}B^2\pi\left(4625\,\mathrm{csgn}\left(\frac{1}{B}\right)\ln\left(41\right)-9250\,\mathrm{csgn}\left(\frac{1}{B}\right)\ln\left(\mathrm{csgn}\left(\frac{1}{B}\right)\left(41\,\mathrm{csgn}\left(\frac{1}{B}\right)+4\sqrt{41}\right)\right)+9250\,\mathrm{csgn}\left(\frac{1}{B}\right)\ln\left(5\right)-824\sqrt{41}-2304\right) \quad (29)$$

**c.**

> $surfaceAreaSym := subs\left(B=8,bowlSurface\right)$

$$surfaceAreaSym := -\frac{1}{100}\pi\left(4625\,\mathrm{csgn}\left(\frac{1}{8}\right)\ln\left(41\right)-9250\,\mathrm{csgn}\left(\frac{1}{8}\right)\ln\left(\mathrm{csgn}\left(\frac{1}{8}\right)\left(41\,\mathrm{csgn}\left(\frac{1}{8}\right)+4\sqrt{41}\right)\right)+9250\,\mathrm{csgn}\left(\frac{1}{8}\right)\ln\left(5\right)-824\sqrt{41}-2304\right) \quad (30)$$

> $surfaceAreaNum := evalf\left(surfaceAreaSym\right)$

(31)

$$surfaceAreaNum := 451.0496332 \quad (31)$$

**d.**

```
> heightP2d := solve(bowlSurface = 500, B)
```

Warning, solutions may have been lost

$$heightP2d := \quad (32)$$

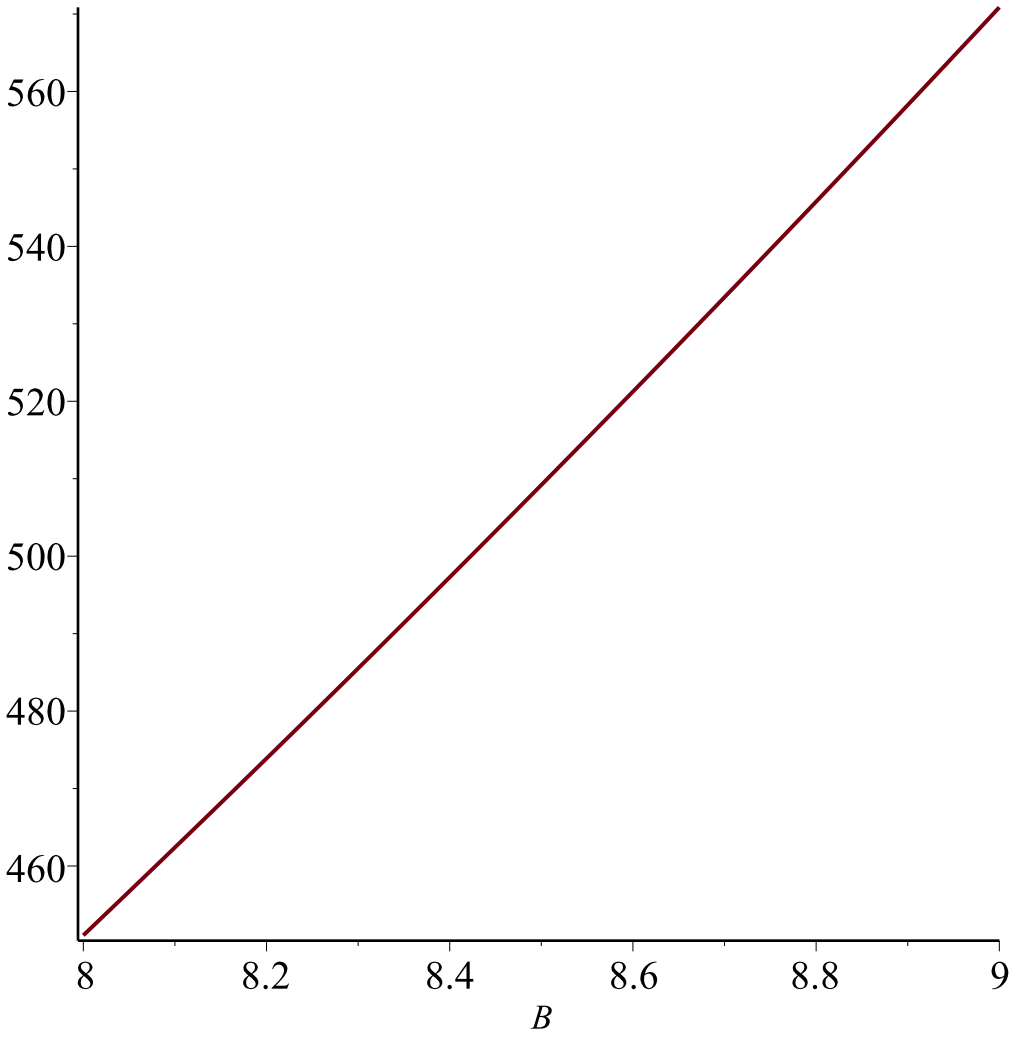
$$-800\sqrt{5}\sqrt{\frac{1}{\pi\left(-9250\ln\left(41-4\sqrt{41}\right)+4625\ln(41)+824\sqrt{41}+9250\ln(5)+2304\right)}},$$

$$800\sqrt{5}\sqrt{\frac{1}{\pi\left(9250\ln\left(41+4\sqrt{41}\right)-4625\ln(41)+824\sqrt{41}-9250\ln(5)+2304\right)}}$$

```
> heightP2d := allvalues(heightP2d)
```

Error, invalid input: too many and/or wrong type of arguments passed to allvalues; first unused argument is 800*5^(1/2)*(1/(Pi*(9250*ln(41+4*41^(1/2))-4625*ln(41)+824*41^(1/2)-9250*ln(5)+2304)))^(1/2)

```
> plot(bowlSurface, B = 8 ..9)
```



> $heightP2d := fsolve(bowlSurface = 500, B = 8.2..8.8)$

$$heightP2d := 8.422922838 \tag{33}$$

***Extra Credit***

> $volBowl := simplify(int(\mathrm{Pi} \cdot bowlShape^2, x = 0..B))$

$$volBowl := \frac{287}{375} B^3 \pi \tag{34}$$

> $heightExtraCreSym := solve(volBowl = 600, B)$

$$heightExtraCreSym := \frac{10}{287} \frac{18533025^{1/3} \left(\pi^2\right)^{1/3}}{\pi}, -\frac{5}{287} \frac{18533025^{1/3} \left(\pi^2\right)^{1/3}}{\pi} \tag{35}$$

$$+ \frac{\frac{5}{287} I\sqrt{3}\ 18533025^{1/3} \left(\pi^2\right)^{1/3}}{\pi}, -\frac{5}{287} \frac{18533025^{1/3} \left(\pi^2\right)^{1/3}}{\pi}$$

$$- \frac{\frac{5}{287} I\sqrt{3}\ 18533025^{1/3} \left(\pi^2\right)^{1/3}}{\pi}$$

> $heightExtraCreNum := fsolve(volBowl = 600)$

$$heightExtraCreNum := 6.295790233 \tag{36}$$

>