CS310 Homework6
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> restart

Problem1

a.

> $odeY1 := diff(y1(t), t) = q1 - \text{beta}\cdot A1\cdot y1(t)$

$$odeY1 := \frac{d}{dt}\, y1(t) = q1 - \beta\, A1\, y1(t) \quad (1)$$

> $odeY2 := diff(y2(t), t) = q2 + \text{beta}\cdot A1\cdot y1(t) - \text{beta}\cdot A2\cdot y2(t) - \text{beta}\cdot A3\cdot y2(t)$

$$odeY2 := \frac{d}{dt}\, y2(t) = q2 + \beta\, A1\, y1(t) - \beta\, A2\, y2(t) - \beta\, A3\, y2(t) \quad (2)$$

> $\text{steadyP1} := \text{solve}(\{rhs(odeY1) = 0, rhs(odeY2) = 0\}, \{y1(t), y2(t)\})$

$$steadyP1 := \left\{y1(t) = \frac{q1}{\beta\, A1}, y2(t) = \frac{q1 + q2}{\beta\, (A2 + A3)}\right\} \quad (3)$$

> $y1steady := rhs(steadyP1[1])$

$$y1steady := \frac{q1}{\beta\, A1} \quad (4)$$

> $y2steady := rhs(steadyP1[2])$

$$y2steady := \frac{q1 + q2}{\beta\, (A2 + A3)} \quad (5)$$

b.

> $initValues1b := q1 = 6, q2 = 2, A1 = 3, A2 = 5, A3 = 3, \text{beta} = \frac{1}{2}$

$$initValues1b := q1 = 6, q2 = 2, A1 = 3, A2 = 5, A3 = 3, \beta = \frac{1}{2} \quad (6)$$

> $y1steadyb := subs(initValues1b, y1steady)$

$$y1steadyb := 4 \quad (7)$$

> $y2steadyb := subs(initValues1b, y2steady)$

$$y2steadyb := 2 \quad (8)$$

c.

> $odeY1c := subs(q1 = 0, odeY1)$

$$odeY1c := \frac{d}{dt}\, y1(t) = -\beta\, A1\, y1(t) \quad (9)$$

> $odeY2c := subs(q2 = 0, \text{beta}\cdot A3\cdot y2(t) = 0, odeY2)$

$$odeY2c := \frac{d}{dt}\, y2(t) = \beta\, A1\, y1(t) - \beta\, A2\, y2(t) \quad (10)$$

d.

> $solnY1 := simplify(dsolve(\{odeY1c, y1(0) = y1steady\}, y1(t)))$

$$solnY1 := y1(t) = \frac{q1\, e^{-\beta\, A1\, t}}{\beta\, A1} \quad (11)$$

> $odeY2d := subs(y1(t) = rhs(solnY1), odeY2c)$

$$odeY2d := \frac{d}{dt}\, y2(t) = q1\, e^{-\beta\, A1\, t} - \beta\, A2\, y2(t) \quad (12)$$

> $solnY2 := simplify(dsolve(\{odeY2d, y2(0) = y2steady\}, y2(t)))$

$$solnY2 := y2(t) = \frac{\left(-q1\, e^{-\beta t (A1 - A2)}\, A2 - q1\, e^{-\beta t (A1 - A2)}\, A3 + q1\, A1 + q2\, A1 - q2\, A2 + q1\, A3\right) e^{-\beta A2 t}}{\beta\, (A1 - A2)\, (A2 + A3)} \quad \textbf{(13)}$$

e.

> $solnY1e := simplify(subs(initValues1b, solnY1))$

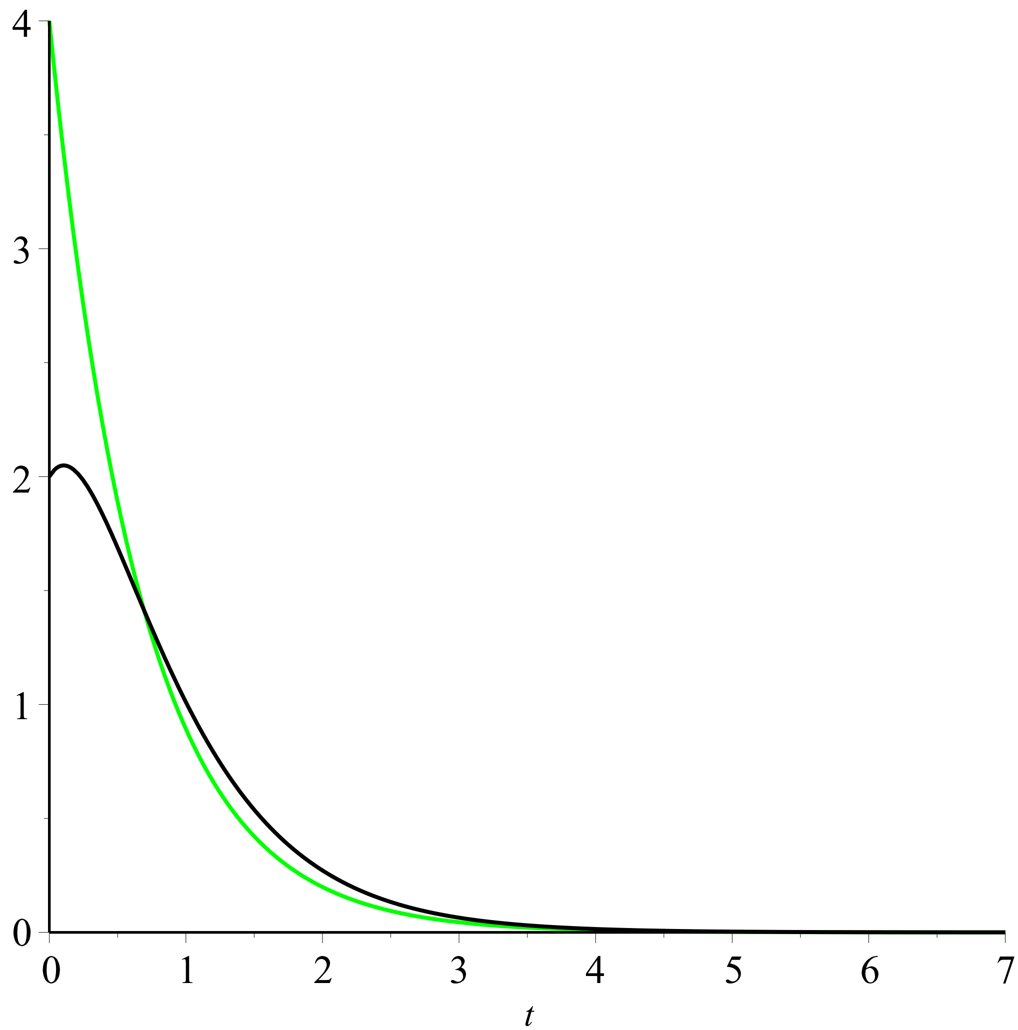
$$solnY1e := y1(t) = 4\, e^{-\frac{3}{2} t} \quad \textbf{(14)}$$

> $solnY2e := simplify(subs(initValues1b, solnY2))$

$$solnY2e := y2(t) = 2\, (3\, e^{t} - 2)\, e^{-\frac{5}{2} t} \quad \textbf{(15)}$$

f.

> $plot([rhs(solnY1e), rhs(solnY2e)], t = 0..7, color = [green, black])$



g.

> $divY2 := diff(rhs(solnY2e), t)$

$$divY2 := 6\, e^{t}\, e^{-\frac{5}{2} t} - 5\, (3\, e^{t} - 2)\, e^{-\frac{5}{2} t} \quad \textbf{(16)}$$

> $tMax := solve(divY2 = 0, t)$

$$tMax := \ln\left(\frac{10}{9}\right) \tag{17}$$

> $y2Max := simplify(rhs(subs(t=tMax, solnY2e)))$

$$y2Max := \frac{81}{125}\sqrt{2}\sqrt{5} \tag{18}$$

Problem 2

a.

> $ode2a := diff(i(t), t\$2) = \frac{v0}{\text{omega}} \cdot \sin(\text{omega} \cdot t) - \left(\frac{R}{L} \cdot diff(i(t), t)\right) - \left(\frac{1}{L \cdot C} \cdot i(t)\right)$

$$ode2a := \frac{d^2}{dt^2}\, i(t) = \frac{v0 \sin(\omega\, t)}{\omega} - \frac{R\left(\frac{d}{dt}\, i(t)\right)}{L} - \frac{i(t)}{L\, C} \tag{19}$$

b.

> $initCond2b := i(0) = i\_0, \mathrm{D}(i)(0) = di\_0$

$$initCond2b := i(0) = i\_0, \mathrm{D}(i)(0) = di\_0 \tag{20}$$

> $ode2SymSoln := dsolve(\{ode2a, initCond2b\}, i(t))$

$$\begin{aligned} ode2SymSoln := i(t) = {} & \Bigg(\mathrm{e}^{-\frac{1}{2}\frac{\left(C R - \sqrt{C^2 R^2 - 4 C L}\right) t}{L C}} \sqrt{C^2 R^2 - 4 C L}\, L \Big(2 C^2 L^2 di\_0\, \omega^2 \\ & + C^2 L R\, i\_0\, \omega^2 + C L \sqrt{C^2 R^2 - 4 C L}\; i\_0\, \omega^2 + 2 C^2 L^2 v0 + C^2 di\_0 R^2 \\ & - C\, di\_0 \sqrt{C^2 R^2 - 4 C L}\; R - 2\, di\_0 L C + C R\, i\_0 - \sqrt{C^2 R^2 - 4 C L}\; i\_0\Big)\Bigg) \Bigg/ \\ & \left(C \left(C R^2 - 4 L\right)\left(2 C L^2 \omega^2 + C R^2 - \sqrt{C^2 R^2 - 4 C L}\; R - 2 L\right)\right) \\ & - \Bigg(\mathrm{e}^{-\frac{1}{2}\frac{\left(C R + \sqrt{C^2 R^2 - 4 C L}\right) t}{L C}} \Big(2 C^2 L^2 di\_0\, \omega^2 + C^2 L R\, i\_0\, \omega^2 - C L \sqrt{C^2 R^2 - 4 C L}\; i\_0\, \omega^2 + 2 C^2 L^2 v0 \\ & + \sqrt{C^2 R^2 - 4 C L}\; R - 2 L\Big) C \left(C R^2 - 4 L\right)\Bigg) \\ & - \frac{v0\, L\, C \left(C L \sin(\omega t)\, \omega^2 + C \cos(\omega t)\, R\, \omega - \sin(\omega t)\right)}{\omega \left(C^2 L^2 \omega^4 + C^2 R^2 \omega^2 - 2 C L \omega^2 + 1\right)} \end{aligned}$$

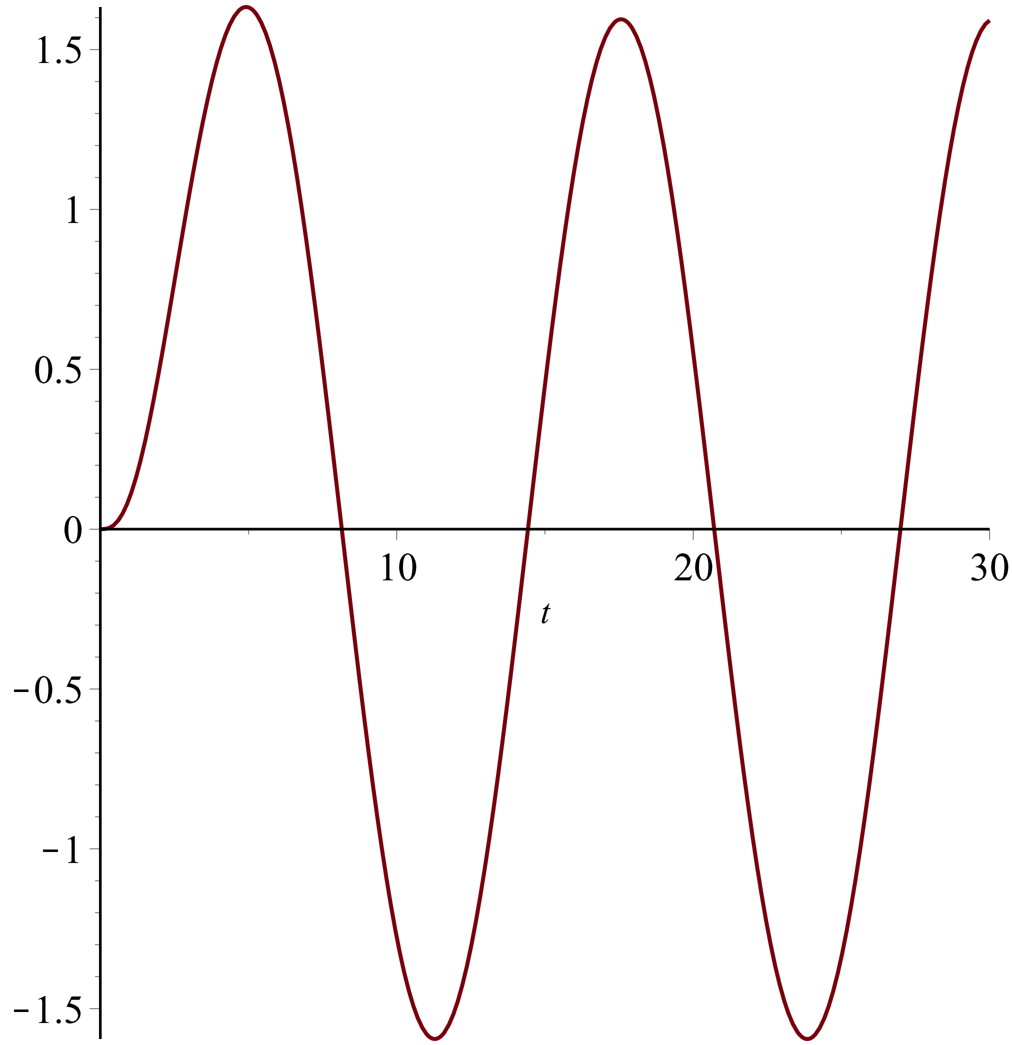
c.

> $ode2NumSoln := subs(i\_0 = 0, di\_0 = 0, L = 1, v0 = 1, C = 1, R = 2.01, \text{omega} = 0.5, ode2SymSoln)$

$$\begin{aligned} ode2NumSoln := i(t) = {} & 4.672311755\, \mathrm{e}^{-0.9048750780\, t} - 3.394112681\, \mathrm{e}^{-1.105124922\, t} \\ & + 0.9538799062 \sin(0.5\, t) - 1.278199074 \cos(0.5\, t) \end{aligned} \tag{22}$$

d.

> $plot(rhs(ode2NumSoln), t = 0..30)$



e.

> $tValue := fsolve(rhs(ode2NumSoln) = -1, t = 5..10)$

$$tValue := 9.499197807 \quad (23)$$

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