Institute of Technology of Cambodia

Mechanic and Industrial Engineer

Group: I_4-GIM(Mechanic)

Assignment: Construction Mechanic

TOPIC: KUKA KR 10 R1100-2

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Table of Contents

I. Introduction

In this homework we learn Forward kinematic and inverse kinematic. For forward kinematic we can find position in the end of robot arm (x, y, z). Inverse kinematic we can find the start point (angle and distance) int this robot arm.

II. Forward Kinematic

We find end point or some point in joint robot arm by forward kinematic by knowing angle and distance.

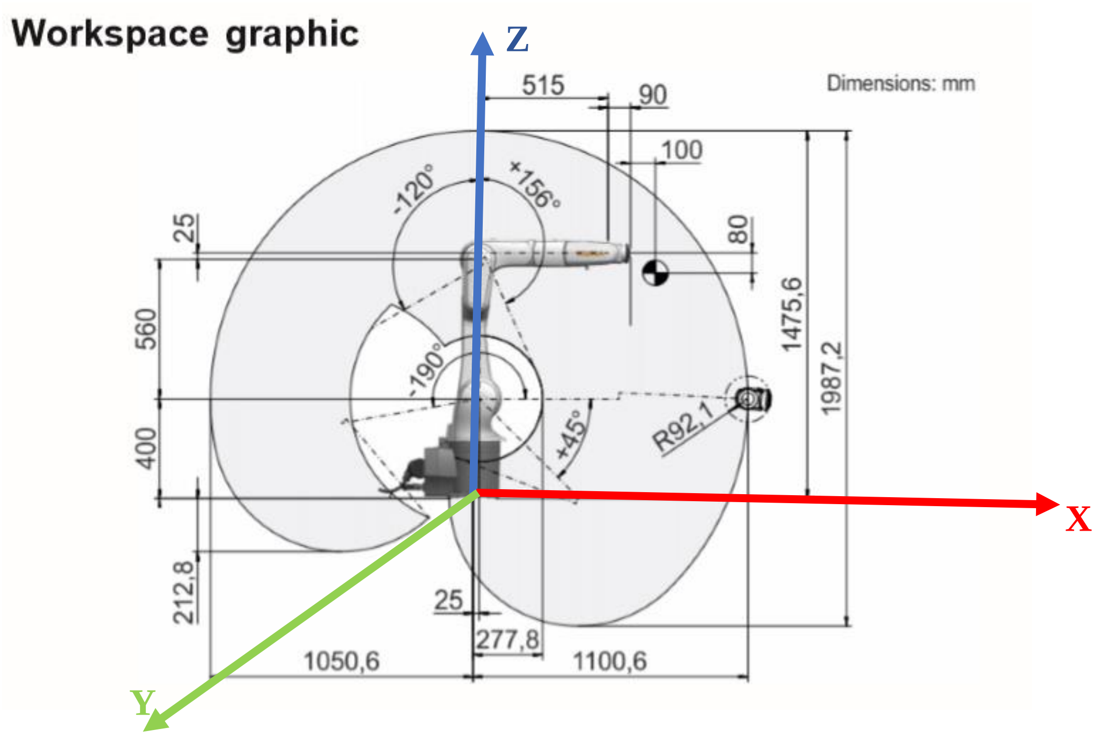


Figure1: KUKA KR 10 R1100-2

Denavit-Hartenberg Parameters

Link(i)	θ_i	α_i	a_i	d_i
1	θ_1	90	a_1	d_1
2	θ_2	0	a_2	0
3	$\theta_3 - 90$	90	a_3	0
4	θ_4	-90	0	d_4
5	θ_5	90	0	0
6	$\theta_6 + 180$	0	0	d_6

Table 1

4 parameters of Denavit-Hartenberg: $\theta, d, a\ and\ \alpha$:

a_i: link length (displacement along x_{i-1} from $z_{i-1}$to z_i)

α_i: link twist (rotation around x_{i-1} from $z_{i-1}$to z_i)

d_i: link offset (displacement along z_i from $x_{i-1}$to x_i)

θ_i: joint angle (rotation around z_i from $x_{i-1}$to x_i)

$$ {}^{i-1}_{i}T = Rot(z, \theta_i)Trans(0,0,\ d_i)Trans(a_i, 0,0)Rot(x, \alpha_i $$

$$ = \begin{bmatrix} cos\theta_i & -cos\theta_i cos\alpha_i & sin\theta_i sin\alpha_i & a_i cos\theta_i \\ sin\theta_i & cos\theta_i cos\alpha_i & -cos\theta_i sin\alpha_i & a_i sin\theta_i \\ 0 & sin\alpha_i & cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Therefore, in general for a series of N kinematic links, the overall transformation between the first frame K_0, the base of the first kinematic link, and the last K_N, the end-effector, would be a matrix multiplication of all the D-H transformation matrices:

$$ {}^{0}_{N}T = {}^{0}_{1}T\,{}^{1}_{2}T \ldots {}^{N-1}_{N}T = \begin{bmatrix} l_x & m_x & n_x & p_x \\ l_y & m_y & n_y & p_y \\ l_z & m_z & n_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} l & m & n & p \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} R & p \\ 0 & 1 \end{bmatrix} $$

Where R - corresponds to a 3x3 matrix, representing rotation; $p^{\rightarrow}$ - corresponds to a 3x1 matrix (vector) that represents translation.

Thus, for a 6-DOF robot as KUKA KR10 R100-2, the overall robot transform would be: ${}^{0}_{6}T = {}^{0}_{1}T\,{}^{1}_{2}T\,{}^{2}_{3}T\,{}^{3}_{4}T\,{}^{4}_{5}T\,{}^{5}_{6}T$

$$ {}^{0}_{1}T = \begin{bmatrix} \cos\theta_1 & 0 & -\sin\theta_1 & a_1\cos\theta_1 \\ \sin\theta_1 & 0 & \cos\theta_i & a_1\sin\theta_1 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{1}_{2}T = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & a_2\cos\theta_2 \\ \sin\theta_2 & \cos\theta_2 & 0 & a_2\sin\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{2}_{3}T = \begin{bmatrix} \cos\theta_3 & 0 & -\sin\theta_3 & a_3\cos\theta_3 \\ \sin\theta_3 & 0 & \cos\theta_3 & a_3\sin\theta_3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}_4^3T = \begin{bmatrix} \cos\theta_4 & 0 & \sin\theta_4 & 0 \\ \sin\theta_4 & 0 & -\cos\theta_i & 0 \\ 0 & 1 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}_5^4T = \begin{bmatrix} \cos\theta_5 & 0 & -\sin\theta_5 & 0 \\ \sin\theta_5 & 0 & \cos\theta_5 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}_6^5T = \begin{bmatrix} \cos\theta_6 & -\sin\theta_6 & 0 & 0 \\ \sin\theta_6 & \cos\theta_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Each of joint angles must be multiplied by '-1', the third angle must have 90° subtracted from it and the sixth angle must have 180° added to it before the multiplication according to D-H parameters (Table 1).

Calculating overall transformation matrix, you will get the following results:

$$ l_x = \mathrm{s_1(s_4c_5c_6 + c_4s_6) + c_1(-s_{23}s_5c_6 + c_{23}(c_4c_5c_6 - s_4s_6))} $$
$$ l_y = \mathrm{-c_1(s_4c_5c_6 + c_4s_6) + s_1(-s_{23}s_5c_6 + c_{23}(c_4c_5c_6 - s_4s_6))} $$
$$ l_z = \mathrm{-c_6(s_{23}c_4c_5 + c_{23}s_5) + s_{23}s_5s_6} $$
$$ m_x = \mathrm{c_6(s_1c_4 - c_1c_{23}s_4) - s_6(s_1s_4c_5 + c_1(c_{23}c_4c_5 - s_{23}s_5))} $$
$$ m_y = \mathrm{c_1(-c_4c_6 + c_5s_4s_6) - s_1(-s_{23}s_5s_6 + c_{23}(s_4c_6 + c_4c_5s_6))} $$
$$ m_z = \mathrm{s_{23}s_4c_6 + s_6(s_{23}c_4c_5 + c_{23}s_5)} $$
$$ n_x = \mathrm{-s_1s_4s_5 - c_1(s_{23}c_5 + c_{23}c_4s_5)} $$
$$ n_y = \mathrm{-s_1s_{23}c_5 + s_5(-s_1c_{23}c_4 + c_1s_4)} $$
$$ n_z = \mathrm{-c_{23}c_5 + c_4s_{23}s_5} $$
$$ p_x = \mathrm{-d_6s_1s_4s_5 + c_1(a_1 + a_2c_2 - s_{23}(d_4 + d_6c_5) + c_{23}(a_3 - d_6c_4s_5))} $$
$$ p_y = \mathrm{d_6c_1s_4s_5 + s_1(a_1 + a_2c_2 - s_{23}(d_4 + d_6c_5) + c_{23}(a_3 - d_6c_4s_5))} $$
$$ p_z = \mathrm{d_1 - c_{23}(d_4 + d_6c_5) - a_2s_2 + s_{23}(-a_3 + d_6c_4s_5)} $$

where $s_i = \sin\theta_i$, $c_i = \cos\theta_i$, $s_{23} = \sin(\theta_2 + \theta_3)$, $c_{23} = \cos(\theta_2 + \theta_3)$

To translate from rotation matrix to Euler angles (Roll, Pitch, Yaw – RPY angles) we have the following formulas for rotation matrix in RPY convention (X, Y', Z" rotation):

$$\boldsymbol{R}_{RPY}(\gamma,\beta,\alpha) = \boldsymbol{R}_z(\alpha)\cdot\boldsymbol{R}_y(\beta)\cdot\boldsymbol{R}_x(\gamma)$$
$$= \begin{bmatrix} \cos\alpha\cos\beta & \cos\alpha\sin\beta\sin\gamma - \sin\alpha\cos\gamma & \cos\alpha\sin\beta\cos\gamma + \sin\alpha\sin\gamma \\ \sin\alpha\cos\beta & \sin\alpha\sin\beta\sin\gamma + \cos\alpha\cos\gamma & \sin\alpha\sin\beta\cos\gamma - \cos\alpha\,\mathrm{ain}\,\gamma \\ -\sin\beta & \cos\beta\sin\gamma & \cos\beta\cos\gamma \end{bmatrix}$$
$$= \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

From the previous rotation matrix, we will get the following equations for RPY angles:

$$\beta = Atan2(-r_{31}, \sqrt{r_{11}{}^2 + r_{21}{}^2})$$
$$\alpha = Atan2(r_{21}, r_{11})$$
$$\gamma = Atan2(r_{32}, r_{33})$$

III. *Inverse Kinematic*

Inverse kinematic we can find the start point (angle and distance) int this robot arm.

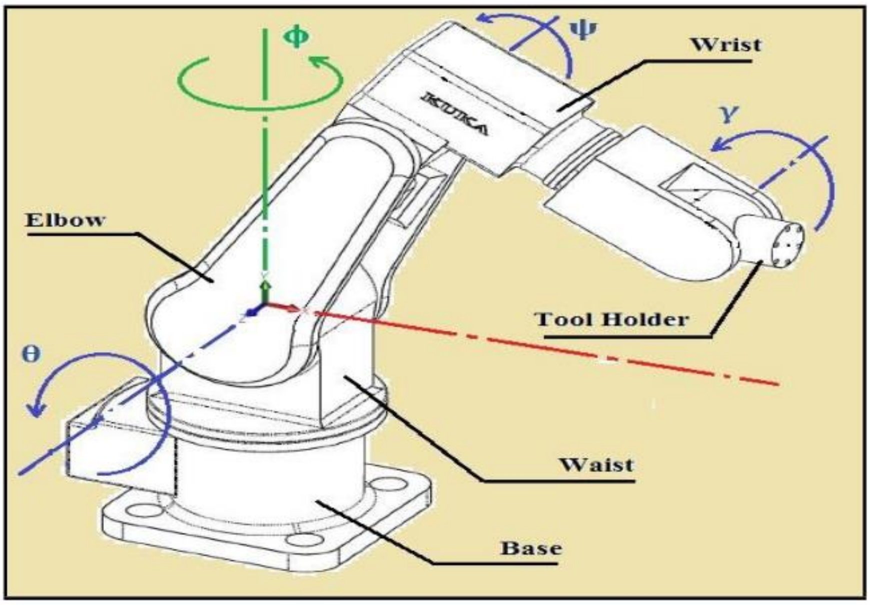


Figure2: KUKA KR 10 R1100-2

In this study, we defined the 4-DOF (RRRR) KUKA manipulator robot parameters as the following:
x: a displacement of end-effector manipulator along x-axis from the rotation point.
y: a displacement of end-effector manipulator along y-axis from the rotation point.
z: a displacement of end-effector manipulator along z-axis from the rotation point.
x1: a displacement of tool holder axis along x-axis from the rotation point.
y1: a displacement of tool holder axis along y-axis from the rotation point.
z1: a displacement of tool holder axis along z-axis from the rotation point.
El: an elbow manipulator length.
Wr: a wrist manipulator length.

Gr: a tool holder manipulator length.
L1: a distance between the reference point (0, 0, 0) and tool holder axis (x1, y1, z1), which measured from the top plane of a manipulator.
M: a distance between the reference point (0, 0, 0) and tool holder axis (x1, y1, z1), which measured from the front plane of a manipulator.
L2: a total length of M and Gr.
L3: a distance between the axis of elbow and the axis of tool holder.
N: a projected length of M in the top plane of a manipulator.
$\varphi 1$: an angle between Gr and x-axis, which measured from top plane of a manipulator.
$\varphi 2$: an angle between L1 and x-axis, which measured from top plane of a manipulator.
θ1: a slope of the M in the xy-plane.
θ2: a slope of the tool holder in the xy-plane.
θ3: a slope of the end-effector in the xy-plane.
β: a slope of the wrist in the xy-plane.
α: an angle between M and Wr, which measured from front plane of a manipulator.
φ : the rotation angle of the waist of a manipulator.
θ: the rotation angle of the elbow of a manipulator.
ψ: the rotation angle of the wrist of a manipulator.
γ: the rotation angle of the tool holder of a manipulator.

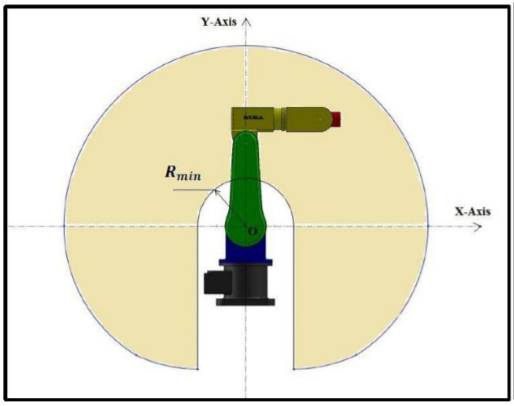
Manipulator Robot Parameters Constraints:

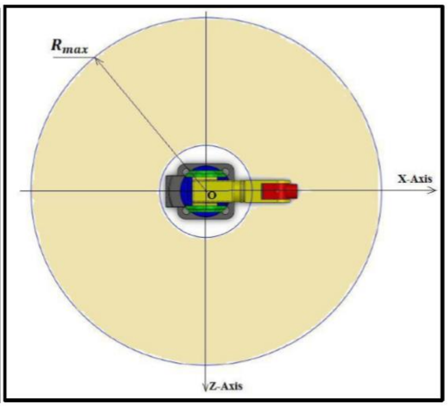
1- El > 0;
2- Wr > 0;
3- Gr > 0;
4- El > Wr > Gr .

Manipulator Robot Work Space Constraints:

1- $\sqrt{x^2 + y^2 + z^2} \geq (Wr + Gr - El)$;
2- $\sqrt{x^2 + y^2 + z^2} \leq (Wr + Gr + El)$;

The 4-DOF (RRRR) robotic arm's manipulator Workspace



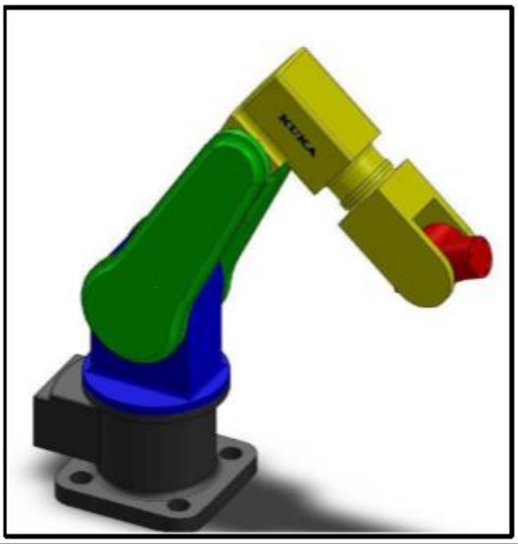


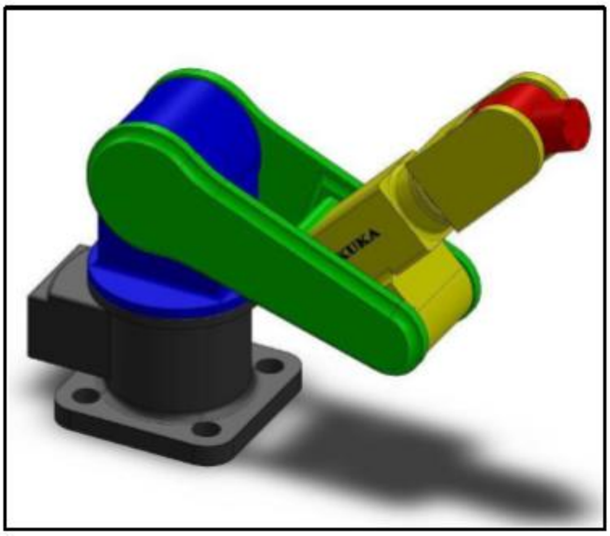
1- The maximum outer diameter will be:

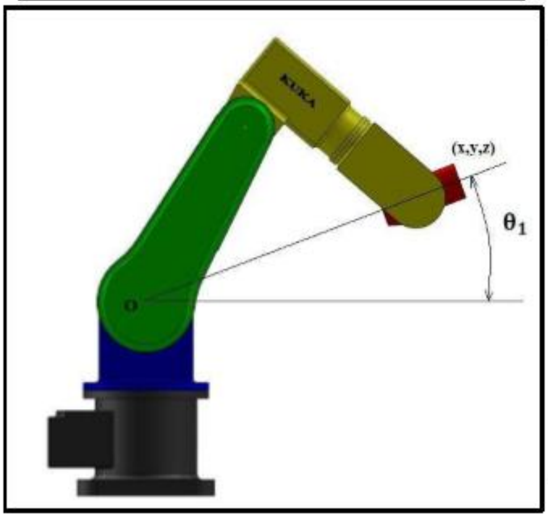
2(El + Wr +Gr).

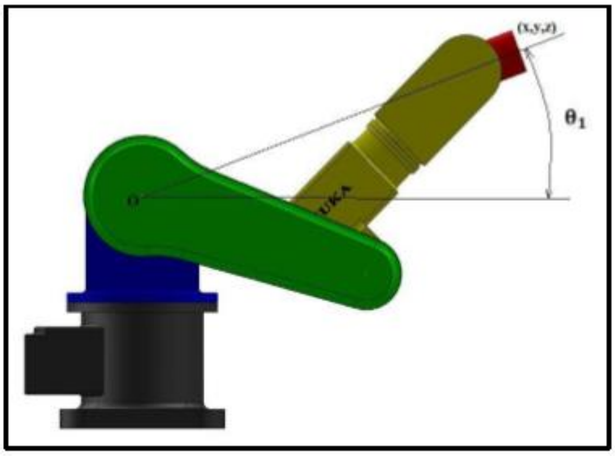
2- The minimum inner diameter will be:

2(Wr +Gr _El).









Define the Parameters:

$L = \sqrt{x^2 + y^2 + z^2}$, $\theta_1 = \tan^{-1}(\frac{y}{\sqrt{x^2+z^2}})$, $M = L_2 - Gr$, $y_1 = M \times \sin(\theta_1)$,

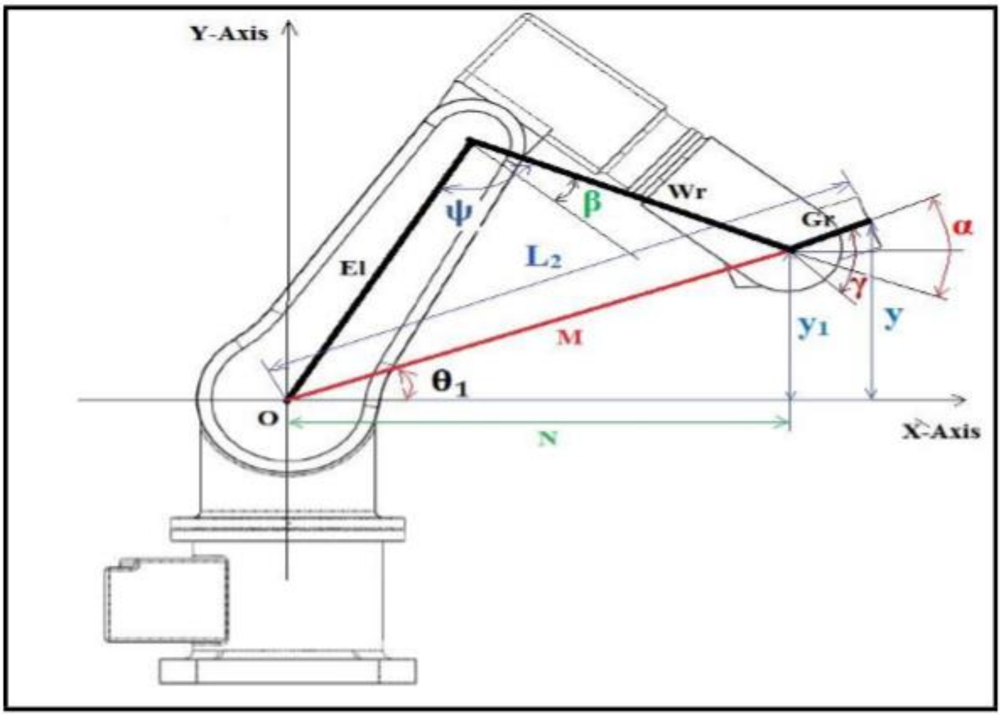
$\varphi_1 = \tan^{-1}(\frac{z}{x})$, $N = M \times \cos(\theta_1)$, $x_1 = N \times \cos(\varphi_1)$, $z_1 = N \times \sin(\varphi_1)$

When $(x \leq 0)\&\&(z > 0)$, then: $x_1 = -N \times \cos(\varphi_1)$, $x_1 = -N \times \cos(\varphi_1)$

When$(x \leq 0)\&\&(z < 0)$, then: $x_1 = -N \times \cos(\varphi_1)$, $x_1 = -N \times \cos(\varphi_1)$

$$L_1 = \sqrt{{x_1}^2 + {z_1}^2}\ , \quad \alpha = \cos^{-1}\left(\frac{Wr^2 + \sqrt{x_1^2 + z_1^2 + y_1^2} - El^2}{2 \times Wr \times \sqrt{x_1^2 + z_1^2 + y_1^2}}\right), \quad \phi_2 = \tan^{-1}\left(\frac{z_1}{x_1}\right), \quad L_3 = \sqrt{{L_1}^2 + {y_1}^2}$$

$$\theta_2 = \tan^{-1}\left(\frac{y_1}{L_1}\right), \quad \theta_3 = \cos^{-1}\left(\frac{{L_3}^2 + El^2 - Wr^2}{2 \times L_3 \times El}\right)$$



$$\theta = (\theta_2 + \theta_3) - 90\ , \quad \psi = \cos^{-1}\left(\frac{Wr^2 + El^2 - {L_3}^2}{2 \times Wr \times El}\right) - (90 + \beta), \quad \gamma = (\alpha + \beta)$$

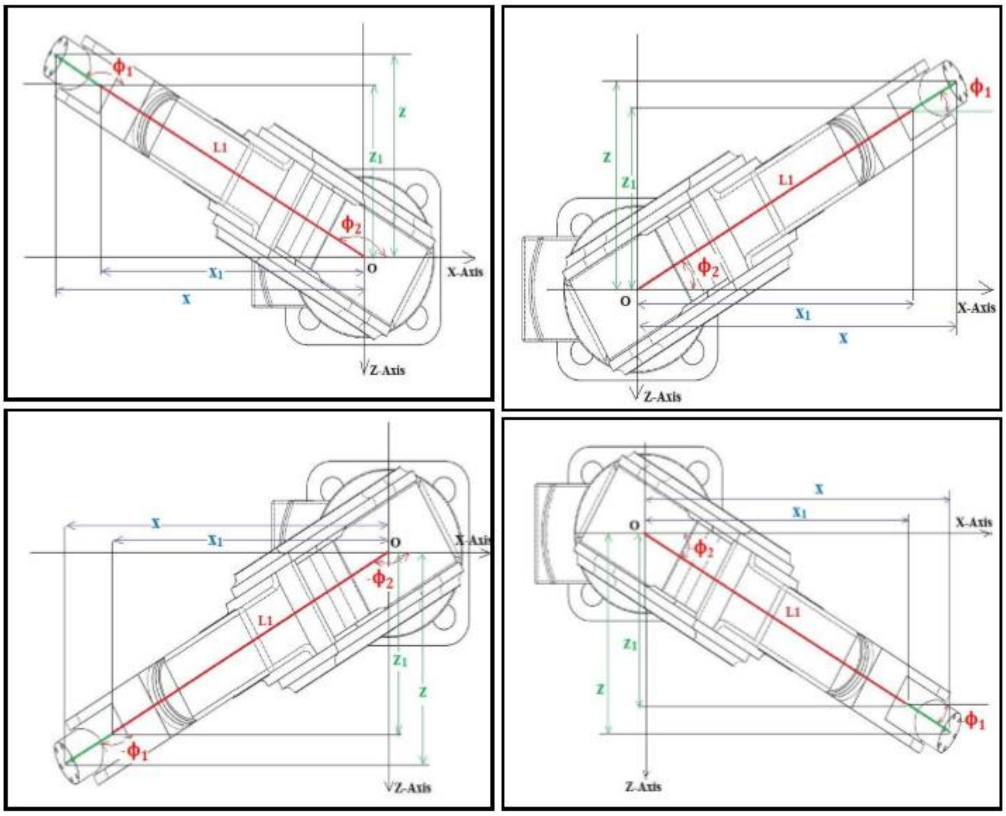
First Position:

1- When (x1 > 0) and (z1 ≥ 0), then:
   $\varphi_1 = -\varphi_2$
2- When (x1 ≥0) and (z1 < 0), then:
   $\varphi_2 = \cos^{-1}(\frac{x_1}{L_1})\ , \varphi = -\varphi_2$
3- When (x1 < 0) and (z1 ≤ 0), then:

   $\varphi = 180 - \varphi_2$
4- When (x1 ≤ 0) and (z1 > 0), then:

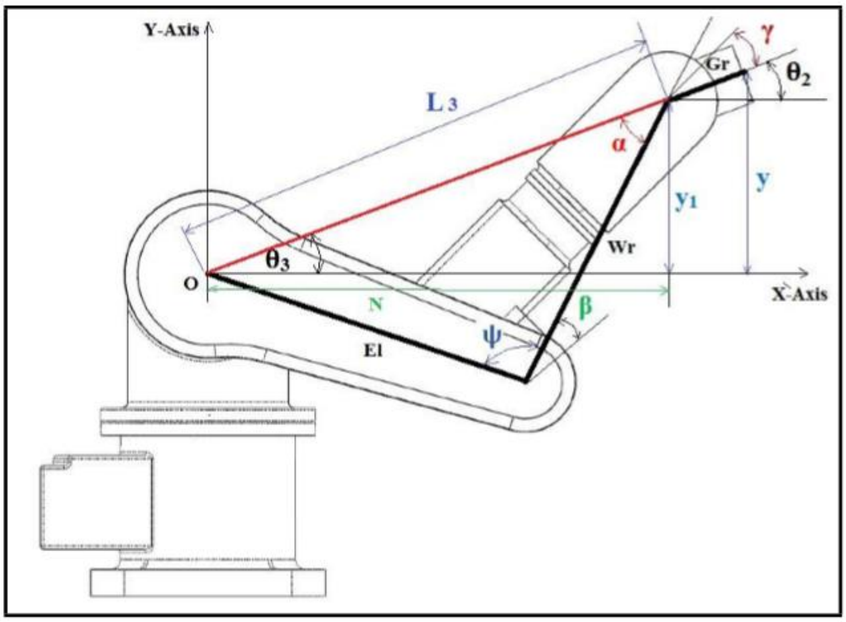
   $\varphi_2 = \cos^{-1}(\frac{z_1}{L_1})\ , \varphi = 180 + \varphi_2$

Top views of upper and down elbow when (x > 0), (z ≥ 0) && (x ≥ 0), (z < 0) && (x < 0), (z ≤ 0) and (x ≤ 0), (z > 0) is as shown below.



Second Position:

$$\theta = (\theta_2 - \theta_3) - 90, \quad \psi = 270 - \left(\cos^{-1}\left(\frac{Wr^2 + El^2 - L_3{}^2}{2 \times Wr \times El} \right) + \beta \right), \quad \gamma = (\beta - \alpha)$$



1- When (x1 > 0) and (z1 ≥ 0), then: $\varphi = -\varphi_2$
2- When (x1 ≥0) and (z1 < 0), then: $\varphi_2 = \cos^{-1}(\frac{x_1}{L_1})$, $\varphi = -\varphi_2$
3- When (x1 < 0) and (z1 ≤ 0), then: $\varphi = 180 - \varphi_2$
4- When (x1 < 0) and (z1 ≤ 0), then: $\varphi_2 = \cos^{-1}(\frac{z_1}{L_1})$, $\varphi = 180 + \varphi_2$

IV. Conclusion

I learned to use how to use its inverse and forward kinematics. So, by defining a concrete robot chain I can now control most of the non-mobile robots. Know more clearly about KUKA KR 10 R1100-2 robot arm.