

1 Momentum equations

$$\frac{\partial u}{\partial t} + \frac{\partial p}{\partial x} = \frac{1}{Re} \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) - \frac{\partial(u^2)}{\partial x} - \frac{\partial(uv)}{\partial y} - \frac{\partial(uw)}{\partial z} + g_x \quad (1)$$

$$\frac{\partial v}{\partial t} + \frac{\partial p}{\partial y} = \frac{1}{Re} \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) - \frac{\partial(uv)}{\partial x} - \frac{\partial(v^2)}{\partial y} - \frac{\partial(vw)}{\partial z} + g_y \quad (2)$$

$$\frac{\partial w}{\partial t} + \frac{\partial p}{\partial z} = \frac{1}{Re} \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) - \frac{\partial(uw)}{\partial x} - \frac{\partial(vw)}{\partial y} - \frac{\partial(w^2)}{\partial z} + g_z \quad (3)$$

2 Continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (4)$$

3 Energy equation

$$\frac{\partial T}{\partial t} = \frac{1}{Re} \frac{1}{Pr} \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) - \frac{\partial(uT)}{\partial x} - \frac{\partial(vT)}{\partial y} - \frac{\partial(wT)}{\partial z} \quad (5)$$

4 F, G, H

4.1 equations

$$\begin{aligned} F_{i,j,k} = & u_{i,j,k} + \delta t \left(\frac{1}{Re} \left(\left[\frac{\partial^2 u}{\partial x^2} \right]_{i,j,k} + \left[\frac{\partial^2 u}{\partial y^2} \right]_{i,j,k} + \left[\frac{\partial^2 u}{\partial z^2} \right]_{i,j,k} \right) - \left[\frac{\partial(u^2)}{\partial x} \right]_{i,j,k} - \left[\frac{\partial(uv)}{\partial y} \right]_{i,j,k} - \left[\frac{\partial(uw)}{\partial z} \right]_{i,j,k} \right. \\ & \left. + \textcolor{red}{g}_x - \frac{\beta}{2} \left(T_{i,j,k}^{(n+1)} + T_{i+1,j,k}^{(n+1)} \right) g_x \right) \\ & i = 1, \dots, imax - 1; \quad j = 1, \dots, jmax; \quad k = 1, \dots, kmax \end{aligned} \quad (6)$$

$$\begin{aligned} G_{i,j,k} = & v_{i,j,k} + \delta t \left(\frac{1}{Re} \left(\left[\frac{\partial^2 v}{\partial x^2} \right]_{i,j,k} + \left[\frac{\partial^2 v}{\partial y^2} \right]_{i,j,k} + \left[\frac{\partial^2 v}{\partial z^2} \right]_{i,j,k} \right) - \left[\frac{\partial(uv)}{\partial x} \right]_{i,j,k} - \left[\frac{\partial(v^2)}{\partial y} \right]_{i,j,k} - \left[\frac{\partial(vw)}{\partial z} \right]_{i,j,k} \right. \\ & \left. + \textcolor{red}{g}_y - \frac{\beta}{2} \left(T_{i,j,k}^{(n+1)} + T_{i,j+1,k}^{(n+1)} \right) g_y \right) \\ & i = 1, \dots, imax; \quad j = 1, \dots, jmax - 1; \quad k = 1, \dots, kmax \end{aligned} \quad (7)$$

$$\begin{aligned}
H_{i,j,k} = & w_{i,j,k} + \delta t \left(\frac{1}{Re} \left(\left[\frac{\partial^2 w}{\partial x^2} \right]_{i,j,k} + \left[\frac{\partial^2 w}{\partial y^2} \right]_{i,j,k} + \left[\frac{\partial^2 w}{\partial z^2} \right]_{i,j,k} \right) - \left[\frac{\partial(uw)}{\partial x} \right]_{i,j,k} - \left[\frac{\partial(vw)}{\partial y} \right]_{i,j,k} - \left[\frac{\partial(w^2)}{\partial z} \right]_{i,j,k} \right. \\
& + \textcolor{red}{g}_z - \frac{\beta}{2} \left(T_{i,j,k}^{(n+1)} + T_{i,j,k+1}^{(n+1)} \right) g_y \Big) \\
& i = 1, \dots, imax; \quad j = 1, \dots, jmax; \quad k = 1, \dots, kmax - 1
\end{aligned} \tag{8}$$

4.2 discretization

4.2.1 F

$$\left[\frac{\partial^2 u}{\partial x^2} \right]_{i,j,k} = \frac{u_{i+1,j,k} - 2u_{i,j,k} + u_{i-1,j,k}}{(\delta x)^2} \tag{9}$$

$$\left[\frac{\partial^2 u}{\partial y^2} \right]_{i,j,k} = \frac{u_{i,j+1,k} - 2u_{i,j,k} + u_{i,j-1,k}}{(\delta y)^2} \tag{10}$$

$$\left[\frac{\partial^2 u}{\partial z^2} \right]_{i,j,k} = \frac{u_{i,j,k+1} - 2u_{i,j,k} + u_{i,j,k-1}}{(\delta z)^2} \tag{11}$$

$$\begin{aligned}
\left[\frac{\partial(u^2)}{\partial x} \right]_{i,j,k} = & \frac{1}{\delta x} \left(\left(\frac{u_{i,j,k} + u_{i+1,j,k}}{2} \right)^2 - \left(\frac{u_{i-1,j,k} + u_{i,j,k}}{2} \right)^2 \right) + \\
& \frac{\gamma}{\delta x} \left(\frac{|u_{i,j,k} + u_{i+1,j,k}|}{2} \frac{(u_{i,j,k} - u_{i+1,j,k})}{2} + \frac{|u_{i-1,j,k} + u_{i,j,k}|}{2} \frac{(u_{i-1,j,k} - u_{i,j,k})}{2} \right)
\end{aligned} \tag{12}$$

$$\begin{aligned}
\left[\frac{\partial(uv)}{\partial y} \right]_{i,j,k} = & \frac{1}{\delta y} \left(\frac{(v_{i,j,k} + v_{i+1,j,k})}{2} \frac{(u_{i,j,k} + u_{i,j+1,k})}{2} - \frac{(v_{i,j-1,k} + v_{i+1,j-1,k})}{2} \frac{(u_{i,j-1,k} + u_{i,j,k})}{2} \right) + \\
& \frac{\gamma}{\delta y} \left(\frac{|v_{i,j,k} + v_{i+1,j,k}|}{2} \frac{(u_{i,j,k} - u_{i,j+1,k})}{2} - \frac{|v_{i,j-1,k} + v_{i+1,j-1,k}|}{2} \frac{(u_{i,j-1,k} - u_{i,j,k})}{2} \right)
\end{aligned} \tag{13}$$

$$\begin{aligned}
\left[\frac{\partial(uw)}{\partial z} \right]_{i,j,k} = & \frac{1}{\delta z} \left(\frac{(w_{i,j,k} + w_{i+1,j,k})}{2} \frac{(u_{i,j,k} + u_{i,j,k+1})}{2} - \frac{(w_{i,j,k-1} + w_{i+1,j,k-1})}{2} \frac{(u_{i,j,k-1} + u_{i,j,k})}{2} \right) + \\
& \frac{\gamma}{\delta z} \left(\frac{|w_{i,j,k} + w_{i+1,j,k}|}{2} \frac{(u_{i,j,k} - u_{i,j,k+1})}{2} - \frac{|w_{i,j,k-1} + w_{i+1,j,k-1}|}{2} \frac{(u_{i,j,k-1} - u_{i,j,k})}{2} \right)
\end{aligned} \tag{14}$$

5 SOR solver

$$p_{i,j,k}^{it+1} = (1 - \omega)p_{i,j,k}^{it} + \frac{\omega}{2\left(\frac{1}{(\delta x)^2} + \frac{1}{(\delta y)^2} + \frac{1}{(\delta z)^2}\right)} \left(\frac{p_{i+1,j,k}^{it} + p_{i-1,j,k}^{it+1}}{(\delta x)^2} + \frac{p_{i,j+1,k}^{it} + p_{i,j-1,k}^{it+1}}{(\delta y)^2} + \frac{p_{i,j,k+1}^{it} + p_{i,j,k-1}^{it+1}}{(\delta z)^2} - rs_{i,j,k} \right)$$

$$it = 1, \dots, itmax; \quad i = 1, \dots, imax; \quad j = 1, \dots, jmax; \quad k = 1, \dots, kmax$$
(15)

$$res := \left(\sum_{i=1}^{imax} \sum_{j=1}^{jmax} \sum_{k=1}^{kmax} \left(\frac{p_{i+1,j,k} - 2p_{i,j,k} + p_{i-1,j,k}}{(\delta x)^2} + \frac{p_{i,j+1,k} - 2p_{i,j,k} + p_{i,j-1,k}}{(\delta y)^2} + \frac{p_{i,j,k+1} - 2p_{i,j,k} + p_{i,j,k-1}}{(\delta z)^2} - rs_{i,j,k} \right)^2 / (imax \cdot jmax \cdot kmax) \right)^{1/2}$$
(16)

6 calculate u,v,w

$$u_{i,j,k}^{(n+1)} = F_{i,j,k}^{(n)} - \frac{\delta t}{\delta x} \left(p_{i+1,j,k}^{(n+1)} - p_{i,j,k}^{(n+1)} \right)$$

$$i = 1, \dots, imax - 1; \quad j = 1, \dots, jmax; \quad k = 1, \dots, kmax$$
(17)

$$v_{i,j,k}^{(n+1)} = G_{i,j,k}^{(n)} - \frac{\delta t}{\delta y} \left(p_{i,j+1,k}^{(n+1)} - p_{i,j,k}^{(n+1)} \right)$$

$$i = 1, \dots, imax; \quad j = 1, \dots, jmax - 1; \quad k = 1, \dots, kmax$$
(18)

$$w_{i,j,k}^{(n+1)} = H_{i,j,k}^{(n)} - \frac{\delta t}{\delta z} \left(p_{i,j,k+1}^{(n+1)} - p_{i,j,k}^{(n+1)} \right)$$

$$i = 1, \dots, imax; \quad j = 1, \dots, jmax; \quad k = 1, \dots, kmax - 1$$
(19)

7 Energy Equation

$$T_{i,j,k}^{(n+1)} = T_{i,j,k}^{(n)} + \delta t \left(\frac{1}{Re} \frac{1}{Pr} \left(\left[\frac{\partial^2 T}{\partial x^2} \right]_{i,j,k} + \left[\frac{\partial^2 T}{\partial y^2} \right]_{i,j,k} + \left[\frac{\partial^2 T}{\partial z^2} \right]_{i,j,k} \right) - \left[\frac{\partial(uT)}{\partial x} \right]_{i,j,k} - \left[\frac{\partial(vT)}{\partial y} \right]_{i,j,k} - \left[\frac{\partial(wT)}{\partial z} \right]_{i,j,k} \right)$$
(20)