1 Momentum equations

$$\frac{\partial u}{\partial t} + \frac{\partial p}{\partial x} = \frac{1}{Re}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) - \frac{\partial(u^2)}{\partial x} - \frac{\partial(uv)}{\partial y} - \frac{\partial(uw)}{\partial z} + g_x \tag{1}$$

$$\frac{\partial v}{\partial t} + \frac{\partial p}{\partial y} = \frac{1}{Re}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) - \frac{\partial(uv)}{\partial x} - \frac{\partial(v^2)}{\partial y} - \frac{\partial(vw)}{\partial z} + g_y \tag{2}$$

$$\frac{\partial w}{\partial t} + \frac{\partial p}{\partial z} = \frac{1}{Re}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right) - \frac{\partial(uw)}{\partial x} - \frac{\partial(vw)}{\partial y} - \frac{\partial(w^2)}{\partial z} + g_z \tag{3}$$

2 Continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{4}$$

3 Energy equation

$$\frac{\partial T}{\partial t} = \frac{1}{Re}\frac{1}{Pr}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) - \frac{\partial(uT)}{\partial x} - \frac{\partial(vT)}{\partial y} - \frac{\partial(wT)}{\partial z} \tag{5}$$

4 F, G, H

4.1 equations

$$
\begin{aligned}
F_{i,j,k} = u_{i,j,k} + \delta t\Big(\frac{1}{Re}&\left(\left[\frac{\partial^2 u}{\partial x^2}\right]_{i,j,k} + \left[\frac{\partial^2 u}{\partial y^2}\right]_{i,j,k} + \left[\frac{\partial^2 u}{\partial z^2}\right]_{i,j,k}\right) - \left[\frac{\partial(u^2)}{\partial x}\right]_{i,j,k} - \left[\frac{\partial(uv)}{\partial y}\right]_{i,j,k} - \left[\frac{\partial(uw)}{\partial z}\right]_{i,j,k}\\
&+ g_x - \frac{\beta}{2}\left(T_{i,j,k}^{(n+1)} + T_{i+1,j,k}^{(n+1)}\right)g_x\Big)\\
&i = 1,\ldots,imax-1;\quad j = 1,\ldots,jmax;\quad k = 1,\ldots,kmax
\end{aligned}
\tag{6}
$$

$$
\begin{aligned}
G_{i,j,k} = v_{i,j,k} + \delta t\Big(\frac{1}{Re}&\left(\left[\frac{\partial^2 v}{\partial x^2}\right]_{i,j,k} + \left[\frac{\partial^2 v}{\partial y^2}\right]_{i,j,k} + \left[\frac{\partial^2 v}{\partial z^2}\right]_{i,j,k}\right) - \left[\frac{\partial(uv)}{\partial x}\right]_{i,j,k} - \left[\frac{\partial(v^2)}{\partial y}\right]_{i,j,k} - \left[\frac{\partial(vw)}{\partial z}\right]_{i,j,k}\\
&+ g_y - \frac{\beta}{2}\left(T_{i,j,k}^{(n+1)} + T_{i,j+1,k}^{(n+1)}\right)g_y\Big)\\
&i = 1,\ldots,imax;\quad j = 1,\ldots,jmax-1;\quad k = 1,\ldots,kmax
\end{aligned}
\tag{7}
$$

$$H_{i,j,k} = w_{i,j,k} + \delta t \Big(\frac{1}{Re} \left(\left[\frac{\partial^2 w}{\partial x^2} \right]_{i,j,k} + \left[\frac{\partial^2 w}{\partial y^2} \right]_{i,j,k} + \left[\frac{\partial^2 w}{\partial z^2} \right]_{i,j,k} \right) - \left[\frac{\partial(uw)}{\partial x} \right]_{i,j,k} - \left[\frac{\partial(vw)}{\partial y} \right]_{i,j,k} - \left[\frac{\partial(w^2)}{\partial z} \right]_{i,j,k}$$
$$+ g_z - \frac{\beta}{2} \left(T_{i,j,k}^{(n+1)} + T_{i,j,k+1}^{(n+1)} \right) g_y \Big)$$
$$i = 1, \ldots, imax; \quad j = 1, \ldots, jmax; \quad k = 1, \ldots, kmax - 1$$

$$(8)$$

4.2 discretization

4.2.1 F

$$\left[\frac{\partial^2 u}{\partial x^2} \right]_{i,j,k} \tag{9}$$

$$\left[\frac{\partial^2 u}{\partial x^2} \right]_{i,j,k} = \frac{u_{i+1,j,k} - 2u_{i,j,k} + u_{i-1,j,k}}{(\delta x)^2} \tag{10}$$

$$\left[\frac{\partial^2 u}{\partial y^2} \right]_{i,j,k} = \frac{u_{i,j+1,k} - 2u_{i,j,k} + u_{i,j-1,k}}{(\delta y)^2} \tag{11}$$

$$\left[\frac{\partial^2 u}{\partial z^2} \right]_{i,j,k} = \frac{u_{i,j,k+1} - 2u_{i,j,k} + u_{i,j,k-1}}{(\delta z)^2} \tag{12}$$

5 calculate uvw

$$u_{i,j,k}^{(n+1)} = F_{i,j,k}^{(n)} - \frac{\delta t}{\delta x} \left(p_{i+1,j,k}^{(n+1)} - p_{i,j,k}^{(n+1)} \right)$$
$$i = 1, \ldots, imax - 1; \quad j = 1, \ldots, jmax; \quad k = 1, \ldots, kmax \tag{13}$$

$$v_{i,j,k}^{(n+1)} = G_{i,j,k}^{(n)} - \frac{\delta t}{\delta y} \left(p_{i,j+1,k}^{(n+1)} - p_{i,j,k}^{(n+1)} \right)$$
$$i = 1, \ldots, imax; \quad j = 1, \ldots, jmax - 1; \quad k = 1, \ldots, kmax \tag{14}$$

$$w_{i,j,k}^{(n+1)} = H_{i,j,k}^{(n)} - \frac{\delta t}{\delta z} \left(p_{i,j,k+1}^{(n+1)} - p_{i,j,k}^{(n+1)} \right)$$
$$i = 1, \ldots, imax; \quad j = 1, \ldots, jmax; \quad k = 1, \ldots, kmax - 1 \tag{15}$$

6 Energy Equation

$$T_{i,j,k}^{(n+1)} = T_{i,j,k}^{(n)} + \delta t \left(\frac{1}{Re} \frac{1}{Pr} \left(\left[\frac{\partial^2 T}{\partial x^2} \right]_{i,j,k} + \left[\frac{\partial^2 T}{\partial y^2} \right]_{i,j,k} + \left[\frac{\partial^2 T}{\partial z^2} \right]_{i,j,k} \right) - \left[\frac{\partial(uT)}{\partial x} \right]_{i,j,k} - \left[\frac{\partial(vT)}{\partial y} \right]_{i,j,k} - \left[\frac{\partial(wT)}{\partial z} \right]_{i,j,k} \right)$$

$$(16)$$