1 Momentum equations

$$\frac{\partial u}{\partial t}+\frac{\partial p}{\partial x}=\frac{1}{Re}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)-\frac{\partial (u^2)}{\partial x}-\frac{\partial (uv)}{\partial y}-\frac{\partial (uw)}{\partial z}+g_x \tag{1}$$

$$\frac{\partial v}{\partial t}+\frac{\partial p}{\partial y}=\frac{1}{Re}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)-\frac{\partial (uv)}{\partial x}-\frac{\partial (v^2)}{\partial y}-\frac{\partial (vw)}{\partial z}+g_y \tag{2}$$

$$\frac{\partial w}{\partial t}+\frac{\partial p}{\partial z}=\frac{1}{Re}\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)-\frac{\partial (uw)}{\partial x}-\frac{\partial (vw)}{\partial y}-\frac{\partial (w^2)}{\partial z}+g_z \tag{3}$$

2 Continuity equation

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{4}$$

3 Energy equation

$$\frac{\partial T}{\partial t}=\frac{1}{Re}\frac{1}{Pr}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)-\frac{\partial (uT)}{\partial x}-\frac{\partial (vT)}{\partial y}-\frac{\partial (wT)}{\partial z} \tag{5}$$

4 F, G, H

4.1 Equations

$$\begin{aligned}F_{i,j,k}=u_{i,j,k}+\delta t\Big(&\frac{1}{Re}\left(\left[\frac{\partial^2 u}{\partial x^2}\right]_{i,j,k}+\left[\frac{\partial^2 u}{\partial y^2}\right]_{i,j,k}+\left[\frac{\partial^2 u}{\partial z^2}\right]_{i,j,k}\right)-\left[\frac{\partial (u^2)}{\partial x}\right]_{i,j,k}-\left[\frac{\partial (uv)}{\partial y}\right]_{i,j,k}-\left[\frac{\partial (uw)}{\partial z}\right]_{i,j,k}\\&+g_x-\frac{\beta}{2}\left(T^{(n+1)}_{i,j,k}+T^{(n+1)}_{i+1,j,k}\right)g_x\Big)\\&i=1,\ldots,imax-1;\quad j=1,\ldots,jmax;\quad k=1,\ldots,kmax\end{aligned} \tag{6}$$

$$\begin{aligned}G_{i,j,k}=v_{i,j,k}+\delta t\Big(&\frac{1}{Re}\left(\left[\frac{\partial^2 v}{\partial x^2}\right]_{i,j,k}+\left[\frac{\partial^2 v}{\partial y^2}\right]_{i,j,k}+\left[\frac{\partial^2 v}{\partial z^2}\right]_{i,j,k}\right)-\left[\frac{\partial (uv)}{\partial x}\right]_{i,j,k}-\left[\frac{\partial (v^2)}{\partial y}\right]_{i,j,k}-\left[\frac{\partial (vw)}{\partial z}\right]_{i,j,k}\\&+g_y-\frac{\beta}{2}\left(T^{(n+1)}_{i,j,k}+T^{(n+1)}_{i,j+1,k}\right)g_y\Big)\\&i=1,\ldots,imax;\quad j=1,\ldots,jmax-1;\quad k=1,\ldots,kmax\end{aligned} \tag{7}$$

$$H_{i,j,k} = w_{i,j,k} + \delta t\Big(\frac{1}{Re}\left(\left[\frac{\partial^2 w}{\partial x^2}\right]_{i,j,k} + \left[\frac{\partial^2 w}{\partial y^2}\right]_{i,j,k} + \left[\frac{\partial^2 w}{\partial z^2}\right]_{i,j,k}\right) - \left[\frac{\partial (uw)}{\partial x}\right]_{i,j,k} - \left[\frac{\partial (vw)}{\partial y}\right]_{i,j,k} - \left[\frac{\partial (w^2)}{\partial z}\right]_{i,j,k}$$
$$+ g_z - \frac{\beta}{2}\left(T^{(n+1)}_{i,j,k} + T^{(n+1)}_{i,j,k+1}\right) g_y\Big)$$
$$i = 1, \ldots, imax; \quad j = 1, \ldots, jmax; \quad k = 1, \ldots, kmax - 1 \tag{8}$$

4.2 Discretization

4.2.1 F

$$\left[\frac{\partial^2 u}{\partial x^2}\right]_{i,j,k} = \frac{u_{i+1,j,k} - 2u_{i,j,k} + u_{i-1,j,k}}{(\delta x)^2} \tag{9}$$

$$\left[\frac{\partial^2 u}{\partial y^2}\right]_{i,j,k} = \frac{u_{i,j+1,k} - 2u_{i,j,k} + u_{i,j-1,k}}{(\delta y)^2} \tag{10}$$

$$\left[\frac{\partial^2 u}{\partial z^2}\right]_{i,j,k} = \frac{u_{i,j,k+1} - 2u_{i,j,k} + u_{i,j,k-1}}{(\delta z)^2} \tag{11}$$

$$\left[\frac{\partial (u^2)}{\partial x}\right]_{i,j,k} = \frac{1}{\delta x}\left(\left(\frac{u_{i,j,k} + u_{i+1,j,k}}{2}\right)^2 - \left(\frac{u_{i-1,j,k} + u_{i,j,k}}{2}\right)^2\right) + \frac{\gamma}{\delta x}\left(\frac{|u_{i,j,k} + u_{i+1,j,k}|}{2}\frac{(u_{i,j,k} - u_{i+1,j,k})}{2} + \frac{|u_{i-1,j,k} + u_{i,j,k}|}{2}\frac{(u_{i-1,j,k} - u_{i,j,k})}{2}\right) \tag{12}$$

$$\left[\frac{\partial (uv)}{\partial y}\right]_{i,j,k} = \frac{1}{\delta y}\left(\frac{(v_{i,j,k} + v_{i+1,j,k})}{2}\frac{(u_{i,j,k} + u_{i,j+1,k})}{2} - \frac{(v_{i,j-1,k} + v_{i+1,j-1,k})}{2}\frac{(u_{i,j-1,k} + u_{i,j,k})}{2}\right) + \frac{\gamma}{\delta y}\left(\frac{|v_{i,j,k} + v_{i+1,j,k}|}{2}\frac{(u_{i,j,k} - u_{i,j+1,k})}{2} - \frac{|v_{i,j-1,k} + v_{i+1,j-1,k}|}{2}\frac{(u_{i,j-1,k} - u_{i,j,k})}{2}\right) \tag{13}$$

$$\left[\frac{\partial (uw)}{\partial z}\right]_{i,j,k} = \frac{1}{\delta z}\left(\frac{(w_{i,j,k} + w_{i+1,j,k})}{2}\frac{(u_{i,j,k} + u_{i,j,k+1})}{2} - \frac{(w_{i,j,k-1} + w_{i+1,j,k-1})}{2}\frac{(u_{i,j,k-1} + u_{i,j,k})}{2}\right) + \frac{\gamma}{\delta z}\left(\frac{|w_{i,j,k} + w_{i+1,j,k}|}{2}\frac{(u_{i,j,k} - u_{i,j,k+1})}{2} - \frac{|w_{i,j,k-1} + w_{i+1,j,k-1}|}{2}\frac{(u_{i,j,k-1} - u_{i,j,k})}{2}\right) \tag{14}$$

4.2.2 G

$$\left[\frac{\partial^2 v}{\partial x^2}\right]_{i,j,k} = \frac{v_{i+1,j,k} - 2v_{i,j,k} + v_{i-1,j,k}}{(\delta x)^2} \tag{15}$$

$$\left[\frac{\partial^2 v}{\partial y^2}\right]_{i,j,k} = \frac{v_{i,j+1,k} - 2v_{i,j,k} + v_{i,j-1,k}}{(\delta y)^2} \tag{16}$$

$$\left[\frac{\partial^2 v}{\partial z^2}\right]_{i,j,k} = \frac{v_{i,j,k+1} - 2v_{i,j,k} + v_{i,j,k-1}}{(\delta z)^2} \tag{17}$$

$$\begin{aligned}\left[\frac{\partial (uv)}{\partial x}\right]_{i,j,k} =& \frac{1}{\delta x}\left(\frac{(u_{i,j,k} + u_{i,j+1,k})}{2}\frac{(v_{i,j,k} + v_{i+1,j,k})}{2} - \frac{(u_{i-1,j,k} + u_{i-1,j+1,k})}{2}\frac{(v_{i-1,j,k} + v_{i,j,k})}{2}\right) + \\ & \frac{1}{\delta x}\left(\frac{|u_{i,j,k} + u_{i,j+1,k}|}{2}\frac{(v_{i,j,k} - v_{i+1,j,k})}{2} - \frac{|u_{i-1,j,k} + u_{i-1,j+1,k}|}{2}\frac{(v_{i-1,j,k} - v_{i,j,k})}{2}\right)\end{aligned} \tag{18}$$

$$\begin{aligned}\left[\frac{\partial (v^2)}{\partial y}\right]_{i,j,k} =& \frac{1}{\delta y}\left(\left(\frac{v_{i,j,k} + v_{i,j+1,k}}{2}\right)^2 - \left(\frac{v_{i,j-1,k} + v_{i,j,k}}{2}\right)^2\right) + \\ & \frac{\gamma}{\delta x}\left(\frac{|v_{i,j,k} + v_{i,j+1,k}|}{2}\frac{(v_{i,j,k} - v_{i,j+1,k})}{2} + \frac{|v_{i,j-1,k} + v_{i,j,k}|}{2}\frac{(v_{i,j-1,k} - u_{i,j,k})}{2}\right)\end{aligned} \tag{19}$$

$$\begin{aligned}\left[\frac{\partial (vw)}{\partial z}\right]_{i,j,k} =& \frac{1}{\delta z}\left(\frac{(w_{i,j,k} + w_{i,j+1,k})}{2}\frac{(v_{i,j,k} + v_{i,j,k+1})}{2} - \frac{(w_{i,j,k-1} + w_{i,j+1,k-1})}{2}\frac{(v_{i,j,k-1} + v_{i,j,k})}{2}\right) + \\ & \frac{\gamma}{\delta z}\left(\frac{|w_{i,j,k} + w_{i,j+1,k}|}{2}\frac{(v_{i,j,k} - v_{i,j,k+1})}{2} - \frac{|w_{i,j,k-1} + w_{i,j+1,k-1}|}{2}\frac{(v_{i,j,k-1} - v_{i,j,k})}{2}\right)\end{aligned} \tag{20}$$

4.2.3 H

$$\left[\frac{\partial^2 w}{\partial x^2}\right]_{i,j,k} = \frac{w_{i+1,j,k} - 2w_{i,j,k} + w_{i-1,j,k}}{(\delta x)^2} \tag{21}$$

$$\left[\frac{\partial^2 w}{\partial y^2}\right]_{i,j,k} = \frac{w_{i,j+1,k} - 2w_{i,j,k} + w_{i,j-1,k}}{(\delta y)^2} \tag{22}$$

$$\left[\frac{\partial^2 w}{\partial z^2}\right]_{i,j,k} = \frac{w_{i,j,k+1} - 2w_{i,j,k} + w_{i,j,k-1}}{(\delta z)^2} \tag{23}$$

$$\left[\frac{\partial(uw)}{\partial x}\right]_{i,j,k} = \frac{1}{\delta x}\left(\frac{(u_{i,j,k}+u_{i,j,k+1})}{2}\frac{(w_{i,j,k}+w_{i+1,j,k})}{2} - \frac{(u_{i-1,j,k}+u_{i-1,j,k+1})}{2}\frac{(w_{i-1,j,k}+w_{i,j,k})}{2}\right) + \frac{1}{\delta x}\left(\frac{|u_{i,j,k}+u_{i,j,k+1}|}{2}\frac{(w_{i,j,k}-w_{i+1,j,k})}{2} - \frac{|u_{i-1,j,k}+u_{i-1,j,k+1}|}{2}\frac{(w_{i-1,j,k}-w_{i,j,k})}{2}\right) \tag{24}$$

$$\left[\frac{\partial(vw)}{\partial y}\right]_{i,j,k} = \frac{1}{\delta y}\left(\frac{(v_{i,j,k}+v_{i,j,k+1})}{2}\frac{(w_{i,j,k}+w_{i,j+1,k})}{2} - \frac{(v_{i,j-1,k}+v_{i,j-1,k+1})}{2}\frac{(w_{i,j-1,k}+w_{i,j,k})}{2}\right) + \frac{\gamma}{\delta y}\left(\frac{|v_{i,j,k}+v_{i,j,k+1}|}{2}\frac{(w_{i,j,k}-w_{i,j+1,k})}{2} - \frac{|v_{i,j-1,k}+v_{i,j-1,k+1}|}{2}\frac{(w_{i,j-1,k}-w_{i,j,k})}{2}\right) \tag{25}$$

$$\left[\frac{\partial(w^2)}{\partial z}\right]_{i,j,k} = \frac{1}{\delta z}\left(\left(\frac{w_{i,j,k}+w_{i,j,k+1}}{2}\right)^2 - \left(\frac{w_{i,j,k-1}+w_{i,j,k}}{2}\right)^2\right) + \frac{\gamma}{\delta x}\left(\frac{|w_{i,j,k}+v_{i,j,k+1}|}{2}\frac{(w_{i,j,k}-w_{i,j,k+1})}{2} + \frac{|w_{i,j,k-1}+w_{i,j,k}|}{2}\frac{(w_{i,j,k-1}-w_{i,j,k})}{2}\right) \tag{26}$$

5 SOR solver

$$p_{i,j,k}^{it+1} = (1-\omega)p_{i,j,k}^{it} + \frac{\omega}{2(\frac{1}{(\delta x)^2}+\frac{1}{(\delta y)^2}+\frac{1}{(\delta z)^2})}\left(\frac{p_{i+1,j,k}^{it}+p_{i-1,j,k}^{it+1}}{(\delta x)^2} + \frac{p_{i,j+1,k}^{it}+p_{i,j-1,k}^{it+1}}{(\delta y)^2} + \frac{p_{i,j,k+1}^{it}+p_{i,j,k-1}^{it+1}}{(\delta z)^2} - rs_{i,j,k}\right)$$
$$it = 1,\ldots,itmax; \quad i = 1,\ldots,imax; \quad j = 1,\ldots,jmax; \quad k = 1,\ldots,kmax \tag{27}$$

$$res := \left(\sum_{i=1}^{imax}\sum_{j=1}^{jmax}\sum_{k=1}^{kmax}\left(\frac{p_{i+1,j,k}-2p_{i,j,k}+p_{i-1,j,k}}{(\delta x)^2} + \frac{p_{i,j+1,k}-2p_{i,j,k}+p_{i,j-1,k}}{(\delta y)^2} + \frac{p_{i,j,k+1}-2p_{i,j,k}+p_{i,j,k-1}}{(\delta z)^2} - rs_{i,j,k}\right)^2/(imax\cdot jmax\cdot kmax)\right)^{1/2} \tag{28}$$

6 calculate uvw

$$u_{i,j,k}^{(n+1)} = F_{i,j,k}^{(n)} - \frac{\delta t}{\delta x}\left(p_{i+1,j,k}^{(n+1)} - p_{i,j,k}^{(n+1)}\right)$$
$$i = 1,\ldots,imax-1; \quad j = 1,\ldots,jmax; \quad k = 1,\ldots,kmax \tag{29}$$

$$v_{i,j,k}^{(n+1)} = G_{i,j,k}^{(n)} - \frac{\delta t}{\delta y}\left(p_{i,j+1,k}^{(n+1)} - p_{i,j,k}^{(n+1)}\right) \qquad (30)$$
$$i = 1, \ldots, imax; \quad j = 1, \ldots, jmax - 1; \quad k = 1, \ldots, kmax$$

$$w_{i,j,k}^{(n+1)} = H_{i,j,k}^{(n)} - \frac{\delta t}{\delta z}\left(p_{i,j,k+1}^{(n+1)} - p_{i,j,k}^{(n+1)}\right) \qquad (31)$$
$$i = 1, \ldots, imax; \quad j = 1, \ldots, jmax; \quad k = 1, \ldots, kmax - 1$$

7 Energy Equation

7.1 equation

$$T_{i,j,k}^{(n+1)} = T_{i,j,k}^{(n)} + \delta t\left(\frac{1}{Re}\frac{1}{Pr}\left(\left[\frac{\partial^2 T}{\partial x^2}\right]_{i,j,k} + \left[\frac{\partial^2 T}{\partial y^2}\right]_{i,j,k} + \left[\frac{\partial^2 T}{\partial z^2}\right]_{i,j,k}\right) - \left[\frac{\partial (uT)}{\partial x}\right]_{i,j,k} - \left[\frac{\partial (vT)}{\partial y}\right]_{i,j,k} - \left[\frac{\partial (wT)}{\partial z}\right]_{i,j,k}\right) \qquad (32)$$

7.2 Discretization

$$\left[\frac{\partial^2 T}{\partial x^2}\right]_{i,j,k} = \frac{T_{i+1,j,k} - 2T_{i,j,k} + T_{i-1,j,k}}{(\delta x)^2} \qquad (33)$$

$$\left[\frac{\partial^2 T}{\partial y^2}\right]_{i,j,k} = \frac{T_{i,j+1,k} - 2T_{i,j,k} + T_{i,j-1,k}}{(\delta y)^2} \qquad (34)$$

$$\left[\frac{\partial^2 T}{\partial z^2}\right]_{i,j,k} = \frac{T_{i,j,k+1} - 2T_{i,j,k} + T_{i,j,k-1}}{(\delta z)^2} \qquad (35)$$

$$\begin{aligned}\left[\frac{\partial (uT)}{\partial x}\right]_{i,j,k} =& \frac{1}{\delta x}\left(u_{i,j,k}\frac{T_{i,j,k} + T_{i+1,j,k}}{2} - u_{i-1,j,k}\frac{T_{i-1,j,k} + T_{i,j,k}}{2}\right) + \\ & \frac{\gamma}{\delta x}\left(|u_{i,j,k}|\frac{T_{i,j,k} - T_{i+1,j,k}}{2} - |u_{i-1,j,k}|\frac{T_{i-1,j,k} - T_{i,j,k}}{2}\right)\end{aligned} \qquad (36)$$

$$\begin{aligned}\left[\frac{\partial (vT)}{\partial y}\right]_{i,j,k} =& \frac{1}{\delta y}\left(v_{i,j,k}\frac{T_{i,j,k} + T_{i,j+1,k}}{2} - v_{i,j-1,k}\frac{T_{i,j-1,k} + T_{i,j,k}}{2}\right) + \\ & \frac{\gamma}{\delta y}\left(|v_{i,j,k}|\frac{T_{i,j,k} - T_{i,j+1,k}}{2} - |v_{i,j-1,k}|\frac{T_{i,j-1,k} - T_{i,j,k}}{2}\right)\end{aligned} \qquad (37)$$

$$\begin{aligned}\left[\frac{\partial (wT)}{\partial z}\right]_{i,j,k} =& \frac{1}{\delta z}\left(w_{i,j,k}\frac{T_{i,j,k} + T_{i,j,k+1}}{2} - w_{i,j,k-1}\frac{T_{i,j,k-1} + T_{i,j,k}}{2}\right) + \\ & \frac{\gamma}{\delta z}\left(|w_{i,j,k}|\frac{T_{i,j,k} - T_{i,j,k+1}}{2} - |w_{i,j,k-1}|\frac{T_{i,j,k-1} - T_{i,j,k}}{2}\right)\end{aligned} \qquad (38)$$