

CHAPTER-1  
RELATIONS AND FUNCTIONS

## EXERCISE - 1.1

1. Determine whether each of the following relations are reflexive, symmetric and transitive:
  - (i) Relation  $R$  in the set  $A = \{1, 2, 3, \dots, 13, 14\}$  defined as  $R = \{(x, y) : 3x - y = 0\}$
  - (ii) Relation  $R$  in the set  $N$  of natural numbers defined as  $R = \{(x, y) : y = x + 5 \text{ and } x \leq 4\}$
  - (iii) Relation  $R$  in the set  $A = 1, 2, 3, 4, 5, 6$  as  $R = \{(x, y) : y \text{ is divisible by } x\}$
  - (iv) Relation  $R$  in the set  $Z$  of all integers defined as  $R = \{(x, y) : x - y \text{ is an integer}\}$
  - (v) Relation  $R$  in the set  $A$  of human beings in a town at a particular time given by
    - (a)  $R = \{(x, y) : x \text{ and } y \text{ work at the same place}\}$
    - (b)  $R = \{(x, y) : x \text{ and } y \text{ live in the same locality}\}$
    - (c)  $R = \{(x, y) : x \text{ is exactly 7 cm taller than } y\}$
    - (d)  $R = \{(x, y) : x \text{ is wife of } y\}$  (e)  $R = \{(x, y) : x \text{ is father of } y\}$
2. Show that the relation  $R$  in the set  $R$  of real numbers, defined as  $R = \{(a, b) : ab^2\}$  is neither reflexive nor symmetric nor transitive.
3. Check whether the relation  $R$  defined in the set  $\{1, 2, 3, 4, 5, 6\}$  as  $R = \{(a, b) : b = a + 1\}$  is reflexive, symmetric or transitive.
4. Show that the relation  $R$  in  $R$  defined as  $R = \{(a, b) : a \leq b\}$ , is reflexive and transitive but not symmetric.
5. Check whether the relation  $R$  in  $R$  defined by  $R = \{(a, b) : a \leq b^3\}$  is reflexive, symmetric or transitive.
6. Show that the relation  $R$  in the set  $\{1, 2, 3\}$  given by  $R = \{(1, 2), (2, 1)\}$  is symmetric but neither reflexive nor transitive.

7. Show that the relation  $R$  in the set  $A$  of all the books in a library of a college, given by  $R = \{(x, y) : x \text{ and } y \text{ have same number of pages}\}$  is an equivalence relation.
8. Show that the relation  $R$  in the set  $A = \{1, 2, 3, 4, 5\}$  given by  $R = \{(a, b) : |a-b| \text{ is even}\}$ , is an equivalence relation. Show that all the elements of  $\{1, 3, 5\}$  are related to each other and all the elements of  $\{2, 4\}$  are related to each other. But no element of  $\{1, 3, 5\}$  is related to any element of  $\{2, 4\}$ .
9. Show that each of the relation  $R$  in the set  $A = \{x \in \mathbb{Z} : 0 \leq x \leq 12\}$ , given by
- (i)  $R = \{(a, b) : |a-b| \text{ is a multiple of } 4\}$
  - (ii)  $R = \{(a, b) : a = b\}$
- is an equivalence relation. Find the set of all elements related to 1 in each case.
10. Give an example of a relation. Which is
- (i) Symmetric but neither reflexive nor transitive.
  - (ii) Transitive but neither reflexive nor symmetric.
  - (iii) Reflexive and symmetric but not transitive.
  - (iv) Reflexive and transitive but not symmetric.
  - (v) Symmetric and transitive but not reflexive.
11. Show that the relation  $R$  in the set  $A$  of points in a plane given by  $R = \{(P, Q) : \text{distance of the point } P \text{ from the origin is same as the distance of the point } Q \text{ from the origin}\}$ , is an equivalence relation. Further, show that the set of all points related to a point  $P \neq (0, 0)$  is the circle passing through  $P$  with origin as centre.
12. Show that the relation  $R$  defined in the set  $A$  of all triangles as  $R = \{(T_1, T_2) : T_1 \text{ is similar to } T_2\}$ , is equivalence relation. Consider three right angle triangles  $T_1$  with sides 3, 4, 5,  $T_2$  with sides 5, 12, 13 and  $T_3$  with sides 6, 8, 10. Which triangles among  $T_1$ ,  $T_2$  and  $T_3$  are related?
13. Show that the relation  $R$  defined in the set  $A$  of all polygons as  $R = \{(P_1, P_2) : P_1 \text{ and } P_2 \text{ have same number of sides}\}$ , is an equivalence

relation. What is the set of all elements in A related to the right angle triangle T with sides 3, 4 and 5?

14. Let L be the set of all lines in XY plane and R be the relation in L defined as  $R = \{(L_1, L_2) : L_1 \text{ is parallel to } L_2\}$ . Show that R is an equivalence relation. Find the set of all lines related to the line  $y = 2x + 4$ .

15. Let R be the relation in the set  $\{1, 2, 3, 4\}$  given by  $R = \{(1, 2), (2, 2), (1, 1), (4, 4), (1, 3), (3, 3), (3, 2)\}$ . Choose the correct answer.

(A) R is reflexive and symmetric but not transitive.

(B) R is reflexive and transitive but not symmetric.

(C) R is symmetric and transitive but not reflexive.

(D) R is an equivalence relation.

16. Let R be the relation in the set N given by  $R = \{(a, b) : a = b - 2, b > 6\}$ . Choose the correct answer?

(A)  $(2, 4) \in R$  (B)  $(3, 8) \in R$  (C)  $(6, 8) \in R$  (D)  $(8, 7) \in R$