

1. (a) For final score set $FS = \{fs_1, fs_2, fs_3, \dots, fs_{1000}\}$, the mean could be computed as $\mu = \frac{1}{1000} \sum_{i=1}^{1000} fs_i = 87.084$. The standard deviation could be computed as $\sigma = \sqrt{\frac{1}{1000} \sum_{i=1}^{1000} (fs_i - \mu)^2} \approx 10.914$

(b) We could sort the final score set $FS = \{fs_1, fs_2, fs_3, \dots, fs_{1000}\}$ in ascending order into sorted final score set $OF S = \{ofs_1, ofs_2, ofs_3, \dots, ofs_{1000}\}$. It is a one-to-one-mapping, where ofs_1 means the minimum of the final score set, ofs_2 means the second minimum of the final score set and so on.

The first quartile Q_1 is 25th percentile, which is $ofs_{250} = 82$. The median is $ofs_{500} = 89$. The third quartile is 75th percentile, which is $ofs_{750} = 96$

(c) Maximum is $ofs_{1000} = 100$. Minimum is $ofs_1 = 35$

(d) Mode is the number with highest frequency. Suppose a set with 101 elements in it: $S = \{0, 0, 0, 0, \dots, 0\}$, where s_j represents the j th element of S . For any fs_i in FS , there exists a s_j in S so that $fs_i = j$. Then $s_{j+} = 1$. After visiting all elements in FS and one thousand time additions, s_k is the largest element of S . k is the mode of S . $k = 97$

(e) YES.

Median is 89. Mean is 87.084. Standard deviation is 10.914.

$$87.084 - 10.914 = 76.17 < 89 < 97.998 = 87.084 + 10.914.$$

(f) NO.

$$(maximum - median) = (100 - 89) = 11 > 10.5 = 1.5 * (Q_3 - median)$$

2. Median is at the middle of the data. Define the hourly pay in SkyNet as $HP = \{x_1, x_2, x_3, \dots, x_{583}\}$ in ascending order, where 583 is calculated by adding all the frequency together. There are $\frac{583}{2} = 291.5$ elements before the median x_{median} . There are $54 + 74 + 89 = 219$ smaller than $x_{219} = 25$. There are $219 + 102 = 321$ elements smaller than $x_{321} = 30$. Therefore, x_{median} is between 25 and 30. $291.5 - 219 = 102 * \frac{x_{median} - 25}{30 - 25}$. Therefore, $x_{median} = 28.554$.

3.(a) z-score could be calculated by $zfs_i = \frac{fs_i - \mu}{\sigma}$, where fs is initial final score, μ is expected value of fs , σ is standard deviation of fs . μ could be calculated by $\mu = \frac{1}{1000} \sum_{i=1}^{1000} fs_i = 87.084$. σ could be computed by $\sigma = \sqrt{\frac{1}{1000} \sum_{i=1}^{1000} (fs_i - \mu)^2} \approx 10.914$, $\mu_z = 0$.

Variance of finals_original could be calculated by $\frac{1}{1000} \sum_{i=1}^{1000} (fs_i - \mu)^2 \approx 119.113$.
Variance of finals_normalized could be calculated by $\frac{1}{1000} \sum_{i=1}^{1000} (zfs_i - \mu_z)^2 = 1$.

$$(b) zfs_{e1} = \frac{fs_{e1} - \mu}{\sigma} = \frac{90 - \mu}{\sigma} \approx 0.267$$

(c) Pearson's correlation coefficient could be calculated by $r_{A,B} = \frac{Cov(A,B)}{\sigma_A \sigma_B}$, where $Cov(A, B) = \sum_{i=1}^n (a_i - \mu_A)(b_i - \mu_B)$. Pearson's correlation coefficient between midterm-original and finals_original could be calculated by

$$r_{mo,fo} = \frac{Cov(mo,fo)}{\sigma_{mo} \sigma_{fo}} = \frac{\sum_{i=1}^{1000} (mo_i - \mu_{mo})(fo_i - \mu_{fo})}{\sqrt{\frac{1}{1000} \sum_{i=1}^{1000} (mo_i - \mu_{mo})^2} \sqrt{\frac{1}{1000} \sum_{i=1}^{1000} (fo_i - \mu_{fo})^2}} \approx 0.544.$$

(d) Pearson's correlation coefficient between midterm-original and finals_normalized could be calculated by

$$r_{mo,fn} = \frac{Cov(mo,fn)}{\sigma_{mo} \sigma_{fn}} = \frac{\sum_{i=1}^{1000} (mo_i - \mu_{mo})(fn_i - \mu_{fn})}{\sqrt{\frac{1}{1000} \sum_{i=1}^{1000} (mo_i - \mu_{mo})^2} \sqrt{\frac{1}{1000} \sum_{i=1}^{1000} (fn_i - \mu_{fn})^2}} \approx 0.544.$$

$$(e) Cov(mo, fo) = \sum_{i=1}^{1000} (mo_i - \mu_{mo})(fo_i - \mu_{fo}) \approx 78.176.$$

4.(a) Minkowski distance could be calculated by

$d(CML, CBL) = \sqrt[h]{|n_{m1} - n_{b1}|^h + |n_{m2} - n_{b2}|^h + \dots + |n_{m100} - n_{b100}|^h}$, where n_{mi} means the number of i th book of CML, n_{bi} means the number of i th book of CBL.

When $h=1$,

$$d_1(CML, CBL) = |n_{m1} - n_{b1}| + |n_{m2} - n_{b2}| + \dots + |n_{m100} - n_{b100}| = 6152.$$

When $h=2$,

$$d_2(CML, CBL) = \sqrt{|n_{m1} - n_{b1}|^2 + |n_{m2} - n_{b2}|^2 + \dots + |n_{m100} - n_{b100}|^2} \approx 715.328.$$

When $h=3$,

$$d_3(CML, CBL) = \sqrt[3]{|n_{m1} - n_{b1}|^3 + |n_{m2} - n_{b2}|^3 + \dots + |n_{m100} - n_{b100}|^3}$$

$$= \max_{i=1}^{\infty} |n_{mi} - n_{bi}| = 170.$$

(b) I don't agree.

$$d_1(CML, CBL) = 6125 > d_2(CML, CBL) = 715.328 > d_3(CML, CBL) = 170.$$

(c) Possibility density function of CML and CBL could be calculated by

$P_{CML}(i) = \frac{n_{mi}}{\sum_{j=1}^{100} n_{mj}}$ and $P_{CBL}(i) = \frac{n_{bi}}{\sum_{j=1}^{100} n_{bj}}$. Assume the i th element of P_{CML} is p_i and the i th element of P_{CBL} is q_i . $D_{KL}(CML||CBL) = \sum_{i=1}^{100} p_i \log(\frac{p_i}{q_i}) \approx 0.207$.

5.(a) Distance for binary attributes could be calculated by $d(i, j) = \frac{r+s}{q+r+s+t}$, where i, j represents two attributes, q, r, s, t represents the same meaning in slides. For Buy Beer and Buy Diaper, the distance could be calculated by $d(BB, BD) = \frac{10+40}{3500} = \frac{1}{70} \approx 0.0143$.

(b) Jaccard coefficient could be calculated by $sim_{Jaccard}(i, j) = \frac{q}{q+r+s}$, where i, j, q, r, s represents the same meaning in slides. For Buy Beer and Buyr Diaper, the Jaccard coefficient could be calculated by $sim_{Jaccard}(BB, BD) = \frac{150}{150+10+40} = 0.75$.

(c) Assume two nominal attributes A and B, where A has value $\{a_1, a_2, \dots, a_c\}$, B has value $\{b_1, b_2, \dots, b_r\}$

e_{ij} represents the expected frequency in each entry (i, j) and could be calculated by $e_{ij} = \frac{count(A=a_i) \times count(B=b_j)}{n}$.

The χ^2 statistic could be calculated by $\chi^2 = \sum_{i=1}^c \sum_{j=1}^r \frac{(o_{ij} - e_{ij})^2}{e_{ij}}$, where o_{ij} is the observed frequency.

$$e_{11} = \frac{160 \times 190}{3500} \approx 8.686$$

$$e_{12} = \frac{190 \times 3340}{3500} \approx 181.314$$

$$e_{21} = \frac{160 \times 3310}{3500} \approx 151.314$$

$$e_{22} = \frac{3310 \times 3340}{3500} \approx 33158.686$$

$$\chi^2 = \frac{(o_{11} - e_{11})^2}{e_{11}} + \frac{(o_{12} - e_{12})^2}{e_{12}} + \frac{(o_{21} - e_{21})^2}{e_{21}} + \frac{(o_{22} - e_{22})^2}{e_{22}} \approx 2547.582$$

(d) Yes

We could know that for one freedom (calculated by $(2-1) \times (2-1)$) and $\alpha = 0.05$, value need to reject null hypothesis is 3.841. $2547.582 > 3.841$. Therefore, we could reject the hypothesis.