CS 412 Intro. to Data Mining

Chapter 5. Mining Frequent Patterns, Association and Correlations: Basic Concepts and Methods

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Chapter 6: Mining Frequent Patterns, Association and Correlations: Basic Concepts and Methods

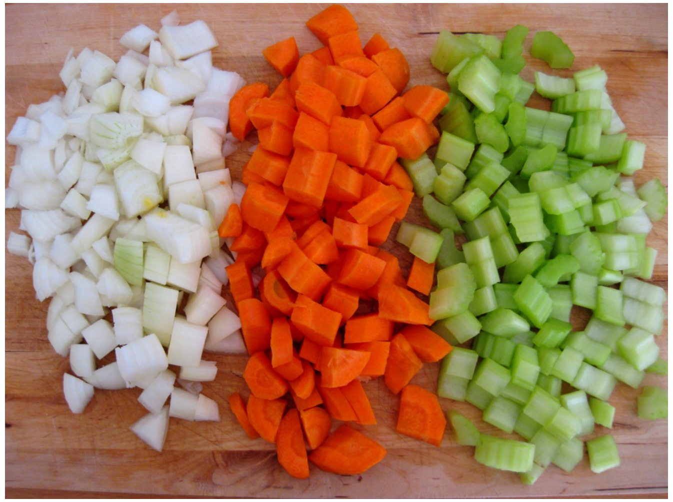
- Basic Concepts
- Efficient Pattern Mining Methods
- Pattern Evaluation
- Summary

Pattern Discovery: Basic Concepts

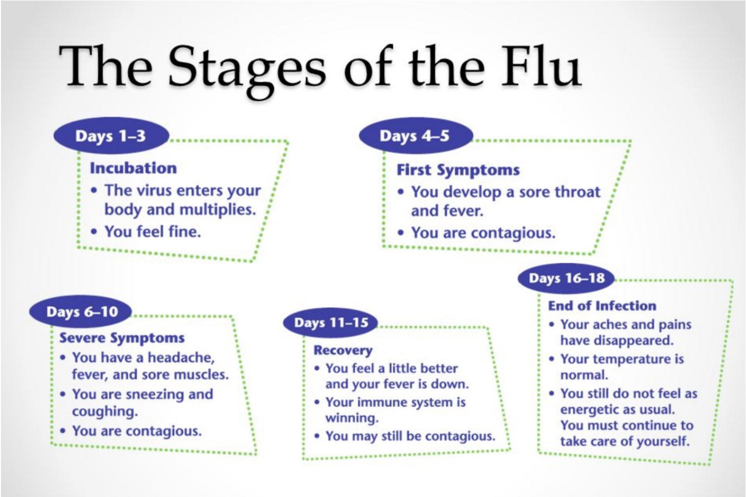
- What Is Pattern Discovery? Why Is It Important?
- Basic Concepts: Frequent Patterns and Association Rules
- Compressed Representation: Closed Patterns and Max-Patterns

What are Patterns?

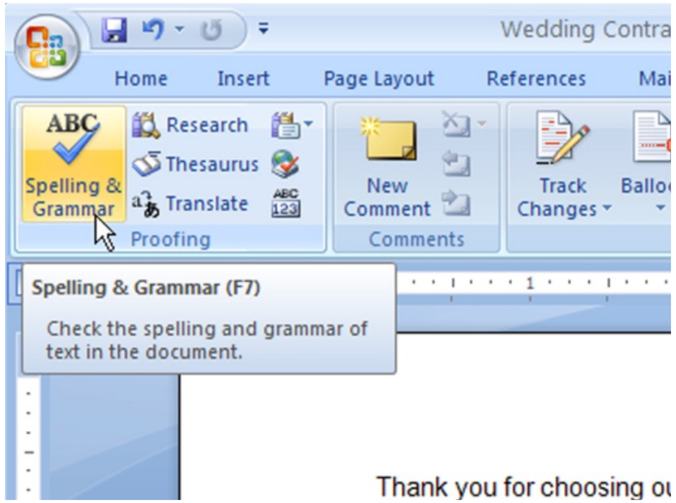
- What are patterns?
  - Patterns: A set of items, subsequences, or substructures that occur frequently together (or strongly correlated) in a data set
  - Patterns represent intrinsic and important properties of datasets



Frequent item set



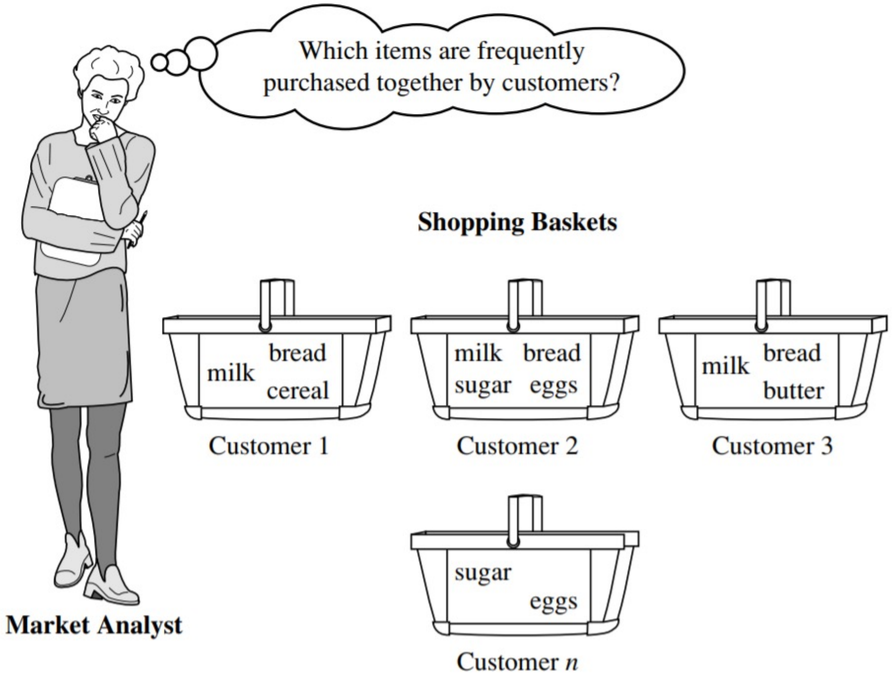
Frequent sequences



Frequent structures

What Is Pattern Discovery?

- Pattern discovery: Uncovering patterns from massive data sets
- It can answer questions such as:
  - What products were often purchased together?
  - What are the subsequent purchases after buying an iPad?



Pattern Discovery: Why Is It Important?

- Foundation for many essential data mining tasks
  - Association, correlation, and causality analysis
  - Mining **sequential**, structural (e.g., sub-graph) patterns
  - **Classification**: Discriminative pattern-based analysis
  - **Cluster** analysis: Pattern-based subspace clustering
- Broad applications
  - Market basket analysis, cross-marketing, catalog design, sale campaign analysis, Web log analysis, biological sequence analysis
  - Many types of data: spatiotemporal, multimedia, time-series, and stream data

Basic Concepts: Transactional Database

- Transactional Database (TDB)
  - Each transaction is associated with an identifier, called a TID.
  - May also have counts associated with each item sold

Tid	Items bought
1	Beer, Nuts, Diaper
2	Beer, Coffee, Diaper
3	Beer, Diaper, Eggs
4	Nuts, Eggs, Milk
5	Nuts, Coffee, Diaper, Eggs, Milk

Basic Concepts: k-Itemsets and Their Supports

- Itemset: A set of one or more items

$$I = \{ I_1, I_2, \cdots, I_m \}$$

- k-itemset: An itemset containing k items:

  $X = \{x_1, \ldots, x_k\}$

  - Ex. {Beer, Nuts, Diaper} is a 3-itemset

- Absolute support (count)
  - sup{X} = occurrences of an itemset X
    - Ex. sup{Beer} = 3
    - Ex. sup{Diaper} = 4
    - Ex. sup{Beer, Diaper} = 3
    - Ex. sup{Beer, Eggs} = 1

Tid	Items bought
1	Beer, Nuts, Diaper
2	Beer, Coffee, Diaper
3	Beer, Diaper, Eggs
4	Nuts, Eggs, Milk
5	Nuts, Coffee, Diaper, Eggs, Milk

- Relative support
  - *s*{*X*} = The fraction of transactions that contains X (i.e., the probability that a transaction contains X)
  - Ex. s{Beer} = 3/5 = 60%
  - Ex. s{Diaper} = 4/5 = 80%
  - Ex. s{Beer, Eggs} = 1/5 = 20%

Basic Concepts: Frequent Itemsets (Patterns)

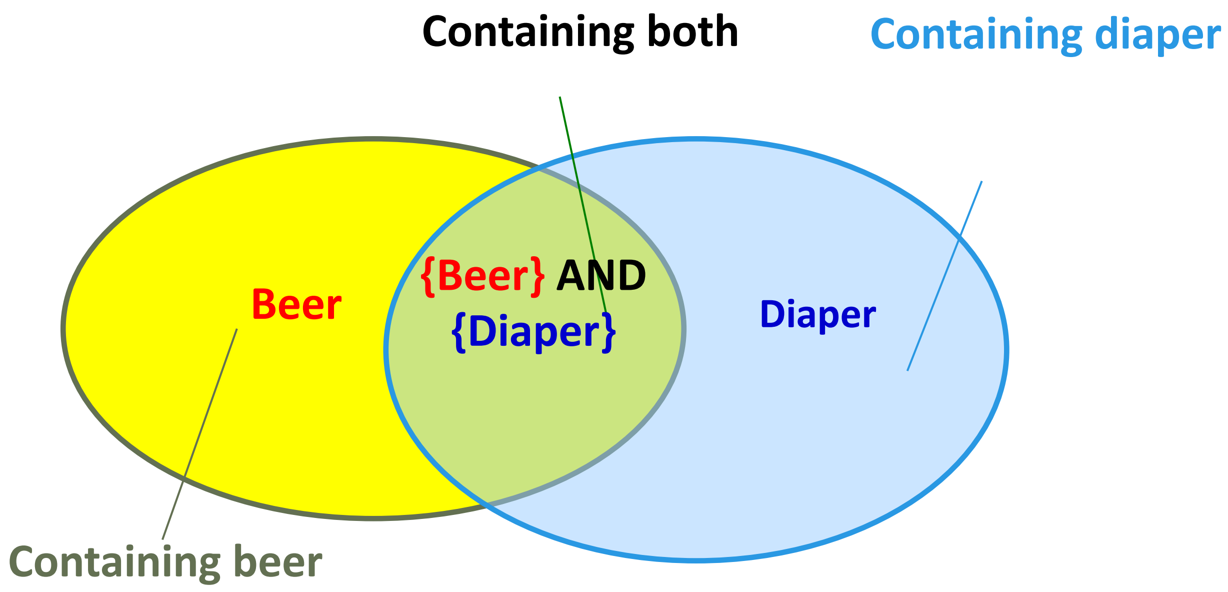
- An itemset (or a pattern) X is *frequent* if the support of X is no less than a *minsup* threshold σ
- Let σ = *50%* (σ: *minsup* threshold) for the given 5-transaction dataset
  - All the frequent 1-itemsets:
    - Beer: 3/5 (60%); Nuts: 3/5 (60%); Diaper: 4/5 (80%); Eggs: 3/5 (60%)
  - All the frequent 2-itemsets:
    - {Beer, Diaper}: 3/5 (60%)
  - All the frequent 3-itemsets?
    - None

Tid	Items bought
1	Beer, Nuts, Diaper
2	Beer, Coffee, Diaper
3	Beer, Diaper, Eggs
4	Nuts, Eggs, Milk
5	Nuts, Coffee, Diaper, Eggs, Milk

- Why do these itemsets (shown on the left) form the complete set of frequent *k*-itemsets (patterns) for any *k*?
- **Observation**: We may need an efficient method to mine a complete set of frequent patterns

From Frequent Itemsets to Association Rules

- Compared with itemsets, association rules can be more telling
  - Ex. *Diaper* → *Beer*
    - *Buying diapers may likely lead to buying beers*



Transactions with {Beer} AND {Diaper} = {Beer, Diaper}

Note: X AND Y:

- Transactions contain both X and Y

Association Rules

- How do we compute the strength of an association rule $X \rightarrow Y$ (Both X and Y are itemsets)?
- We first compute the following two metrics, s and c.
  - Support of X AND Y
    - Ex. s{Diaper, Beer} = 3/5 = 0.6 (i.e., 60%)
  - Confidence of $X \rightarrow Y$
    - The *conditional probability* that a transaction containing X also contains *Y:*

      c = sup(X, Y) / sup(X)
    - Ex. c = sup{Diaper, Beer}/sup{Diaper} = ¾ = 0.75

Tid	Items bought
1	Beer, Nuts, Diaper
2	Beer, Coffee, Diaper
3	Beer, Diaper, Eggs
4	Nuts, Eggs, Milk
5	Nuts, Coffee, Diaper, Eggs, Milk

- In pattern analysis, we are often interested in those rules that dominate the database, and these two metrics ensure the popularity and correlation of X and Y.

Mining Frequent Itemsets and Association Rules

- **Association rule mining**
  - Given two thresholds: *minsup, minconf*
  - Find all of the rules, $X \rightarrow Y$ (s, c) such that s ≥ *minsup* and *c* ≥ *minconf*
- Let *minsup = 50%*
  - Freq. 1-itemsets: Beer: 3, Nuts: 3, Diaper: 4, Eggs: 3
  - Freq. 2-itemsets: {Beer, Diaper}: 3
- Let *minconf = 50%*
  - *Beer* → *Diaper* (60%, 100%)
  - *Diaper* → *Beer* (60%, 75%)

(Q: Are these all the rules?)

Tid	Items bought
1	Beer, Nuts, Diaper
2	Beer, Coffee, Diaper
3	Beer, Diaper, Eggs
4	Nuts, Eggs, Milk
5	Nuts, Coffee, Diaper, Eggs, Milk

- **Observations:**
  - Mining association rules and mining frequent patterns are very close problems
  - Scalable methods are needed for mining large datasets

Challenge: There Are Too Many Frequent Patterns!

- A long pattern contains a combinatorial number of sub-patterns
- How many frequent itemsets does the following TDB_1 contain (minsup = 1)?
  - $TDB_{1:}$ T_1: $\{a_1, \ldots, a_{50}\}$; T_2: $\{a_1, \ldots, a_{100}\}$
  - All frequent patterns with minsup = 1:

1-itemsets: $\{a_1\}$: 2, $\{a_2\}$: 2, …, $\{a_{50}\}$: 2, $\{a_{51}\}$: 1, …, $\{a_{100}\}$: 1,

2-itemsets: $\{a_1, a_2\}$: 2, …, $\{a_1, a_{50}\}$: 2, $\{a_1, a_{51}\}$: 1 …, …, $\{a_{99}, a_{100}\}$: 1,

…, …, …, …

99-itemsets: $\{a_1, a_2, \ldots, a_{99}\}$: 1, …, $\{a_2, a_3, \ldots, a_{100}\}$: 1

100-itemset: $\{a_1, a_2, \ldots, a_{100}\}$: 1

- The total number of frequent itemsets:

$$\binom{100}{1} + \binom{100}{2} + \binom{100}{3} + \cdots + \binom{100}{100} = 2^{100} - 1$$

Too huge set for any one to compute or store!

Expressing Patterns in Compressed Form

- How to reduce the redundancy of the list of all the frequent itemsets?
  - If $\{a_1, \ldots, a_{99}\}$ and $\{a_1, \ldots, a_{100}\}$ have the same support in the database, then we don't need to list both of them
- Solution 1: **Closed patterns**: A pattern (itemset) X is closed if X is *frequent,* and there exists *no super-pattern* Y ɔ X, *with the same support* as X
  - Ex. TDB_1: T_1: $\{a_1, \ldots, a_{50}\}$; T_2: $\{a_1, \ldots, a_{100}\}$
  - Suppose *minsup* = 1. How many closed patterns does TDB_1 contain?
    - Two: P_1: "$\{a_1, \ldots, a_{50}\}$: 2"; P_2: "$\{a_1, \ldots, a_{100}\}$: 1"

Expressing Patterns in Compressed Form: Closed Patterns

- Closed pattern is a lossless compression of frequent patterns
  - Reduces the # of patterns but does not lose the support information!
  - Given P_1: "$\{a_1, \ldots, a_{50}\}$: 2"; P_2: "$\{a_1, \ldots, a_{100}\}$: 1"
  - You will still be able to say: "$\{a_2, \ldots, a_{40}\}$: 2", "$\{a_5, a_{51}\}$: 1"

Expressing Patterns in Compressed Form: Maximal Patterns

- Solution 2: **Max-patterns**: A pattern X is a maximal pattern if X is frequent and there exists no frequent super-pattern Y ɔ X
- Difference from close-patterns?
  - Do not care the real support of the sub-patterns of a max-pattern
  - Let Transaction DB TDB_1: T_1: $\{a_1, \ldots, a_{50}\}$; T_2: $\{a_1, \ldots, a_{100}\}$
  - Suppose *minsup* = 1. How many max-patterns does TDB_1 contain?
    - One: P: "$\{a_1, \ldots, a_{100}\}$: 1"

Expressing Patterns in Compressed Form: Maximal Patterns

- Maximal pattern is a lossy compression!
  - We only know a subset of the max-pattern P, $\{a_1, \ldots, a_{40}\}$, is frequent
  - But we do not know the real support of $\{a_1, \ldots, a_{40}\}$, …, any more!
- Thus in many applications, mining closed-patterns is more desirable than mining maximal patterns

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Efficient Pattern Mining Methods

- The Downward Closure Property of Frequent Patterns
  - The Apriori Algorithm
  - Extensions or Improvements of Apriori
- Mining Frequent Patterns by Exploring Vertical Data Format
- FPGrowth: A Frequent Pattern-Growth Approach
- Mining Closed Patterns

The Downward Closure Property of Frequent Patterns

- **Frequent** itemset: $\{a_1, \ldots, a_{50}\}$
  - Subsets are all **frequent**: $\{a_1\}$, $\{a_2\}$, …, $\{a_{50}\}$, $\{a_1, a_2\}$, …, $\{a_1, \ldots, a_{49}\}$, …
- Downward closure (Apriori): Any subset of a frequent itemset must be frequent
  - If **{beer, diaper, nuts}** is frequent, so is **{beer, diaper}**
  - If any subset of an itemset S is **infrequent**, then there is no chance for S to be frequent.

A sharp knife for pruning!

Apriori Pruning and Scalable Mining Methods

- Apriori pruning principle: If there is any itemset which is infrequent, its superset should not even be generated! (Agrawal & Srikant @VLDB'94, Mannila, et al. @ KDD' 94)
- Scalable mining Methods: Three major approaches
  - Level-wise, join-based approach: Apriori (Agrawal & Srikant@VLDB'94)
  - Vertical data format approach: Eclat (Zaki, Parthasarathy, Ogihara, Li @KDD'97)
  - Frequent pattern projection and growth: FPgrowth (Han, Pei, Yin @SIGMOD'00)

Apriori: A Candidate Generation & Test Approach

- Outline of Apriori (level-wise, candidate generation and test)
  - Scan DB once to get frequent 1-itemset
  - Repeat
    - Generate length-(k+1) candidate itemsets from length-k frequent itemsets
    - Test the candidates against DB to find frequent (k+1)-itemsets
    - Set k := k +1
  - Until no frequent or candidate set can be generated
  - Return all the frequent itemsets derived

The Apriori Algorithm (Pseudo-Code)

C_k: Candidate itemset of size k

F_k : Frequent itemset of size k

K := 1;

F_k := {frequent items}; // frequent 1-itemset

While (F_k != $\emptyset$) **do {** // when F_k is non-empty

C_{k+1} := candidates generated from F_k; // candidate generation

Derive F_{k+1} by counting candidates in C_{k+1} with respect to *TDB* at minsup;

k := k + 1

}

return $\cup_k F_k$ // return F_k generated at each level

The Apriori Algorithm—An Example

minsup = 2

Database TDB

Tid	Items
10	A, C, D
20	B, C, E
30	A, B, C, E
40	B, E

1^{st} scan

C_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{D}	1
{E}	3

F_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{E}	3

C_2

Itemset
{A, B}
{A, C}
{A, E}
{B, C}
{B, E}
{C, E}

2^{nd} scan

C_2

Itemset	sup
{A, B}	1
{A, C}	2
{A, E}	1
{B, C}	2
{B, E}	3
{C, E}	2

F_2

Itemset	sup
{A, C}	2
{B, C}	2
{B, E}	3
{C, E}	2

C_3

Itemset
{B, C, E}

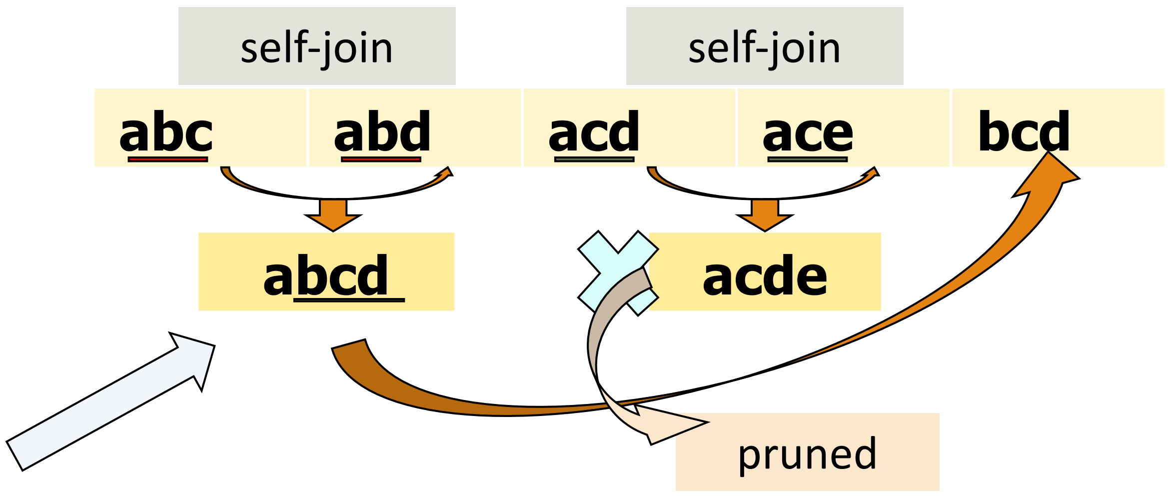
3^{rd} scan

F_3

Itemset	sup
{B, C, E}	2

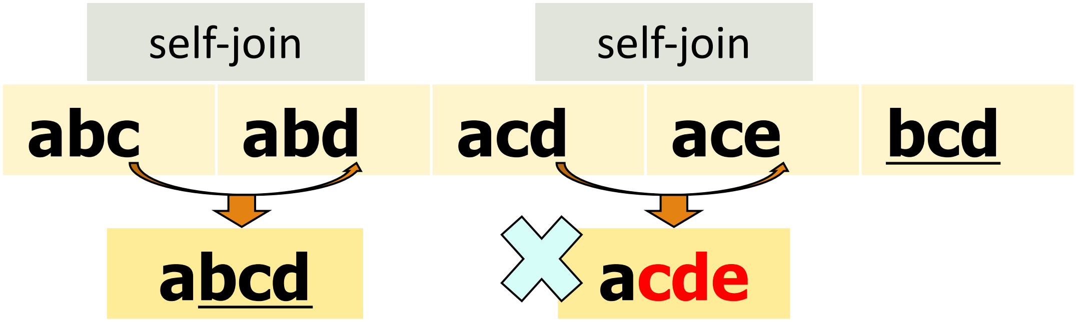
Apriori: Implementation Tricks

- How to generate candidates?
  - Step 1: self-joining F_k
  - Step 2: pruning
- Example of candidate-generation
  - $F_3 = \{abc, abd, acd, ace, bcd\}$
  - Self-joining: $F_3 * F_3$
    - *abcd* from *abc* and *abd*
    - *acde* from *acd* and *ace*
  - Pruning:
    - *acde* is removed because *ade* is not in F_3
  - $C_4 = \{abcd\}$



Candidate Generation (Pseudo-Code)

- Suppose the items in F_{k-1} are listed in an order
- // Step 1: Joining



for each p **in** F_{k-1}

    for each q **in** F_{k-1}

        if $p.item_1 = q.item_1, \ldots, p.item_{k-2} = q.item_{k-2}, p.item_{k-1} < q.item_{k-1}$ **{**

            c = **join**(p, q)

- // Step 2: pruning

            if has_infrequent_subset(c, F_{k-1})

                continue // prune

            else add c **to** C_k

        }

Apriori: Improvements and Alternatives

- Reduce passes of transaction database scans
  - **Partitioning** (e.g., Savasere, et al., 1995)
  - Dynamic itemset counting (Brin, et al., 1997)
- Shrink the number of candidates
  - **Hashing** (e.g., DHP: Park, et al., 1995)
  - Pruning by support lower bounding (e.g., Bayardo 1998)
  - Sampling (e.g., Toivonen, 1996)
- Exploring special data structures
  - Tree projection (Agarwal, et al., 2001)
  - H-miner (Pei, et al., 2001)
  - Hypercube decomposition (e.g., LCM: Uno, et al., 2004)

To be discussed in subsequent slides

To be discussed in subsequent slides

Partitioning: Scan Database Only Twice

- Theorem: *Any itemset that is potentially frequent in TDB must be frequent in at least one of the partitions of TDB*

Here is the proof!

TDB_1 + TDB_2 + . . . + TDB_k = TDB

$sup_1(X) > \sigma|TDB_1|$ or $sup_2(X) > \sigma|TDB_2|$ or . . . $sup_k(X) > \sigma|TDB_k|$ $sup(X) > \sigma|TDB|$

Partitioning: Scan Database Only Twice

- Method: Scan DB **twice** (A. Savasere, E. Omiecinski and S. Navathe, *VLDB'95*)
  - Scan 1: Partition database so that each partition can fit in main memory (why?)
    - Mine local frequent patterns in this partition
  - Scan 2: Consolidate global frequent patterns
    - Find global frequent itemset candidates (those frequent in at least one partition)
    - Find the true frequency of those candidates, by scanning TDB_i one more time

Direct Hashing and Pruning (DHP)

- **Hashing**: v = *hash*(itemset)
- **1st scan**: When counting the 1-itemset, hash 2-itemset to calculate the bucket count
- Example: At the 1st scan of TDB, count 1-itemset, and hash 2-itemsets in the transaction to its bucket
  - {ab, ad, ce}
  - {bd, be, de}
  - …
- At the end of the first scan,
  - *if minsup = 80, remove ab, ad, ce, since count{ab, ad, ce}* < 80

Itemsets	Count
{ab, ad, ce}	35
{bd, be, de}	298
……	…
{yz, qs}	58

V might be same for different itemset

Hash Table

Check the minsup

Direct Hashing and Pruning (DHP)

Table 5.1: Transactional Data for an *AllElectronics* Branch

TID	*List of item_IDs*
T100	I1, I2, I5
T200	I2, I4
T300	I2, I3
T400	I1, I2, I4
T500	I1, I3
T600	I2, I3
T700	I1, I3
T800	I1, I2, I3, I5
T900	I1, I2, I3

L_1

Itemset	Sup. count
{I1}	6
{I2}	7
{I3}	6
{I4}	2
{I5}	2

Hash for {Ix,Iy}

Create hash table H_2 using hash function $h(x, y) = ((order\ of\ x) \times 10 + (order\ of\ y))\ mod\ 7$

H_2

bucket address	0	1	2	3	4	5	6
bucket count	2	2	4	2	2	4	4
bucket contents	{I1, I4} {I3, I5}	{I1, I5} {I1, I5}	{I2, I3} {I2, I3} {I2, I3} {I2, I3}	{I2, I4} {I2, I4}	{I2, I5} {I2, I5}	{I1, I2} {I1, I2} {I1, I2} {I1, I2}	{I1, I3} {I1, I3} {I1, I3} {I1, I3}

Exploring Vertical Data Format: ECLAT

- ECLAT (Equivalence Class Transformation): A **depth-first search** algorithm using set intersection [Zaki et al. @KDD'97]
- Vertical format
- Properties of Tid-Lists
  - $t(X) = t(Y)$: X and Y always happen together (e.g., $t(ac) = t(d)$)
  - $t(X) \subset t(Y)$: transaction having X always has Y (e.g., $t(ac) \subset t(ce)$)
- Frequent patterns: vertical set intersections
- Using diffset to accelerate mining
  - Only keep track of differences of tids
  - $t(e) = \{T_{10}, T_{20}, T_{30}\}$, $t(ce) = \{T_{10}, T_{30}\} \rightarrow$ Diffset $(ce, e) = \{T_{20}\}$

A transaction DB in Horizontal Data Format

Tid	Itemset
10	a, c, d, e
20	a, b, e
30	b, c, e

The transaction DB in Vertical Data Format

Item	TidList
a	10, 20
b	20, 30
c	10, 30
d	10
e	10, 20, 30

$t(e) = \{T_{10}, T_{20}, T_{30}\}$;
$t(a) = \{T_{10}, T_{20}\}$;
$t(ae) = \{T_{10}, T_{20}\}$

Exploring Vertical Data Format: ECLAT

Table 5.3: The Vertical Data Format of the Transaction Data Set D of Table 5.1

itemset	*TID_set*
I1	{T100, T400, T500, T700, T800, T900}
I2	{T100, T200, T300, T400, T600, T800, T900}
I3	{T300, T500, T600, T700, T800, T900}
I4	{T200, T400}
I5	{T100, T800}

Table 5.1: Transactional Data for an *AllElectronics* Branch

TID	*List of item_IDs*
T100	I1, I2, I5
T200	I2, I4
T300	I2, I3
T400	I1, I2, I4
T500	I1, I3
T600	I2, I3
T700	I1, I3
T800	I1, I2, I3, I5
T900	I1, I2, I3

Table 5.4: 2-Itemsets in Vertical Data Format

itemset	*TID_set*
{I1, I2}	{T100, T400, T800, T900}
{I1, I3}	{T500, T700, T800, T900}
{I1, I4}	{T400}
{I1, I5}	{T100, T800}
{I2, I3}	{T300, T600, T800, T900}
{I2, I4}	{T200, T400}
{I2, I5}	{T100, T800}
{I3, I5}	{T800}

Table 5.5: 3-Itemsets in Vertical Data Format

itemset	*TID_set*
{I1, I2, I3}	{T800, T900}
{I1, I2, I5}	{T100, T800}

Why Mining Frequent Patterns by Pattern Growth?

- Apriori: A ***breadth-first search*** mining algorithm
  - First find the complete set of frequent k-itemsets
  - Then derive frequent (k+1)-itemset candidates
  - Scan DB again to find true frequent (k+1)-itemsets

Why Mining Frequent Patterns by Pattern Growth?

- Motivation for a different mining methodology
  - Can we develop a *depth-first search* mining algorithm?
  - For a frequent itemset ρ, can subsequent search be confined to only those transactions that containing ρ?
- Such thinking leads to a frequent pattern growth approach:
  - **FPGrowth** (J. Han, J. Pei, Y. Yin, "Mining Frequent Patterns without Candidate Generation," SIGMOD 2000)

Prerequisite: Find frequent 1-itemset

TID	Items in the Transaction
100	*{f, a, c, d, g, i, m, p}*
200	*{a, b, c, f, l, m, o}*
300	*{b, f, h, j, o, w}*
400	*{b, c, k, s, p}*
500	*{a, f, c, e, l, p, m, n}*

1. Scan DB once, find single item frequent pattern:

   Let min_support = 3

   f:4, a:3, c:4, b:3, m:3, p:3

2. Sort frequent items in frequency descending order, F-list

   F-list = f-c-a-b-m-p

Example: Construct FP-tree from a Transaction DB

TID	Items in the Transaction	Ordered, frequent itemlist
100	$\{f, a, c, d, g, i, m, p\}$	f, c, a, m, p
200	$\{a, b, c, f, l, m, o\}$	f, c, a, b, m
300	$\{b, f, h, j, o, w\}$	f, b
400	$\{b, c, k, s, p\}$	c, b, p
500	$\{a, f, c, e, l, p, m, n\}$	f, c, a, m, p

3. Scan DB again, find the ordered frequent itemlist for each transaction

Example: Construct FP-tree from a Transaction DB

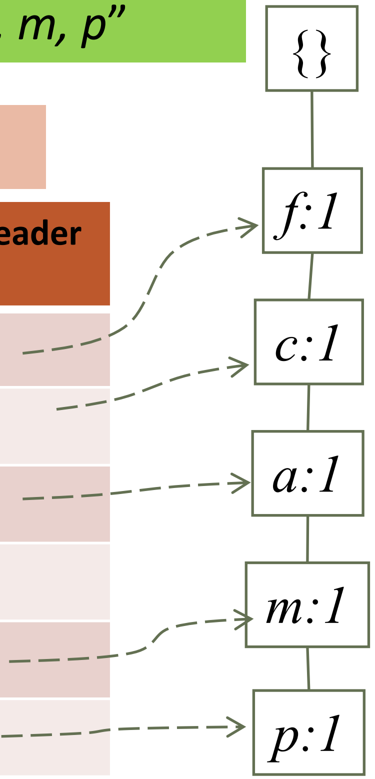
TID	Items in the Transaction	Ordered, frequent itemlist
100	*{f, a, c, d, g, i, m, p}*	*f, c, a, m, p*
200	*{a, b, c, f, l, m, o}*	*f, c, a, b, m*
300	*{b, f, h, j, o, w}*	*f, b*
400	*{b, c, k, s, p}*	*c, b, p*
500	*{a, f, c, e, l, p, m, n}*	*f, c, a, m, p*

4. For each transaction, insert the ordered frequent itemlist into an FP-tree, with shared sub-branches merged, counts accumulated

After inserting the 1st frequent Itemlist: *"f, c, a, m, p"*

Header Table

Item	Frequency	header
f	4	
c	4	
a	3	
b	3	
m	3	
p	3	



Example: Construct FP-tree from a Transaction DB

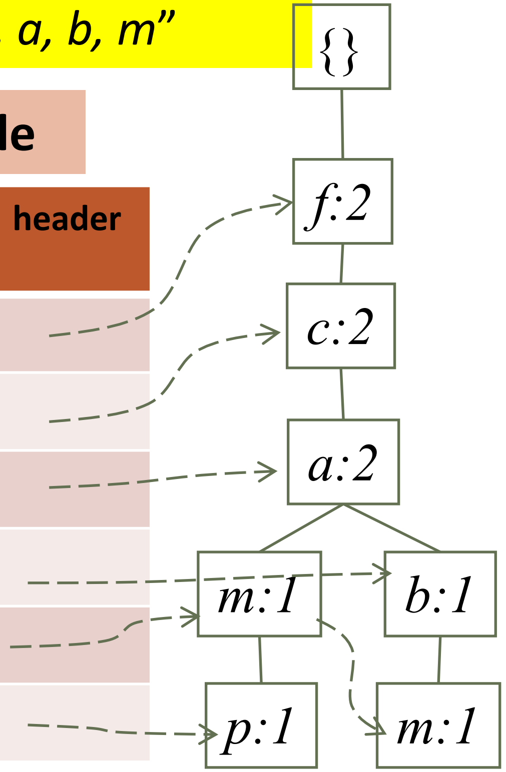
TID	Items in the Transaction	Ordered, frequent itemlist
100	*{f, a, c, d, g, i, m, p}*	*f, c, a, m, p*
200	*{a, b, c, f, l, m, o}*	*f, c, a, b, m*
300	*{b, f, h, j, o, w}*	*f, b*
400	*{b, c, k, s, p}*	*c, b, p*
500	*{a, f, c, e, l, p, m, n}*	*f, c, a, m, p*

4. For each transaction, insert the ordered frequent itemlist into an FP-tree, with shared sub-branches merged, counts accumulated

After inserting the 2nd frequent itemlist *"f, c, a, b, m"*

Header Table

Item	Frequency	header
f	4	
c	4	
a	3	
b	3	
m	3	
p	3	



Example: Construct FP-tree from a Transaction DB

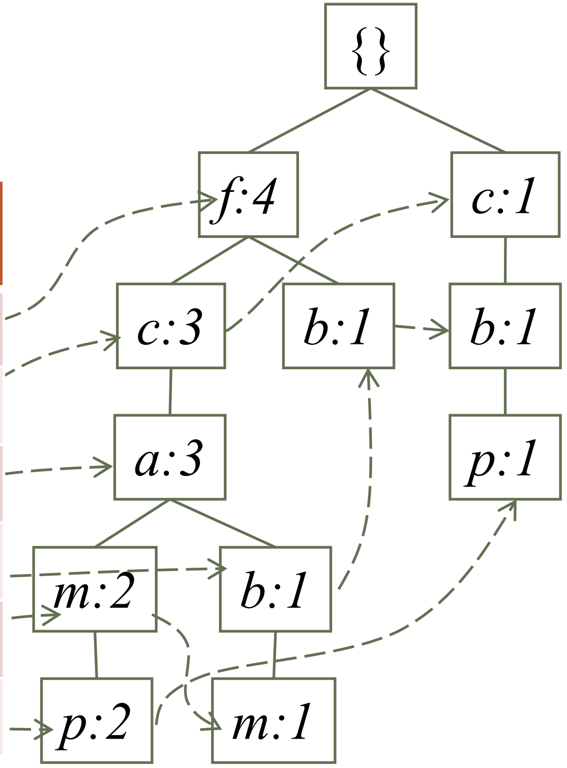
TID	Items in the Transaction	Ordered, frequent itemlist
100	*{f, a, c, d, g, i, m, p}*	*f, c, a, m, p*
200	*{a, b, c, f, l, m, o}*	*f, c, a, b, m*
300	*{b, f, h, j, o, w}*	*f, b*
400	*{b, c, k, s, p}*	*c, b, p*
500	*{a, f, c, e, l, p, m, n}*	*f, c, a, m, p*

4. For each transaction, insert the ordered frequent itemlist into an FP-tree, with shared sub-branches merged, counts accumulated

After inserting all the frequent itemlists

Header Table

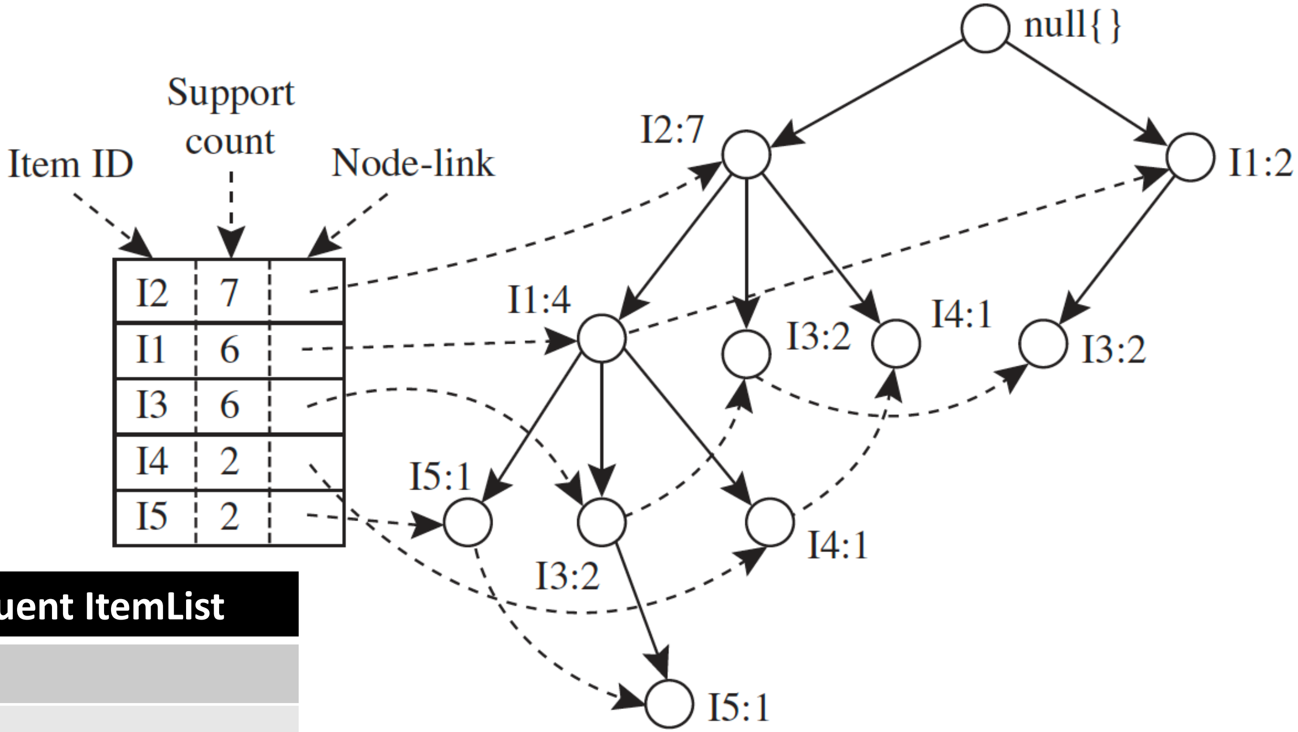
Item	Frequency	header
f	4	
c	4	
a	3	
b	3	
m	3	
p	3	



Example: Construct FP-tree from a Transaction DB

Table 5.1: Transactional Data for an *AllElectronics* Branch

TID	*List of item_IDs*
T100	I1, I2, I5
T200	I2, I4
T300	I2, I3
T400	I1, I2, I4
T500	I1, I3
T600	I2, I3
T700	I1, I3
T800	I1, I2, I3, I5
T900	I1, I2, I3



TID	Ordered, Frequent ItemList
T100	I2, I1, I5
T200	I2, I4
T300	I2, I3
T400	I2, I1, I4
T500	I1, I3
T600	I2, I3
T700	I1, I3
T800	I2, I1, I3, I5
T900	I2, I1, I3

Example: Construct FP-tree from a Transaction DB

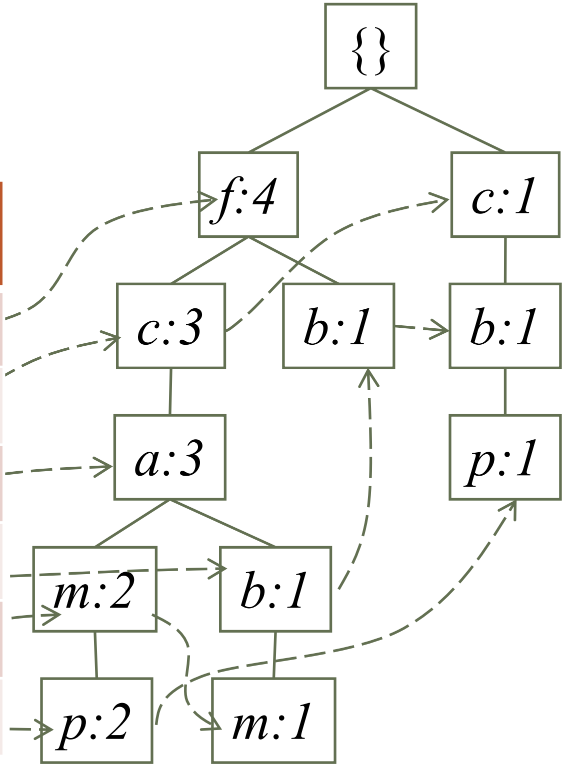
TID	Items in the Transaction	Ordered, frequent itemlist
100	*{f, a, c, d, g, i, m, p}*	*f, c, a, m, p*
200	*{a, b, c, f, l, m, o}*	*f, c, a, b, m*
300	*{b, f, h, j, o, w}*	*f, b*
400	*{b, c, k, s, p}*	*c, b, p*
500	*{a, f, c, e, l, p, m, n}*	*f, c, a, m, p*

4. For each transaction, insert the ordered frequent itemlist into an FP-tree, with shared sub-branches merged, counts accumulated

After inserting all the frequent itemlists

Header Table

Item	Frequency	header
f	4	
c	4	
a	3	
b	3	
m	3	
p	3	

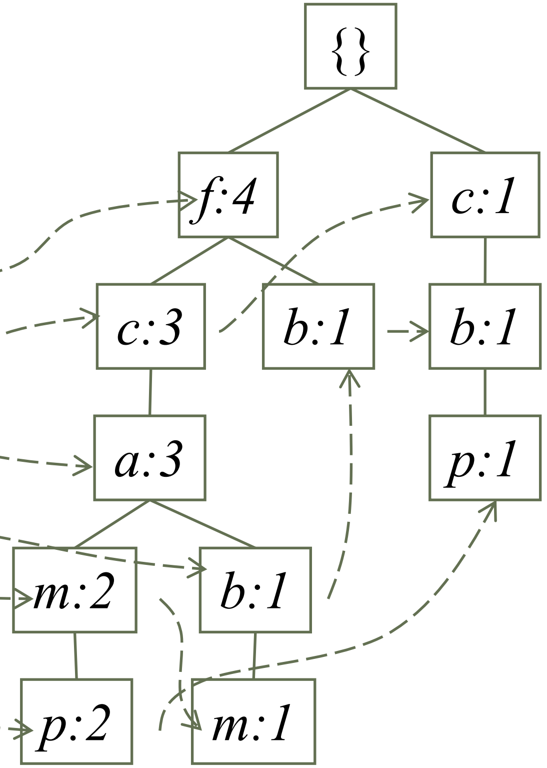


Mining FP-Tree: Divide and Conquer Based on Patterns and Data

- Pattern mining can be partitioned according to current patterns
  - We start to calculate the conditional database from bottom to top (from the least frequent item)
  - Conditional database: the database under the condition that *p* exists
    - *p*'s conditional database (Patterns containing *p*): *fcam:2, cb:1*

min_support = 3

Item	Frequency	Header
f	4	
c	4	
a	3	
b	3	
m	3	
p	3	



Conditional database of each pattern

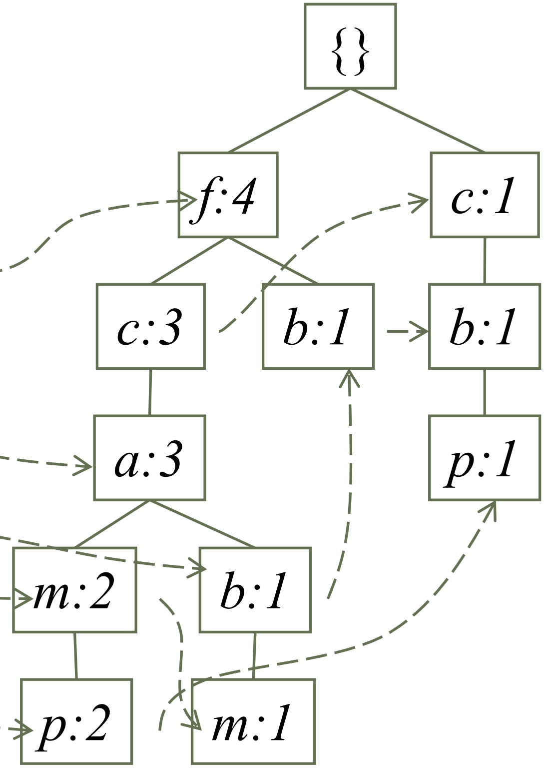
Item	*Conditional database*
p	***fcam:2, cb:1***

Mining FP-Tree: Divide and Conquer Based on Patterns and Data

- *p*'s conditional database (Patterns containing *p*): *fcam:2, cb:1*
- *After calculating p's conditional database, we calculate m's conditional database*

min_support = 3

Item	Frequency	Header
f	4	
c	4	
a	3	
b	3	
m	3	
p	3	



Conditional database of each pattern

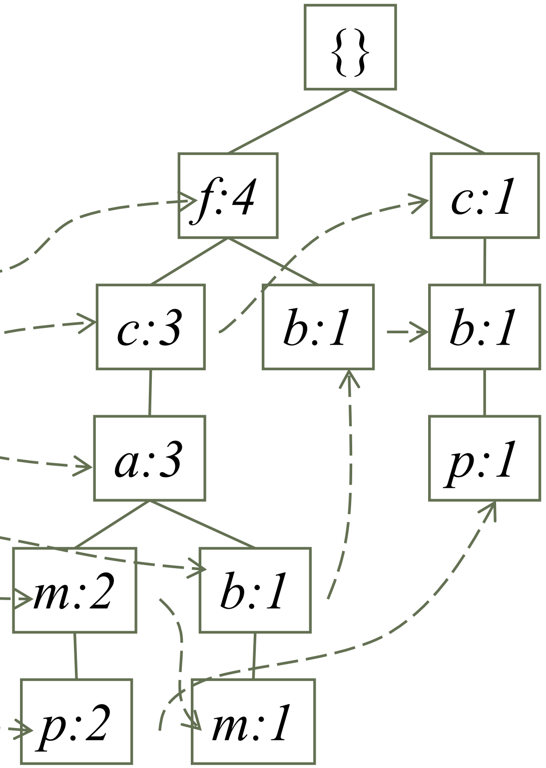
Item	***Conditional database***
m	***fca:2, fcab:1***
p	***fcam:2, cb:1***

Mining FP-Tree: Divide and Conquer Based on Patterns and Data

- *Repeat and calculate the conditional database of b, a, and c*
- *Since f is the most frequent item, we don't need to compute its conditional dataset*

min_support = 3

Item	Frequency	Header
f	4	
c	4	
a	3	
b	3	
m	3	
p	3	



Conditional database of each pattern

Item	*Conditional database*
c	***f:3***
a	***fc:3***
b	***fca:1, f:1, c:1***
m	***fca:2, fcab:1***
p	***fcam:2, cb:1***

Mine Each Conditional Database Recursively

min_support = 3

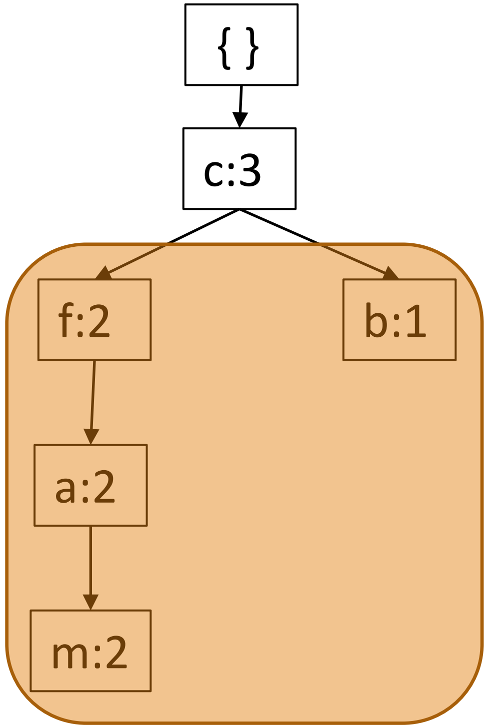
p   fcam:2, cb: 1

Constructing p's 1 itemlist, sorted by frequency

Item	Frequency
c	3
f	2
a	2
m	2
b	1

Original	*Ordered*
f, c, a, m	*c, f, a, m*
c, b	*c, b*

Conditional FP-tree for p



- For each conditional database
  - Mine single-item patterns
  - Construct its FP-tree & mine it

Frequent pattern   cp: 3

Mine Each Conditional Database Recursively

min_support = 3

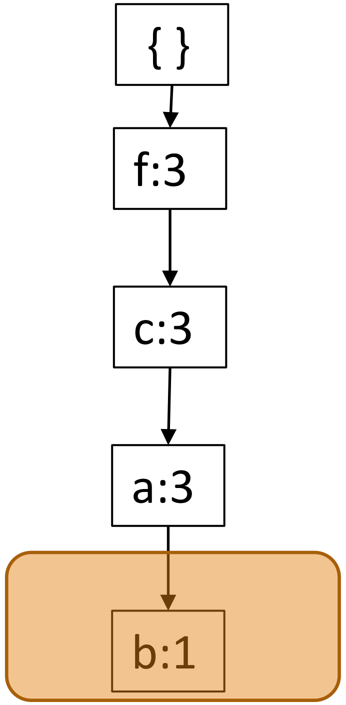
m fca:2, fcab: 1

Constructing m's 1 itemlist, sorted by frequency

Item	Frequency
f	3
c	3
a	3
b	1

Original	*Ordered*
f, c, a	*f, c, a*
f, c, a, b	*f, c, a, b*

Conditional FP-tree for m



- For each conditional database
  - Mine single-item patterns
  - Construct its FP-tree & mine it

Frequent pattern fcam: 3

Form conditional database:
am fc: 3

Mine Each Conditional Database Recursively

min_support = 3

Conditional Data Bases

item	*cond. data base*
c	*f:3*
a	*fc:3*
b	*fca:1, f:1, c:1*
m	*fca:2, fcab:1*
p	*fcam:2, cb:1*

- For each conditional database
  - Mine single-item patterns
  - Construct its FP-tree & mine it

e.g., mining m's FP-tree

{}
|
f:3
|
c:3
|
a:3
m's FP-tree

{}
|
f:3
|
c:3
am's FP-tree

{}
|
f:3
cm's FP-tree

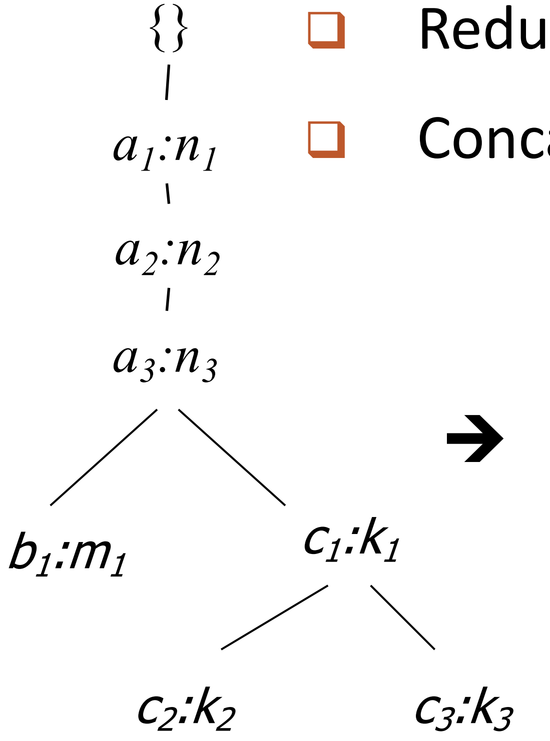
{}
|
f:3
cam's FP-tree

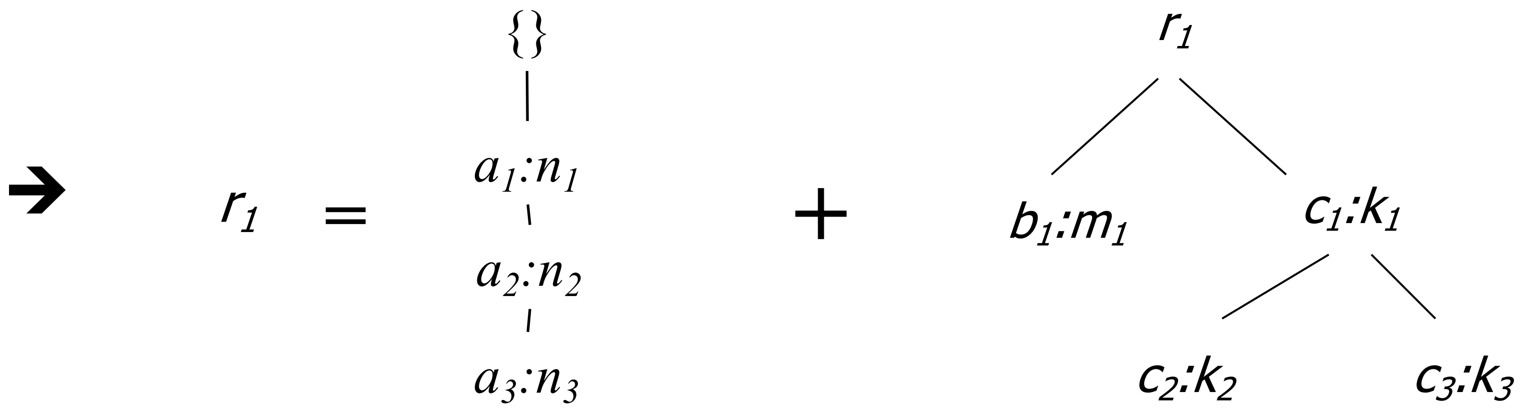
Actually, for single branch FP-tree, all the frequent patterns can be generated in one shot

m: 3
fm: 3, cm: 3, am: 3
fcm: 3, fam:3, cam: 3
fcam: 3

A Special Case: Single Prefix Path in FP-tree

- Suppose a (conditional) FP-tree T has a shared single prefix-path P
- Mining can be decomposed into two parts
  - Reduction of the single prefix path into one node
  - Concatenation of the mining results of the two parts

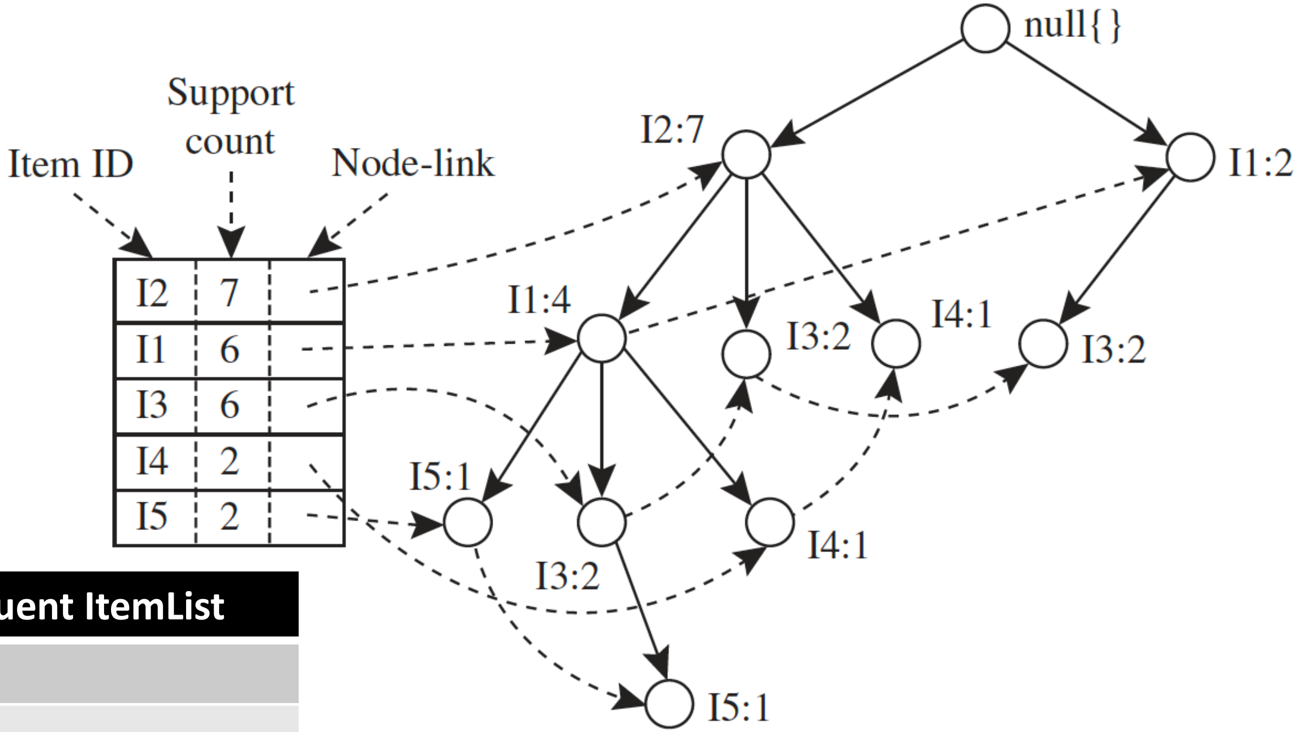




Example: Construct FP-tree from a Transaction DB

Table 5.1: Transactional Data for an *AllElectronics* Branch

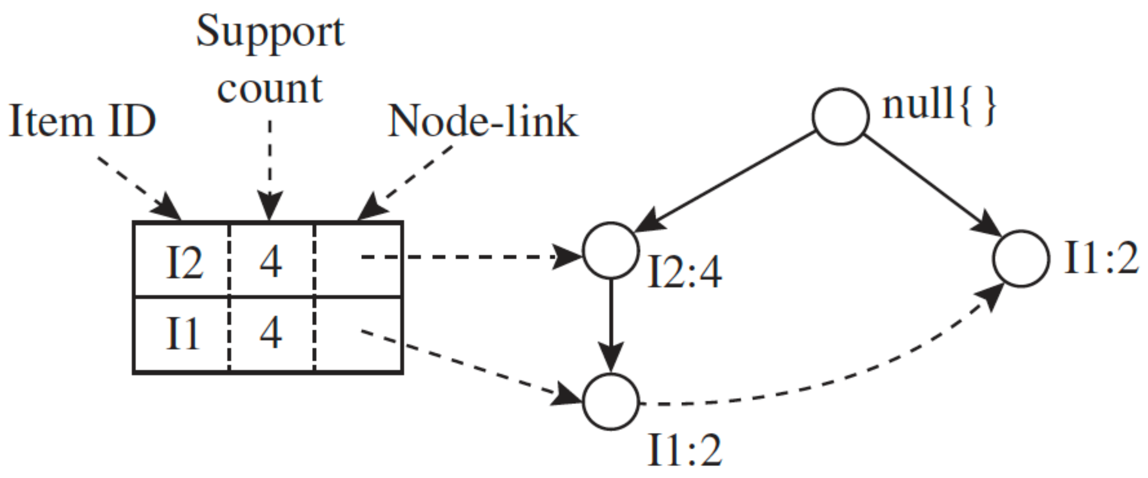
TID	*List of item_IDs*
T100	I1, I2, I5
T200	I2, I4
T300	I2, I3
T400	I1, I2, I4
T500	I1, I3
T600	I2, I3
T700	I1, I3
T800	I1, I2, I3, I5
T900	I1, I2, I3



TID	Ordered, Frequent ItemList
T100	I2, I1, I5
T200	I2, I4
T300	I2, I3
T400	I2, I1, I4
T500	I1, I3
T600	I2, I3
T700	I1, I3
T800	I2, I1, I3, I5
T900	I2, I1, I3

Example: Mining with Conditional FP-trees

Item	*Conditional Pattern Base*	*Conditional FP-tree*	*Frequent Patterns Generated*
I5	{{I2, I1: 1}, {I2, I1, I3: 1}}	⟨I2: 2, I1: 2⟩	{I2, I5: 2}, {I1, I5: 2}, {I2, I1, I5: 2}
I4	{{I2, I1: 1}, {I2: 1}}	⟨I2: 2⟩	{I2, I4: 2}
I3	{{I2, I1: 2}, {I2: 2}, {I1: 2}}	⟨I2: 4, I1: 2⟩, ⟨I1: 2⟩	{I2, I3: 4}, {I1, I3: 4}, {I2, I1, I3: 2}
I1	{{I2: 4}}	⟨I2: 4⟩	{I2, I1: 4}



Item	*Conditional database*
I1	*I2: 2*

Figure 5.8: The conditional FP-tree associated with the conditional node I3.

FPGrowth: Mining Frequent Patterns by Pattern Growth

- Essence of frequent pattern growth (FPGrowth) methodology
  - Find frequent single items and partition the database based on each such single item pattern
  - Recursively grow frequent patterns by doing the above for each *partitioned database* (also called the pattern's *conditional database*)
  - To facilitate efficient processing, an efficient data structure, FP-tree, can be constructed

FPGrowth: Mining Frequent Patterns by Pattern Growth

- Mining becomes
  - Recursively construct and mine (conditional) FP-trees
  - Until the resulting FP-tree is empty, or until it contains only one path—single path will generate all the combinations of its sub-paths, each of which is a frequent pattern

Mining Closed and Maximal Patterns

- Closed frequent itemsets
  - Find all frequent itemsets (FI), then remove itemsets which are (i) proper subsets of, and (ii) has same support as another FI
  - Can be time consuming
- More efficient approaches
  - If all transactions containing FI X also contains an itemset Y but not any proper superset of Y, then X U Y is a frequent closed itemset
  - No need to search for any itemset containing X and Y
- Maximal frequent itemsets
  - Extensions of approaches for Closed FIs

Chapter 6: Mining Frequent Patterns, Association and Correlations: Basic Concepts and Methods

- Basic Concepts
- Efficient Pattern Mining Methods
- Pattern Evaluation
- Summary

Pattern Evaluation

- Limitation of the Support-Confidence Framework
- Interestingness Measures: Lift and χ^2
- Null-Invariant Measures
- Comparison of Interestingness Measures

How to Judge if a Rule/Pattern Is Interesting?

- Pattern-mining will generate a large set of patterns/rules
  - Not all the generated patterns/rules are interesting
- Interestingness measures: Objective vs. subjective
  - Objective interestingness measures
    - Support, confidence, correlation, …
  - Subjective interestingness measures:
      - Different users may judge interestingness differently
    - Let a user specify
      - Query-based: Relevant to a user's particular request
    - Judge against one's knowledge-base
      - unexpected, freshness, timeliness

Limitation of the Support-Confidence Framework

- Are *s* and *c* interesting in association rules: "A $\Rightarrow$ B" [*s*, *c*]? Be careful!
- Example: Suppose one school may have the following statistics on # of students who may play basketball and/or eat cereal:

	play-basketball	not play-basketball	sum (row)
eat-cereal	400	350	750
not eat-cereal	200	50	250
sum(col.)	600	400	1000

2-way contingency table

- Association rule mining may generate the following:
  - *play-basketball* $\Rightarrow$ *eat-cereal* [40%, 66.7%] (higher s & c)
- But this strong association rule is misleading: The overall % of students eating cereal is 75% > 66.7%, a more telling rule:
  - ¬ *play-basketball* $\Rightarrow$ *eat-cereal* [35%, 87.5%] (high s & c)

Interestingness Measure: Lift

- Measure of dependent/correlated events: **lift**

$$lift(B,C) = \frac{c(B \rightarrow C)}{s(C)} = \frac{s(B \cup C)}{s(B) \times s(C)}$$

- Lift(B, C) may tell how B and C are correlated
  - Lift(B, C) = 1: B and C are independent
  - > 1: positively correlated
  - < 1: negatively correlated
- For our example,

$$lift(B,C) = \frac{400/1000}{600/1000 \times 750/1000} = 0.89$$

$$lift(B,\neg C) = \frac{200/1000}{600/1000 \times 250/1000} = 1.33$$

- Thus, B and C are negatively correlated since lift(B, C) < 1;
  - B and ¬C are positively correlated since lift(B, ¬C) > 1

Lift is more telling than s & c

	B	¬B	Σ_{row}
C	400	350	750
¬C	200	50	250
$\Sigma_{col.}$	600	400	1000

Interestingness Measure: χ^2

- Another measure to test correlated events: χ^2

$$\chi^2 = \sum \frac{(Observed - Expected)^2}{Expected}$$

- For the table on the right,

$$\chi^2 = \frac{(400-450)^2}{450} + \frac{(350-300)^2}{300} + \frac{(200-150)^2}{150} + \frac{(50-100)^2}{100} = 55.56$$

	B	¬B	Σ_{row}
C	400 (450)	350 (300)	750
¬C	200 (150)	50 (100)	250
Σ_{col}	600	400	1000

Expected value

Observed value

- Lookup χ^2 distribution table => B, C are correlated
- χ^2-test shows B and C are negatively correlated since the expected value is 450 but the observed is only 400
- Thus, χ^2 is also more telling than the support-confidence framework

Lift and χ^2: Are They Always Good Measures?

- Null transactions: Transactions that contain neither B nor C
- Let's examine the new dataset D
  - BC (100) is much rarer than B¬C (1000) and ¬BC (1000), but there are many ¬B¬C (100000)
  - Unlikely B & C will happen together!
- But, Lift(B, C) = 8.44 >> 1 (Lift shows B and C are strongly positively correlated!)
- χ^2 = 670: Observed(BC) >> expected value (11.85)
- *Too many null transactions may "spoil the soup"!*

	B	¬B	Σ_{row}
C	100	1000	1100
¬C	1000	100000	101000
$\Sigma_{col.}$	1100	101000	102100

null transactions

Contingency table with expected values added

	B	¬B	Σ_{row}
C	100 (11.85)	1000	1100
¬C	1000 (988.15)	100000	101000
$\Sigma_{col.}$	1100	101000	102100

Interestingness Measures & Null-Invariance

- *Null invariance* means: The number of null transactions does not matter. Does not change the measure value.
- A few interestingness measures: Some are null invariant

Measure	Definition	Range	Null-Invariant?
$\chi^2(A,B)$	$\sum_{i,j} \frac{(e(a_i,b_j)-o(a_i,b_j))^2}{e(a_i,b_j)}$	$[0, \infty]$	No
$Lift(A,B)$	$\frac{s(A \cup B)}{s(A) \times s(B)}$	$[0, \infty]$	No
$Allconf(A,B)$	$\frac{s(A \cup B)}{max\{s(A), s(B)\}}$	$[0, 1]$	Yes
$Jaccard(A,B)$	$\frac{s(A \cup B)}{s(A)+s(B)-s(A \cup B)}$	$[0, 1]$	Yes
$Cosine(A,B)$	$\frac{s(A \cup B)}{\sqrt{s(A) \times s(B)}}$	$[0, 1]$	Yes
$Kulczynski(A,B)$	$\frac{1}{2}\left(\frac{s(A \cup B)}{s(A)} + \frac{s(A \cup B)}{s(B)}\right)$	$[0, 1]$	Yes
$MaxConf(A,B)$	$max\{\frac{s(A \cup B)}{s(A)}, \frac{s(A \cup B)}{s(B)}\}$	$[0, 1]$	Yes

Let

$$p = \frac{s(A \cup B)}{s(A)} = P(B|A)$$

$$q = \frac{s(A \cup B)}{s(B)} = P(A|B)$$

p, q are null invariant

Essentially min, max, mean variants of p, q

Null Invariance: An Important Property

- Why is null invariance crucial for the analysis of massive transaction data?
  - Many transactions may contain neither milk nor coffee!

milk vs. coffee contingency table

	$milk$	$\neg milk$	Σ_{row}
$coffee$	mc	$\neg mc$	c
$\neg coffee$	$m\neg c$	$\neg m \neg c$	$\neg c$
Σ_{col}	m	$\neg m$	Σ

- Lift and χ^2 are not null-invariant: not good to evaluate data that contain too many or too few null transactions!
- Many measures are not null-invariant!

Data set	mc	$\neg mc$	$m\neg c$	$\neg m\neg c$	χ^2	$Lift$
D_1	10,000	1,000	1,000	100,000	90557	9.26
D_2	10,000	1,000	1,000	100	0	1
D_3	100	1,000	1,000	100,000	670	8.44
D_4	1,000	1,000	1,000	100,000	24740	25.75
D_5	1,000	100	10,000	100,000	8173	9.18
D_6	1,000	10	100,000	100,000	965	1.97

Comparison of Null-Invariant Measures

- Not all null-invariant measures are created equal
- Which one is better?
  - D_4—D_6 differentiate the null-invariant measures
  - Kulc (Kulczynski 1927) holds firm and is in balance of both directional implications

2-variable contingency table

	$milk$	$\neg milk$	Σ_{row}
$coffee$	mc	$\neg mc$	c
$\neg coffee$	$m\neg c$	$\neg m\neg c$	$\neg c$
Σ_{col}	m	$\neg m$	Σ

All 5 are null-invariant

Data set	mc	$\neg mc$	$m\neg c$	$\neg m\neg c$	$AllConf$	$Jaccard$	$Cosine$	$Kulc$	$MaxConf$
D_1	10,000	1,000	1,000	100,000	0.91	0.83	0.91	0.91	0.91
D_2	10,000	1,000	1,000	100	0.91	0.83	0.91	0.91	0.91
D_3	100	1,000	1,000	100,000	0.09	0.05	0.09	0.09	0.09
D_4	1,000	1,000	1,000	100,000	0.5	0.33	0.5	0.5	0.5
D_5	1,000	100	10,000	100,000	0.09	0.09	0.29	0.5	0.91
D_6	1,000	10	100,000	100,000	0.01	0.01	0.10	0.5	0.99

Imbalance Ratio with Kulczynski Measure

- IR (Imbalance Ratio): measure the imbalance of two itemsets A and B in rule implications:

$$IR(A,B) = \frac{|s(A)-s(B)|}{s(A)+s(B)-s(A \cup B)}$$

- Kulczynski and Imbalance Ratio (IR) together present a clear picture for all the three datasets D_4 through D_6
  - D_4 is neutral & balanced; D_5 is neutral but imbalanced
  - D_6 is neutral but very imbalanced

Data set	mc	$\neg mc$	$m \neg c$	$\neg m \neg c$	Jaccard	$Cosine$	$Kulc$	IR
D_1	10,000	1,000	1,000	100,000	0.83	0.91	0.91	0
D_2	10,000	1,000	1,000	100	0.83	0.91	0.91	0
D_3	100	1,000	1,000	100,000	0.05	0.09	0.09	0
D_4	1,000	1,000	1,000	100,000	0.33	0.5	0.5	0
D_5	1,000	100	10,000	100,000	0.09	0.29	0.5	0.89
D_6	1,000	10	100,000	100,000	0.01	0.10	0.5	0.99

Example: Analysis of DBLP Coauthor Relationships

- DBLP: Computer science research publication bibliographic database
  - > 3.8 million entries on authors, paper, venue, year, and other information

ID	Author A	Author B	$s(A \cup B)$	$s(A)$	$s(B)$	Jaccard	Cosine	Kulc
1	Hans-Peter Kriegel	Martin Ester	28	146	54	0.163 (2)	0.315 (7)	0.355 (9)
2	Michael Carey	Miron Livny	26	104	58	0.191 (1)	0.335 (4)	0.349 (10)
3	Hans-Peter Kriegel	Joerg Sander	24	146	36	0.152 (3)	0.331 (5)	0.416 (8)
4	Christos Faloutsos	Spiros Papadimitriou	20	162	26	0.119 (7)	0.308 (10)	0.446 (7)
5	Hans-Peter Kriegel	Martin Pfeifle	18	146	18	0.123 (6)	0.351 (2)	0.562 (2)
6	Hector Garcia-Molina	Wilburt Labio	16	144	18	0.110 (9)	0.314 (8)	0.500 (4)
7	Divyakant Agrawal	Wang Hsiung	16	120	16	0.133 (5)	0.365 (1)	0.567 (1)
8	Elke Rundensteiner	Murali Mani	16	104	20	0.148 (4)	0.351 (3)	0.477 (6)
9	Divyakant Agrawal	Oliver Po	12	120	12	0.100 (10)	0.316 (6)	0.550 (3)
10	Gerhard Weikum	Martin Theobald	12	106	14	0.111 (8)	0.312 (9)	0.485 (5)

Advisor-advisee relation: Kulc: high, Jaccard: low, cosine: middle

- Which pairs of authors are strongly related? Is A the advisor, or the advisee?
  - Use Kulc to find Advisor-advisee, close collaborators

What Measures to Choose for Effective Pattern Evaluation?

- Null value cases are predominant in many large datasets
  - Neither milk nor coffee is in most of the baskets; neither Mike nor Jim is an author in most of the papers; ……
- *Null-invariance* is an important property
- Lift, χ^2 and cosine are good measures if null transactions are not predominant
  - Otherwise, *Kulczynski* + *Imbalance Ratio* should be used to judge the interestingness of a pattern

What Measures to Choose for Effective Pattern Evaluation?

- Exercise: Mining research collaborations from research bibliographic data
  - Find a group of frequent collaborators from research bibliographic data (e.g., DBLP)
  - Can you find the likely advisor-advisee relationship and during which years such a relationship happened?
  - Ref.: C. Wang, J. Han, Y. Jia, J. Tang, D. Zhang, Y. Yu, and J. Guo, "Mining Advisor-Advisee Relationships from Research Publication Networks", KDD'10

Chapter 6: Mining Frequent Patterns, Association and Correlations: Basic Concepts and Methods

- Basic Concepts
- Efficient Pattern Mining Methods
- Pattern Evaluation
- Summary

Summary

- Basic Concepts
  - What Is Pattern Discovery?   Why Is It Important?
  - Basic Concepts: Frequent Patterns and Association Rules
  - Compressed Representation: Closed Patterns and Max-Patterns
- Efficient Pattern Mining Methods
  - The Downward Closure Property of Frequent Patterns
  - The Apriori Algorithm
  - Extensions or Improvements of Apriori
  - Mining Frequent Patterns by Exploring Vertical Data Format
  - FPGrowth: A Frequent Pattern-Growth Approach
  - Mining Closed Patterns
- Pattern Evaluation
  - Interestingness Measures in Pattern Mining
  - Interestingness Measures: Lift and χ^2
  - Null-Invariant Measures
  - Comparison of Interestingness Measures

Recommended Readings (Basic Concepts)

- R. Agrawal, T. Imielinski, and A. Swami, “Mining association rules between sets of items in large databases”, in Proc. of SIGMOD'93
- R. J. Bayardo, “Efficiently mining long patterns from databases”, in Proc. of SIGMOD'98
- N. Pasquier, Y. Bastide, R. Taouil, and L. Lakhal, “Discovering frequent closed itemsets for association rules”, in Proc. of ICDT'99
- J. Han, H. Cheng, D. Xin, and X. Yan, “Frequent Pattern Mining: Current Status and Future Directions”, Data Mining and Knowledge Discovery, 15(1): 55-86, 2007

Recommended Readings (Efficient Pattern Mining Methods)

- R. Agrawal and R. Srikant, “Fast algorithms for mining association rules”, VLDB'94
- A. Savasere, E. Omiecinski, and S. Navathe, “An efficient algorithm for mining association rules in large databases”, VLDB'95
- J. S. Park, M. S. Chen, and P. S. Yu, “An effective hash-based algorithm for mining association rules”, SIGMOD'95
- S. Sarawagi, S. Thomas, and R. Agrawal, “Integrating association rule mining with relational database systems: Alternatives and implications”, SIGMOD'98
- M. J. Zaki, S. Parthasarathy, M. Ogihara, and W. Li, “Parallel algorithm for discovery of association rules”, Data Mining and Knowledge Discovery, 1997
- J. Han, J. Pei, and Y. Yin, “Mining frequent patterns without candidate generation”, SIGMOD’00
- M. J. Zaki and Hsiao, “CHARM: An Efficient Algorithm for Closed Itemset Mining”, SDM'02
- J. Wang, J. Han, and J. Pei, “CLOSET+: Searching for the Best Strategies for Mining Frequent Closed Itemsets”, KDD'03
- C. C. Aggarwal, M.A., Bhuiyan, M. A. Hasan, “Frequent Pattern Mining Algorithms: A Survey”, in Aggarwal and Han (eds.): Frequent Pattern Mining, Springer, 2014

Recommended Readings (Pattern Evaluation)

- C. C. Aggarwal and P. S. Yu. A New Framework for Itemset Generation. PODS'98
- S. Brin, R. Motwani, and C. Silverstein. Beyond market basket: Generalizing association rules to correlations. SIGMOD'97
- M. Klemettinen, H. Mannila, P. Ronkainen, H. Toivonen, and A. I. Verkamo. Finding interesting rules from large sets of discovered association rules. CIKM'94
- E. Omiecinski. Alternative Interest Measures for Mining Associations. TKDE'03
- P.-N. Tan, V. Kumar, and J. Srivastava. Selecting the Right Interestingness Measure for Association Patterns. KDD'02
- T. Wu, Y. Chen and J. Han, Re-Examination of Interestingness Measures in Pattern Mining: A Unified Framework, Data Mining and Knowledge Discovery, 21(3):371-397, 2010