Algorithmic Trading

Dr. Javier Sabio González

Head of Advanced Analytics & Algorithmic Trading, BBVA C&IB

Topics

1. Markets Macrostructure
2. Markets Microstructure
3. Algorithmic Trading Fundamentals
4. Algorithmic Execution
5. **Algorithmic Market-Making**
6. Algorithmic Investment
7. The Future of Algorithmic Trading

5. Algorithmic Market-Making

Market Making

Definition: a market maker is a liquidity provider: a trader who, on a continuous and regular basis, proposes prices at which he stands ready to buy and sell a given asset.

Remarks:

- They are present at almost every market
- They play an essential role in price formation
- On order-driven markets:
  - There are “official market makers”, which have an agreement with the Exchange to maintain orderly markets; and “unofficial” and provide liquidity on a discretionary way, for example high frequency traders but also broker-dealers
  - For example the Designated Market Makers on the NYSE have to participate in auctions and quote at the NBBO a specified percentage of time.
  - Examples: market-making of equity futures and options in MEFF, CBOE, …
- On quote-driven markets the market makers are the broker-dealers
  - Examples: market-making of corporate bonds in Tradeweb, MarketAxess, …

Market Making

The rationale of market making: by providing t**he service of liquidity** or “**immediacy to trade at favourable prices**”, the MM expects to capture the bid-offer spread. Alternatively, the bid-offer spread can be seen as a compensation for the **costs** and **risks** of the activity:

- The **costs** are those of **order handling**: labor and capital needed to provide quote information, order routing, execution and clearing.

The **risks** are the following

- **Holding an inventory**, since rarely two opposing trades will happen simultaneously. A non-zero inventory means having a price risk due to the market volatility, that should be hedged if holding times are expected to be long enough
- **Asymmetry of information** between the market-maker and other traders: the market-maker stands ready to buy and sell at the prices quoted, but standing against the flow of more informed traders means that on average prices will move against the market-maker, who would have quoted more conservatively with the full information

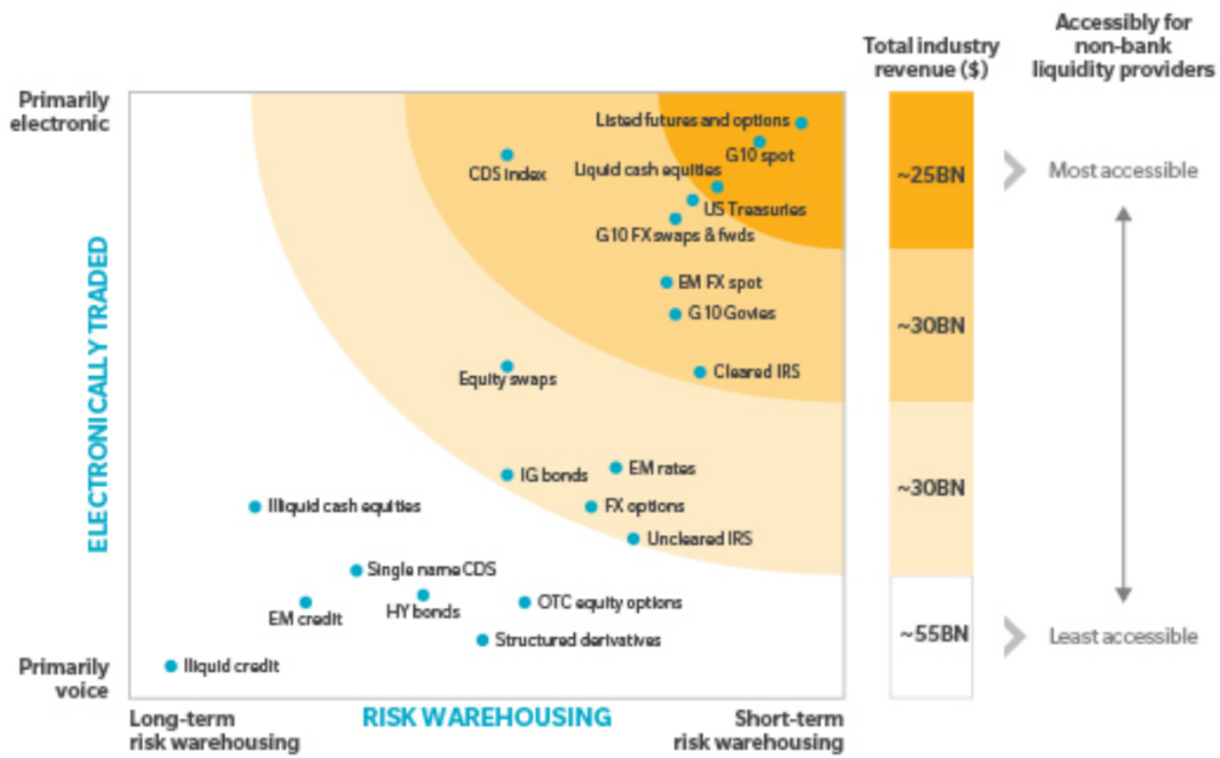
Market Making

Which agents make markets?

- **Investment banks acting as broker-dealers** make markets in most of the financial instruments, both in order-driven and quote-driven markets. In many cases they act as official market makers, having contracts with the markets.
  - Examples: Goldman Sachs, J.P. Morgan, Citi, Morgan Stanley, BNP, BBVA, Santander, …
- **Non-bank (new) liquidity providers**, which are typically companies with a strong technological basis and focus on very liquid instruments traded in electronic platforms. They operate similarly as banks. They are becoming dominant in retail execution using internalisation and payment for order flow.
  - Examples: Citadel, Virtu Financial, …
- **High-frequency trading firms**, that focus on market-making in electronic and very liquid markets with very short-term periods of holding and small inventories. They don't act as official market makers, since typically they don't want to quote in stressed market conditions.
  - Examples: Virtu Financial, Jump Trading, Two Sigma, …

Market Making

NON-BANK LIQUIDITY PROVIDERS ARE ACTIVE IN THE LOWEST MARGIN PARTS OF THE MOST LIQUID PRODUCTS, BUT ARE LOOKING TO EXTEND THEIR BUSINESS MODEL



Source: Oliver Wyman

Optimal Market Making

The optimal market-making problem: propose / quote bid and offer prices in an optimal way for making money out of the bid-ask spread, while mitigating the inventory and information asymmetry risks.

Remarks:

- The market maker faces various **trade-offs**:
  - **Static**: large bid-ask spreads lead to profitable trades but less transactions, whereas small bid-ask spreads are less profitable per trade but more often
  - **Dynamic**: quoting conservatively leads to inventory risk, but quoting aggressively intensifies information asymmetry risk.
- The market-making typically has the following strategies to mitigate risks:
  - **Resize** the bid-ask spread: for instance increase it in case of high volatility / uncertainty to reduce the information asymmetry risk
  - **Skew** the quotes: for instance quote more aggressively on the bid than in the ask with large long inventories, and more aggressively in the ask than the bid with large short inventories. This way the inventory will tend to rebalance
  - **Hedge** its inventory using correlated instruments that are very liquid and cheap to execute (small bid-offer). This way

Components of a Market Making system

1. Pricing engine
2. Analytics server: market and client analytics
3. Market making server:
    - Quotes determination (bid-ask size and skew)
    - Inventory hedging
    - Performance monitoring
4. Risk management system

1) Pricing Engine

- It handles the calculation of the "fair" price of the instrument(s) and the sensitivity of the price to risk factors, necessary to assess inventory risk and for hedging
- To estimate the fair price, there are various typical alternatives:
  - In **very liquid markets**, directly reference it to a liquid source:
    - in an order-driven market, the mid-price of the LOB
    - in a quote-driven market, a mid-price from indicative quotes in an electronic market of directly from large brokers (e.g. ICAP)
  - In **less liquid and/or fragmented markets**, a model is typically needed:
    - Statistical models (e.g. Bayesian models like the Kalman filter) to build a mid-price out of asynchronous observations of trades and bid-offers in different markets (d2c, d2d) and even correlated instruments
    - Pricing models that try to estimate a fair-price based on relative pricing or non-arbitrage models (e.g. risk-neutral pricing like the Black-Scholes model)
    - A combination of both models can be used, for instance using the pricing model as an extra input for the statistical model
- For sensitivities, bump the pricing mode if prices are constructed out of risk factors (eg in risk neutral models or in factor models), or directly model the sensitivities

2) Analytics server: market and client analytics

- Market analytics:
  - Price trend predictions
  - Volatility estimations and predictions
  - Information content of market flows
  - Regime change alerts
  - Predictions of correlations and/or cointegration between instruments
- Client analytics (in quote-driven markets where client identity is known)
  - Client segmentation based on behaviour and information asymmetry
  - Client elasticity of demand
  - Client behavioural patterns

3) Market making server: quotes determination

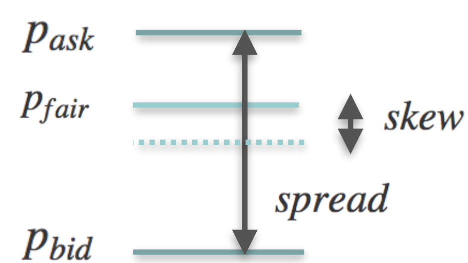
- Quotes are the prices (bid and ask) that the market-maker publishes
- In **quote-driven markets**, quotes:
  - Are **indicative**, unless the market is hybrid with executable quotes
  - In electronic markets they are **streamed** by the market maker for **standard sizes**
  - In order to execute at these prices, the client needs to send a **RfQ or RfS**
- In **order-driven markets**:
  - Quotes are **passive limit orders**, and therefore always **executable**
- It is useful to decompose bid and ask quotes in the following **components**:

$$p_{bid} = p_{fair} + \frac{1}{2}(-\text{spread} + \text{skew})$$

$$p_{ask} = p_{fair} + \frac{1}{2}(\text{spread} + \text{skew})$$

$$\text{spread} \equiv p_{ask} - p_{bid}$$

$$\text{skew} \equiv p_{ask} + p_{bid} - 2p_{fair}$$



- This way, once the fair price is estimated, the quote determination problem can be **decomposed in the determination of the spread and the skew**
  - Optimal spreads and skews will be analysed later using different models
  - They typically depend on **price volatility, inventory levels, risk aversion +** client segmentation based on behaviour (if client is known), flow “toxicity” signals, etc

3) Market making server: inventory hedging

- Market-makers **need to keep an inventory of the instrument** in order to stand ready to buy or sell at any time, since typically two opposing trades won't happen necessarily consecutively and in a short time window
- As we have seen, having an inventory carries a **risk of depreciation** due to price volatility
- Market-makers can handle inventory risk ("**hedge**") in two ways:
  - **Skewing their prices** when inventory accumulates in order to increase the probability of a trade that will rebalance the inventory to small levels
  - **Taking a position in the same instrument in another market** (typically inter-dealer markets) **or in a correlated instrument** with high liquidity (small bid-offers, high volumes of negotiation), in order to neutralise the market risk of the resulting portfolio.
    - In this case, the **execution server** is used to take the position in a cost-efficient way, using algorithmic execution strategies
    - Examples: futures traded in regulated markets, liquid issues of safe government bonds like US Treasuries and German bonds

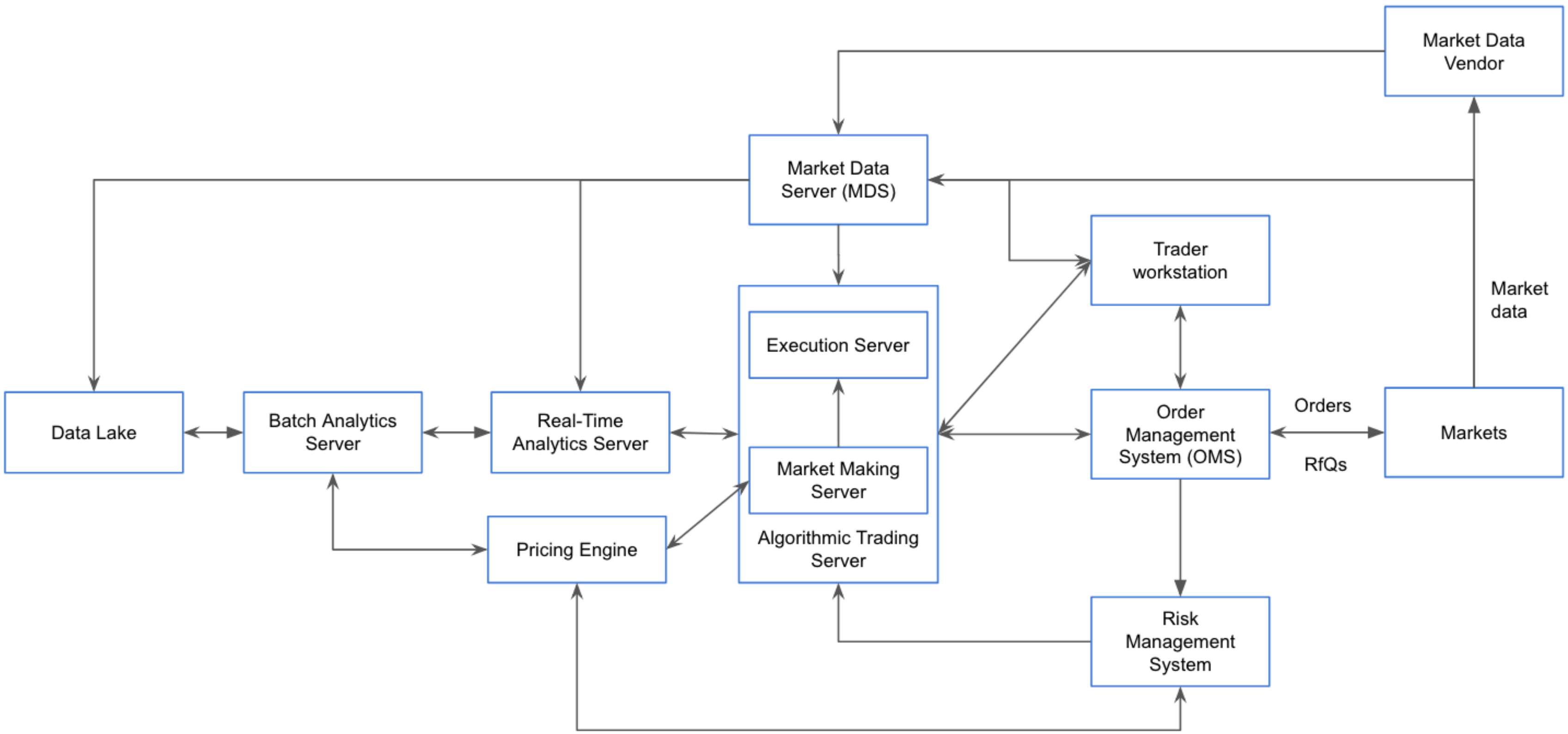
3) Market making server: performance monitoring

- Typically P&L metrics. It is important to define time-horizon for profitability (round trip, end of day, …)
- “Flow value”: track average profitability of flows including hedging costs over different time windows. It can be a measure of flow toxicity, if on average the market maker loses money on certain flows (based on clients, groups of clients, market conditions, time of day, …)

4) Risk management system

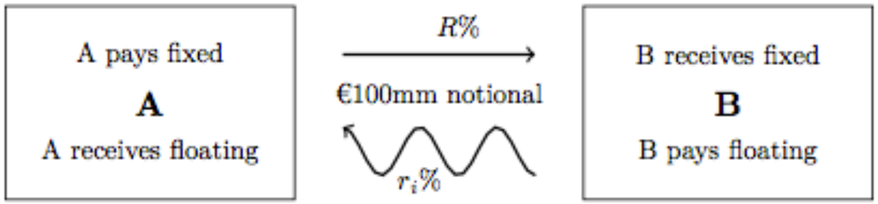
- Maintains the inventory position and assess its risk by using different measures, for example:
  - Sensitivity of inventory to risk factors
  - Risk limits
  - Value at risk

Full infrastructure



<u>Example 1</u>: Market-making in a quote-driven market: IR Swaps

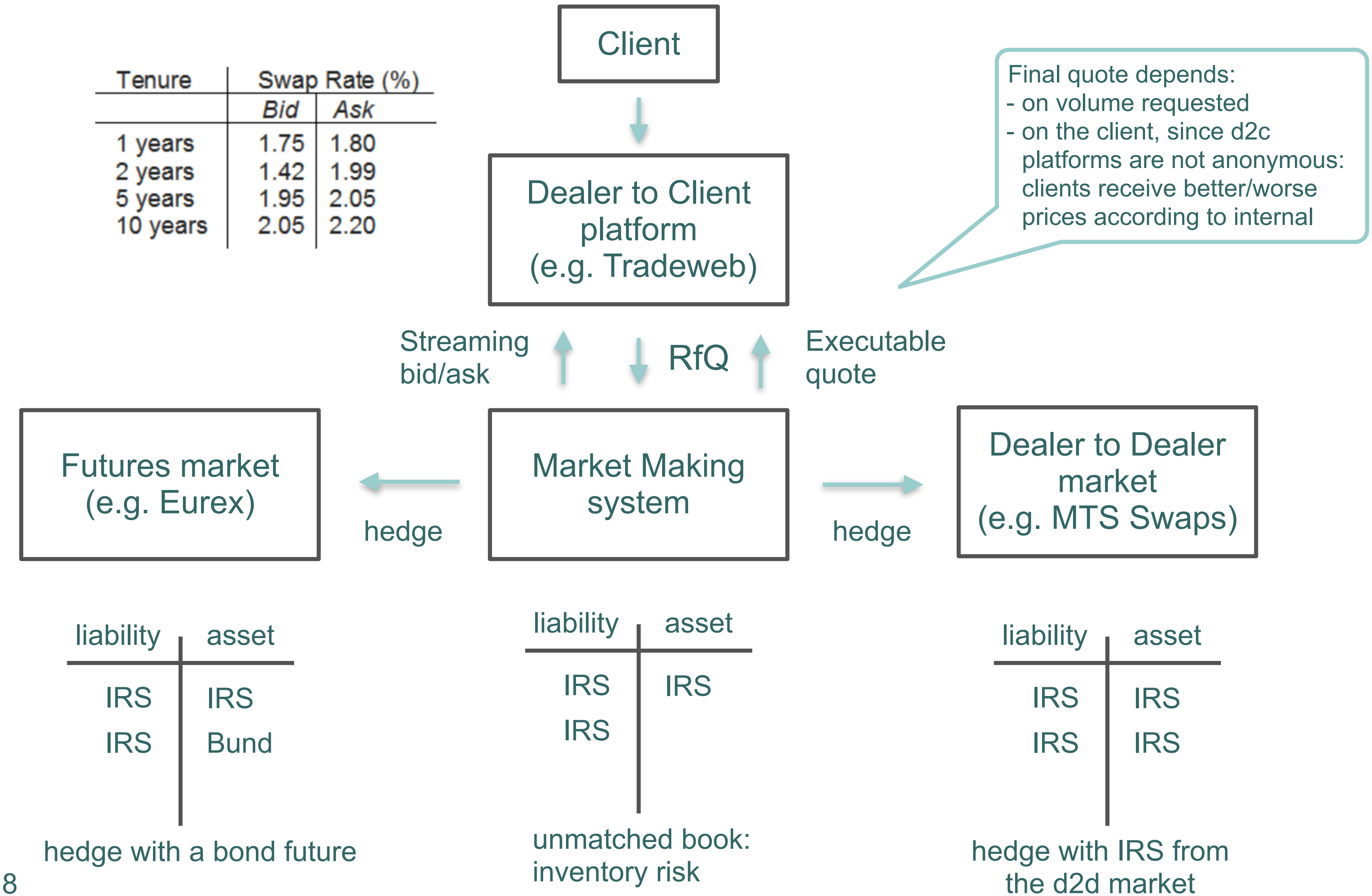
- Interest Rate Swaps are one of the most negotiated Fixed Income derivatives with an outstanding notional amount of around 300 trillion dollars worldwide
- In a standard IRS, two parties agree to exchange cashflows linked to a fixed interest rate against a variable (floating in the jargon) interest rate



- IRS are OTC instruments (over the counter) traded in a quote-driven way, both via voice or in electronic platforms (dealer-to-dealer and dealer-to-client) where the RfQ mechanism is mostly used
- Examples of electronic platforms are:
  - Dealer-to-client platforms: Bloomberg, Tradeweb
  - Inter-dealer platforms: ICAP i-Swap, CME SwapStream, MTS Swaps
- The most liquid contracts are fixed-to-float with standard tenors: 1Y, 2Y, 3Y, 5Y, 10Y, 30Y, which roll at the standard IMM dates (20th March, June, September, December)

Example 1: Market-making in a quote-driven market: IR Swaps

Tenure	Swap Rate (%)	
	Bid	*Ask*
1 years	1.75	1.80
2 years	1.42	1.99
5 years	1.95	2.05
10 years	2.05	2.20



<u>Example 2:</u> Market-making in an order-driven market: IBEX 35 Futures at MEFF

- The **IBEX 35 index** is a capitalisation-weighted index comprising the **35 most liquid Spanish stocks** traded in the Spanish Continuous Market. It is published by BME.
- **Futures contracts on the IBEX 35** can be traded at the **regulated market MEFF** (Mercados Españoles de Futuros Financieros)
- They have **expiration dates** at the third Friday of the two nearest calendar months and the ten nearest quarterly expiries of the March, June, September and December cycle
- They are **quoted** as whole index points. The nominal value of the contract is calculated multiplying the price of the future times a multiplier of 10. Example: a future contract of value 10.000 has a nominal value of 100.000€
- As other futures contracts, they are **settled in cash daily** until the expiration date
  - A **daily settlement price** (for any day before expiration) is calculated as a VWAP of trades between 17:29 and 17:30. At the expiration date, the settlement price is calculated using and arithmetic average of the IBEX 35 index between 16:15 and 16:45, taking a value per minute.
  - **Daily settlement** is calculated as the difference with the previous settlement price or the price at which it was traded on the trading day

Example 2: Market-making in an order-driven market: IBEX 35 Futures at MEFF

- Market mechanics:
  - Opening auction from 7:55 to 8:00 am
  - Continuous market from 8:00 am to 20:00 pm on a Limit Order Book
  - Order types allowed: market order, limit order, immediate or kill, stop order, stop limit order, fill or kill, auction price order
- There are the following official market makers:

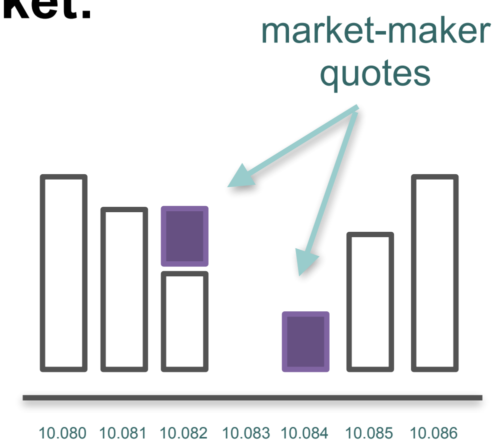
MARKET MAKERS

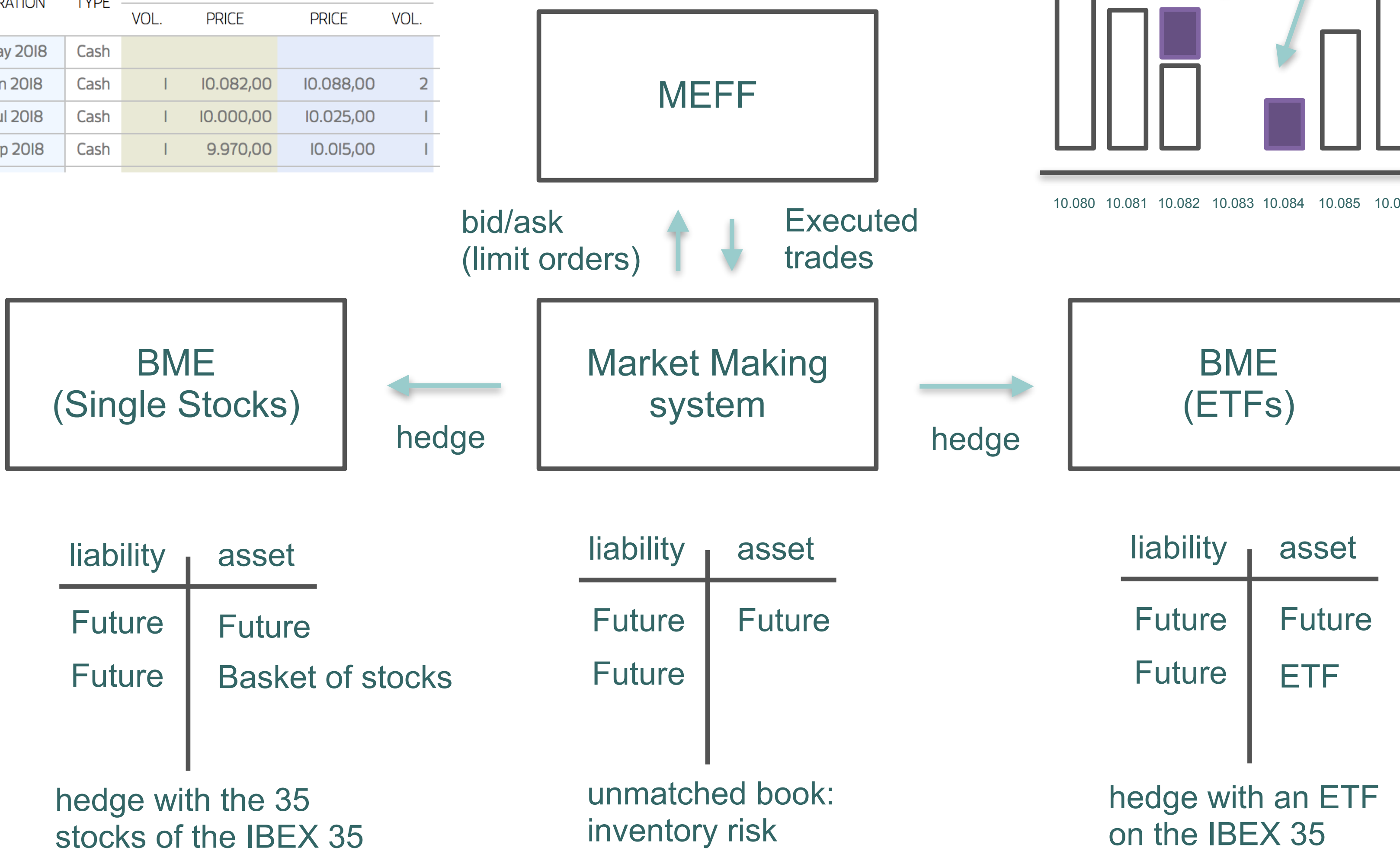
IBEX 35 Futures

BANKIA, S.A.	Eladio Sala esala@bankia.es
BNP PARIBAS ARBITRAGE SNC	Christophe Zacharaiasen christophe.zachariasen@bnpparibas.com
SOCIÉTÉ GÉNÉRALE, SUCURSAL EN ESPAÑA	Fadh el Bakkali fadh.elbakkali@sgcib.com
TOWER RESEARCH CAPITAL EUROPE LIMITED.	Angela Cappuccino angela.cappuccino@tower-research.com
VIRTU FINANCIAL IRELAND LTD	Sinead Lynch slynch@virtufinancial.com

Example 2: Market-making in an order-driven market: IBEX 35 Futures at MEFF

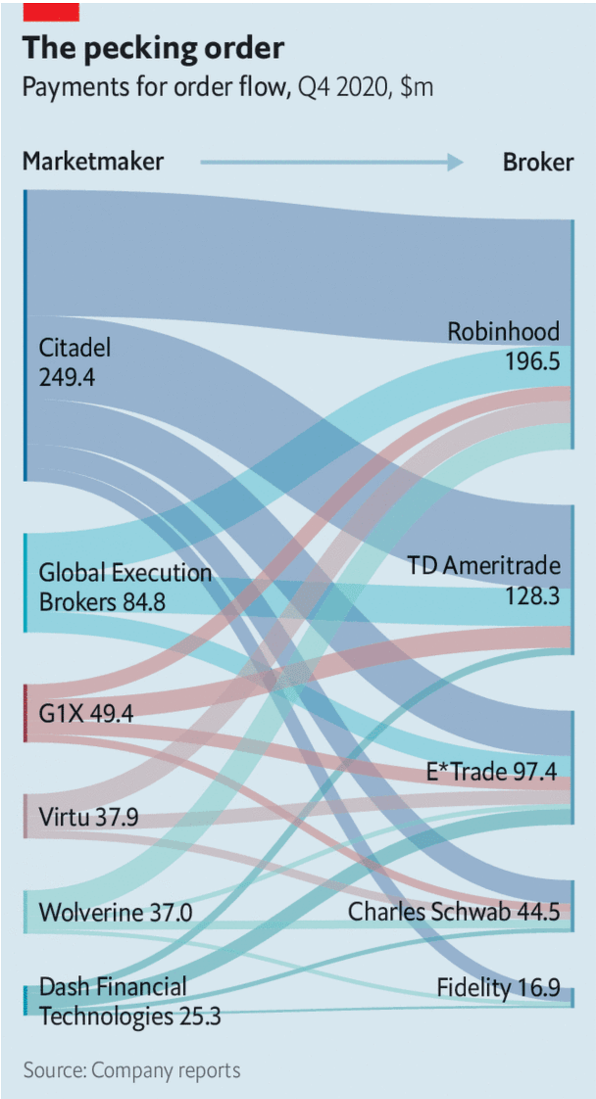
EXPIRATION	TYPE	BID		ASK	
		VOL.	PRICE	PRICE	VOL.
18 may 2018	Cash				
15 jun 2018	Cash	1	10.082,00	10.088,00	2
20 jul 2018	Cash	1	10.000,00	10.025,00	1
21 sep 2018	Cash	1	9.970,00	10.015,00	1





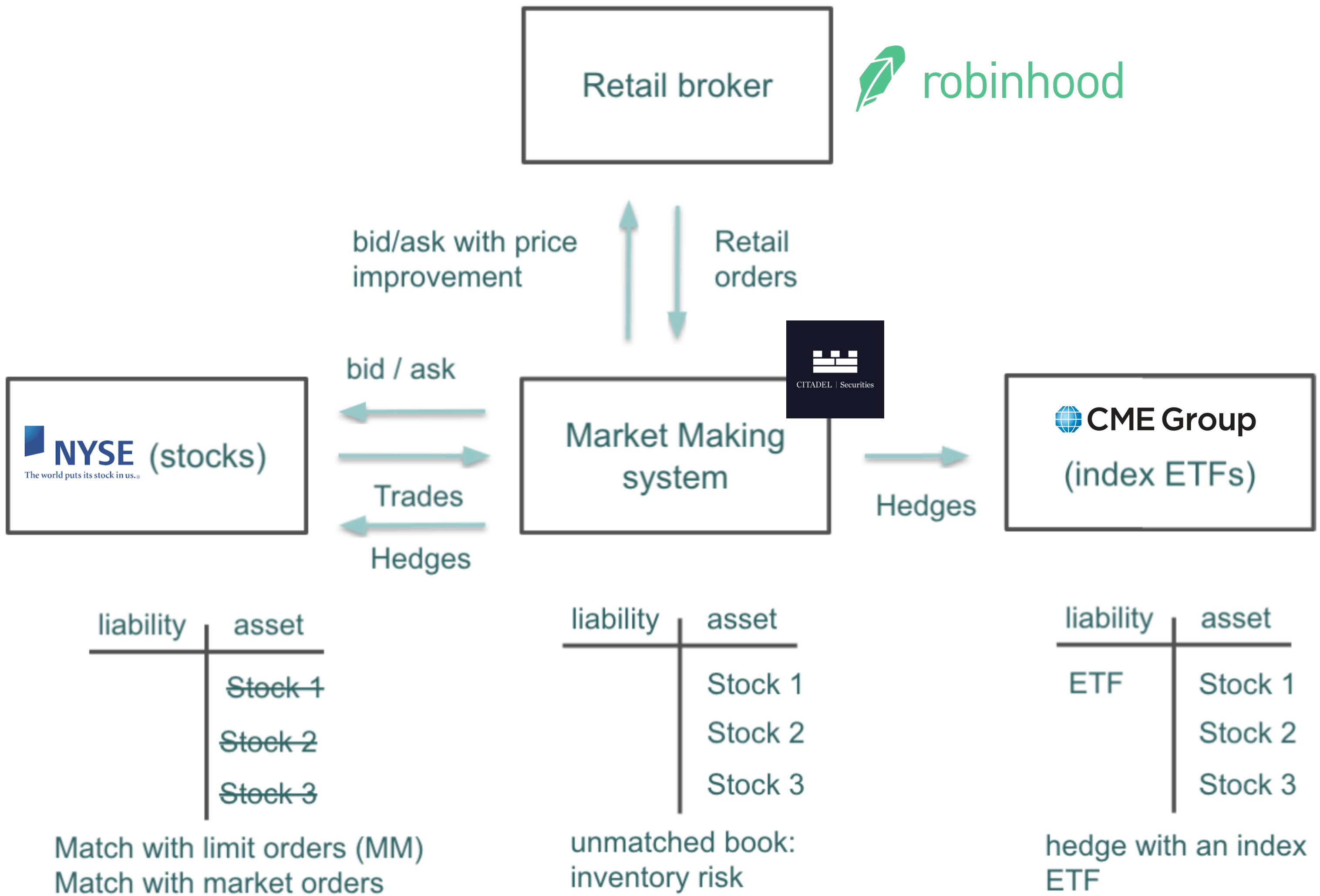
<u>Example 3:</u> Market-making for an internalised in NYSE Stocks

- **Robinhood** is a popular broker that gives access to retail investors to trade **stocks, ETFs, options**, etc
- It revolutionized the retail investor in 2015 with its accessible app and trading **without trading fees**, which would force the competition to change their business model
- Instead of making money from fees as traditional brokers, it uses "**Payment for Order Flow"** (PFOF), by sending retail orders typically to an HFT Market Making or new liquidity provider, for example Citadel Securities, Virtu or Two Sigma
- HFT Market Makers offer a **price improvement** in bid/ asks with respect to a Trading Venue, since they benefit from a flow of orders that typically are information asymmetry and adverse selection risk than those from large investors
- The retail broker keeps part or the totality of the price improvement as an **implicit fee** to the customer



The Economist

<u>Example 3:</u> Market-making for an internalised in NYSE Stocks



Models for the determinants of the bid-offer spread

- The research on **market microstructure** has devoted a large corpus to explain the **existence of a bid-offer spread** in markets “in equilibrium”
- Most of the explanations come from models that try to explain the bid-offer spread as a **compensation for the costs and risks of the market-making** activity.
  - They typically model market-makers as rational “optimising” agents
  - This might not be a realistic assumption to understand real “human” market-makers, but they provide a good basis to build market-making algorithms
- We will analyse two of the **most important models**:
  - Grossman-Miller model (1988): simplified model to understand the influence of the inventory risk in the bid-offer spread
  - Glosten-Milgrom model (1985): simplified model to understand the influence of asymmetric information (more informed traders than market makers) in the bid-offer spread

Grossman-Miller model

- Toy model to **analyze the effect that inventory risk has over the quoted spread**
- Set-up of the model:
  - There are *n* identical market-makers (MMs) for a given asset competing with each other, and liquidity traders (LTs) who consume liquidity from the market-makers
  - There are only three discrete time-steps to quote and trade: t = {1, 2, 3}
  - There is no uncertainty about the arrival of liquidity traders:
    - at t = 1 a liquidity trader (LT1) comes to the market to sell *i* units of the asset
    - at t = 2 another liquidity trader (LT2) comes to the market to buy *i* units
  - There are no trading costs or direct costs for holding the inventory
  - The MMs start with a cash amount of W_0 but no assets
  - Uncertainty comes from the price of the asset, S_t, which has dynamics:

$$S_3 = \mu + \epsilon_2 + \epsilon_3$$

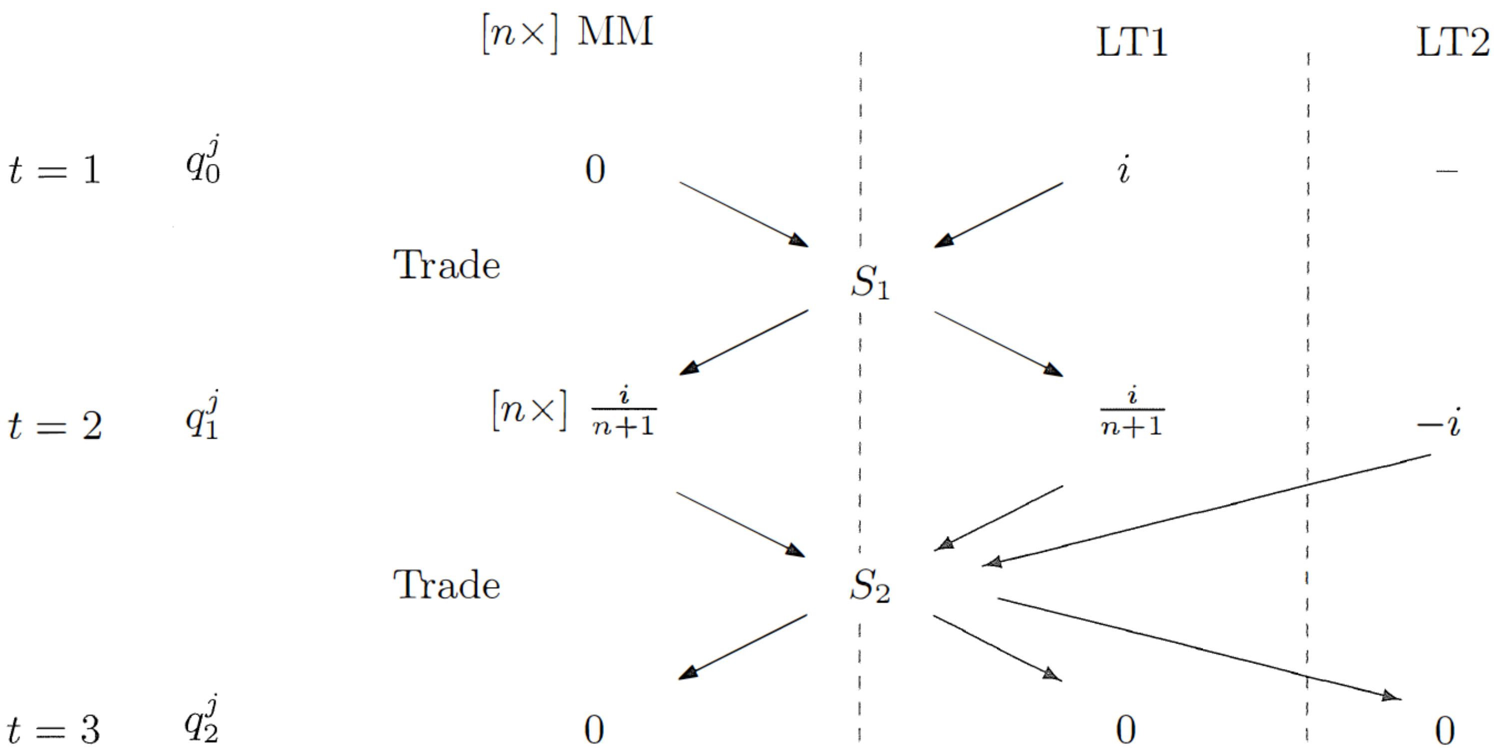
$$\mu \sim constant \qquad \epsilon_2 \text{ announced between t} = 1 \text{ and t} = 2$$

$$\epsilon_i \sim N(0, \sigma^2) \qquad \epsilon_3 \text{ announced between t} = 2 \text{ and t} = 3$$

  - MMs and LTs are risk averse, which is modelled by an exponential utility function on the cash position X_i:

$$U(X) = -exp(-\gamma X)$$

Grossman-Miller model



Sketch of the model. Source: Cartea et al (2015)

Grossman-Miller model

Solving the model:

- Each agent, the n MMs and the liquidity traders LT1 and LT2 **maximise**, at each period, their **expected utility** given the current information:

$$max_{q_t^j} E_t[U(X_{t+1}^j)]\,, \quad j \in \{MM, LT1, LT2\}$$
$$X_{t+1}^j = X_t^j + q_t^j S_{t+1}$$

where the cash accounts evolves with the asset price

- The model is solved **backwards in time**. The goal is to find quantities *q* and prices *S* such that supply and demand are in equilibrium at each period

Grossman-Miller model

Result:

- The equilibrium price for the asset quoted by the market-makers is:

$$S_1 = \mu - \gamma\sigma^2 \frac{i}{n+1} < E_1[S_2] = \mu$$

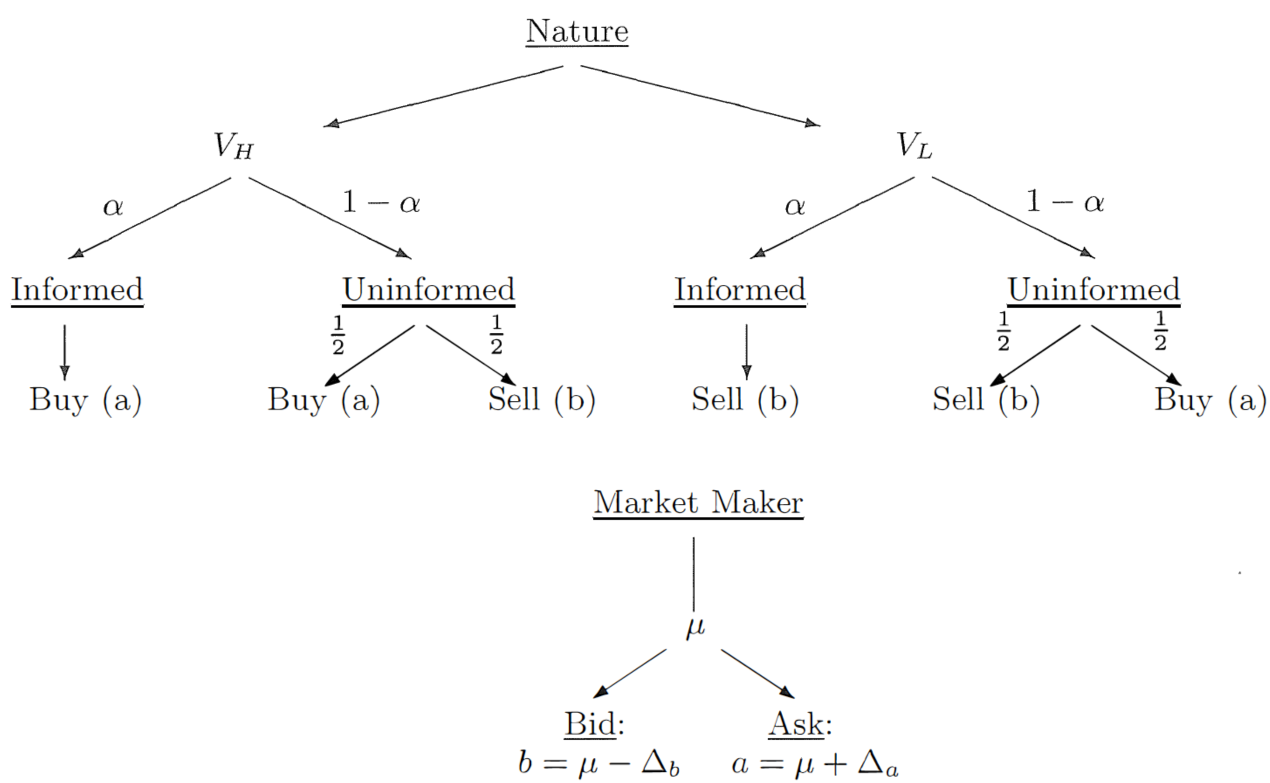
- The price quoted differs from the fair or efficient price by the "liquidity premium" (bid-offer spread), which depends on:
  - **risk aversion coefficient**: more risk averse market makers quote larger spreads
  - **volatility of the asset**: market makers quote larger bid-offer spreads when volatilities of the assets are larger, since inventory risk is higher
  - **volume that the liquidity traders want to trade** in the market: the larger the volume, the larger the spread since inventory holds will increase, and hence overall inventory risk
  - **number of market makers**: since they compete with each other, the more market-makers the smaller the spread. When the number of market-makers is very large, the spread tends to zero and the price tends to the efficient one due to competition

Glosten-Milgrom model

- Toy model to **analize the effect on the quoted spreads of the information asymmetry between the market-maker and some informed traders**
- Set-up of the model:
  - There are *n* competitive market-makers, liquidity traders and informed traders
  - The population of liquidity traders and informed traders is normalised to 1, and a proportion of α are and informed and $1-\alpha$ are non-informed (liquidity traders)
  - The asset has a future value of v which can be either high, V_H, with probability p, and low, V_L, with probability $1-p$
  - Market makers quote firm bid and asks: bid at price a and ask at price b
  - Liquidity traders are price insensitive and will buy or sell with probability 1/2 irrespective of the quoted prices from the market-makers
  - Informed traders know the future value of the asset but for simplicity are only allowed to trade one unit:
    - If $v = V_H$ they will buy one unit if $a < V_H$, and do nothing otherwise
    - if $v = V_L$ they will sell one unit if $b < V_L$, and do nothing otherwise

Glosten-Milgrom model

- Trading happens in **only one time-step**, where “nature” chooses randomly:
  - whether the market maker trades with a liquidity trader or an informed trader
    - in case of a liquidity trader, whether it buys or sells
  - whether the asset has a high or low future value
- The probability tree looks as follows:



Source: Cartea et al (2015)

Glosten-Milgrom model

Solving the model:

- The market-makers must choose their bid and ask quotes to maximise their expected profit. However, in a fully **competitive** setting, **the market-maker expected profits tend to zero in equilibrium**
- Therefore, the condition to solver the optimal bid/ask quotes is zero expected profit:

$$p\left(\alpha(\mu+\Delta_a-V_H)+\frac{1-\alpha}{2}(\mu+\Delta_a-V_H)\right)+(1-p)(\frac{1-\alpha}{2})(\mu+\Delta_a-V_L)=0$$

$$p(\frac{1-\alpha}{2})(\mu-\Delta_b+V_H)+(1-p)\left(\alpha(\mu-\Delta_b+V_L)+\frac{1-\alpha}{2}(\mu-\Delta_b+V_L)\right)=0$$

Solution

- Quoted bid/asks:

$$\Delta_b=\frac{1}{1+\frac{1-\alpha}{\alpha}\frac{1}{2(1-p)}}(\mu-V_L)$$

$$\Delta_a=\frac{1}{1+\frac{1-\alpha}{\alpha}\frac{1}{2p}}(V_H-\mu)$$

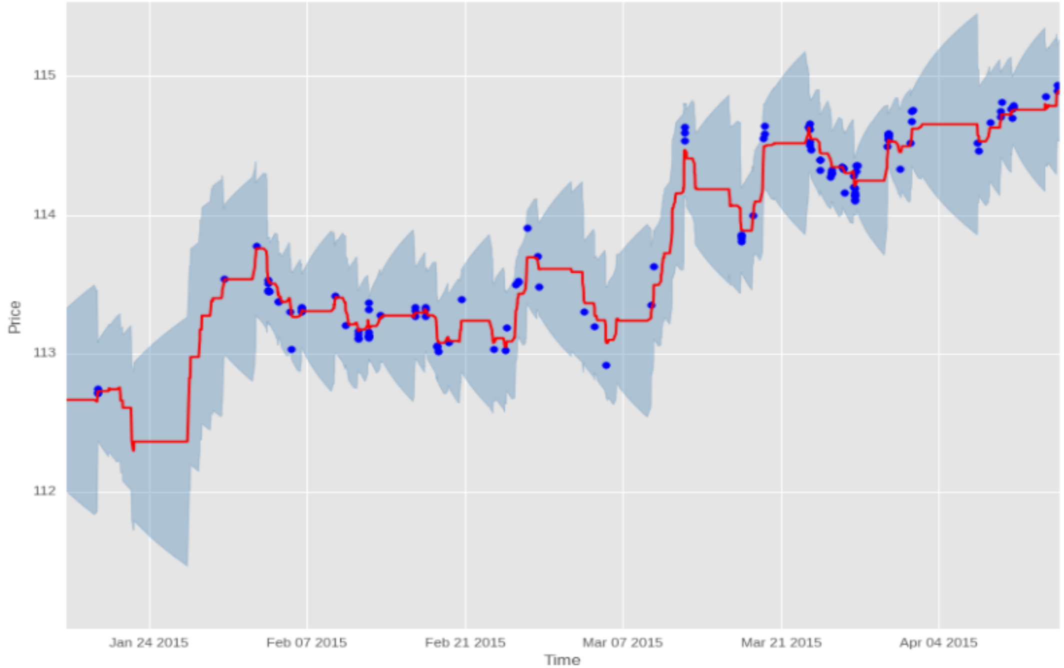
- The larger the probability of an informed or **"toxic" flow**, α, the larger the bid/ask spread: this way the market-makers protect themselves against the toxic flow
- For the same reason, the larger the **magnitude of the toxicity**, which is given by the spreads $V_L-\mu$ and $\mu-V_H$, the larger is consequentially the bid/ask spread

Market-making algorithms

- Algorithms can be used to automate the three components of a market-making system: **fair price determination, quoting (spread & skew) and hedging**
- There are two **paradigms** of algorithmic market-making systems:
  - **Partially automated system**: some of the components are automated. Examples:
    - Fair prices, quotes and hedges are automated, but a human trader adjusts them according to his/her estimation of market conditions, inventory, risk aversion…
    - Fair prices and hedges are automated, but human traders quote manually:
      - typically in quote-driven markets with a relatively small number of requests (< 100 per trader).
      - with more requests per trader or in order-driven markets it is more difficult to keep the pace necessary to answer RfQs or update limit orders
  - **Full automated system**: the algorithm quotes automatically and handles its own inventory. It adjusts the spread and skew depending on inventory levels and market conditions, auto-hedging the residual when necessary
    - In high-frequency-trading firms this is the most common case
    - Banks are tending to fully automate market-making for small notionals

Market-making algorithms

- **Algorithmic fair-price determination.** As discussed already:
  - For very liquid markets, the market "mid" price is typically used
  - For less liquid and/or fragmented markets, statistical models (e.g. Bayesian models) are used to build fair-prices out of observations at different times / markets
  - For derivatives in not very liquid markets, risk-neutral models are typically used



Example of Bayesian model to estimate a fair price

Market-making algorithms

- **Algorithmic quoting**:
  - For **partially automated systems**, where the algorithms don't handle their own inventory, simple "auto-negotiation" rules are commonly used. Examples:
    - a pre-calculated bid-offer spread on top of the fair price
    - place limit orders at the latest best bid/ask
  - For **fully automated systems**, models based on rational agents are typically used as a basic component of the algorithmic strategy:
    - They maximise the utility of the final inventory for a given level of risk aversion
    - Stochastic Control Theory is typically used to derive the optimal strategy. The basic framework was proposed by Avellaneda & Stoikov (2008). See later.
  - Client analysis is used to **differentiate the quotes** (if their ID is known or inferred)
- **Algorithmic hedging.** It has two main sub-components:
  - Automated choice of the hedging instrument(s) according to correlation, liquidity …
  - Automated execution of the hedge in the market, for which algorithmic execution algorithms can be used
- Quoting and hedging are linked via the risk evaluation of the inventory

Optimal market-making quotes

- The optimal strategy to quote bid-offer quotes for a market-maker who wants to profit from round trips buy/sell was addressed initially by **Holl & Stoll in their 1981** influential paper: “**Optimal dealer pricing under transactions and return uncertainty**”
- They use **Stochastic Dynamic Programming** to solve the problem of an agent, the market-maker, who must choose optimal bid-ask quotes to maximise his/her expected utility of terminal wealth. The fair price of the instrument traded is stochastic, as are the arrival of liquidity consuming orders from other agents.
- The model can be used in principle for **optimal market-making in quote-driven and order-driven markets,** although a careful adaptation must be done when prices are discrete, as in Limit Order Books (tick sizes)
- In **2008**, **Avellaneda and Stoikov** followed up on the formalism introduced by Ho & Stoll and tackled the problem of optimal market-making in the context of high-frequency-trading of Stocks.
    - Their paper re-popularized the Ho & Stoll analysis and introduced simplifications
    - In **2012**, **Gueant, Lehalle and Fernández-Tapia** extended the Avellaneda and Stoikov results and obtained useful closed-form approximations for optimal bid-asks

Avellaneda & Stoikov model

Set-up of the model:

- There is a single market-maker quoting bid and ask for a given instrument
- The "fair" price (or "mid") of the instrument follows a Brownian dynamics:

$$dS_t = \sigma dW_t$$

- The market-maker quotes continuously bid and asks prices, S_t^b and S_t^a, and we can define the following half bid-ask spreads:

$$\delta_t^a = S_t^a - S_t \text{ and } \delta_t^b = S_t - S_t^b$$

- From these we define the quoted bid-ask spread and the skew as:

$$\psi_t = \delta_t^b + \delta_t^a$$
$$\eta_t = \delta_t^b - \delta_t^a$$

- Transactions happen at random times, and only one unit of the asset is transacted per transaction.

Avellaneda & Stoikov model

- The total number of transactions at a given time t at the bid and the ask is modelled by point processes N_t^a and N_t^b with intensities λ_t^a and λ_t^b which depend exponentially on the half-spreads quoted by the market-maker:

$$\lambda_t^a = Ae^{-k\delta_t^a}$$

$$\lambda_t^b = Ae^{-k\delta_t^b}$$

- The market-maker inventory is, at any given time:

$$q_t = N_t^b - N_t^a$$

- And the cash amount the market-maker holds evolves with the following dynamics:

$$dX_t = S_t^a dN_t^a - S_t^b dN_t^b = (S_t + \delta_t^a)dN_t^a - (S_t - \delta_t^b)dN_t^b$$

- The market-maker sets its preferences based on expected utility of terminal cash at a time horizon T with exponential utility and risk aversion coefficient ɣ.

$$u = -\exp(-\gamma(X_T + q_T S_T - l(q_T))]$$

where we assume the market-maker liquidates the remaining inventory at T with a liquidity penalty *l(q)*

Avellaneda & Stoikov model

Optimal market-making problem:

$$max_{\{\delta_t^a,\delta_t^b\}} E_0[-\exp(-\gamma(X_T + q_T S_T - l(q_T))]$$

Solution: optimal bid/ask are given by:

$$S_t^b = r_t^b(q_t, S_t) - \frac{1}{\gamma}\log\left(1 - \frac{\gamma}{k}\right)$$
$$S_t^a = r_t^a(q_t, S_t) + \frac{1}{\gamma}\log\left(1 - \frac{\gamma}{k}\right)$$

They depend on:

- **Reservation prices** r_t, which are the highest/lowest prices at which the market maker is willing to buy/sell the instrument. They don't have a closed-form in the model, but generally depend on inventory, liquidity penalty, fair price, volatility, etc
- **Risk aversion** of the market-maker: the more risk-averse, the larger the spreads
- The **probability of closing a trade far from the fair price**, given by the parameter *k*, the larger it is, the smaller the probability, reducing the bid-ask spreads

Avellaneda & Stoikov model

Simulations (Avellaneda & Stoikov, 2007):

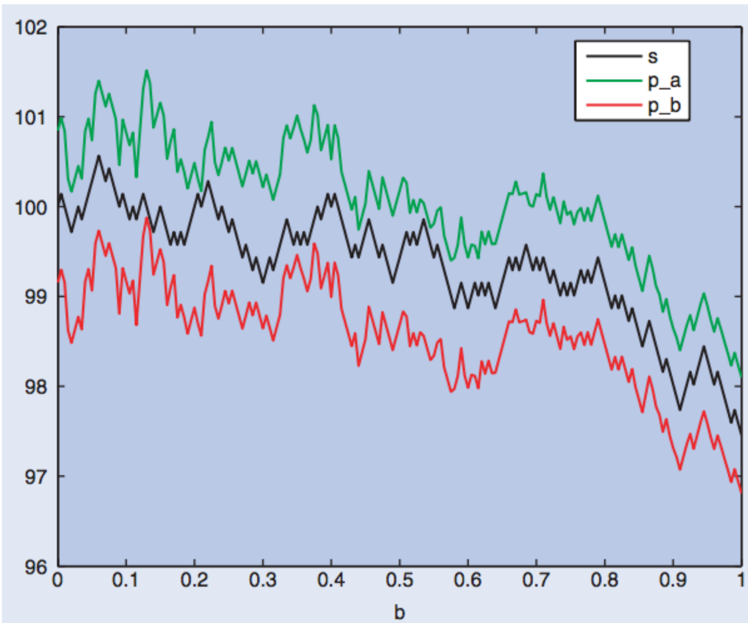


Figure 1. The mid-price and the optimal bid and ask quotes.

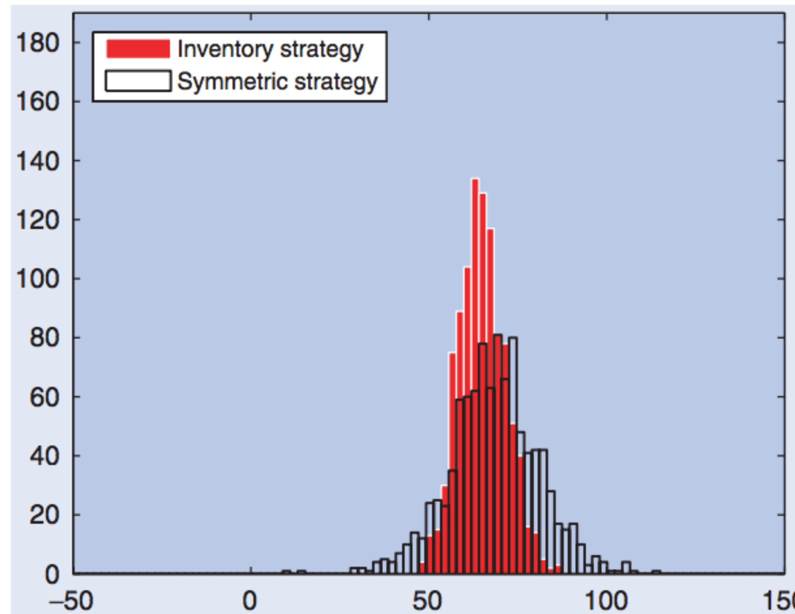


Figure 2. $\gamma = 0.1$.

Table 1. 1000 simulations with $\gamma = 0.1$.

Strategy	Average spread	Profit	Std (Profit)	Final q	Std (Final q)
Inventory	1.49	65.0	6.6	0.08	2.9
Symmetric	1.49	68.4	12.7	0.26	8.4

Table 2. 1000 simulations with $\gamma = 0.01$.

Strategy	Average Spread	Profit	Std (Profit)	Final q	Std (Final q)
Inventory	1.35	68.6	8.7	0.12	5.1
Symmetric	1.35	68.8	12.8	0.09	8.7

Avellaneda & Stoikov model

Simulations (Avellaneda & Stoikov, 2007):

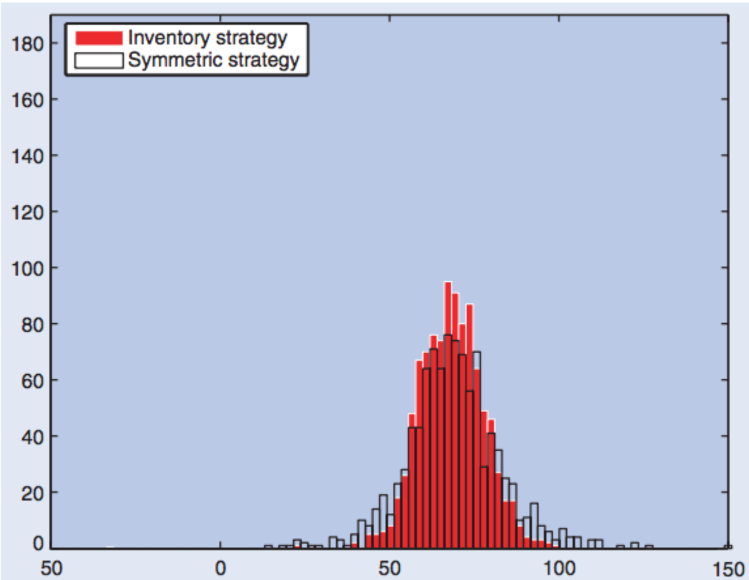


Figure 3. $\gamma = 0.01$.

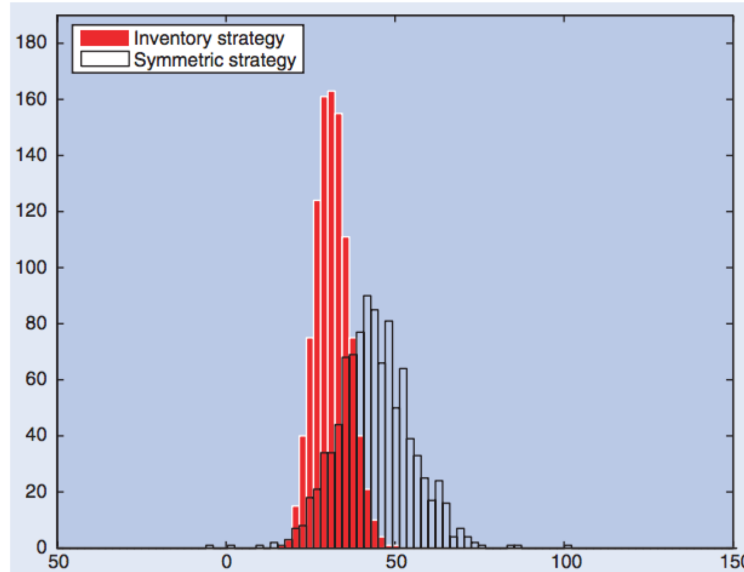


Figure 4. $\gamma = 1$.

Table 3. 1000 simulations with $\gamma = 1$.

Strategy	Average spread	Profit	Std (Profit)	Final q	Std (Final q)
Inventory	3.02	31.4	5.0	0.02	1.7
Symmetric	3.02	44.0	11.0	0.00	5.1

Avellaneda & Stoikov model

Guéant - Lehalle-Fernandez-Tapia approximation for infinite horizon $(T \to \infty)$

- Optimal bid-asks:

$$S_t^b = S_t + \frac{2q+1}{2}\sqrt{\frac{\sigma^2\gamma}{2kA}(1+\frac{\gamma}{k})^{1+\frac{k}{\gamma}}} - \frac{1}{\gamma}\log\left(1-\frac{\gamma}{k}\right)$$

$$S_t^a = S_t + \frac{2q-1}{2}\sqrt{\frac{\sigma^2\gamma}{2kA}(1+\frac{\gamma}{k})^{1+\frac{k}{\gamma}}} + \frac{1}{\gamma}\log\left(1-\frac{\gamma}{k}\right)$$

- Spread and skew:

$$\psi_\infty = \frac{2}{\gamma}\log\left(1-\frac{\gamma}{k}\right) + \sqrt{\frac{\sigma^2\gamma}{2kA}(1+\frac{\gamma}{k})^{1+\frac{k}{\gamma}}}$$

$$\eta_\infty = 2q\sqrt{\frac{\sigma^2\gamma}{2kA}(1+\frac{\gamma}{k})^{1+\frac{k}{\gamma}}}$$

Avellaneda & Stoikov model

This approximation allows to understand the quotes of the market-maker as a result of two risks of different nature:

- A static risk associated with **transaction uncertainty**, which exists even if price volatility is zero, where the spread is:

$$\psi_{\infty}(\sigma = 0) = \frac{2}{\gamma}\log\left(1 + \frac{\gamma}{k}\right)$$

  - This spread **balances the frequency of trades with the margin of trades**: the farther from the "fair" price the quotes are, the larger the margin but the smaller the probability of closing a trade at that price.
  - It only affects the spread, not the skew, since it affects equally bids and asks
- **Price (inventory) risk**, which can be seen as the **risk that the prices moves adversely without the market-maker being able to unwind the position quickly** enough because of trade uncertainty
  - This risk affects both spreads and skews, but the effect on **the skew depends on the inventory size**: the market-maker will skew its quotes more aggressively with inventory size, to avoid holding large inventories
  - Both spread and skew depend positively on risk aversion and volatility, and negatively on the probability of closing a trade quoting far from the "fair" price