HW 1

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(a) associative: $A(BC)=(AB)C$

set $A_{n\times n}=(a_{ij})$ $B_{n\times n}=(b_{ij})$ $C_{n\times n}=(c_{ij})$

$AB=(d_{ij})$ $BC=(e_{ij})$

$ABC=(f_{ij})$ $A(BC)=(g_{ij})$

$\Rightarrow d_{ij}=a_{i1}b_{1j}+a_{i2}b_{2j}+\cdots+a_{in}+b_{nj}, j,i=1,2,\cdots n$

$e_{ij}=b_{i1}c_{1j}+b_{i2}c_{2j}+\cdots+b_{in}c_{nj}, i,j=1,2,\cdots n$

$f_{ij}=d_{i1}c_{1j}+d_{i2}c_{2j}+\cdots+d_{in}c_{nj}, i,j=1,2,\cdots n$

$g_{ij}=a_{i1}e_{1j}+a_{i2}e_{2j}+\cdots+a_{in}e_{nj}, i,j=1,2,\cdots n$

$\Rightarrow \forall i,j=1,2,\cdots n$

$$
\begin{aligned}
f_{ij} &= d_{i1}c_{1j}+d_{i2}c_{2j}\cdots+d_{in}c_{nj} \\
&=(a_{i1}b_{11}+a_{i2}b_{21}\cdots+a_{in}b_{n1})c_{1j}+ \\
&\quad (a_{i1}b_{12}+a_{i2}b_{22}\cdots+a_{in}b_{n2})c_{2j}+ \\
&\quad \cdots(a_{i1}b_{1n}+a_{i2}b_{2n}\cdots+a_{in}b_{nn})c_{nj} \\
&=a_{i1}(b_{11}c_{1j}+b_{12}c_{2j}+\cdots+b_{1n}c_{nj})+ \\
&\quad a_{i2}(b_{21}c_{1j}+\cdots+b_{2n}c_{nj})\cdots \\
&\quad +a_{in}(b_{n1}c_{1j}+\cdots b_{nn}c_{nj}) \\
&= a_{i1}e_{1j}+a_{i2}e_{2j}+\cdots a_{in}e_{nj} \\
&= g_{ij}
\end{aligned}
$$

$\Rightarrow (AB)C=A(BC)$

distributive: $A(B+C) = AC+BC$

$(A+B)C = AC+BC$

$$[(A+B)C]_{ij} = \sum_k (A_{ik} + B_{ik}) C_{kj}$$

$$= \sum_k A_{ik} C_{kj} + \sum_k B_{ik} C_{kj}$$

$$= (AC)_{ij} + (BC)_{ij}$$

$\Rightarrow (A+B)C = AC+BC$

works similarly $A(B+C) = AB+AC$

Commtative: $AB = BA$

set $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$AB = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $BA = \begin{bmatrix} 1 & 4 & 9 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$AB \neq BA$

commtative is incorrect.

1b) $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 6 \\ 0 & 0 & 1 \end{bmatrix}$

$[A\ E] = \begin{bmatrix} 1 & 2 & 3 & 1 & 0 & 0 \\ 1 & 4 & 6 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$

$\xrightarrow{r_2 - r_1} \begin{bmatrix} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 2 & 3 & -1 & 1 & 0 \end{bmatrix} \xrightarrow{r_1 - r_2} \begin{bmatrix} 1 & 0 & 0 & 2 & -1 & 0 \\ 0 & 2 & 3 & -1 & 1 & 0 \end{bmatrix}$

$$\xrightarrow{r_2 - r_1} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 2 & 3 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{r_1 - r_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & -1 & 0 \\ 0 & 2 & 3 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{r_2 - 3r_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & -1 & 0 \\ 0 & 2 & 0 & -1 & 1 & -3 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{\frac{1}{2}r_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & -1 & 0 \\ 0 & 1 & 0 & -\frac{1}{2} & \frac{1}{2} & -\frac{3}{2} \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 2 & -1 & 0 \\ -\frac{1}{2} & \frac{1}{2} & -\frac{3}{2} \\ 0 & 0 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 6 \\ 1 & 0 & 0 \end{bmatrix} \quad \det(A) = \begin{vmatrix} 1 & 2 & 3 \\ 1 & 4 & 6 \\ 1 & 0 & 0 \end{vmatrix}$$

$$= 1\times4\times0 + 2\times6\times1 + 3\times1\times0$$
$$- 3\times4\times1 - 2\times1\times0 - 1\times6\times0$$
$$= 12 - 12 = 0$$

$\Rightarrow$ A is a singular matrix $\Rightarrow$ A is not invertible

1(c) $L = (A^T A)^{-1} A^T$

$R = A^T (A A^T)^{-1}$

$Ax = I$
$A^T A x = A^T I$
$x = (A^T A)^{-1} A^T$

$xA = I$
$x A A^T = I A^T$
$x = A^T (A A^T)^{-1}$

$A^T \in R^{2\times3}$, $(A^T A)^{-1} \in R^{3\times3}$
hance A^T can mutiply $(A^T A)^{-1}$

$(A^T A)^{-1} \in R^{3\times3}$, $A^T \in R^{2\times3}$, hance the size of $(A^T A)^{-1}$ can't mutiply A^T

$\therefore$ A has only Right-Pseudo Inverse

$\therefore$ A has only Right-Pseudo Inverse

(d) set matrix W and vectors $\vec{v}$

if $W * \vec{v} = \lambda \vec{v}$ $(\lambda \neq 0)$

these vectors are called eigenvectors and λ is called eigenvalues

A squared symmetric matrix $A = QDQ^T$, where the columns of Q are the eigenvectors of A and D is a diagonal matrix where the entries are the corresponding eignvalues.

That can help to reduce Dimension, so that we can use less data and the speed can be improved.