

Napatau
or
Tk-Lifetime-Analysis

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1 Introduction

Napatau is a lifetime analysis tool which has a graphical user interface. It supports the DDCM analysis of direct and indirect gated spectra measured in RDDS measurements. In the DDCM analysis the time derivative of the doppler shifted intensities ($\frac{d}{dt}I_{sh}$) has to be determined out of the measured intensities I_{sh} . In other programs (e.g. apatau) used in the IKP this was achieved by fitting a few smoothly connected polynomials of second order at the measured datapoints I_{sh} . This procedure is kept in Napatau, but besides the fit of I_{sh} now the unshifted datapoints I_{us} are taken into account additionally. The datapoints I_{us} are fitted simultaneously with the time derivative of the second order polynomials fitted at I_{sh} multiplied by a factor, called taufactor, which should be equal to tau. Furthermore the datapoints outside the region of sensitivity can be left out to reduce the systematical errors. Furthermore the classical method can be applied, by switching off the fit of the datapoints I_{us} .

2 Basic Setup

The variable `$DSAMINPUTHPATH` should be set to the directory in which the files `distances.dat`, `v_c` and `norm.fac` are located. The structure of these files can be read in detail in the Hitch Hiker's Guide of Lifetime Analysis at:

<http://www.ikp.uni-koeln.de/lifetime/HHG2ALTM/HHG2ALTM.html>

After this has been done one can start `napatau`. Fig. 1 is the first you will see. As next one wants to analyse the measured intensities I_{us} and I_{sh} , and in the indirect case additionally the intensities of the feeding transition. The file with the intensities should have the structure described in the Hitch Hiker's Guide. Napatau automatically determines if it's a direct or indirect case. In the indirect case after loading the intensity file a window with the alpha plot appears, which can be saved as postscript file (see Fig. 2).

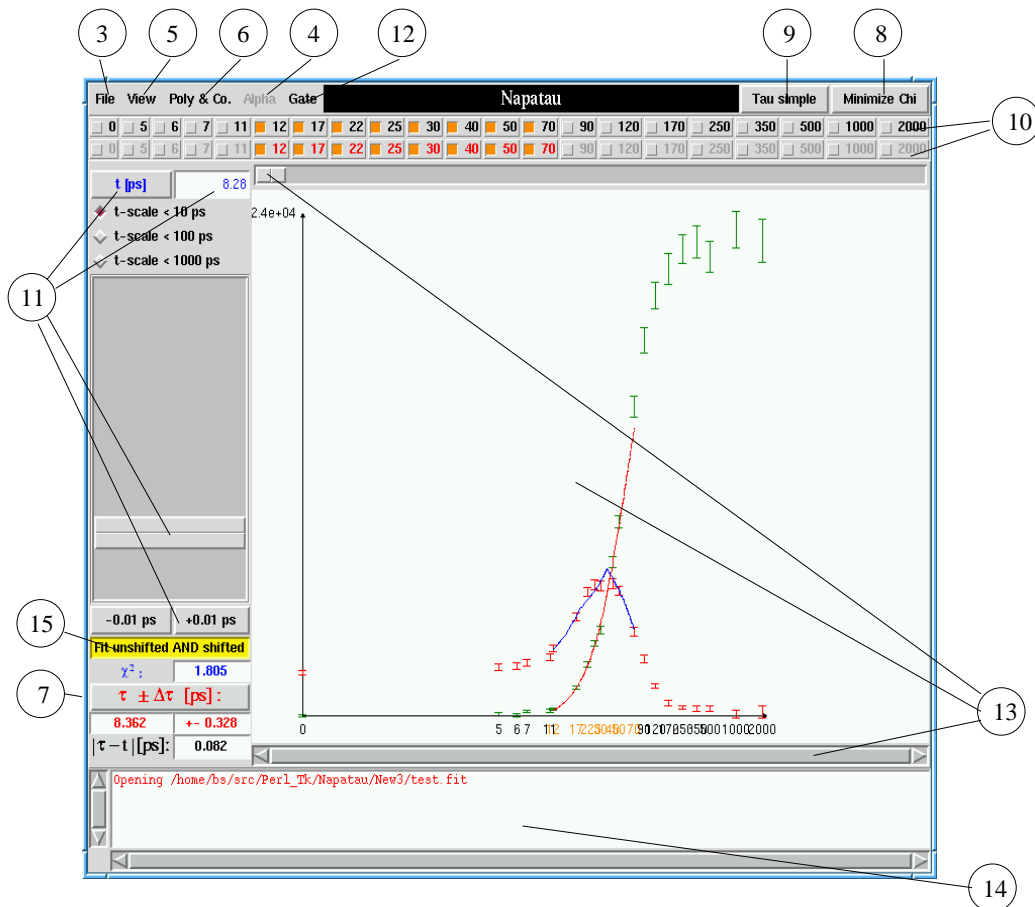


Figure 1: The initial napatau window. Numbers refer to the sections of this manual.

3 File opening and saving

The File menu has four buttons:

- Open: Opening file with measured intensities.
- Save: Saving the present analysis. The plots which can be saved as post-script files appear in additional windows. All data is stored in the filename .napadat

file. Further a file `filename.napaset` is created which saves the inter-actively done adjustments, and which can be reread:

- Read Setup: Reads the file `filename.napaset` from a previous analysis, which contains information about:
 - Taufactor
 - Region of Sensitivity
 - Number of Polynomials
 - Samplingpoints
- Exit: Quit the Programm, without saving the present analysis!

4 Alpha calculation

In the indirect gating case the required alpha can be calculated in two ways or given manually. Alpha menu:

- Sum Ratio: $\alpha = \frac{\sum_i I_{us}^i + I_{sh}^i}{\sum_i I_{us}^i + I_{sh}^i}$
- Weigted Mean (Default): $\alpha_i = \frac{I_{us}^i + I_{sh}^i}{I_{us}^i + I_{sh}^i}$
- Manual: A alpha calculated in some other way can be given manually.

After changing the alpha the new value will be displayed in a new window (see Fig. 2). While analysing direct gated data the alpha menu will be disabled.

5 View

In this menu one can choose the items which should be displayed on the canvas and one can view the tauplot and the alpha window. Just press an find out what happens!

6 Poly & Co

This menu enables to choose the number of polynomials fitted to the datapoints. Furtheron one can move the sampling points of these polynomials (see Fig. 3).

- 1 - 10 (Default: 3): Number of Polynomials!
- Equidistant: The sampling points of the polynomials are placed equidistant in the region of sensitivity.
- Exponential (Default): The distances of the sampling points to each other increase exponential to larger distances.
- Manual: The sampling points can be moved manual.(Fig. 3)

7 Error calculation

By pressing the button 7 in Fig. 1 the error and the mean τ value is calculated and shown in the labels below. Furthermore the difference of τ and t is calculated and displayed in the corresponding label. The formular for the calculation of the error is presented in section 17.

8 Minimize Chi

The button `Minimize Chi` sets the tau factor to the value for which the χ^2 is minimal.

9 Tau Simple

This button uses a simple algorithm to calculate a first estimate of τ . For a detailed descripten see section 17.

10 Region of Sensitivity

The second row of buttons in the main window enables the choice of the region of sensitivity. The fit of the data will only take the points into account, which are selected with this buttons. In the row below one can choose the points which should be taken into account for the calculation of τ and $\Delta\tau$.

11 Taufactor

The taufactor (see sections 1,17) can be changed in three ways:

1. The large scrollbar at the left of the main window.
2. The buttons just below the large taufactor scrollbar.
3. Type in the taufactor in the textwidet above the taufactor scrollbar and press the button `tau[ps]` to calculate a new fit.

12 Gate

With the Gate menubutton one must select the gating methode before loading the fit-file. The columns of the fit-file should have the following order:

- direct gate from above:
 $distance_LABEL \quad I^{sh} \quad \Delta I^{sh} \quad I^{us} \quad \Delta I^{us}$
 \dots
- indirect gates from above:
 $distance_LABEL \quad \underbrace{I^{sh} \quad \Delta I^{sh} \quad I^{us} \quad \Delta I^{us}}_{depop.transitions} \quad \underbrace{I^{sh} \quad \Delta I^{sh} \quad I^{us} \quad \Delta I^{us}}_{directfeeder}$
 \dots
- direct gate from below:
 $distance_LABEL \quad \underbrace{(I^{us} + I^{sh}) \quad \Delta(I^{us} + I^{sh}) \quad I^{sh} \quad \Delta I^{sh}}_{directfeeder}$
 \dots

In the case of a gate from below the intensities of the component $(I^{us} + I^{sh})_i$ of the direct feeding transition will be transformed in the following way:

$$(\widetilde{I^{us} + I^{sh}})_i = 1.05((I^{us} + I^{sh})_{max} + \Delta(I^{us} + I^{sh})_{max}) - (I^{us} + I^{sh})_i \quad (1)$$

13 Canvas

The canvas displays the data and further information. The scrollbar above enables a horizontal scaling of the canvas, the scrollbar below a horizontal movement. The information displayed can be chosen with the View menu (see section 5).

14 Information

At the bottom of the main window is a scrollable text widget, which informs about errors, file opening etc.

15 Weight of the Fit

With the yellow button below the tauscale one can change the type of fit concerning the two different components I_{us} and I_{sh} . If the button is pressed (yellow) both of the components I_{us} and I_{sh} will be taken into account for the fit. If the button is not pressed (grey) just I_{sh} is taken into account, which is the classical method used by apatau. The value which will be changed through this button is w ($= 1$ if button pressed, $= 0$ if button not pressed) used in the equations (5) and (6) in the appendix.

16 Installation

To install NAPATAU you will need the following:

- Perl 5.004 or newer: available at <http://www.perl.com>
- Perl/Tk 8 or newer: available at <http://www.cpan.org>
- The Perl Random.pm module: <http://www.cpan.org>
- The Perl Matrix.pm module: <http://www.cpan.org>
- The Perl Tau.pm module included in the lifetime package: <http://www.ikp.uni-koeln.de/downloads/lifetime.tar.gz>
- The Napatau Source: <http://www.ikp.uni-koeln.de/downloads/napatau.gz>

Install Perl and Perl/Tk according to their description. Check if they are installed properly:

```
perl -MTk -e 'print $Tk::VERSION, "\n";'
```

Install the Matrix.pm and Random.pm modules from cpan.org. Put Tau.pm in a place where Perl can find it. (See the PERL5LIB environment variable or try

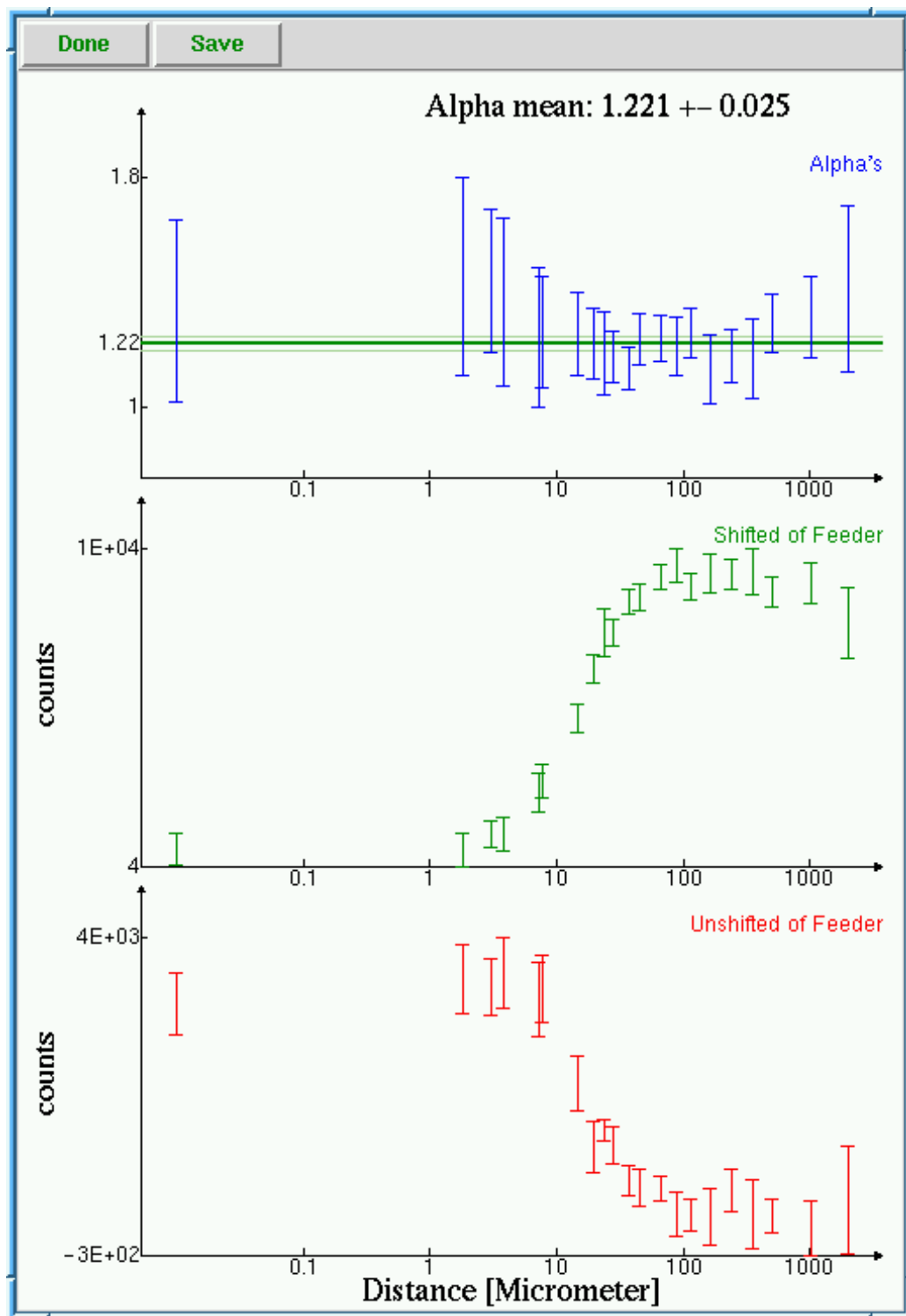
```
perl -e 'print map \{"$_:"} @INC, "\n";'
```

and see the path there.)

Unzip and tar the *napatau.tar.gz* source and copy the Perl file *napatau* to the desired directory and the *.ppm files to the directory:

```
/ikp/share/leben/bilder/NAPATAU
```

Napatau should now be installed properly. One further problem could occur because of the missing of some fonts. If this is the case one should remove/change the fonts in the top of the *napatau* file.



10

Figure 2: The alpha plot window

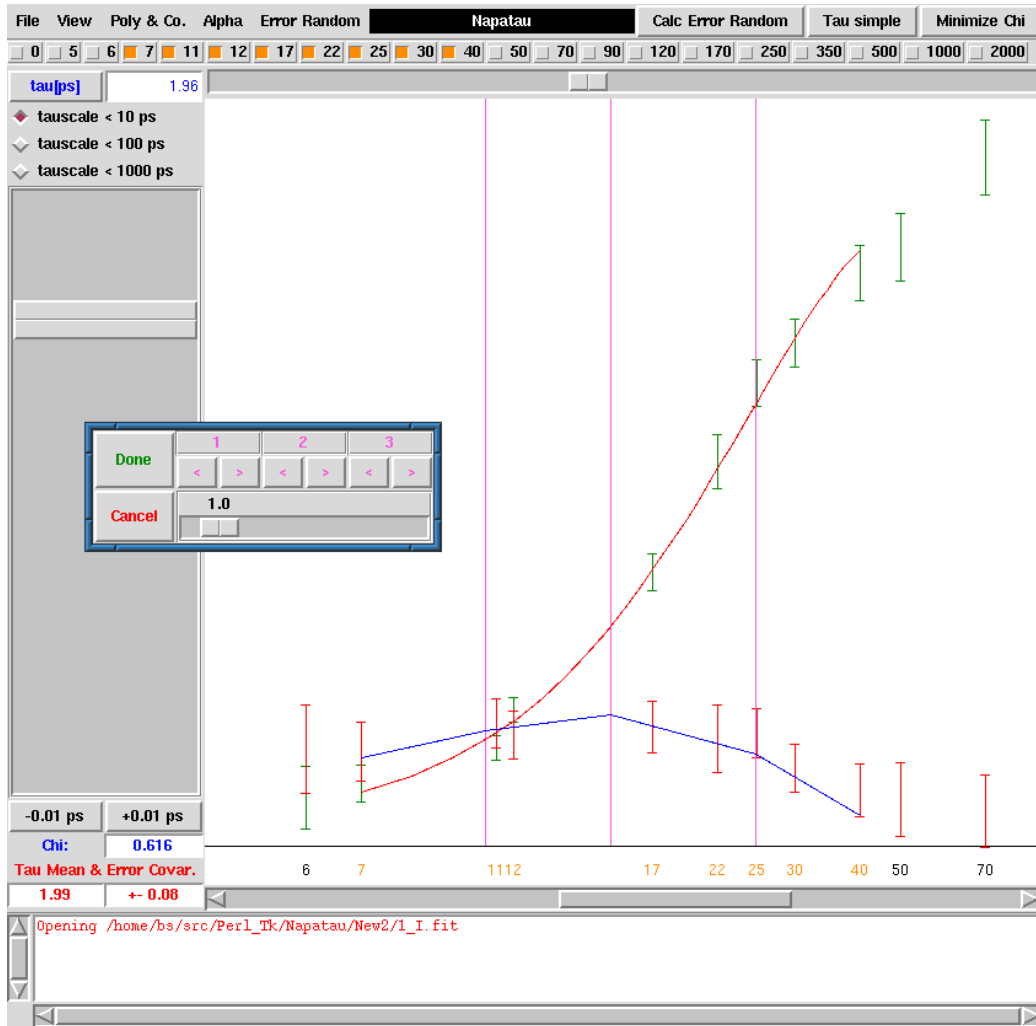


Figure 3: Moving the sampling points of 4 polynomials

17 Appendix

The differential decay-curve (DDC) method is a well known analysing procedure for Recoil-distance Doppler-shift (RDDS) [1] data, which is a powerful technique for the determination of lifetimes of excited nuclear states in the picosecond region. The theoretical background of the DDC method is presented in detail in ref. [2, 3] and the works quoted therein. In the present letter, we report on a new fitting procedure introduced into the DDC analysis, which can be adopted to other related methods using similar fitting algorithms. For a simple explanation of the new approach the DDC method will be used in the case of a gate set on the direct feeder of the investigated level. In this case the lifetime τ_i of the level of interest at one target to stopper (T2S) distance i depends on the intensities of the Doppler shifted I_i^{sh} and unshifted I_i^{us} components of the depopulating transition in the simple way [2, 3]:

$$\tau_i = \frac{I_i^{us}}{\frac{d}{dt} I_i^{sh}}. \quad (2)$$

Under the presumption that $\tau_i = \tilde{t}$ for all T2S distances one can write:

$$I_i^{us} = \tilde{t} \frac{d}{dt} I_i^{sh}. \quad (3)$$

The main problem in calculating τ_i is that the denominator in eq. (2) is a time derivative of a measured quantity and not a direct measurand. This time derivative was evaluated in the case of the DDC method by fitting second order polynomials piecewise at the intensities I_i^{sh} measured at every T2S distance regarding the mean time of flight for the recoils between the target and stopper. Fitting means that the χ^2 :

$$\chi^2 = \sum_i \left(\frac{I_i^{sh} - f^{(a_1, \dots, a_n)}(t_i)}{\Delta I_i^{sh}} \right)^2 \quad (4)$$

is minimized for a fitting function $f(t)$ which depends linear from the free parameters a_1 to a_n , where ΔI_i^{sh} denotes the statistical error of I_i^{sh} . Assuming a hypothetical \tilde{t}^{hyp} the measured intensities I_i^{us} can be included into the fit using eq. (3):

$$\chi^2 = \sum_i \left(\left(\frac{I_i^{sh} - f^{(a_1, \dots, a_n)}(t_i)}{\Delta I_i^{sh}} \right)^2 + w \left(\frac{I_i^{us} - \tilde{t}^{hyp} \frac{d}{dt} f^{(a_1, \dots, a_n)}(t_i)}{\Delta I_i^{us}} \right)^2 \right). \quad (5)$$

The term $\tilde{t}^{hyp} \frac{d}{dt} f^{(a_1, \dots, a_n)}(t_i)$ in eq. (5) must not be linear if \tilde{t}^{hyp} (= taufactor) is seen as a free parameter of the fit. But assuming that \tilde{t}^{hyp} is a constant eq. (5) can

be solved for this \tilde{t}^{hyp} by solving the set of linear equations:

$$\frac{\partial}{\partial a_j} \chi^2 = 0 = \sum_i \left(\left(\frac{I_i^{sh} - \frac{\partial}{\partial a_j} f^{(a_1, \dots, a_n)}(t_i)}{\Delta I_i^{sh}} \right)^2 + w \left(\frac{I_i^{us} - \tilde{t}^{hyp} \frac{d}{dt} \frac{\partial}{\partial a_j} f^{(a_1, \dots, a_n)}(t_i)}{\Delta I_i^{us}} \right)^2 \right). \quad (6)$$

Through variation of \tilde{t}^{hyp} the absolute minimum of eq. (5) can be calculated. Let us denote \tilde{t}^{opt} as the value of \tilde{t}^{hyp} for which the fit is best and $f^{opt}(t)$ the respective fit function. Then we can calculate for every T2S distance a τ_i according to eq. (2):

$$\tau_i = \frac{I_i^{us}}{\frac{d}{dt} I_i^{sh}} = \frac{I_i^{us}}{\frac{d}{dt} f^{opt}(t_i)}, \quad (7)$$

where t_i denotes the mean time of flight corresponding to the T2S distance i . From the τ_i of every T2S distance one can calculate the weighted mean τ_{final} . The absolute difference between τ_{final} and \tilde{t}^{opt} can be interpreted as a systematic error of the fitting procedure.

To conclude, this new approach of fitting DDC data can include the double amount of datapoints and though reduce the statistical error of the fit. Besides this the systematical error of the fitting procedure can be evaluated through comparison of the theoretical hypothesis with the analysed result.

17.1 Error Calculation

In difference to the classical method the error can not be calculated in the simple way, because the denominator of eq. (7) is now dependent from the nominator. For the calculation of the error one has to take into account the covariance between the nominator and denominator of eq. (7).

Now $\frac{d}{dt} f^{opt}(t_i)$ is the derivative of the j -th polynomial P_j , which depends on the free parameters (a_1, \dots, a_n) of the fit. So we can simply write:

$$\frac{d}{dt} f^{opt}(t_i) = \dot{P}_j^{(a_1, \dots, a_n)}(t_i) = \dot{P}_{j(i)} \quad (8)$$

That means that eq. (7) can be written as:

$$\tau_i = \frac{I_i^{us}}{\dot{P}_{j(i)}} \quad (9)$$

Now $\Delta\tau_i$ can be calculated in the following way:

$$\begin{aligned}\Delta\tau_i^2 = & \sum_{k,l} \frac{\partial\tau_i}{\partial I_k^{us}} \frac{\partial\tau_i}{\partial I_l^{us}} cov(I_k^{us}, I_l^{us}) \\ & + \sum_{k,l} \frac{\partial\tau_i}{\partial a_k} \frac{\partial\tau_i}{\partial a_l} cov(a_k, a_l) \\ & + \sum_{k,l} \frac{\partial\tau_i}{\partial I_k^{us}} \frac{\partial\tau_i}{\partial a_l} cov(I_k^{us}, a_l) \quad (10)\end{aligned}$$

The first two sums on the right side of eq. (10) are the ones known from the Gaussian error propagation:

$$\sum_{k,l} \frac{\partial\tau_i}{\partial I_k^{us}} \frac{\partial\tau_i}{\partial I_l^{us}} cov(I_k^{us}, I_l^{us}) = \frac{\Delta I_i^{us2}}{\dot{P}_{j(i)}^2} \quad (11)$$

$$\sum_{k,l} \frac{\partial\tau_i}{\partial a_k} \frac{\partial\tau_i}{\partial a_l} cov(a_k, a_l) = \frac{I_i^{us2}}{\dot{P}_{j(i)}^4} \Delta \dot{P}_{j(i)}^2 \quad (12)$$

with

$$\Delta \dot{P}_{j(i)}^2 = \sum_{k,l} \frac{\partial \dot{P}_{j(i)}}{\partial a_k} \frac{\partial \dot{P}_{j(i)}}{\partial a_l} cov(a_k, a_l) \quad (13)$$

In eq. (13) the term $cov(a_k, a_l)$ is equal to the kl -th matrix element C_{kl}^{-1} of the inverse fit matrix. The third sum in eq. (10) can be calculated in the following way:

$$\begin{aligned}\sum_{k,l} \frac{\partial\tau_i}{\partial I_k^{us}} \frac{\partial\tau_i}{\partial a_l} cov(I_k^{us}, a_l) &= \sum_l \frac{\partial\tau_i}{\partial I_i^{us}} \frac{\partial\tau_i}{\partial a_l} cov(I_i^{us}, a_l) \\ &= \frac{1}{\dot{P}_{j(i)}} \sum_l \frac{\partial\tau_i}{\partial a_l} cov(I_i^{us}, a_l) \\ &= -\frac{I_i^{us}}{\dot{P}_{j(i)}^3} \sum_l \frac{\partial \dot{P}_{j(i)}}{\partial a_l} cov(I_i^{us}, a_l) \quad (14)\end{aligned}$$

Let us write the measured intensities and the taufactor as:

$$(I_1^{us}, \dots, I_N^{us}, I_1^{sh}, \dots, I_N^{sh}, \tilde{t}) = (Y_1, \dots, Y_{2N+1}) \quad (15)$$

Then we can derive $cov(I_i^{us}, a_l)$ in the following way:

$$\begin{aligned}
cov(I_i^{us}, a_l) &= cov(Y_i, a_l) = \sum_{j,k}^{2N+1} \frac{\partial Y_i}{\partial Y_j} \frac{\partial a_l}{\partial Y_k} cov(Y_j, Y_k) \\
&= \sum_k^{2N+1} \frac{\partial a_l}{\partial Y_k} cov(Y_i, Y_k) = \frac{\partial a_l}{\partial Y_i} cov(Y_i, Y_i) \\
&= \frac{\partial a_l}{\partial Y_i} \Delta Y_i^2 = \frac{\partial a_l}{\partial I_i^{us}} \Delta I_i^{us2} \\
&= \left(- \sum_k C_{lk}^{-1} \frac{1}{\Delta I_i^{us2}} \frac{\partial(\tilde{t} \dot{P}_{j(i)})}{\partial a_k} \right) \Delta I_i^{us2} \\
&= -\tilde{t} \sum_k C_{lk}^{-1} \frac{\partial \dot{P}_{j(i)}}{\partial a_k}
\end{aligned} \tag{16}$$

Inserting this in eq. (14) gives us:

$$\begin{aligned}
\sum_{k,l} \frac{\partial \tau_i}{\partial I_k^{us}} \frac{\partial \tau_i}{\partial a_l} cov(I_k^{us}, a_l) &= \frac{I_i^{us} \tilde{t}}{\dot{P}_{j(i)}^3} \sum_{l,k} \frac{\partial \dot{P}_{j(i)}}{\partial a_l} \frac{\partial \dot{P}_{j(i)}}{\partial a_k} C_{lk}^{-1} \\
&= \frac{I_i^{us} \tilde{t}}{\dot{P}_{j(i)}^3} \Delta \dot{P}_{j(i)}^2
\end{aligned} \tag{17}$$

Finally with equations (11,12,13) we get for $\Delta \tau_i$:

$$\Delta \tau_i^2 = \frac{\Delta I_i^{us2}}{\dot{P}_{j(i)}^2} + \frac{I_i^{us2}}{\dot{P}_{j(i)}^4} \Delta \dot{P}_{j(i)}^2 + \frac{I_i^{us} \tilde{t}}{\dot{P}_{j(i)}^3} \Delta \dot{P}_{j(i)}^2 \tag{18}$$

References

- [1] T.K.Alexander and J.S.Forster, in *Advances in Nuclear Physics* v.10, p.197, Eds. M.Baranger and E.Vogt, Plenum Press, 1978
- [2] A.Dewald, S.Harissopulos and P. von Brentano, *Z. Phys.* A334 (1989)163
- [3] G.Böhm, A.Dewald, P.Petkov and P. von Brentano, *Nucl. Instr. Meth. Phys. Res. A* 329 (1993) 248