



Mechanical, Automotive, & Materials Engineering

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CONTENTS

1	Introduction	1
1.1	System Description	1
2	Analysis	4
2.1	Eigenvalue Analysis	4
2.2	Frequency Response Plots	6
2.3	Steady State Gains	7
2.4	Equilibrium Analysis	7
3	Conclusion	9
A	Equations of Motion	10

LIST OF FIGURES

2.1 Frequency response: Steer torque 6

LIST OF TABLES

1.1	Body CG Locations and Mass	1
1.2	Body Inertia Properties	2
1.3	Connection Location and Direction	2
1.4	Connection Locations	2
1.5	Connection Properties	3
2.1	Eigenvalues	4
2.2	Eigenvalue Analysis	5
2.3	Steady State Gains	7
2.4	System Static Deflections	7
2.5	System Preloads	7

CHAPTER 1

INTRODUCTION

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1.1 System Description

The properties of the bodies are given in Tables 1.1 and 1.2. The properties of the connections are given in Tables 1.3, 1.4, and 1.5.

Table 1.1: Body CG Locations and Mass

No.	Body Name	Location [m]	Mass [kg]
1	Frame	0.678, 0.000, 0.472	165.130
2	Upper body	0.415, 0.000, 1.140	33.680
3	Upper fork	1.164, 0.000, 0.770	9.990
4	Lower fork	1.365, 0.000, 0.324	7.250
5	Swing arm	0.196, 0.000, 0.311	8.000
6	Bell crank	0.493, 0.000, 0.173	0.000
7	Front wheel, bike	1.410, 0.000, 0.282	11.900
8	Rear wheel, bike	0.000, 0.000, 0.297	14.700

Table 1.2: Body Inertia Properties

No.	Body Name	Inertia [kg·m ²] (I_{xx} , I_{yy} , I_{zz} ; I_{xy} , I_{yz} , I_{zx})
1	Frame	11.085, 22.013, 14.982; 0.000, 0.000, 3.691
2	Upper body	1.428, 1.347, 0.916; 0.000, 0.000, -0.443
3	Upper fork	1.341, 1.548, 0.413; 0.000, 0.000, 0.000
4	Lower fork	0.000, 0.000, 0.000; 0.000, 0.000, 0.000
5	Swing arm	0.020, 0.259, 0.259; 0.000, 0.000, 0.000
6	Bell crank	0.000, 0.000, 0.000; 0.000, 0.000, 0.000
7	Front wheel, bike	0.270, 0.484, 0.270; 0.000, 0.000, 0.000
8	Rear wheel, bike	0.383, 0.638, 0.383; 0.000, 0.000, 0.000

Note: inertias are defined as the positive integral over the body, e.g.,
 $I_{xy} = + \int r_x r_y dm$.

Table 1.3: Connection Location and Direction

No.	Connection Name	Location [m]	Unit Axis
1	Rear axle	0.000, 0.000, 0.297	0.000, 1.000, 0.000
2	Swing arm pivot	0.549, 0.000, 0.361	0.000, 1.000, 0.000
3	Bell crank pivot	0.539, 0.000, 0.188	0.000, 1.000, 0.000
4	Seat	0.364, 0.000, 0.844	0.000, 1.000, 0.000
5	Steering head	1.173, 0.000, 0.749	0.407, 0.000, -0.914
6	Front axle	1.410, 0.000, 0.282	0.000, 1.000, 0.000
7	Fork slider	1.342, 0.000, 0.426	-0.427, 0.000, 0.904
8	Front tire, lateral	1.410, 0.000, 0.000	0.000, 1.000, 0.000
9	Rear tire, lateral	0.000, 0.000, 0.000	0.000, 1.000, 0.000
10	Front tire, longitudinal	1.410, 0.000, 0.000	1.000, 0.000, 0.000
11	Rear tire, longitudinal	0.000, 0.000, 0.000	1.000, 0.000, 0.000
12	Front tire, vertical	1.410, 0.000, 0.000	0.000, 0.000, 1.000
13	Rear tire, vertical	0.000, 0.000, 0.000	0.000, 0.000, 1.000

Table 1.4: Connection Locations

No.	Connection Name	Location [m]	Location [m]
1	Rear spring	0.487, 0.000, 0.489	0.444, 0.000, 0.178
2	Right front spring	1.226, -0.100, 0.671	1.396, -0.100, 0.311
3	Left front spring	1.226, 0.100, 0.671	1.396, 0.100, 0.311
4	Pull rod (dogbone)	0.372, 0.000, 0.275	0.495, 0.000, 0.152

Table 1.5: Connection Properties

No.	Connection Name	Stiffness [N/m]	Damping [Ns/m]
1	Front tire, lateral	0	40,000
2	Rear tire, lateral	0	40,000
3	Front tire, longitudinal	0	60,000
4	Rear tire, longitudinal	0	60,000
5	Front tire, vertical	130,000	0
6	Rear tire, vertical	141,000	0
1	Rear spring	58,570	11,650
2	Right front spring	12,500	1,067
3	Left front spring	12,500	1,067

CHAPTER 2

ANALYSIS

Replace this text with the body of your report. Add sections or subsections as appropriate.

2.1 Eigenvalue Analysis

The eigenvalue properties are given in Tables 2.1 and 2.2.

Table 2.1: Eigenvalues

No.	Real [rad/s]	Imaginary [rad/s]	Real [Hz]	Imaginary [Hz]
1	$-1.0962033417 \times 10^4$	0.0000000000×10^0	$-1.7446618047 \times 10^3$	0.0000000000×10^0
2	$-8.5684873940 \times 10^3$	0.0000000000×10^0	$-1.3637171236 \times 10^3$	0.0000000000×10^0
3	$-1.5355793775 \times 10^3$	0.0000000000×10^0	$-2.4439504844 \times 10^2$	0.0000000000×10^0
4	$-1.0307598362 \times 10^3$	0.0000000000×10^0	$-1.6405052308 \times 10^2$	0.0000000000×10^0
5	$-1.9151454160 \times 10^2$	0.0000000000×10^0	$-3.0480485970 \times 10^1$	0.0000000000×10^0
6	$-4.9341537145 \times 10^1$	2.7178675088×10^1	$-7.8529495364 \times 10^0$	4.3256204869×10^0
7	$-4.9341537145 \times 10^1$	$-2.7178675088 \times 10^1$	$-7.8529495364 \times 10^0$	$-4.3256204869 \times 10^0$
8	$-1.6117128744 \times 10^1$	4.1587849975×10^1	$-2.5651207080 \times 10^0$	6.6189118960×10^0
9	$-1.6117128744 \times 10^1$	$-4.1587849975 \times 10^1$	$-2.5651207080 \times 10^0$	$-6.6189118960 \times 10^0$
10	$-1.1952951897 \times 10^1$	1.9646894268×10^1	$-1.9023713789 \times 10^0$	3.1269003392×10^0
11	$-1.1952951897 \times 10^1$	$-1.9646894268 \times 10^1$	$-1.9023713789 \times 10^0$	$-3.1269003392 \times 10^0$
12	$-1.9371828845 \times 10^1$	0.0000000000×10^0	$-3.0831223174 \times 10^0$	0.0000000000×10^0
13	$-7.0156106805 \times 10^0$	0.0000000000×10^0	$-1.1165691186 \times 10^0$	0.0000000000×10^0
14	3.7935644393×10^0	$9.9342545811 \times 10^{-1}$	$6.0376453245 \times 10^{-1}$	$1.5810857225 \times 10^{-1}$
15	3.7935644393×10^0	$-9.9342545811 \times 10^{-1}$	$6.0376453245 \times 10^{-1}$	$-1.5810857225 \times 10^{-1}$
16	$-3.4323923367 \times 10^0$	0.0000000000×10^0	$-5.4628220701 \times 10^{-1}$	0.0000000000×10^0
17	$-3.7971003929 \times 10^{-7}$	0.0000000000×10^0	$-6.0432729695 \times 10^{-8}$	0.0000000000×10^0
18	$3.7970968818 \times 10^{-7}$	0.0000000000×10^0	$6.0432673813 \times 10^{-8}$	0.0000000000×10^0
19	$3.8710444747 \times 10^{-12}$	$1.9719713858 \times 10^{-5}$	$6.1609586308 \times 10^{-13}$	$3.1384899369 \times 10^{-6}$
20	$3.8710444747 \times 10^{-12}$	$-1.9719713858 \times 10^{-5}$	$6.1609586308 \times 10^{-13}$	$-3.1384899369 \times 10^{-6}$
21	$2.7652959658 \times 10^{-12}$	0.0000000000×10^0	$4.4011052207 \times 10^{-13}$	0.0000000000×10^0
22	$-1.3716291571 \times 10^{-11}$	0.0000000000×10^0	$-2.1830156044 \times 10^{-12}$	0.0000000000×10^0

Note: oscillatory roots appear as complex conjugates.

Table 2.2: Eigenvalue Analysis

No.	Frequency (ω_n) [Hz]	Damping Ratio (ζ)	Time Constant (τ) [s]	Wavelength (λ) [s]
1	—	—	$9.1223951062 \times 10^{-5}$	—
2	—	—	$1.1670671310 \times 10^{-4}$	—
3	—	—	$6.5121999854 \times 10^{-4}$	—
4	—	—	$9.7015809584 \times 10^{-4}$	—
5	—	—	$5.2215356163 \times 10^{-3}$	—
6	8.9654787389×10^0	$8.7590967143 \times 10^{-1}$	$2.0266900017 \times 10^{-2}$	$2.3118070645 \times 10^{-1}$
7	8.9654787389×10^0	$8.7590967143 \times 10^{-1}$	$2.0266900017 \times 10^{-2}$	$2.3118070645 \times 10^{-1}$
8	7.0985800647×10^0	$3.6135687485 \times 10^{-1}$	$6.2045790904 \times 10^{-2}$	$1.5108223462 \times 10^{-1}$
9	7.0985800647×10^0	$3.6135687485 \times 10^{-1}$	$6.2045790904 \times 10^{-2}$	$1.5108223462 \times 10^{-1}$
10	3.6601260353×10^0	$5.1975570256 \times 10^{-1}$	$8.3661342288 \times 10^{-2}$	$3.1980552353 \times 10^{-1}$
11	3.6601260353×10^0	$5.1975570256 \times 10^{-1}$	$8.3661342288 \times 10^{-2}$	$3.1980552353 \times 10^{-1}$
12	—	—	$5.1621352223 \times 10^{-2}$	—
13	—	—	$1.4253926644 \times 10^{-1}$	—
14	$6.2412333017 \times 10^{-1}$	$-9.6738016873 \times 10^{-1}$	$-2.6360432675 \times 10^{-1}$	6.3247677577×10^0
15	$6.2412333017 \times 10^{-1}$	$-9.6738016873 \times 10^{-1}$	$-2.6360432675 \times 10^{-1}$	6.3247677577×10^0
16	—	—	$2.9134198597 \times 10^{-1}$	—
17	—	—	2.6335885189×10^6	—
18	—	—	$-2.6335909542 \times 10^6$	—
19	$3.1384899369 \times 10^{-6}$	$-1.9630327816 \times 10^{-7}$	$-2.5832821259 \times 10^{11}$	3.1862456790×10^5
20	$3.1384899369 \times 10^{-6}$	$-1.9630327816 \times 10^{-7}$	$-2.5832821259 \times 10^{11}$	3.1862456790×10^5
21	—	—	$-3.6162494444 \times 10^{11}$	—
22	—	—	$7.2906003407 \times 10^{10}$	—

Notes: a) oscillatory roots are listed twice, b) negative time constants denote unstable roots.

There are 11 degrees of freedom.

There are 5 oscillatory modes, 13 damped modes, 4 unstable modes, and 0 rigid body modes.

2.2 Frequency Response Plots

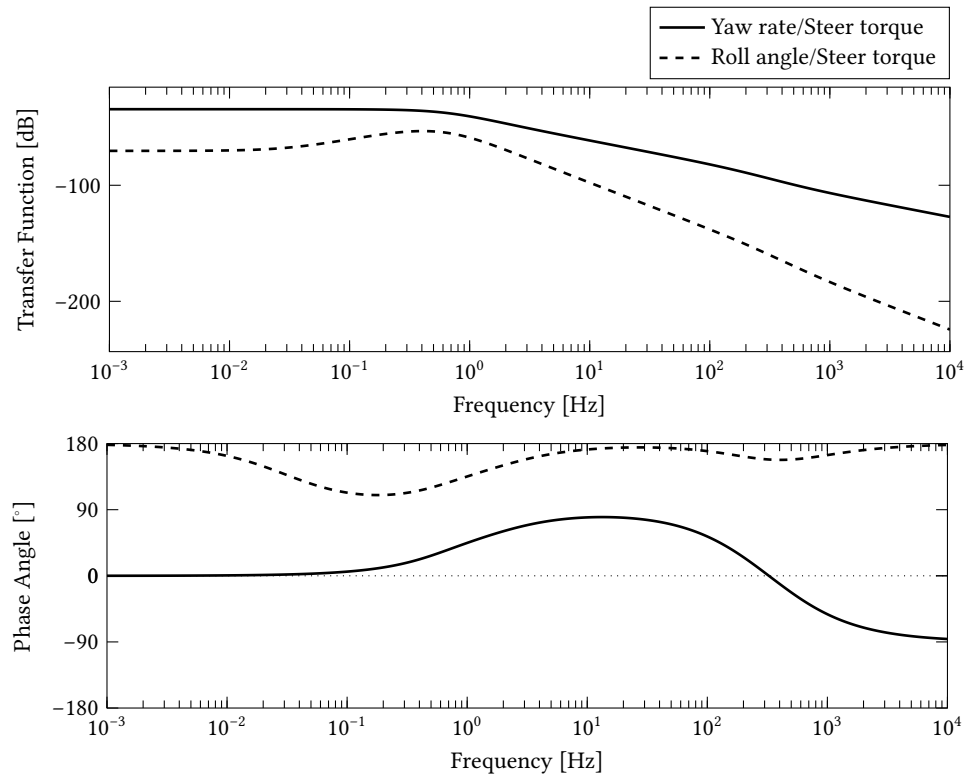


Figure 2.1: Frequency response: Steer torque

2.3 Steady State Gains

The steady state gains are given in Table 2.3.

Table 2.3: Steady State Gains

No.	Output/Input	Gain
1	Yaw rate/Steer torque	$1.2439450000 \times 10^{-2}$
2	Roll angle/Steer torque	$-3.9505000000 \times 10^{-4}$

2.4 Equilibrium Analysis

The results of the equilibrium load analysis are given in Tables 2.4 and 2.5.

Table 2.4: System Static Deflections

No.	Body Name	Type	Deflection [m] or [rad]
1	Frame	translation	5.9952×10^{-3} , 0.0000×10^0 , -5.6380×10^{-2}
–	–	rotation	0.0000×10^0 , -2.1752×10^{-2} , 0.0000×10^0
2	Upper body	translation	-8.5266×10^{-3} , 0.0000×10^0 , -6.2098×10^{-2}
–	–	rotation	0.0000×10^0 , -2.1752×10^{-2} , 0.0000×10^0
3	Upper fork	translation	-4.7826×10^{-4} , 0.0000×10^0 , -4.5806×10^{-2}
–	–	rotation	0.0000×10^0 , -2.1752×10^{-2} , 0.0000×10^0
4	Lower fork	translation	-5.6905×10^{-3} , 0.0000×10^0 , -9.8518×10^{-3}
–	–	rotation	0.0000×10^0 , -2.1752×10^{-2} , 0.0000×10^0
5	Swing arm	translation	3.9213×10^{-3} , 0.0000×10^0 , -2.7082×10^{-2}
–	–	rotation	0.0000×10^0 , 9.0939×10^{-2} , 0.0000×10^0
6	Bell crank	translation	6.9350×10^{-3} , 0.0000×10^0 , -4.3242×10^{-2}
–	–	rotation	0.0000×10^0 , 3.4851×10^{-1} , 0.0000×10^0
7	Front wheel, bike	translation	-4.7770×10^{-3} , 0.0000×10^0 , -8.8729×10^{-3}
–	–	rotation	0.0000×10^0 , 0.0000×10^0 , 0.0000×10^0
8	Rear wheel, bike	translation	2.6209×10^{-3} , 0.0000×10^0 , -9.2581×10^{-3}
–	–	rotation	0.0000×10^0 , 0.0000×10^0 , 0.0000×10^0

Table 2.5: System Preloads

No.	Connector Name	Type	Load [N] or [Nm] (Components; Magnitude)
1	Rear axle	force	0.0000×10^0 , 0.0000×10^0 , 1.1612×10^3 ; 1.1612×10^3
2	Swing arm pivot	force	-2.3178×10^3 , 0.0000×10^0 , 1.2389×10^3 ; 2.6281×10^3
3	Bell crank pivot	force	2.5910×10^3 , 0.0000×10^0 , -3.3410×10^2 ; 2.6125×10^3
4	Seat	force	0.0000×10^0 , 0.0000×10^0 , 3.3040×10^2 ; 3.3040×10^2
–	–	moment	0.0000×10^0 , -1.6850×10^1 , 0.0000×10^0 ; 1.6850×10^1
5	Steering head	force	0.0000×10^0 , 0.0000×10^0 , -8.6761×10^2 ; 8.6761×10^2
–	–	moment	0.0000×10^0 , 2.3293×10^2 , 0.0000×10^0 ; 2.3293×10^2
6	Front axle	force	0.0000×10^0 , 0.0000×10^0 , 1.0367×10^3 ; 1.0367×10^3
7	Fork slider	force	3.7284×10^2 , 0.0000×10^0 , 1.7607×10^2 ; 4.1232×10^2
–	–	moment	0.0000×10^0 , -6.8862×10^1 , 0.0000×10^0 ; 6.8862×10^1
8	Front tire, lateral	force	0.0000×10^0 , 0.0000×10^0 , 0.0000×10^0 ; 0.0000×10^0
9	Rear tire, lateral	force	0.0000×10^0 , 0.0000×10^0 , 0.0000×10^0 ; 0.0000×10^0

10	Front tire, longitudinal	force	0.0000×10^0 ,	0.0000×10^0 ,	0.0000×10^0 ;	0.0000×10^0
11	Rear tire, longitudinal	force	0.0000×10^0 ,	0.0000×10^0 ,	0.0000×10^0 ;	0.0000×10^0
12	Front tire, vertical	force	0.0000×10^0 ,	0.0000×10^0 ,	1.1535×10^3 ;	1.1535×10^3
13	Rear tire, vertical	force	0.0000×10^0 ,	0.0000×10^0 ,	1.3054×10^3 ;	1.3054×10^3
14	Rear spring	force	2.7323×10^2 ,	0.0000×10^0 ,	1.9875×10^3 ;	-2.0062×10^3
15	Right front spring	force	-1.8642×10^2 ,	0.0000×10^0 ,	3.9478×10^2 ;	-4.3658×10^2
16	Left front spring	force	-1.8642×10^2 ,	0.0000×10^0 ,	3.9478×10^2 ;	-4.3658×10^2
17	Pull rod (dogbone)	force	2.3178×10^3 ,	0.0000×10^0 ,	-2.3216×10^3 ;	3.2806×10^3

CHAPTER 3

CONCLUSION

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APPENDIX A

EQUATIONS OF MOTION

The equations of motion are of the form

$$\begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{M} & -\mathbf{G} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{p}} \\ \dot{\mathbf{w}} \\ \dot{\mathbf{u}} \end{Bmatrix} + \begin{bmatrix} \mathbf{V} & -\mathbf{I} & \mathbf{0} \\ \mathbf{K} & \mathbf{L} & -\mathbf{F} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{Bmatrix} \mathbf{p} \\ \mathbf{w} \\ \mathbf{u} \end{Bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{I} \end{bmatrix} \{\mathbf{u}\}$$

The mass matrix of the system is

Row	Column	Value	Row	Column	Value
1	1	1.65130000×10^2	19	19	7.25000000×10^0
2	2	1.65130000×10^2	20	20	7.25000000×10^0
3	3	1.65130000×10^2	21	21	7.25000000×10^0
4	4	1.10850000×10^1	25	25	8.00000000×10^0
6	4	-3.69100000×10^0	26	26	8.00000000×10^0
5	5	2.20130000×10^1	27	27	8.00000000×10^0
4	6	-3.69100000×10^0	28	28	$2.00000000 \times 10^{-2}$
6	6	1.49820000×10^1	29	29	$2.59000000 \times 10^{-1}$
7	7	3.36800000×10^1	30	30	$2.59000000 \times 10^{-1}$
8	8	3.36800000×10^1	37	37	1.19000000×10^1
9	9	3.36800000×10^1	38	38	1.19000000×10^1
10	10	1.42800000×10^0	39	39	1.19000000×10^1
12	10	$4.43000000 \times 10^{-1}$	40	40	$2.70000000 \times 10^{-1}$
11	11	1.34700000×10^0	41	41	$4.84000000 \times 10^{-1}$
10	12	$4.43000000 \times 10^{-1}$	42	42	$2.70000000 \times 10^{-1}$
12	12	$9.16000000 \times 10^{-1}$	43	43	1.47000000×10^1
13	13	9.99000000×10^0	44	44	1.47000000×10^1
14	14	9.99000000×10^0	45	45	1.47000000×10^1
15	15	9.99000000×10^0	46	46	$3.83000000 \times 10^{-1}$
16	16	1.34100000×10^0	47	47	$6.38000000 \times 10^{-1}$
17	17	1.54800000×10^0	48	48	$3.83000000 \times 10^{-1}$
18	18	$4.12500000 \times 10^{-1}$	–	–	–

The damping matrix is

A. EQUATIONS OF MOTION

Row	Column	Value	Row	Column	Value
1	1	2.16096323×10^2	17	21	2.71428644×10^1
3	1	1.57188566×10^3	19	21	-8.23979811×10^2
5	1	3.03616953×10^2	21	21	1.74489842×10^3
31	1	-2.16096323×10^2	23	21	-4.39132770×10^1
33	1	-1.57188566×10^3	16	22	-1.74489842×10^1
35	1	-7.71558003×10^1	18	22	-8.23979811×10^0
1	3	1.57188566×10^3	22	22	1.74489842×10^1
3	3	1.14339037×10^4	24	22	8.23979811×10^0
5	3	2.20851114×10^3	13	23	-2.07368252×10^1
31	3	-1.57188566×10^3	15	23	4.39132770×10^1
33	3	-1.14339037×10^4	17	23	$-6.83095420 \times 10^{-1}$
35	3	-5.61231653×10^2	19	23	2.07368252×10^1
1	5	3.03616953×10^2	21	23	-4.39132770×10^1
3	5	2.20851114×10^3	23	23	1.10515080×10^0
5	5	4.26584094×10^2	16	24	-8.23979811×10^0
31	5	-3.03616953×10^2	18	24	-3.89101577×10^0
33	5	-2.20851114×10^3	22	24	8.23979811×10^0
35	5	-1.08404478×10^2	24	24	3.89101577×10^0
13	13	3.89101577×10^2	1	31	-2.16096323×10^2
15	13	-8.23979811×10^2	3	31	-1.57188566×10^3
17	13	1.28174637×10^1	5	31	-3.03616953×10^2
19	13	-3.89101577×10^2	31	31	2.16096323×10^2
21	13	8.23979811×10^2	33	31	1.57188566×10^3
23	13	-2.07368252×10^1	35	31	7.71558003×10^1
13	15	-8.23979811×10^2	1	33	-1.57188566×10^3
15	15	1.74489842×10^3	3	33	-1.14339037×10^4
17	15	-2.71428644×10^1	5	33	-2.20851114×10^3
19	15	8.23979811×10^2	31	33	1.57188566×10^3
21	15	-1.74489842×10^3	33	33	1.14339037×10^4
23	15	4.39132770×10^1	35	33	5.61231653×10^2
16	16	1.74489842×10^1	1	35	-7.71558003×10^1
18	16	8.23979811×10^0	3	35	-5.61231653×10^2
22	16	-1.74489842×10^1	5	35	-1.08404478×10^2
24	16	-8.23979811×10^0	31	35	7.71558003×10^1
13	17	1.28174637×10^1	33	35	5.61231653×10^2
15	17	-2.71428644×10^1	35	35	2.75479816×10^1
17	17	$4.22222334 \times 10^{-1}$	37	37	6.00000000×10^4
19	17	-1.28174637×10^1	41	37	-1.69200000×10^4
21	17	2.71428644×10^1	38	38	4.00000000×10^4
23	17	$-6.83095420 \times 10^{-1}$	40	38	1.12800000×10^4
16	18	8.23979811×10^0	38	40	1.12800000×10^4
18	18	3.89101577×10^0	40	40	3.18096000×10^3
22	18	-8.23979811×10^0	37	41	-1.69200000×10^4
24	18	-3.89101577×10^0	41	41	4.77144000×10^3
13	19	-3.89101577×10^2	43	43	6.00000000×10^4
15	19	8.23979811×10^2	47	43	-1.78200000×10^4
17	19	-1.28174637×10^1	44	44	4.00000000×10^4
19	19	3.89101577×10^2	46	44	1.18800000×10^4
21	19	-8.23979811×10^2	44	46	1.18800000×10^4
23	19	2.07368252×10^1	46	46	3.52836000×10^3
13	21	8.23979811×10^2	43	47	-1.78200000×10^4
15	21	-1.74489842×10^3	47	47	5.29254000×10^3

The stiffness matrix is

A. EQUATIONS OF MOTION

Row	Column	Value	Row	Column	Value
1	1	-5.19379154×10^3	14	24	6.88664390×10^1
3	1	8.76598165×10^3	16	24	-9.81667937×10^1
5	1	1.95582148×10^3	18	24	-2.33532988×10^1
31	1	5.19379154×10^3	20	24	-6.88664390×10^1
33	1	-8.76598165×10^3	22	24	1.51090408×10^2
35	1	-3.95296386×10^2	24	24	3.71954531×10^1
2	2	-6.39890235×10^3	40	24	4.66532616×10^1
4	2	-2.62631771×10^2	5	25	-1.23888803×10^3
6	2	1.49478359×10^3	25	25	9.48364498×10^3
32	2	6.39890235×10^3	27	25	9.46817411×10^3
34	2	-3.49806662×10^1	29	25	-2.01444532×10^3
36	2	-3.09280280×10^2	31	25	-9.48364498×10^3
1	3	8.76598165×10^3	33	25	-9.46817411×10^3
3	3	5.73648892×10^4	35	25	2.13351586×10^2
5	3	1.08214863×10^4	47	25	-1.16119135×10^3
31	3	-8.76598165×10^3	4	26	1.23888803×10^3
33	3	-5.73648892×10^4	6	26	2.31781211×10^3
35	3	-2.82055701×10^3	26	26	1.89363735×10^4
2	4	1.04941999×10^2	28	26	6.91177631×10^2
4	4	-5.56810615×10^1	30	26	3.33658900×10^3
6	4	1.53029840×10^2	32	26	-1.89363735×10^4
32	4	-1.04941999×10^2	34	26	-3.88826868×10^2
34	4	$5.73682925 \times 10^{-1}$	36	26	-3.72415345×10^1
36	4	5.07219660×10^0	46	26	1.16119135×10^3
1	5	1.58824771×10^3	5	27	-2.31781211×10^3
3	5	1.10947194×10^4	25	27	9.46817411×10^3
5	5	2.12446398×10^3	27	27	9.45272847×10^3
31	5	-1.58824771×10^3	29	27	-2.01115911×10^3
33	5	-1.10947194×10^4	31	27	-9.46817411×10^3
35	5	-5.44927194×10^2	33	27	-9.45272847×10^3
2	6	1.22155046×10^3	35	27	2.13003541×10^2
4	6	3.37691642×10^2	4	28	-6.13249576×10^1
6	6	-1.98799190×10^2	6	28	-1.14731699×10^2
32	6	-1.22155046×10^3	26	28	6.91177631×10^2
34	6	6.67780917×10^0	28	28	1.54686282×10^2
36	6	5.90416055×10^1	30	28	3.21117340×10^2
5	7	-3.30400800×10^2	32	28	-6.91177631×10^2
4	8	3.30400800×10^2	34	28	-1.41921807×10^1
4	10	9.78647170×10^1	36	28	-1.35931601×10^0
10	10	-9.78647170×10^1	46	28	1.66050364×10^1
12	10	1.68504408×10^1	5	29	7.56862718×10^2
5	11	9.78647170×10^1	25	29	-2.01444532×10^3
11	11	-9.78647170×10^1	27	29	-2.01115911×10^3
4	12	-1.68504408×10^1	29	29	1.47562607×10^2
5	13	8.67614747×10^2	31	29	2.01444532×10^3
13	13	2.76505646×10^3	33	29	2.01115911×10^3
15	13	-1.04998344×10^4	35	29	-4.53185568×10^1
17	13	3.80896068×10^2	47	29	1.66050364×10^1
19	13	-2.76505646×10^3	4	30	4.37327475×10^2
21	13	1.04998344×10^4	6	30	8.18187675×10^2
23	13	-4.69261174×10^2	26	30	3.33658900×10^3
4	14	-8.67614747×10^2	28	30	3.21117340×10^2
14	14	-2.19319869×10^3	30	30	1.78117801×10^2
16	14	-2.17565310×10^2	32	30	-3.33658900×10^3
18	14	-1.36855598×10^2	34	30	-6.85112942×10^1
20	14	2.19319869×10^3	36	30	-6.56195837×10^0

A. EQUATIONS OF MOTION

22	14	2.05015340×10^2	46	30	-2.27593505×10^2
24	14	-3.03977339×10^2	1	31	5.19379154×10^3
13	15	-1.04998344×10^4	3	31	-8.76598165×10^3
15	15	2.00417448×10^4	5	31	-1.25414740×10^3
17	15	-2.09021301×10^2	25	31	-9.48364498×10^3
19	15	1.04998344×10^4	27	31	-9.46817411×10^3
21	15	-2.00417448×10^4	29	31	2.01444532×10^3
23	15	8.63556751×10^2	31	31	4.28985344×10^3
4	16	-1.82199097×10^1	33	31	1.82341558×10^4
14	16	-2.17565310×10^2	35	31	1.81944800×10^2
16	16	1.44719965×10^2	2	32	6.39890235×10^3
18	16	5.71621391×10^1	4	32	-4.39042314×10^2
20	16	2.17565310×10^2	6	32	-3.81259570×10^3
22	16	-1.36979186×10^2	26	32	-1.89363735×10^4
24	16	-2.26425053×10^2	28	32	-6.91177631×10^2
5	17	-1.82199097×10^1	30	32	-3.33658900×10^3
13	17	3.80896068×10^2	32	32	1.25374711×10^4
15	17	-2.09021301×10^2	34	32	4.23807534×10^2
17	17	-1.02311222×10^2	36	32	3.46521815×10^2
19	17	-3.80896068×10^2	1	33	-8.76598165×10^3
21	17	2.09021301×10^2	3	33	-5.73648892×10^4
23	17	-7.33523285×10^0	5	33	-8.50367420×10^3
4	18	-7.80853272×10^0	25	33	-9.46817411×10^3
14	18	-1.36855598×10^2	27	33	-9.45272847×10^3
16	18	-1.17003054×10^1	29	33	2.01115911×10^3
18	18	-2.43434814×10^1	31	33	1.82341558×10^4
20	18	1.36855598×10^2	33	33	6.68176177×10^4
22	18	-7.18522597×10^1	35	33	2.60755347×10^3
24	18	-8.97194913×10^1	2	34	-3.49806662×10^1
13	19	-2.76505646×10^3	4	34	5.60746102×10^0
15	19	1.04998344×10^4	6	34	4.57162241×10^1
17	19	-3.80896068×10^2	26	34	-3.88826868×10^2
19	19	2.76505646×10^3	28	34	-1.41921807×10^1
21	19	-1.04998344×10^4	30	34	-6.85112942×10^1
23	19	4.69261174×10^2	32	34	4.23807534×10^2
41	19	-1.03673915×10^3	34	34	-5.57762630×10^1
14	20	2.19319869×10^3	36	34	-8.60631886×10^1
16	20	2.17565310×10^2	1	35	-3.95296386×10^2
18	20	1.36855598×10^2	3	35	-2.82055701×10^3
20	20	-2.19319869×10^3	5	35	-6.60031547×10^2
22	20	-2.05015340×10^2	25	35	2.13351586×10^2
24	20	3.03977339×10^2	27	35	2.13003541×10^2
40	20	1.03673915×10^3	29	35	-4.53185568×10^1
13	21	1.04998344×10^4	31	35	1.81944800×10^2
15	21	-2.00417448×10^4	33	35	2.60755347×10^3
17	21	2.09021301×10^2	35	35	2.08504690×10^2
19	21	-1.04998344×10^4	2	36	-3.09280280×10^2
21	21	2.00417448×10^4	4	36	-1.04189214×10^1
23	21	-8.63556751×10^2	6	36	-6.10965255×10^1
14	22	2.89502227×10^1	26	36	-3.72415345×10^1
16	22	-1.97545586×10^2	28	36	-1.35931601×10^0
18	22	-1.03191851×10^2	30	36	-6.56195837×10^0
20	22	-2.89502227×10^1	32	36	3.46521815×10^2
22	22	1.66914342×10^2	34	36	-8.60631886×10^1
24	22	4.01484009×10^1	36	36	1.13910730×10^2
40	22	4.35430442×10^1	41	37	1.03673915×10^3
13	23	-2.93196056×10^2	40	38	-1.03673915×10^3
15	23	4.90712973×10^2	39	39	1.30000000×10^5
17	23	-1.53544075×10^0	40	40	-3.25280837×10^2

A. EQUATIONS OF MOTION

19	23	2.93196056×10^2	47	43	1.16119135×10^3
21	23	-4.90712973×10^2	46	44	-1.16119135×10^3
23	23	-9.87546993×10^0	45	45	1.41000000×10^5
41	23	4.35430442×10^1	46	46	-3.87703311×10^2

The velocity matrix is

Row	Column	Value	Row	Column	Value
3	5	1.00000000×10^0	27	29	1.00000000×10^0
2	6	-1.00000000×10^0	26	30	-1.00000000×10^0
9	11	1.00000000×10^0	33	35	1.00000000×10^0
8	12	-1.00000000×10^0	32	36	-1.00000000×10^0
15	17	1.00000000×10^0	39	41	1.00000000×10^0
14	18	-1.00000000×10^0	38	42	-1.00000000×10^0
21	23	1.00000000×10^0	45	47	1.00000000×10^0
20	24	-1.00000000×10^0	44	48	-1.00000000×10^0

The input force matrix is

Row	Column	Value	Row	Column	Value
4	1	$-4.06755809 \times 10^{-1}$	16	1	$4.06755809 \times 10^{-1}$
6	1	$9.13536924 \times 10^{-1}$	18	1	$-9.13536924 \times 10^{-1}$

The input force rate matrix is

Row	Column	Value	Row	Column	Value
1	1	0.00000000×10^0	-	-	-

The system is subject to constraints

$$\begin{bmatrix} \mathbf{J}_h & \mathbf{0} & \mathbf{0} \\ -\mathbf{J}_h \mathbf{V} & \mathbf{J}_h & \mathbf{0} \\ \mathbf{0} & \mathbf{J}_{nh} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{p}} & \mathbf{p} \\ \dot{\mathbf{w}} & \mathbf{w} \\ \dot{\mathbf{u}} & \mathbf{u} \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}$$

Row	Column	Value	Row	Column	Value
7	1	-1.00000000×10^0	50	36	1.00000000×10^0
12	1	-1.00000000×10^0	28	37	-1.00000000×10^0
17	1	-1.00000000×10^0	29	38	-1.00000000×10^0
23	1	-1.00000000×10^0	30	39	-1.00000000×10^0
8	2	-1.00000000×10^0	31	40	1.00000000×10^0
13	2	-1.00000000×10^0	67	41	1.00000000×10^0
18	2	-1.00000000×10^0	32	42	-1.00000000×10^0

A. EQUATIONS OF MOTION

24	2	-1.00000000×10^0	66	42	-1.00000000×10^0
9	3	-1.00000000×10^0	2	43	-1.00000000×10^0
14	3	-1.00000000×10^0	3	44	-1.00000000×10^0
19	3	-1.00000000×10^0	4	45	-1.00000000×10^0
25	3	-1.00000000×10^0	5	46	1.00000000×10^0
8	4	$-1.11600000 \times 10^{-1}$	41	47	1.00000000×10^0
10	4	1.00000000×10^0	6	48	-1.00000000×10^0
13	4	$-2.84600000 \times 10^{-1}$	40	48	-1.00000000×10^0
15	4	1.00000000×10^0	44	49	-1.00000000×10^0
18	4	$3.71400000 \times 10^{-1}$	49	49	-1.00000000×10^0
20	4	-1.00000000×10^0	54	49	-1.00000000×10^0
24	4	$2.76600000 \times 10^{-1}$	60	49	-1.00000000×10^0
27	4	$-9.13536924 \times 10^{-1}$	45	50	-1.00000000×10^0
7	5	$1.11600000 \times 10^{-1}$	50	50	-1.00000000×10^0
9	5	$-1.28900000 \times 10^{-1}$	55	50	-1.00000000×10^0
12	5	$2.84600000 \times 10^{-1}$	61	50	-1.00000000×10^0
14	5	$-1.38900000 \times 10^{-1}$	46	51	-1.00000000×10^0
17	5	$-3.71400000 \times 10^{-1}$	51	51	-1.00000000×10^0
19	5	$-3.13900000 \times 10^{-1}$	56	51	-1.00000000×10^0
21	5	-1.00000000×10^0	62	51	-1.00000000×10^0
23	5	$-2.76600000 \times 10^{-1}$	45	52	$-1.11600000 \times 10^{-1}$
25	5	$4.95100000 \times 10^{-1}$	47	52	1.00000000×10^0
26	5	-1.00000000×10^0	50	52	$-2.84600000 \times 10^{-1}$
46	5	1.00000000×10^0	52	52	1.00000000×10^0
51	5	1.00000000×10^0	55	52	$3.71400000 \times 10^{-1}$
56	5	1.00000000×10^0	57	52	-1.00000000×10^0
62	5	1.00000000×10^0	61	52	$2.76600000 \times 10^{-1}$
8	6	$1.28900000 \times 10^{-1}$	64	52	$-9.13536924 \times 10^{-1}$
11	6	-1.00000000×10^0	44	53	$1.11600000 \times 10^{-1}$
13	6	$1.38900000 \times 10^{-1}$	46	53	$-1.28900000 \times 10^{-1}$
16	6	-1.00000000×10^0	49	53	$2.84600000 \times 10^{-1}$
18	6	$3.13900000 \times 10^{-1}$	51	53	$-1.38900000 \times 10^{-1}$
22	6	-1.00000000×10^0	54	53	$-3.71400000 \times 10^{-1}$
24	6	$-4.95100000 \times 10^{-1}$	56	53	$-3.13900000 \times 10^{-1}$
27	6	$-4.06755809 \times 10^{-1}$	58	53	-1.00000000×10^0
45	6	-1.00000000×10^0	60	53	$-2.76600000 \times 10^{-1}$
50	6	-1.00000000×10^0	62	53	$4.95100000 \times 10^{-1}$
55	6	-1.00000000×10^0	63	53	-1.00000000×10^0
61	6	-1.00000000×10^0	45	54	$1.28900000 \times 10^{-1}$
17	7	1.00000000×10^0	48	54	-1.00000000×10^0
18	8	1.00000000×10^0	50	54	$1.38900000 \times 10^{-1}$
19	9	1.00000000×10^0	53	54	-1.00000000×10^0
18	10	$2.96200000 \times 10^{-1}$	55	54	$3.13900000 \times 10^{-1}$
20	10	1.00000000×10^0	59	54	-1.00000000×10^0
17	11	$-2.96200000 \times 10^{-1}$	61	54	$-4.95100000 \times 10^{-1}$
19	11	$5.10000000 \times 10^{-2}$	64	54	$-4.06755809 \times 10^{-1}$
21	11	1.00000000×10^0	54	55	1.00000000×10^0
56	11	-1.00000000×10^0	55	56	1.00000000×10^0
18	12	$-5.10000000 \times 10^{-2}$	56	57	1.00000000×10^0
22	12	1.00000000×10^0	55	58	$2.96200000 \times 10^{-1}$
55	12	1.00000000×10^0	57	58	1.00000000×10^0
23	13	1.00000000×10^0	54	59	$-2.96200000 \times 10^{-1}$
33	13	$9.04248647 \times 10^{-1}$	56	59	$5.10000000 \times 10^{-2}$
24	14	1.00000000×10^0	58	59	1.00000000×10^0
34	14	1.00000000×10^0	55	60	$-5.10000000 \times 10^{-2}$
25	15	1.00000000×10^0	59	60	1.00000000×10^0
33	15	$4.27006305 \times 10^{-1}$	60	61	1.00000000×10^0
24	16	$2.10000000 \times 10^{-2}$	70	61	$9.04248647 \times 10^{-1}$
27	16	$9.13536924 \times 10^{-1}$	61	62	1.00000000×10^0

A. EQUATIONS OF MOTION

34	16	$3.44000000 \times 10^{-1}$	71	62	1.00000000×10^0
35	16	1.00000000×10^0	62	63	1.00000000×10^0
23	17	$-2.10000000 \times 10^{-2}$	70	63	$4.27006305 \times 10^{-1}$
25	17	$-9.00000000 \times 10^{-3}$	61	64	$2.10000000 \times 10^{-2}$
26	17	1.00000000×10^0	64	64	$9.13536924 \times 10^{-1}$
33	17	$-3.87068657 \times 10^{-1}$	71	64	$3.44000000 \times 10^{-1}$
36	17	1.00000000×10^0	72	64	1.00000000×10^0
62	17	-1.00000000×10^0	60	65	$-2.10000000 \times 10^{-2}$
70	17	$-4.27006305 \times 10^{-1}$	62	65	$-9.00000000 \times 10^{-3}$
24	18	$9.00000000 \times 10^{-3}$	63	65	1.00000000×10^0
27	18	$4.06755809 \times 10^{-1}$	70	65	$-3.87068657 \times 10^{-1}$
34	18	$1.78000000 \times 10^{-1}$	73	65	1.00000000×10^0
37	18	1.00000000×10^0	61	66	$9.00000000 \times 10^{-3}$
61	18	1.00000000×10^0	64	66	$4.06755809 \times 10^{-1}$
71	18	1.00000000×10^0	71	66	$1.78000000 \times 10^{-1}$
28	19	1.00000000×10^0	74	66	1.00000000×10^0
33	19	$-9.04248647 \times 10^{-1}$	65	67	1.00000000×10^0
29	20	1.00000000×10^0	70	67	$-9.04248647 \times 10^{-1}$
34	20	-1.00000000×10^0	66	68	1.00000000×10^0
30	21	1.00000000×10^0	71	68	-1.00000000×10^0
33	21	$-4.27006305 \times 10^{-1}$	67	69	1.00000000×10^0
29	22	$4.20000000 \times 10^{-2}$	70	69	$-4.27006305 \times 10^{-1}$
31	22	-1.00000000×10^0	66	70	$4.20000000 \times 10^{-2}$
34	22	$1.02000000 \times 10^{-1}$	68	70	-1.00000000×10^0
35	22	-1.00000000×10^0	71	70	$1.02000000 \times 10^{-1}$
28	23	$-4.20000000 \times 10^{-2}$	72	70	-1.00000000×10^0
30	23	$-4.50000000 \times 10^{-2}$	65	71	$-4.20000000 \times 10^{-2}$
33	23	$-1.02054507 \times 10^{-1}$	67	71	$-4.50000000 \times 10^{-2}$
36	23	-1.00000000×10^0	70	71	$-1.02054507 \times 10^{-1}$
67	23	-1.00000000×10^0	73	71	-1.00000000×10^0
70	23	$4.27006305 \times 10^{-1}$	66	72	$4.50000000 \times 10^{-2}$
29	24	$4.50000000 \times 10^{-2}$	69	72	1.00000000×10^0
32	24	1.00000000×10^0	71	72	$2.30000000 \times 10^{-2}$
34	24	$2.30000000 \times 10^{-2}$	74	72	-1.00000000×10^0
37	24	-1.00000000×10^0	38	73	$7.06529316 \times 10^{-1}$
66	24	1.00000000×10^0	39	73	1.00000000×10^0
71	24	-1.00000000×10^0	44	73	1.00000000×10^0
1	25	$7.06529316 \times 10^{-1}$	40	74	1.00000000×10^0
2	25	1.00000000×10^0	45	74	1.00000000×10^0
7	25	1.00000000×10^0	38	75	$-7.07683775 \times 10^{-1}$
3	26	1.00000000×10^0	41	75	1.00000000×10^0
8	26	1.00000000×10^0	46	75	1.00000000×10^0
1	27	$-7.07683775 \times 10^{-1}$	40	76	$1.43000000 \times 10^{-2}$
4	27	1.00000000×10^0	42	76	-1.00000000×10^0
9	27	1.00000000×10^0	45	76	$-4.95000000 \times 10^{-2}$
3	28	$1.43000000 \times 10^{-2}$	47	76	-1.00000000×10^0
5	28	-1.00000000×10^0	38	77	$9.89055612 \times 10^{-2}$
8	28	$-4.95000000 \times 10^{-2}$	39	77	$-1.43000000 \times 10^{-2}$
10	28	-1.00000000×10^0	41	77	$1.96000000 \times 10^{-1}$
1	29	$9.89055612 \times 10^{-2}$	44	77	$4.95000000 \times 10^{-2}$
2	29	$-1.43000000 \times 10^{-2}$	46	77	$-3.53000000 \times 10^{-1}$
4	29	$1.96000000 \times 10^{-1}$	40	78	$-1.96000000 \times 10^{-1}$
7	29	$4.95000000 \times 10^{-2}$	43	78	1.00000000×10^0
9	29	$-3.53000000 \times 10^{-1}$	45	78	$3.53000000 \times 10^{-1}$
38	29	$7.07683775 \times 10^{-1}$	48	78	1.00000000×10^0
41	29	-1.00000000×10^0	38	79	$-7.06529316 \times 10^{-1}$
46	29	-1.00000000×10^0	49	79	1.00000000×10^0
3	30	$-1.96000000 \times 10^{-1}$	50	80	1.00000000×10^0
6	30	1.00000000×10^0	38	81	$7.07683775 \times 10^{-1}$

A. EQUATIONS OF MOTION

8	30	$3.53000000 \times 10^{-1}$	51	81	1.00000000×10^0
11	30	1.00000000×10^0	50	82	$-1.50666667 \times 10^{-2}$
40	30	1.00000000×10^0	52	82	-1.00000000×10^0
45	30	1.00000000×10^0	38	83	$1.31156239 \times 10^{-2}$
1	31	$-7.06529316 \times 10^{-1}$	49	83	$1.50666667 \times 10^{-2}$
12	31	1.00000000×10^0	51	83	$-4.63666667 \times 10^{-2}$
13	32	1.00000000×10^0	50	84	$4.63666667 \times 10^{-2}$
1	33	$7.07683775 \times 10^{-1}$	53	84	1.00000000×10^0
14	33	1.00000000×10^0	65	85	-1.00000000×10^0
13	34	$-1.50666667 \times 10^{-2}$	66	86	-1.00000000×10^0
15	34	-1.00000000×10^0	67	87	-1.00000000×10^0
1	35	$1.31156239 \times 10^{-2}$	68	88	1.00000000×10^0
12	35	$1.50666667 \times 10^{-2}$	69	90	-1.00000000×10^0
14	35	$-4.63666667 \times 10^{-2}$	39	91	-1.00000000×10^0
38	35	$-7.07683775 \times 10^{-1}$	40	92	-1.00000000×10^0
51	35	-1.00000000×10^0	41	93	-1.00000000×10^0
13	36	$4.63666667 \times 10^{-2}$	42	94	1.00000000×10^0
16	36	1.00000000×10^0	43	96	-1.00000000×10^0

The full state space equations:

$$\begin{bmatrix} \mathbf{E} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{x}} \\ \mathbf{y} \end{Bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} \begin{Bmatrix} \mathbf{x} \\ \mathbf{u} \end{Bmatrix}$$

$$\begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} = \begin{bmatrix} -3.72984500 \times 10^3 & 8.93828589 \times 10^2 & 1.19919442 \times 10^3 & -1.13841060 \times 10^2 & -1.81318497 \times 10^3 & 5.78121360 \times 10^1 & -3.13669694 \times 10^2 \\ 6.35959731 \times 10^3 & 6.22306240 \times 10^2 & -5.91065451 \times 10^3 & -2.37342843 \times 10^3 & 7.01938918 \times 10^2 & -5.73029773 \times 10^3 & -4.79852399 \times 10^3 \\ -1.87968210 \times 10^3 & 6.98420610 \times 10^2 & -1.12760308 \times 10^3 & -1.47506916 \times 10^3 & -5.46567166 \times 10^2 & 5.41560007 \times 10^2 & -3.00826333 \times 10^1 \\ -6.49772380 \times 10^3 & 6.25412304 \times 10^2 & 9.90571697 \times 10^2 & -4.32751679 \times 10^2 & -7.29879235 \times 10^2 & 5.42226831 \times 10^3 & 5.68897528 \times 10^3 \\ -4.96682361 \times 10^2 & 5.87025003 \times 10^2 & 1.92219521 \times 10^3 & 2.55429476 \times 10^3 & -6.68887421 \times 10^2 & 8.20211611 \times 10^2 & 9.08718174 \times 10^2 \\ -1.67521990 \times 10^3 & -3.16558472 \times 10^1 & 1.91393056 \times 10^3 & 8.30874304 \times 10^2 & -1.15203043 \times 10^3 & -3.03437548 \times 10^3 & -6.59424493 \times 10^2 \\ -6.92649640 \times 10^3 & 1.24809801 \times 10^3 & 1.60284032 \times 10^3 & -1.54167597 \times 10^3 & -3.22626865 \times 10^3 & 1.13356787 \times 10^2 & -1.45387832 \times 10^3 \\ -1.87374869 \times 10^3 & 1.28882537 \times 10^3 & 1.91065651 \times 10^3 & 2.65046133 \times 10^3 & -1.65425463 \times 10^3 & 8.46611468 \times 10^2 & 4.73965714 \times 10^2 \\ -3.13479228 \times 10^3 & 1.10763216 \times 10^3 & 6.49137612 \times 10^2 & -4.39529079 \times 10^2 & -1.17337669 \times 10^3 & 6.13155983 \times 10^2 & -1.05457773 \times 10^3 \\ 4.20540154 \times 10^2 & 1.95900194 \times 10^3 & -1.98332574 \times 10^3 & -2.35887113 \times 10^3 & -2.51996606 \times 10^3 & -3.69157194 \times 10^3 & -6.73311800 \times 10^3 \\ 1.78524691 \times 10^2 & 2.29862192 \times 10^3 & 2.88774779 \times 10^2 & 1.59685331 \times 10^3 & -3.00660264 \times 10^3 & -2.85829661 \times 10^3 & -5.12004285 \times 10^3 \\ -1.34879561 \times 10^3 & 1.29811181 \times 10^3 & 1.83939576 \times 10^3 & 1.72863696 \times 10^3 & -7.30488470 \times 10^2 & -2.19962530 \times 10^2 & 3.37649590 \times 10^3 \\ -6.92215057 \times 10^3 & -1.61326107 \times 10^3 & 8.61088241 \times 10^3 & 6.38964638 \times 10^3 & 1.26992016 \times 10^3 & 8.60443617 \times 10^3 & 1.31105735 \times 10^4 \\ -1.59148342 \times 10^3 & 6.45379776 \times 10^2 & 7.13392735 \times 10^2 & -8.79568925 \times 10^2 & -1.44467328 \times 10^3 & -4.01234238 \times 10^3 & -1.31308652 \times 10^3 \\ 3.96370263 \times 10^2 & -6.16243338 \times 10^2 & -2.63060876 \times 10^3 & -4.00225657 \times 10^3 & 1.33392434 \times 10^3 & 1.71416128 \times 10^2 & 1.06370331 \times 10^3 \\ 8.96456916 \times 10^2 & -9.79648611 \times 10^2 & -1.29370991 \times 10^2 & -7.35592419 \times 10^2 & 1.15121728 \times 10^3 & 1.81558856 \times 10^3 & -6.98548904 \times 10^0 \\ -2.09143381 \times 10^3 & 1.31064029 \times 10^3 & 2.27481485 \times 10^3 & 2.80561103 \times 10^3 & -1.84895985 \times 10^3 & 1.23068770 \times 10^3 & 1.10962512 \times 10^2 \\ -5.25861243 \times 10^3 & -2.92572507 \times 10^2 & 2.77131207 \times 10^2 & -3.21597042 \times 10^3 & -1.35508312 \times 10^3 & -4.77880414 \times 10^3 & 8.46488651 \times 10^2 \\ -9.20364355 \times 10^3 & 2.32570513 \times 10^3 & 1.64490156 \times 10^3 & -1.60342202 \times 10^3 & -4.73344655 \times 10^3 & 7.04145784 \times 10^2 & -2.43778253 \times 10^3 \\ -6.89681713 \times 10^3 & -1.43899899 \times 10^2 & 9.86090268 \times 10^2 & -1.44769166 \times 10^3 & -8.68927291 \times 10^2 & 4.25051563 \times 10^3 & 3.53238195 \times 10^3 \\ -8.61555997 \times 10^2 & -1.91351486 \times 10^3 & -1.33347262 \times 10^2 & -1.72278607 \times 10^3 & 1.19345529 \times 10^3 & -1.90593147 \times 10^2 & 2.58329575 \times 10^1 \\ -1.11056787 \times 10^3 & 1.57231945 \times 10^2 & 2.49133202 \times 10^2 & -3.51943273 \times 10^3 & -7.04468621 \times 10^2 & -1.13925870 \times 10^3 & -2.74287093 \times 10^3 \\ 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 \\ -2.37841681 \times 10^{-4} & -8.00040341 \times 10^{-2} & 3.54659577 \times 10^{-2} & 1.48460827 \times 10^{-1} & 3.79226092 \times 10^{-2} & 6.47765002 \times 10^{-3} & 4.89748278 \times 10^{-2} \\ 2.50873954 \times 10^{-2} & -1.04212663 \times 10^{-1} & 2.29350552 \times 10^{-2} & -8.13384193 \times 10^{-2} & 3.00364477 \times 10^{-2} & 1.11878680 \times 10^{-1} & 8.70452117 \times 10^{-3} \end{bmatrix}$$

A. EQUATIONS OF MOTION

$$\mathbf{E} = \begin{bmatrix} 3.01414747 \times 10^0 & 1.03861033 \times 10^0 & 2.74323374 \times 10^0 & -1.48091997 \times 10^0 & 9.98619432 \times 10^{-1} & 7.91768155 \times 10^{-1} & 3.01694572 \times 10^{-1} & -2.05833 \\ -4.97897103 \times 10^0 & -1.72444107 \times 10^0 & -2.16080638 \times 10^0 & 2.32279824 \times 10^{-2} & -1.60347586 \times 10^0 & 2.61018623 \times 10^0 & -1.06854444 \times 10^{-1} & -2.72387 \\ -8.97487715 \times 10^{-1} & 8.47109092 \times 10^{-2} & 1.32047069 \times 10^{-2} & 2.06535279 \times 10^{-2} & -7.16900671 \times 10^{-1} & 2.08352468 \times 10^{-1} & 4.93196951 \times 10^{-2} & -4.60010 \\ -5.25287813 \times 10^{-2} & 6.05618462 \times 10^{-1} & 3.64790553 \times 10^0 & -1.31434305 \times 10^{-1} & 2.09527792 \times 10^{-1} & 2.21007400 \times 10^0 & 4.21761516 \times 10^{-1} & -1.55793 \\ -1.05622652 \times 10^0 & -1.22184843 \times 10^0 & 6.06042236 \times 10^{-1} & -3.01193181 \times 10^0 & 1.59932609 \times 10^0 & -2.73991061 \times 10^{-1} & 1.93336284 \times 10^{-1} & -7.08541 \\ 3.43794479 \times 10^0 & -2.95600069 \times 10^{-1} & 1.49204313 \times 10^0 & 8.43482790 \times 10^{-1} & 2.65254235 \times 10^0 & 6.94375878 \times 10^0 & -7.21965369 \times 10^{-2} & -1.48409 \\ 7.53844897 \times 10^0 & 4.69966032 \times 10^0 & 4.19554196 \times 10^0 & -1.78380464 \times 10^0 & -1.26282325 \times 10^{-1} & -1.76360941 \times 10^0 & 7.74261616 \times 10^{-2} & 8.64460 \\ -1.19880813 \times 10^{-1} & -6.79202936 \times 10^{-1} & 1.49464627 \times 10^0 & -5.89057463 \times 10^0 & -2.22676177 \times 10^{-1} & -4.75819424 \times 10^0 & -3.44880709 \times 10^{-1} & 7.41822 \\ -1.63499604 \times 10^0 & -1.07426083 \times 10^0 & -1.38604648 \times 10^0 & 1.04560562 \times 10^0 & 3.78342580 \times 10^{-2} & 1.58092608 \times 10^{-1} & 1.43962233 \times 10^{-1} & -3.99184 \\ 3.16730458 \times 10^0 & 1.74272185 \times 10^0 & 1.05906935 \times 10^0 & -2.31556963 \times 10^0 & -3.23421959 \times 10^{-2} & -2.05534118 \times 10^0 & 1.69053537 \times 10^{-1} & 6.04122 \\ 1.30597122 \times 10^0 & -1.47718609 \times 10^0 & 2.15632003 \times 10^0 & -6.22052031 \times 10^0 & 2.09273868 \times 10^0 & -2.86627463 \times 10^{-1} & 7.76110039 \times 10^{-1} & -1.55526 \\ -3.57969927 \times 10^0 & -5.57838187 \times 10^0 & 1.01362569 \times 10^0 & 2.86964791 \times 10^{-2} & 3.29243316 \times 10^0 & 1.08218552 \times 10^1 & 7.04039701 \times 10^{-1} & -3.25596 \\ -3.37475941 \times 10^{-1} & -2.73043255 \times 10^0 & 1.29113310 \times 10^0 & -5.71916594 \times 10^{-1} & 2.53212888 \times 10^0 & 7.12903023 \times 10^{-1} & -7.63206026 \times 10^{-1} & -5.62211 \\ 2.74389765 \times 10^0 & 8.03154386 \times 10^{-1} & 4.81962869 \times 10^{-1} & 2.33926230 \times 10^{-1} & 7.73583284 \times 10^{-1} & 1.67868819 \times 10^0 & -7.25071061 \times 10^{-2} & 2.00195 \\ 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000 \\ 1.06455425 \times 10^0 & 2.30662400 \times 10^0 & -1.99253277 \times 10^0 & 9.08598294 \times 10^{-1} & -2.86166203 \times 10^0 & -8.53190439 \times 10^0 & -2.82497318 \times 10^{-1} & 3.36787 \\ 5.85476619 \times 10^{-1} & -7.98151909 \times 10^{-1} & 1.91712390 \times 10^0 & -5.47581991 \times 10^0 & 1.21161541 \times 10^0 & -2.04833384 \times 10^0 & 4.87766314 \times 10^{-1} & -7.69673 \\ 6.51208752 \times 10^0 & 2.26279856 \times 10^0 & 2.72473238 \times 10^0 & 6.32945668 \times 10^0 & 2.66033846 \times 10^0 & 1.35752211 \times 10^1 & -4.21067875 \times 10^{-1} & -1.14151 \\ 9.19831332 \times 10^0 & 5.75028061 \times 10^0 & 5.75897337 \times 10^0 & -4.26956170 \times 10^0 & 1.41360686 \times 10^{-1} & -3.57051843 \times 10^0 & 4.63641637 \times 10^{-1} & 2.31397 \\ 4.60750433 \times 10^0 & 5.43307602 \times 10^0 & 2.91134929 \times 10^0 & -7.22423711 \times 10^{-1} & -2.90611723 \times 10^0 & -8.07882634 \times 10^0 & -4.97916859 \times 10^{-1} & 2.31076 \\ 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000 \\ 4.79596820 \times 10^0 & 2.68463893 \times 10^0 & -7.00733526 \times 10^{-1} & 4.00775685 \times 10^0 & -3.13502035 \times 10^{-1} & -1.22041876 \times 10^0 & -1.99232381 \times 10^{-1} & 2.80657 \\ 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000 \end{bmatrix}$$

The reduced state space equations:

$$\begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} = \begin{bmatrix} -5.39442564 \times 10^2 & -9.43587795 \times 10^1 & 1.64405008 \times 10^2 & -4.02632119 \times 10^1 & 4.69358894 \times 10^{-3} & -3.26432602 \times 10^{-6} & -4.41001658 \times 10^{-11} & - \\ 2.95065621 \times 10^2 & -1.00842960 \times 10^3 & -1.94918423 \times 10^1 & 3.29522485 \times 10^1 & -3.84133061 \times 10^{-3} & 2.67165587 \times 10^{-6} & 3.97460109 \times 10^{-12} & - \\ 2.95710921 \times 10^3 & 2.05365041 \times 10^1 & -8.93677686 \times 10^2 & 2.19885910 \times 10^2 & 2.69688140 \times 10^{-1} & -1.87569859 \times 10^{-4} & 2.37901096 \times 10^{-10} & - \\ -1.76212533 \times 10^3 & 8.30329227 \times 10^1 & 5.32903873 \times 10^2 & -1.27664238 \times 10^2 & 1.22034529 \times 10^0 & 3.97275072 \times 10^{-5} & -1.41574866 \times 10^{-10} & - \\ -8.93391374 \times 10^1 & -1.09274272 \times 10^3 & 1.10756420 \times 10^2 & 6.22703938 \times 10^0 & -3.49600828 \times 10^0 & 7.62658531 \times 10^0 & -3.07793528 \times 10^{-11} & - \\ -2.61423175 \times 10^3 & -3.05260817 \times 10^2 & 8.11125065 \times 10^2 & -1.92105798 \times 10^2 & -2.63779297 \times 10^{-1} & 3.50999269 \times 10^0 & -1.89700202 \times 10^{-5} & - \\ 1.50186221 \times 10^3 & 1.65854460 \times 10^2 & -4.64490229 \times 10^2 & 1.11594014 \times 10^2 & 5.01754228 \times 10^{-1} & -1.29428794 \times 10^0 & 8.50457054 \times 10^{-2} & - \\ 9.03878828 \times 10^2 & -1.09552023 \times 10^2 & -2.64180042 \times 10^2 & 6.81060862 \times 10^1 & -5.37799194 \times 10^{-1} & 5.71836953 \times 10^{-1} & 2.46184396 \times 10^{-2} & - \\ 1.70549381 \times 10^3 & 3.97927554 \times 10^2 & -5.43544416 \times 10^2 & 1.23224381 \times 10^2 & 1.54190734 \times 10^0 & -2.50593969 \times 10^0 & -4.25798372 \times 10^{-3} & - \\ 2.40453312 \times 10^{-3} & -2.01438832 \times 10^{-1} & 1.47072075 \times 10^{-2} & 1.81387869 \times 10^{-3} & -8.73210434 \times 10^{-4} & 8.91526847 \times 10^{-4} & 1.44505148 \times 10^{-4} & - \\ 9.84681648 \times 10^2 & 1.65084383 \times 10^2 & -3.05867086 \times 10^2 & 7.44888279 \times 10^1 & 7.02363715 \times 10^{-1} & -1.61173776 \times 10^0 & -1.85180486 \times 10^{-1} & - \\ -3.46835590 \times 10^{-3} & 1.21136142 \times 10^{-2} & 1.20120899 \times 10^{-2} & 1.02305127 \times 10^{-2} & 2.96476338 \times 10^{-3} & -2.45450551 \times 10^{-3} & 2.04445464 \times 10^{-8} & - \\ -6.50443350 \times 10^{-3} & 2.40866862 \times 10^{-3} & 1.04523985 \times 10^{-2} & -9.90053307 \times 10^{-5} & -5.29788994 \times 10^{-3} & 1.36773798 \times 10^{-2} & -8.82544153 \times 10^{-9} & - \end{bmatrix}$$