



Mechanical, Automotive, & Materials Engineering

401 Sunset Avenue

Windsor, Ontario, Canada N9B 3P4

519 253 3000 (2616)

www.uwindsor.ca/mame

EoM Analysis
GSXR 1100 multilink

John Smith: ID 12345678

Jane Smith: ID 87654321

January 18, 2020

CONTENTS

LIST OF FIGURES

LIST OF TABLES

CHAPTER 1

INTRODUCTION

Replace this text with the introduction of your report.

1.1 System Description

The properties of the bodies are given in Tables ?? and ?. The properties of the connections are given in Tables ??, ??, and ??.

Table 1.1: Body CG Locations and Mass

No.	Body Name	Location [m]	Mass [kg]
1	Frame	0.678, 0.000, 0.472	165.130
2	Upper body	0.415, 0.000, 1.140	33.680
3	Upper fork	1.164, 0.000, 0.770	9.990
4	Lower fork	1.365, 0.000, 0.324	7.250
5	Swing arm	0.196, 0.000, 0.311	8.000
6	Bell crank	0.493, 0.000, 0.173	0.000
7	Front wheel, bike	1.410, 0.000, 0.282	11.900
8	Rear wheel, bike	0.000, 0.000, 0.297	14.700
9	Front contact patch	1.410, 0.000, 0.000	0.000
10	Rear contact patch	0.000, 0.000, 0.000	0.000
11	Lower left arm	1.140, 0.156, 0.200	2.000
12	Upper left arm	1.140, 0.175, 0.300	2.000
13	Lower right arm	1.140, -0.156, 0.200	2.000
14	Upper right arm	1.140, -0.175, 0.300	2.000

Table 1.2: Body Inertia Properties

No.	Body Name	Inertia [†] [kg m ²] (I_{xx} , I_{yy} , I_{zz} ; I_{xy} , I_{yz} , I_{zx})
1	Frame	11.085, 22.013, 14.982; 0.000, 0.000, 3.691
2	Upper body	1.428, 1.347, 0.916; 0.000, 0.000, -0.443
3	Upper fork	1.341, 1.548, 0.413; 0.000, 0.000, 0.000

4	Lower fork	0.000, 0.000, 0.000;	0.000, 0.000, 0.000
5	Swing arm	0.020, 0.259, 0.259;	0.000, 0.000, 0.000
6	Bell crank	0.000, 0.000, 0.000;	0.000, 0.000, 0.000
7	Front wheel, bike	0.270, 0.484, 0.270;	0.000, 0.000, 0.000
8	Rear wheel, bike	0.383, 0.638, 0.383;	0.000, 0.000, 0.000
9	Front contact patch	0.000, 0.000, 0.000;	0.000, 0.000, 0.000
10	Rear contact patch	0.000, 0.000, 0.000;	0.000, 0.000, 0.000
11	Lower left arm	0.002, 0.007, 0.009;	-0.004, 0.000, 0.000
12	Upper left arm	0.004, 0.007, 0.010;	-0.005, 0.000, 0.000
13	Lower right arm	0.002, 0.007, 0.009;	0.004, 0.000, 0.000
14	Upper right arm	0.004, 0.007, 0.010;	0.005, 0.000, 0.000

[†]Inertias are defined as the positive integral over the body.

For example, $I_{xy} = + \int r_x r_y \, dm$.

Table 1.3: Connection Location and Direction

No.	Connection Name	Location [m]	Unit Axis
1	Rear axle	0.000, 0.000, 0.297	0.000, 1.000, 0.000
2	Swing arm pivot	0.549, 0.000, 0.361	0.000, 1.000, 0.000
3	Bell crank pivot	0.539, 0.000, 0.188	0.000, 1.000, 0.000
4	Seat	0.364, 0.000, 0.844	0.000, 1.000, 0.000
5	Steering head	1.173, 0.000, 0.749	0.407, 0.000, -0.914
6	Front axle	1.410, 0.000, 0.282	0.000, 1.000, 0.000
7	Front contact patch constraint	1.410, 0.000, 0.000	0.000, 1.000, 0.000
8	Rear contact patch constraint	0.000, 0.000, 0.000	0.000, 1.000, 0.000
9	Front tire, sidewall	1.410, 0.000, 0.000	0.000, 1.000, 0.000
10	Rear tire, sidewall	0.000, 0.000, 0.000	0.000, 1.000, 0.000
11	Front tire, vertical	1.410, 0.000, 0.000	0.000, 0.000, 1.000
12	Rear tire, vertical	0.000, 0.000, 0.000	0.000, 0.000, 1.000
13	Lower left rear joint	1.040, 0.212, 0.200	1.000, 0.000, 0.000
14	Lower left front joint	1.240, 0.100, 0.200	- , - , -
15	Upper left rear joint	1.040, 0.250, 0.300	1.000, 0.000, 0.000
16	Upper left front joint	1.240, 0.100, 0.300	- , - , -
17	Lower right rear joint	1.040, -0.212, 0.200	1.000, 0.000, 0.000
18	Lower right front joint	1.240, -0.100, 0.200	- , - , -
19	Upper right rear joint	1.040, -0.250, 0.300	1.000, 0.000, 0.000
20	Upper right front joint	1.240, -0.100, 0.300	- , - , -
21	Front tire, lateral	1.410, 0.000, 0.000	0.000, 1.000, 0.000
22	Rear tire, lateral	0.000, 0.000, 0.000	0.000, 1.000, 0.000
23	Front tire, longitudinal	1.410, 0.000, 0.000	1.000, 0.000, 0.000
24	Rear tire, longitudinal	0.000, 0.000, 0.000	1.000, 0.000, 0.000

Table 1.4: Connection Locations

No.	Connection Name	Location [m]	Location [m]
1	Rear spring	0.487, 0.000, 0.489	0.444, 0.000, 0.178
2	Right front spring	1.206, -0.100, 0.714	1.276, -0.100, 0.550
3	Left front spring	1.206, 0.100, 0.714	1.276, 0.100, 0.550
4	Pull rod (dogbone)	0.372, 0.000, 0.275	0.495, 0.000, 0.152
5	Steering shaft	1.276, 0.000, 0.550	1.164, 0.000, 0.770

Table 1.5: Connection Properties

No.	Connection Name	Stiffness [N/m]	Damping [Ns/m]
1	Front tire, lateral	0	667
2	Rear tire, lateral	0	667
3	Front tire, longitudinal	0	2,000
4	Rear tire, longitudinal	0	2,000
1	Rear spring	58,570	11,650
2	Right front spring	14,970	1,278
3	Left front spring	14,970	1,278

CHAPTER 2

ANALYSIS

Replace this text with the body of your report. Add sections or subsections as appropriate.

2.1 Eigenvalue Analysis

The eigenvalue properties are given in Tables ?? and ??.

Table 2.1: Eigenvalues

No.	Real [rad/s]	Imaginary [rad/s]	Real [Hz]	Imaginary [Hz]
1	$-3.4130489912 \times 10^2$	0.0000000000×10^0	$-5.4320361796 \times 10^1$	0.0000000000×10^0
2	$-2.8401165460 \times 10^2$	0.0000000000×10^0	$-4.5201858725 \times 10^1$	0.0000000000×10^0
3	$-2.8254923434 \times 10^1$	1.2508855673×10^2	$-4.4969107312 \times 10^0$	1.9908462128×10^1
4	$-2.8254923434 \times 10^1$	$-1.2508855673 \times 10^2$	$-4.4969107312 \times 10^0$	$-1.9908462128 \times 10^1$
5	$-1.6906423815 \times 10^0$	1.2069100644×10^1	$-2.6907409201 \times 10^{-1}$	1.9208570262×10^0
6	$-1.6906423815 \times 10^0$	$-1.2069100644 \times 10^1$	$-2.6907409201 \times 10^{-1}$	$-1.9208570262 \times 10^0$
7	$-1.9575156340 \times 10^1$	0.0000000000×10^0	$-3.1154828932 \times 10^0$	0.0000000000×10^0
8	$-1.7234372733 \times 10^1$	0.0000000000×10^0	$-2.7429356116 \times 10^0$	0.0000000000×10^0
9	$-1.5306103891 \times 10^1$	1.1751628794×10^1	$-2.4360420937 \times 10^0$	1.8703298119×10^0
10	$-1.5306103891 \times 10^1$	$-1.1751628794 \times 10^1$	$-2.4360420937 \times 10^0$	$-1.8703298119 \times 10^0$
11	3.7298793837×10^0	0.0000000000×10^0	$5.9362874106 \times 10^{-1}$	0.0000000000×10^0
12	$-2.6234605271 \times 10^0$	0.0000000000×10^0	$-4.1753671090 \times 10^{-1}$	0.0000000000×10^0
13	$-1.2976123174 \times 10^{-12}$	$6.9185143822 \times 10^{-7}$	$-2.0652141453 \times 10^{-13}$	$1.1011157628 \times 10^{-7}$
14	$-1.2976123174 \times 10^{-12}$	$-6.9185143822 \times 10^{-7}$	$-2.0652141453 \times 10^{-13}$	$-1.1011157628 \times 10^{-7}$
15	$3.9093673943 \times 10^{-12}$	0.0000000000×10^0	$6.2219514517 \times 10^{-13}$	0.0000000000×10^0
16	$-6.8747258428 \times 10^{-13}$	0.0000000000×10^0	$-1.0941466003 \times 10^{-13}$	0.0000000000×10^0
17	$9.9065881394 \times 10^{-13}$	0.0000000000×10^0	$1.5766824716 \times 10^{-13}$	0.0000000000×10^0
18	$-3.7744386440 \times 10^{-13}$	0.0000000000×10^0	$-6.0072056759 \times 10^{-14}$	0.0000000000×10^0

Note: oscillatory roots appear as complex conjugates.

Table 2.2: Eigenvalue Analysis

No.	Frequency (ω_n) [Hz]	Damping Ratio (ζ)	Time Constant (τ) [s]	Wavelength (λ) [s]
1	—	—	$2.9299315731 \times 10^{-3}$	—
2	—	—	$3.5209822689 \times 10^{-3}$	—
3	2.0410023774×10^1	$2.2032853960 \times 10^{-1}$	$3.5392061930 \times 10^{-2}$	$5.0229896892 \times 10^{-2}$
4	2.0410023774×10^1	$2.2032853960 \times 10^{-1}$	$3.5392061930 \times 10^{-2}$	$5.0229896892 \times 10^{-2}$
5	1.9396114513×10^0	$1.3872576996 \times 10^{-1}$	$5.9149114618 \times 10^{-1}$	$5.2060095382 \times 10^{-1}$
6	1.9396114513×10^0	$1.3872576996 \times 10^{-1}$	$5.9149114618 \times 10^{-1}$	$5.2060095382 \times 10^{-1}$
7	—	—	$5.1085160325 \times 10^{-2}$	—
8	—	—	$5.8023579707 \times 10^{-2}$	—
9	3.0712269026×10^0	$7.9318206401 \times 10^{-1}$	$6.5333412549 \times 10^{-2}$	$5.3466505941 \times 10^{-1}$
10	3.0712269026×10^0	$7.9318206401 \times 10^{-1}$	$6.5333412549 \times 10^{-2}$	$5.3466505941 \times 10^{-1}$
11	—	—	$-2.6810518441 \times 10^{-1}$	—
12	—	—	$3.8117592762 \times 10^{-1}$	—
13	$1.1011157628 \times 10^{-7}$	$1.8755649634 \times 10^{-6}$	$7.7064619886 \times 10^{11}$	9.0816972547×10^6
14	$1.1011157628 \times 10^{-7}$	$1.8755649634 \times 10^{-6}$	$7.7064619886 \times 10^{11}$	9.0816972547×10^6
15	—	—	$-2.5579586136 \times 10^{11}$	—
16	—	—	$1.4546034604 \times 10^{12}$	—
17	—	—	$-1.0094292666 \times 10^{12}$	—
18	—	—	$2.6494005979 \times 10^{12}$	—

Notes: a) oscillatory roots are listed twice, b) negative time constants denote unstable roots.

There are 9 degrees of freedom. There are 4 oscillatory modes, 11 damped modes, 3 unstable modes, and 0 rigid body modes.

2.2 Frequency Response Plots

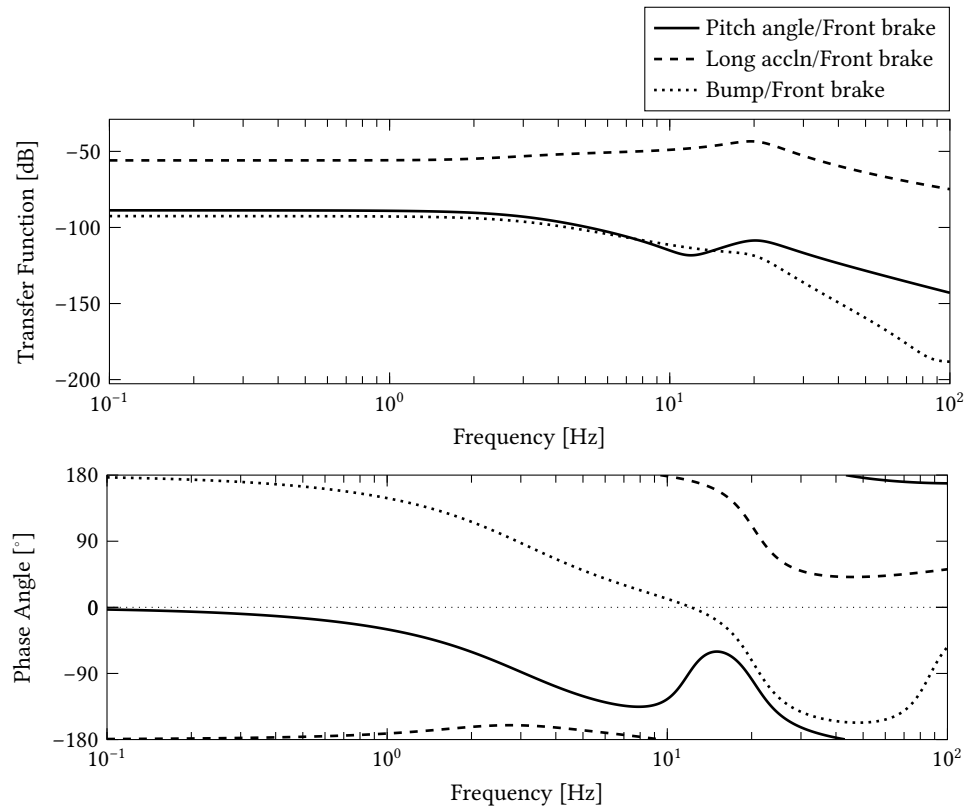


Figure 2.1: Frequency response: Front brake

2.3 Steady State Gains

The steady state gains are given in Table ??.

Table 2.3: Steady State Gains

No.	Output/Input	Gain
1	Pitch angle/Front brake	$3.6580000000 \times 10^{-5}$
2	Long accln/Front brake	$-1.6011800000 \times 10^{-3}$
3	Bump/Front brake	$-2.3620000000 \times 10^{-5}$

2.4 Hankel Singular Value Analysis

The Hankel singular values are given in Table ??.

Table 2.4: Hankel Singular Values

No.	Hankel SV
1	$3.9272905233 \times 10^{-3}$
2	$2.6538947560 \times 10^{-3}$
3	$5.5223551621 \times 10^{-4}$
4	$8.2746255651 \times 10^{-5}$
5	$2.6772220148 \times 10^{-6}$
6	$3.3260106573 \times 10^{-9}$
7	$3.7032899677 \times 10^{-20}$
8	$1.7593485115 \times 10^{-21}$
9	$7.4027414638 \times 10^{-22}$
10	$3.4351626390 \times 10^{-22}$
11	$7.2860712201 \times 10^{-23}$

2.5 Equilibrium Analysis

The results of the equilibrium load analysis are given in Tables ?? and ??.

Table 2.5: System Static Deflections

No.	Body Name	Type	Deflection [m] or [rad]
1	Frame	translation	3.3610×10^{-4} , 0.0000×10^0 , -5.1377×10^{-2}
–	–	rotation	0.0000×10^0 , -1.6710×10^{-2} , 0.0000×10^0
2	Upper body	translation	-1.0820×10^{-2} , 0.0000×10^0 , -5.5770×10^{-2}
–	–	rotation	0.0000×10^0 , -1.6710×10^{-2} , 0.0000×10^0
3	Upper fork	translation	-4.6368×10^{-3} , 0.0000×10^0 , -4.3254×10^{-2}
–	–	rotation	0.0000×10^0 , -1.6710×10^{-2} , 0.0000×10^0
4	Lower fork	translation	2.8159×10^{-3} , 0.0000×10^0 , -7.5195×10^{-4}
–	–	rotation	0.0000×10^0 , -1.6710×10^{-2} , 0.0000×10^0
5	Swing arm	translation	-2.6256×10^{-3} , 0.0000×10^0 , -1.9111×10^{-2}
–	–	rotation	0.0000×10^0 , 9.7506×10^{-2} , 0.0000×10^0
6	Bell crank	translation	-3.1054×10^{-4} , 0.0000×10^0 , -3.7072×10^{-2}

–	–	rotation	0.0000×10^0 , 3.5856×10^{-1} , 0.0000×10^0
7	Front wheel, bike	translation	3.5177×10^{-3} , 0.0000×10^0 , 0.0000×10^0
–	–	rotation	0.0000×10^0 , 4.7703×10^{-4} , 0.0000×10^0
8	Rear wheel, bike	translation	-4.0199×10^{-3} , 0.0000×10^0 , 0.0000×10^0
–	–	rotation	0.0000×10^0 , -5.7174×10^{-4} , 0.0000×10^0
9	Front contact patch	translation	3.3832×10^{-3} , 0.0000×10^0 , 0.0000×10^0
–	–	rotation	0.0000×10^0 , 4.7703×10^{-4} , 0.0000×10^0
10	Rear contact patch	translation	-3.8501×10^{-3} , 0.0000×10^0 , 0.0000×10^0
–	–	rotation	0.0000×10^0 , -5.7174×10^{-4} , 0.0000×10^0
11	Lower left arm	translation	4.8879×10^{-3} , 0.0000×10^0 , -2.4083×10^{-2}
–	–	rotation	0.0000×10^0 , -2.1243×10^{-1} , 0.0000×10^0
12	Upper left arm	translation	3.2169×10^{-3} , 0.0000×10^0 , -2.4083×10^{-2}
–	–	rotation	0.0000×10^0 , -2.1243×10^{-1} , 0.0000×10^0
13	Lower right arm	translation	4.8879×10^{-3} , 0.0000×10^0 , -2.4083×10^{-2}
–	–	rotation	0.0000×10^0 , -2.1243×10^{-1} , 0.0000×10^0
14	Upper right arm	translation	3.2169×10^{-3} , 0.0000×10^0 , -2.4083×10^{-2}
–	–	rotation	0.0000×10^0 , -2.1243×10^{-1} , 0.0000×10^0

Table 2.6: System Preloads

No.	Connector Name	Type	Load [N] or [Nm] (Components; Magnitude)		
1	Rear axle	force	0.0000×10^0 , 0.0000×10^0 , 1.1762×10^3	1.1762×10^3	
–	–	moment	0.0000×10^0 , 0.0000×10^0 , 0.0000×10^0	0.0000×10^0	
2	Swing arm pivot	force	-2.3492×10^3 , 0.0000×10^0 , 1.2553×10^3	2.6635×10^3	
–	–	moment	0.0000×10^0 , 0.0000×10^0 , 0.0000×10^0	0.0000×10^0	
3	Bell crank pivot	force	2.6261×10^3 , 0.0000×10^0 , -3.3862×10^2	2.6478×10^3	
–	–	moment	0.0000×10^0 , 0.0000×10^0 , 0.0000×10^0	0.0000×10^0	
4	Seat	force	0.0000×10^0 , 0.0000×10^0 , 3.3040×10^2	3.3040×10^2	
–	–	moment	0.0000×10^0 , -1.6850×10^1 , 0.0000×10^0	1.6850×10^1	
5	Steering head	force	4.2458×10^2 , 0.0000×10^0 , -8.9183×10^2	9.8774×10^2	
–	–	moment	0.0000×10^0 , 1.8686×10^1 , 0.0000×10^0	1.8686×10^1	
6	Front axle	force	0.0000×10^0 , 0.0000×10^0 , 1.1002×10^3	1.1002×10^3	
–	–	moment	0.0000×10^0 , 0.0000×10^0 , 0.0000×10^0	0.0000×10^0	
7	Front contact patch constraint	force	0.0000×10^0 , 0.0000×10^0 , 0.0000×10^0	0.0000×10^0	
–	–	moment	0.0000×10^0 , 0.0000×10^0 , 0.0000×10^0	0.0000×10^0	
8	Rear contact patch constraint	force	0.0000×10^0 , 0.0000×10^0 , 0.0000×10^0	0.0000×10^0	
–	–	moment	0.0000×10^0 , 0.0000×10^0 , 0.0000×10^0	0.0000×10^0	
9	Front tire, sidewall	force	0.0000×10^0 , 0.0000×10^0 , 0.0000×10^0	0.0000×10^0	
–	–	moment	0.0000×10^0 , 0.0000×10^0 , 0.0000×10^0	0.0000×10^0	
10	Rear tire, sidewall	force	0.0000×10^0 , 0.0000×10^0 , 0.0000×10^0	0.0000×10^0	
–	–	moment	0.0000×10^0 , 0.0000×10^0 , 0.0000×10^0	0.0000×10^0	
11	Front tire, vertical	force	0.0000×10^0 , 0.0000×10^0 , 1.2169×10^3	1.2169×10^3	
–	–	moment	0.0000×10^0 , 0.0000×10^0 , 0.0000×10^0	0.0000×10^0	
12	Rear tire, vertical	force	0.0000×10^0 , 0.0000×10^0 , 1.3204×10^3	1.3204×10^3	
–	–	moment	0.0000×10^0 , 0.0000×10^0 , 0.0000×10^0	0.0000×10^0	
13	Lower left rear joint	force	-1.8010×10^2 , 1.0129×10^2 , 9.8100×10^0	2.0686×10^2	
–	–	moment	0.0000×10^0 , 0.0000×10^0 , 0.0000×10^0	0.0000×10^0	
14	Lower left front joint	force	1.8010×10^2 , -1.0129×10^2 , 9.8100×10^0	2.0686×10^2	
–	–	moment	0.0000×10^0 , 0.0000×10^0 , 0.0000×10^0	0.0000×10^0	
15	Upper left rear joint	force	-3.2189×10^1 , 2.4142×10^1 , 9.8100×10^0	4.1415×10^1	
–	–	moment	0.0000×10^0 , 0.0000×10^0 , 0.0000×10^0	0.0000×10^0	
16	Upper left front joint	force	3.2189×10^1 , -2.4142×10^1 , 9.8100×10^0	4.1415×10^1	
–	–	moment	0.0000×10^0 , 0.0000×10^0 , 0.0000×10^0	0.0000×10^0	
17	Lower right rear joint	force	-1.8010×10^2 , -1.0129×10^2 , 9.8100×10^0	2.0686×10^2	
–	–	moment	0.0000×10^0 , 0.0000×10^0 , 0.0000×10^0	0.0000×10^0	

18	Lower right front joint	force	1.8010×10^2 ,	1.0129×10^2 ,	9.8100×10^0 ; 2.0686×10^2
-	-	moment	0.0000×10^0 ,	0.0000×10^0 ,	0.0000×10^0 ; 0.0000×10^0
19	Upper right rear joint	force	-3.2189×10^1 ,	-2.4142×10^1 ,	9.8100×10^0 ; 4.1415×10^1
-	-	moment	0.0000×10^0 ,	0.0000×10^0 ,	0.0000×10^0 ; 0.0000×10^0
20	Upper right front joint	force	3.2189×10^1 ,	2.4142×10^1 ,	9.8100×10^0 ; 4.1415×10^1
-	-	moment	0.0000×10^0 ,	0.0000×10^0 ,	0.0000×10^0 ; 0.0000×10^0
21	Front tire, lateral	force	0.0000×10^0 ,	0.0000×10^0 ,	0.0000×10^0 ; 0.0000×10^0
-	-	moment	0.0000×10^0 ,	0.0000×10^0 ,	0.0000×10^0 ; 0.0000×10^0
22	Rear tire, lateral	force	0.0000×10^0 ,	0.0000×10^0 ,	0.0000×10^0 ; 0.0000×10^0
-	-	moment	0.0000×10^0 ,	0.0000×10^0 ,	0.0000×10^0 ; 0.0000×10^0
23	Front tire, longitudinal	force	0.0000×10^0 ,	0.0000×10^0 ,	0.0000×10^0 ; 0.0000×10^0
-	-	moment	0.0000×10^0 ,	0.0000×10^0 ,	0.0000×10^0 ; 0.0000×10^0
24	Rear tire, longitudinal	force	0.0000×10^0 ,	0.0000×10^0 ,	0.0000×10^0 ; 0.0000×10^0
-	-	moment	0.0000×10^0 ,	0.0000×10^0 ,	0.0000×10^0 ; 0.0000×10^0
25	Rear spring	force	2.7693×10^2 ,	0.0000×10^0 ,	2.0144×10^3 ; -2.0333×10^3
26	Right front spring	force	-2.1229×10^2 ,	0.0000×10^0 ,	4.9491×10^2 ; -5.3852×10^2
27	Left front spring	force	-2.1229×10^2 ,	0.0000×10^0 ,	4.9491×10^2 ; -5.3852×10^2
28	Pull rod (dogbone)	force	2.3492×10^3 ,	0.0000×10^0 ,	-2.3530×10^3 ; 3.3249×10^3
29	Steering shaft	moment	0.0000×10^0 ,	0.0000×10^0 ,	0.0000×10^0 ; 0.0000×10^0

CHAPTER 3

CONCLUSION

Replace this text with the conclusion to your report.

APPENDIX A

EQUATIONS OF MOTION

The equations of motion are of the form

$$\begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{M} & -\mathbf{G} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{p}} \\ \dot{\mathbf{w}} \\ \dot{\mathbf{u}} \end{Bmatrix} + \begin{bmatrix} \mathbf{V} & -\mathbf{I} & \mathbf{0} \\ \mathbf{K} & \mathbf{L} & -\mathbf{F} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{Bmatrix} \mathbf{p} \\ \mathbf{w} \\ \mathbf{u} \end{Bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{I} \end{bmatrix} \{ \mathbf{u} \}$$

The system is subject to constraints

$$\begin{bmatrix} \mathbf{J}_h & \mathbf{0} & \mathbf{0} \\ -\mathbf{J}_h \mathbf{V} & \mathbf{J}_h & \mathbf{0} \\ \mathbf{0} & \mathbf{J}_{nh} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{p}} & \mathbf{p} \\ \dot{\mathbf{w}} & \mathbf{w} \\ \dot{\mathbf{u}} & \mathbf{u} \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}$$

The full state space equations:

$$\begin{bmatrix} \mathbf{E} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{x}} \\ \mathbf{y} \end{Bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} \begin{Bmatrix} \mathbf{x} \\ \mathbf{u} \end{Bmatrix}$$

$$\begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} = \begin{bmatrix} -8.97497577 \times 10^1 & 2.34983430 \times 10^1 & -3.56770060 \times 10^1 & 4.08683640 \times 10^1 & -1.18963615 \times 10^2 & 5.75526627 \times 10^1 & 5.39587200 \times 10^0 & 5 \\ 5.44079190 \times 10^1 & -2.41153469 \times 10^1 & -1.12304938 \times 10^1 & 8.44986821 \times 10^1 & -8.12318600 \times 10^1 & 3.81047262 \times 10^1 & 3.31125592 \times 10^1 & -9 \\ 5.40462138 \times 10^1 & -2.66257365 \times 10^1 & 3.60052319 \times 10^1 & -4.78253278 \times 10^0 & 9.24223754 \times 10^1 & -6.63982650 \times 10^1 & -2.41329112 \times 10^1 & -9 \\ -1.00010242 \times 10^1 & -3.58186901 \times 10^1 & 1.84422800 \times 10^1 & -9.15192185 \times 10^0 & -4.04552920 \times 10^1 & 2.62440476 \times 10^1 & -3.94986061 \times 10^1 & -3 \\ -4.21692041 \times 10^1 & 3.57200745 \times 10^0 & 1.74550805 \times 10^1 & -7.21013642 \times 10^1 & 6.00645609 \times 10^1 & -9.74602053 \times 10^1 & -1.46306343 \times 10^2 & -1 \\ 3.29258912 \times 10^0 & -2.27641452 \times 10^1 & -2.66000413 \times 10^0 & 1.86620076 \times 10^1 & -1.70481069 \times 10^1 & 2.58231828 \times 10^1 & 2.72808776 \times 10^1 & 2 \\ 2.47060980 \times 10^1 & 2.60330365 \times 10^{-1} & -9.55039919 \times 10^0 & 7.19019118 \times 10^1 & -9.12328418 \times 10^1 & 2.82517320 \times 10^1 & 8.72431140 \times 10^0 & -7 \\ 7.62662668 \times 10^1 & -1.11693438 \times 10^{-1} & 4.95508488 \times 10^1 & 9.54867797 \times 10^0 & 2.47532559 \times 10^1 & -6.78950474 \times 10^1 & -8.10497115 \times 10^1 & -1 \\ -6.24036587 \times 10^0 & 4.10451848 \times 10^1 & -8.27442459 \times 10^1 & 2.49225334 \times 10^2 & -3.55206395 \times 10^2 & 1.38701845 \times 10^2 & 7.16567307 \times 10^1 & 7 \\ -3.73919514 \times 10^1 & -1.64453301 \times 10^1 & 2.79213576 \times 10^1 & -2.23318894 \times 10^2 & 2.12255958 \times 10^2 & -1.22373375 \times 10^1 & 1.22177031 \times 10^1 & 9 \\ -7.65316472 \times 10^1 & -1.21063416 \times 10^1 & -1.47192143 \times 10^1 & -1.31800227 \times 10^2 & 9.92784982 \times 10^1 & -1.71045076 \times 10^1 & -2.59763744 \times 10^1 & 3 \\ -5.29937459 \times 10^1 & 3.57915827 \times 10^1 & -2.97978784 \times 10^1 & 1.44286827 \times 10^1 & -7.32122284 \times 10^1 & 2.66043839 \times 10^1 & -9.53859478 \times 10^{-2} & 4 \\ -8.44358213 \times 10^1 & 4.61249815 \times 10^1 & -3.91064934 \times 10^1 & 9.90757275 \times 10^1 & -2.19052076 \times 10^2 & 4.59443823 \times 10^1 & -5.84306120 \times 10^1 & -1 \\ 5.44614902 \times 10^1 & -3.79642700 \times 10^1 & 5.04669516 \times 10^1 & 2.31396336 \times 10^0 & 3.24045252 \times 10^1 & -4.43193543 \times 10^1 & -6.23121746 \times 10^1 & -1 \\ -1.18592456 \times 10^2 & 3.57672328 \times 10^1 & -3.50452850 \times 10^1 & 1.51441694 \times 10^1 & -1.31441636 \times 10^2 & 5.85156249 \times 10^1 & -2.02302733 \times 10^1 & 6 \\ 4.54336953 \times 10^1 & -3.21573787 \times 10^0 & -2.78488494 \times 10^1 & 1.17377338 \times 10^2 & -1.48320073 \times 10^2 & 1.11439074 \times 10^2 & 1.06716164 \times 10^2 & 1 \\ 7.81475224 \times 10^1 & -4.58926099 \times 10^0 & -1.78639992 \times 10^1 & 7.28573818 \times 10^1 & -8.73472460 \times 10^1 & 4.27492722 \times 10^1 & 4.02436877 \times 10^1 & 1 \\ 1.43272483 \times 10^1 & -3.09109136 \times 10^1 & 2.51876147 \times 10^1 & -2.91992013 \times 10^1 & 5.64770830 \times 10^1 & -2.14579789 \times 10^1 & -1.73327294 \times 10^1 & -3 \\ 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0 \\ -2.35492388 \times 10^{-4} & -6.19683548 \times 10^{-4} & -5.13569241 \times 10^{-4} & -1.79811225 \times 10^{-3} & 2.23738223 \times 10^{-3} & -7.70703935 \times 10^{-4} & -2.75946269 \times 10^{-4} & 9 \\ 1.48110504 \times 10^{-1} & 5.23293127 \times 10^{-1} & -5.90161698 \times 10^{-1} & 3.60390609 \times 10^0 & -4.52119117 \times 10^0 & 1.63025639 \times 10^0 & 1.00963030 \times 10^0 & 8 \\ 8.29144616 \times 10^{-5} & 9.66752941 \times 10^{-5} & 1.01655603 \times 10^{-3} & -6.92968049 \times 10^{-4} & 1.11625614 \times 10^{-3} & -5.35811111 \times 10^{-4} & -2.97125075 \times 10^{-4} & -4 \end{bmatrix}$$

A. EQUATIONS OF MOTION

$$\mathbf{E} = \begin{bmatrix} 4.96293822 \times 10^0 & -7.52092801 \times 10^{-1} & 3.21455912 \times 10^{-3} & -8.98852233 \times 10^{-1} & -3.50615812 \times 10^{-1} & 6.07694476 \times 10^{-1} & 4.28296820 \times 10^0 & -1.1303377 \times 10^0 \\ -3.03085078 \times 10^0 & 8.06818582 \times 10^{-1} & -6.57991790 \times 10^{-1} & -4.89402582 \times 10^{-3} & 8.19785357 \times 10^{-1} & -5.49747905 \times 10^{-1} & -2.76269396 \times 10^0 & 1.2984368 \times 10^0 \\ -1.62136797 \times 10^0 & 1.72985754 \times 10^{-1} & 5.15640306 \times 10^{-1} & 1.98814294 \times 10^0 & 2.05110838 \times 10^0 & 1.34725815 \times 10^0 & -3.18563140 \times 10^0 & 1.7437836 \times 10^0 \\ 2.69182098 \times 10^0 & -6.93074145 \times 10^{-1} & 4.39478315 \times 10^{-1} & 1.46589314 \times 10^0 & 1.54864229 \times 10^0 & 1.29874889 \times 10^0 & 6.06997436 \times 10^{-1} & 7.5921492 \times 10^0 \\ 1.59975486 \times 10^0 & -3.57677674 \times 10^{-1} & 7.23781351 \times 10^{-1} & 2.80773817 \times 10^0 & 4.33964746 \times 10^0 & 2.38073399 \times 10^0 & -2.58139300 \times 10^0 & 2.0631027 \times 10^0 \\ -1.71883175 \times 10^0 & 3.46514501 \times 10^{-1} & -5.65396176 \times 10^{-1} & -1.21965634 \times 10^0 & -1.42044997 \times 10^0 & -7.75033086 \times 10^{-1} & 3.21228705 \times 10^{-1} & -4.6723158 \times 10^0 \\ 1.12332706 \times 10^0 & -5.13551744 \times 10^{-1} & -4.01738799 \times 10^{-1} & 4.36911172 \times 10^{-1} & 1.26901819 \times 10^0 & 1.60420236 \times 10^0 & 3.89195197 \times 10^{-1} & 6.8675833 \times 10^0 \\ 2.34153000 \times 10^0 & -8.99791039 \times 10^{-1} & 8.26862511 \times 10^{-1} & 4.36588787 \times 10^0 & 5.35035076 \times 10^0 & 4.29772791 \times 10^0 & -2.99578026 \times 10^0 & 3.0521431 \times 10^0 \\ 1.89667821 \times 10^{-1} & -2.12192376 \times 10^{-1} & -1.10781357 \times 10^0 & -1.89815385 \times 10^0 & 2.66499578 \times 10^{-1} & -1.98201974 \times 10^{-1} & 7.89972577 \times 10^{-1} & -6.7442856 \times 10^0 \\ -1.14529306 \times 10^0 & 5.97770393 \times 10^{-1} & 4.93596337 \times 10^{-2} & -2.39783161 \times 10^0 & -5.00190641 \times 10^0 & -3.11439857 \times 10^0 & 3.82060777 \times 10^0 & -2.5760745 \times 10^0 \\ 3.68708780 \times 10^0 & -5.52158844 \times 10^{-1} & 3.39280137 \times 10^{-1} & -1.23197319 \times 10^0 & -7.21803139 \times 10^{-1} & 2.96489469 \times 10^{-1} & 4.59645434 \times 10^0 & -1.1019047 \times 10^0 \\ 2.09184927 \times 10^0 & -4.49958865 \times 10^{-1} & -1.92107653 \times 10^{-1} & -1.16314709 \times 10^0 & -1.19874723 \times 10^0 & 5.56182476 \times 10^{-1} & 3.67790177 \times 10^0 & -7.5148355 \times 10^0 \\ 4.16883340 \times 10^0 & -1.01835862 \times 10^0 & 1.10082457 \times 10^{-1} & 4.31397484 \times 10^{-1} & 2.09372292 \times 10^0 & 2.15316193 \times 10^0 & 2.32795841 \times 10^0 & 3.6885256 \times 10^0 \\ -1.00020000 \times 10^0 & -9.95152853 \times 10^{-3} & 1.34909401 \times 10^{-3} & 2.99357829 \times 10^0 & 2.45305255 \times 10^0 & 1.99594725 \times 10^0 & -3.58236264 \times 10^0 & 2.3209987 \times 10^0 \\ 3.99721436 \times 10^0 & -7.76594477 \times 10^{-1} & -9.82466916 \times 10^{-2} & -1.50101631 \times 10^0 & -1.50704932 \times 10^0 & 2.78450882 \times 10^{-1} & 5.30130713 \times 10^0 & -1.3095541 \times 10^0 \\ -1.45587346 \times 10^0 & 8.18344259 \times 10^{-1} & -7.16528581 \times 10^{-1} & -1.33237781 \times 10^0 & -1.51488680 \times 10^0 & -2.14093428 \times 10^0 & -2.01695980 \times 10^{-1} & -1.6503308 \times 10^0 \\ -2.97591907 \times 10^0 & 1.79287705 \times 10^{-1} & -8.89771964 \times 10^{-1} & -5.66786487 \times 10^{-1} & 7.25487165 \times 10^{-2} & -7.14943988 \times 10^{-1} & -2.31148210 \times 10^0 & 3.5077245 \times 10^0 \\ -1.05005828 \times 10^0 & 7.63978844 \times 10^{-2} & 6.99698193 \times 10^{-2} & 3.93135102 \times 10^{-1} & 2.89111208 \times 10^{-2} & 2.60998624 \times 10^{-1} & -7.84425007 \times 10^{-1} & 5.4711696 \times 10^0 \\ 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 \end{bmatrix}$$

The reduced state space equations:

$$\begin{Bmatrix} \dot{\mathbf{x}} \\ \mathbf{y} \end{Bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} \begin{Bmatrix} \mathbf{x} \\ \mathbf{u} \end{Bmatrix}$$

$$\begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} = \begin{bmatrix} -3.82090103 \times 10^{-2} & -2.18777941 \times 10^{-1} & 3.88244324 \times 10^0 & -5.41646992 \times 10^0 & -8.27601238 \times 10^{-1} & -3.73578201 \times 10^0 & 4.89885628 \times 10^0 \\ -4.18120041 \times 10^{-2} & 3.65118202 \times 10^{-1} & -4.86652517 \times 10^0 & 6.56738556 \times 10^0 & 1.37613571 \times 10^0 & 2.70644141 \times 10^0 & -7.94503997 \times 10^0 \\ 4.36609937 \times 10^{-1} & 1.05739449 \times 10^0 & -2.24975690 \times 10^1 & 3.39497650 \times 10^1 & 2.80302886 \times 10^0 & 2.34634592 \times 10^1 & -2.75926965 \times 10^1 \\ 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & -1.98752901 \times 10^1 & 2.34736176 \times 10^1 & 2.67536488 \times 10^0 & 2.66042781 \times 10^1 & -1.03604570 \times 10^1 \\ -2.83727388 \times 10^{-1} & 9.58223682 \times 10^{-2} & -5.60769085 \times 10^{-1} & 2.34694621 \times 10^0 & -2.00195629 \times 10^0 & -1.85125913 \times 10^0 & 5.66608081 \times 10^0 \\ 0.00000000 \times 10^0 & 6.18578219 \times 10^{-1} & 1.27176308 \times 10^0 & 3.12611885 \times 10^{-1} & -2.24211948 \times 10^0 & -1.99240075 \times 10^1 & -7.64328907 \times 10^0 \\ 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 3.08188904 \times 10^0 & -2.02424821 \times 10^0 & -1.96378826 \times 10^1 \\ 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & -1.33442805 \times 10^1 & -1.76193612 \times 10^1 \\ 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & -1.08900000 \times 10^{-9} & 6.90000000 \times 10^{-10} & -6.04000000 \times 10^{-10} & 3.90160000 \times 10^{-8} \\ 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 \\ 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 \\ 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 \\ 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 \\ 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 \\ 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 \\ 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 \\ 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 \\ 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 \\ 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 \\ 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 \\ 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 \\ 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 & 0.00000000 \times 10^0 \end{bmatrix}$$