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EoM Analysis
Spring Mass Damper

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January 11, 2017

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Chapter 1

Introduction

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1.1 System Description

The properties of the bodies are given in Tables 1.1 and 1.2. The properties of the connecctions are given in Tables 1.3, 1.4, and 1.5.

Table 1.1: Body CG Locations and Mass

No.	Body Name	Location [m]	Mass [kg]
1	block	0.000, 0.000, 1.000	1.000

Table 1.2: Body Inertia Properties

No.	Body Name	Inertia† [kg m^2] (I_{xx}, I_{yy}, I_{zz}; I_{xy}, I_{yz}, I_{zx})
1	block	0.000, 0.000, 0.000; 0.000, 0.000, 0.000

†Inertias are defined as the positive integral over the body, e.g., $I_{xy} = +\int r_x r_y \, dm$.

Table 1.3: Connection Location and Direction

No.	Connection Name	Location [m]	Unit Axis
1	slider 1	0.000, 0.000, 1.000	0.000, 0.000, 1.000
2	spring 1	0.000, 0.000, 0.500	0.000, 0.000, 1.000

Table 1.4: Connection Locations

No.	Connection Name	Location [m]	Location [m]

Table 1.5: Connection Properties

No.	Connection Name	Stiffness [N/m]	Damping [Ns/m]
1	spring 1	100	2

Chapter 2

Analysis

Replace this text with the body of your report. Add sections or subsections as appropriate.

2.1 Eigenvalue Analysis

The eigenvalue properties are given in Tables 2.1 and 2.2.

Table 2.1: Eigenvalues

No.	Real [rad/s]	Imaginary [rad/s]	Real [Hz]	Imaginary [Hz]
1	$-1.0000000000 \times 10^{0}$	$9.9498743711 \times 10^{0}$	$-1.5915494309 \times 10^{-1}$	$1.5835716893 \times 10^{0}$
2	$-1.0000000000 \times 10^{0}$	$-9.9498743711 \times 10^{0}$	$-1.5915494309 \times 10^{-1}$	$-1.5835716893 \times 10^{0}$

Note: oscillatory roots appear as complex conjugates.

Table 2.2: Eigenvalue Analysis

No.	Frequency (ω_n) [Hz]	Damping Ratio (ζ)	Time Constant (τ) [s]	Wavelength (λ) [s]
1	$1.5915494309 \times 10^{0}$	$1.0000000000 \times 10^{-1}$	$1.0000000000 \times 10^{0}$	$6.3148388340 \times 10^{-1}$
2	$1.5915494309 \times 10^{0}$	$1.0000000000 \times 10^{-1}$	$1.0000000000 \times 10^{0}$	$6.3148388340 \times 10^{-1}$

Notes: a) oscillatory roots are listed twice, b) negative time constants denote unstable roots.

There are 1 degrees of freedom. There are 1 oscillatory modes, 1 damped modes, 0 unstable modes, and 0 rigid body modes.

2.2 Frequency Response Plots

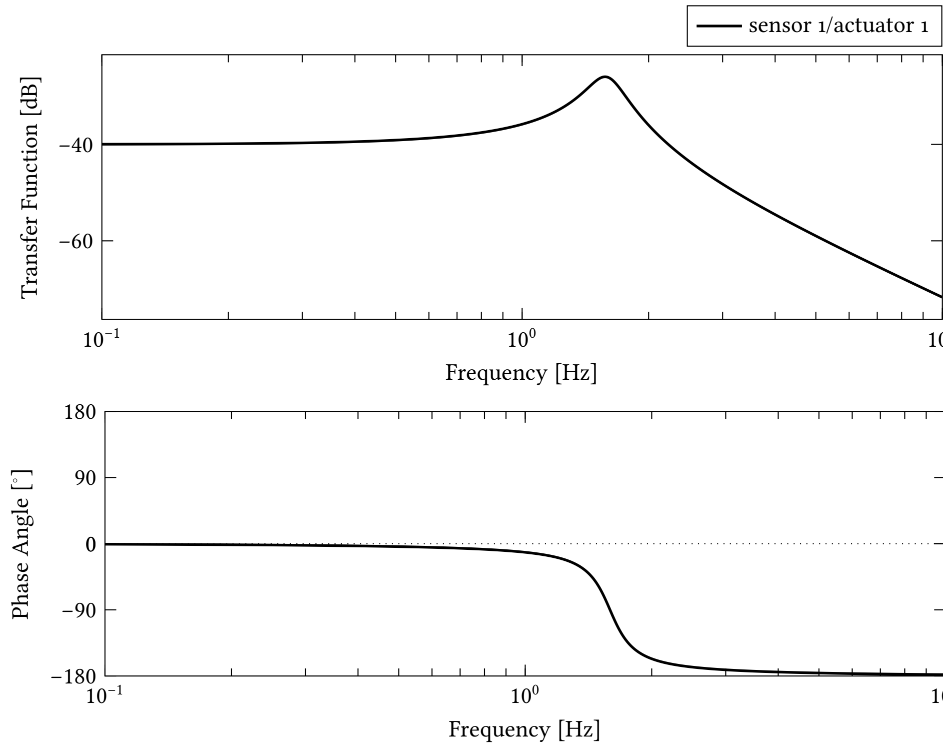

Figure 2.1: Frequency response: actuator 1

2.3 Steady State Gains

The steady state gains are given in Table 2.3.

Table 2.3: Steady State Gains

No.	Output/Input	Gain
1	sensor 1/actuator 1	$1.0000000000 \times 10^{-2}$

2.4 Hankel Singular Value Analysis

The Hankel singular values are given in Table 2.4.

Table 2.4: Hankel Singular Values

No.	Hankel SV
1	$2.7624689053 \times 10^{-2}$
2	$2.2624689053 \times 10^{-2}$
3	$0.0000000000 \times 10^{0}$

2.5 Equilibrium Analysis

The results of the equilibrium load analysis are given in Tables 2.5 and 2.6.

Table 2.5: System Static Deflections

No.	Body Name	Type	Deflection [m] or [rad]
1	block	translation	0.0000×10^{0}, 0.0000×10^{0}, -9.8100×10^{-2}
–	–	rotation	0.0000×10^{0}, 0.0000×10^{0}, 0.0000×10^{0}

Table 2.6: System Preloads

No.	Connector Name	Type	Load [N] or [Nm] (Components; Magnitude)
1	slider 1	force	0.0000×10^{0}, 0.0000×10^{0}, 0.0000×10^{0}; 0.0000×10^{0}
2	spring 1	force	0.0000×10^{0}, 0.0000×10^{0}, 9.8100×10^{0}; 9.8100×10^{0}

Chapter 3

Conclusion

Replace this text with the conclusion to your report.

Appendix A

Equations of Motion

The equations of motion are of the form

$$\begin{bmatrix} \mathrm{I} & 0 & 0 \\ 0 & \mathrm{M} & -\mathrm{G} \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \dot{\boldsymbol{p}} \\ \dot{\boldsymbol{w}} \\ \dot{\boldsymbol{u}} \end{Bmatrix} + \begin{bmatrix} \mathrm{V} & -\mathrm{I} & 0 \\ \mathrm{K} & \mathrm{L} & -\mathrm{F} \\ 0 & 0 & \mathrm{I} \end{bmatrix} \begin{Bmatrix} \boldsymbol{p} \\ \boldsymbol{w} \\ \boldsymbol{u} \end{Bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \mathrm{I} \end{bmatrix} \left\{ \boldsymbol{u} \right\}$$

The system is subject to constraints

$$\begin{bmatrix} \mathrm{J_h} & 0 & 0 \\ -\mathrm{J_h V} & \mathrm{J_h} & 0 \\ 0 & \mathrm{J_{nh}} & 0 \end{bmatrix} \begin{bmatrix} \dot{\boldsymbol{p}} & \boldsymbol{p} \\ \dot{\boldsymbol{w}} & \boldsymbol{w} \\ \dot{\boldsymbol{u}} & \boldsymbol{u} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

The mass matrix of the system is

$$\mathbf{M} = \begin{bmatrix} 1.000000 \times 10^{0} & 0.000000 \times 10^{0} & 0.000000 \times 10^{0} & 0.000000 \times 10^{0} & 0.000000 \times 10^{0} & 0.000000 \times 10^{0} \\ 0.000000 \times 10^{0} & 1.000000 \times 10^{0} & 0.000000 \times 10^{0} & 0.000000 \times 10^{0} & 0.000000 \times 10^{0} & 0.000000 \times 10^{0} \\ 0.000000 \times 10^{0} & 0.000000 \times 10^{0} & 1.000000 \times 10^{0} & 0.000000 \times 10^{0} & 0.000000 \times 10^{0} & 0.000000 \times 10^{0} \\ 0.000000 \times 10^{0} & 0.000000 \times 10^{0} & 0.000000 \times 10^{0} & 0.000000 \times 10^{0} & 0.000000 \times 10^{0} & 0.000000 \times 10^{0} \\ 0.000000 \times 10^{0} & 0.000000 \times 10^{0} & 0.000000 \times 10^{0} & 0.000000 \times 10^{0} & 0.000000 \times 10^{0} & 0.000000 \times 10^{0} \\ 0.000000 \times 10^{0} & 0.000000 \times 10^{0} & 0.000000 \times 10^{0} & 0.000000 \times 10^{0} & 0.000000 \times 10^{0} & 0.000000 \times 10^{0} \end{bmatrix}$$

The damping matrix is

$$\mathbf{L} = \begin{bmatrix} 0.000000 \times 10^{0} & 0.000000 \times 10^{0} & 0.000000 \times 10^{0} & 0.000000 \times 10^{0} & 0.000000 \times 10^{0} & 0.000000 \times 10^{0} \\ 0.000000 \times 10^{0} & 0.000000 \times 10^{0} & 0.000000 \times 10^{0} & 0.000000 \times 10^{0} & 0.000000 \times 10^{0} & 0.000000 \times 10^{0} \\ 0.000000 \times 10^{0} & 0.000000 \times 10^{0} & 2.000000 \times 10^{0} & 0.000000 \times 10^{0} & 0.000000 \times 10^{0} & 0.000000 \times 10^{0} \\ 0.000000 \times 10^{0} & 0.000000 \times 10^{0} & 0.000000 \times 10^{0} & 0.000000 \times 10^{0} & 0.000000 \times 10^{0} & 0.000000 \times 10^{0} \\ 0.000000 \times 10^{0} & 0.000000 \times 10^{0} & 0.000000 \times 10^{0} & 0.000000 \times 10^{0} & 0.000000 \times 10^{0} & 0.000000 \times 10^{0} \\ 0.000000 \times 10^{0} & 0.000000 \times 10^{0} & 0.000000 \times 10^{0} & 0.000000 \times 10^{0} & 0.000000 \times 10^{0} & 0.000000 \times 10^{0} \end{bmatrix}$$

The stiffness matrix is

$$\mathbf{K} = \begin{bmatrix} 0.000000 \times 10^{0} & 0.000000 \times 10^{0} & 0.000000 \times 10^{0} & 0.000000 \times 10^{0} & 0.000000 \times 10^{0} & 0.000000 \times 10^{0} \\ 0.000000 \times 10^{0} & 0.000000 \times 10^{0} & 0.000000 \times 10^{0} & 0.000000 \times 10^{0} & 0.000000 \times 10^{0} & 0.000000 \times 10^{0} \\ 0.000000 \times 10^{0} & 0.000000 \times 10^{0} & 1.000000 \times 10^{2} & 0.000000 \times 10^{0} & 0.000000 \times 10^{0} & 0.000000 \times 10^{0} \\ 0.000000 \times 10^{0} & 0.000000 \times 10^{0} & 0.000000 \times 10^{0} & -4.905000 \times 10^{0} & 0.000000 \times 10^{0} & 0.000000 \times 10^{0} \\ 0.000000 \times 10^{0} & 0.000000 \times 10^{0} & 0.000000 \times 10^{0} & 0.000000 \times 10^{0} & -4.905000 \times 10^{0} & 0.000000 \times 10^{0} \\ 0.000000 \times 10^{0} & 0.000000 \times 10^{0} & 0.000000 \times 10^{0} & 0.000000 \times 10^{0} & 0.000000 \times 10^{0} & 0.000000 \times 10^{0} \end{bmatrix}$$

The velocity matrix is

$$\mathbf{V} = \begin{bmatrix} 0.000000\times10^{0} & 0.000000\times10^{0} & 0.000000\times10^{0} & 0.000000\times10^{0} & 0.000000\times10^{0} & 0.000000\times10^{0} \\ 0.000000\times10^{0} & 0.000000\times10^{0} & 0.000000\times10^{0} & 0.000000\times10^{0} & 0.000000\times10^{0} & 0.000000\times10^{0} \\ 0.000000\times10^{0} & 0.000000\times10^{0} & 0.000000\times10^{0} & 0.000000\times10^{0} & 0.000000\times10^{0} & 0.000000\times10^{0} \\ 0.000000\times10^{0} & 0.000000\times10^{0} & 0.000000\times10^{0} & 0.000000\times10^{0} & 0.000000\times10^{0} & 0.000000\times10^{0} \\ 0.000000\times10^{0} & 0.000000\times10^{0} & 0.000000\times10^{0} & 0.000000\times10^{0} & 0.000000\times10^{0} & 0.000000\times10^{0} \\ 0.000000\times10^{0} & 0.000000\times10^{0} & 0.000000\times10^{0} & 0.000000\times10^{0} & 0.000000\times10^{0} & 0.000000\times10^{0} \end{bmatrix}$$

The input force matrix is

$$\mathbf{F} = \begin{bmatrix} 0.000000\times10^{0} \\ 0.000000\times10^{0} \\ 1.000000\times10^{0} \\ 0.000000\times10^{0} \\ -5.000000\times10^{-2} \\ 0.000000\times10^{0} \end{bmatrix}$$

The input force rate matrix is

$$\mathbf{G} = \begin{bmatrix} 0.000000\times10^{0} \\ 0.000000\times10^{0} \\ 0.000000\times10^{0} \\ 0.000000\times10^{0} \\ 0.000000\times10^{0} \\ 0.000000\times10^{0} \end{bmatrix}$$

Row	Column	Value	Row	Column	Value
2	1	-1.00000000×10^{0}	7	7	-1.00000000×10^{0}
1	2	1.00000000×10^{0}	6	8	1.00000000×10^{0}
3	4	1.00000000×10^{0}	8	10	1.00000000×10^{0}
4	5	1.00000000×10^{0}	9	11	1.00000000×10^{0}
5	6	1.00000000×10^{0}	10	12	1.00000000×10^{0}

The full state space equations:

$$\begin{bmatrix} \mathrm{E} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{Bmatrix} \dot{\boldsymbol{x}} \\ \boldsymbol{y} \end{Bmatrix} = \begin{bmatrix} \mathrm{A} & \mathrm{B} \\ \mathrm{C} & \mathrm{D} \end{bmatrix} \begin{Bmatrix} \boldsymbol{x} \\ \boldsymbol{u} \end{Bmatrix}$$

$$\begin{bmatrix} \mathrm{A} & \mathrm{B} \\ \mathrm{C} & \mathrm{D} \end{bmatrix} = \left[\begin{array}{ccc|c} -2.00000000\times10^{0} & 1.00000000\times10^{2} & -1.00000000\times10^{0} & 0.00000000\times10^{0} \\ -1.00000000\times10^{0} & 0.00000000\times10^{0} & 0.00000000\times10^{0} & 0.00000000\times10^{0} \\ 0.00000000\times10^{0} & 0.00000000\times10^{0} & -1.00000000\times10^{0} & 1.00000000\times10^{0} \\ \hline 0.00000000\times10^{0} & 1.00000000\times10^{0} & 0.00000000\times10^{0} & 0.00000000\times10^{0} \end{array}\right]$$

$$\mathrm{E} = \begin{bmatrix} 1.00000000\times10^{0} & 0.00000000\times10^{0} & 0.00000000\times10^{0} \\ 0.00000000\times10^{0} & 1.00000000\times10^{0} & 0.00000000\times10^{0} \\ 0.00000000\times10^{0} & 0.00000000\times10^{0} & 0.00000000\times10^{0} \end{bmatrix}$$

The reduced state space equations:

$$\begin{Bmatrix} \dot{\boldsymbol{x}} \\ \boldsymbol{y} \end{Bmatrix} = \begin{bmatrix} \mathrm{A} & \mathrm{B} \\ \mathrm{C} & \mathrm{D} \end{bmatrix} \begin{Bmatrix} \boldsymbol{x} \\ \boldsymbol{u} \end{Bmatrix}$$

$$\begin{bmatrix} \mathrm{A} & \mathrm{B} \\ \mathrm{C} & \mathrm{D} \end{bmatrix} = \left[\begin{array}{cc|c} -2.00000000\times10^{0} & 1.25000000\times10^{1} & -5.00000000\times10^{-1} \\ -8.00000000\times10^{0} & 0.00000000\times10^{0} & 0.00000000\times10^{0} \\ \hline 0.00000000\times10^{0} & 2.50000000\times10^{-1} & 0.00000000\times10^{0} \end{array}\right]$$