Segment Tree

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The Problem

Range Sum Query

You are given an integer array a. Answer q queries of the following form:

- $sum(i, j)$: returns $\sum_{k=i}^{j} a[k]$

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$$sum(3, 6) = -5 + 1 + 7 + (-6) = -3$$

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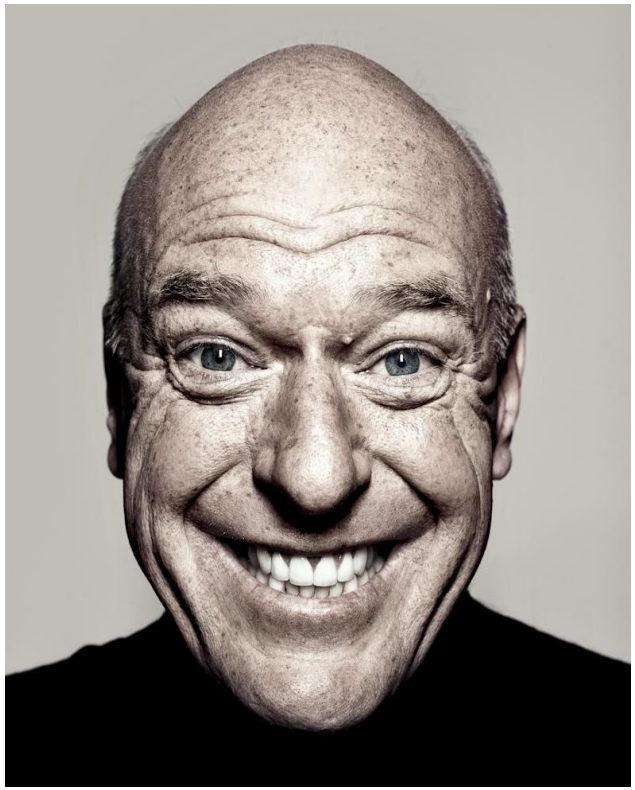
$$sum(3,6) = sum(0,6) - sum(0,2) = 3 - 6 = -3$$

That was easy!



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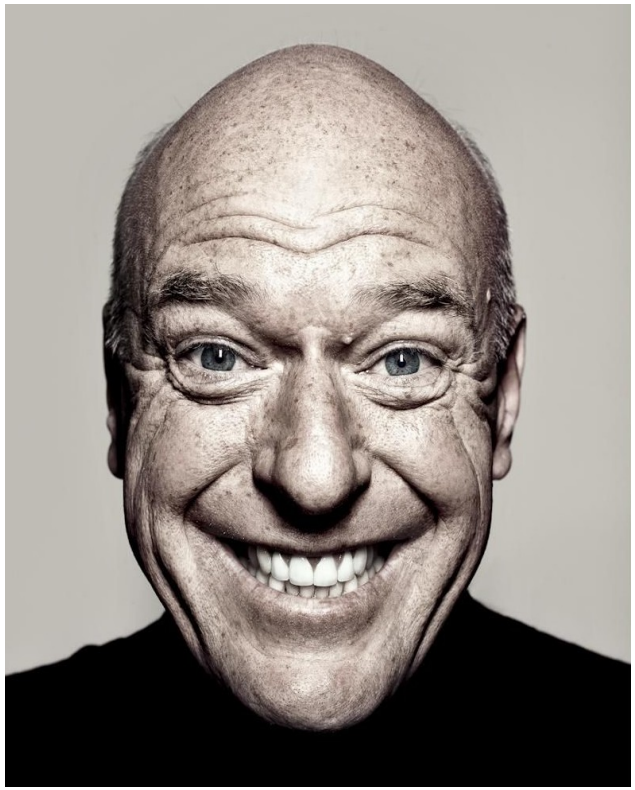
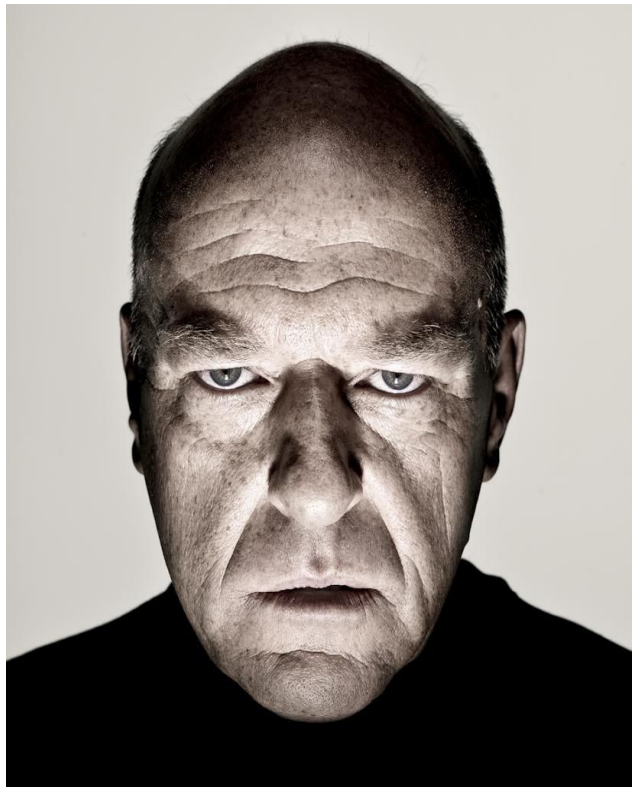


Let's see a hard version



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Will Prefix Sum Work Well Now?

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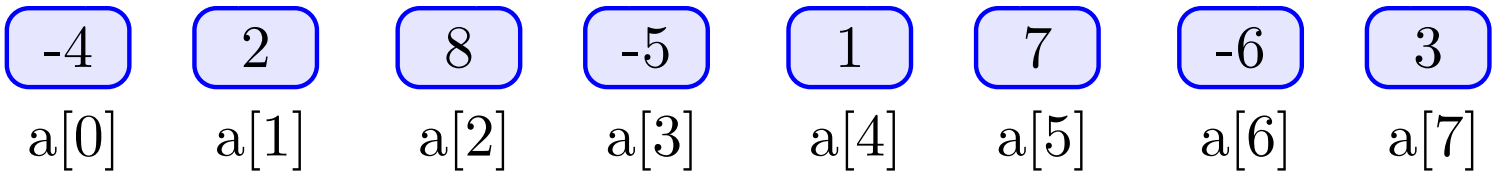
Will Prefix Sum Work Well Now?

No, update makes it $O(n)$. Look for something better!

Construction of Segment Tree



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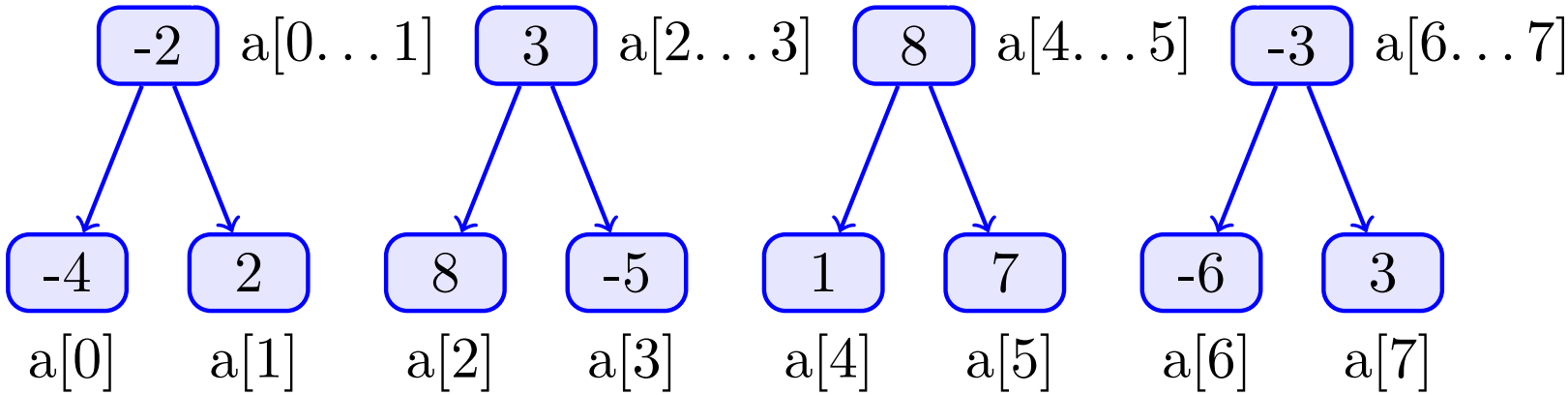

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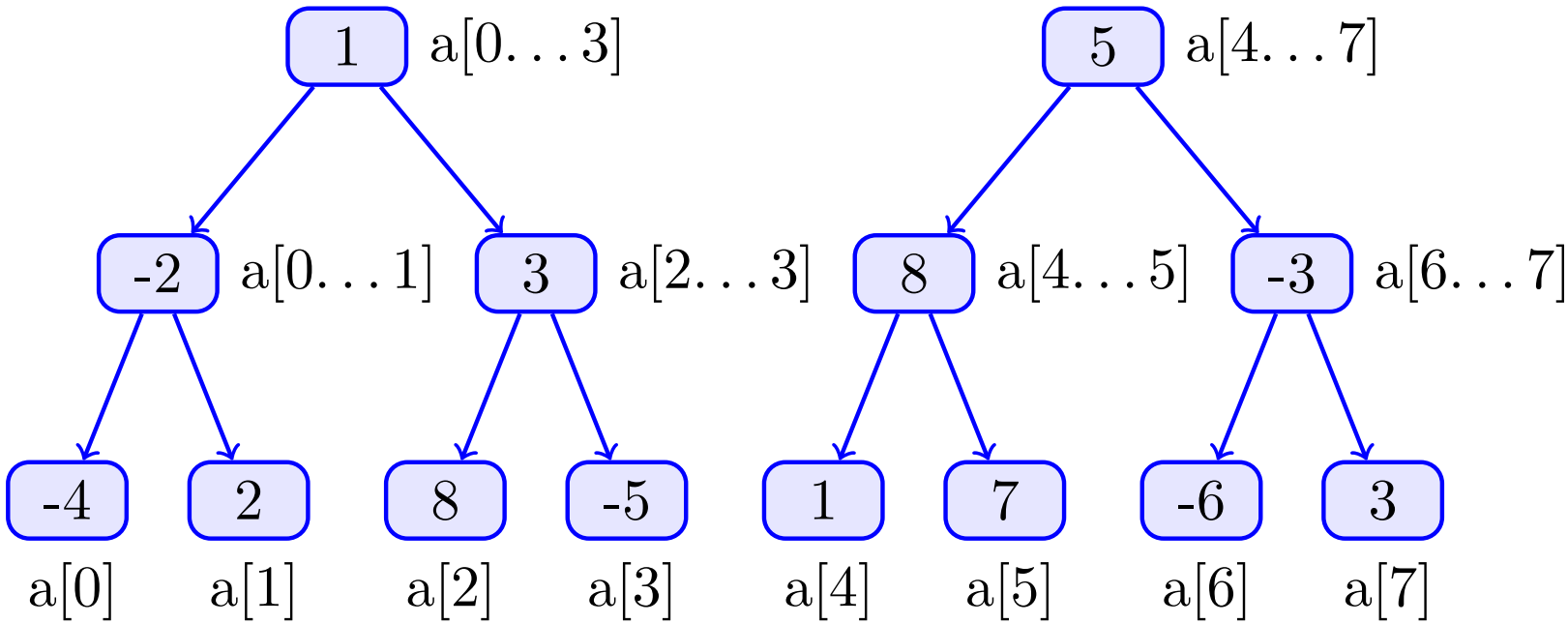

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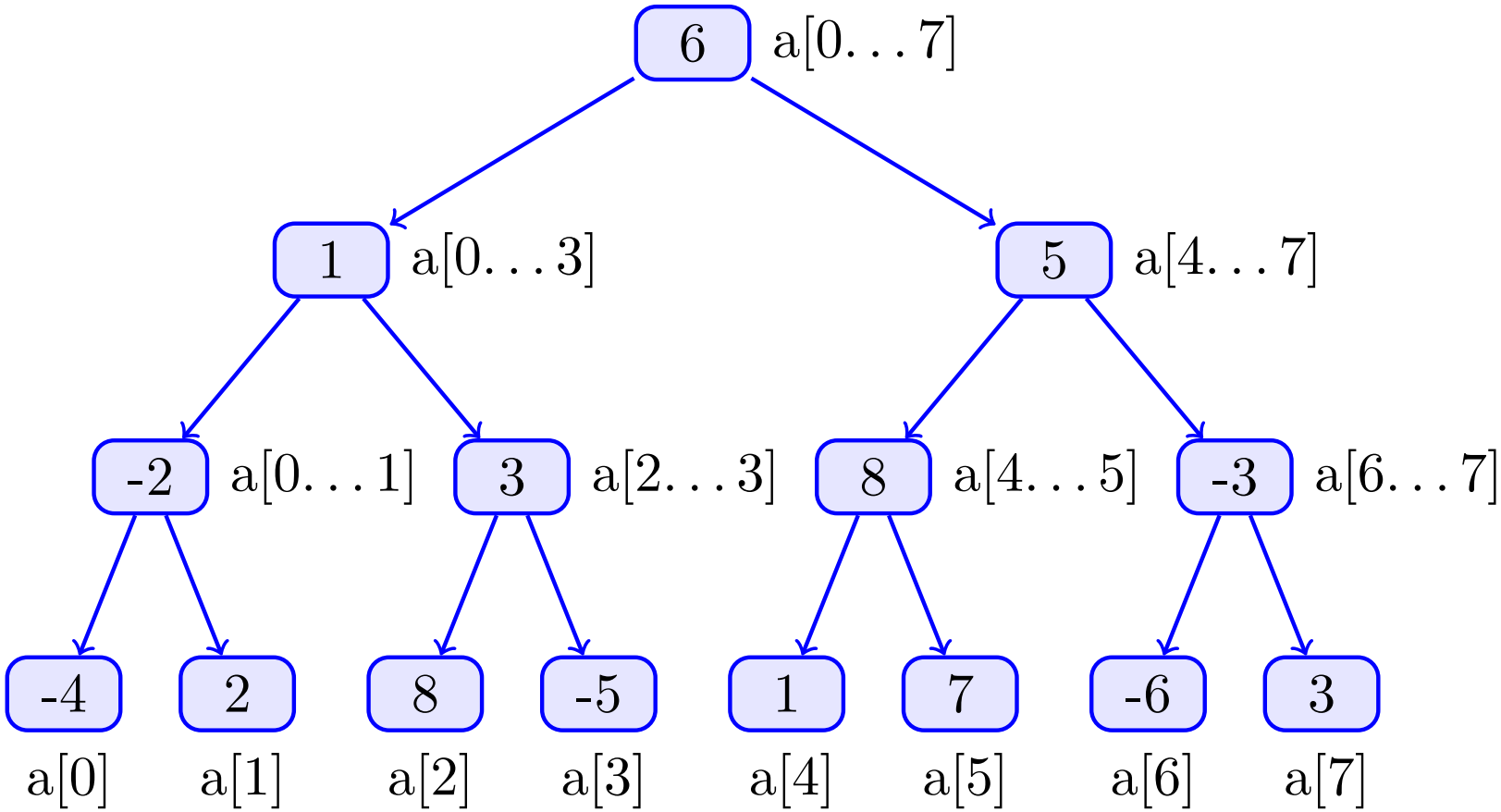
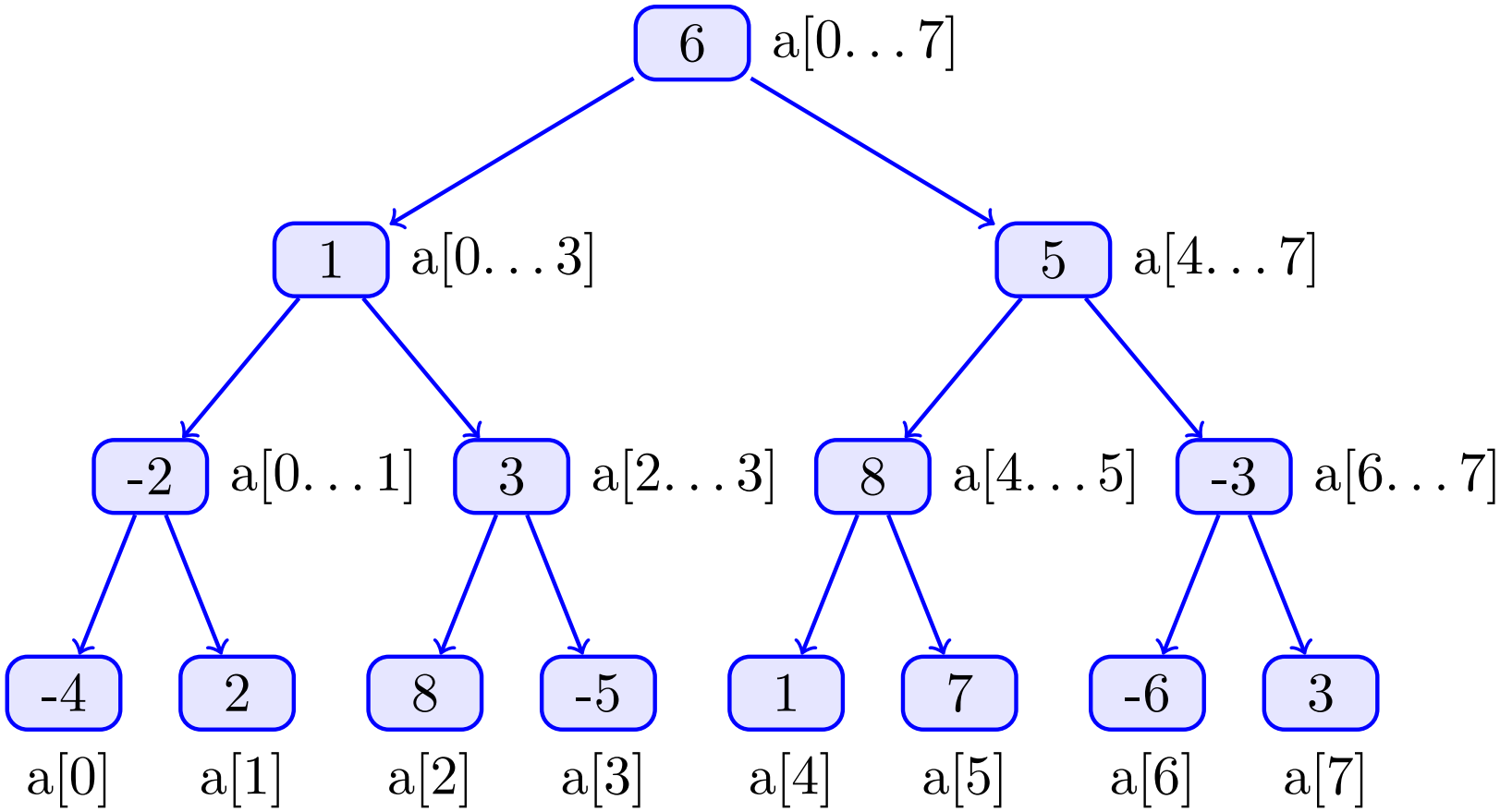


Build Code

```
void build(int arr[], int idx, int l, int r) {
    if (l == r) tree[idx] = arr[l];
    else {
        int mid = (l + r) / 2;
        build(arr, idx * 2, l, mid);
        build(arr, idx * 2 + 1, mid + 1, r);
        tree[idx] = tree[idx * 2] + tree[idx * 2 + 1];
    }
}
```

Update Segment Tree



$$update(2, -7) : a[2] = -7$$

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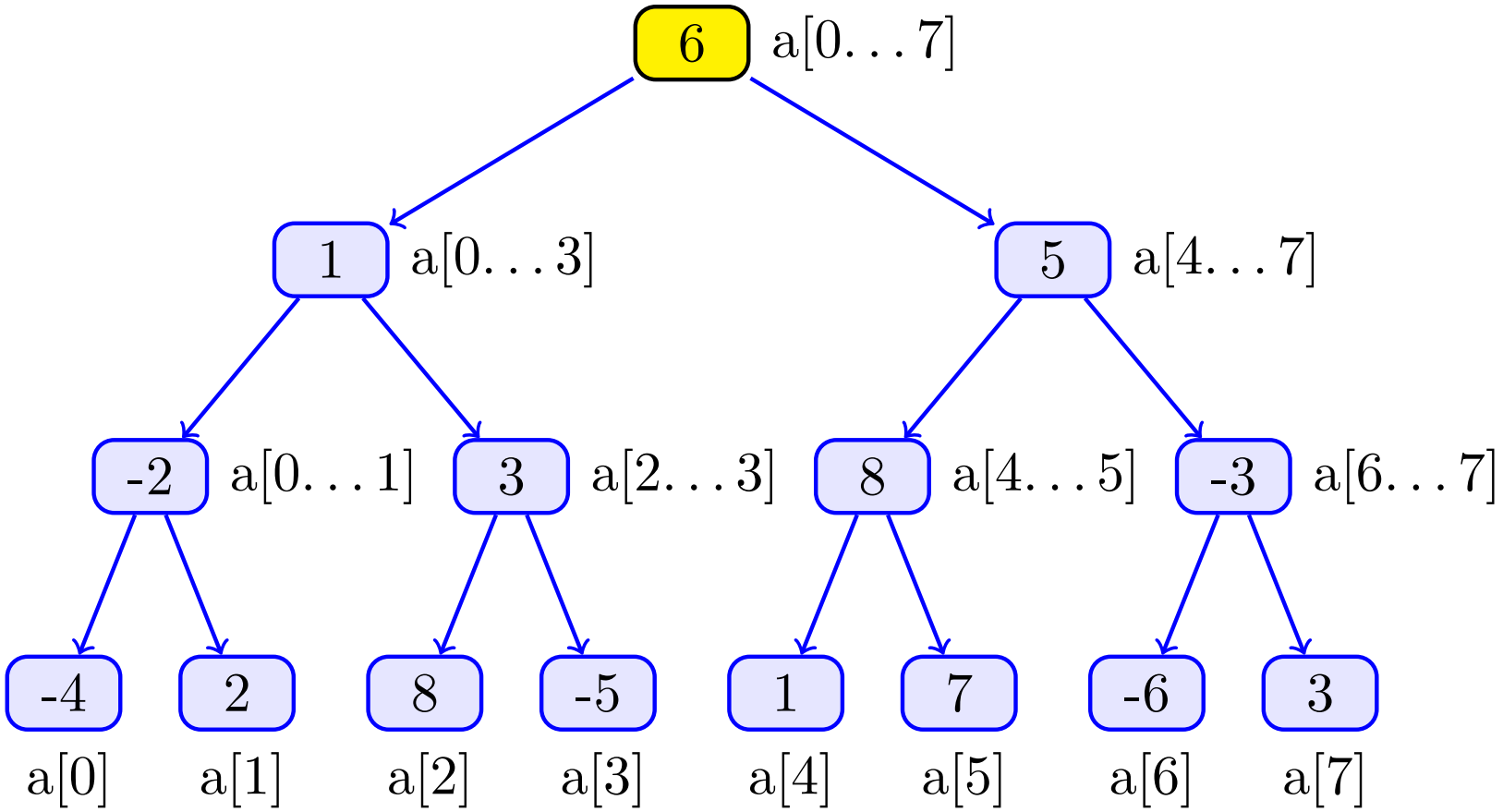

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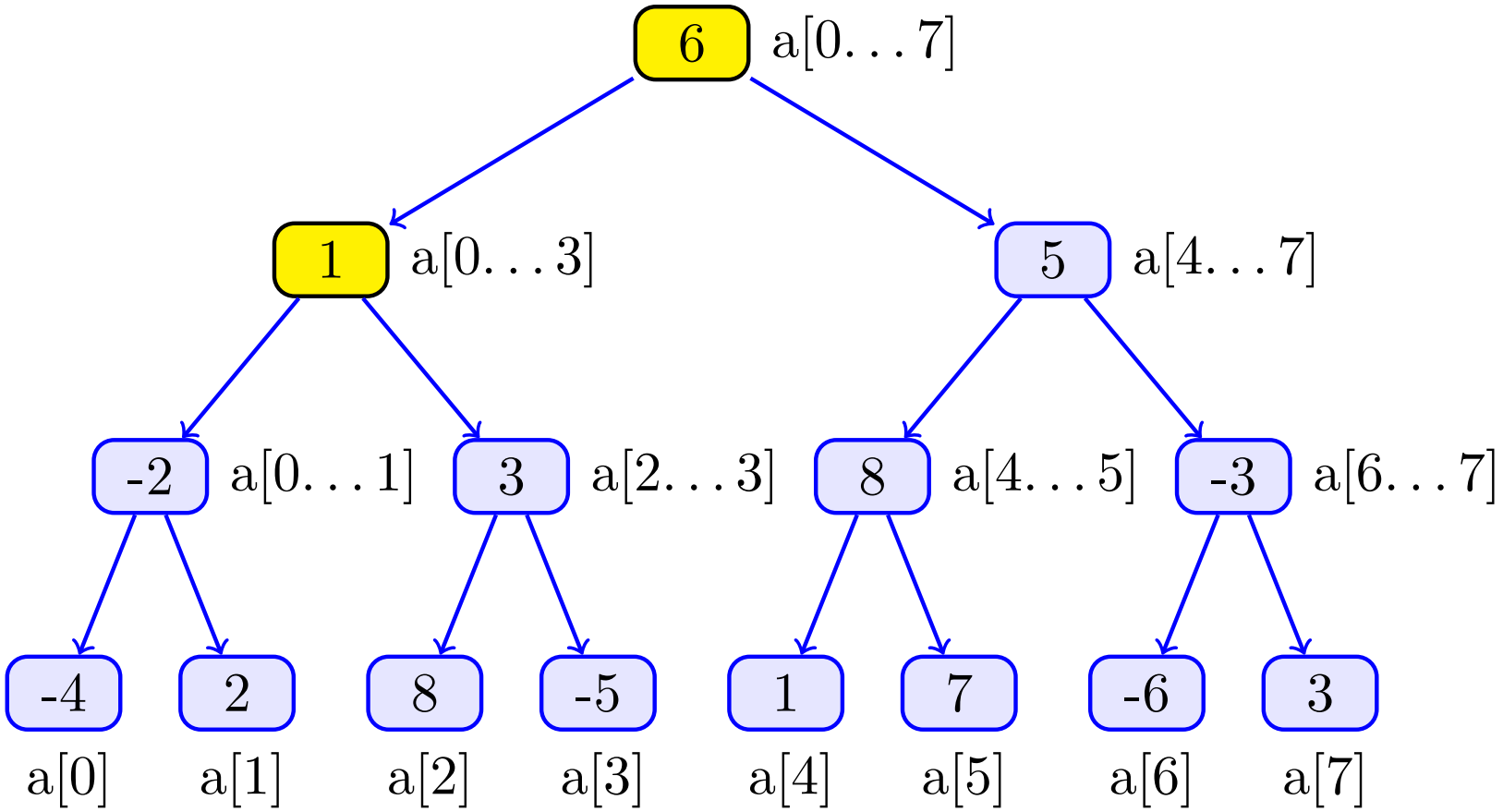

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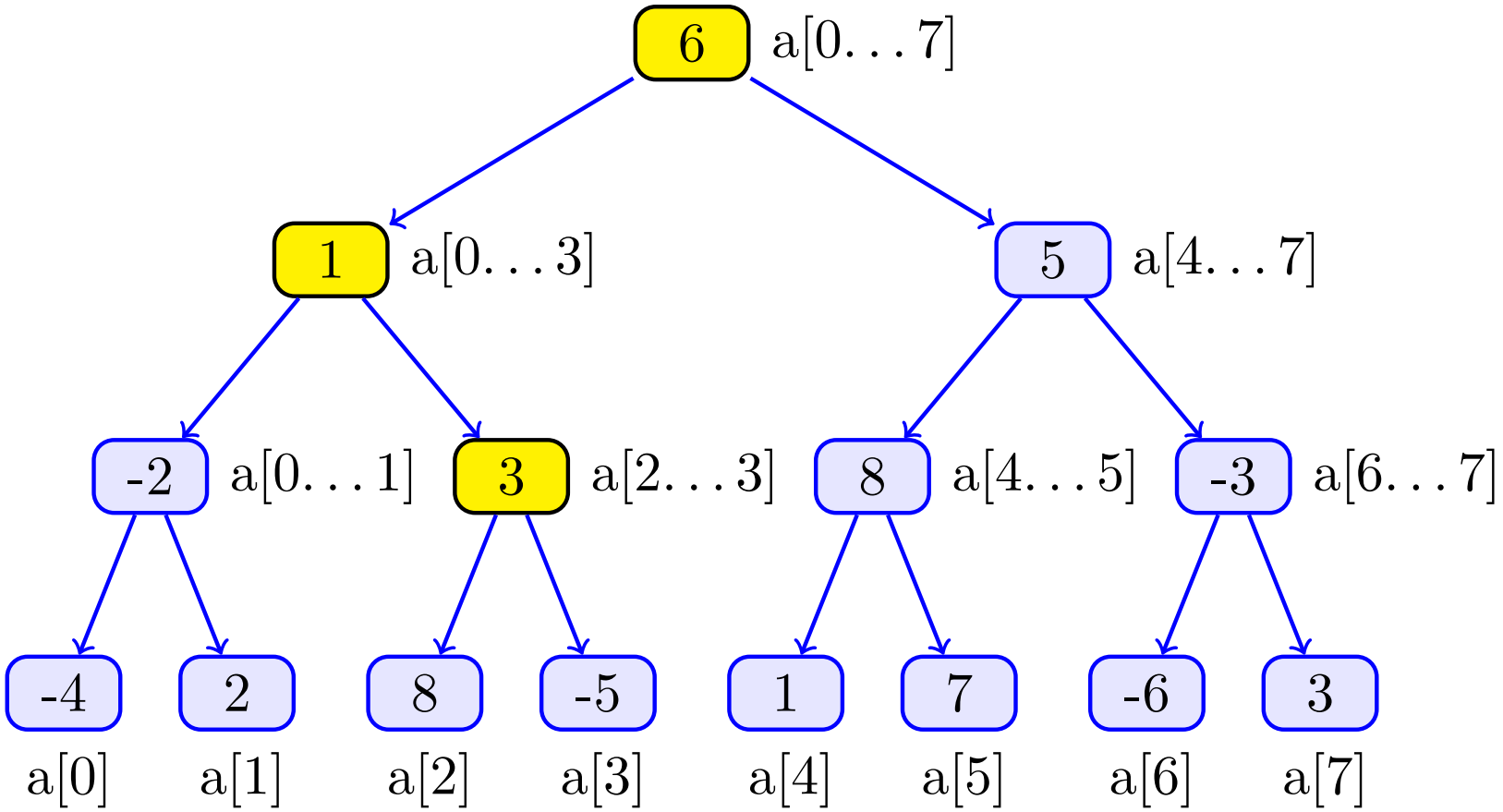

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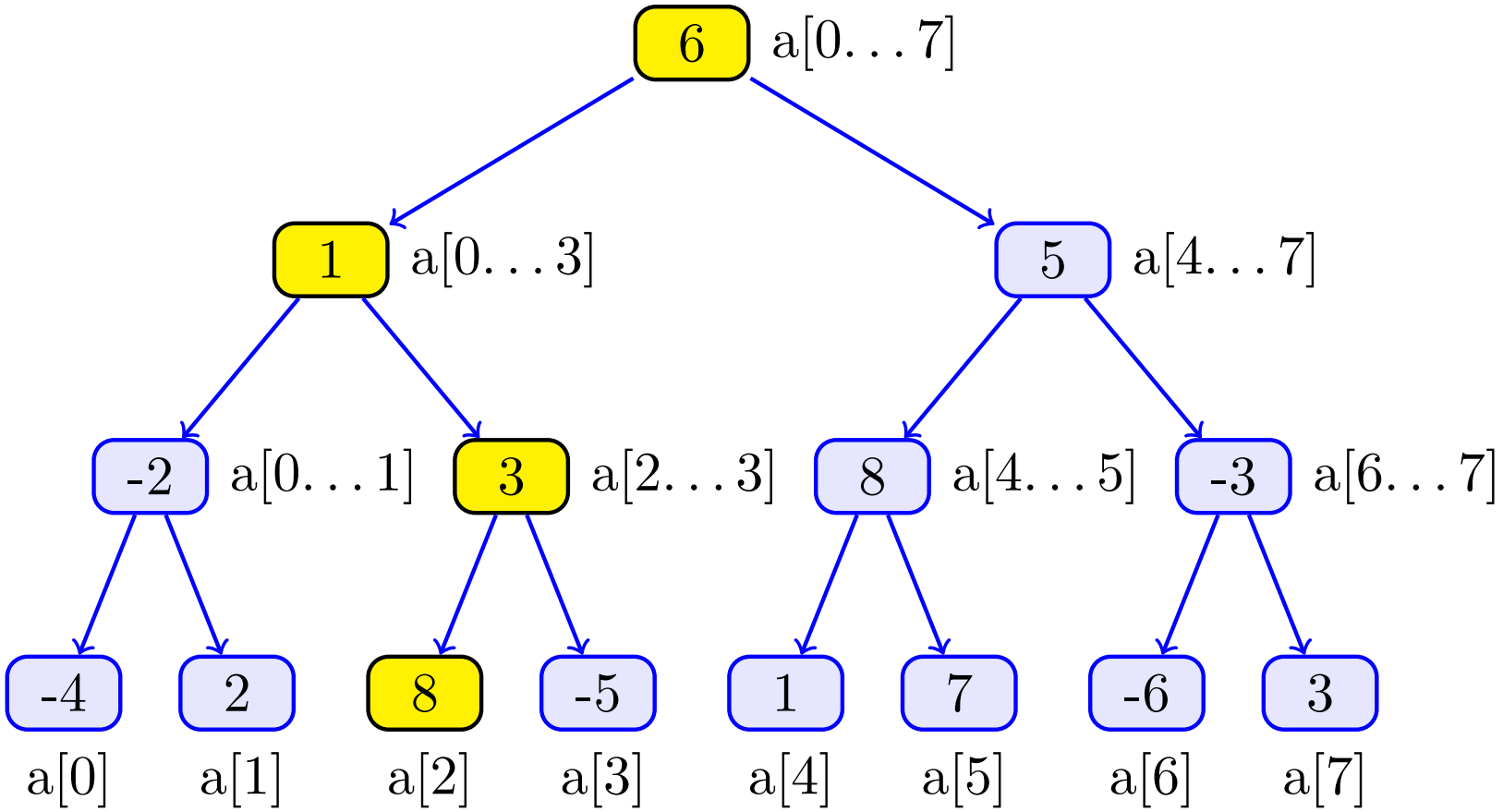

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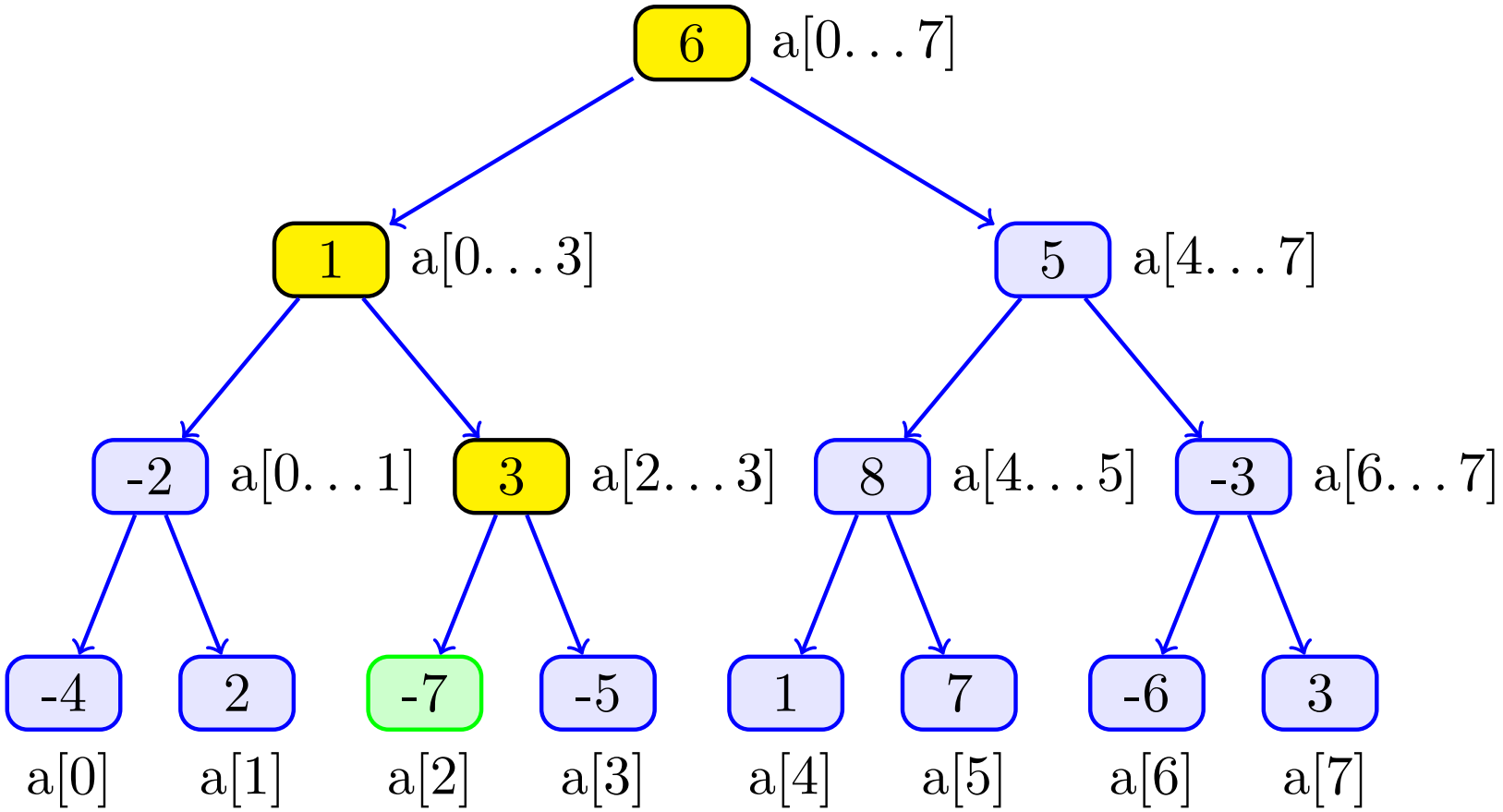

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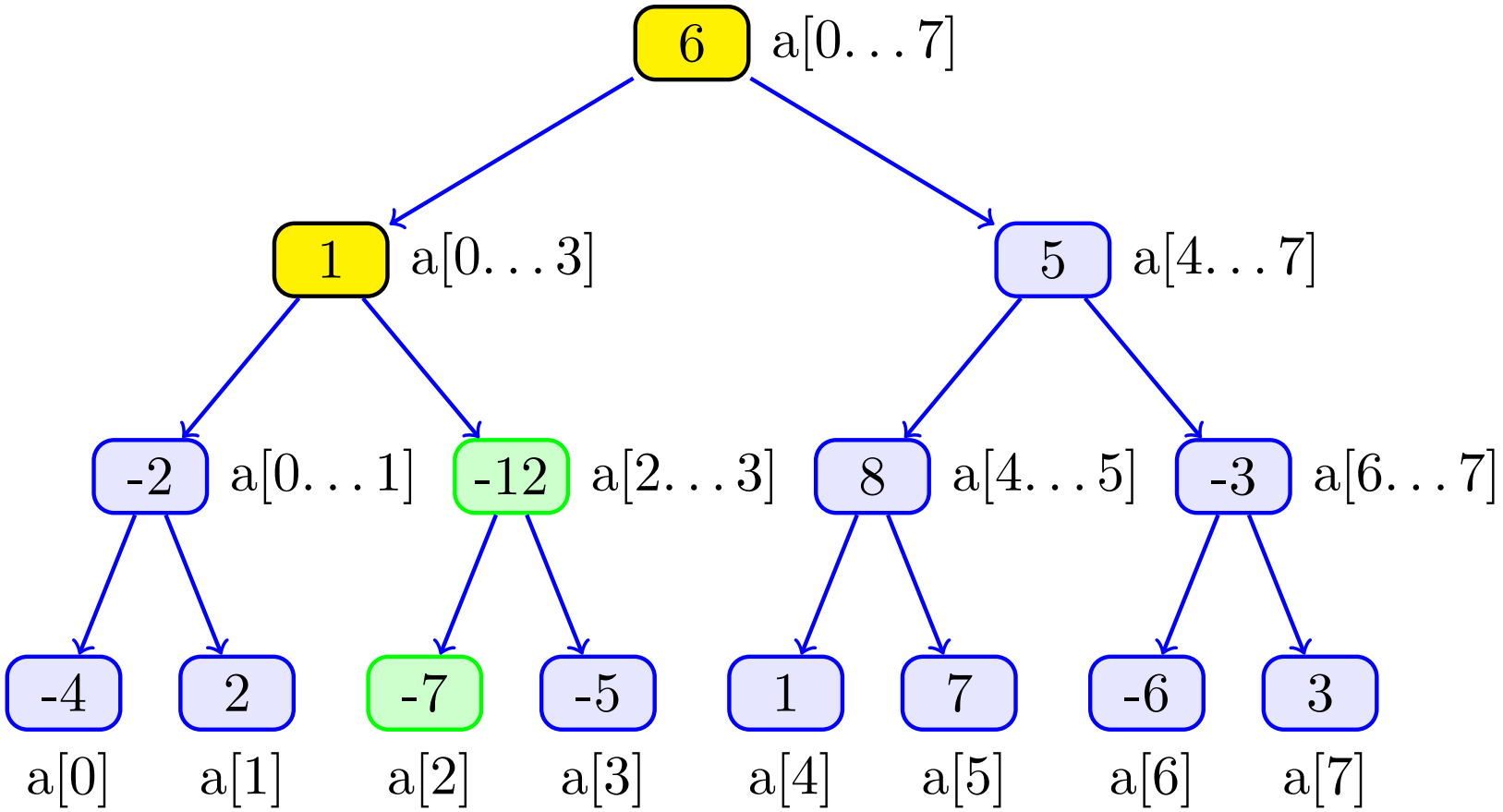

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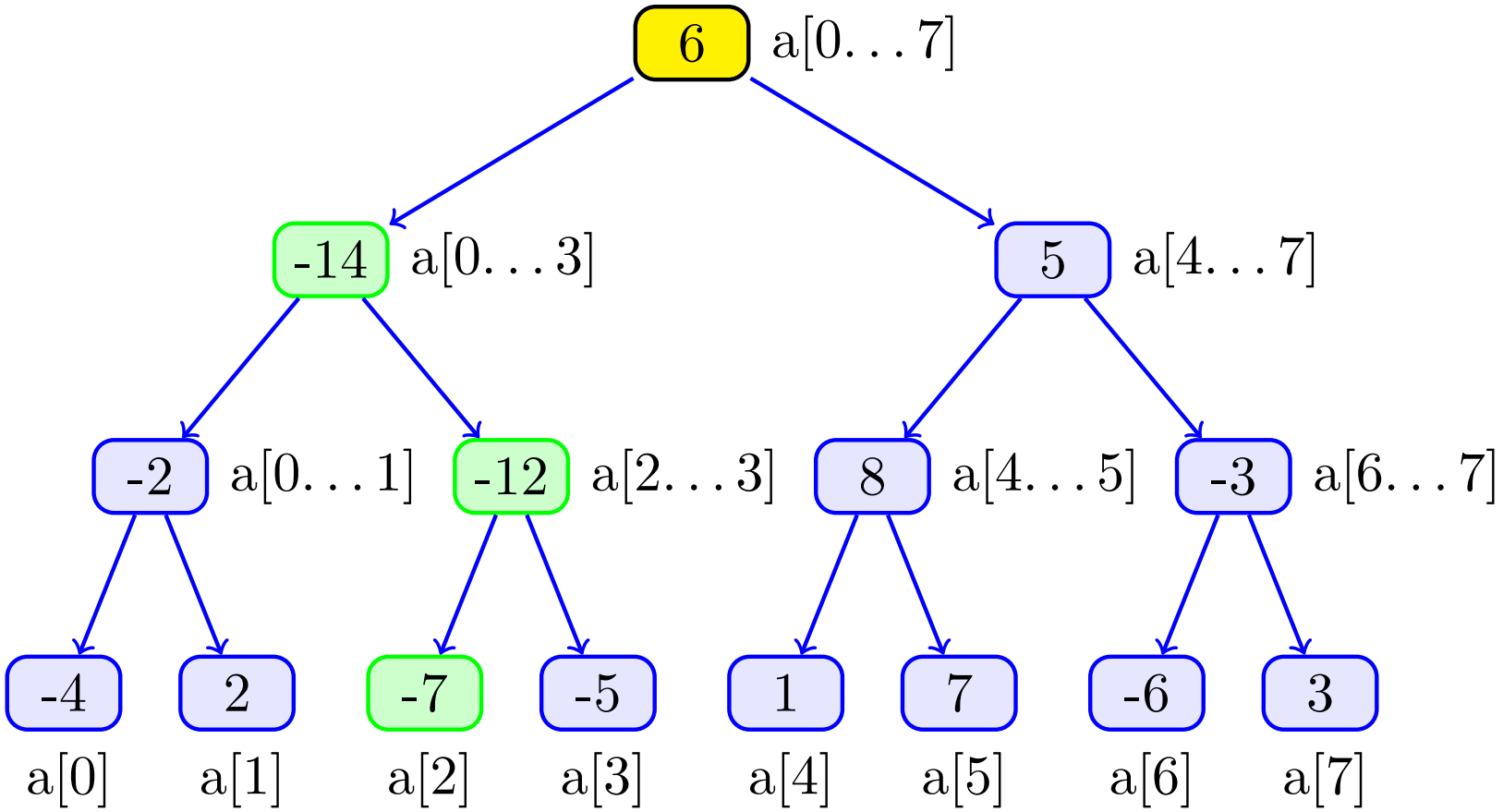

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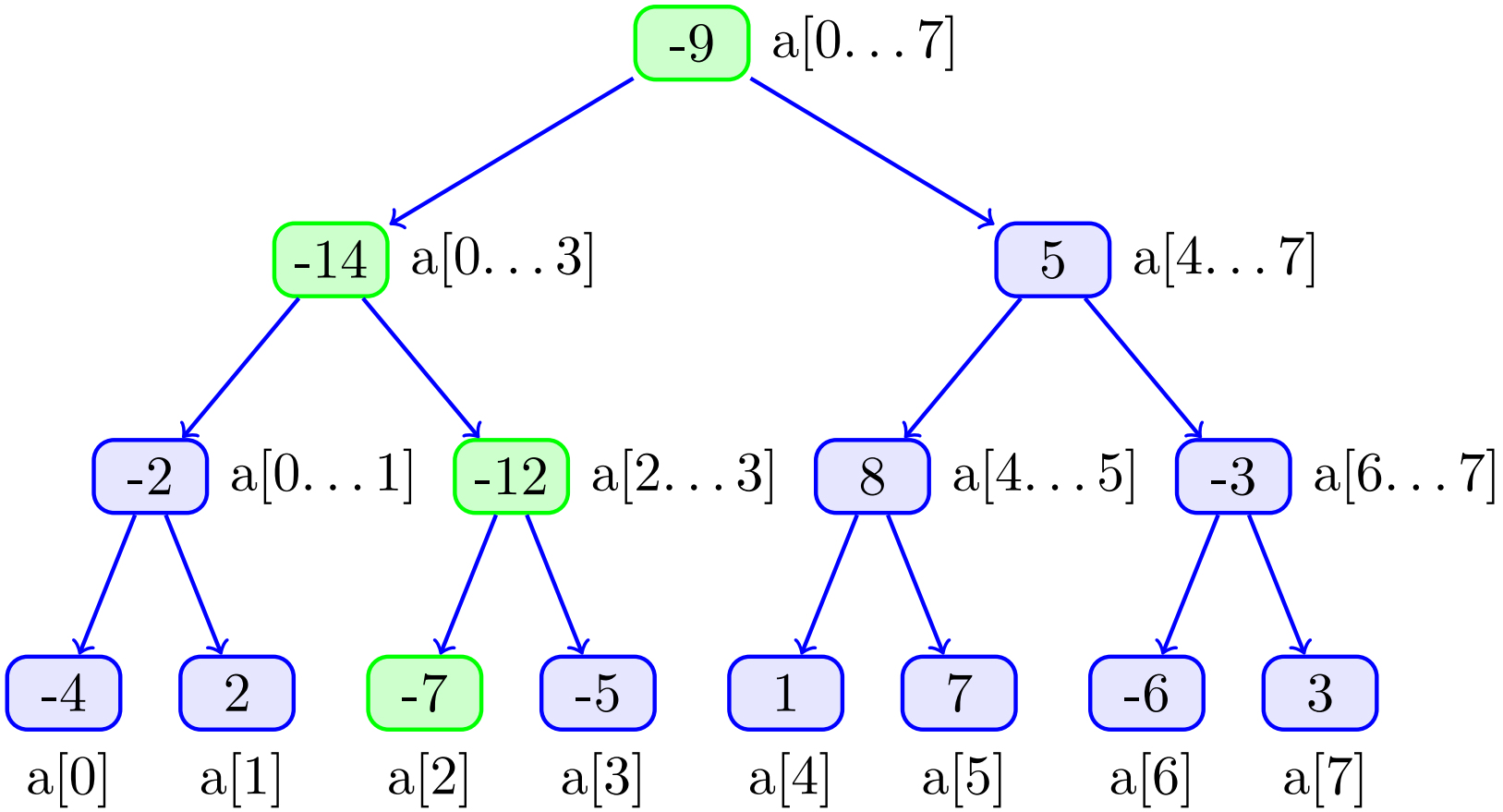

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Update Code

```
void update(int idx, int l, int r, int pos, int new_val) {
    if (l == r)
        tree[idx] = new_val;
    else {
        int mid = (l + r) / 2;
        if (pos <= mid)
            update(idx * 2, l, mid, pos, new_val);
        else
            update(idx * 2 + 1, mid + 1, r, pos, new_val);
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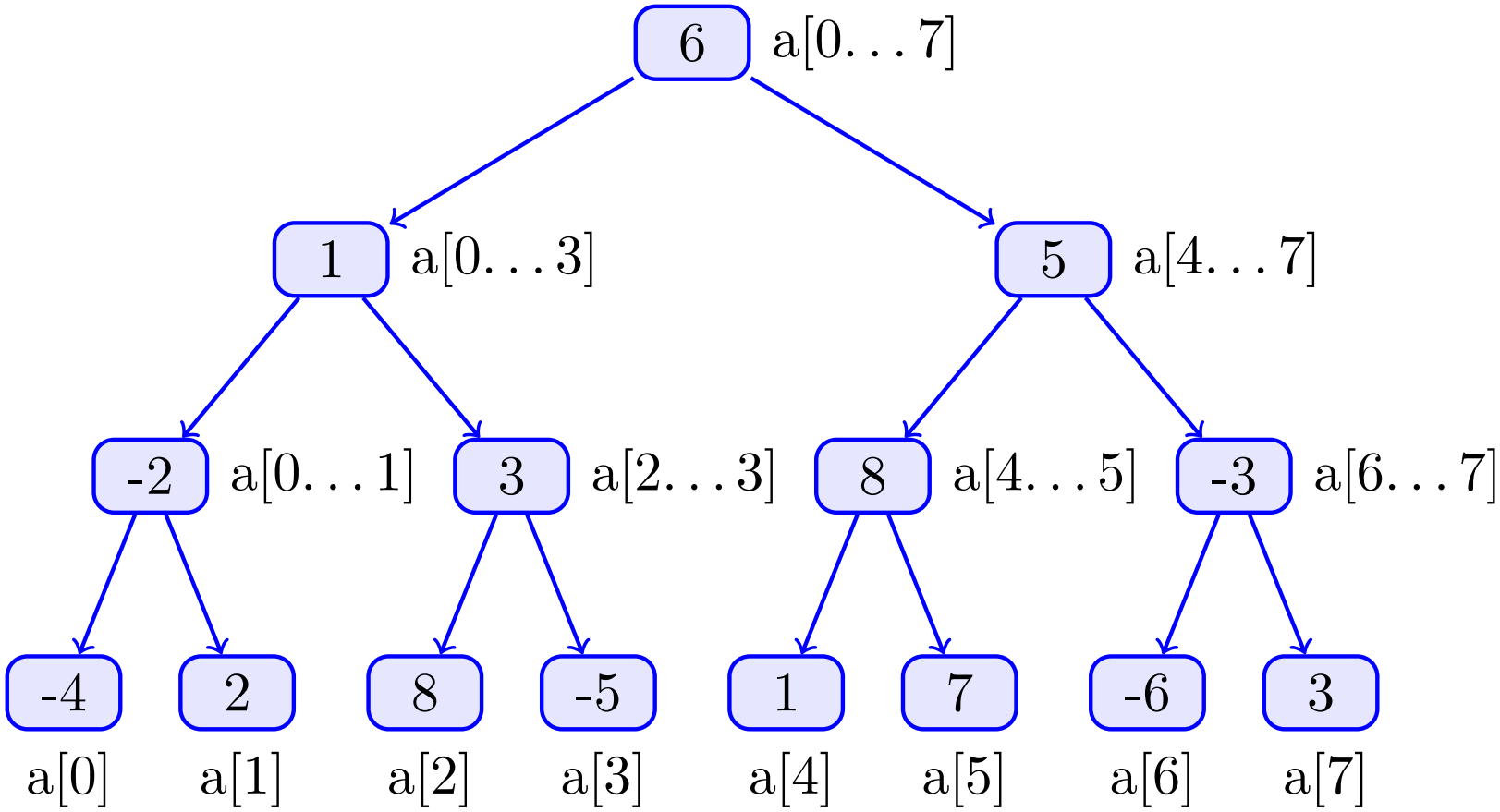
Answering Query From Segment Tree



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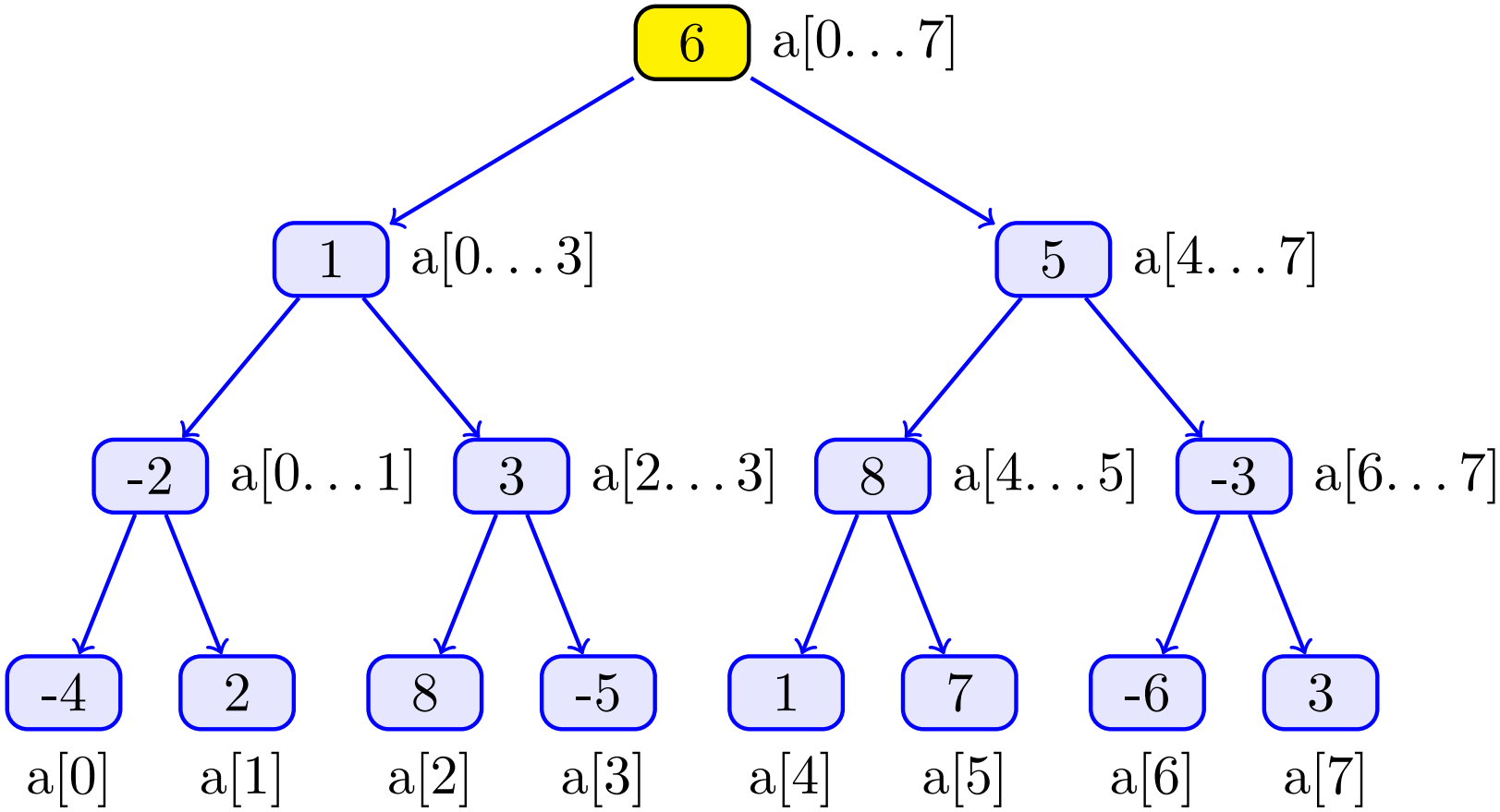

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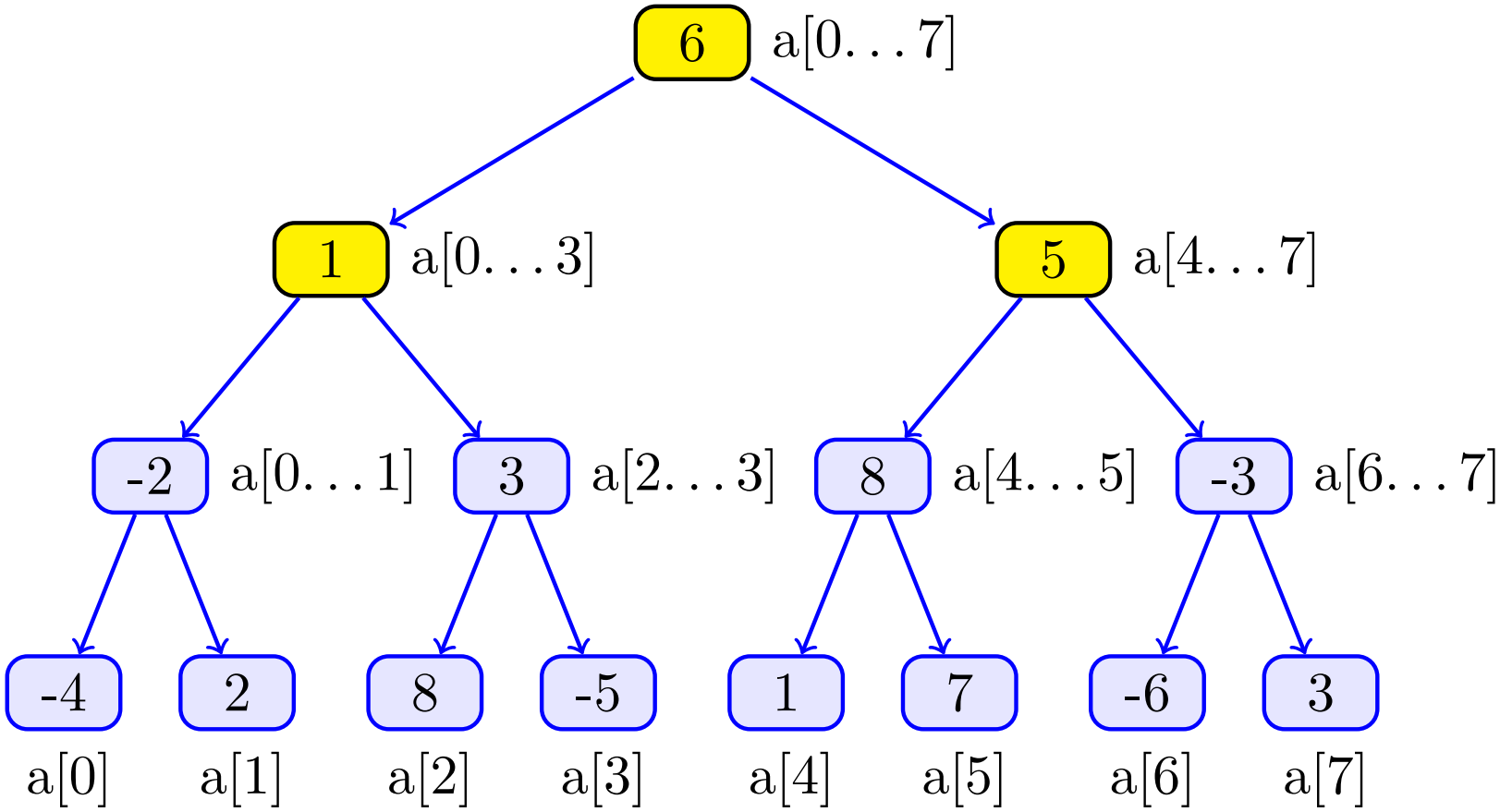

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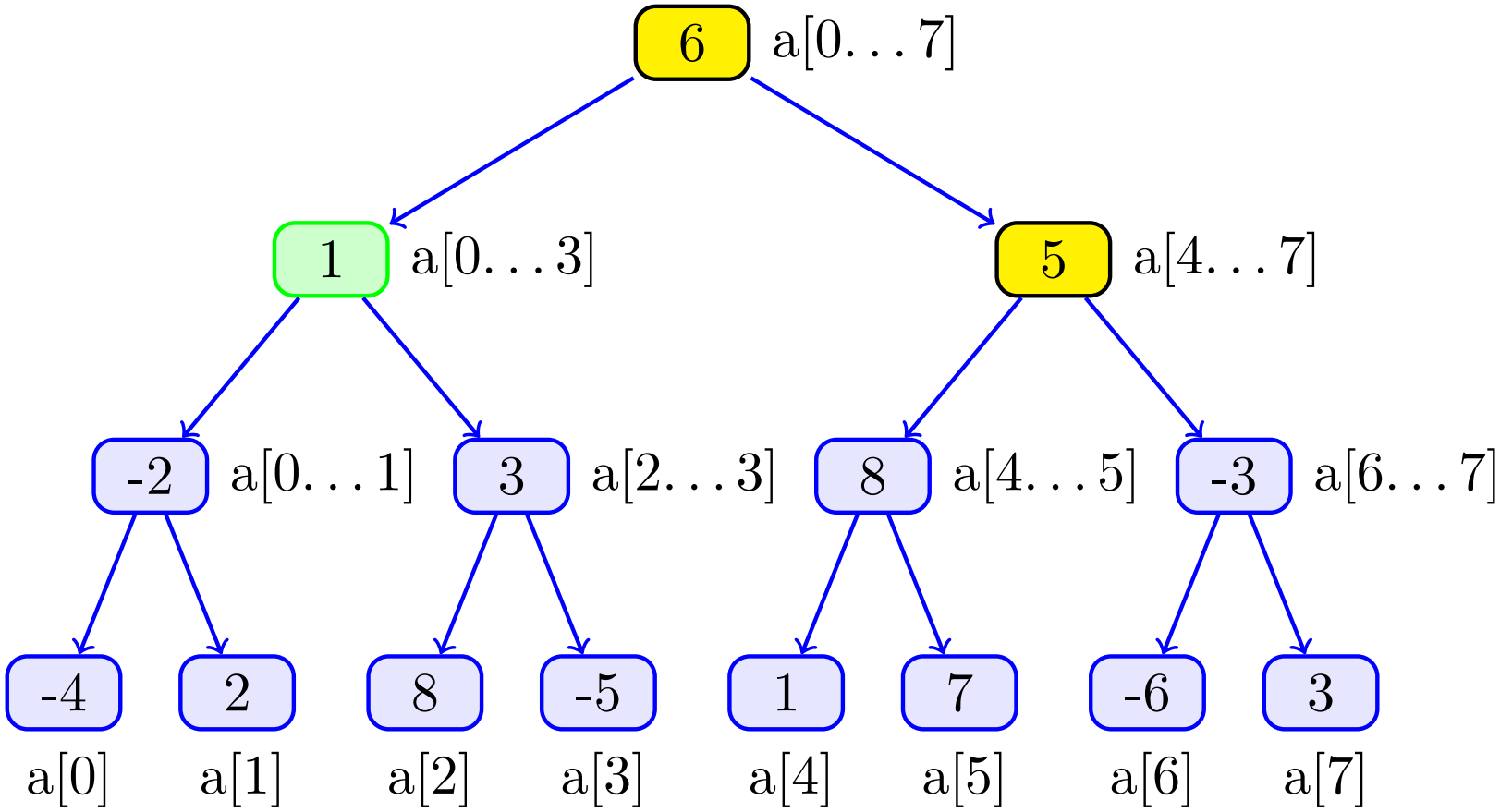

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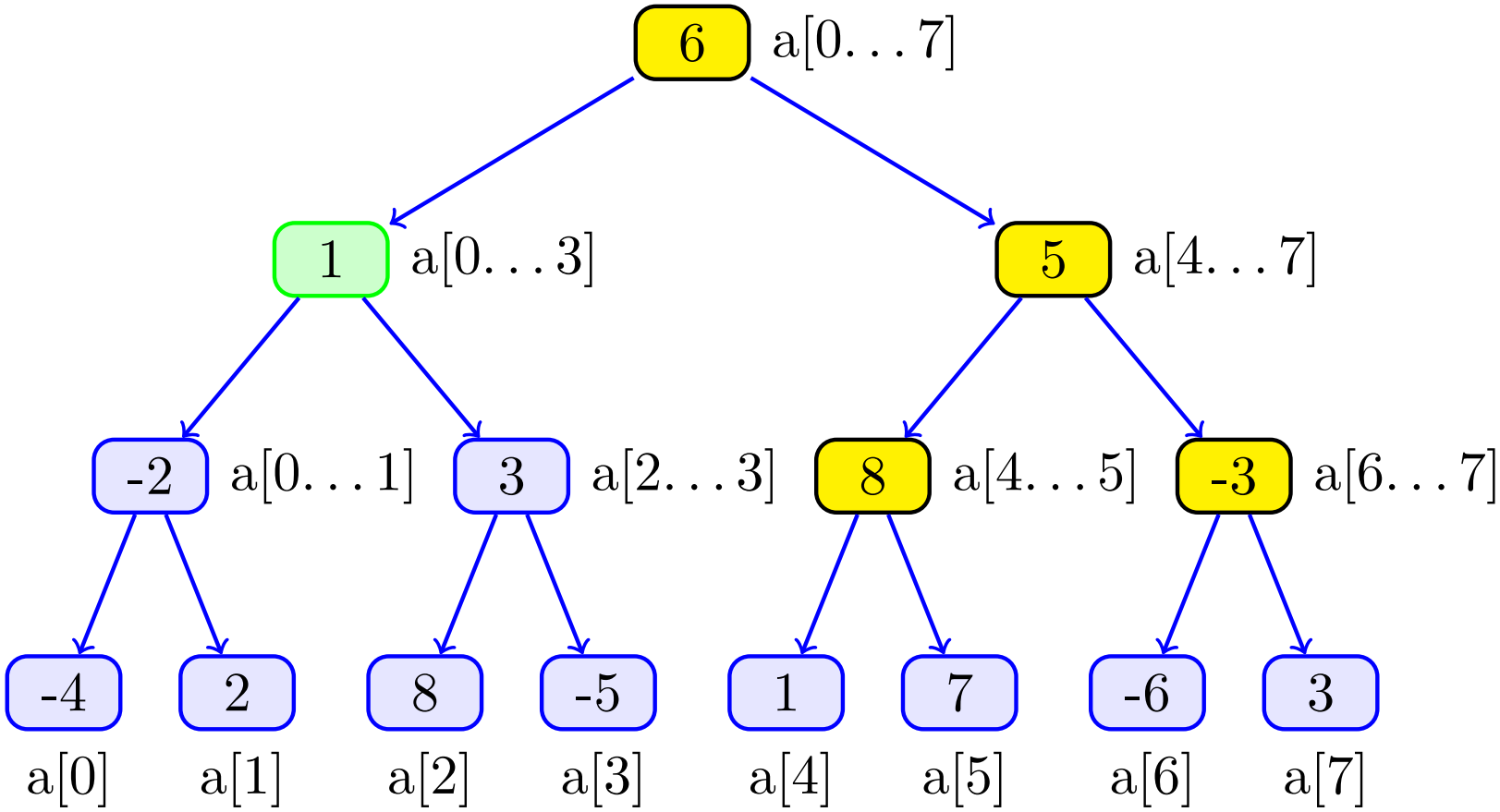

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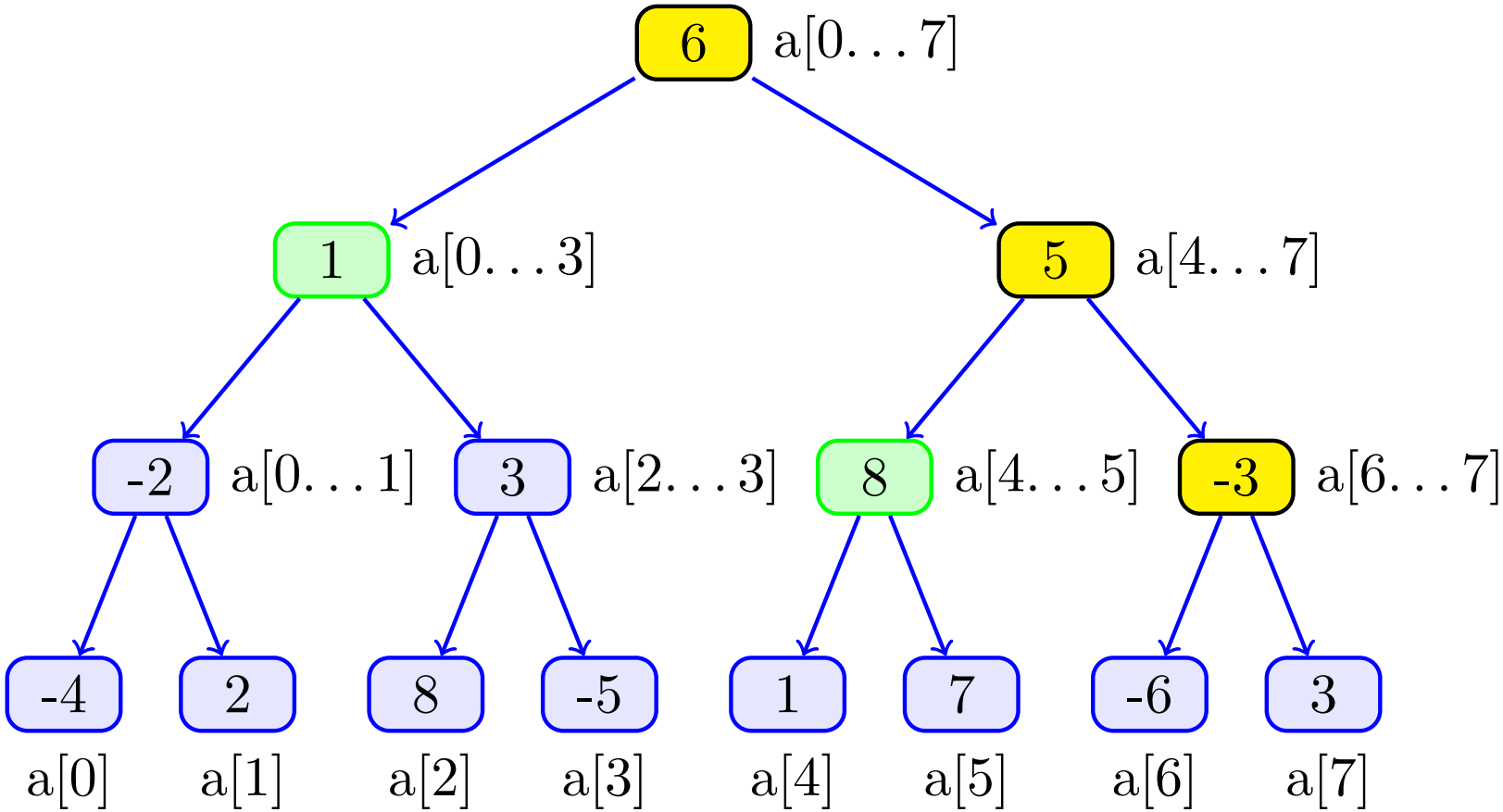

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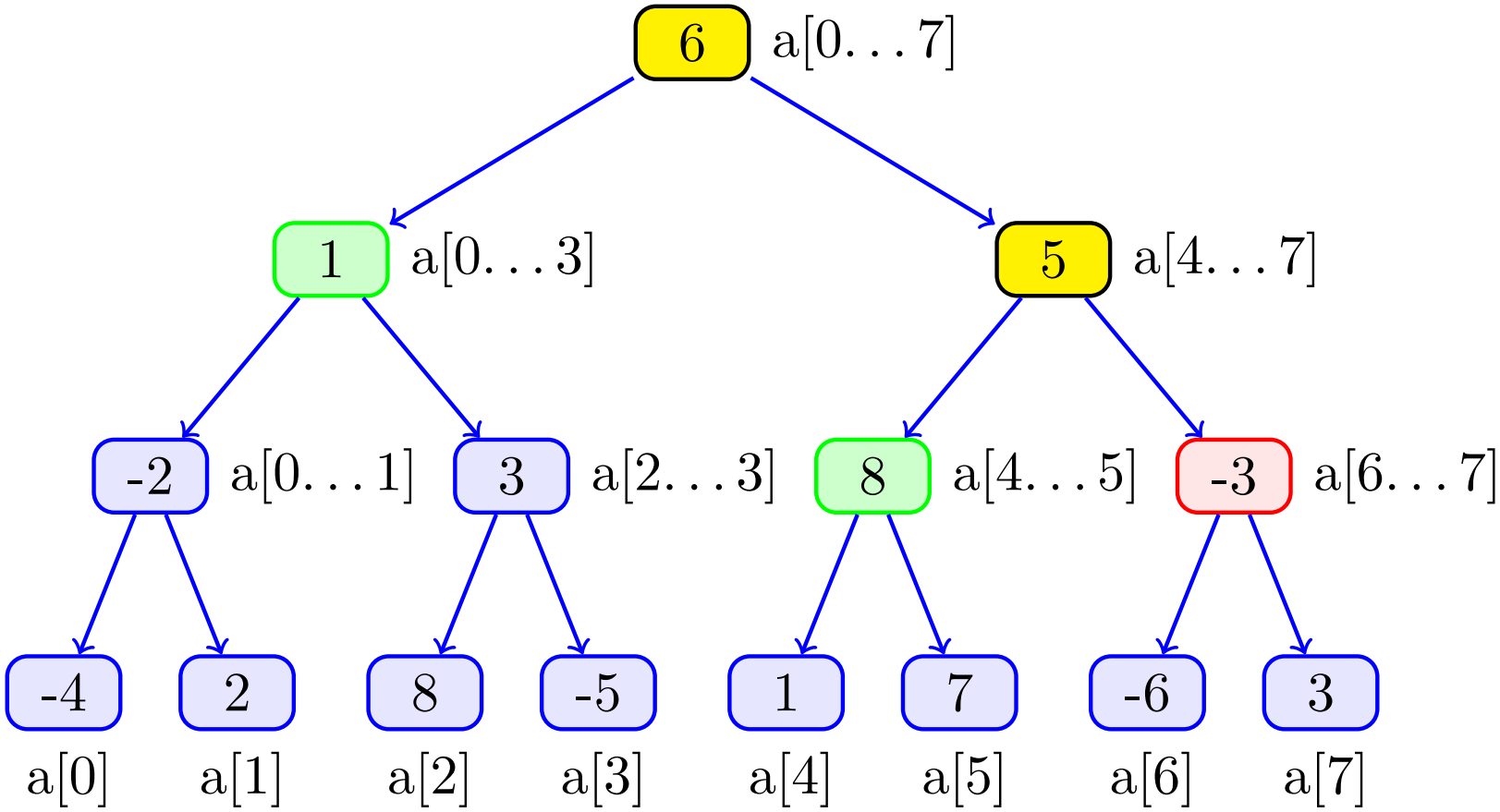

$$sum(0, 5) = 1 + 8 = 9$$

Query Code

```
int query(int idx, int l, int r, int ql, int qr) {
    if (ql > qr) return 0;
    if (ql == l && qr == r) return tree[idx];
    int mid = (l + r) / 2;
    return sum(idx * 2, l, mid, ql, min(qr, mid))
            + sum(idx * 2 + 1, mid + 1, r, max(ql, mid + 1), qr);
}
```

Complexity Comparison

Data Structure	Preprocessing	Query	Update
Segment Tree	$O(n)$	$O(logn)$	$O(logn)$
Prefix Sum Array	$O(n)$	$O(1)$	$O(n)$
Sqrt Decomposition	$O(n)$	$O(\sqrt{n})$	$O(\sqrt{n})$

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- Range Update : Lazy Propagation

Thanks For Your Time and Attention

Keep Using Segment Trees...