SDP + Cones Mathematical Optimization

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Second Order Conic Programming

SOCP

$$\min_{x \in \mathbb{R}^n} c^\top x$$
s.t. $Ax = b$

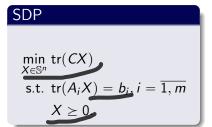
$$\|G_i x - h_i\|_2 \leqslant e_i^\top x + f_i, i = \overline{1, M}$$

$$(or \|G_i x - h_i\|_2 \leqslant t).$$

where $A \in \mathbb{R}^{m \times n}$, $G_i \in \mathbb{R}^{k_i \times n}$, $i = \overline{1, M}$.



Semi-Defined Programming



Spectral Radius Minimisation

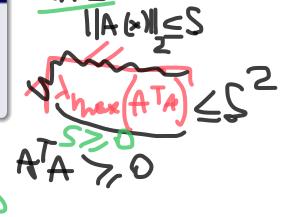
Statement

Consider

$$A(x) := A_0 + \sum_{i=1}^{n} A_i$$
, with

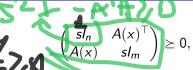
$$A_j \in \mathbb{R}^{m \times n}, j = 1,$$

 $\min_{x\in\mathbb{R}^n}\|A(x)\|_2.$



Schur complement

SOCP LSPY



Shur Complement

For a block matrix $M \in \mathbb{R}^{(p+q)\times(p+q)}$:

$$(M/D) = A - BD^{-1}C \in \mathbb{R}^{p \times p}$$





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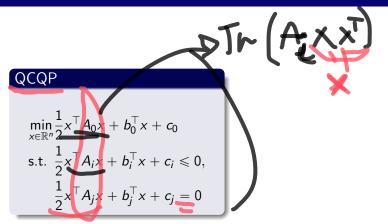
Final SRM statement



Final SRM statement

$$\min_{\mathbf{x} \in \mathbb{R}^n, t \in \mathbb{R}} t$$
s.t.
$$\left(\begin{array}{cc} s I_n & A(\mathbf{x})^\top \\ A(\mathbf{x}) & s I_m \end{array} \right) \geq 0.$$

Non-Convex QCQP to SDP



Non-Convex QCQP to SDP

QCQP

$$\min_{x \in \mathbb{R}^{n}} \frac{1}{2} x^{\top} A_{0} x + b_{0}^{\top} x + c_{0}$$
s.t.
$$\frac{1}{2} x^{\top} A_{i} x + b_{i}^{\top} x + c_{i} \leq 0,$$

$$\frac{1}{2} x^{\top} A_{j} x + b_{j}^{\top} x + c_{j} = 0$$

Equivalent

$$\begin{aligned} \min_{\mathbf{x} \in \mathbb{R}^n, X \in \mathbb{S}^n} & \frac{1}{2} \mathrm{tr} \left(A_i X \right) + b_0^\top x + c_0, \\ \text{s.t.} & \frac{1}{2} \mathrm{tr} \left(A_i X \right) + b_i^\top x + c_i \leqslant 0 \\ & \frac{1}{2} \mathrm{tr} \left(A_i X \right) + b_j^\top x + c_j = 0, \\ & X = x x^\top \end{aligned}$$

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The Greatest Shur again...



Equivalent

$$\min_{x \in \mathbb{R}^n, X \in \mathbb{S}^n} \frac{1}{2} \mathsf{tr} \left(A_i X \right) + b_0^\top x + c_0,$$

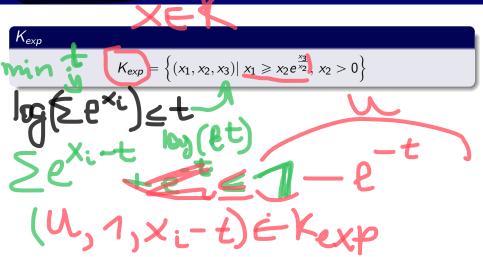
s.t.
$$\frac{1}{2}\operatorname{tr}(A_iX) + b_i^{\top}x + c_i \leq 0$$

$$\frac{1}{2}\operatorname{tr}(A_jX) + b_j^{\top}x + c_j = 0$$





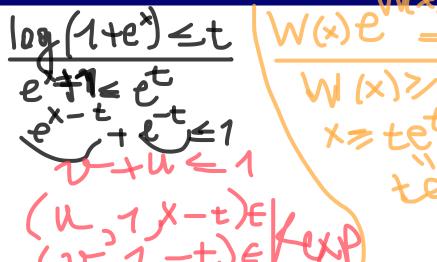
Exponential Cone



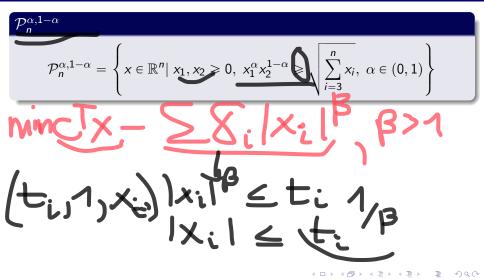
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Exponential Cone. Application



Power Cone



Quasiconvex Programming

