Note: this document is for your reference. The materials presented here are not require in this course.

1. Number Systems

There are infinite ways to represent a number. The four commonly associated with modern computers and digital electronics are: Decimal, Binary, Octal, and Hexadecimal.

Decimal (base 10) is the way most human beings represent numbers.

Decimal counting goes:
0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21 and so on.

Binary (base 2) is the natural way most digital circuits represent and manipulate numbers.

Binary counting goes:
0, 1, 10, 11, 100, 101, 110, 111, 1000, 1001, 1010, 1011, 1100, 1101, 1110, 1111, 10000, 10001, and so on.

Octal (base 8) was previously a popular choice for representing digital circuit numbers in a form that is more compact than binary. Octal is sometimes abbreviated as **oct**.

Octal counting goes:
0, 1, 2, 3, 4, 5, 6, 7,   10, 11, 12, 13, 14, 15, 16, 17,   20, 21, and so on.

Hexadecimal (base 16) is currently the most popular choice for representing digital circuit numbers in a form that is more compact than binary. Hexadecimal numbers are sometimes represented by preceding the value with '0x', or '0X', as in 0x1B84. Hexadecimal is sometimes abbreviated as **hex**.

Hexadecimal counting goes:
0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F,   10, 11, 12,13,14,15,16,17,18,19,1A,1B,1C,1D,1E,1F, 20, 21 and so on.   Note: A~F can be substituted with a~f.

All four number systems are equally capable of representing any number. Furthermore, a number can be perfectly converted between the various number systems without loss of numeric value.

Two observations: For a number system of base *b*, where *b* is 10 (decimal), 2 (binary), 8 (oct) or 16 (hex), and others

- Starting from the right most position, the 1st position has base value b^0, the 2nd position has base value b^1, the 3rd position has base value b^2 and so on, i.e., … $b^3\ b^2\ b^1\ b^0$. Thus a decimal literal 134 represents one 10^2 and three 10^1 and four 10^0. That is, $1\times10^2 + 3\times10^1 + 4\times10^0 = 100+ 30+ 4 = 134$.
- For each position, there are *b* possible values, ranging from 0 to *b*-1. So each position of decimal has 10 possible values (0-9), binary has 2 values (0-1), oct has 8 values (0-7), and hex has 16 values (0-9 and then A-F -- we cannot use 10-15 as each position can has only one letter). In general, for a number system of base *b*, there are *b* digit symbols.

2. Conversion from Binary, Oct, Hex to Decimal

2.1 Binary to Decimal

They say there are only 10 people in this world: those that understand binary and those that don’t. Ha ha.

If you don’t get that joke, you'll need a method to convert from binary to decimal.

The conversion can be performed in the conventional mathematical way, by showing each digit place as an increasing power of 2.

$11100001 = (1 \times 2^7) + (1 \times 2^6) + (1 \times 2^5) + (0 \times 2^4) + (0 \times 2^3) + (0 \times 2^2) + (0 \times 2^1) + (1 \times 2^0)$

$= 225$ decimal

[Optional] One another method involves addition and multiplication.

1. Start the decimal result at 0.
2. Remove the most significant binary digit (leftmost) and add it to the result.
3. If all binary digits have been removed, you’re done. Stop.
4. **Otherwise, multiply the result by 2.**
5. Go to step 2.

Here is an example of converting 11100001 binary to decimal:

Binary Digits	Operation	Decimal Result	Operation	Decimal Result
11100001	+1	1	× 2	2
1100001	+1	3	× 2	6
100001	+1	7	× 2	14
00001	+0	14	× 2	28
0001	+0	28	× 2	56
001	+0	56	× 2	112
01	+0	112	× 2	224
1	+1	225	done	

2.2 Octal to Decimal

The conversion can be performed in the conventional mathematical way, by showing each digit place as an increasing power of 8.

345 octal = $(3 \times 8^2) + (4 \times 8^1) + (5 \times 8^0)$

$= (3 \times 64) + (4 \times 8) + (5 \times 1) = 229$ decimal

[Optional] Converting octal to decimal can be done with addition and multiplication.

1. Start the decimal result at 0.
2. Remove the most significant octal digit (leftmost) and add it to the result.
3. If all octal digits have been removed, you're done. Stop.
4. Otherwise, **multiply the result by 8.**
5. Go to step 2.

Octal Digits	Operation	Decimal Result	Operation	Decimal Result
345	+3	3	× 8	24
45	+4	28	× 8	224
5	+5	229	done.	

2.3 Hexadecimal to Decimal

Converting hexadecimal to decimal can be performed in the conventional mathematical way, by showing each digit place as an increasing power of 16. Of course, hexadecimal letter values need to be converted to decimal values before performing the math.

Hexadecimal:	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
Decimal:	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

34F hexadecimal:
$= (3 \times 16^2) + (4 \times 16^1) + (F \times 16^0)$

$= (3 \times 256) + (4 \times 16) + (15 \times 1) = 847$ decimal

[Optional] Converting hex to decimal can also be done with addition and multiplication.

1. Start the decimal result at 0.
2. Remove the most significant hex digit (leftmost) and add it to the result.
3. If all hex digits have been removed, you're done. Stop.
4. Otherwise, **multiply the result by 16.**
5. Go to step 2.

Of course, hexadecimal letter values (i.e., A - F) need to be converted to decimal values before performing the math.

Hex Digits	Operation	Decimal Result	Operation	Decimal Result
34F	+3	3	× 16	48
4F	+4	52	× 16	832
F	+15	847	done.	

3. Conversion from Decimal to Binary, Oct and Hex

A repeated division and remainder algorithm can convert decimal to binary, octal, or hexadecimal.

1. Divide the decimal number by the desired target radix (2, 8, or 16).
2. Append the remainder as the next most significant digit.
3. Repeat until the decimal number has reached zero.

3.1 Decimal to Binary

Here is an example of using repeated division to convert 1792 decimal to binary:

Decimal Number	Operation	Quotient	Remainder	Binary Result
1792	÷ 2 =	896	0	0
896	÷ 2 =	448	0	00
448	÷ 2 =	224	0	000
224	÷ 2 =	112	0	0000
112	÷ 2 =	56	0	00000
56	÷ 2 =	28	0	000000
28	÷ 2 =	14	0	0000000
14	÷ 2 =	7	0	00000000
7	÷ 2 =	3	1	100000000
3	÷ 2 =	1	1	1100000000
1	÷ 2 =	0	1	**11100000000**
0	done.			

3.2 Decimal to Octal

Here is an example of using repeated division to convert 1792 decimal to octal:

Decimal Number	Operation	Quotient	Remainder	Octal Result
1792	÷ 8 =	224	0	0
224	÷ 8 =	28	0	00
28	÷ 8 =	3	4	400
3	÷ 8 =	0	3	**3400**
0	done.			

3.3 Decimal to Hexadecimal

Here is an example of using repeated division to convert 1792 decimal to hexadecimal:

Decimal Number	Operation	Quotient	Remainder	Hexadecimal Result
1792	÷ 16 =	112	0	0
112	÷ 16 =	7	0	00
7	÷ 16 =	0	7	**700**
0	done.			

Another example, where letters are involved.

Decimal Number	Operation	Quotient	Remainder	Hexadecimal Result
48879	÷ 16 =	3054	15	**F**
3054	÷ 16 =	190	14	EF
190	÷ 16 =	11	14	EEF
11	÷ 16 =	0	11	**BEEF**
0	done.			

4. Conversion from Binary to Oct and Hex

4.1 Binary to Octal

An easy way to convert from binary to octal is to group binary digits into sets of **three**, starting with the least significant (rightmost) digits.

Binary: 11100101 =	11 100 101	
	011 100 101	Pad the most significant digits with zeros if necessary to complete a group of three.

Then, look up the decimal value of each group, as shown in the table:

Binary:	**000**	**001**	**010**	**011**	**100**	**101**	**110**	**111**
Octal:	0	1	2	3	4	5	6	7

Binary = 011 100 101

Octal = 3 4 5 = 345

4.2 Binary to Hexadecimal

An equally easy way to convert from binary to hexadecimal is to group binary digits into sets of **four**, starting with the least significant (rightmost) digits.

Binary: 11100101 = <u>1110</u> <u>0101</u>

Then, look up the decimal value of each group, as shown in the table:

Binary:	**0000**	**0001**	**0010**	**0011**	**0100**	**0101**	**0110**	**0111**
Hexadecimal:	0	1	2	3	4	5	6	7
Binary:	**1000**	**1001**	**1010**	**1011**	**1100**	**1101**	**1110**	**1111**
Hexadecimal:	8	9	A	B	C	D	E	F

Binary = <u>1110</u> <u>0101</u>
Hexadecimal = E 5 = E5

5. Conversion from Oct and Hex to Binary

5.1 Octal to Binary

Converting from octal to binary is as easy as converting from binary to octal (just the opposite operation). Simply look up each octal digit to obtain the equivalent group of three binary digits.

Octal:	0	1	2	3	4	5	6	7
Binary:	**000**	**001**	**010**	**011**	**100**	**101**	**110**	**111**

Octal = 3 4 5

Binary = 011 100 101 = 011100101 binary

5.2 Hexadecimal to Binary

Converting from hexadecimal to binary is as easy as converting from binary to hexadecimal (just the opposite operation). Simply look up each hexadecimal digit to obtain the equivalent group of four binary digits.

Hexadecimal:	0	1	2	3	4	5	6	7
Binary:	**0000**	**0001**	**0010**	**0011**	**0100**	**0101**	**0110**	**0111**
Hexadecimal:	8	9	A	B	C	D	E	F
Binary:	**1000**	**1001**	**1010**	**1011**	**1100**	**1101**	**1110**	**1111**

Hexadecimal = A 2 D E
Binary = 1010 0010 1101 1110 = 1010001011011110 binary

Decimal	Binary	Octal	Hex
0	0	0	0
1	1	1	1
2	10	2	2
3	11	3	3
4	100	3	3
5	101	5	5
6	110	6	6
7	111	7	7
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F
16	10000	20	10
20	10100	24	14
30	11110	36	1E
40	101000	50	28
50	110010	62	32
60	111100	74	3C
70	1000110	106	46
80	1010000	120	50
90	1011010	132	5A
100	1100100	144	64
1000	1111101000	1750	3E8
2989	101110101101	5655	BAD

6. What else? Conversion between Oct and Hex

6.1 Octal to Hexadecimal

When converting from octal to hexadecimal, it is often easier to first convert the octal number into binary and then from binary into hexadecimal. For example, to convert 345 octal into hex:

Octal = 3 4 5

Binary = 011 100 101 = 011100101 binary

Binary = 1110 0101

Hexadecimal =E 5 = E5 hex

Therefore, through a two-step conversion process, octal 345 equals binary 011100101 equals hexadecimal E5.

6.2 Hexadecimal to Octal

When converting from hexadecimal to octal, it is often easier to first convert the hexadecimal number into binary and then from binary into octal. For example, to convert A2DE hex into octal:

Hexadecimal = A 2 D E

Binary = 1010 0010 1101 1110 = 1010001011011110 binary

Add leading zeros or remove leading zeros to group into sets of three binary digits.

Binary: 1010001011011110 = 001 010 001 011 011 110

Binary = 001 010 001 011 011 110

Octal = 1 2 1 3 3 6 = 121336 octal

Therefore, through a two-step conversion process, hexadecimal A2DE equals binary 1010001011011110 equals octal 121336.