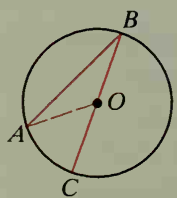
The next theorem compares the measure of an inscribed angle with the measure of its intercepted arc. Its proof requires us to consider three possible cases.

Theorem 9-7

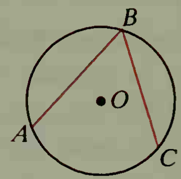
The measure of an inscribed angle is equal to half the measure of its intercepted arc.

Given: $\angle ABC$ inscribed in $\odot O$

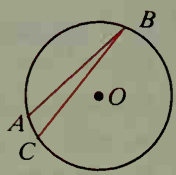
Prove: $m\angle ABC = \frac{1}{2}m\overset{\frown}{AC}$



Case I:
Point O lies on $\angle ABC$.



Case II:
Point O lies inside $\angle ABC$.



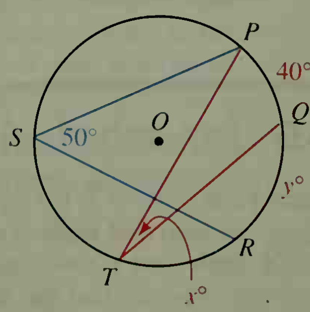
Case III:
Point O lies outside $\angle ABC$.

Key steps of proof of Case I:

1. Draw radius $\overline{OA}$ and let $m\angle ABC = x$.
2. $m\angle A = x$ (Why?)
3. $m\angle AOC = 2x$ (Why?)
4. $m\overset{\frown}{AC} = 2x$ (Why?)
5. $m\angle ABC = \frac{1}{2}m\overset{\frown}{AC}$ (Substitution Prop.)

Now that Case I has been proved, it can be used to prove Case II and Case III. An auxiliary line will be used in those proofs, which are left as Classroom Exercises 12 and 13.

Example 1 Find the values of x and y in $\odot O$.



Solution $m\angle PTQ = \frac{1}{2}m\overset{\frown}{PQ}$, so

$x = \frac{1}{2} \cdot 40 = 20.$

$m\angle PSR = \frac{1}{2}m\overset{\frown}{PR}$, so

$50 = \frac{1}{2}(40 + y)$ and $y = 60.$