RELATIVE VELOCITY

The case of the faster car overtaking your car was easy to solve with a minimum of thought and effort, but you will encounter many situations in which a more systematic method of solving such problems is beneficial. To develop this method, write down all the information that is given and that you want to know in the form of velocities with subscripts appended.

$\mathbf{v_{se}}$ = +80 km/h north (Here the subscript *se* means the velocity of the *slower* car with respect to *Earth*.)

$\mathbf{v_{fe}}$ = +90 km/h north (The subscript *fe* means the velocity of the *fast* car with respect to *Earth*.)

We want to know $\mathbf{v_{fs}}$, which is the velocity of the fast car with respect to the slower car. To find this, we write an equation for $\mathbf{v_{fs}}$ in terms of the other velocities, so on the right side of the equation the subscripts start with *f* and eventually end with *s*. Also, each velocity subscript starts with the letter that ended the preceding velocity subscript.

$$\mathbf{v_{fs}} = \mathbf{v_{fe}} + \mathbf{v_{es}}$$

The boldface notation indicates that velocity is a vector quantity. This approach to adding and monitoring subscripts is similar to vector addition, in which vector arrows are placed head to tail to find a resultant.

We know that $\mathbf{v_{es}} = -\mathbf{v_{se}}$ because an observer in the slow car perceives Earth as moving south at a velocity of 80 km/h while a stationary observer on the ground (Earth) views the car as moving north at a velocity of 80 km/h. Thus, this problem can be solved as follows:

$$\mathbf{v_{fs}} = \mathbf{v_{fe}} + \mathbf{v_{es}} = \mathbf{v_{fe}} - \mathbf{v_{se}}$$

$$\mathbf{v_{fs}} = (+90 \text{ km/h north}) - (+80 \text{ km/h north}) = +10 \text{ km/h north}$$

When solving relative velocity problems, follow the above technique for writing subscripts. The particular subscripts will vary depending on the problem, but the method for ordering the subscripts does not change. A general form of the relative velocity equation is $\mathbf{v_{ac}} = \mathbf{v_{ab}} + \mathbf{v_{bc}}$. This general form may help you remember the technique for writing subscripts.

Did you know?

Like velocity, displacement and acceleration depend on the frame in which they are measured. In some cases, it is instructive to visualize gravity as the ground accelerating toward a projectile rather than the projectile accelerating toward the ground.

ADVANCED TOPICS

See "Special Relativity and Velocities" in **Appendix J: Advanced Topics** to learn about how velocities are added in Einstein's special theory of relativity.

Why it Matters

Conceptual Challenge

1. Elevator Acceleration A boy bounces a rubber ball in an elevator that is going down. If the boy drops the ball as the elevator is slowing down, is the magnitude of the ball's acceleration relative to the elevator less than or greater than the magnitude of its acceleration relative to the ground?

2. Aircraft Carrier Is the velocity of a plane relative to an aircraft carrier slower when it approaches from the stern (rear) or from the bow (front)?