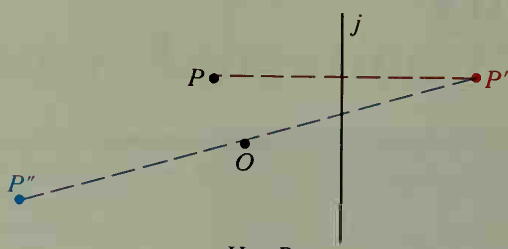
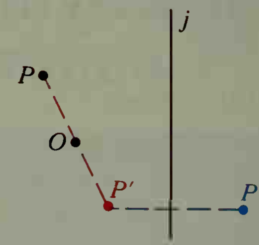
Example 2 Show that $H_O \circ R_j \neq R_j \circ H_O$.

Solution Study the two diagrams below.



$H_O \circ R_j$

Here R_j, the reflection of P in line j, is carried out first, mapping P to P'. Then H_O maps P' to P''. Thus P'' is the image of P under the composite $H_O \circ R_j$.



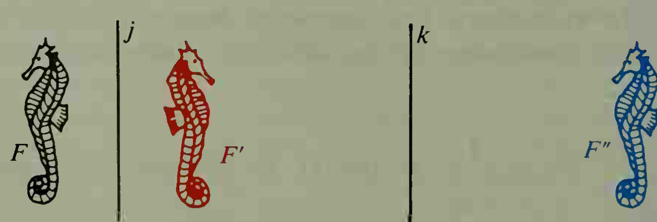
$R_j \circ H_O$

With the order changed in the composite, the half-turn is carried out first, followed by the reflection in line j. The image point P'' is now in a different place.

Notice that the two composites map P to different image points, so the composites are not equal.

Example 2 shows that the order in a composite of transformations can be very important, but this is not always true. For example, if S and T are two translations, then order is not important, since $S \circ T = T \circ S$ (see Exercise 10).

Example 2 above shows the effect of a composite of mappings on a single point P. The diagram below shows a composite of reflections acting on a whole figure, F. F is reflected in line j to F', and F' is reflected in line k to F''. Thus $R_k \circ R_j$ maps F to F''. Again notice that the first reflection, R_j, is written on the right.



The final image F'' is the same size and shape as F. Also, F'' is the image of F under a translation. This illustrates our next two theorems. First, the composite of any two isometries is an isometry. Second, the composite of reflections in two parallel lines is a translation.