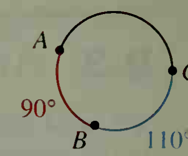
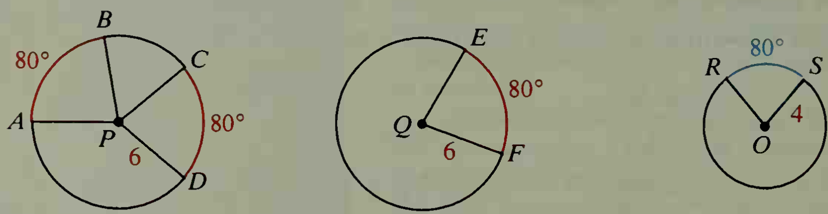
Applying the Arc Addition Postulate to the circle shown at the right, we have



$$m\overset{\frown}{AB} + m\overset{\frown}{BC} = m\overset{\frown}{ABC}$$
$$90 + 110 = 200$$

Congruent arcs are arcs, in the same circle or in congruent circles, that have equal measures. In the diagram below, $\odot P$ and $\odot Q$ are congruent circles and $\overset{\frown}{AB} \cong \overset{\frown}{CD} \cong \overset{\frown}{EF}$. However, $\overset{\frown}{EF}$ is not congruent to $\overset{\frown}{RS}$ even though both arcs have the same degree measure, because $\odot Q$ is not congruent to $\odot O$.



Notice that each of the congruent arcs above has an 80° central angle, so these congruent arcs have congruent central angles. The relationship between congruence of minor arcs and congruence of their central angles is stated in Theorem 9-3 below. This theorem follows immediately from the definition of congruent arcs.

Theorem 9-3

In the same circle or in congruent circles, two minor arcs are congruent if and only if their central angles are congruent.

Example The radius of the Earth is about 6400 km. The latitude of the Arctic Circle is 66.6° North. (That is, in the figure, $m\overset{\frown}{BE} = 66.6$.) Find the radius of the Arctic Circle.

Solution Let N be the North Pole and let $\overline{ON}$ intersect $\overline{AB}$ in M. Since $m\overset{\frown}{NE} = 90$, $m\overset{\frown}{NB} = 90 - 66.6 = 23.4$ and $m\angle NOB = 23.4$. Similarly, $m\angle NOA = 23.4$. Since $\triangle AOB$ is isosceles and $\overline{OM}$ bisects the vertex $\angle AOB$, (1) M is the midpoint of $\overline{AB}$ (and thus the center of the Arctic Circle) and (2) $\overline{OM} \perp \overline{AB}$. Using trigonometry in right $\triangle MOB$:

$$\sin 23.4° = \frac{MB}{OB}$$
$$MB = OB \cdot \sin 23.4°$$
$$MB \approx 6400(0.3971)$$
$$MB \approx 2500 \text{ km}$$