Angular Kinematics

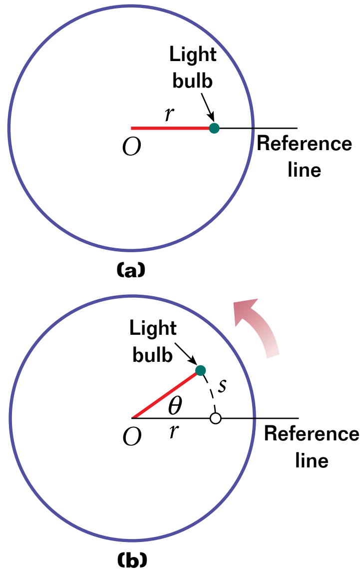

Figure 1
A light bulb on a rotating Ferris wheel **(a)** begins at a point along a reference line and **(b)** moves through an arc length s and therefore through the angle θ.

A point on an object that rotates about a fixed axis undergoes circular motion around that axis. The linear quantities introduced in the chapter "Motion in One Dimension" cannot be used for circular motion because we are considering the rotational motion of an extended object rather than the linear motion of a particle. For this reason, circular motion is described in terms of the change in angular position. All points on a rigid rotating object, except the points on the axis, move through the same angle during any time interval.

Measuring angles with radians

Many of the equations that describe circular motion require that angles be measured in **radians** (rad) rather than in degrees. To see how radians are measured, consider **Figure 1,** which illustrates a light bulb on a rotating Ferris wheel. At $t = 0$, the bulb is on a fixed reference line, as shown in **Figure 1(a).** After a time interval Δt, the bulb advances to a new position, as shown in **Figure 1(b).** In this time interval, the line from the center to the bulb (depicted with a red line in both diagrams) moved through the angle θ with respect to the reference line. Likewise, the bulb moved a distance s, measured along the circumference of the circle; s is the *arc length.*

In general, any angle θ measured in radians is defined by the following equation:

$$\theta = \frac{\text{arc length}}{\text{radius}} = \frac{s}{r}$$

Note that if the arc length, s, is equal to the length of the radius, r, the angle θ swept by r is equal to 1 rad. Because θ is the ratio of an arc length (a distance) to the length of the radius (also a distance), the units cancel and the abbreviation *rad* is substituted in their place. In other words, the radian is a pure number, with no dimensions.

When the bulb on the Ferris wheel moves through an angle of 360° (one revolution of the wheel), the arc length s is equal to the circumference of the circle, or $2\pi r$. Substituting this value for s into the equation above gives the corresponding angle in radians.

$$\theta = \frac{s}{r} = \frac{2\pi r}{r} = 2\pi \text{ rad}$$

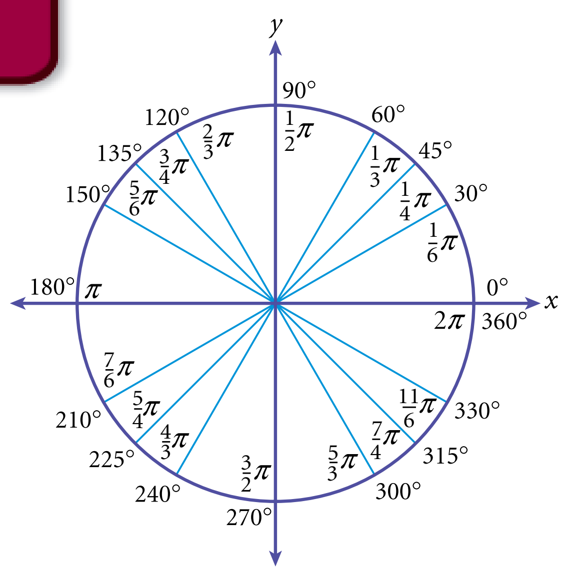

Figure 2
Angular motion is measured in units of radians. Because there are 2π radians in a full circle, radians are often expressed as a multiple of π.