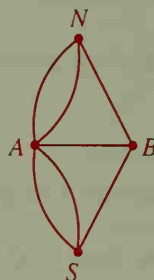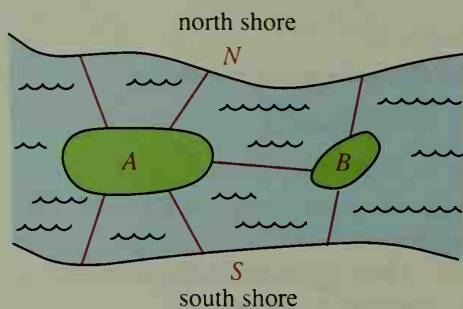**Extra**   *Networks*

The Pregel River flows through the old city of Koenigsberg, now Kaliningrad. Once, seven bridges joined the shores and the two islands in the river as shown in the diagram at the left below. A popular problem of that time was to try to walk across all seven bridges without crossing any bridge more than once. Can you find a way to do it?


Mathematician Leonard Euler analyzed this problem using a diagram called a *network*, shown at the right above. He represented each land mass by a point (called a vertex) and each bridge by an arc. He then classified each vertex with an odd number of arcs coming from it as *odd* and each vertex with an even number of arcs as *even*. From here Euler discovered which networks can be traced without backtracking, that is, without drawing over an arc twice.
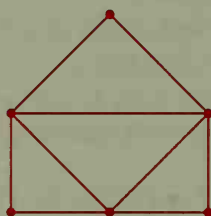
Exercises

Find the number of odd and even vertices in each network. Imagine traveling each network to see if it can be traced without backtracking.
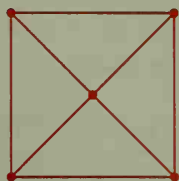
1.   2.   3.

The number of odd vertices will tell you whether or not a network can be traced without backtracking. Do you see how? If not, read on.

4. Suppose that a given network can be traced without backtracking.
   a. Consider a vertex that is neither the start nor end of a journey through this network. Is such a vertex odd or even?
   b. Now consider the two vertices at the start and finish of a journey through this network. Can both of these vertices be odd? even?
   c. Can just one of the start and finish vertices be odd?

5. Tell why it is impossible to walk across the seven bridges of Koenigsberg without crossing any bridge more than once.