5-3 *Theorems Involving Parallel Lines*

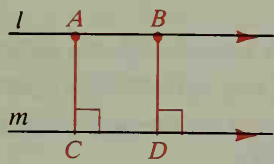
In this section we will prove four useful theorems about parallel lines. The first theorem uses the definition of the distance from a point to a line. (See page 154.)

Theorem 5-8

If two lines are parallel, then all points on one line are equidistant from the other line.

Given: $l \parallel m$; A and B are any points on l;
$\overline{AC} \perp m$; $\overline{BD} \perp m$

Prove: $AC = BD$



Proof:

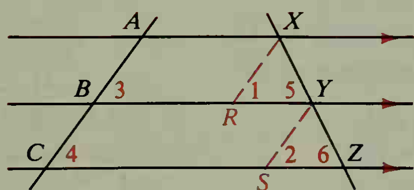
Since $\overline{AB}$ and $\overline{CD}$ are contained in parallel lines, $\overline{AB} \parallel \overline{CD}$. Since $\overline{AC}$ and $\overline{BD}$ are coplanar and are both perpendicular to m, they are parallel. Thus $ABDC$ is a parallelogram, by the definition of a parallelogram. Since opposite sides $\overline{AC}$ and $\overline{BD}$ are congruent, $AC = BD$.

Theorem 5-9

If three parallel lines cut off congruent segments on one transversal, then they cut off congruent segments on every transversal.

Given: $\overleftrightarrow{AX} \parallel \overleftrightarrow{BY} \parallel \overleftrightarrow{CZ}$;
$\overline{AB} \cong \overline{BC}$

Prove: $\overline{XY} \cong \overline{YZ}$



Proof:

Through X and Y draw lines parallel to $\overleftrightarrow{AC}$, intersecting $\overleftrightarrow{BY}$ at R and $\overleftrightarrow{CZ}$ at S, as shown. Then $AXRB$ and $BYSC$ are parallelograms, by the definition of a parallelogram. Since the opposite sides of a parallelogram are congruent, $\overline{XR} \cong \overline{AB}$ and $\overline{BC} \cong \overline{YS}$. It is given that $\overline{AB} \cong \overline{BC}$, so using the Transitive Property twice gives $\overline{XR} \cong \overline{YS}$. Parallel lines are cut by transversals to form the following pairs of congruent corresponding angles:

$\angle 1 \cong \angle 3 \qquad \angle 3 \cong \angle 4 \qquad \angle 4 \cong \angle 2 \qquad \angle 5 \cong \angle 6$

Then $\angle 1 \cong \angle 2$ (Transitive Property), and $\triangle XYR \cong \triangle YZS$ by AAS. Since $\overline{XY}$ and $\overline{YZ}$ are corresponding parts of these triangles, $\overline{XY} \cong \overline{YZ}$.