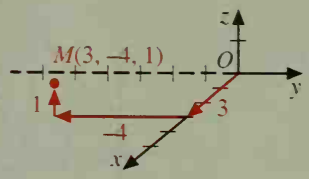
Example 2 Graph point M with coordinates $(3, -4, 1)$.

Solution



Finding the midpoint of a segment drawn in three dimensions is similar to the method used in two dimensions.

The Midpoint Formula in Three Dimensions

The midpoint of the segment that joins points (x_1, y_1, z_1) and (x_2, y_2, z_2) is the point

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right).$$

Example 3 Find the coordinates of the midpoint of the segment that joins $(-6, 4, -2)$ and $(2, 6, -4)$.

Solution The coordinates of the midpoint are

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right) = \left(\frac{-6 + 2}{2}, \frac{4 + 6}{2}, \frac{-2 + (-4)}{2}\right)$$
$$= \left(\frac{-4}{2}, \frac{10}{2}, \frac{-6}{2}\right)$$
$$= (-2, 5, -3).$$

The distance between two points in three dimensions can be found using a formula similar to the Distance Formula for two dimensions.

The Distance Formula in Three Dimensions

The distance d between points (x_1, y_1, z_1) and (x_2, y_2, z_2) is given by:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Example 4 Find the distance between points $(-2, 3, -7)$ and $(2, -6, 9)$.

Solution Let (x_1, y_1, z_1) be $(-2, 3, -7)$ and (x_2, y_2, z_2) be $(2, -6, 9)$.

Then $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

$= \sqrt{(2 - (-2))^2 + (-6 - 3)^2 + (9 - (-7))^2}$

$= \sqrt{16 + 81 + 256}$

$= \sqrt{353}$

The distance between the points is $\sqrt{353}$, or about 18.8.