SECTION 3

Harmonics

SECTION OBJECTIVES

- Differentiate between the harmonic series of open and closed pipes.
- Calculate the harmonics of a vibrating string and of open and closed pipes.
- Relate harmonics and timbre.
- Relate the frequency difference between two waves to the number of beats heard per second.

STANDING WAVES ON A VIBRATING STRING

As discussed in the chapter "Vibrations and Waves," a variety of standing waves can occur when a string is fixed at both ends and set into vibration. The vibrations on the string of a musical instrument, such as the violin in **Figure 13,** usually consist of many standing waves together at the same time, each of which has a different wavelength and frequency. So, the sounds you hear from a stringed instrument, even those that sound like a single pitch, actually consist of multiple frequencies.

Table 3, on the next page, shows several possible vibrations on an idealized string. The ends of the string, which cannot vibrate, must always be nodes (N). The simplest vibration that can occur is shown in the first row of **Table 3.** In this case, the center of the string experiences the most displacement, and so it is an antinode (A). Because the distance from one node to the next is always half a wavelength, the string length (L) must equal $\lambda_1/2$. Thus, the wavelength is twice the string length ($\lambda_1 = 2L$).

As described in the chapter on waves, the speed of a wave equals the frequency times the wavelength, which can be rearranged as shown.

$$v = f\lambda, \text{ so } f = \frac{v}{\lambda}$$

By substituting the value for wavelength found above into this equation for frequency, we see that the frequency of this vibration is equal to the speed of the wave divided by twice the string length.

$$\text{fundamental frequency} = f_1 = \frac{v}{\lambda_1} = \frac{v}{2L}$$

Figure 13
The vibrating strings of a violin produce standing waves whose frequencies depend on the string lengths.

This frequency of vibration is called the **fundamental frequency** of the vibrating string. Because frequency is inversely proportional to wavelength and because we are considering the greatest possible wavelength, the fundamental frequency is the lowest possible frequency of a standing wave on this string.

fundamental frequency

the lowest frequency of vibration of a standing wave

Harmonics are integral multiples of the fundamental frequency

The next possible standing wave for a string is shown in the second row of **Table 3.** In this case, there are three nodes instead of two, so the string length is equal to one wavelength. Because this wavelength is half the previous wavelength, the frequency of this wave is twice that of the fundamental frequency.

$$f_2 = 2f_1$$