

Example 2 Find the area of a regular polygon with 9 sides inscribed in a circle with radius 10.

Solution $m\angle AOB = \frac{360}{9} = 40$; $m\angle AOX = 20$

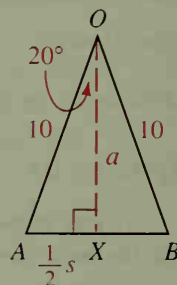
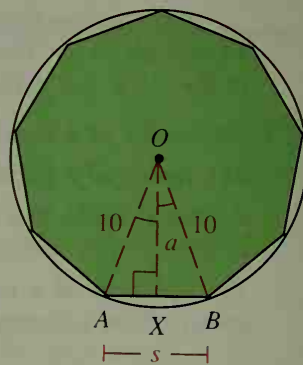
Use trigonometry to find a and s :

$$\begin{aligned}\cos 20^\circ &= \frac{a}{10} & \sin 20^\circ &= \frac{\frac{1}{2}s}{10} \\ a &= 10 \cdot \cos 20^\circ & \frac{1}{2}s &= 10 \sin 20^\circ \\ a &\approx 10(0.9397) & s &\approx 20(0.3420) \\ a &\approx 9.397 & s &\approx 6.840\end{aligned}$$

To find the area of the polygon, use either of two methods:

Method 1 Area of polygon $= 9 \cdot \text{area of } \triangle AOB$
 $= 9 \cdot \frac{1}{2}sa$
 $\approx \frac{9}{2}(6.840)(9.397)$
 ≈ 289

Method 2 Area of polygon $= \frac{1}{2}ap$
 $\approx \frac{1}{2}(9.397)(9 \cdot 6.840)$
 ≈ 289



Classroom Exercises

- Find the measure of a central angle of a regular polygon with (a) 10 sides, (b) 15 sides, (c) 360 sides, and (d) n sides.

Find the perimeter and the area of each regular polygon described.

- A regular octagon with side 4 and apothem a
- A regular pentagon with side s and apothem 3
- A regular decagon with side s and apothem a
- Explain why the apothem of a regular polygon must be less than the radius.

For each regular polygon shown, find (a) the perimeter, (b) the measure of a central angle, (c) the apothem a , (d) the radius r , and (e) the area A .

