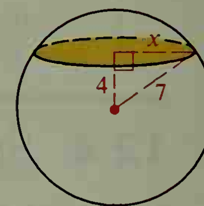
Example 2 The area of a sphere is 256π. Find the volume.

Solution To find the volume, first find the radius.

$$\begin{aligned}(1)\quad A = 256\pi &= 4\pi r^2 \\ 64 &= r^2 \\ 8 &= r\end{aligned} \qquad\qquad (2)\quad \begin{aligned} V = \frac{4}{3}\pi r^3 &= \frac{4}{3}\pi \cdot 8^3 \\ &= \frac{2048\pi}{3}\end{aligned}$$

Example 3 A plane passes 4 cm from the center of a sphere with radius 7 cm. Find the area of the circle of intersection.



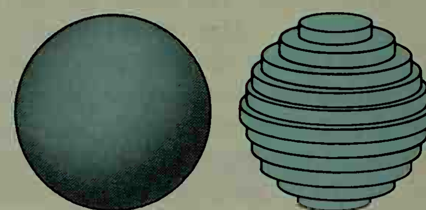
Solution Let x = radius of the circle.

$x = \sqrt{7^2 - 4^2} = \sqrt{33}$

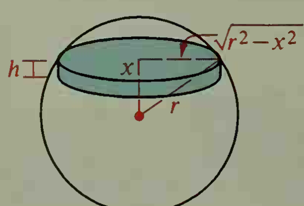
Area $= \pi x^2 = \pi(\sqrt{33})^2 = 33\pi\ (\text{cm}^2)$

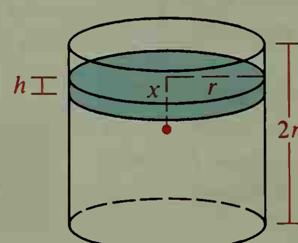
Justification of the Volume Formula (Optional)

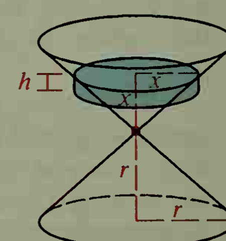
Any solid can be approximated by a stack of thin circular discs of equal thickness, as shown by the sphere drawn at the right. Each disc is actually a cylinder with height h.



The sphere, the cylinder, and the double cone below all have radius r and height $2r$. Look at the disc that is x units above the center of each solid.





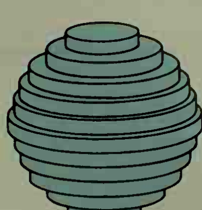


Disc volume:
$\pi(\sqrt{r^2 - x^2})^2 h = \pi(r^2 - x^2)h$
$= \pi r^2 h - \pi x^2 h$

Disc volume: $\pi r^2 h$

Disc volume: $\pi x^2 h$

Note from the calculations above that no matter what x is, the volume of the first disc equals the difference between the volumes of the other two discs.



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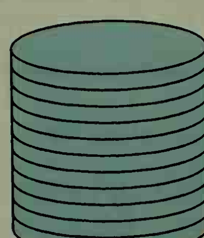

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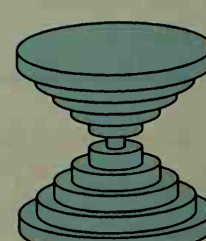

Total volume of discs in sphere = Total volume of discs in cylinder − Total volume of discs in double cone