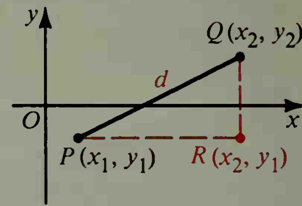
Using a method suggested by Example 1, you can find a formula for the distance between points $P(x_1, y_1)$ and $Q(x_2, y_2)$. First draw a right triangle as shown. The coordinates of R are (x_2, y_1).

$$\begin{aligned} PR &= |x_2 - x_1|;\ QR = |y_2 - y_1| \\ d^2 &= (PR)^2 + (QR)^2 \\ &= |x_2 - x_1|^2 + |y_2 - y_1|^2 \\ &= (x_2 - x_1)^2 + (y_2 - y_1)^2 \\ d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \end{aligned}$$



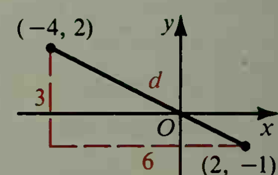
Since d represents distance, d cannot be negative.

Theorem 13-1 *The Distance Formula*

The distance d between points (x_1, y_1) and (x_2, y_2) is given by:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Example 2 Find the distance between points $(-4, 2)$ and $(2, -1)$.



Solution 1 Draw a right triangle. The legs have lengths 6 and 3.

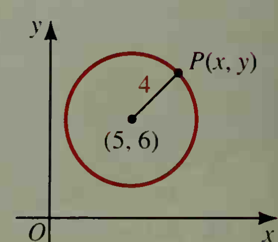
$$d^2 = 6^2 + 3^2 = 36 + 9 = 45$$
$$d = \sqrt{45} = \sqrt{9} \cdot \sqrt{5} = 3\sqrt{5}$$

Solution 2 Let (x_1, y_1) be $(-4, 2)$ and (x_2, y_2) be $(2, -1)$.

$$\begin{aligned} \text{Then } d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(2 - (-4))^2 + ((-1) - 2)^2} \\ &= \sqrt{6^2 + (-3)^2} = \sqrt{36 + 9} = \sqrt{45} = 3\sqrt{5} \end{aligned}$$

You can use the distance formula to find an equation of a circle. Example 3 shows how to do this.

Example 3 Find an equation of a circle with center $(5, 6)$ and radius 4.



Solution Let $P(x, y)$ represent any point on the circle. Since the distance from P to the center is 4,

$$\sqrt{(x - 5)^2 + (y - 6)^2} = 4,$$
$$\text{or } (x - 5)^2 + (y - 6)^2 = 16.$$

Either of these two equations is an equation of the circle, but the second equation is the one usually used.

Example 3 can be generalized to give the theorem at the top of the next page.