Suppose p stands for "Hawks swoop," and q stands for "Gulls glide." Express in symbolic form each of the following statements.

11. Hawks swoop or gulls glide.

12. Gulls do not glide.

13. It is not true that "Hawks swoop or gulls glide."

14. Hawks do not swoop and gulls do not glide.

15. It is not true that "Hawks swoop and gulls glide."

16. Hawks do not swoop or gulls do not glide.

17. Do the statements in Exercises 13 and 14 mean the same thing?

18. Do the statements in Exercises 15 and 16 mean the same thing?

Make a truth table for each of the following statements.

19. $p \vee \sim q$

20. $\sim p \vee q$

21. $\sim(\sim p)$

22. $\sim(p \wedge q)$

23. $p \vee \sim p$

24. $p \wedge \sim p$

25. $p \wedge (q \vee r)$

26. $(p \wedge q) \vee (p \wedge r)$

Truth Tables for Conditionals

The conditional statement "If p, then q," which is discussed in Lesson 2–1, is symbolized as $p \rightarrow q$. This is also read as "p implies q" and as "q follows from p." The truth table for $p \rightarrow q$ is shown at the right. Notice that the only time a conditional is false is when the hypothesis p is true and the conclusion q is false. The example below will show why this is a reasonable way to make out the truth table.

Truth table for conditionals

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Example Mom promises, "If I catch the early train home I'll take you swimming." Consider the four possibilities of the truth table.

1. Mom catches the early train home and takes you swimming. She kept her promise; her statement was *true*.
2. Mom catches the early train home but does not take you swimming. She broke her promise; her statement was *false*.
3. Mom does not catch the early train home but still takes you swimming. She has not broken her promise; her statement was *true*.
4. Mom does not catch the early train home and does not take you swimming. She has not broken her promise; her statement was *true*.

The tables on the next page show the converse and contrapositive of $p \rightarrow q$. Make sure that you understand how these tables were made. Notice that the last column of the table for the contrapositive $\sim q \rightarrow \sim p$ is identical with the last column of the table for the conditional on this page. In other words, the contrapositive of a statement is true (or false) if and only if the statement itself is true (or false). This is what we mean when we say that a statement and its contrapositive are logically equivalent (see Lesson 6–2). On the other hand, a statement and its converse are not logically equivalent. Can you see why?