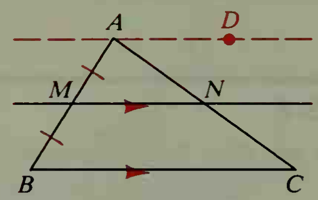
Theorem 5-10

A line that contains the midpoint of one side of a triangle and is parallel to another side passes through the midpoint of the third side.

Given: M is the midpoint of $\overline{AB}$;
$\overleftrightarrow{MN} \parallel \overline{BC}$

Prove: N is the midpoint of $\overline{AC}$.



Proof:

Let $\overleftrightarrow{AD}$ be the line through A parallel to $\overleftrightarrow{MN}$. Then $\overleftrightarrow{AD}$, $\overleftrightarrow{MN}$, and $\overleftrightarrow{BC}$ are three parallel lines that cut off congruent segments on transversal $\overleftrightarrow{AB}$. By Theorem 5-9 they also cut off congruent segments on $\overleftrightarrow{AC}$. Thus $\overline{AN} \cong \overline{NC}$ and N is the midpoint of $\overline{AC}$.

The next theorem has two parts, the first of which is closely related to Theorem 5-10.

Theorem 5-11

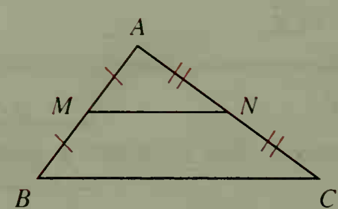
The segment that joins the midpoints of two sides of a triangle

(1) is parallel to the third side;

(2) is half as long as the third side.

Given: M is the midpoint of $\overline{AB}$;
N is the midpoint of $\overline{AC}$.

Prove: (1) $\overline{MN} \parallel \overline{BC}$
(2) $MN = \frac{1}{2}BC$



Proof of (1):

There is exactly one line through M parallel to $\overline{BC}$. By Theorem 5-10 that line passes through N, the midpoint of $\overline{AC}$. Thus $\overline{MN} \parallel \overline{BC}$.

Proof of (2):

Let L be the midpoint of $\overline{BC}$, and draw $\overline{NL}$. By part (1), $\overline{MN} \parallel \overline{BC}$ and also $\overline{NL} \parallel \overline{AB}$. Thus quad. $MNLB$ is a parallelogram. Since its opposite sides are congruent, $MN = BL$. Since L is the midpoint of $\overline{BC}$, $BL = \frac{1}{2}BC$. Therefore $MN = \frac{1}{2}BC$.