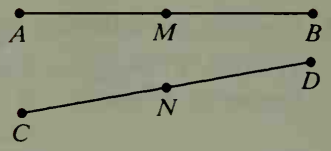
Example 1 Given: M is the midpoint of $\overline{AB}$;
N is the midpoint of $\overline{CD}$;
$AB = CD$
What can you deduce?



Solution Because M and N are midpoints, you know that $AM = MB$ and $CN = ND$.
From the Midpoint Theorem, you know that $AM = \frac{1}{2}AB$ and $CN = \frac{1}{2}CD$.
Since $AB = CD$, you know that $\frac{1}{2}AB = \frac{1}{2}CD$.
By substitution, you get $AM = CN$.
Thus you can deduce that AM, MB, CN, and ND are all equal.

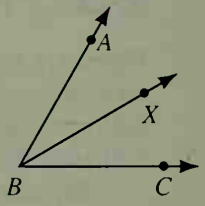
The next theorem is similar to the Midpoint Theorem. It proves properties of the angle bisector that are not given in the definition. The proof is left as Classroom Exercise 10.

Theorem 2-2 *Angle Bisector Theorem*

If $\overrightarrow{BX}$ is the bisector of $\angle ABC$, then

$$m\angle ABX = \tfrac{1}{2}m\angle ABC \text{ and } m\angle XBC = \tfrac{1}{2}m\angle ABC.$$

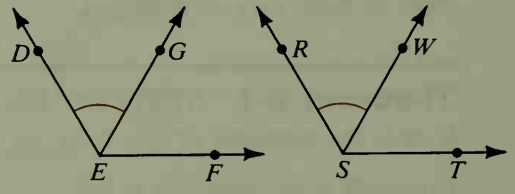
Given: $\overrightarrow{BX}$ is the bisector of $\angle ABC$.
Prove: $m\angle ABX = \frac{1}{2}m\angle ABC$; $m\angle XBC = \frac{1}{2}m\angle ABC$



In addition to postulates and definitions, theorems may be used to justify steps in a proof. Notice the use of the Angle Bisector Theorem in Example 2.

Example 2

Given: $\overrightarrow{EG}$ is the bisector of $\angle DEF$;
$\overrightarrow{SW}$ is the bisector of $\angle RST$;
$m\angle DEG = m\angle RSW$
Prove: $m\angle DEF = m\angle RST$



Proof:

Statements	Reasons
1. $\overrightarrow{EG}$ is the bisector of $\angle DEF$; $\overrightarrow{SW}$ is the bisector of $\angle RST$.	1. Given
2. $m\angle DEG = \frac{1}{2}m\angle DEF$; $m\angle RSW = \frac{1}{2}m\angle RST$	2. Angle Bisector Theorem
3. $m\angle DEG = m\angle RSW$	3. Given
4. $\frac{1}{2}m\angle DEF = \frac{1}{2}m\angle RST$	4. Substitution Prop. (Steps 2 and 3)
5. $m\angle DEF = m\angle RST$	5. Multiplication Prop. of =