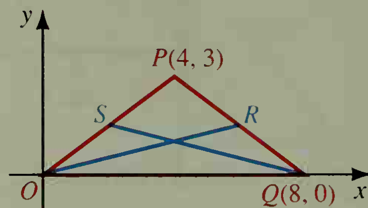
13-9 Coordinate Geometry Proofs

It is easy to verify that $\triangle OPQ$ is an isosceles triangle. Knowing this, we can deduce that medians $\overline{OR}$ and $\overline{QS}$ are congruent by using the midpoint and distance formulas.



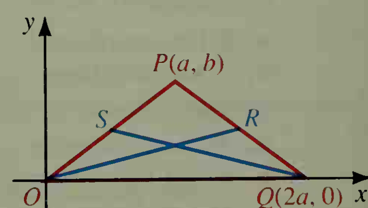
In order to give a coordinate proof that the medians to the legs are congruent for *any* isosceles triangle, and not just for the specific isosceles triangle above, you could use the figure below. Compare the general coordinates given in the figure below with the specific coordinates given for the triangle above. A coordinate proof follows.-

Example 1

Prove that the medians to the legs of an isosceles triangle are congruent.

Proof:

Let OPQ be any isosceles triangle with $PO = PQ$. Choose convenient axes and coordinates as shown.



By the midpoint formula,

S has coordinates $\left(\frac{a}{2}, \frac{b}{2}\right)$ and R has coordinates $\left(\frac{3a}{2}, \frac{b}{2}\right)$.

By the distance formula,

$$OR = \sqrt{\left(\frac{3a}{2} - 0\right)^2 + \left(\frac{b}{2} - 0\right)^2}$$

$$= \sqrt{\frac{9a^2}{4} + \frac{b^2}{4}}$$

and $$QS = \sqrt{\left(\frac{a}{2} - 2a\right)^2 + \left(\frac{b}{2} - 0\right)^2}$$

$$= \sqrt{\frac{9a^2}{4} + \frac{b^2}{4}}$$

Therefore, $\overline{OR} \cong \overline{QS}$.

It is possible to prove many theorems of geometry by using coordinate methods rather than the noncoordinate methods involving congruent triangles and angles formed by parallel lines. Coordinate proofs are sometimes, but not always, much easier than noncoordinate proofs. For example, compare the proof in Example 2 with the proof of Theorem 5-11 on page 178.