7-5 *Theorems for Similar Triangles*

You can prove two triangles similar by using the definition of similar polygons or by using the AA Postulate. Of course, in practice you would always use the AA Postulate instead of the definition. (Why?) Two additional methods are established in the theorems below. The proofs involve proportions, congruence, and similarity.

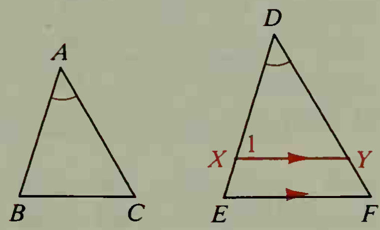
Theorem 7-1 *SAS Similarity Theorem*

If an angle of one triangle is congruent to an angle of another triangle and the sides including those angles are in proportion, then the triangles are similar.

Given: $\angle A \cong \angle D$;

$\frac{AB}{DE} = \frac{AC}{DF}$

Prove: $\triangle ABC \sim \triangle DEF$



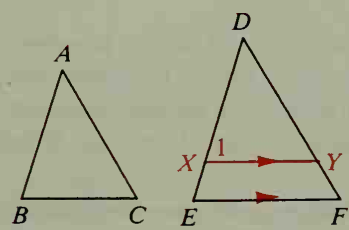
Plan for Proof: Take X on $\overline{DE}$ so that $DX = AB$. Draw a line through X parallel to $\overleftrightarrow{EF}$. Then $\triangle DEF \sim \triangle DXY$ and $\frac{DX}{DE} = \frac{DY}{DF}$. Since $\frac{AB}{DE} = \frac{AC}{DF}$ and $DX = AB$, you can deduce that $DY = AC$. Thus $\triangle ABC \cong \triangle DXY$ by SAS. Therefore, $\triangle ABC \sim \triangle DXY$, and $\triangle ABC \sim \triangle DEF$ by the Transitive Property of Similarity.

Theorem 7-2 *SSS Similarity Theorem*

If the sides of two triangles are in proportion, then the triangles are similar.

Given: $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

Prove: $\triangle ABC \sim \triangle DEF$



Plan for Proof: Take X on $\overline{DE}$ so that $DX = AB$. Draw a line through X parallel to $\overleftrightarrow{EF}$. Then $\triangle DEF \sim \triangle DXY$ and $\frac{DX}{DE} = \frac{XY}{EF} = \frac{DY}{DF}$. With the given proportion and $DX = AB$, you can deduce that $AC = DY$ and $BC = XY$. Thus $\triangle ABC \cong \triangle DXY$ by SSS. Therefore, $\triangle ABC \sim \triangle DXY$, and $\triangle ABC \sim \triangle DEF$ by the Transitive Property of Similarity.