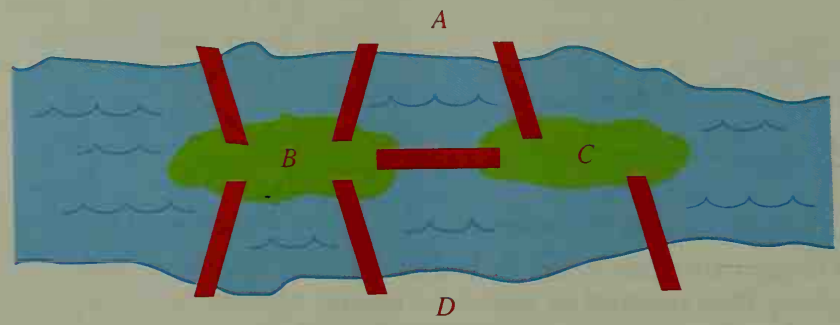
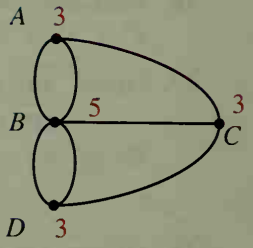
Discrete Mathematics

A Famous Bridge Problem

In the 1700's the European city of Koenigsberg (now called Kaliningrad) had seven bridges connecting both banks of the Pregel River to two islands in the river. Villagers of the city developed this problem: Is it possible to start at some point in the city, travel over each of the seven bridges exactly once, and return to your starting point? Do you think it is possible?



To analyze this problem, the Swiss mathematician Leonhard Euler (1707–1783) drew a *graph* like the one at the right. This graph is not like the graphs you have studied in algebra. It is a diagram in which the 4 land masses of the city (A, B, C, and D) are represented by *vertices* (points), and the 7 bridges are represented by *edges* (lines and arcs). Next to each vertex is a number indicating how many edges are attached to the vertex. This number is called the *valence* of the vertex.



Euler reasoned that in order for a person to travel over every bridge once and return to the starting point, every vertex must have an even valence. This is because a person traveling *into* a vertex must also *leave* it. So the edges must be paired, one "in" with one "out."

In the 7-bridges problem, none of the vertices has an even valence, so a circuit over all 7 bridges is impossible. However, if two more bridges are added, giving the 9 bridges shown at the right, then every vertex has an even valence, and a circuit over all 9 bridges is possible. Such a circuit, which traces every edge of a graph exactly once and returns to its starting point, is called an *Euler circuit*. Can you find an Euler circuit for the graph of the 9-bridges problem?