Example 1 Function g maps every number to a number that is six more than its double.

a. Express this fact using function notation.
b. Find the image of 7.
c. Find the preimage of 8.

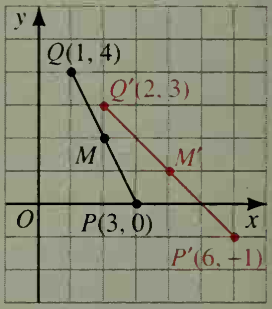
Solution

a. $g:x \rightarrow 2x + 6$, or $g(x) = 2x + 6$
b. $g:7 \rightarrow 2 \cdot 7 + 6 = 20$. Thus the image of 7 is 20.
c. $g:x \rightarrow 2x + 6 = 8$. Therefore $x = 1$, so 1 is the preimage of 8.

Example 2 Mapping G maps each point (x, y) to the point $(2x, y - 1)$.

a. Express this fact using mapping notation.
b. Find P' and Q', the images of $P(3, 0)$ and $Q(1, 4)$.
c. Decide whether G maps M, the midpoint of $\overline{PQ}$, to M', the midpoint of $\overline{P'Q'}$.
d. Decide whether $PQ = P'Q'$.

Solution



a. $G:(x, y) \rightarrow (2x, y - 1)$

b. $G:(3, 0) \rightarrow (2 \cdot 3, 0 - 1) = (6, -1) = P'$
$G:(1, 4) \rightarrow (2 \cdot 1, 4 - 1) = (2, 3) = Q'$

c. $M = \left(\frac{3 + 1}{2}, \frac{0 + 4}{2}\right) = (2, 2)$

$M' = \left(\frac{6 + 2}{2}, \frac{-1 + 3}{2}\right) = (4, 1)$

$G:(2, 2) \rightarrow (2 \cdot 2, 2 - 1) = (4, 1)$
Thus G does map midpoint M to midpoint M'.

d. Use the distance formula to show that

$$PQ = \sqrt{(1 - 3)^2 + (4 - 0)^2}$$
$$= \sqrt{(-2)^2 + 4^2} = \sqrt{20} = 2\sqrt{5}$$
$$P'Q' = \sqrt{(2 - 6)^2 + (3 - (-1))^2}$$
$$= \sqrt{(-4)^2 + 4^2} = \sqrt{32} = 4\sqrt{2}$$

Thus $PQ \neq P'Q'$.

Although the diagram for Example 2 shows only points of $\overline{PQ}$ and their image points, you should understand that mapping G maps *every* point of the plane to an image point. Also, every point of the plane has a preimage point. A one-to-one mapping from the whole plane to the whole plane is called a **transformation.** Moreover, if a transformation maps every segment to a congruent segment, it is called an **isometry.** The transformation in Example 2 is *not* an isometry because $PQ \neq P'Q'$.

By definition, an isometry maps any segment to a congruent segment, so we can say that an isometry *preserves* distance. The next theorem states that an isometry also maps any triangle to a congruent triangle. For this reason, an isometry is sometimes called a **congruence mapping.**