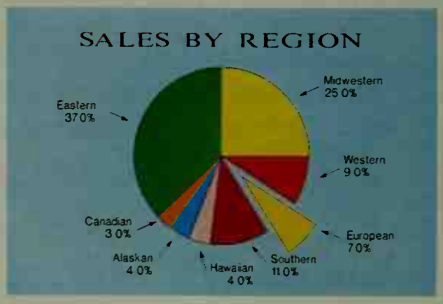
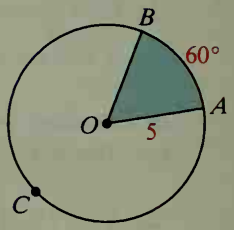
11-6 Arc Lengths and Areas of Sectors

A *pie chart* is often used to analyze data or to help plan business strategy. The radii of a pie chart divide the interior of the circle into regions called sectors, whose areas represent the relative sizes of particular items. A **sector of a circle** is a region bounded by two radii and an arc of the circle. The shaded region of the diagram at the right below is called sector AOB. The unshaded region is also a sector.



The length of $\overset{\frown}{AB}$ in circle O is part of the circumference of the circle. Since $m\overset{\frown}{AB} = 60$ and $\frac{60}{360} = \frac{1}{6}$, the length of $\overset{\frown}{AB}$ is $\frac{1}{6}$ of the circumference. Thus,



$$\text{Length of } \overset{\frown}{AB} = \frac{1}{6}(2\pi \cdot 5) = \frac{5}{3}\pi.$$

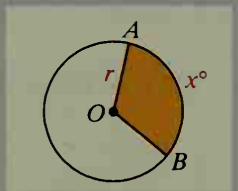
Similarly, the area of sector AOB is $\frac{1}{6}$ of the area of the circle. Thus,

$$\text{Area of sector } AOB = \frac{1}{6}(\pi \cdot 5^2) = \frac{25}{6}\pi.$$

In general, if $m\overset{\frown}{AB} = x$:

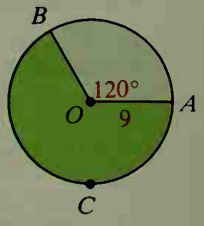
$$\text{Length of } \overset{\frown}{AB} = \frac{x}{360} \cdot 2\pi r$$

$$\text{Area of sector } AOB = \frac{x}{360} \cdot \pi r^2$$



Example 1 In $\odot O$ with radius 9, $m\angle AOB = 120$. Find the lengths of the arcs $\overset{\frown}{AB}$ and $\overset{\frown}{ACB}$ and the areas of the two sectors shown.

Solution $m\overset{\frown}{AB} = 120$, and $m\overset{\frown}{ACB} = 240$.



Minor arc $\overset{\frown}{AB}$:

$$\text{Length of } \overset{\frown}{AB} = \frac{120}{360} \cdot (2\pi \cdot 9) = \frac{1}{3}(18\pi) = 6\pi$$

$$\text{Area of sector } AOB = \frac{120}{360} \cdot (\pi \cdot 9^2) = \frac{1}{3}(81\pi) = 27\pi$$

Major arc $\overset{\frown}{ACB}$:

$$\text{Length of } \overset{\frown}{ACB} = \frac{240}{360} \cdot (2\pi \cdot 9) = \frac{2}{3}(18\pi) = 12\pi$$

$$\text{Area of sector} = \frac{240}{360} \cdot (\pi \cdot 9^2) = \frac{2}{3}(81\pi) = 54\pi$$