Calculating $[H_3O^+]$ and $[OH^-]$ from pH

You have now learned to calculate the pH of a solution, given its $[H_3O^+]$. Suppose that you are given the pH of a solution instead. How can you determine its hydronium ion concentration?

You already know the following equation.

$$pH = -\log [H_3O^+]$$

Remember that the base of common logarithms is 10. Therefore, the antilog of a common logarithm is 10 raised to that number.

$$\begin{aligned} \log [H_3O^+] &= -pH \\ [H_3O^+] &= \text{antilog } (-pH) \\ [H_3O^+] &= 10^{-pH} \end{aligned}$$

The simplest cases are those in which pH values are integers. The exponent of 10 that gives the $[H_3O^+]$ is the negative of the pH. For an aqueous solution that has a pH of 2, for example, the $[H_3O^+]$ is equal to 10^{-2} M. Likewise, when the pH is 0, the $[H_3O^+]$ is 1 M because $10^0 = 1$. Sample Problem D shows how to convert a pH value that is a positive integer. Sample Problem E shows how to use a calculator to convert a pH that is not an integral number.

SAMPLE PROBLEM D

For more help, go to the Math Tutor at the end of this chapter.

Determine the hydronium ion concentration of an aqueous solution that has a pH of 4.0.

SOLUTION

1 ANALYZE

Given: pH = 4.0
Unknown: $[H_3O^+]$

2 PLAN

$$pH \longrightarrow [H_3O^+]$$

This problem requires that you rearrange the pH equation and solve for the $[H_3O^+]$. Because 4.0 has one digit to the right of the decimal, the answer must have one significant figure.

$$\begin{aligned} pH &= -\log [H_3O^+] \\ \log [H_3O^+] &= -pH \\ [H_3O^+] &= \text{antilog } (-pH) \\ [H_3O^+] &= 1 \times 10^{-pH} \end{aligned}$$

3 COMPUTE

$$\begin{aligned} [H_3O^+] &= 1 \times 10^{-pH} \\ [H_3O^+] &= 1 \times 10^{-4} \text{ M} \end{aligned}$$

4 EVALUATE

A solution with a pH of 4.0 is acidic. The answer, 1×10^{-4} M, is greater than 1.0×10^{-7} M, which is correct for an acidic solution.