The Equivalence of Mass and Energy

Einstein's $E_R = mc^2$ is one of the most famous equations of the twentieth century. Einstein discovered this equation through his work with relative velocity and kinetic energy.

Relativistic kinetic energy

In the appendix feature "Special Relativity and Velocities," you learned how Einstein's special theory of relativity modifies the classical addition of velocities. The classical equation for kinetic energy ($KE = \frac{1}{2}mv^2$) must also be modified for relativity. In 1905, Einstein derived a new equation for kinetic energy based on the principles of special relativity:

$$KE = \frac{mc^2}{\sqrt{1 - \left(\frac{v^2}{c^2}\right)}} - mc^2$$

In this equation, m is the mass of the object, v is the velocity of the object, and c is the speed of light. Although it isn't immediately obvious, this equation reduces to the classical equation $KE = \frac{1}{2}mv^2$ for speeds that are small relative to the speed of light, as shown in **Figure 1.** The graph also illustrates that velocity can never be greater than $1.0c$ in the theory of special relativity.

Einstein's relativistic expression for kinetic energy has been confirmed by experiments in which electrons are accelerated to extremely high speeds in particle accelerators. In all cases, the experimental data correspond to Einstein's equation rather than to the classical equation. Nonetheless, the difference between the two theories at low speeds (relative to c) is so minimal that the classical equation can be used in all such cases when the speed is much less than c.

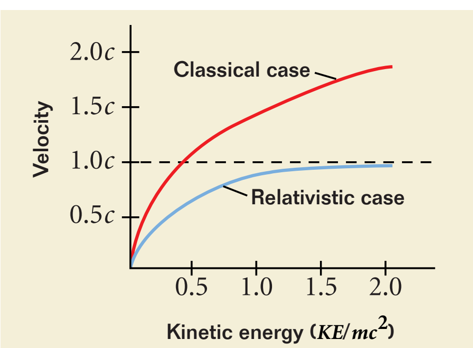

Figure 1
This graph of velocity versus kinetic energy for both the classical and relativistic equations shows that the two theories are in agreement when v is much less than c. Note that v is always less than c in the relativistic case.

Rest energy

The second term of Einstein's equation for kinetic energy, $-mc^2$, is required so that $KE = 0$ when $v = 0$. Note that this term is independent of velocity. This suggests that the *total* energy of an object equals its kinetic energy plus some additional form of energy equal to mc^2. The mathematical expression of this additional energy is the familiar Einstein equation:

$$E_R = mc^2$$

This equation shows that an object has a certain amount of energy (E_R), known as *rest energy,* simply by virtue of its mass. The