

Application

Passive Solar Design

Passive solar homes are designed to let the sun heat the house during the winter but to prevent the sun from heating the house during the summer. Because the Earth's axis is not perpendicular to the *ecliptic* (the plane of the Earth's orbit around the sun), the sun is lower in the sky in the winter than it is in the summer.

From the latitude of the homesite the architect can determine the elevation angle of the sun (the angle at which a person has to look up from the horizontal to see the sun at noon) during the winter and during the summer. The architect can then design an overhang for windows that will let sunlight in the windows during the winter, but will shade the windows during the summer.

The Earth's axis makes an angle of $23\frac{1}{2}^\circ$ with a perpendicular to the ecliptic plane. So for places in the northern hemisphere between the Tropic of Cancer and the Arctic Circle, the angle of elevation of the sun at noon on the longest day of the year, at the summer solstice, is $90^\circ - \text{the latitude} + 23\frac{1}{2}^\circ$. Its angle of elevation at noon on the shortest day, at the winter solstice, is $90^\circ - \text{the latitude} - 23\frac{1}{2}^\circ$. For example, in Terre Haute, Indiana, at latitude $39\frac{1}{2}^\circ$ north, the angle of elevation of the sun at noon on the longest day is 74° ($90 - 39\frac{1}{2} + 23\frac{1}{2} = 74$), and at noon on the shortest day it is 27° ($90 - 39\frac{1}{2} - 23\frac{1}{2} = 27$).



Exercises

Find the angle of elevation of the sun at noon on the longest day and at noon on the shortest day in the following cities. The approximate north latitudes are in parentheses.

1. Seattle, Washington ($47\frac{1}{2}^\circ$)
2. Chicago, Illinois (42°)
3. Houston, Texas (30°)
4. Los Angeles, California (34°)
5. Nome, Alaska ($64\frac{1}{2}^\circ$)
6. Miami, Florida (26°)
7. For a city south of the Tropic of Cancer, such as San Juan, Puerto Rico (18°N), the formula gives a summer solstice angle greater than 90° . What does this mean?
8. For a place north of the Arctic Circle, such as Prudhoe Bay, Alaska (70°N), the formula gives a negative value for the angle of elevation of the sun at noon at the winter solstice. What does this mean?