A conditional whose truth table contains only Ts in the last column is a tautology that represents a **valid argument.** A valid argument is true no matter what the truth or falsity of the components is. Its validity is independent of the real world. For example, the conditional "If p, then p" is always true, even if p stands for "Unicorns exist."

The geometrical theorems in this text are all valid (if you accept the Postulates), and you can use them with confidence in any logical argument. Every theorem in this text can be written as a tautology in the form of a conditional whose hypothesis is a conjunction of givens, definitions, and theorems, and whose conclusion is the statement that is to be proved.

Theorem: $[a \wedge b \wedge (c \rightarrow d) \wedge (e \rightarrow f)] \rightarrow g$
givens + definition + theorem = conclusion

In everyday situations, though, you must be careful to inspect the logic of arguments. Even though the reasoning is logically correct, the conclusion may be wrong. The problem is usually that some of the given premises or conditionals are wrong.

Example The following is a logically valid argument. Is the conclusion true?

1. The weather is sunny.
2. If the weather is sunny, the plane will arrive on time.
3. If the plane arrives on time, we will be able to ski today.
4. Therefore, we will be able to ski today.

Solution We cannot evaluate the truth of the conclusion unless we investigate all the premises. The first statement, a given, may not be accurate. Perhaps it is cloudy. Also, one or more of the remaining conditional statements might be wrong. Perhaps the plane will malfunction and be late even though the weather is sunny. Perhaps we won't be able to get to the ski area, even if the plane lands on time. Or maybe we don't even know how to ski! It is important to investigate the truth of every premise before you can draw meaningful conclusions.

Exercises

1. Make a truth table for each statement. Which is a tautology?

a. $(p \vee q) \rightarrow p$ **b.** $(p \rightarrow q) \rightarrow p$
c. $(p \wedge q) \rightarrow p$ **d.** $(p \rightarrow q) \rightarrow q$

2. Show that $(p \vee q \vee r) \vee \sim(p \wedge q \wedge r)$ is a tautology by making an eight-row truth table.

3. Show that the argument $(p \wedge q \wedge r) \rightarrow (p \vee q \vee r)$ is valid by making an eight-row truth table.