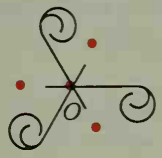
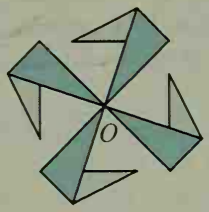
3. Make a group table for the three symmetries of this figure.



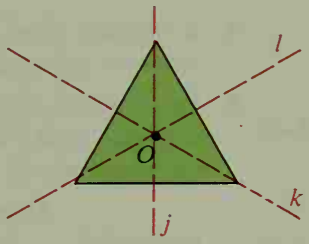
4. Make a group table for the four symmetries of this figure.



5. A transformation that is its own inverse is called a *self-inverse*.
- **a.** How many of the four symmetries of the figure in Exercise 4 are self-inverses?
- **b.** How many of the four symmetries of the rectangle are self-inverses?

6. A symmetry group is called *commutative* if $A \circ B = B \circ A$ for every pair of symmetries A and B in the group. The symmetry group of the rectangle is commutative, as you can see from the completed table. (For example, $H_O \circ R_j$ and $R_j \circ H_O$ are both equal to R_k.) Tell whether the groups in Exercises 3 and 4 are commutative or not.

7. An equilateral triangle has three rotational symmetries (I, $\mathcal{R}_{O,\,120}$, and $\mathcal{R}_{O,\,240}$) and three line symmetries (R_j, R_k, and R_l).
- **a.** Make a group table for these six symmetries.
- **b.** Give an example which shows that this group is *not* commutative.



8. A square has four rotational symmetries (including the identity) and four line symmetries. Make a group table for these eight symmetries. Is this a commutative group?

9. The four rotational symmetries of the square satisfy the four requirements for a group, and so they are called a *subgroup* of the full symmetry group. (Notice that the identity is one of these rotational symmetries and that the product of two rotations is another rotation in the subgroup.)
- **a.** Do the four line symmetries of the square form a subgroup?
- **b.** Does the symmetry group of the equilateral triangle have a subgroup?
- **c.** Which two symmetries of the figure in Exercise 4 form a subgroup?

10. The tessellation with fish on page 610 has translational symmetry. Let S be the horizontal translation mapping each fish to the fish of the same color to its right, and let T be the vertical translation mapping each fish to the fish of the same color directly above.
- **a.** Describe the mapping S^3. Is it a symmetry of the pattern?
- **b.** Describe T^{-1}. Is it a symmetry?
- **c.** Describe $S \circ T$. Is it a symmetry?
- **d.** How many symmetries does the tessellation have?
- **e.** Does this set of symmetries satisfy the four requirements for a group?