In the formula $(PH)^2 = AP \cdot BP$, PH is the distance from the observer to the horizon, and BP is the observer's height above the surface of the Earth. If the height is small compared to the diameter, AB, of the Earth, then $AP \approx AB$ in the formula. Using 12,800,000 m for AB, you can rewrite the formula as:

$$(\text{distance})^2 \approx (12{,}800{,}000)(\text{height})$$

Taking square roots, you get:

$$\text{distance} \approx \sqrt{12{,}800{,}000} \cdot \sqrt{\text{height}} \approx 3600\sqrt{\text{height}}$$

So the approximate distance (in meters) to the horizon is 3600 times the square root of your height (in meters) above the surface of the Earth. If your height is less than 400 km, the error in this approximation will be less than one percent.

Exercises

In Exercises 1 and 2 give your answer to the nearest kilometer, in Exercises 3 and 5 to the nearest 10 km, and in Exercise 4 to the nearest meter.

1. If you stand on a dune with your eyes about 16 m above sea level, how far out to sea can you look?
2. A lookout climbs high in the rigging of a sailing ship to a point 36 m above the water line. About how far away is the horizon?
3. From a balloon floating 10 km above the ocean, how far away is the farthest point you can see on the Earth's surface?
4. How high must a lookout be to see an object on the horizon 8 km away?
5. You are approaching the coast of Japan in a small sailboat. The highest point on the central island of Honshu is the cone of Mount Fuji, 3776 m above sea level. Roughly how far away from the mountain will you be when you can first see the top? (Assume that the sky is clear!)

Chapter Summary

1. Many of the terms used with circles and spheres are discussed on pages 329 and 330.
2. If a line is tangent to a circle, then the line is perpendicular to the radius drawn to the point of tangency. The converse is also true.
3. Tangents to a circle from a point are congruent.