

## SAMPLE PROBLEM C

### STRATEGY Adding Vectors Algebraically

#### PROBLEM

A hiker walks 27.0 km from her base camp at  $35^\circ$  south of east. The next day, she walks 41.0 km in a direction  $65^\circ$  north of east and discovers a forest ranger's tower. Find the magnitude and direction of her resultant displacement between the base camp and the tower.

#### SOLUTION

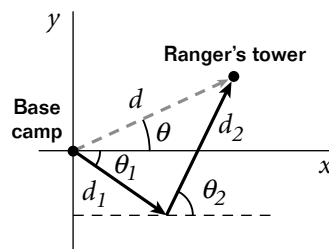
- 1. Select a coordinate system. Then sketch and label each vector.**

**Given:**  $d_1 = 27.0 \text{ km}$      $\theta_1 = -35^\circ$   
 $d_2 = 41.0 \text{ km}$      $\theta_2 = 65^\circ$

**Unknown:**  $d = ?$      $\theta = ?$



$\theta_1$  is negative, because clockwise angles from the positive  $x$ -axis are conventionally considered to be negative.



- 2. Find the  $x$  and  $y$  components of all vectors.**

Make a separate sketch of the displacements for each day. Use the cosine and sine functions to find the displacement components.

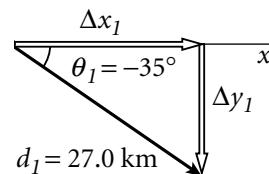
$$\cos \theta = \frac{\Delta x}{d} \qquad \sin \theta = \frac{\Delta y}{d}$$

(a) For day 1:  $\Delta x_1 = d_1 \cos \theta_1 = (27.0 \text{ km}) [\cos (-35^\circ)] = 22 \text{ km}$

$\Delta y_1 = d_1 \sin \theta_1 = (27.0 \text{ km}) [\sin (-35^\circ)] = -15 \text{ km}$

(b) For day 2:  $\Delta x_2 = d_2 \cos \theta_2 = (41.0 \text{ km}) (\cos 65^\circ) = 17 \text{ km}$

$\Delta y_2 = d_2 \sin \theta_2 = (41.0 \text{ km}) (\sin 65^\circ) = 37 \text{ km}$



- 3. Find the  $x$  and  $y$  components of the total displacement.**

$$\Delta x_{tot} = \Delta x_1 + \Delta x_2 = 22 \text{ km} + 17 \text{ km} = 39 \text{ km}$$

$$\Delta y_{tot} = \Delta y_1 + \Delta y_2 = -15 \text{ km} + 37 \text{ km} = 22 \text{ km}$$

- 4. Use the Pythagorean theorem to find the magnitude of the resultant vector.**

$$d^2 = (\Delta x_{tot})^2 + (\Delta y_{tot})^2$$

$$d = \sqrt{(\Delta x_{tot})^2 + (\Delta y_{tot})^2} = \sqrt{(39 \text{ km})^2 + (22 \text{ km})^2} = 45 \text{ km}$$

- 5. Use a suitable trigonometric function to find the angle.**

$$\theta = \tan^{-1} \left( \frac{\Delta y_{tot}}{\Delta x_{tot}} \right) = \tan^{-1} \left( \frac{22 \text{ km}}{39 \text{ km}} \right) = 29^\circ \text{ north of east}$$

