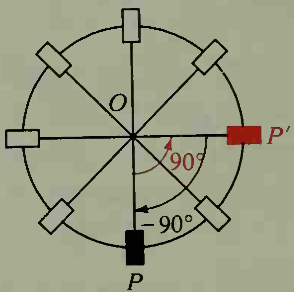
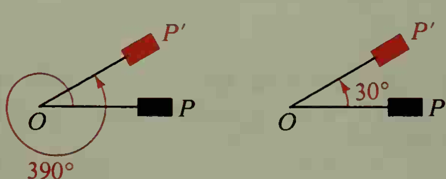
14-4 *Rotations*

A *rotation* is a transformation suggested by rotating a paddle wheel. When the wheel moves, each paddle rotates to a new position. When the wheel stops, the new position of a paddle (P') can be referred to mathematically as the image of the initial position of the paddle (P).

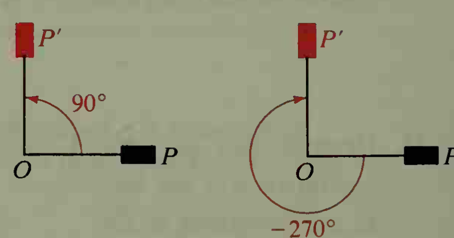


For the counterclockwise rotation shown about point O through 90°, we write $\mathcal{R}_{O,\,90}$. A counterclockwise rotation is considered positive, and a clockwise rotation is considered negative. If the red paddle is rotated about O clockwise until it moves into the position of the black paddle, the rotation is denoted by $\mathcal{R}_{O,\,-90}$. (Note that to avoid confusion with the R used for reflections we use a script $\mathcal{R}$ for rotations.)

A full revolution, or 360° rotation about point O, rotates any point P around to itself so that $P' = P$. The diagram at the left below shows a rotation of 390° about O. Since 390° is 30° more than one full revolution, the image of any point P under a 390° rotation is the same as its image under a 30° rotation, and the two rotations are said to be equal. Similarly, the diagram at the right below shows that a 90° counterclockwise rotation is equal to a 270° clockwise rotation because both have the same effect on any point P.



$\mathcal{R}_{O,\,390} = \mathcal{R}_{O,\,30}$
Notice: $390 - 360 = 30$



$\mathcal{R}_{O,\,90} = \mathcal{R}_{O,\,-270}$
Notice: $90 - 360 = -270$

In the following definition of a rotation, the angle measure x can be positive or negative and can be more than 180 in absolute value.

A **rotation** about point O through $x°$ is a transformation such that:

(1) If a point P is different from O, then $OP' = OP$ and $m\angle POP' = x$.
(2) If point P is the point O, then $P' = P$.