A fractal will not have dimension of 1, 2, or 3. But you can use your knowledge of shapes having these dimensions to form an idea of the dimension of a fractal. You can estimate that the dimension of a curve (or a coastline) that looks almost like a line will have dimension close to 1, while the dimension of a very jagged curve (or a coastline) that tends toward filling a plane will have dimension closer to 2.

In this next example, the dimension of the Koch curve is found.

Example 2 A Koch curve can be replaced with 4 curves similar to itself with scale factor $\frac{1}{3}$. In this case, $N = 4$ and $R = 3$. Then we must solve $4 = 3^D$, where D is the self-similarity dimension.

Solution 1 Find an approximation by using a calculator and the $\boxed{y^x}$ key.
Since $3^1 = 3 \leq 4 \leq 9 = 3^2$, $1 \leq D \leq 2$.
Since $3^{1.2} = 3.73 \leq 4 \leq 4.17 = 3^{1.3}$, $1.2 \leq D \leq 1.3$.
Since $3^{1.26} = 3.99 \leq 4 \leq 4.04 = 3^{1.27}$, $1.26 \leq D \leq 1.27$.
Thus $D = 1.26$ to two decimal places.

Solution 2 Find a more accurate answer using a calculator and logarithms. If $4 = 3^D$, then $D = \dfrac{\log 4}{\log 3}$. Using the $\boxed{\log}$ key on a calculator, you will find that D is approximately 1.2618595071429.

Fractals are geometric shapes that are often defined by the following ideas of similarity and dimension:
1. the shape is "similar to itself," or self-similar;
2. the shape has self-similarity dimension that is not an integer.

Exercises

Find the self-similarity dimension of the fractals defined in Exercises 1–5 on pages 688–689. Values for N and R are listed below. Express your answer to the nearest hundredth.

1. $N = 8, R = 4$ **2.** $N = 5, R = 3$ **3.** $N = 4, R = \frac{9}{4}$

4. $N = 4, R = \frac{5}{2}$ **5.** $N = 9, R = 3$

6. Find the self-similarity dimension of the Sierpiński gasket. First try to predict between which two integers the dimension will be.

7. Find the self-similarity dimension of the Cantor set. First try to predict between which two integers the dimension will be.

8. Create your own fractal and find its self-similarity dimension.