

Chapter Summary

1. A transformation is a one-to-one mapping from the whole plane to the whole plane. If the transformation S maps P to P' , we write $S:P \rightarrow P'$ or $S(P) = P'$.
2. The word "mapping" is used in geometry as the word "function" is used in algebra. If the function f maps every number to its square we write $f:x \rightarrow x^2$ or $f(x) = x^2$.
3. An isometry is a transformation that preserves distance. An isometry maps any figure to a congruent figure.

4. Some basic isometries are:

Reflection in a line. R_j is a reflection in line j .

Translation or glide. $T:(x, y) \rightarrow (x + a, y + b)$ is a translation.

Rotation about a point. $\mathcal{R}_{O,x}$ is a rotation counterclockwise about O through x° . H_O is a half-turn about O .

Glide reflection. A glide followed by a reflection in a line parallel to the glide yields a glide reflection.

5. A dilation maps any figure to a similar figure. $D_{O,k}$ is a dilation with center O and nonzero scale factor k . A dilation is an isometry if $|k| = 1$.
6. Properties of figures that are preserved by a transformation are said to be invariant under that transformation. Invariant properties are checked in the table below.

	Distance	Angle Measure	Parallelism	Ratio of distances	Area
Isometry:	✓	✓	✓	✓	✓
Dilation:		✓	✓	✓	

7. The combination of one mapping followed by another is called a composite or product of mappings. The mapping A followed by B is written $B \circ A$.
8. A composite of isometries is an isometry.
 - A composite of reflections in two parallel lines is a translation.
 - A composite of reflections in two intersecting lines is a rotation.
 - A composite of reflections in two perpendicular lines is a half-turn.
9. The identity transformation I keeps all points fixed. A transformation S followed by its inverse S^{-1} is equal to the identity.
10. A symmetry of a figure is an isometry that maps the figure onto itself. Figures can have line symmetry, point symmetry, and rotational symmetry. A tessellation, or covering of the plane with congruent figures, may also have translational and glide reflection symmetry. Solid figures in space can have planes of symmetry and rotational symmetry about an axis.