The greater the spring constant (k), the stiffer the spring; hence a greater force is required to stretch or compress the spring. When force is greater, acceleration is greater and the amount of time required for a single cycle should decrease (assuming that the amplitude remains constant). Thus, for a given amplitude, a stiffer spring will take less time to complete one cycle of motion than one that is less stiff.

As with the pendulum, the equation for the period of a mass-spring system can be derived mathematically or found experimentally.

PERIOD OF A MASS-SPRING SYSTEM IN SIMPLE HARMONIC MOTION

$$T = 2\pi\sqrt{\frac{m}{k}}$$

period = 2π × square root of (mass divided by spring constant)

Note that changing the amplitude of the vibration does not affect the period. This statement is true only for systems and circumstances in which the spring obeys Hooke's law.

SAMPLE PROBLEM C

Simple Harmonic Motion of a Mass-Spring System

PROBLEM

The body of a 1275 kg car is supported on a frame by four springs. Two people riding in the car have a combined mass of 153 kg. When driven over a pothole in the road, the frame vibrates with a period of 0.840 s. For the first few seconds, the vibration approximates simple harmonic motion. Find the spring constant of a single spring.

SOLUTION

Given: $m = \dfrac{(1275\text{ kg} + 153\text{ kg})}{4} = 357\text{ kg}$ $\quad T = 0.840\text{ s}$

Unknown: $k = ?$

Use the equation for the period of a mass-spring system, and solve for k.

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$T^2 = 4\pi^2\left(\frac{m}{k}\right)$$

$$k = \frac{4\pi^2 m}{T^2} = \frac{4\pi^2(357\text{ kg})}{(0.840\text{ s})^2}$$

$$\boxed{k = 2.00 \times 10^4\text{ N/m}}$$