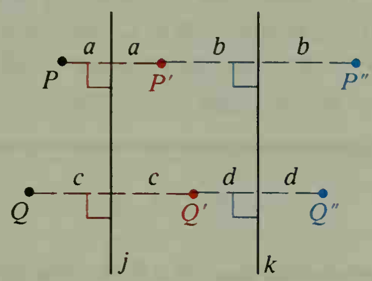
Theorem 14-6

The composite of two isometries is an isometry.

Theorem 14-7

A composite of reflections in two parallel lines is a translation. The translation glides all points through twice the distance from the first line of reflection to the second.

Although we will not present a formal proof of Theorem 14-7, the following argument should convince you that it is true. Assume that $j \parallel k$ and that R_j maps P to P' and Q to Q', and that R_k maps P' to P'' and Q' to Q''. To show that the composite $R_k \circ R_j$ is a translation we will demonstrate that $PP'' = QQ''$ and that $\overleftrightarrow{PP''}$ and $\overleftrightarrow{QQ''}$ are parallel.



The letters a, b, c, and d in the diagram label pairs of distances that are equal according to the definition of a reflection. P, P', and P'' are collinear and

$$PP'' = 2a + 2b = 2(a + b)$$

Similarly,

$$QQ'' = 2c + 2d = 2(c + d)$$

But $(a + b) = (c + d)$, since by Theorem 5-8, the distance between the parallel lines j and k is constant. Therefore $PP'' = QQ'' =$ twice the distance from j to k.

That $\overleftrightarrow{PP''}$ and $\overleftrightarrow{QQ''}$ are parallel follows from the fact that both lines are perpendicular to j and k. Theorem 3-7 guarantees that if two lines in a plane are perpendicular to the same line, then the two lines are parallel.

You should make diagrams for the case when P is on j or k, when P is located between j and k, and when P is to the right of k. Convince yourself that $PP'' = 2(a + b)$ in these cases also. In every case, the glide is perpendicular to j and k and goes in the direction from j to k (that is, from the first line of reflection toward the second line of reflection).

Theorem 14-7 shows that when lines j and k are parallel, $R_k \circ R_j$ translates points through twice the distance between the lines. If lines j and k intersect, $R_k \circ R_j$ rotates points through twice the measure of the angle between the lines. This is our next theorem.