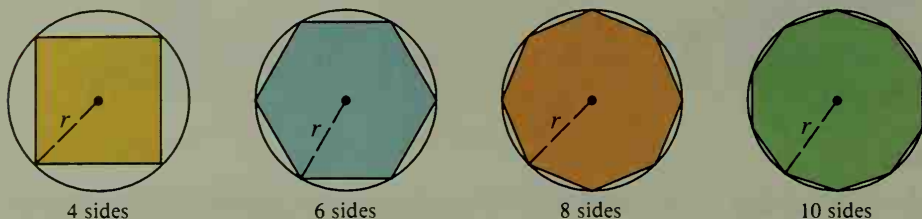


First consider a sequence of regular polygons inscribed in a circle with radius  $r$ . Four such polygons are shown below. Imagine that the number of sides of the regular polygons continues to increase. As you can see in the diagrams, the more sides a regular polygon has, the closer it approximates (or “fits”) the curve of the circle.



Now consider the perimeters and the areas of this sequence of regular polygons. The table below contains values that are approximations (using trigonometry) of the perimeters and the areas of regular polygons in terms of the radius,  $r$ .

As the table suggests, these perimeters give us a sequence of numbers that get closer and closer to a limiting number. This limiting number is defined to be the perimeter, or **circumference**, of the circle.

The area of a circle is defined in a similar way. The areas of the inscribed regular polygons get closer and closer to a limiting number, defined to be the **area** of the circle.

The results in the table suggest that the circumference and the area of a circle with radius  $r$  are *approximately*  $6.28r$  and  $3.14r^2$ .

Number of Sides of Polygon	Perimeter	Area
4	$5.66r$	$2.00r^2$
6	$6.00r$	$2.60r^2$
8	$6.12r$	$2.83r^2$
10	$6.18r$	$2.93r^2$
20	$6.26r$	$3.09r^2$
30	$6.27r$	$3.12r^2$
100	$6.28r$	$3.14r^2$

The exact values are given by the formulas below. (Proofs are suggested in Classroom Exercises 13 and 14 and Written Exercise 33.)

Circumference,  $C$ , of circle with radius  $r$ :  $C = 2\pi r$

Circumference,  $C$ , of circle with diameter  $d$ :  $C = \pi d$

Area,  $A$ , of circle with radius  $r$ :  $A = \pi r^2$

These formulas involve a famous number denoted by the Greek letter  $\pi$  (*pi*), which is the first letter in a Greek word that means “measure around.” The number  $\pi$  is the ratio of the circumference of a circle to the diameter. This ratio is a constant for *all* circles. Because  $\pi$  is an irrational number, there isn’t any decimal or fraction that expresses the constant number  $\pi$  exactly. Here are some common approximations for  $\pi$ :

$$3.14 \qquad \frac{22}{7} \qquad 3.1416 \qquad 3.14159$$