★ **11.** If you apply the Koch curve construction to an equilateral triangle, you get the sequence of pre-fractals shown below. Calculate the area of each pre-fractal. If you can, calculate the area of the fractal. (*Hint*: Let the area of the original equilateral triangle be 1 square unit.)

Dimension of Fractals

The dimension of a fractal is usually not an integer. Measuring the dimension of a fractal involves finding its *self-similarity dimension*, defined below.

In general, if a shape can be replaced, or covered, by N shapes similar to itself with scale factor $\frac{1}{R}$, then it has **self-similarity dimension** D, where $N = R^D$.

The example below demonstrates the use of this definition with shapes whose dimensions you already know.

Example 1

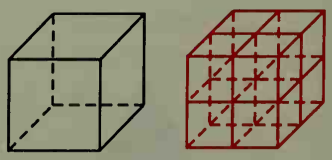
a. A *segment* can be replaced by 2 segments similar to itself with scale factor $\frac{1}{2}$. In this case, $N = 2$ and $R = 2$. Since $2 = (2)^1$, we will say that the segment has self-similarity dimension 1.

Dimension 1

b. A *square* can be replaced by 4 squares similar to itself with scale factor $\frac{1}{2}$. In this case, $N = 4$ and $R = 2$. Since $4 = 2^2$, we will say that the square has self-similarity dimension 2.

Dimension 2

c. A *cube* can be replaced by 8 cubes similar to itself with scale factor $\frac{1}{2}$. In this case, $N = 8$ and $R = 2$. Since $8 = 2^3$, we will say that the cube has self-similarity dimension 3.



Dimension 3