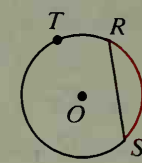
9-4 Arcs and Chords

In $\odot O$ shown at the right, $\overline{RS}$ cuts off two arcs, $\overset{\frown}{RS}$ and $\overset{\frown}{RTS}$. We speak of $\overset{\frown}{RS}$, the minor arc, as being *the arc of chord* $\overline{RS}$.



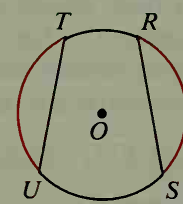
Theorem 9-4

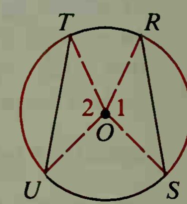
In the same circle or in congruent circles:

(1) Congruent arcs have congruent chords.

(2) Congruent chords have congruent arcs.

Here is a paragraph proof of part (1) for one circle. You will be asked to write a paragraph proof of part (2) in Written Exercise 16.





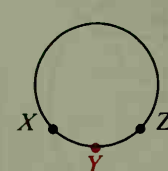
Given: $\odot O$; $\overset{\frown}{RS} \cong \overset{\frown}{TU}$

Prove: $\overline{RS} \cong \overline{TU}$

Proof:

Draw radii $\overline{OR}$, $\overline{OS}$, $\overline{OT}$, and $\overline{OU}$. $\overline{OR} \cong \overline{OT}$ and $\overline{OS} \cong \overline{OU}$ because they are all radii of the same circle. Since $\overset{\frown}{RS} \cong \overset{\frown}{TU}$, central angles 1 and 2 are congruent. Then $\triangle ROS \cong \triangle TOU$ by SAS and corresponding parts $\overline{RS}$ and $\overline{TU}$ are congruent.

A point Y is called the *midpoint* of $\overset{\frown}{XYZ}$ if $\overset{\frown}{XY} \cong \overset{\frown}{YZ}$. Any line, segment, or ray that contains Y bisects $\overset{\frown}{XYZ}$.



Theorem 9-5

A diameter that is perpendicular to a chord bisects the chord and its arc.

Given: $\odot O$; $\overline{CD} \perp \overline{AB}$

Prove: $\overline{AZ} \cong \overline{BZ}$; $\overset{\frown}{AD} \cong \overset{\frown}{BD}$

Plan for Proof: Draw $\overline{OA}$ and $\overline{OB}$. Then use the HL Theorem to prove that $\triangle OZA \cong \triangle OZB$. Then use corresponding parts of congruent triangles to show that $\overline{AZ} \cong \overline{BZ}$ and $\angle 1 \cong \angle 2$. Finally, apply the theorem that congruent central angles have congruent arcs.