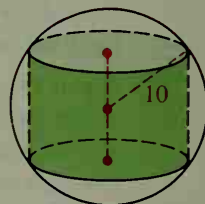**29.** A circle with diameter 9 in. is rotated about a diameter. Find the area and volume of the solid formed.

30. A cylinder with height 12 is inscribed in a sphere with radius 10. Find the volume of the cylinder.

C **31.** A cylinder with height $2x$ is inscribed in a sphere with radius 10.

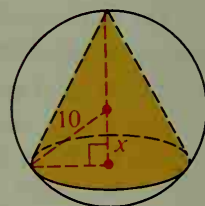**a.** Show that the volume of the cylinder, V, is
$$2\pi x(100 - x^2).$$

b. By using calculus, one can show that V is maximum when $x = \dfrac{10\sqrt{3}}{3}$. Substitute this value for x to find the maximum volume V.

Exs. 30, 31

c. (Optional) Use a calculator or a computer to evaluate $V = 2\pi x(100 - x^2)$ for various values of x between 0 and 10. Show that the maximum volume V occurs when $x \approx 5.77$.

32. A cone is inscribed in a sphere with radius 10, as shown.

a. Show that the volume of the cone, V, is
$$-\tfrac{1}{3}\pi(100 - x^2)(10 + x).$$

b. By using calculus, one can show that V is maximum when $x = \tfrac{10}{3}$. Substitute this value for x to find the maximum volume V.

c. (Optional) Use a calculator or a computer to evaluate the volume for various values of x between 0 and 10. Show that the maximum volume V occurs when $x = \tfrac{10}{3}$.

33. Sketch two intersecting spheres with radii 15 cm and 20 cm, respectively. The centers of the spheres are 25 cm apart. Find the area of the circle that is formed by the intersection. (*Hint*: Use Exercise 42 on page 289.)

34. A sphere is inscribed in a cone with radius 6 cm and height 8 cm. Find the volume of the sphere.

Challenge

In a training exercise, two research submarines are assigned to take any two positions in the ocean at a depth between 100 m and 500 m and within a square area with 6000 m sides. If one submarine takes its position in the center of the square area at the 100-meter depth, what is the probability that the other submarine is within 400 m? Use $\pi \approx 3.14$. (*Hint*: Consider each submarine to be a point in a rectangular solid. Use the ideas of geometric probability.)

6000 m

6000 m

100 m depth

500 m depth

(Not to scale)