Using Deductive Reasoning

Objectives

- 1. Recognize the hypothesis and the conclusion of an if-then statement.
- 2. State the converse of an if-then statement.
- 3. Use a counterexample to disprove an if-then statement.
- 4. Understand the meaning of if and only if.
- 5. Use properties from algebra and properties of congruence in proofs.
- 6. Use the Midpoint Theorem and the Angle Bisector Theorem.
- 7. Know the kinds of reasons that can be used in proofs.

2-1 *If-Then Statements; Converses*

Your friend says, "If it rains after school, then I will give you a ride home."

A geometry student reads, "If B is between A and C, then AB + BC = AC."

These are examples of if-then statements, which are also called conditional statements or simply conditionals.

To represent an if-then statement symbolically, let p represent the **hypothesis**, shown in red, and let q represent the **conclusion**, shown in blue. Then we have the basic form of an if-then statement shown below:

The converse of a conditional is formed by interchanging the hypothesis and the conclusion.

Statement: If p, then q. Converse: If q, then p.

A statement and its converse say different things. In fact, some true statements have false converses.

Statement: If Ed lives in Texas, then he lives south of Canada. False Converse: If Ed lives south of Canada, then he lives in Texas.

An if-then statement is false if an example can be found for which the hypothesis is true and the conclusion is false. Such an example is called a counterexample. It takes only one counterexample to disprove a statement. We know the converse above is false because we can find a counterexample: Ed could live in Kansas City, which is south of Canada and is not in Texas.

Some true statements have true converses.

Statement: If 4x = 20, then x = 5. True Converse: If x = 5, then 4x = 20.