

Method 2 Show that both pairs of opposite sides are parallel.

$$\text{Slope of } \overline{AB} = \frac{4 - 1}{6 - 2} = \frac{3}{4};$$

$$\text{slope of } \overline{DC} = \frac{7 - 4}{3 - (-1)} = \frac{3}{4}.$$

$$\text{Slope of } \overline{AD} = \frac{4 - 1}{-1 - 2} = \frac{3}{-3} = -1;$$

$$\text{slope of } \overline{BC} = \frac{7 - 4}{3 - 6} = \frac{3}{-3} = -1.$$

Method 3 Show that the diagonals bisect each other.

$$\text{Midpoint of } \overline{AC} = \left(\frac{3 + 2}{2}, \frac{1 + 7}{2} \right) = \left(\frac{5}{2}, 4 \right);$$

$$\text{midpoint of } \overline{BD} = \left(\frac{-1 + 6}{2}, \frac{4 + 4}{2} \right) = \left(\frac{5}{2}, 4 \right).$$

Since the diagonals have the same midpoint, they bisect each other.

You could also use the work in Methods 1 and 2 to show that one pair of opposite sides is both congruent and parallel.

Exercises

The coordinates of the vertices of quadrilateral $ABCD$ are given. Show that $ABCD$ is a parallelogram by using each of the three methods shown in the example.

1. $A(5, 7)$, $B(0, 3)$, $C(1, -3)$, $D(6, 1)$
2. $A(-2, 6)$, $B(-3, 2)$, $C(2, -4)$, $D(3, 0)$

Decide whether quadrilateral $DEFG$ is a parallelogram.

3. $D(3, 5)$, $E(5, 7)$, $F(3, 4)$, $G(0, 1)$
4. $D(3, -2)$, $E(-2, 5)$, $F(5, 6)$, $G(10, -1)$

The coordinates of three vertices of $\square PQRS$ are given. Find the coordinates of the missing vertex.

5. $P(0, 0)$, $Q(5, 2)$, $R(8, 4)$
6. $P(-2, 0)$, $Q(2, 1)$, $S(0, 5)$
7.
 - a. Draw the triangle with vertices $O(0, 0)$, $I(4, 2)$, and $J(2, 6)$.
 - b. Find the coordinates of M and N , the midpoints of \overline{OJ} and \overline{IJ} , respectively.
 - c. Find the slopes of \overline{MN} and \overline{OI} . What do your results tell you about \overline{MN} and \overline{OI} ? What kind of quadrilateral is $OMNI$?
8. Repeat Exercise 7 for the general triangle with vertices $O(0, 0)$, $I(a, 0)$, and $J(b, c)$.
9. Given the quadrilateral $ABCD$ with vertices $A(-4, 5)$, $B(4, -1)$, $C(7, 3)$, and $D(-1, 9)$.
 - a. Use slopes to show that opposite sides are parallel and adjacent sides are perpendicular.
 - b. What kind of quadrilateral is $ABCD$?