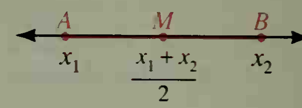
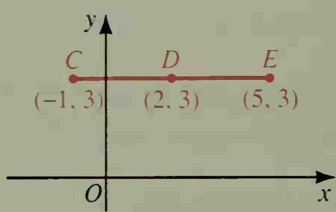
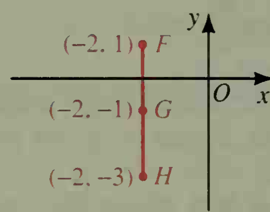
13-5 *The Midpoint Formula*

On a number line, if points A and B have coordinates x_1 and x_2, then the midpoint of $\overline{AB}$ has coordinate $\frac{x_1 + x_2}{2}$, the average of x_1 and x_2. (See Exercise 19 on page 47.)



This idea can be used to find the midpoint of any horizontal or vertical segment.



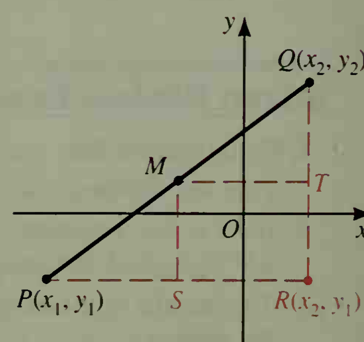


If a segment $\overline{PQ}$ is neither horizontal nor vertical, then the coordinates of its midpoint M can be found by drawing horizontal and vertical auxiliary lines as shown.

Since M is the midpoint of $\overline{PQ}$ and $\overline{MS} \parallel \overline{QR}$, S is the midpoint of $\overline{PR}$ (Theorem 5-10). Thus both S and M have x-coordinate $\frac{x_1 + x_2}{2}$.

Similarly, $\overline{MT} \parallel \overline{PR}$, so T is the midpoint of $\overline{QR}$. Thus both T and M have y-coordinate $\frac{y_1 + y_2}{2}$.

This discussion leads to the following theorem.



Theorem 13-5 *The Midpoint Formula*

The midpoint of the segment that joins points (x_1, y_1) and (x_2, y_2) is the point

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

Example 1 Find the midpoint of the segment that joins $(-11, 3)$ and $(8, -7)$.

Solution The x-coordinate of the midpoint is

$$\frac{x_1 + x_2}{2} = \frac{-11 + 8}{2} = \frac{-3}{2}, \text{ or } -\frac{3}{2}.$$

The y-coordinate of the midpoint is

$$\frac{y_1 + y_2}{2} = \frac{3 - 7}{2} = \frac{-4}{2} = -2.$$

The midpoint is $\left(-\frac{3}{2}, -2\right)$.