Bernoulli's equation

Imagine a fluid moving through a pipe of varying cross-sectional area and elevation, as shown in **Figure 2.** When the cross-sectional area changes, the pressure and speed of the fluid can change. This change in kinetic energy may be compensated for by a change in gravitational potential energy or by a change in pressure (so energy is still conserved). The expression for the conservation of energy in fluids is called *Bernoulli's equation*. Bernoulli's equation is expressed mathematically as follows:

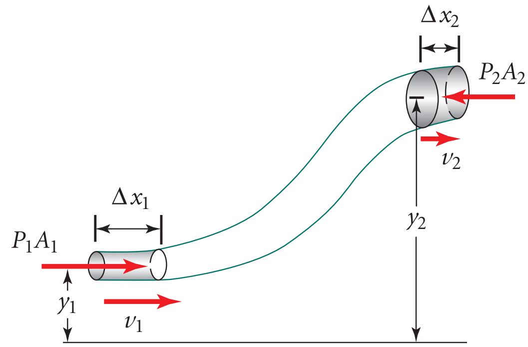

Figure 2
As a fluid flows through this pipe, it may change velocity, pressure, and elevation.

BERNOULLI'S EQUATION

$$P + \tfrac{1}{2}\rho v^2 + \rho g h = \text{constant}$$

pressure + kinetic energy per unit volume + gravitational potential energy per unit volume = constant along a given streamline

Bernoulli's equation differs slightly from the law of conservation of energy. For example, two of the terms on the left side of the equation look like the terms for kinetic energy and gravitational potential energy, but they contain density, ρ, instead of mass, m. The reason is that the conserved quantity in Bernoulli's equation is energy per unit volume, not just energy. This statement of the conservation of energy in fluids also includes an additional term: pressure, P. If you wish to compare the energy in a given volume of fluid at two different points, Bernoulli's equation takes the following equivalent form:

$$P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g h_2$$

extension

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Comparing Bernoulli's principle and Bernoulli's equation

Two special cases of Bernoulli's equation are worth mentioning here. First, if the fluid is at rest, then both speeds are zero. This case is a static situation, such as a column of water in a cylinder. If the height at the top of the column, h_1, is defined as zero and h_2 is the depth, then Bernoulli's equation reduces to the equation for pressure as a function of depth, introduced in the chapter on fluids:

$$P_1 = P_2 + \rho g h_2 \qquad \text{(static fluid)}$$

Second, imagine again a fluid flowing through a horizontal pipe with a constriction. Because the height of the fluid is constant, the gravitational potential energy does not change. Bernoulli's equation then reduces to the following:

$$P_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \tfrac{1}{2}\rho v_2^2 \qquad \text{(horizontal pipe)}$$

This equation suggests that if v_1 is greater than v_2 at two different points in the flow, then P_1 must be less than P_2. In other words, the pressure decreases as speed increases—Bernoulli's principle. Thus, Bernoulli's principle is a special case of Bernoulli's equation and is strictly true only when elevation is constant.