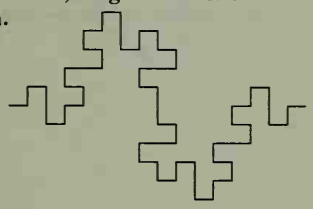
# Fractal Geometry

## Exercises, Pages 685–686

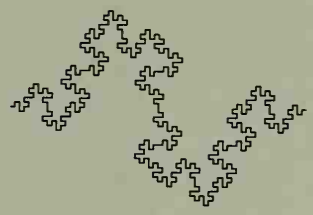
**1.** For ruler lengths 1 in., $\frac{1}{2}$ in., $\frac{1}{4}$ in.: number of sides 7, 15, 30; perimeters 7 in., $7\frac{1}{4}$ in., $7\frac{1}{2}$ in. **3. a.** Yes; a typical sequence of approximations is: 17 in., $18\frac{1}{4}$ in., $18\frac{3}{4}$ in. **b.** The approximate length of the circumference will always be less than $6\pi$, or 18.85, in. **c.** The circumference of a circle has a measurable length; a jagged coastline does not.

## Exercises, Pages 688–690

**1. a.**

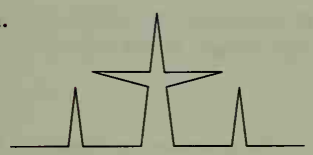


**b.**

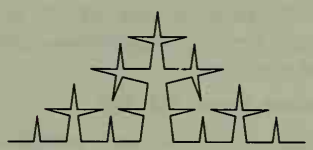


**c.** For levels 0, 1, 2, 3:
edge lengths 1, $\frac{1}{4}$, $\frac{1}{16}$, $\frac{1}{64}$;
number of edges 1, 8, 64, 512;
total lengths 1, 2, 4, 8

**3. a.**

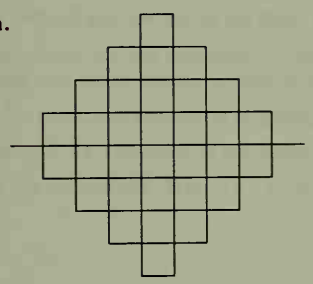


**b.**

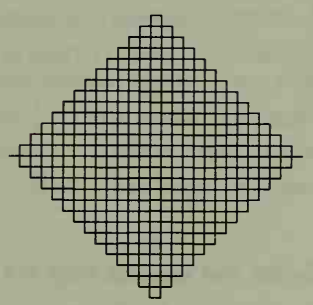


**c.** For levels 0, 1, 2, 3:
edge lengths 1, $\frac{4}{9}$, $\frac{16}{81}$, $\frac{64}{729}$;
number of edges 1, 4, 16, 64;
total lengths 1, $1\frac{7}{9}$, $3\frac{13}{81}$, $5\frac{451}{729}$

**5. a.**



**b.**



**c.** For levels 0, 1, 2, 3:
edge lengths 1, $\frac{1}{3}$, $\frac{1}{9}$, $\frac{1}{27}$;
number of edges 1, 9, 81, 729;
total lengths 1, 3, 9, 27

**7. a.** As the pre-fractal level increases, the edge length for the Cantor set decreases by a factor of $\frac{1}{3}$. **b.** The number of edges increases by a factor of 2. **c.** The sum of the lengths decreases by a factor of $\frac{2}{3}$.

**9.** The level 3 pre-fractal for the Cantor set is defined by the following 16 endpoints. (The boldface points were given as endpoints of the level 1 pre-fractal.)

$(\mathbf{0}, \mathbf{0})$, $\left(\frac{1}{27}, 0\right)$, $\left(\frac{2}{27}, 0\right)$, $\left(\frac{1}{9}, 0\right)$, $\left(\frac{2}{9}, 0\right)$, $\left(\frac{7}{27}, 0\right)$, $\left(\frac{8}{27}, 0\right)$, $\left(\mathbf{\frac{1}{3}}, \mathbf{0}\right)$, $\left(\mathbf{\frac{2}{3}}, \mathbf{0}\right)$, $\left(\frac{19}{27}, 0\right)$, $\left(\frac{20}{27}, 0\right)$, $\left(\frac{7}{9}, 0\right)$, $\left(\frac{8}{9}, 0\right)$, $\left(\frac{25}{27}, 0\right)$, $\left(\frac{26}{27}, 0\right)$, $(\mathbf{1}, \mathbf{0})$. **11.** The area of the snowflake fractal is $1 + \frac{1}{3} + \frac{4}{27} + \frac{16}{243} + \ldots = 1.6$ times the area of the original equilateral triangle.

## Exercises, Page 691

**1.** 1.5 **3.** 1.71 **5.** 2 **7.** $N = 2$, $R = 3$, $D \approx 0.63$