SAMPLE PROBLEM D

Conservation of Momentum

PROBLEM

A 76 kg boater, initially at rest in a stationary 45 kg boat, steps out of the boat and onto the dock. If the boater moves out of the boat with a velocity of 2.5 m/s to the right, what is the final velocity of the boat?

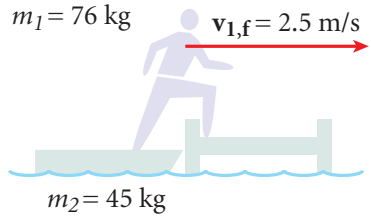
SOLUTION

1. DEFINE

Given: $m_1 = 76\text{ kg}$ $m_2 = 45\text{ kg}$
$\mathbf{v_{1,i}} = 0$ $\mathbf{v_{2,i}} = 0$
$\mathbf{v_{1,f}} = 2.5\text{ m/s to the right}$

Unknown: $\mathbf{v_{2,f}} = ?$

Diagram:



2. PLAN

Choose an equation or situation: Because the total momentum of an isolated system remains constant, the total initial momentum of the boater and the boat will be equal to the total final momentum of the boater and the boat.

$$m_1\mathbf{v_{1,i}} + m_2\mathbf{v_{2,i}} = m_1\mathbf{v_{1,f}} + m_2\mathbf{v_{2,f}}$$

Because the boater and the boat are initially at rest, the total initial momentum of the system is equal to zero. Therefore, the final momentum of the system must also be equal to zero.

$$m_1\mathbf{v_{1,f}} + m_2\mathbf{v_{2,f}} = 0$$

Rearrange the equation to solve for the final velocity of the boat.

$$m_2\mathbf{v_{2,f}} = -m_1\mathbf{v_{1,f}}$$

$$\mathbf{v_{2,f}} = -\frac{m_1}{m_2}\mathbf{v_{1,f}}$$

3. CALCULATE

Substitute the values into the equation and solve:

$$\mathbf{v_{2,f}} = -\frac{76\text{ kg}}{45\text{ kg}}\,(2.5\text{ m/s to the right})$$

$$\mathbf{v_{2,f}} = -4.2\text{ m/s to the right}$$

4. EVALUATE

The negative sign for $\mathbf{v_{2,f}}$ indicates that the boat is moving to the left, in the direction *opposite* the motion of the boater. Therefore,

$$\boxed{\mathbf{v_{2,f}} = 4.2\text{ m/s to the left}}$$