♦ Calculator Key-In

The number π is an irrational number. It cannot be expressed exactly as the ratio of two integers. Decimal *approximations* of π have been computed to thousands of decimal places. We can easily look up values in reference books, but such was not always the case. In the past, mathematicians had to rely on their cleverness to compute an approximate value of π. One of the earliest approximations was that of Archimedes, who found that $3\frac{1}{7} > \pi > 3\frac{10}{71}$.

Exercises

1. Find decimal approximations of $3\frac{1}{7}$ and $3\frac{10}{71}$. Did Archimedes approximate π correct to hundredths?

In Exercises 2–4, find approximations for π. The more terms or factors you use, the better your approximations will be.

2. $\pi \approx 2\sqrt{3}\left(1 - \frac{1}{3 \cdot 3} + \frac{1}{3^2 \cdot 5} - \frac{1}{3^3 \cdot 7} + \frac{1}{3^4 \cdot 9} - \frac{1}{3^5 \cdot 11} + \cdots\right)$ (Sharpe, 18th century)

3. $\pi \approx 2 \cdot \frac{2}{1} \cdot \frac{2}{3} \cdot \frac{4}{3} \cdot \frac{4}{5} \cdot \frac{6}{5} \cdot \frac{6}{7} \cdot \frac{8}{7} \cdot \frac{8}{9} \cdot \ldots$ (Wallis, 17th century)

4. This exercise is for calculators that have a square root function and a memory.

$$\pi \approx 2 \div \left(\sqrt{0.5} \cdot \sqrt{0.5 + 0.5\sqrt{0.5}} \cdot \sqrt{0.5 + 0.5\sqrt{0.5 + 0.5\sqrt{0.5}}} \cdot \ldots\right)$$

(Vieta, 16th century)

Algebra Review: *Evaluating Expressions*

Find the value of each expression using the given values of the variables.

Example $2\pi r$ when $r = \frac{5}{4}$ **Solution** $2 \cdot \pi \cdot \frac{5}{4} = \left(2 \cdot \frac{5}{4}\right)\pi = \frac{5}{2}\pi$

1. πr^2 when $r = \frac{2}{3}\sqrt{3}$
2. πrl when $r = 4\frac{1}{5}$ and $l = 15$
3. $\frac{1}{3}\pi r^2 h$ when $r = 2\sqrt{6}$ and $h = 4$
4. $\frac{4}{3}\pi r^3$ when $r = 6$
5. $\pi r\sqrt{r^2 + h^2}$ when $r = h = \sqrt{5}$
6. $2\pi r^2 + 2\pi rh$ when $r = 10$ and $h = 6$
7. $\pi r^2 + \pi r\sqrt{r^2 + h^2}$ when $r = 2$ and $h = 2\sqrt{3}$
8. $\pi(r_1^2 - r_2^2)$ when $r_1 = 6$ and $r_2 = 3\sqrt{2}$