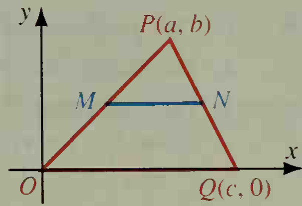
Example 2

Prove that the segment joining the midpoints of two sides of a triangle is parallel to the third side and is half as long as the third side.

Proof:



Let OPQ be any triangle. Choose convenient axes and coordinates as shown. By the midpoint formula, M has coordinates $\left(\frac{a}{2}, \frac{b}{2}\right)$ and N has coordinates $\left(\frac{a+c}{2}, \frac{b}{2}\right)$.

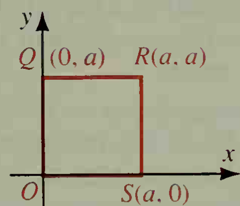
Slope of $\overline{MN} = 0$ and slope of $\overline{OQ} = 0$. (Why?)

Since $\overline{MN}$ and $\overline{OQ}$ have equal slopes, $\overline{MN} \parallel \overline{OQ}$.

Since $MN = \frac{a+c}{2} - \frac{a}{2} = \frac{c}{2}$ and $OQ = c - 0 = c$, $MN = \frac{1}{2}OQ$.

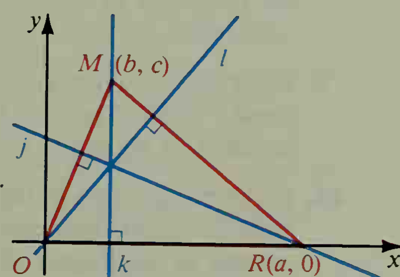
Classroom Exercises

In Exercises 1–4 use the diagram at the right.



1. What kind of figure is quad. $OQRS$? Why?
2. Show that $\overline{OR} \cong \overline{QS}$.
3. Show that $\overline{OR} \perp \overline{QS}$.
4. Show that $\overline{OR}$ bisects $\overline{QS}$.
5. The purpose of this exercise is to prove that the lines that contain the altitudes of a triangle intersect in a point (called the *orthocenter*).

   Given $\triangle ROM$, with lines j, k, and l containing the altitudes, we choose axes and coordinates as shown.



   a. The equation of line k is __?__.

   b. Since the slope of $\overline{MR}$ is $\frac{c}{b-a}$, the slope of line l is __?__.

   c. Show that an equation of line l is $y = \left(\frac{a-b}{c}\right)x$.

   d. Show that lines k and l intersect where $x = b$ and $y = \frac{ab - b^2}{c}$.

   e. Since the slope of $\overline{OM}$ = __?__, the slope of line j is __?__.

   f. Show that an equation of line j is $y = -\frac{b}{c}(x - a)$.

   g. Show that lines k and j intersect where $x = b$ and $y = \frac{ab - b^2}{c}$.

   h. From parts (d) and (g) we see that the three altitude lines intersect in a point. Name the coordinates of that point.