11-2 *Areas of Parallelograms, Triangles, and Rhombuses*

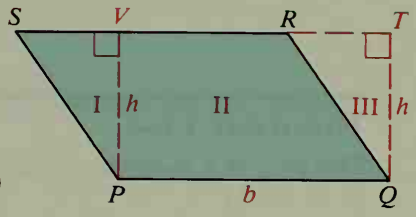
Although proofs of most area formulas are easy to understand, detailed formal proofs are long and time consuming. Therefore, we will show the key steps of each proof.

Theorem 11-2

The area of a parallelogram equals the product of a base and the height to that base. ($A = bh$)

Given: $\square PQRS$

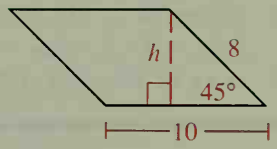
Prove: $A = bh$



Key steps of proof:

1. Draw altitudes $\overline{PV}$ and $\overline{QT}$, forming two rt. $\triangle$.
2. Area I = Area III ($\triangle PSV \cong \triangle QRT$ by HL or AAS)
3. Area of $\square PQRS$ = Area II + Area I
   = Area II + Area III
   = Area of rect. $PQTV$
   = bh

Example 1 Find the area of the parallelogram shown.



Solution Notice the 45°-45°-90° triangle.

$$h = \frac{8}{\sqrt{2}} = \frac{8}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{8\sqrt{2}}{2} = 4\sqrt{2}$$

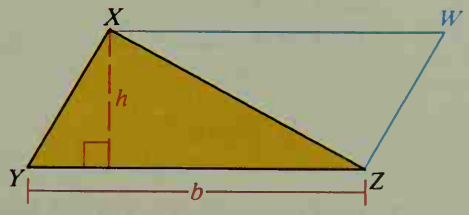
$$A = bh = 10 \cdot 4\sqrt{2} = 40\sqrt{2}$$

Theorem 11-3

The area of a triangle equals half the product of a base and the height to that base. ($A = \frac{1}{2}bh$)

Given: $\triangle XYZ$

Prove: $A = \frac{1}{2}bh$



Key steps of proof:

1. Draw $\overline{XW} \parallel \overline{YZ}$ and $\overline{ZW} \parallel \overline{YX}$, forming $\square WXYZ$.
2. $\triangle XYZ \cong \triangle ZWX$ (SAS or SSS)
3. Area of $\triangle XYZ$ = $\frac{1}{2}$ · Area of $\square WXYZ$
   = $\frac{1}{2}bh$