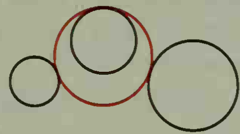
The Problem of Appolonius (Chapter 10)

Materials: Compass and straightedge

In the third century B.C., the Greek geometer Appolonius gave a complete solution of the following famous construction problem:

Given three coplanar circles, construct a fourth circle tangent to all three.

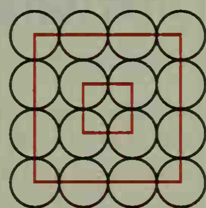


1. Three coplanar circles have up to *eight* circles tangent to all three. In the diagram, the red circle is tangent to the three black circles. Copy the diagram and sketch (don't construct) as many other circles as you can that appear to be tangent to all three black circles.
2. Draw three *congruent* circles whose centers are not collinear. Devise a compass and straightedge construction of two different circles tangent to the three congruent circles. Explain the steps of your construction. Demonstrate that your method works for other configurations of three congruent circles.

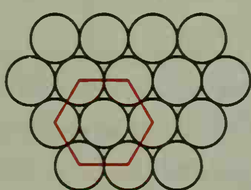
Covering the Plane (Chapter 11)

In this project you will investigate the percentage of the plane that can be covered by different arrangements of non-overlapping circles.

1. The diagram below shows a *square arrangement* of congruent circles.



   a. Given that the radius of each circle is 1 unit, find the area of the small square and the area of the portion of the small square that is covered by the circles. Then find the percentage of the small square's interior that is covered by the circles. Repeat these calculations for the larger square.
   b. Repeat part (a) given that the radius of each circle is 2 units.
   c. Make a conjecture about the percentage of the plane that would be covered if the square arrangement of circles extended without end in all directions.



2. Find the percentage of the plane covered by a *hexagonal arrangement* of congruent circles, as shown at the right. Show that you get the same percentage regardless of the radius of the circles. How does this percentage compare with the percentage of the plane covered by the square arrangement of circles in Exercise 1?
3. Suppose you fit one smaller circle into each existing gap in the square and hexagonal arrangements shown above. Find the largest percentage of the plane that can be covered in each case. Show your method.