SAMPLE PROBLEM F

Relative Velocity

PROBLEM

A boat heading north crosses a wide river with a velocity of 10.00 km/h relative to the water. The river has a uniform velocity of 5.00 km/h due east. Determine the boat's velocity with respect to an observer on shore.

SOLUTION

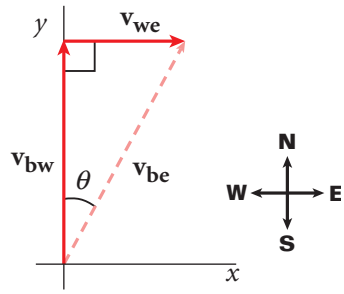
1. DEFINE

Given: $\mathbf{v_{bw}} = 10.00$ km/h due north (velocity of the *boat, b,* with respect to the *water, w*)

$\mathbf{v_{we}} = 5.00$ km/h due east (velocity of the *water, w,* with respect to *Earth, e*)

Unknown: $\mathbf{v_{be}} = ?$

Diagram: See the diagram on the right.



2. PLAN

Choose an equation or situation:

To find $\mathbf{v_{be}}$, write the equation so that the subscripts on the right start with *b* and end with *e*.

$$\mathbf{v_{be}} = \mathbf{v_{bw}} + \mathbf{v_{we}}$$

As in Section 2, we use the Pythagorean theorem to calculate the magnitude of the resultant velocity and the tangent function to find the direction.

$$(v_{be})^2 = (v_{bw})^2 + (v_{we})^2$$

$$\tan\theta = \frac{v_{we}}{v_{bw}}$$

Rearrange the equations to isolate the unknowns:

$$v_{be} = \sqrt{(v_{bw})^2 + (v_{we})^2}$$

$$\theta = \tan^{-1}\left(\frac{v_{we}}{v_{bw}}\right)$$

3. CALCULATE

Substitute the known values into the equations and solve:

$$v_{be} = \sqrt{(10.00\text{ km/h})^2 + (5.00\text{ km/h})^2}$$

$$\boxed{v_{be} = 11.18\text{ km/h}}$$

$$\theta = \tan^{-1}\left(\frac{5.00\text{ km/h}}{10.0\text{ km/h}}\right)$$

$$\boxed{\theta = 26.6°}$$

4. EVALUATE

The boat travels at a speed of 11.18 km/h in the direction 26.6° east of north with respect to Earth.