9-2 *Tangents*

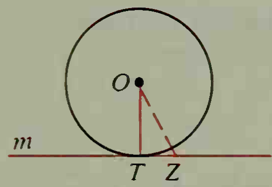
In Written Exercise 2 on page 330 you had the chance to preview the next theorem about tangents and radii.

Theorem 9-1

If a line is tangent to a circle, then the line is perpendicular to the radius drawn to the point of tangency.

Given: m is tangent to $\odot O$ at T.

Prove: $\overline{OT} \perp m$



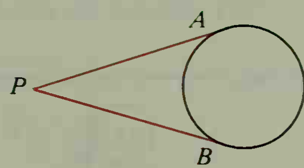
Proof:

Assume temporarily that $\overline{OT}$ is not perpendicular to m. Then the perpendicular segment from O to m intersects m in some other point Z. Draw $\overline{OZ}$. By Corollary 1, page 220, the perpendicular segment from O to m is the shortest segment from O to m, so $OZ < OT$. Because tangent m intersects $\odot O$ only in point T, Z lies outside $\odot O$, and $OZ > OT$. The statements $OZ < OT$ and $OZ > OT$ are contradictory. Thus the temporary assumption must be false. It follows that $\overline{OT} \perp m$.

Corollary

Tangents to a circle from a point are congruent.

In the figure, $\overline{PA}$ and $\overline{PB}$ are tangent to the circle at A and B. By the corollary, $\overline{PA} \cong \overline{PB}$. For a proof, see Classroom Exercise 4.



Theorem 9-2 is the converse of Theorem 9-1. Its proof is left as Exercise 22.

Theorem 9-2

If a line in the plane of a circle is perpendicular to a radius at its outer endpoint, then the line is tangent to the circle.

Given: Line l in the plane of $\odot Q$;
$l \perp$ radius $\overline{QR}$ at R

Prove: l is tangent to $\odot Q$.