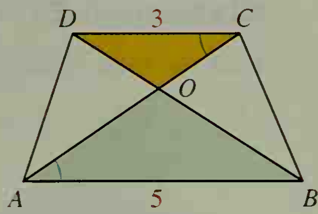
Example 2 $ABCD$ is a trapezoid. Find the ratio of the areas of:

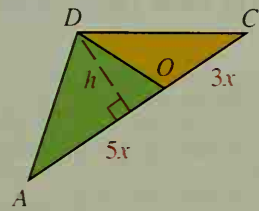
a. $\triangle COD$ and $\triangle AOB$
b. $\triangle COD$ and $\triangle AOD$
c. $\triangle OAB$ and $\triangle DAB$



Solution $\triangle COD \sim \triangle AOB$ by the AA Similarity Postulate, with a scale factor of 3:5. Thus each of the corresponding sides and heights of these triangles has a 3:5 ratio.

a. Since $\triangle COD \sim \triangle AOB$,

$$\frac{\text{area of }\triangle COD}{\text{area of }\triangle AOB} = \left(\frac{3}{5}\right)^2 = \frac{9}{25}.$$

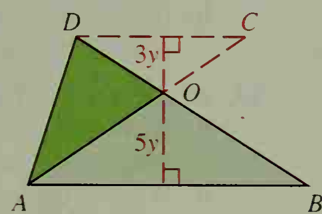


b. Since $\triangle COD$ and $\triangle AOD$ have the same height, h, their area ratio equals their base ratio.

$$\frac{\text{area of }\triangle COD}{\text{area of }\triangle AOD} = \frac{CO}{AO} = \frac{3x}{5x} = \frac{3}{5}$$

c. Since $\triangle OAB$ and $\triangle DAB$ have the same base, $\overline{AB}$, their area ratio equals their height ratio. Notice that the height of $\triangle DAB$ is $3y + 5y$, or $8y$.

$$\frac{\text{area of }\triangle OAB}{\text{area of }\triangle DAB} = \frac{5y}{8y} = \frac{5}{8}$$



You know that the ratios of the perimeters and areas of two similar triangles are related to their scale factor. These relationships can be generalized to any two similar figures.

Theorem 11-7

If the scale factor of two similar figures is $a:b$, then

(1) the ratio of the perimeters is $a:b$.

(2) the ratio of the areas is $a^2:b^2$.

Example 3 Find the ratio of the perimeters and the ratio of the areas of the two similar figures.

Solution The scale factor is 8:12, or 2:3. Therefore, the ratio of the perimeters is 2:3. The ratio of the areas is $2^2:3^2$, or 4:9.