

Simple Harmonic Motion

SECTION OBJECTIVES

- Identify the conditions of simple harmonic motion.
- Explain how force, velocity, and acceleration change as an object vibrates with simple harmonic motion.
- Calculate the spring force using Hooke's law.

HOOKE'S LAW

A repeated motion, such as that of an acrobat swinging on a trapeze, is called a periodic motion. Other periodic motions include those made by a child on a playground swing, a wrecking ball swaying to and fro, and the pendulum of a grandfather clock or a metronome. In each of these cases, the periodic motion is back and forth over the same path.

One of the simplest types of back-and-forth periodic motion is the motion of a mass attached to a spring, as shown in **Figure 1**. Let us assume that the mass moves on a frictionless horizontal surface. When the spring is stretched or compressed and then released, it vibrates back and forth about its unstretched position. We will begin by considering this example, and then we will apply our conclusions to the swinging motion of a trapeze acrobat.

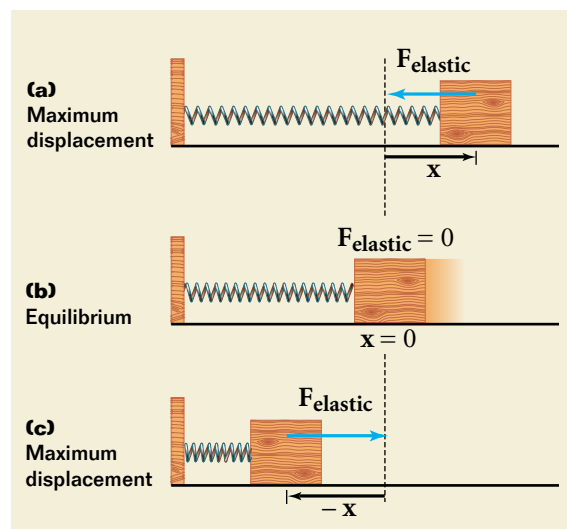


Figure 1

The direction of the force acting on the mass (F_{elastic}) is always opposite the direction of the mass's displacement from equilibrium ($x = 0$). (a) When the spring is stretched to the right, the spring force pulls the mass to the left. (b) When the spring is unstretched, the spring force is zero. (c) When the spring is compressed to the left, the spring force is directed to the right.

At the equilibrium position, speed reaches a maximum

In **Figure 1(a)**, the spring is stretched away from its unstretched, or equilibrium, position ($x = 0$). In this stretched position, the spring exerts a force on the mass toward the equilibrium position. This spring force decreases as the spring moves toward the equilibrium position, and it reaches zero at equilibrium, as illustrated in **Figure 1(b)**. The mass's acceleration also becomes zero at equilibrium.

Though the spring force and acceleration decrease as the mass moves toward the equilibrium position, the speed of the mass increases. At the equilibrium position, when acceleration reaches zero, the speed reaches a maximum. At that point, although the spring force is zero, the mass's momentum causes it to overshoot the equilibrium position and compress the spring.

At maximum displacement, spring force and acceleration reach a maximum

As the mass moves beyond equilibrium, the spring force and the acceleration increase. But the direction of the spring force and of the acceleration (toward equilibrium) is opposite the mass's direction of motion (away from equilibrium), and the mass begins to slow down.