Example 1 Find the image of $\triangle ABC$ under the expansion $D_{O,\,2}$.

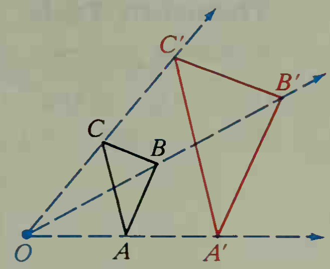
Solution

$D_{O,\,2}: \triangle ABC \to \triangle A'B'C'$

$OA' = 2 \cdot OA$

$OB' = 2 \cdot OB$

$OC' = 2 \cdot OC$



Example 2 Find the image of $\triangle RST$ under the contraction $D_{O,\,\frac{2}{3}}$.

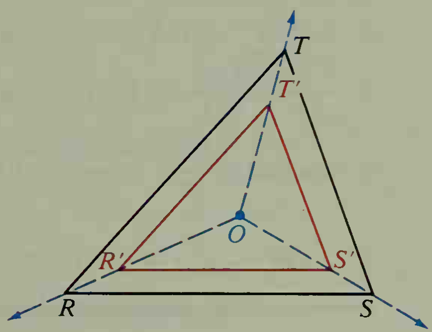
Solution

$D_{O,\,\frac{2}{3}}: \triangle RST \to \triangle R'S'T'$

$OR' = \frac{2}{3} \cdot OR$

$OS' = \frac{2}{3} \cdot OS$

$OT' = \frac{2}{3} \cdot OT$



In the examples above, can you prove that the two triangles are similar? How are the areas of each pair of triangles related?

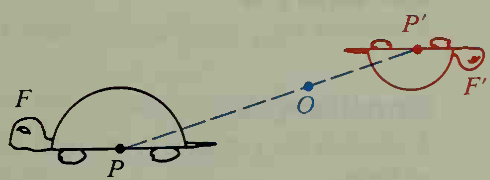
Example 3 Find the image of figure F under the contraction $D_{O,\,-\frac{1}{2}}$.

Solution

$D_{O,\,-\frac{1}{2}}$: figure $F \to$ figure F'

$\overrightarrow{OP}$ is opposite to $\overrightarrow{OP'}$.

$OP' = |-\frac{1}{2}| \cdot OP = \frac{1}{2} \cdot OP$



If the scale factor in Example 3 was -1 instead of $-\frac{1}{2}$, the figure F' would be congruent to the figure F, and the transformation would be an isometry, equivalent to a half-turn. In general, however, as these examples illustrate, dilations do not preserve distance. Therefore a dilation is not an isometry (unless $k = 1$ or $k = -1$).

But a dilation always maps any geometric figure to a similar figure. In the examples above, $\triangle ABC \sim \triangle A'B'C'$, $\triangle RST \sim \triangle R'S'T'$ and the figure F is similar to the figure F'. For this reason, a dilation is an example of a **similarity mapping.**