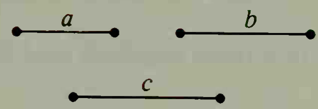
Construction 13

Given three segments, construct a fourth segment so that the four segments are in proportion.

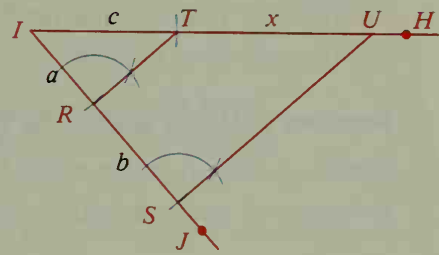


Given: Segments with lengths a, b, and c

Construct: A segment of length x such that $\frac{a}{b} = \frac{c}{x}$

Procedure:

1. Draw an $\angle HIJ$.
2. On $\overrightarrow{IJ}$, mark off $IR = a$ and $RS = b$.
3. On $\overrightarrow{IH}$, mark off $IT = c$.
4. Draw $\overline{RT}$.
5. At S, construct a parallel to $\overline{RT}$, intersecting $\overrightarrow{IH}$ in a point U.



$\overline{TU}$ has length x such that $\frac{a}{b} = \frac{c}{x}$.

Justification: Because $\overline{RT}$ is parallel to side $\overline{SU}$ of $\triangle SIU$, $\overline{RT}$ divides the other two sides of the triangle proportionally. Therefore, $\frac{a}{b} = \frac{c}{x}$.

Construction 14

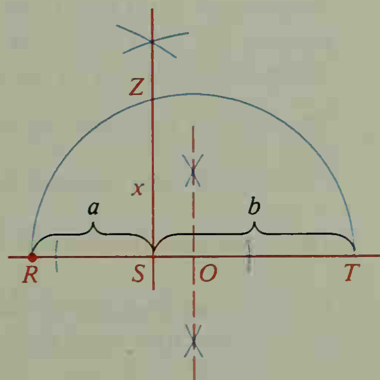
Given two segments, construct their geometric mean.

Given: Segments with lengths a and b

Construct: A segment of length x such that $\frac{a}{x} = \frac{x}{b}$ (or $x = \sqrt{ab}$)

Procedure:

1. Draw a line and mark off $RS = a$ and $ST = b$.
2. Locate the midpoint O of $\overline{RT}$ by constructing the perpendicular bisector of $\overline{RT}$.
3. Using O as center draw a semicircle with a radius equal to OR.
4. At S, construct a perpendicular to $\overline{RT}$. The perpendicular intersects the semicircle at a point Z.



ZS, or x, is the geometric mean between a and b.

Justification: $\overset{\frown}{RZT}$ is a semicircle. If you draw $\overline{RZ}$ and $\overline{ZT}$, then $\triangle RZT$ is a right triangle. Since $\overline{ZS}$ is the altitude to the hypotenuse of rt. $\triangle RZT$, $\frac{a}{x} = \frac{x}{b}$.