## PRINCIPLES OF FLUID FLOW

Fluid behavior is often very complex. Several general principles describing the flow of fluids can be derived relatively easily from basic physical laws.

### The continuity equation results from mass conservation

Imagine that an ideal fluid flows into one end of a pipe and out the other end, as shown in **Figure 9.** The diameter of the pipe is different at each end. How does the speed of fluid flow change as the fluid passes through the pipe?

Because mass is conserved and because the fluid is incompressible, we know that the mass flowing into the bottom of the pipe, $m_1$, must equal the mass flowing out of the top of the pipe, $m_2$, during any given time interval:

$$m_1 = m_2$$

This simple equation can be expanded by recalling that $m = \rho V$ and by using the formula for the volume of a cylinder, $V = A\Delta x$.

$$\rho_1 V_1 = \rho_2 V_2$$
$$\rho_1 A_1 \Delta x_1 = \rho_2 A_2 \Delta x_2$$

The length of the cylinder, $\Delta x$, is also the distance the fluid travels, which is equal to the speed of flow multiplied by the time interval ($\Delta x = v\Delta t$).

$$\rho_1 A_1 v_1 \Delta t = \rho_2 A_2 v_2 \Delta t$$

The time interval and, for an ideal fluid, the density are the same on each side of the equation, so they cancel each other out. The resulting equation is called the continuity equation:

**CONTINUITY EQUATION**

$$A_1 v_1 = A_2 v_2$$

**area × speed in region 1 = area × speed in region 2**

### The speed of fluid flow depends on cross-sectional area

Note in the continuity equation that $A_1$ and $A_2$ can represent any two different cross-sectional areas of the pipe, not just the ends. This equation implies that the fluid speed is faster where the pipe is narrow and slower where the pipe is wide. The product $Av$, which has units of volume per unit time, is called the *flow rate.* The flow rate is constant throughout the pipe.

The continuity equation explains an effect you may have observed as water flows slowly from a faucet, as shown in **Figure 10.** Because the water speeds up due to gravity as it falls, the stream narrows, satisfying the continuity equation. The continuity equation also explains why a river tends to flow more rapidly in places where the river is shallow or narrow than in places where the river is deep and wide.

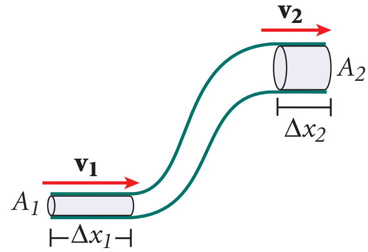


**Figure 9**
The mass flowing into the pipe must equal the mass flowing out of the pipe in the same time interval.

**Figure 10**
The width of a stream of water narrows as the water falls and speed up.