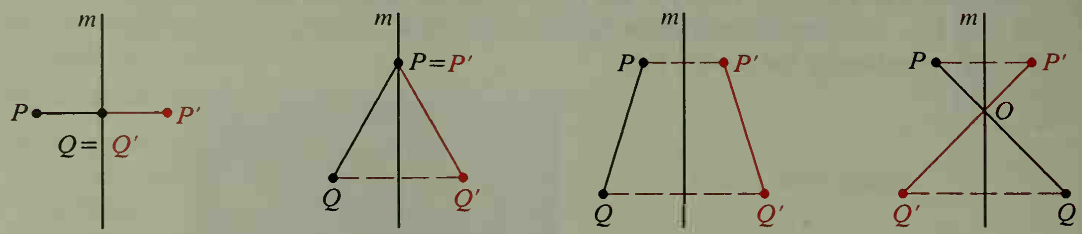
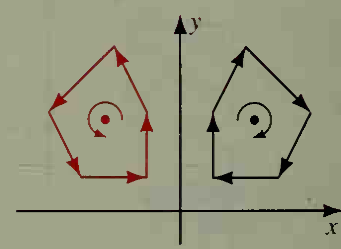
Theorem 14-2 can also be proved without the use of coordinates. If coordinates are not used, we must show that $PQ = P'Q'$ for all choices of P and Q. Four of the possible cases are shown below. In Written Exercises 18–20 you will prove Theorem 14-2 for Cases 2–4, using the fact that the line of reflection, m, is the perpendicular bisector of $\overline{PP'}$ and $\overline{QQ'}$.



Case 1 Case 2 Case 3 Case 4

Since a reflection is an isometry, it preserves distance, angle measure, and the area of a polygon. Another way to say this is that distance, angle measure, and area are *invariant* under a reflection. On the other hand, the orientation of a figure is *not* invariant under a reflection because a reflection changes a clockwise orientation to a counterclockwise one, as shown at the right.



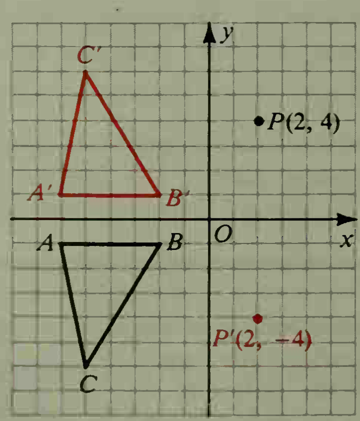
Example Find the image of point $P(2, 4)$ and $\triangle ABC$ under each reflection.

a. The line of reflection is the x-axis.

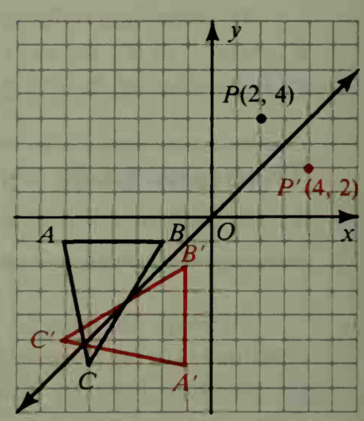
b. The line of reflection is the line $y = x$.

Solution The images are shown in red.

a.



b.



Notice that under reflection in the line $y = x$, the point (x, y) is mapped to the point (y, x).