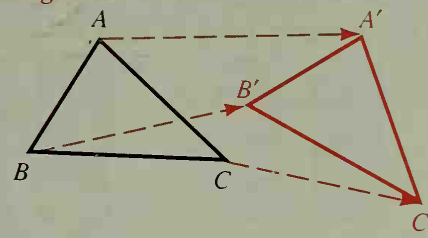
Theorem 14-1

An isometry maps a triangle to a congruent triangle.

Given: Isometry $T: \triangle ABC \to \triangle A'B'C'$

Prove: $\triangle ABC \cong \triangle A'B'C'$



Proof:

Statements	Reasons
1. $\overline{AB} \cong \overline{A'B'}$, $\overline{BC} \cong \overline{B'C'}$, $\overline{AC} \cong \overline{A'C'}$	1. Definition of isometry
2. $\triangle ABC \cong \triangle A'B'C'$	2. SSS Postulate

Corollary 1

An isometry maps an angle to a congruent angle.

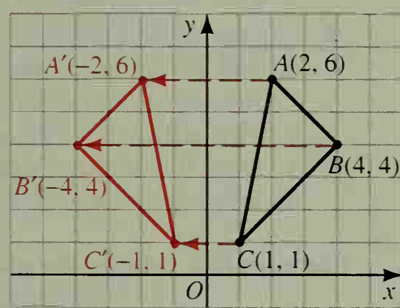
Corollary 2

An isometry maps a polygon to a polygon with the same area.

Example 3 Mapping R maps each point (x, y) to an image point $(-x, y)$.

a. Decide if $BA = B'A'$, $CB = C'B'$, and $CA = C'A'$.

b. Does R *appear* to be an isometry? Does part (a) *prove* that R is an isometry? Explain.



Solution **a.** Use the distance formula to show that

$$BA = \sqrt{(2-4)^2 + (6-4)^2} = \sqrt{(-2)^2 + 2^2} = \sqrt{8} = 2\sqrt{2}$$
$$B'A' = \sqrt{(-2-(-4))^2 + (6-4)^2} = \sqrt{2^2 + 2^2} = \sqrt{8} = 2\sqrt{2}$$
$$CB = \sqrt{(4-1)^2 + (4-1)^2} = \sqrt{3^2 + 3^2} = \sqrt{18} = 3\sqrt{2}$$
$$C'B' = \sqrt{(-4-(-1))^2 + (4-1)^2} = \sqrt{(-3)^2 + 3^2} = \sqrt{18} = 3\sqrt{2}$$
$$CA = \sqrt{(2-1)^2 + (6-1)^2} = \sqrt{1^2 + 5^2} = \sqrt{26}$$
$$C'A' = \sqrt{(-2-(-1))^2 + (6-1)^2} = \sqrt{(-1)^2 + 5^2} = \sqrt{26}$$

We have $BA = B'A'$, $CB = C'B'$, and $CA = C'A'$.

b. R *appears* to be an isometry because part (a) shows that three segments are mapped to congruent segments. Part (a) *does not prove* that R is an isometry because a proof must show that the image of *every* segment is a congruent segment.