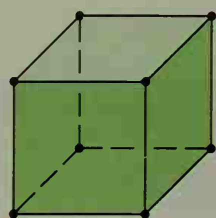
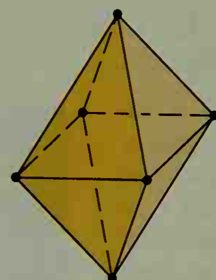
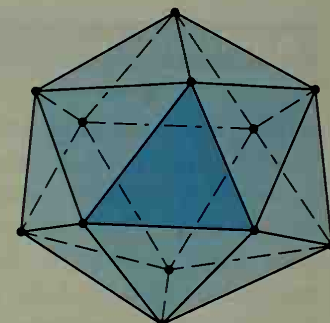
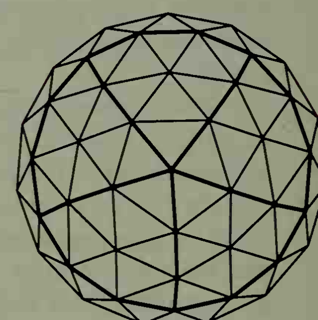
Exercises

For any solid figure with polygons for faces, Euler's formula, $F + V - E = 2$, must hold. In this formula, F, V, and E stand for the number of faces, vertices, and edges, respectively, that the figure has.

1. Verify Euler's formula for each figure below.
   a. Cube   b. Octahedron   c. Icosahedron (20 faces)

2. If each edge of the icosahedron above is trisected and the trisection points are "popped out" to the surface of the circumscribing sphere, one of the many possible geodesic domes is formed. By subdividing the edges of the icosahedron into more than three parts, the resulting geodesic dome is even more spherelike, as shown. In the diagram, find a group of equilateral triangles that cluster to form **(a)** a hexagon, and **(b)** a pentagon.



3. In this exercise, Euler's formula will be used to show that **(a)** the dome's framework *cannot* consist of hexagons only, and **(b)** the framework *can* consist of hexagons plus exactly 12 pentagons.
   a. Use an indirect proof and assume that the framework has n faces, all hexagons. Thus $F = n$. To find V, the number of vertices on the framework, notice that each hexagon contributes 6 vertices, but each vertex is shared by 3 hexagons. Thus $V = \frac{6n}{3}$. To find E, the number of edges of the framework, notice that each hexagon contributes 6 edges, but each edge is shared by 2 hexagons. Thus $E = \frac{6n}{2}$. According to Euler's Formula: $F + V - E$ must equal 2. Does it? What does this contradiction tell you?
   b. Suppose that 12 of the n faces of the framework are pentagons. Show that $V = \frac{6n - 12}{3}$ and that $E = \frac{6n - 12}{2}$. Then use algebra to show that $F + V - E = 2$. Since Euler's formula is satisfied, a dome framework can be constructed when n faces consist of 12 pentagons and $n - 12$ hexagons.