The table below shows the ratios of the perimeters, areas, and volumes for both pairs of similar solids shown on page 508. Notice the relationship between the scale factor and the ratios in each column.

	Cylinders I and II	Pyramids I and II
Scale factor	$\frac{3}{2}$	$\frac{2}{1}$
$\frac{\text{Base perimeter (I)}}{\text{Base perimeter (II)}}$	$\frac{2\pi \cdot 6}{2\pi \cdot 4} = \frac{6}{4}$, or $\frac{3}{2}$	$\frac{4 \cdot 12}{4 \cdot 6} = \frac{12}{6}$, or $\frac{2}{1}$
$\frac{\text{L.A. (I)}}{\text{L.A. (II)}}$	$\frac{2\pi \cdot 6 \cdot 12}{2\pi \cdot 4 \cdot 8} = \frac{9}{4}$, or $\frac{3^2}{2^2}$	$\frac{\frac{1}{2} \cdot 48 \cdot 10}{\frac{1}{2} \cdot 24 \cdot 5} = \frac{4}{1}$, or $\frac{2^2}{1^2}$
$\frac{\text{Volume (I)}}{\text{Volume (II)}}$	$\frac{\pi \cdot 6^2 \cdot 12}{\pi \cdot 4^2 \cdot 8} = \frac{27}{8}$, or $\frac{3^3}{2^3}$	$\frac{\frac{1}{3} \cdot 12^2 \cdot 8}{\frac{1}{3} \cdot 6^2 \cdot 4} = \frac{8}{1}$, or $\frac{2^3}{1^3}$

The results shown in the table above are generalized in the following theorem. (See Exercises 22–27 for proofs.)

Theorem 12-11

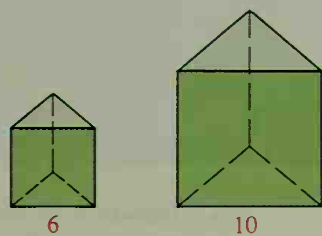
If the scale factor of two similar solids is $a:b$, then

(1) the ratio of corresponding perimeters is $a:b$.

(2) the ratio of the base areas, of the lateral areas, and of the total areas is $a^2:b^2$.

(3) the ratio of the volumes is $a^3:b^3$.

Example For the similar solids shown, find the ratios of the **(a)** base perimeters, **(b)** lateral areas, and **(c)** volumes.



Solution The scale factor is 6:10, or 3:5.

a. Ratio of base perimeters = 3:5

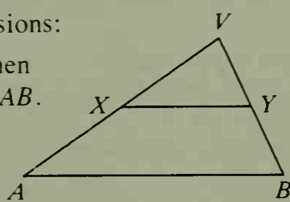
b. Ratio of lateral areas = $3^2:5^2$ = 9:25

c. Ratio of volumes = $3^3:5^3$ = 27:125

Theorem 12-11 is the three-dimensional counterpart of Theorem 11-7 on page 457. (Take a minute to compare these theorems.) There is a similar relationship between the two cases shown below.

In two dimensions:

If $\overline{XY} \parallel \overline{AB}$, then $\triangle VXY \sim \triangle VAB$.



In three dimensions:

If plane $XYZ \parallel$ plane ABC, then $V\text{-}XYZ \sim V\text{-}ABC$.