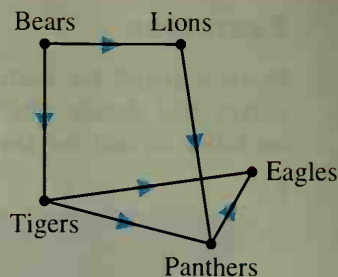


7. Five teams in a tournament play each of the other four teams exactly once. The results after a few games are shown in the graph. An arrow from one team to another indicates that the first team defeated the second.

- Which two teams have the most wins?
- Copy the graph and add arrows to show that the Lions beat both the Eagles and the Tigers, while the Panthers lost to the Bears.
- At the end of the competition, which team do you think should be ranked first, and which team last? Why?



## A Traveler's Puzzle

A business traveler located in Denver must travel to each of the cities shown in the graph below. The airline that the traveler prefers to use connects certain pairs of cities, but not all pairs. Is it possible to make a circuit that starts and ends in Denver and visits each of the other cities exactly once?

This kind of circuit is called a *Hamilton circuit* in honor of William Rowan Hamilton (1805–1865), an Irish mathematician who worked on such circuits. In a Hamilton circuit, each *vertex* is visited once while in an Euler circuit each *edge* is traveled once.

In the graph there are 10 vertices representing the 10 cities. Any other points where two edges intersect are not vertices. The graph has a Hamilton circuit. Can you find it?



The next example is a specific case of the *traveling salesperson problem*.

**Example** The graph shows the distances between four cities, A, B, C, and D. A traveling salesperson is to leave city A to visit the three other cities exactly once, and return to city A. What is the shortest circuit?

**Solution** The salesperson can visit any of the 3 cities first, then go to either of the 2 remaining cities, then go to the 1 unvisited city. So there are  $3 \times 2 \times 1 = 6$  possible circuits. The total distance for each circuit is shown on the next page.

