

## 7-2 Properties of Proportions

The first and last terms of a proportion are called the *extremes*. The middle terms are the *means*. In the proportions below, the extremes are shown in red. The means are shown in black.

$$a:b = c:d \qquad 6:9 = 2:3 \qquad \frac{6}{9} = \frac{2}{3}$$

Notice that  $6 \cdot 3 = 9 \cdot 2$ . This illustrates a property of all proportions, called the *means-extremes* property of proportions:

The product of the extremes equals the product of the means.

$$\frac{a}{b} = \frac{c}{d} \text{ is equivalent to } ad = bc.$$

The two equations are equivalent because we can change either of them into the other by multiplying (or dividing) each side by  $bd$ . Try this yourself.

It is often necessary to replace one proportion by an equivalent proportion. When you do so in a proof, you can use the reason "A property of proportions." The following properties will be justified in the exercises.

### Properties of Proportions

1.  $\frac{a}{b} = \frac{c}{d}$  is equivalent to:

a.  $ad = bc$

b.  $\frac{a}{c} = \frac{b}{d}$

c.  $\frac{b}{a} = \frac{d}{c}$

d.  $\frac{a+b}{b} = \frac{c+d}{d}$

2. If  $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \dots$ , then  $\frac{a+c+e+\dots}{b+d+f+\dots} = \frac{a}{b} = \dots$ .

**Example** Use the proportion  $\frac{x}{y} = \frac{5}{2}$  to complete each statement.

a.  $5y = \underline{\hspace{1cm}}$

b.  $\frac{x+y}{y} = \frac{?}{?}$

c.  $\frac{2}{5} = \frac{?}{?}$

d.  $\frac{x}{5} = \frac{?}{?}$

**Solution** a.  $5y = 2x$

b.  $\frac{x+y}{y} = \frac{7}{2}$

c.  $\frac{2}{5} = \frac{y}{x}$

d.  $\frac{x}{5} = \frac{y}{2}$