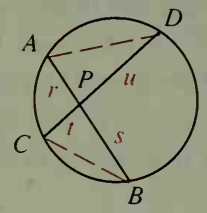
Theorem 9-11

When two chords intersect inside a circle, the product of the segments of one chord equals the product of the segments of the other chord.

Given: $\overline{AB}$ and $\overline{CD}$ intersect at P.

Prove: $r \cdot s = t \cdot u$



Proof:

Statements	Reasons
1. Draw chords $\overline{AD}$ and $\overline{CB}$.	1. Through any two points there is exactly one line.
2. $\angle A \cong \angle C$; $\angle D \cong \angle B$	2. If two inscribed angles intercept ___?___.
3. $\triangle APD \sim \triangle CPB$	3. ___?___
4. $\frac{r}{t} = \frac{u}{s}$	4. ___?___
5. $r \cdot s = t \cdot u$	5. A property of proportions

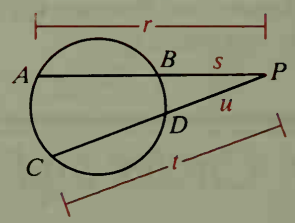
For a proof of the following theorem, see Classroom Exercise 7. In the diagram for the theorem, $\overline{AP}$ and $\overline{CP}$ are *secant segments*. $\overline{BP}$ and $\overline{DP}$ are exterior to the circle and are referred to as *external segments*. The terms "secant segment" and "external segment" can refer to the length of a segment as well as to the segment itself.

Theorem 9-12

When two secant segments are drawn to a circle from an external point, the product of one secant segment and its external segment equals the product of the other secant segment and its external segment.

Given: $\overline{PA}$ and $\overline{PC}$ drawn to the circle from point P

Prove: $r \cdot s = t \cdot u$



Study the diagrams at the top of the next page from left to right. As $\overline{PC}$ approaches a position of tangency, C and D move closer together until they merge. Then $\overline{PC}$ becomes a tangent, and $t = u$.