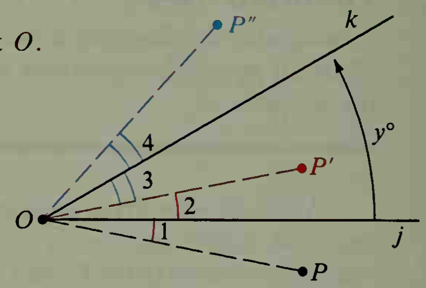
Theorem 14-8

A composite of reflections in two intersecting lines is a rotation about the point of intersection of the two lines. The measure of the angle of rotation is twice the measure of the angle from the first line of reflection to the second.

Given: j intersects k, forming an angle of measure y at O.
Prove: $R_k \circ R_j = \mathcal{R}_{O,\,2y}$



Proof:

The diagram shows an arbitrary point P and its image P' by reflection in j. The image of P' by reflection in k is P''. According to the definition of a rotation we must prove that $OP = OP''$ and $m\angle POP'' = 2y$.

R_j and R_k are isometries, so they preserve both distance and angle measure. Therefore $OP = OP'$, $OP' = OP''$, $m\angle 1 = m\angle 2$, and $m\angle 3 = m\angle 4$. Thus $OP = OP''$ and the measure of the angle of rotation equals

$$m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 = 2m\angle 2 + 2m\angle 3 = 2y.$$

Corollary

A composite of reflections in perpendicular lines is a half-turn about the point where the lines intersect.

Classroom Exercises

1. If $f(x) = x + 1$ and $g(x) = 3x$, find the following.

a. $f(4)$ **b.** $(g \circ f)(4)$ **c.** $(g \circ f)(x)$

d. $g(2)$ **e.** $(f \circ g)(2)$ **f.** $(f \circ g)(x)$

2. Repeat Exercise 1 if $f: x \to \sqrt{x}$ and $g: x \to x + 7$.

Complete the following. R_x and R_y are reflections in the x- and y-axes.

3. $R_x \circ R_y: A \to$ _?_ **4.** $R_x \circ R_y: D \to$ _?_

5. $H_O \circ R_y: B \to$ _?_ **6.** $R_y \circ H_O: B \to$ _?_

7. $H_O \circ H_O: A \to$ _?_ **8.** $R_y \circ R_y: C \to$ _?_