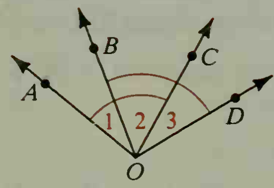
Example 3

Given: $m\angle AOC = m\angle BOD$

Prove: $m\angle 1 = m\angle 3$



Proof:

Statements	Reasons
1. $m\angle AOC = m\angle BOD$	1. Given
2. $m\angle AOC = m\angle 1 + m\angle 2$; $m\angle BOD = m\angle 2 + m\angle 3$	2. Angle Addition Postulate
3. $m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$	3. Substitution Prop.
4. $m\angle 2 = m\angle 2$	4. Reflexive Prop.
5. $m\angle 1 = m\angle 3$	5. Subtraction Prop. of =

Notice that the reason given for Step 4 is "Reflexive Property" rather than "Reflexive Property of Equality." Since the reflexive, symmetric, and transitive properties of equality are so closely related to the corresponding properties of congruence, we will simply use "Reflexive Property" to justify either

$$m\angle BOC = m\angle BOC \quad \text{or} \quad \angle BOC \cong \angle BOC.$$

Suppose, in a proof, you have made the statement that

$$m\angle R = m\angle S$$

and also the statement that

$$m\angle S = m\angle T.$$

You can then deduce that $m\angle R = m\angle T$ and use as your reason either "Transitive Property" or "Substitution Property." Similarly, if you know that

(1) $m\angle R = m\angle S$
(2) $m\angle S = m\angle T$
(3) $m\angle T = m\angle V$

you can go on to write (4) $m\angle R = m\angle V$

and use either "Transitive Property" or "Substitution Property" as your reason. Actually, you use the Transitive Property twice or else make a double substitution.

There are times when the Substitution Property is the simplest one to use. If you know that

(1) $m\angle 4 + m\angle 2 + m\angle 5 = 180$
(2) $m\angle 4 = m\angle 1$; $m\angle 5 = m\angle 3$

you can make a double substitution and get

(3) $m\angle 1 + m\angle 2 + m\angle 3 = 180.$

Note that you can't use the Transitive Property here.