8-4 *Special Right Triangles*

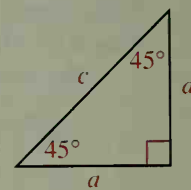
An isosceles right triangle is also called a 45°-45°-90° triangle, because the measures of the angles are 45, 45, and 90.

Theorem 8-6 *45°-45°-90° Theorem*

In a 45°-45°-90° triangle, the hypotenuse is $\sqrt{2}$ times as long as a leg.

Given: A 45°-45°-90° triangle

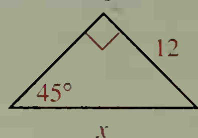
Prove: hypotenuse = $\sqrt{2} \cdot$ leg



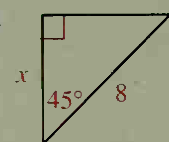
Plan for Proof: Let the sides of the given triangle be a, a, and c. Apply the Pythagorean Theorem and solve for c in terms of a.

Example 1 Find the value of x.

a.



b.



Solution

a. $\text{hyp} = \sqrt{2} \cdot \text{leg}$
$x = \sqrt{2} \cdot 12$
$x = 12\sqrt{2}$

b. $\text{hyp} = \sqrt{2} \cdot \text{leg}$
$8 = \sqrt{2} \cdot x$
$x = \dfrac{8}{\sqrt{2}} = \dfrac{8}{\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{8\sqrt{2}}{2}$
$x = 4\sqrt{2}$

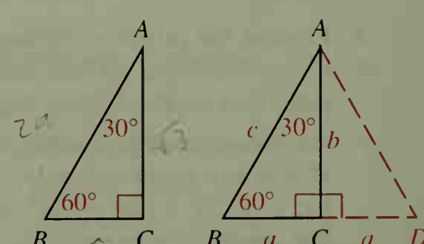
Another special right triangle has acute angles measuring 30 and 60.

Theorem 8-7 *30°-60°-90° Theorem*

In a 30°-60°-90° triangle, the hypotenuse is twice as long as the shorter leg, and the longer leg is $\sqrt{3}$ times as long as the shorter leg.

Given: $\triangle ABC$, a 30°-60°-90° triangle

Prove: hypotenuse = $2 \cdot$ shorter leg
longer leg = $\sqrt{3} \cdot$ shorter leg



Plan for Proof: Build onto $\triangle ABC$ as shown. $\triangle ADC \cong \triangle ABC$, so $\triangle ABD$ is equiangular and equilateral with $c = 2a$. Since $\triangle ABC$ is a right triangle, $a^2 + b^2 = c^2$. By substitution, $a^2 + b^2 = 4a^2$, so $b^2 = 3a^2$ and $b = a\sqrt{3}$.