A network of least possible cost that permits travel from each vertex to any other vertex is called a *minimal spanning tree.* It is called "minimal" because the cost is least; it is called "spanning" because the network spans out to touch every vertex; and it is called a "tree" because it resembles a tree with branches.

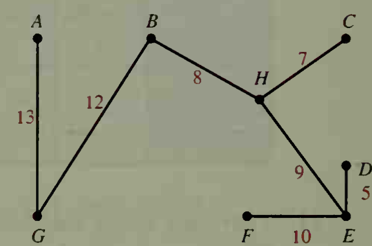
Finding a Minimal Spanning Tree

1. Build the least expensive road (edge) first.
2. Then build the road next lowest in cost.
3. At each stage, build the road that is next lowest in cost and *does not form any circuit.* Stop when all vertices have been reached.

Example Use the steps above to find the minimal spanning tree for the roads in the left-hand graph on page 681.

Solution

1. The least expensive road is *DE*.
2. The roads next lowest in cost are *HC*, *HB*, *HE*, and *EF*.
3. The road next lowest in cost is *HF* or *CD*. These roads are not built because they form circuits. Therefore, go to the road next lowest in cost, *BG*, and finally to *GA*.

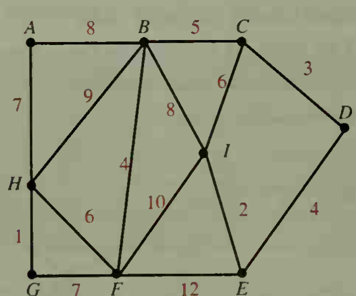


The total cost of the minimal spanning tree is \$5000 + \$7000 + \$8000 + \$9000 + \$10,000 + \$12,000 + \$13,000 = \$64,000.

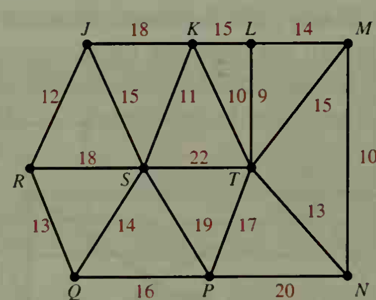
Exercises

Find a minimal spanning tree for each graph.

1.



2.



3. If a graph has *n* vertices, how many edges does a minimal spanning tree have?