SAMPLE PROBLEM B

Harmonics

PROBLEM

What are the first three harmonics in a 2.45 m long pipe that is open at both ends? What are the first three harmonics of this pipe when one end of the pipe is closed? Assume that the speed of sound in air is 345 m/s.

SOLUTION

1. DEFINE

Given: $L = 2.45$ m $\quad v = 345$ m/s

Unknown: Pipe open at both ends: $f_1 \quad f_2 \quad f_3$

Pipe closed at one end: $f_1 \quad f_3 \quad f_5$

2. PLAN

Choose an equation or situation:

When the pipe is open at both ends, the fundamental frequency can be found by using the equation for the entire harmonic series:

$$f_n = n\frac{v}{2L},\ n = 1, 2, 3, \ldots$$

When the pipe is closed at one end, use the following equation:

$$f_n = n\frac{v}{4L},\ n = 1, 3, 5, \ldots$$

In both cases, the second two harmonics can be found by multiplying the harmonic numbers by the fundamental frequency.

3. CALCULATE

Substitute the values into the equations and solve:

For a pipe open at both ends:

$$f_1 = n\frac{v}{2L} = (1)\left(\frac{345 \text{ m/s}}{(2)(2.45 \text{ m})}\right) = \boxed{70.4 \text{ Hz}}$$

The next two harmonics are the second and the third:

$$f_2 = 2f_1 = (2)(70.4 \text{ Hz}) = \boxed{141 \text{ Hz}}$$

$$f_3 = 3f_1 = (3)(70.4 \text{ Hz}) = \boxed{211 \text{ Hz}}$$

For a pipe closed at one end:

$$f_1 = n\frac{v}{4L} = (1)\left(\frac{345 \text{ m/s}}{(4)(2.45 \text{ m})}\right) = \boxed{35.2 \text{ Hz}}$$

The next possible harmonics are the third and the fifth:

$$f_3 = 3f_1 = (3)(35.2 \text{ Hz}) = \boxed{106 \text{ Hz}}$$

$$f_5 = 5f_1 = (5)(35.2 \text{ Hz}) = \boxed{176 \text{ Hz}}$$

TIP

Be sure to use the correct harmonic numbers for each situation. For a pipe open at both ends, $n = 1, 2, 3$, etc. For a pipe closed at one end, only odd harmonics are present, so $n = 1, 3, 5$, etc.