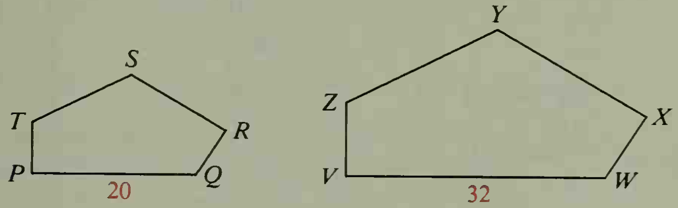
Two polygons are **similar** if their vertices can be paired so that:

(1) Corresponding angles are congruent.

(2) Corresponding sides are in proportion. (Their lengths have the same ratio.)

When you refer to similar polygons, their corresponding vertices must be listed in the same order. If polygon $PQRST$ is similar to polygon $VWXYZ$, you write polygon $PQRST \sim$ polygon $VWXYZ$.



From the definition of similar polygons, we have:

(1) $\angle P \cong \angle V \qquad \angle Q \cong \angle W \qquad \angle R \cong \angle X \qquad \angle S \cong \angle Y \qquad \angle T \cong \angle Z$

(2) $\dfrac{PQ}{VW} = \dfrac{QR}{WX} = \dfrac{RS}{XY} = \dfrac{ST}{YZ} = \dfrac{TP}{ZV}$

Similarity has some of the same properties as equality and congruence (page 37). Similarity is reflexive, symmetric, and transitive.

If two polygons are similar, then the ratio of the lengths of two corresponding sides is called the **scale factor** of the similarity. The scale factor of pentagon $PQRST$ to pentagon $VWXYZ$ is $\dfrac{PQ}{VW} = \dfrac{20}{32} = \dfrac{5}{8}$.

The example that follows shows one convenient way to label corresponding vertices: A and A' (read A prime), B and B', and so on.

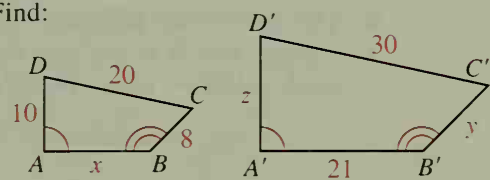
Example Quad. $ABCD \sim$ quad. $A'B'C'D'$. Find:

a. their scale factor

b. the values of x, y, and z

c. the perimeters of the two quadrilaterals

d. the ratio of the perimeters



Solution

a. The scale factor is $\dfrac{DC}{D'C'} = \dfrac{20}{30} = \dfrac{2}{3}$.

b. $\dfrac{DC}{D'C'} = \dfrac{AB}{A'B'} \qquad \dfrac{DC}{D'C'} = \dfrac{BC}{B'C'} \qquad \dfrac{DC}{D'C'} = \dfrac{AD}{A'D'}$

$\dfrac{2}{3} = \dfrac{x}{21} \qquad \dfrac{2}{3} = \dfrac{8}{y} \qquad \dfrac{2}{3} = \dfrac{10}{z}$

$x = 14 \qquad y = 12 \qquad z = 15$

c. The perimeter of quad. $ABCD$ is $10 + 20 + 8 + 14 = 52$.
The perimeter of quad. $A'B'C'D'$ is $15 + 30 + 12 + 21 = 78$.

d. The ratio of the perimeters is $\dfrac{52}{78}$, or $\dfrac{2}{3}$.

If you compare the ratio of the perimeters with the scale factor of the similarity, you discover they are the same. This property will be discussed further in Exercise 23 on page 251 and in Theorem 11-7.