7-2 Properties of Proportions

The first and last terms of a proportion are called the *extremes*. The middle terms are the *means*. In the proportions below, the extremes are shown in red. The means are shown in black.

$$a:b = c:d \qquad 6:9 = 2:3 \qquad \frac{6}{9} = \frac{2}{3}$$

Notice that $6 \cdot 3 = 9 \cdot 2$. This illustrates a property of all proportions, called the *means-extremes* property of proportions:

The product of the extremes equals the product of the means.

$\frac{a}{b} = \frac{c}{d}$ is *equivalent* to $ad = bc$.

The two equations are equivalent because we can change either of them into the other by multiplying (or dividing) each side by bd. Try this yourself.

It is often necessary to replace one proportion by an equivalent proportion. When you do so in a proof, you can use the reason "A property of proportions." The following properties will be justified in the exercises.

Properties of Proportions

1. $\frac{a}{b} = \frac{c}{d}$ is equivalent to:

a. $ad = bc$ **b.** $\frac{a}{c} = \frac{b}{d}$ **c.** $\frac{b}{a} = \frac{d}{c}$ **d.** $\frac{a+b}{b} = \frac{c+d}{d}$

2. If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \cdots$, then $\frac{a+c+e+\cdots}{b+d+f+\cdots} = \frac{a}{b} = \cdots$.

Example Use the proportion $\frac{x}{y} = \frac{5}{2}$ to complete each statement.

a. $5y = \underline{\ ?\ }$ **b.** $\frac{x+y}{y} = \frac{?}{?}$

c. $\frac{2}{5} = \frac{?}{?}$ **d.** $\frac{x}{5} = \frac{?}{?}$

Solution **a.** $5y = 2x$ **b.** $\frac{x+y}{y} = \frac{7}{2}$

c. $\frac{2}{5} = \frac{y}{x}$ **d.** $\frac{x}{5} = \frac{y}{2}$