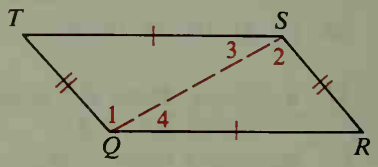
5-2 *Ways to Prove that Quadrilaterals Are Parallelograms*

If both pairs of opposite sides of a quadrilateral are parallel, then by definition the quadrilateral is a parallelogram. The following theorems will give you additional ways to prove that a quadrilateral is a parallelogram.

Theorem 5-4

If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.



Given: $\overline{TS} \cong \overline{QR}$; $\overline{TQ} \cong \overline{SR}$

Prove: Quad. $QRST$ is a $\square$.

Plan for Proof: Draw $\overline{QS}$ and prove that $\triangle TSQ \cong \triangle RQS$. Then $\angle 1 \cong \angle 2$ and $\angle 3 \cong \angle 4$, and opposite sides are parallel.

Theorem 5-5

If one pair of opposite sides of a quadrilateral are both congruent and parallel, then the quadrilateral is a parallelogram.

Theorem 5-6

If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

Theorem 5-7

If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

Five Ways to Prove that a Quadrilateral Is a Parallelogram

1. Show that *both* pairs of opposite sides are parallel.
2. Show that *both* pairs of opposite sides are congruent.
3. Show that *one* pair of opposite sides are both congruent and parallel.
4. Show that both pairs of opposite angles are congruent.
5. Show that the diagonals bisect each other.