The proof of Theorem 8-1 is left as Exercise 40. The altitude to the hypotenuse divides the hypotenuse into two segments. Corollaries 1 and 2 of Theorem 8-1 deal with geometric means and the lengths of these segments.

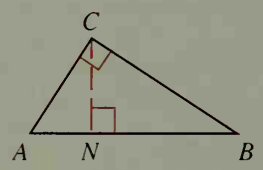
For simplicity in stating these corollaries, the words *segment*, *side*, *leg*, and *hypotenuse* are used to refer to the *length* of a segment rather than the segment itself. We will use this convention throughout the book when the context makes this meaning clear.

Corollary 1

When the altitude is drawn to the hypotenuse of a right triangle, the length of the altitude is the geometric mean between the segments of the hypotenuse.

Given: $\triangle ABC$ with rt. $\angle ACB$; altitude $\overline{CN}$

Prove: $\dfrac{AN}{CN} = \dfrac{CN}{BN}$



Proof:

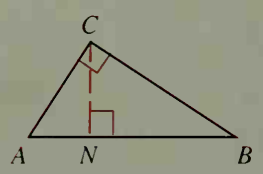
By Theorem 8-1, $\triangle ANC \sim \triangle CNB$. Because corresponding sides of similar triangles are in proportion, $\dfrac{AN}{CN} = \dfrac{CN}{BN}$.

Corollary 2

When the altitude is drawn to the hypotenuse of a right triangle, each leg is the geometric mean between the hypotenuse and the segment of the hypotenuse that is adjacent to that leg.

Given: $\triangle ABC$ with rt. $\angle ACB$; altitude $\overline{CN}$

Prove: (1) $\dfrac{AB}{AC} = \dfrac{AC}{AN}$ and (2) $\dfrac{AB}{BC} = \dfrac{BC}{BN}$



Proof of (1):

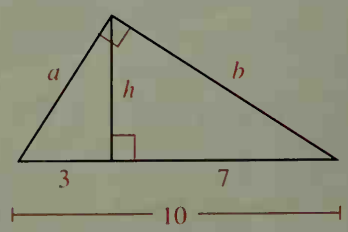
By Theorem 8-1, $\triangle ACB \sim \triangle ANC$. Because corresponding sides of similar triangles are in proportion, $\dfrac{AB}{AC} = \dfrac{AC}{AN}$. The proof of (2) is very similar.

Example 2 Use the diagram to find the values of h, a, and b.

Solution First determine what parts of the "big" triangle are labeled h, a, and b:

h is the altitude to the hypotenuse,
a is a leg, and b is a leg.

By Corollary 1, $\dfrac{3}{h} = \dfrac{h}{7}$ and $h = \sqrt{21}$.

By Corollary 2, $\dfrac{10}{a} = \dfrac{a}{3}$ and $a = \sqrt{30}$.

By Corollary 2, $\dfrac{10}{b} = \dfrac{b}{7}$ and $b = \sqrt{70}$.