Relating Algebra and Geometry

For products of numbers	*For composites of mappings*
1 is the identity.	I is the identity.
$a \cdot 1 = a$ and $1 \cdot a = a$	$S \circ I = S$ and $I \circ S = S$
The inverse of a is written a^{-1}, or $\frac{1}{a}$.	The inverse of S is written S^{-1}.
$a \cdot a^{-1} = 1$ and $a^{-1} \cdot a = 1$	$S \circ S^{-1} = I$ and $S^{-1} \circ S = I$

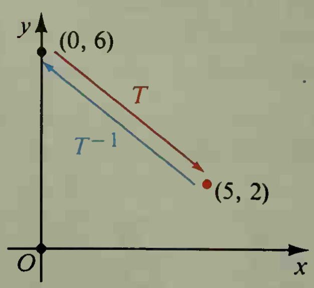
In general, the **inverse** of a transformation T is defined as the transformation S such that $S \circ T = I$. The inverses of some other transformations are illustrated below.

Example 1 Find the inverses of **(a)** translation $T:(x, y) \to (x + 5, y - 4)$, **(b)** rotation $\mathcal{R}_{O,x}$, and **(c)** dilation $D_{O,2}$.

Solution

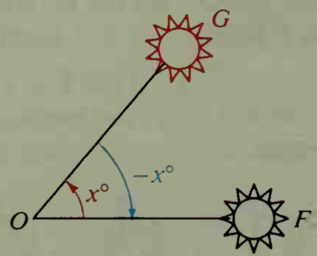
a. $T^{-1}:(x, y) \to (x - 5, y + 4)$

$T:(0, 6) \to (5, 2)$
$T^{-1}:(5, 2) \to (0, 6)$



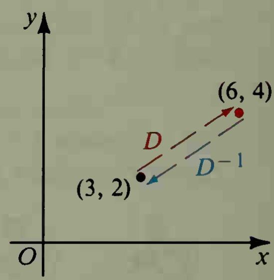
b. The inverse of $\mathcal{R}_{O,x}$ is $\mathcal{R}_{O,-x}$.

$\mathcal{R}_{O,x}:F \to G$
$\mathcal{R}_{O,-x}:G \to F$

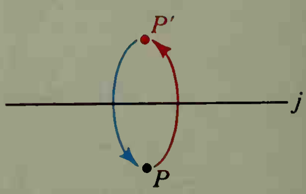


c. The inverse of $D_{O,2}$ is $D_{O,\frac{1}{2}}$.

$D_{O,2}:(3, 2) \to (6, 4)$
$D_{O,\frac{1}{2}}:(6, 4) \to (3, 2)$



Example 2 What is the inverse of R_j? (Refer to the diagram at right.)



Solution Since $R_j \circ R_j = I$, the inverse of R_j is R_j itself. In symbols, $R_j^{-1} = R_j$. Do you see that the inverse of any reflection is that same reflection?