35. Find the center of the circle that passes through (2, 10), (10, 6), and (−6, −6).

Exercises 36–39 refer to $\triangle QRS$ with vertices $Q(-6, 0)$, $R(12, 0)$, and $S(0, 12)$.

C 36. a. Find the equations of the three lines that contain the medians.
b. Show that the three medians meet in a point G (called the *centroid*). (*Hint*: Solve two equations simultaneously and show that their solution satisfies the third equation.)
c. Show that the length QG is $\frac{2}{3}$ of the length of the median from Q.

37. a. Find the equations of the three perpendicular bisectors of the sides of $\triangle QRS$.
b. Show that the three perpendicular bisectors meet in a point C (called the *circumcenter*). (See the hint from Exercise 36(b).)
c. Show that C is equidistant from Q, R, and S by using the distance formula.
d. Find the equation of the circle that can be circumscribed about $\triangle QRS$.

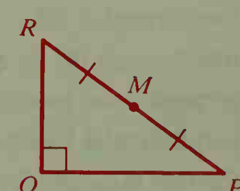
38. a. Find the equations of the three lines that contain the altitudes of $\triangle QRS$.
b. Show that the three altitudes meet in a point H (called the *orthocenter*).

39. a. Refer to Exercises 36, 37, and 38. Use slopes to show that the points C, G, and H are collinear. (The line through these points is called *Euler's Line*.)
b. Show that $GH = 2GC$.

13-8 Organizing Coordinate Proofs

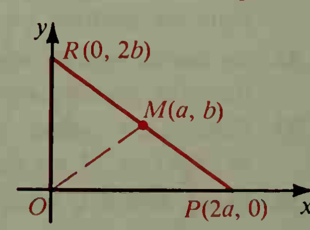
We will illustrate coordinate geometry methods by proving Theorem 5-15:

The midpoint of the hypotenuse of a right triangle is equidistant from the three vertices.



Proof:

Let $\overleftrightarrow{OP}$ and $\overleftrightarrow{OR}$ be the x-axis and y-axis. Let P and R have the coordinates shown.



Then the coordinates of M are (a, b).

$MO = \sqrt{(a-0)^2 + (b-0)^2} = \sqrt{a^2 + b^2}$
$MP = \sqrt{(a-2a)^2 + (b-0)^2} = \sqrt{a^2 + b^2}$
Thus $MO = MP$.

By the definition of midpoint, $MP = MR$.

Hence $MO = MP = MR$.