Trigonometry

Objectives

1. Define the tangent, sine, and cosine ratios for an acute angle.
2. Solve right triangle problems by correct selection and use of the tangent, sine, and cosine ratios.

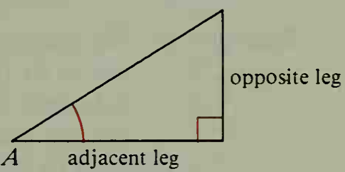
8-5 *The Tangent Ratio*

The word trigonometry comes from Greek words that mean "triangle measurement." In this book our study will be limited to the trigonometry of right triangles. In the right triangle shown, one acute angle is marked. The leg opposite this angle and the leg adjacent to this angle are labeled.

The following ratio of the lengths of the legs is called the *tangent ratio*.

tangent of $\angle A = \dfrac{\text{leg opposite } \angle A}{\text{leg adjacent to } \angle A}$

In abbreviated form: $\tan A = \dfrac{\text{opposite}}{\text{adjacent}}$

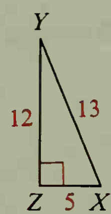


Example 1 Find tan X and tan Y.

Solution

$$\tan X = \frac{\text{leg opposite } \angle X}{\text{leg adjacent to } \angle X} = \frac{12}{5}$$

$$\tan Y = \frac{\text{leg opposite } \angle Y}{\text{leg adjacent to } \angle Y} = \frac{5}{12}$$

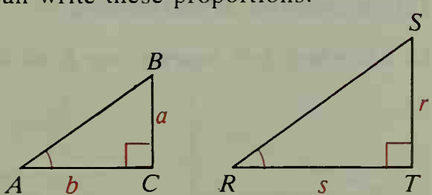


In the right triangles shown below, $m\angle A = m\angle R$. Then by the AA Similarity Postulate, the triangles are similar. We can write these proportions:

$\dfrac{a}{r} = \dfrac{b}{s}$ (Why?)

$\dfrac{a}{b} = \dfrac{r}{s}$ (A property of proportions)

$\tan A = \tan R$ (Def. of tangent ratio)



We have shown that if $m\angle A = m\angle R$, then $\tan A = \tan R$. Thus, we have shown that the value of the tangent of an angle depends only on the size of the angle, not on the size of the right triangle. It is also true that if $\tan A = \tan R$ for acute angles A and R, then $m\angle A = m\angle R$.