

11-7 Ratios of Areas

In this section you will learn to compare the areas of figures by finding ratios.

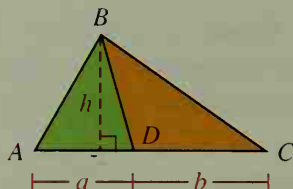
Example 1 Find the ratios of the areas of two triangles:

- with equal heights
- with equal bases
- that are similar

Solution

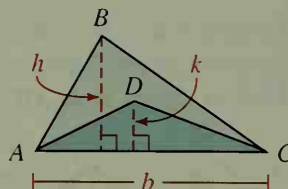
$$\text{a. } \frac{\text{area of } \triangle ABD}{\text{area of } \triangle DBC} = \frac{\frac{1}{2}ah}{\frac{1}{2}bh} = \frac{a}{b}$$

ratio of areas = ratio of bases



$$\text{b. } \frac{\text{area of } \triangle ABC}{\text{area of } \triangle ADC} = \frac{\frac{1}{2}bh}{\frac{1}{2}bk} = \frac{h}{k}$$

ratio of areas = ratio of heights



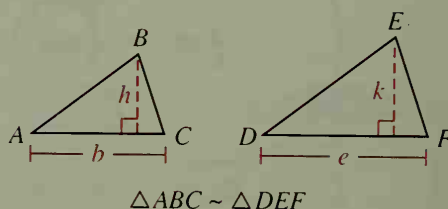
$$\text{c. } \frac{\text{area of } \triangle ABC}{\text{area of } \triangle DEF} = \frac{\frac{1}{2}bh}{\frac{1}{2}ek} = \frac{bh}{ek} = \frac{b}{e} \cdot \frac{h}{k}$$

It follows from Exercise 25 on page 259 that if

$$\triangle ABC \sim \triangle DEF, \text{ then } \frac{h}{k} = \frac{b}{e}.$$

$$\text{Thus, } \frac{\text{area of } \triangle ABC}{\text{area of } \triangle DEF} = \frac{b}{e} \cdot \frac{h}{k} = \frac{b}{e} \cdot \frac{b}{e} = \left(\frac{b}{e}\right)^2 = (\text{scale factor})^2.$$

ratio of areas = square of scale factor



Example 1 justifies the following properties.

Comparing Areas of Triangles

- If two triangles have equal heights, then the ratio of their areas equals the ratio of their bases.
- If two triangles have equal bases, then the ratio of their areas equals the ratio of their heights.
- If two triangles are similar, then the ratio of their areas equals the square of their scale factor.