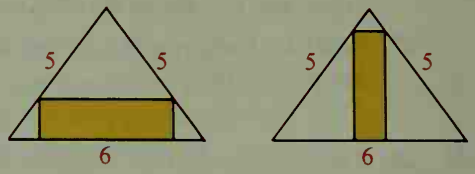
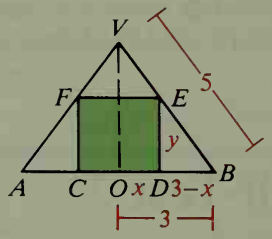
♦ Calculator Key-In

Each diagram shows a rectangle inscribed in an isosceles triangle with legs 5 and base 6. There are many more such rectangles. Which one has the greatest area?



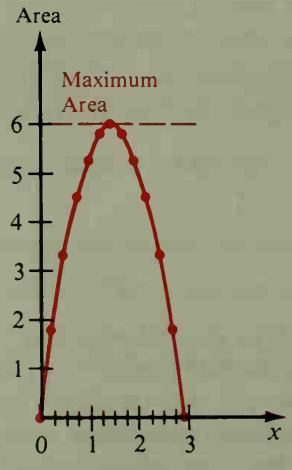
To solve the problem, let $CDEF$ represent any rectangle inscribed in isosceles $\triangle ABV$ with legs 5 and base 6. If we let $OD = x$ and $ED = y$, then the area of the rectangle is $2xy$. Our goal is to express this area in terms of x alone. Then we can find out how the area changes as x changes.



1. In right $\triangle VOB$, $OB = 3$ and $VB = 5$. Thus $VO = 4$ by the Pythagorean Theorem.
2. $\triangle EDB \sim \triangle VOB$ (Why?)
3. $\dfrac{ED}{VO} = \dfrac{DB}{OB}$ (Why?)
4. $\dfrac{y}{4} = \dfrac{3 - x}{3}$ (By substitution in Step 3)
5. $y = \dfrac{4}{3}(3 - x)$ (Multiplication Property of $=$)
6. Area of rectangle: $A = 2xy = 2x \cdot \dfrac{4}{3}(3 - x) = \dfrac{8x(3 - x)}{3}$

Use the formula in Step 6 and a calculator to find the area for many values of x. Calculate $3 - x$ first, then multiply by x, then multiply by 8, and divide by 3.

x	Area
0	0
0.25	1.83333
0.5	3.33333
0.75	4.5
1	5.33333
1.25	5.83333
1.5	6
1.75	5.83333
2	5.33333
2.25	4.5
2.50	3.33333
2.75	1.83333
3	0



The table was used to make a graph showing how the area varies with x. Both the table and the graph suggest that the greatest area, 6 square units, occurs when $x = 1.5$.