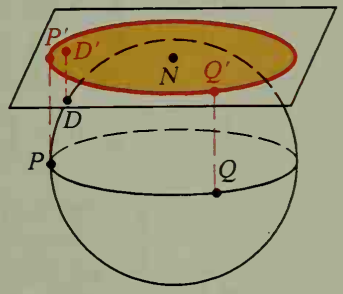
Some Basic Mappings

Objectives

1. Recognize and use the terms *image*, *preimage*, *mapping*, *one-to-one mapping*, *transformation*, *isometry*, and *congruence mapping*.
2. Locate images of figures by reflection, translation, glide reflection, rotation, and dilation.
3. Recognize the properties of the basic mappings.

14-1 *Mappings and Functions*

Have you ever wondered how maps of the round Earth can be made on flat paper? The diagram illustrates the idea behind a *polar map* of the northern hemisphere. A plane is placed tangent to a globe of the Earth at its North Pole N. Every point P of the globe is projected straight upward to exactly one point, called P', in the plane. P' is called the **image** of P, and P is called the **preimage** of P'. The diagram shows the images of two points P and Q on the globe's equator. It also shows D', the image of a point D not on the equator.



This correspondence between points of the globe's northern hemisphere and points in the plane is an example of a *mapping*. If we call this mapping M, then we could indicate that M maps P to P' by writing $M:P \rightarrow P'$. Notice that since the North Pole N is mapped to itself, we can write $M:N \rightarrow N$.

The word *mapping* is used in geometry as the word *function* is used in algebra. While a **mapping** is a correspondence between sets of points, a **function** is a correspondence between sets of numbers. Each number in the first set corresponds to exactly one number in the second set. For example, the squaring function f maps each real number x to its square x^2. We can write $f:x \rightarrow x^2$. Another way to indicate that the value of the function at x is x^2 is to write $f(x) = x^2$ (read "f of x equals x^2"). Similarly, for the mapping M, above, we can write $M(P) = P'$ to indicate that the image of P is P'. With all of these similarities, it should not surprise you that mathematicians often use the words *function* and *mapping* interchangeably.

A mapping (or a function) from set A to set B is called a **one-to-one mapping** (or a one-to-one function) if every member of B has exactly one preimage in A. The polar projection illustrated at the top of the page is a one-to-one mapping of the northern hemisphere of the globe onto a circular region in the tangent plane (the shaded area in the diagram). However, the squaring function $f:x \rightarrow x^2$ is *not* one-to-one because, for example, 9 has two preimages, 3 and -3.