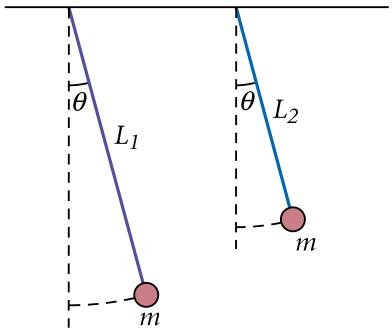

Figure 7
When the length of one pendulum is decreased, the distance that the pendulum travels to equilibrium is also decreased. Because the accelerations of the two pendulums are equal, the shorter pendulum will have a smaller period.

Why does the period of a pendulum depend on pendulum length and free-fall acceleration? When two pendulums have different lengths but the same amplitude, the shorter pendulum will have a smaller arc to travel through, as shown in **Figure 7.** Because the distance from maximum displacement to equilibrium is less while the acceleration caused by the restoring force remains the same, the shorter pendulum will have a shorter period.

Why don't mass and amplitude affect the period of a pendulum? When the bobs of two pendulums differ in mass, the heavier mass provides a larger restoring force, but it also needs a larger force to achieve the same acceleration. This is similar to the situation for objects in free fall, which all have the same acceleration regardless of their mass. Because the acceleration of both pendulums is the same, the period for both is also the same.

For small angles (<15°), when the amplitude of a pendulum increases, the restoring force also increases proportionally. Because force is proportional to acceleration, the initial acceleration will be greater. However, the distance this pendulum must cover is also greater. For small angles, the effects of the two increasing quantities cancel and the pendulum's period remains the same.

SAMPLE PROBLEM B

Simple Harmonic Motion of a Simple Pendulum

PROBLEM

You need to know the height of a tower, but darkness obscures the ceiling. You note that a pendulum extending from the ceiling almost touches the floor and that its period is 12 s. How tall is the tower?

SOLUTION

Given: $T = 12\ \text{s}$ $\quad a_g = g = 9.81\ \text{m/s}^2$

Unknown: $L = ?$

Use the equation for the period of a simple pendulum, and solve for L.

$$T = 2\pi\sqrt{\frac{L}{a_g}}$$

$$\frac{T\sqrt{a_g}}{2\pi} = \sqrt{L}$$

$$\frac{T^2 a_g}{4\pi^2} = L$$

$$L = \frac{(12\ \text{s})^2(9.81\ \text{m/s}^2)}{4\pi^2}$$

$$\boxed{L = 36\ \text{m}}$$

TIP *Remember that on Earth's surface, $a_g = g = 9.81\ m/s^2$. Use this value in the equation for the period of a pendulum if a problem does not specify otherwise. At higher altitudes or on different planets, use the given value of a_g instead.*