Right Triangles

Objectives

1. Determine the geometric mean between two numbers.
2. State and apply the relationships that exist when the altitude is drawn to the hypotenuse of a right triangle.
3. State and apply the Pythagorean Theorem.
4. State and apply the converse of the Pythagorean Theorem and related theorems about obtuse and acute triangles.
5. Determine the lengths of two sides of a 45°-45°-90° or a 30°-60°-90° triangle when the length of the third side is known.

8-1 *Similarity in Right Triangles*

Recall that in the proportion $\frac{a}{x} = \frac{y}{b}$, the terms shown in red are called the *means*. If a, b, and x are positive numbers and $\frac{a}{x} = \frac{x}{b}$, then x is called the **geometric mean** between a and b. If you solve this proportion for x, you will find that $x = \sqrt{ab}$, a positive number. (The other solution, $x = -\sqrt{ab}$, is discarded because x is defined to be positive.)

Example 1 Find the geometric mean between 5 and 11.

Solution 1 Solve the proportion $\frac{5}{x} = \frac{x}{11}$; $x^2 = 5 \cdot 11$; $x = \sqrt{55}$.

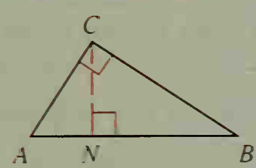
Solution 2 Use the equation $x = \sqrt{ab} = \sqrt{5 \cdot 11} = \sqrt{55}$.

Theorem 8-1

If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle and to each other.

Given: $\triangle ABC$ with rt. $\angle ACB$;
altitude $\overline{CN}$

Prove: $\triangle ACB \sim \triangle ANC \sim \triangle CNB$



Plan for Proof: Begin by redrawing the three triangles you want to prove similar. Mark off congruent angles and apply the AA Similarity Postulate.