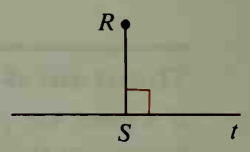
Example Suppose you know that line l is the perpendicular bisector of $\overline{RS}$. What can you deduce if you also know that

a. P lies on l?
b. there is a point Q such that $QR = 7$ and $QS = 7$?

Solution **a.** $PR = PS$ (Theorem 4-5)
b. Q lies on l. (Theorem 4-6)

The **distance from a point to a line** (or plane) is defined to be the length of the perpendicular segment from the point to the line (or plane). Since $\overline{RS} \perp t$, RS is the distance from R to line t.



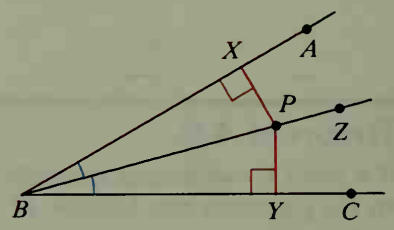
In Exercises 16 and 17 you will prove the following theorems, which are similar to Theorems 4-5 and 4-6.

Theorem 4-7

If a point lies on the bisector of an angle, then the point is equidistant from the sides of the angle.

Given: $\overrightarrow{BZ}$ bisects $\angle ABC$; P lies on $\overrightarrow{BZ}$;
$\overline{PX} \perp \overrightarrow{BA}$; $\overline{PY} \perp \overrightarrow{BC}$

Prove: $PX = PY$

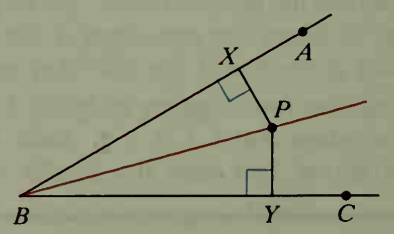


Theorem 4-8

If a point is equidistant from the sides of an angle, then the point lies on the bisector of the angle.

Given: $\overline{PX} \perp \overrightarrow{BA}$; $\overline{PY} \perp \overrightarrow{BC}$;
$PX = PY$

Prove: $\overrightarrow{BP}$ bisects $\angle ABC$.



Theorem 4-5 and its converse, Theorem 4-6, can be combined into a single biconditional statement. The same is true for Theorems 4-7 and 4-8.

> A point is on the perpendicular bisector of a segment if and only if it is equidistant from the endpoints of the segment.

> A point is on the bisector of an angle if and only if it is equidistant from the sides of the angle.