KEPLER'S LAWS OF PLANETARY MOTION

First Law: Each planet travels in an elliptical orbit around the sun, and the sun is at one of the focal points.

Second Law: An imaginary line drawn from the sun to any planet sweeps out equal areas in equal time intervals.

Third Law: The square of a planet's orbital period (T^2) is proportional to the cube of the average distance (r^3) between the planet and the sun, or $T^2 \propto r^3$.

Kepler's first law states that the planets' orbits are ellipses rather than circles. Kepler discovered this law while working with Brahe's data for the orbit of Mars. While trying to explain the data, Kepler experimented with 70 different circular orbits and generated numerous pages of calculations. He finally realized that if the orbit is an ellipse rather than a circle and the sun is at one focal point of the ellipse, the data fit perfectly.

Kepler's second law states that an imaginary line from the sun to any planet sweeps out equal areas in equal times, as shown in **Figure 12.** In other words, if the time a planet takes to travel the arc on the left (Δt_1) is equal to the time the planet takes to cover the arc on the right (Δt_2), then the area A_1 is equal to the area A_2. Thus, the planets travel faster when they are closer to the sun.

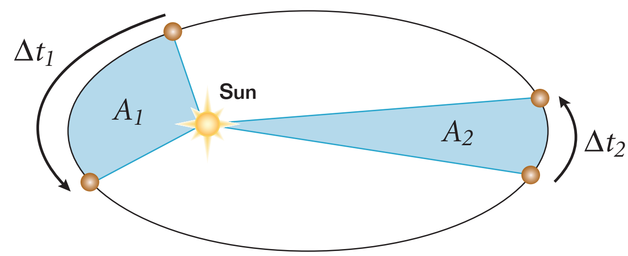

Figure 12
This diagram illustrates a planet moving in an elliptical orbit around the sun. If Δt_1 equals Δt_2, then the two shaded areas are equal. Thus, the planet travels faster when it is closer to the sun and slower when it is farther away.

While Kepler's first two laws describe the motion of each planet individually, his third law relates the orbital periods and distances of one planet to those of another planet. The orbital period (T) is the time a planet takes to finish one full revolution, and the distance (r) is the mean distance between the planet and the sun. Kepler's third law relates the orbital period and mean distance for two orbiting planets as follows:

$$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}, \text{ or } T^2 \propto r^3$$

This law also applies to satellites orbiting Earth, including our moon. In that case, r is the distance between the orbiting satellite and Earth. The proportionality constant depends on the mass of the central object.

Kepler's laws are consistent with Newton's law of gravitation

Newton used Kepler's laws to support his law of gravitation. For example, Newton proved that if force is inversely proportional to distance squared, as stated in the law of universal gravitation, the resulting orbit must be an ellipse or a circle. He also demonstrated that his law of gravitation could be used to derive Kepler's third law. (Try a similar derivation yourself in the Quick Lab at right.) The fact that Kepler's laws closely matched observations gave additional support for Newton's theory of gravitation.

Quick Lab

Kepler's Third Law

You can mathematically show how Kepler's third law can be derived from Newton's law of universal gravitation (assuming circular orbits). To begin, recall that the centripetal force is provided by the gravitational force. Set the equations for gravitational and centripetal force equal to one another, and solve for v_t^2. Because speed equals distance divided by time and because the distance for one period is the circumference ($2\pi r$), $v_t = 2\pi r/T$. Square this value, substitute the squared value into your previous equation, and then isolate T^2. How does your result relate to Kepler's third law?