**C** **15.** Consider the following statements:

*Reflexive Property:* Robot $A$ is as rusty as itself.

*Symmetric Property:* If Robot $A$ is as rusty as Robot $B$, then Robot $B$ is as rusty as Robot $A$.

*Transitive Property:* If Robot $A$ is as rusty as Robot $B$ and Robot $B$ is as rusty as Robot $C$, then Robot $A$ is as rusty as Robot $C$.

A *relation* such as "is as rusty as" that is reflexive, symmetric, and transitive is an *equivalence relation*. Which of the following are equivalence relations?

**a.** is rustier than      **b.** has the same length as

**c.** is opposite (for rays)      **d.** is coplanar with (for lines)
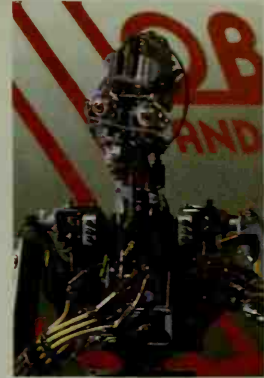
# 2-3 *Proving Theorems*

Chapter 1 included three *theorems*, statements that are proved. The theorems were deduced from *postulates*, statements that are accepted without proof. We will prove additional theorems throughout the book. When writing proofs, we will treat properties from algebra as postulates.

Suppose you are told that $Y$ is the midpoint of $\overline{XZ}$ and that $XZ = 12$. You probably realize that $XY = 6$. Your conclusion about one particular situation suggests the general statement shown below as Theorem 2-1. The theorem uses the definition of a midpoint to prove additional properties of a midpoint that are not explicitly included in the definition. In this case, the theorem states something obvious. Later theorems may not be so obvious. In fact, some of them may surprise you.

## Theorem 2-1 *Midpoint Theorem*

If $M$ is the midpoint of $\overline{AB}$, then $AM = \frac{1}{2}AB$ and $MB = \frac{1}{2}AB$.

Given: $M$ is the midpoint of $\overline{AB}$.

Prove: $AM = \frac{1}{2}AB$; $MB = \frac{1}{2}AB$
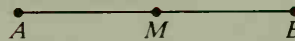
**Proof:**

| Statements | Reasons |
|---|---|
| 1. $M$ is the midpoint of $\overline{AB}$. | 1. Given |
| 2. $\overline{AM} \cong \overline{MB}$, or $AM = MB$ | 2. Definition of midpoint |
| 3. $AM + MB = AB$ | 3. Segment Addition Postulate |
| 4. $AM + AM = AB$, or $2AM = AB$ | 4. Substitution Prop. (Steps 2 and 3) |
| 5. $AM = \frac{1}{2}AB$ | 5. Division Prop. of = |
| 6. $MB = \frac{1}{2}AB$ | 6. Substitution Prop. (Steps 2 and 5) |