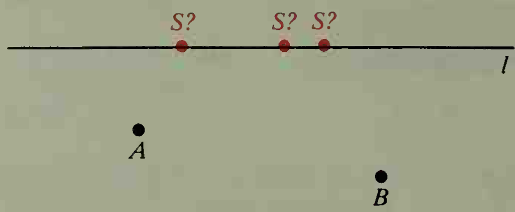
Application — Finding the Shortest Path

The owners of pipeline l plan to construct a pumping station at a point S on line l in order to pipe oil to two major customers, located at A and B. To minimize the cost of constructing lines from S to A and B, they wish to locate S along l so that the distance $SA + SB$ is as small as possible.



The construction engineer uses the following method to locate S:

1. Draw a line through B perpendicular to l, intersecting l at point P.
2. On this perpendicular, locate point C so that $PC = PB$.
3. Draw $\overline{AC}$.
4. Locate S at the intersection of $\overline{AC}$ and l.

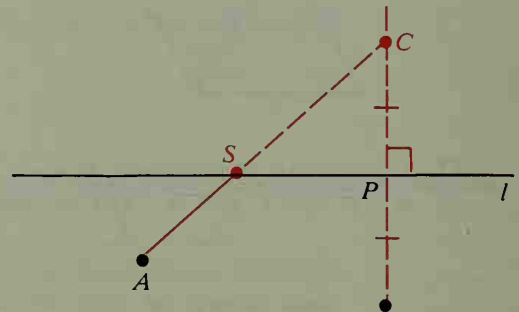


Figure 1

Figure 2 shows the path of the new pipelines through the pumping station located at S, and an alternative path going through a different point, X, on l. You can use Theorem 6-4 (the Triangle Inequality) to show that if X is any point on l other than S, then $AX + XB > AS + SB$. So any alternative path is longer than the path through S.

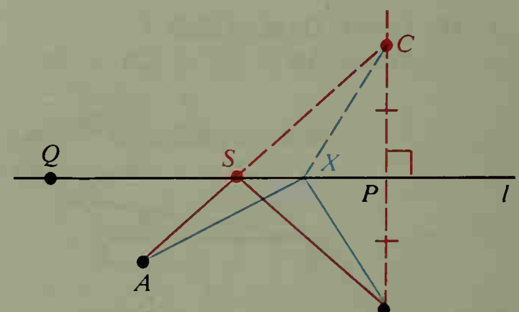


Figure 2