Special Relativity and Velocities

In Section 4 of the chapter "Two-Dimensional Motion and Vectors," you learned that velocity measurements are not absolute; every velocity measurement depends on the frame of reference of the observer with respect to the moving object. For example, imagine that someone riding a bike toward you at 25 m/s (v) throws a softball toward you. If the bicyclist measures the softball's speed (u') to be 15 m/s, you would perceive the ball to be moving toward you at 40 m/s (u) because you have a different frame of reference than the bicyclist does. This is expressed mathematically by the equation $u = v + u'$, which is also known as the classical addition of velocities.

The speed of light

As stated in the appendix feature "Special Relativity and Time Dilation," according to Einstein's special theory of relativity, the speed of light is absolute, or independent of all frames of reference. If, instead of a softball, the bicyclist were to shine a beam of light toward you, both you and the bicyclist would measure the light's speed as 3.0×10^8 m/s. This would remain true even if the bicyclist were moving toward you at 99 percent of the speed of light. Thus, Einstein's theory requires a different approach to the addition of velocities. Einstein's modification of the classical formula, which he derived in his 1905 paper on special relativity, covers both the case of the softball and the case of the light beam.

$$u = \frac{v + u'}{1 + (vu'/c^2)}$$

In the equation, u is the velocity of an object in a reference frame, u' is the velocity of the same object in another reference frame, v is the velocity of one reference frame relative to another, and c is the speed of light.

Figure 1
According to Einstein's relativistic equation for the addition of velocities, material particles can never reach the speed of light.

The universality of Einstein's equation

How does Einstein's equation cover both cases? First we shall consider the bicyclist throwing a softball. Because c^2 is such a large number, the vu'/c^2 term in the denominator is very small for velocities typical of our everyday experience. As a result, the denominator of the equation is essentially equal to 1. Hence, for speeds that are small compared with c, the two theories give nearly the same result, $u = v + u'$, and the classical addition of velocities can be used.

However, when speeds approach the speed of light, vu'/c^2 increases, and the denominator becomes greater than 1 but never more than 2. When this occurs, the difference between the two theories becomes significant. For example, if a bicyclist