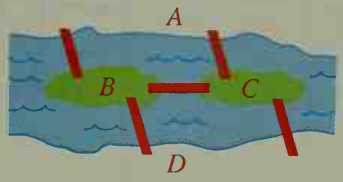
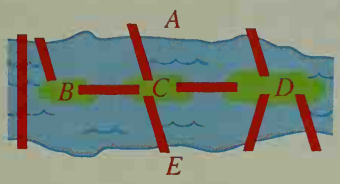
Exercises

Draw a graph for each bridge problem below. Find the valence of each vertex and decide whether an Euler circuit is possible. If possible, find an Euler circuit for the graph.

1.

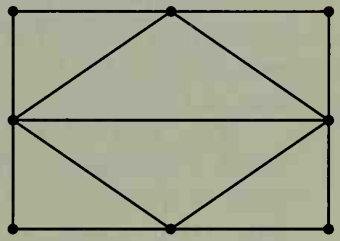


2.



3. An Euler circuit is not possible for the bridge problem in Exercise 1. But it *is* possible to start at one point of the city, travel over each bridge once, and end at a point of the city other than the starting point. Show how this is possible.

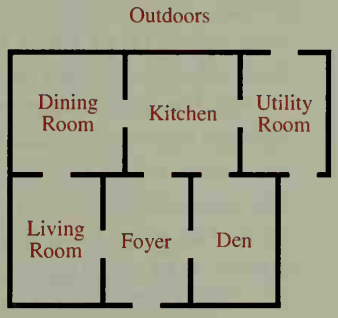
4. A network of city streets is shown at the right.



a. Could a member of the highway department start at an intersection, inspect each street, and return to the starting point without traveling over any portion of a street twice?

b. Can part **a** be done if the inspector starts and ends at different points?

5. The first floor plan of a house is shown at the right.

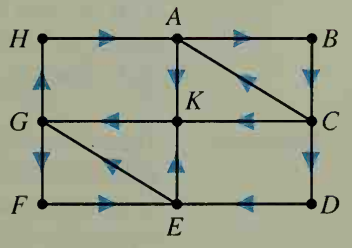


a. Can you enter the house from outside, travel through each doorway exactly once, and return to your starting point? Make a graph in which the outdoors is represented by one vertex and each room is represented by a vertex.

b. Can you travel through each door exactly once and end at a place different from where you started?

c. Examine your answers to Exercises **1–4** more closely. Can you make some conjectures about what is or is not possible with different numbers of vertices with odd valences?

6. The graph at the right represents a network of one-way streets. The arrowheads indicate the direction of travel. Since two streets lead into vertex A, we say that the *invalence* of A is 2. Since two streets lead out of vertex A, we say that the *outvalence* of A is 2.



a. Find the invalence and outvalence of each of the other vertices.

b. Is an Euler circuit possible? Why or why not?

c. Can you change the direction of just one arrowhead so that an Euler circuit is possible? Name the vertices in one order in which they could be visited.