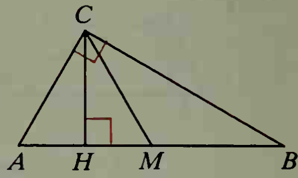
45. The *arithmetic mean* between two numbers r and s is defined to be $\frac{r+s}{2}$.

a. $\overline{CM}$ is the median and $\overline{CH}$ is the altitude to the hypotenuse of right $\triangle ABC$. Show that CM is the arithmetic mean between AH and BH, and that CH is the geometric mean between AH and BH. Then use the diagram to show that the arithmetic mean is greater than the geometric mean.



b. Show algebraically that the arithmetic mean between two different numbers r and s is greater than the geometric mean. (*Hint*: The geometric mean is $\sqrt{rs}$. Work backward from $\frac{r+s}{2} > \sqrt{rs}$ to $(r - s)^2 > 0$ and then reverse the steps.)

8-2 *The Pythagorean Theorem*

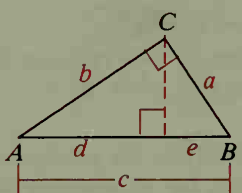
One of the best known and most useful theorems in all of mathematics is the *Pythagorean Theorem.* It is believed that Pythagoras, a Greek mathematician and philosopher, proved this theorem about twenty-five hundred years ago. Many different proofs exist, including one by President Garfield (Exercise 32, page 438) and the proof suggested by the Challenge on page 294.

Theorem 8-2 *Pythagorean Theorem*

In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the legs.

Given: $\triangle ABC$; $\angle ACB$ is a rt. $\angle$.

Prove: $c^2 = a^2 + b^2$



Proof:

Statements	Reasons
1. Draw a perpendicular from C to $\overline{AB}$.	1. Through a point outside a line, there is exactly one line __?__.
2. $\frac{c}{a} = \frac{a}{e}$; $\frac{c}{b} = \frac{b}{d}$	2. When the altitude is drawn to the hypotenuse of a rt. $\triangle$, each leg is the geometric mean between __?__.
3. $ce = a^2$; $cd = b^2$	3. A property of proportions
4. $ce + cd = a^2 + b^2$	4. Addition Property of =
5. $c(e + d) = a^2 + b^2$	5. Distributive Property
6. $c^2 = a^2 + b^2$	6. Substitution Property