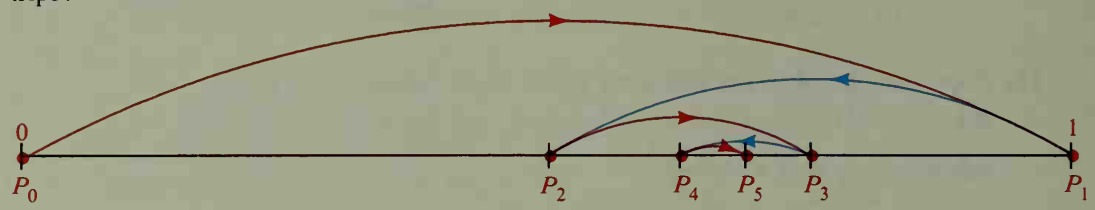
♦ Computer Key-In

A bee starts at point P_0, flies to point P_1, and lands. The bee then returns half of the way to P_0, landing at P_2. From P_2, the bee returns half of the way to P_1, landing at P_3, and so forth. Can you predict the bee's location after 10 trips?



Assuming that P_0 and P_1 have coordinates 0 and 1, respectively, the BASIC program below will compute and print the bee's location at the end of trips 2 through 10. P_n represents the position of the bee after n trips. Since P_n is the midpoint of the bee's previous two positions, P_{n-1} and P_{n-2}, line 50 calculates $P(N)$ by using the statement proved in Exercise 19, page 47.

```
10  DIM P(50)
20  LET P(0) = 0
30  LET P(1) = 1
40  FOR N = 2 TO 10
50  LET P(N) = (1/2) * (P(N - 2) + P(N - 1))
60  PRINT N, P(N)
70  NEXT N
80  END
```

Exercises

1. Enter the program on your computer and RUN it. Do you notice any patterns or trends in the coordinates? Change line 40 so that the computer will print the coordinates up to P_{40}. What simple fraction is approximated by P_{40}?
2. In line 50, $P(n)$ could instead be computed from the *series*
$$1 - \tfrac{1}{2} + \tfrac{1}{4} - \tfrac{1}{8} + \cdots + (-\tfrac{1}{2})^{n-1}$$
where each term of the series reflects the bee's return half of the way from P_{n-1} to P_{n-2}. Replace line 50 with the line below and RUN the new program.
```
50  LET P(N) = P(N - 1) + (-1/2) ↑ (N - 1)
```
Check that both programs produce the same results. (Some slight variations will be expected, due to rounding off.)
3. Suppose that on each trip the bee returned one third of the way to the previous point instead of half of the way. How would the series in Exercise 2 be modified? How would line 50 of Exercise 2 be modified? RUN a modified program for 30 trips and determine what point the bee seems to be approaching.