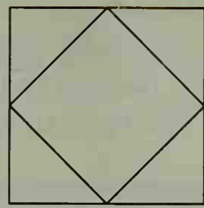
10. The inner square in the diagram is formed by connecting the midpoints of the outer square. It is possible to map the outer square to the inner square by performing a rotation followed by a dilation.



   a. Give the center of the rotation and the amount of the rotation.

   b. Give the center of the dilation and the scale factor.

11–21. Work Exercises 32, 42 on page 527; Classroom Exercise 7 on page 532; Exercises 28–31, 33 on pages 533–534; Classroom Exercise 11 on page 536; Exercise 23 on page 538; and Classroom Exercise 5 on page 541.

Circles (Chapter 9)

Objective: Write the equations of tangent lines, and make observations about circles and their symmetry. (Requires understanding of Lessons 13-1 through 13-7 and 14-1 through 14-5.)

Many relationships among circles and their chords and tangents can be investigated by using coordinates and transformations. Before studying the example below, you should understand how the equation of a circle is used in Examples 3 and 4 of Lesson 13-1.

Example

a. Sketch the circle $x^2 + y^2 = 10$ and the line $y = 3x + 10$.

b. Solve the two equations simultaneously and show that there is just one solution for x.

c. Find the corresponding value for y. Label the solution point, T, on your sketch. What does this tell you about the line and the circle?

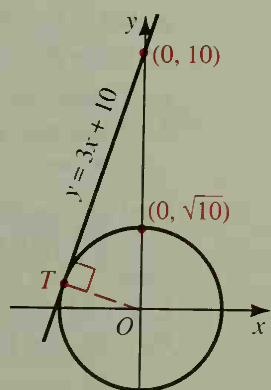
d. Use the slopes to show that $\overline{OT}$ is perpendicular to the line $y = 3x + 10$.

Solution

a. The circle has center O and radius $\sqrt{10}$.

b.

$$\begin{aligned} x^2 + y^2 &= 10 \\ x^2 + (3x + 10)^2 &= 10 \\ x^2 + 9x^2 + 60x + 100 &= 10 \\ 10x^2 + 60x + 90 &= 0 \\ x^2 + 6x + 9 &= 0 \\ (x + 3)(x + 3) &= 0 \\ x + 3 &= 0 \\ x &= -3 \end{aligned}$$



c. $y = 3x + 10$; $y = 3(-3) + 10$; $y = 1$. Point T is $(-3, 1)$. The line is tangent to the circle at point T.

d. The slope of $\overline{OT}$ is $\frac{1 - 0}{-3 - 0} = -\frac{1}{3}$. The slope of $y = 3x + 10$ is 3. Since the slopes are negative reciprocals, the lines are perpendicular.