It is also true that if you are given any regular polygon, you can circumscribe a circle about it. This relationship between circles and regular polygons leads to the following definitions:

The **center of a regular polygon** is the center of the circumscribed circle.

The **radius of a regular polygon** is the distance from the center to a vertex.

A **central angle of a regular polygon** is an angle formed by two radii drawn to consecutive vertices.

The **apothem of a regular polygon** is the (perpendicular) distance from the center of the polygon to a side.

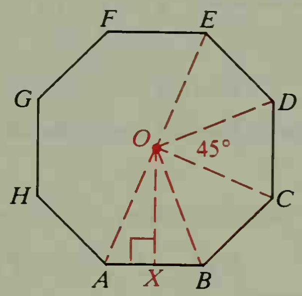
Center of regular octagon: O

Radius: OA, OB, OC, and so on

Central angle: $\angle AOB$, $\angle BOC$, and so on

Measure of central angle: $\frac{360}{8} = 45$

Apothem: OX



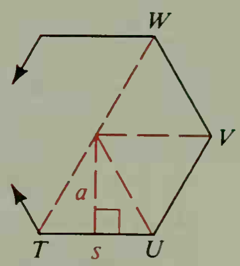
If you know the apothem and the perimeter of a regular polygon, you can use the next theorem to find the area of the polygon.

Theorem 11-6

The area of a regular polygon is equal to half the product of the apothem and the perimeter. ($A = \frac{1}{2}ap$)

Given: Regular n-gon $TUVW$ . . . ; apothem a; side s; perimeter p; area A

Prove: $A = \frac{1}{2}ap$



Key steps of proof:

1. If all radii are drawn, n congruent triangles are formed.
2. Area of each $\triangle = \frac{1}{2}sa$
3. $A = n(\frac{1}{2}sa) = \frac{1}{2}a(ns)$
4. Since $ns = p$, $A = \frac{1}{2}ap$.

Example 1 Find the area of a regular hexagon with apothem 9.

Solution Use 30°-60°-90° $\triangle$ relationships.

$$\frac{1}{2}s = \frac{9}{\sqrt{3}} = 3\sqrt{3}$$

$$s = 6\sqrt{3};\quad p = 36\sqrt{3}$$

$$A = \frac{1}{2}ap = \frac{1}{2}\cdot 9\cdot 36\sqrt{3}$$

$$= 162\sqrt{3}$$