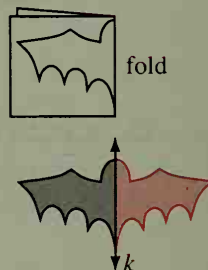


A piece of paper is wrapped around a globe of the Earth to form a cylinder as shown.  $O$  is the center of the Earth and a point  $P$  of the globe is projected along  $\overrightarrow{OP}$  to a point  $P'$  of the cylinder.



16. Describe the image of the globe's equator.
17. Is the image of the Arctic Circle congruent to the image of the equator?
18. Are distances near the equator distorted more than or less than distances near the Arctic Circle?
19. Does the North Pole (point  $N$ ) have an image?
20. Consider the mapping  $S: (x, y) \rightarrow (x, 0)$ .
  - a. Plot the points  $P(4, 5)$ ,  $Q(-3, 2)$ , and  $R(-3, -1)$  and their images.
  - b. Does  $S$  appear to be an isometry? Explain.
  - c. Is  $S$  a transformation? Explain.
21. Mapping  $M$  maps points  $A$  and  $B$  to the same image point. Explain why the mapping  $M$  does not preserve distance.
22. Fold a piece of paper. Cut a design connecting the top and bottom point of the fold, as shown. Unfold the shape. Consider a mapping  $M$  of the points in the gray region to the corresponding points in the red region.
  - a. Does  $M$  appear to be an isometry?
  - b. If a point  $P$  is on line  $k$ , what is the image of  $P$ ?
  - c. If a point  $Q$  is not on line  $k$ , and  $M(Q) = Q'$ , what is the relationship between line  $k$  and  $\overline{QQ'}$ ?



- C** 23. a. Plot the points  $A(6, 1)$ ,  $B(3, 4)$ , and  $C(1, -3)$  and their images  $A'$ ,  $B'$ , and  $C'$  under the transformation  $R: (x, y) \rightarrow (-x, y)$ .
- b. Prove that  $R$  is an isometry. (Hint: Let  $P(x_1, y_1)$  and  $Q(x_2, y_2)$  be any two points. Find  $P'$  and  $Q'$ , and use the distance formula to show that  $PQ = P'Q'$ .)

## Explorations

These exploratory exercises can be done using a computer with a program that draws and measures geometric figures.

As you will learn in the next lesson, a *reflection* is a mapping in the plane across a mirror line, just as your reflection in a mirror is a mapping in space across a mirror plane.

Draw any  $\triangle ABC$ . Reflect  $C$  in  $\overleftrightarrow{AB}$  to locate point  $D$ . Draw  $\overline{AD}$  and  $\overline{BD}$ . What do you notice about  $\triangle ABC$  and  $\triangle ABD$ ?

Draw  $\overline{CD}$ . Label the intersection of  $\overleftrightarrow{AB}$  and  $\overleftrightarrow{CD}$  as  $E$ . Compare  $CE$  and  $DE$ . What do you notice? Measure the angles with vertex  $E$ . What do you notice?

Repeat the construction with other types of triangles.

