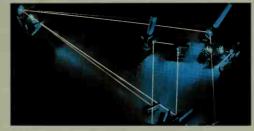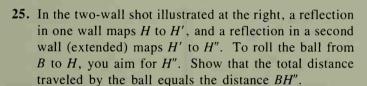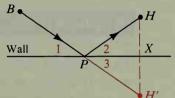**23.** If a transformation maps two parallel lines to two image lines that are also parallel, we say that parallelism is invariant under the transformation. Is parallelism invariant under a reflection?
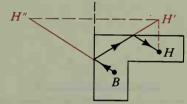
The photograph shows a reflected beam of laser light. Exercises 24–28 deal with the similar reflected path of a golf ball bouncing off the walls of a miniature golf layout. These exercises show how the geometry of reflections can be used to solve the problem of aiming a reflected path at a particular target.
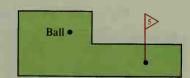

24. A rolling ball that does not have much spin will bounce off a wall so that the two angles that the path forms with the wall are congruent. Thus, to roll the ball from B off the wall shown and into hole H, you need to aim the ball so that $\angle 1 \cong \angle 2$.

   a. Let H' be the image of H by reflection in the wall. $\overline{BH'}$ intersects the wall at P. Why is $\angle 1 \cong \angle 3$? Why is $\angle 3 \cong \angle 2$? Why is $\angle 1 \cong \angle 2$? You can conclude that if you aim for H', the ball will roll to H.

   b. Show that the distance traveled by the ball equals the distance BH'.

25. In the two-wall shot illustrated at the right, a reflection in one wall maps H to H', and a reflection in a second wall (extended) maps H' to H''. To roll the ball from B to H, you aim for H''. Show that the total distance traveled by the ball equals the distance BH''.
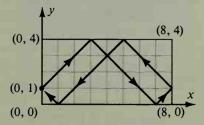

26. Show how to score a hole in one on the fifth hole of the golf course shown by rolling the ball off one wall.

27. Repeat Exercise 26 but roll the ball off two walls.

28. Repeat Exercise 26 but roll the ball off three walls.

29. A ball rolls at a 45° angle away from one side of a billiard table that has a coordinate grid on it. If the ball starts at the point (0, 1) it will eventually return to its starting point. Would this happen if the ball started from other points on the y-axis between (0, 0) and (0, 4)?

30. The line with equation $y = 2x + 3$ is reflected in the y-axis. Find an equation of the image line.

31. The line with equation $y = x + 5$ is reflected in the x-axis. Find an equation of the image line.