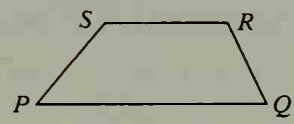
Example 2

Prove that the bases of a trapezoid have unequal lengths.



Given: Trap. $PQRS$ with bases $\overline{PQ}$ and $\overline{SR}$

Prove: $PQ \neq SR$

Proof:

Assume temporarily that $PQ = SR$. We know that $\overline{PQ} \parallel \overline{SR}$ by the definition of a trapezoid. Since quadrilateral $PQRS$ has two sides that are both congruent and parallel, it must be a parallelogram, and $\overline{PS}$ must be parallel to $\overline{QR}$. But this contradicts the fact that, by definition, trapezoid $PQRS$ can have only one pair of parallel sides. The temporary assumption that $PQ = SR$ must be false. It follows that $PQ \neq SR$.

Can you see how proving a statement by proving its contrapositive is related to *indirect proof*? If you want to prove the statement "If p, then q," you could prove the contrapositive "If not q, then not p." Or you could write an indirect proof—assume that q is false and show that this assumption implies that p is false.

Classroom Exercises

1. An indirect proof is to be used to prove the following:

If $AB = AC$, then $\triangle ABD \cong \triangle ACD$.

Which one of the following is the correct way to begin?

a. Assume temporarily that $AB \neq AC$.

b. Assume temporarily that $\triangle ABD \ncong \triangle ACD$.

What is the first sentence of an indirect proof of the statement shown?

2. $\triangle ABC$ is equilateral.

3. Doug is a Canadian.

4. $a \geq b$

5. Kim isn't a violinist.

6. $m\angle X > m\angle Y$

7. $\overline{CX}$ isn't a median of $\triangle ABC$.

8. Planning to write an indirect proof that $\angle A$ is an obtuse angle, Becky began by saying "Assume temporarily that $\angle A$ is an acute angle." What has Becky overlooked?

9. Wishing to prove that l and m are skew lines, John began an indirect proof by supposing that l and m are intersecting lines. What possibility has John overlooked?