6-3 *Indirect Proof*

Until now, the proofs you have written have been *direct* proofs. Sometimes it is difficult or even impossible to find a direct proof. In that case it may be possible to reason *indirectly*. Indirect reasoning is commonplace in everyday life. Suppose, for example, that after walking home, Sue enters the house carrying a dry umbrella. You can conclude that it is not raining outside. Why? Because if it were raining, then her umbrella would be wet. The umbrella is not wet. Therefore, it is not raining.

In an **indirect proof** you begin by assuming temporarily that the desired conclusion is not true. Then you reason logically until you reach a contradiction of the hypothesis or some other known fact. Because you've reached a contradiction, you know that the temporary assumption is impossible and therefore the desired conclusion must be true.

The procedure for writing an indirect proof is summarized below. Notice how these steps are applied in the examples of indirect proof that follow.

How to Write an Indirect Proof

1. Assume temporarily that the conclusion is not true.
2. Reason logically until you reach a contradiction of a known fact.
3. Point out that the temporary assumption must be false, and that the conclusion must then be true.

Example 1

Given: n is an integer and n^2 is even.

Prove: n is even.

Proof:

Assume temporarily that n is not even. Then n is odd, and

$$\begin{aligned} n^2 &= n \times n \\ &= \text{odd} \times \text{odd} = \text{odd}. \end{aligned}$$

But this contradicts the given information that n^2 is even. Therefore the temporary assumption that n is not even must be false. It follows that n is even.