

Converse of  $p \rightarrow q$ 

| $p$ | $q$ | $q \rightarrow p$ |
|-----|-----|-------------------|
| T   | T   | T                 |
| T   | F   | T                 |
| F   | T   | F                 |
| F   | F   | T                 |

Contrapositive of  $p \rightarrow q$ 

| $p$ | $q$ | $\sim q$ | $\sim p$ | $\sim q \rightarrow \sim p$ |
|-----|-----|----------|----------|-----------------------------|
| T   | T   | F        | F        | T                           |
| T   | F   | T        | F        | F                           |
| F   | T   | F        | T        | T                           |
| F   | F   | T        | T        | T                           |

## Exercises

Suppose  $p$  represents “You like to paint,”  $q$  represents “You are an artist,” and  $r$  represents “You draw landscapes.” Express in words each of the following statements.

1.  $p \rightarrow q$
2.  $q \rightarrow r$
3.  $\sim q \rightarrow \sim r$
4.  $\sim(p \rightarrow q)$
5.  $(p \wedge q) \rightarrow r$
6.  $p \wedge (q \rightarrow r)$
7.  $(r \vee q) \rightarrow p$
8.  $r \vee (q \rightarrow p)$

Let  $b$ ,  $s$ , and  $k$  represent the following statements.

$b$ : Bonnie bellows.  $s$ : Sheila shouts.  $k$ : Keiko cackles.

Express in symbolic form each of the following statements.

9. If Bonnie bellows, then Keiko cackles.
10. If Keiko cackles, then Sheila does not shout.
11. If Bonnie does not bellow or Keiko does not cackle, then Sheila shouts.
12. Sheila shouts, and if Bonnie bellows, then Keiko cackles.
13. It is not true that Sheila shouts if Bonnie bellows.
14. If Bonnie does not bellow, then Keiko cackles and Sheila shouts.
15. a. Make a truth table for  $\sim p \rightarrow \sim q$  (the inverse of  $p \rightarrow q$ ). Your first two columns should be the same as the first two columns of the table for  $p \rightarrow q$ . The last columns of the two tables should be different. Are they? Is  $\sim p \rightarrow \sim q$  logically equivalent to  $p \rightarrow q$ ?  
b. Compare the truth table for  $\sim p \rightarrow \sim q$  (the inverse of  $p \rightarrow q$ ) with the truth table for  $q \rightarrow p$  (the converse of  $p \rightarrow q$ ). Are the last columns the same? Are the inverse and the converse logically equivalent?

Make truth tables for the following statements.

16.  $p \rightarrow \sim q$
17.  $\sim(p \rightarrow q)$
18.  $p \wedge \sim q$
19. By comparing the truth tables in Exercises 16–18, you should find that two of the three statements are logically equivalent. Which two?
20. The biconditional statement “ $p$  if and only if  $q$ ” is defined as  $(p \rightarrow q) \wedge (q \rightarrow p)$ . Make a truth table for this statement.