6-5 *Inequalities for Two Triangles*

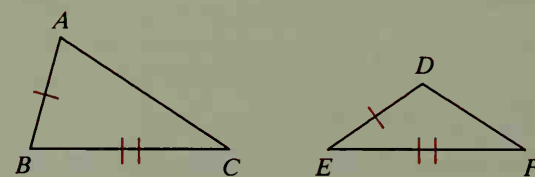
Begin with two matched pairs of sticks joined loosely at B and E. Open them so that $m\angle B > m\angle E$ and you find that $AC > DF$. Conversely, if you open them so that $AC > DF$, you see that $m\angle B > m\angle E$. Two theorems are suggested by these examples. The first theorem is surprisingly difficult to prove. The second theorem has an indirect proof.

Theorem 6-5 *SAS Inequality Theorem*

If two sides of one triangle are congruent to two sides of another triangle, but the included angle of the first triangle is larger than the included angle of the second, then the third side of the first triangle is longer than the third side of the second triangle.

Given: $\overline{BA} \cong \overline{ED}$; $\overline{BC} \cong \overline{EF}$;
$m\angle B > m\angle E$

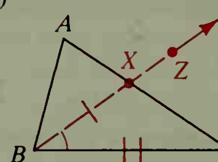
Prove: $AC > DF$



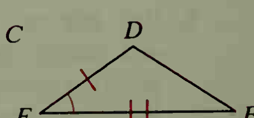
Proof:

Draw $\overrightarrow{BZ}$ so that $m\angle ZBC = m\angle E$. On $\overrightarrow{BZ}$ take point X so that $BX = ED$.
Then either X is on $\overline{AC}$ or X is not on $\overline{AC}$.
In both cases $\triangle XBC \cong \triangle DEF$ by SAS, and $XC = DF$.

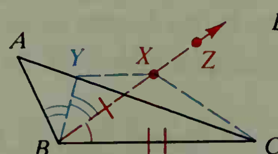
Case 1: X is on $\overline{AC}$.
$AC > XC$ (Seg. Add. Post. and a Prop. of Ineq.)
$AC > DF$ (Substitution Property, using the equation in red above)



Case 1



Case 2: X is not on $\overline{AC}$.
Draw the bisector of $\angle ABX$, intersecting $\overline{AC}$ at Y.
Draw $\overline{XY}$ and $\overline{XC}$.
$BA = ED = BX$
Since $\triangle ABY \cong \triangle XBY$ (SAS), $AY = XY$.
$XY + YC > XC$ (Why?)
$AY + YC > XC$ (Why?), or $AC > XC$
$AC > DF$ (Substitution Property)



Case 2