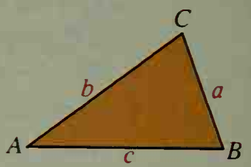
♦ Calculator Key-In

More than 2000 years ago, Heron, a mathematician from Alexandria, Egypt, derived a formula for finding the area of a triangle when the lengths of its sides are known. This formula is known as **Heron's Formula.** To find the area of $\triangle ABC$ using this formula:



Step 1 Find the *semiperimeter* $s = \frac{1}{2}(a + b + c)$.

Step 2 Area $= A = \sqrt{s(s - a)(s - b)(s - c)}$

Example If $a = 5$, $b = 6$, and $c = 7$, find the area of $\triangle ABC$.

Solution *Step 1* $s = \frac{1}{2}(5 + 6 + 7) = 9$

Step 2
$$\begin{aligned} A &= \sqrt{s(s - a)(s - b)(s - c)} \\ &= \sqrt{9(9 - 5)(9 - 6)(9 - 7)} \\ &= \sqrt{9 \cdot 4 \cdot 3 \cdot 2} \\ &= 6\sqrt{6} \end{aligned}$$

It is convenient to use a calculator when evaluating areas by using Heron's Formula. A calculator gives 14.7 as the approximate area of the triangle in the example above.

Exercises

The lengths of the sides of a triangle are given. Use a calculator to find the area and the three heights of the triangle, each correct to three significant digits. (*Hint*: $h = \frac{2A}{b}$.)

1. 9, 10, 11 **2.** 5, 7, 8 **3.** 6, 11, 13 **4.** 15, 16, 17

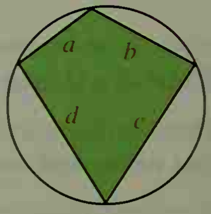
5. 6.3, 7.2, 10.1 **6.** 68, 77, 105 **7.** 5.5, 6.5, 10 **8.** 12, 18, 27

Use two different methods to find the exact area of each triangle whose sides are given.

9. 3, 4, 5 **10.** 6, 6, 6 **11.** 13, 13, 10 **12.** 29, 29, 42

13. Something strange happens when Heron's Formula is used with $a = 47$, $b = 38$, and $c = 85$. Why does this occur?

14. Heron also derived the following formula for the area of an inscribed quadrilateral with sides a, b, c, and d:



$A = \sqrt{(s - a)(s - b)(s - c)(s - d)}$,
where the semiperimeter $s = \frac{1}{2}(a + b + c + d)$

Use this formula to find the area of an isosceles trapezoid with sides 10, 10, 10, and 20 that is inscribed in a circle.