8-3 *The Converse of the Pythagorean Theorem*

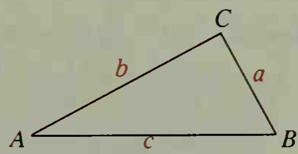
We have seen that the converse of a theorem is not necessarily true. However, the converse of the Pythagorean Theorem *is* true. It is stated below as Theorem 8-3.

Theorem 8-3

If the square of one side of a triangle is equal to the sum of the squares of the other two sides, then the triangle is a right triangle.

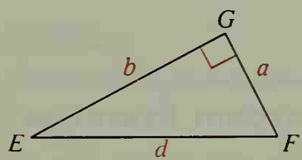
Given: $\triangle ABC$ with $c^2 = a^2 + b^2$

Prove: $\triangle ABC$ is a right triangle.



Key steps of proof:

1. Draw rt. $\triangle EFG$ with legs a and b.
2. $d^2 = a^2 + b^2$ (Pythagorean Theorem)
3. $c^2 = a^2 + b^2$ (Given)
4. $c = d$ (Substitution)
5. $\triangle ABC \cong \triangle EFG$ (SSS Postulate)
6. $\angle C$ is a rt. $\angle$. (Corr. parts of $\cong$ $\triangle$s are $\cong$.)
7. $\triangle ABC$ is a rt. $\triangle$. (Def. of a rt. $\triangle$)



A triangle with sides 3 units, 4 units, and 5 units long is called a 3-4-5 triangle. The numbers 3, 4, and 5 satisfy the equation $a^2 + b^2 = c^2$, so we can apply Theorem 8-3 to conclude that a 3-4-5 triangle is a right triangle. The side lengths shown in the table all satisfy the equation $a^2 + b^2 = c^2$, so the triangles formed are right triangles.

Some Common Right Triangle Lengths

3, 4, 5	5, 12, 13	8, 15, 17	7, 24, 25
6, 8, 10	10, 24, 26		
9, 12, 15			
12, 16, 20			
15, 20, 25			

Theorem 8-3 is restated on the next page, along with Theorems 8-4 and 8-5. If you know the lengths of the sides of a triangle, you can use these theorems to determine whether the triangle is right, acute, or obtuse. In each theorem, *c is the length of the longest side* of $\triangle ABC$. Exercises 20 and 19 ask you to state Theorems 8-4 and 8-5 more formally and then prove them.