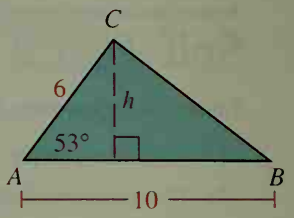
Example 1 Given the SAS information shown for $\triangle ABC$, find its area.

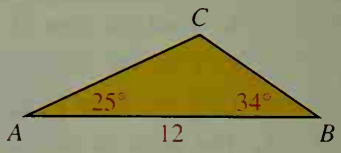


Solution Draw the altitude from C.

Then $\frac{h}{6} = \sin 53° \approx 0.7986$; $h \approx 4.79$.

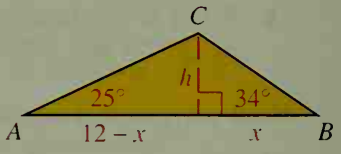
Area $= \frac{1}{2}bh \approx \frac{1}{2}(10)(4.79) \approx 24.0$

Example 2 Given the ASA information shown for $\triangle ABC$, find its area.



Solution

Step 1 Draw the altitude from C.



Then $\tan 25° = \dfrac{h}{12 - x}$ and $\tan 34° = \dfrac{h}{x}$.

$(12 - x) \tan 25° = h$ and $x \tan 34° = h$

$(12 - x) \tan 25° = x \tan 34°$

$(12 - x)(0.4663) \approx x(0.6745)$

$5.5956 - 0.4663x \approx 0.6745x$

$5.5956 \approx 1.1408x$

$4.905 \approx x$

Step 2 Knowing x, we can find h:

$h = x \tan 34° \approx (4.905)(0.6745) \approx 3.308$

Step 3 Area $= \frac{1}{2}bh \approx \frac{1}{2}(12)(3.308) \approx 19.8$

Exercises

Use the given information to find the approximate area of $\triangle ABC$. In Exercises 6 and 7 the altitude from C lies outside the triangle.

1. (SAS) $AB = 8$, $m\angle B = 67$, $BC = 15$
2. (HL) $m\angle C = 90$, $AB = 30$, $BC = 20$ (Use $\sqrt{5} \approx 2.236$.)
3. (SSS) $AB = 10$, $BC = 12$, $CA = 8$ (*Hint*: Use Heron's Formula.)
4. (ASA) $m\angle A = 28$, $AB = 10$, $m\angle B = 42$
5. (AAS) $m\angle A = 36$, $m\angle B = 80$, $BC = 10$ (*Hint*: Find the measure of $\angle C$. Then proceed as in Example 2.)
6. (SAS) $AB = 12$, $m\angle A = 118$, $AC = 20$
7. (ASA) $m\angle A = 107$, $AB = 20$, $m\angle B = 35$

★ 8. The two triangles shown have two pairs of congruent corresponding sides and one pair of congruent corresponding non-included angles (SSA). Of course, they are *not* congruent. Find the area of each triangle.