18. If $T:(x, y) \to (x + 2, y)$, then $T^2:(x, y) \to (\underline{?}, \underline{?})$.

19. If $T:(x, y) \to (x + 3, y - 4)$, then $T^2:(x, y) \to (\underline{?}, \underline{?})$.

20. If R_x is reflection in the x-axis, then $(R_x)^2:P \to \underline{?}$.

In each exercise, a rule is given for a mapping S. Write the rule for S^{-1}.

B 21. $S:(x, y) \to (x + 5, y + 2)$

22. $S:(x, y) \to (x - 3, y - 1)$

23. $S:(x, y) \to (3x, -\frac{1}{2}y)$

24. $S:(x, y) \to (\frac{1}{4}x, \frac{1}{4}y)$

25. $S:(x, y) \to (x - 4, 4y)$

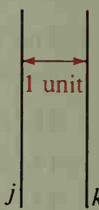
26. $S:(x, y) \to (y, x)$

27. If $S:(x, y) \to (x + 12, y - 3)$, find a translation T such that $T^6 = S$.

28. Find a transformation S (other than the identity) for which $S^5 = I$.

C 29. a. j and k are vertical lines 1 unit apart. According to Theorem 14-7, $R_k \circ R_j$ and $R_j \circ R_k$ are both translations. Describe in words the distance and direction of each translation.

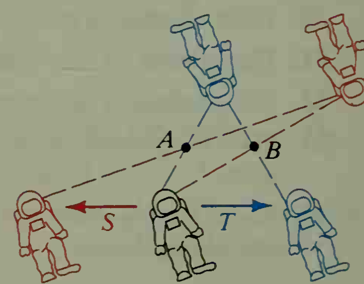
b. Show that $R_k \circ R_j$ and $R_j \circ R_k$ are inverses by showing that their composite is I. Note: Forming composites of transformations is an associative operation, so $(R_k \circ R_j) \circ (R_j \circ R_k) = R_k \circ (R_j \circ R_j) \circ R_k$.



30. The blue lines in the diagram illustrate the statement $H_B \circ H_A$ = translation T. The red lines show that $H_A \circ H_B$ = translation S.

a. How is translation S related to translation T?

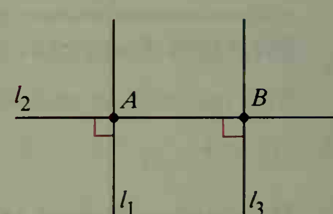
b. Prove your answer correct by showing that $(H_A \circ H_B) \circ (H_B \circ H_A) = I$. (*Hint:* See Exercise 29, part (b).)



31. Complete the proof by giving a reason for each step.

Given: $l_1 \perp l_2$; $l_3 \perp l_2$; R_1, R_2, and R_3 denote reflections in l_1, l_2, and l_3.

Prove: $H_B \circ H_A$ is a translation.



Proof:

Statements	Reasons
1. $H_A = R_2 \circ R_1$	1. $\underline{?}$
2. $H_B = R_3 \circ R_2$	2. $\underline{?}$
3. $H_B \circ H_A = (R_3 \circ R_2) \circ (R_2 \circ R_1)$	3. $\underline{?}$
4. $H_B \circ H_A = (R_3 \circ (R_2 \circ R_2)) \circ R_1$	4. Composition is associative.
5. $H_B \circ H_A = (R_3 \circ I) \circ R_1$	5. $\underline{?}$
6. $H_B \circ H_A = R_3 \circ R_1$	6. $\underline{?}$
7. $H_B \circ H_A$ is a translation.	7. $\underline{?}$