Mapping the rectangle by R_j and then by H_O has the same effect as the single symmetry R_k, so $H_O \circ R_j = R_k$. We can record this fact in a table resembling a multiplication table. Follow the row for R_j to where it meets the column for H_O, and enter the product $H_O \circ R_j$, which is R_k.

$\circ$	I	R_j	R_k	H_O
I				
R_j				R_k
R_k				
H_O				

We can determine other products of symmetries in the same way, but sometimes short cuts can be used. For example, we know that

(1) $R_j \circ R_j = I$ and $R_k \circ R_k = I$ (Why?)
(2) $H_O \circ H_O = I$ (Why?)
(3) $R_j \circ R_k = H_O$ and $R_k \circ R_j = H_O$
(Corollary to Theorem 14-8)

Also we know that the product of any symmetry and the identity is that same symmetry. The completed table is shown at the right.

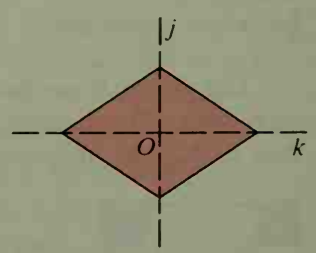
$\circ$	I	R_j	R_k	H_O
I	I	R_j	R_k	H_O
R_j	R_j	I	H_O	R_k
R_k	R_k	H_O	I	R_j
H_O	H_O	R_k	R_j	I

By studying the table you can see that the symmetries of the rectangle have these four properties, similar to the properties of nonzero real numbers under multiplication:

(1) The product of two symmetries is another symmetry.
(2) The set of symmetries contains the identity.
(3) Each symmetry has an inverse that is also a symmetry. (In this example each symmetry is its own inverse.)
(4) Forming products of transformations is an associative operation: $A \circ (B \circ C) = (A \circ B) \circ C$ for any three symmetries A, B, and C.

A set of symmetries with these four properties is called a symmetry *group*. Symmetry groups are used in crystallography, and more general groups are important in physics and advanced mathematics. The exercises that follow illustrate the fact that the symmetries of any figure form a group.

Exercises

1. An isosceles triangle has just two symmetries, including the identity. Make a 2 by 2 group table showing how these symmetries combine.

2. **a.** List the four symmetries of the rhombus shown. (Include the identity.)
b. Make a group table showing all products of two symmetries.
c. Is your table in part (b) identical to the table of symmetries for the rectangle?