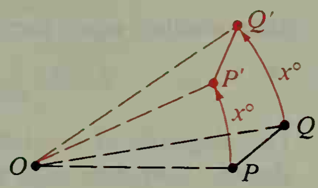
Theorem 14-4

A rotation is an isometry.

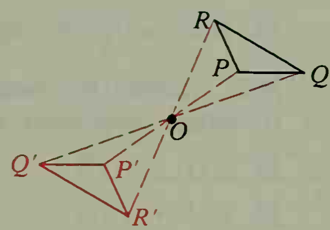


Given: $\mathcal{R}_{O,x}$ maps P to P' and Q to Q'.
Prove: $\overline{PQ} \cong \overline{P'Q'}$

Key steps of proof:

1. $OP = OP'$, $OQ = OQ'$ (Definition of rotation)
2. $m\angle POP' = m\angle QOQ' = x$ (Definition of rotation)
3. $m\angle POQ = m\angle P'OQ'$ (Subtraction Property of $=$: subtract $m\angle QOP'$.)
4. $\triangle POQ \cong \triangle P'OQ'$ (SAS Postulate)
5. $\overline{PQ} \cong \overline{P'Q'}$ (Corr. parts of $\cong$ ▵ are $\cong$.)

A rotation about point O through 180° is called a **half-turn** about O and is usually denoted by H_O. The diagram shows $\triangle PQR$ and its image $\triangle P'Q'R'$ by H_O. Notice that O is the midpoint of $\overline{PP'}$, $\overline{QQ'}$, and $\overline{RR'}$.



Using coordinates, a half-turn H_O about the origin can be written

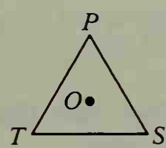
$$H_O:(x, y) \rightarrow (-x, -y).$$

Classroom Exercises

State another name for each rotation.

1. $\mathcal{R}_{O,50}$ 2. $\mathcal{R}_{O,-40}$ 3. $\mathcal{R}_{O,-90}$ 4. $\mathcal{R}_{O,400}$ 5. $\mathcal{R}_{O,-180}$

In the diagram for Exercises 6–11, O is the center of equilateral $\triangle PST$. State the images of points P, S, and T for each rotation.



Exs. 6–11

6. $\mathcal{R}_{O,120}$ 7. $\mathcal{R}_{O,-120}$ 8. $\mathcal{R}_{O,360}$

Name each image point.

9. $\mathcal{R}_{T,60}(S)$ 10. $\mathcal{R}_{T,-60}(P)$ 11. $\mathcal{R}_{O,240}(P)$

12. Draw a coordinate grid on the chalkboard. Plot the origin and $A(4, 1)$. Give the coordinates of **(a)** $H_O(A)$, **(b)** $\mathcal{R}_{O,90}(A)$, and **(c)** $\mathcal{R}_{O,-90}(A)$.
13. Repeat Exercise 12 if A has coordinates $(-3, 5)$.
14. Is congruence invariant under a half-turn mapping? Explain.
15. Read each expression aloud.
    a. $R_k(A) = A'$ **b.** $H_O:(-2, 0) \rightarrow (2, 0)$
    c. $T:(x, y) \rightarrow (x - 1, y + 3)$ **d.** $\mathcal{R}_{P,10}$