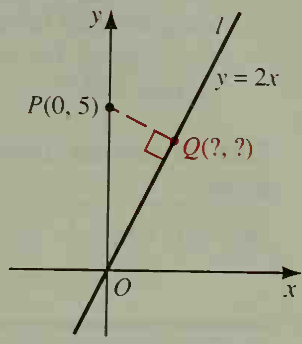
Example 2 Find the shortest distance from $P(0, 5)$ to the line l with equation $y = 2x$.

Solution The shortest segment is the perpendicular $\overline{PQ}$.
Its length can be found in three steps.

Step 1 *Find the equation of* $\overleftrightarrow{PQ}$.



Slope of line $y = 2x$ is 2.

Then the slope of $\overleftrightarrow{PQ}$ is $-\frac{1}{2}$.

The equation of $\overleftrightarrow{PQ}$ is $y = -\frac{1}{2}x + 5$.

Step 2 *Find Q by solving the equations for l and* $\overleftrightarrow{PQ}$ *simultaneously.*

$y = 2x$ and $y = -\frac{1}{2}x + 5$

$2x = -\frac{1}{2}x + 5$

$\frac{5}{2}x = 5$

$x = 2$

If $x = 2$, then $y = 2x = 4$. Thus Q is $(2, 4)$.

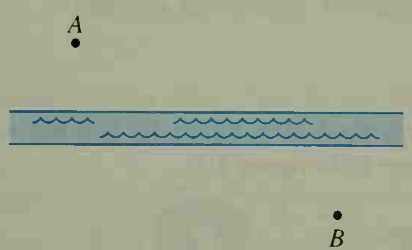
Step 3 *Find PQ by using the distance formula.*

$PQ = \sqrt{(0 - 2)^2 + (5 - 4)^2} = \sqrt{5}$

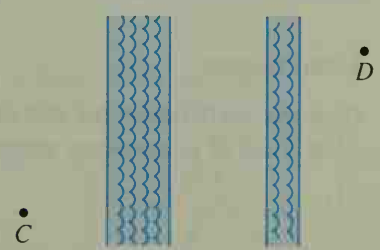
Exercises

In Exercises 1–5 assume that each bridge must be perpendicular to the two river banks it joins.

1. Copy the figure shown and find the location of a bridge across the river that will allow the path from A to B to be minimum.



★ 2. Two bridges are to be built over the parallel rivers shown. Find where they should be built if the total distance from C to D, including the distances across the bridges, is to be minimum.



3. A river flows between the lines $x = 3$ and $x = 4$. Where should a bridge be constructed to minimize the path from $O(0, 0)$ to $P(5, 4)$?

4. A river flows between the lines $y = x$ and $y = x + 2$. Where should a bridge be constructed to minimize the path from $Q(0, 6)$ to $R(8, 5)$?