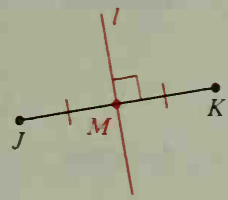
A **perpendicular bisector** of a segment is a line (or ray or segment) that is perpendicular to the segment at its midpoint. In the figure at the right, line l is a perpendicular bisector of $\overline{JK}$.



In a given plane, there is exactly one line perpendicular to a segment at its midpoint. We speak of *the* perpendicular bisector of a segment in such a case.

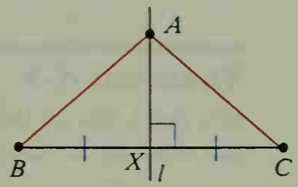
Proofs of the following theorems are left as Exercises 14 and 15.

Theorem 4-5

If a point lies on the perpendicular bisector of a segment, then the point is equidistant from the endpoints of the segment.

Given: Line l is the perpendicular bisector of $\overline{BC}$; A is on l.

Prove: $AB = AC$

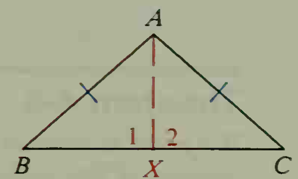


Theorem 4-6

If a point is equidistant from the endpoints of a segment, then the point lies on the perpendicular bisector of the segment.

Given: $AB = AC$

Prove: A is on the perpendicular bisector of $\overline{BC}$.



Plan for Proof: The perpendicular bisector of $\overline{BC}$ must contain the midpoint of $\overline{BC}$ and be perpendicular to $\overline{BC}$. Draw an auxiliary line containing A that has one of these properties and prove that it has the other property as well. For example, first draw a segment from A to the midpoint X of $\overline{BC}$. You can show that $\overline{AX} \perp \overline{BC}$ if you can show that $\angle 1 \cong \angle 2$. Since these angles are corresponding parts of two triangles, first show that $\triangle AXB \cong \triangle AXC$.

In the proof of Theorem 4-6 other auxiliary lines could have been chosen instead. For example, we can draw the altitude to $\overline{BC}$ from A, meeting $\overline{BC}$ at a point Y as shown in the diagram at the right. Here, since $\overline{AY} \perp \overline{BC}$ we need to prove that $\overline{YB} \cong \overline{YC}$. Either method can be used to prove Theorem 4-6.