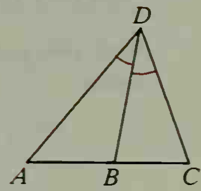
18. Prove the corollary of the Triangle Proportionality Theorem.

19. Prove the Triangle Angle-Bisector Theorem.



Complete.

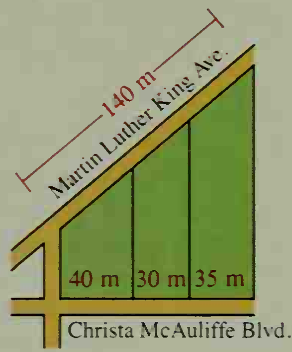
20. $AD = 21$, $DC = 14$, $AC = 25$, $AB = \underline{\ ?\ }$

21. $AC = 60$, $CD = 30$, $AD = 50$, $BC = \underline{\ ?\ }$

22. $AB = 27$, $BC = x$, $CD = \frac{4}{3}x$, $AD = x$, $AC = \underline{\ ?\ }$

23. $AB = 2x - 12$, $BC = x$, $CD = x + 5$, $AD = 2x - 4$, $AC = \underline{\ ?\ }$

24. Three lots with parallel side boundaries extend from the avenue to the boulevard as shown. Find, to the nearest tenth of a meter, the frontages of the lots on Martin Luther King Avenue.



Ex. 24

25. The lengths of the sides of $\triangle ABC$ are $BC = 12$, $CA = 13$, and $AB = 14$. If M is the midpoint of $\overline{CA}$, and P is the point where $\overline{CA}$ is cut by the bisector of $\angle B$, find MP.

26. Prove: If a line bisects both an angle of a triangle and the opposite side, then the triangle is isosceles.

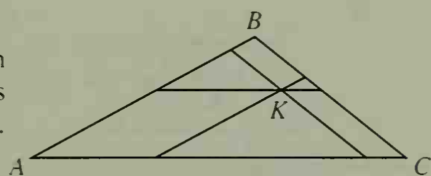
C **27.** Discover and prove a theorem about planes and transversals suggested by the corollary of the Triangle Proportionality Theorem.

28. Prove that there cannot be a triangle in which the trisectors of an angle also trisect the opposite side.

29. Can there exist a $\triangle ROS$ in which the trisectors of $\angle O$ intersect $\overline{RS}$ at D and E, with $RD = 1$, $DE = 2$, and $ES = 4$? Explain.

30. Angle E of $\triangle ZEN$ is obtuse. The bisector of $\angle E$ intersects $\overline{ZN}$ at X. J and K lie on $\overline{ZE}$ and $\overline{NE}$ with $ZJ = ZX$ and $NK = NX$. Discover and prove something about quadrilateral $ZNKJ$.

★ **31.** In $\triangle ABC$, $AB = 8$, $BC = 6$, and $AC = 12$. Each of the three segments drawn through point K has length x and is parallel to a side of the triangle. Find the value of x.



★ **32.** In $\triangle RST$, U lies on $\overline{TS}$ with $TU:US = 2:3$. M is the midpoint of $\overline{RU}$. $\overrightarrow{TM}$ intersects $\overline{RS}$ in V. Find the ratio $RV:RS$.

★ **33.** Prove *Ceva's Theorem:* If P is any point inside $\triangle ABC$, then $\frac{AX}{XB} \cdot \frac{BY}{YC} \cdot \frac{CZ}{ZA} = 1$.

(*Hint:* Draw lines parallel to $\overline{CX}$ through A and B. Apply the Triangle Proportionality Theorem to $\triangle ABM$. Show that $\triangle APN \sim \triangle MPB$, $\triangle BYM \sim \triangle CYP$, and $\triangle CZP \sim \triangle AZN$.)