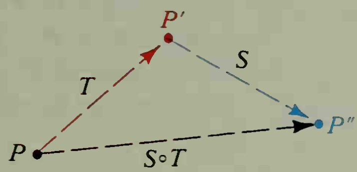
Composition and Symmetry

Objectives

1. Locate the images of figures by composites of mappings.
2. Recognize and use the terms *identity* and *inverse* in relation to mappings.
3. Describe the symmetry of figures and solids.

14-6 *Composites of Mappings*

Suppose a transformation T maps point P to P' and then a transformation S maps P' to P''. Then T and S can be combined to produce a new transformation that maps P directly to P''. This new transformation is called the **composite** of S and T and is written $S \circ T$. Notice in the diagram that $P'' = S(P') = S(T(P)) = (S \circ T)(P)$.



We reduce the number of parentheses needed to indicate that the composite of S and T maps P to P'' by writing $S \circ T: P \rightarrow P''$. Notice that T, the transformation that is applied first, is written closer to P, and S, the transformation that is applied second, is written farther from P. For this reason, the composite $S \circ T$ is often read "S after T," or "T followed by S."

The operation that combines two mappings (or functions) to produce the composite mapping (or composite function) is called *composition*. We shall see that composition has many characteristics similar to multiplication, but there is one important exception. For multiplication, it makes no difference in which order you multiply two numbers. For composition, however, the order of the mappings or functions usually *does* make a difference. Examples 1 and 2 illustrate this.

Example 1 If $f(x) = x^2$ and $g(x) = 2x$, find **(a)** $(g \circ f)(x)$ and **(b)** $(f \circ g)(x)$.

Solution

a. $(g \circ f)(x) = g(f(x)) = g(x^2) = 2x^2$

b. $(f \circ g)(x) = f(g(x)) = f(2x) = (2x)^2$, or $4x^2$

In mapping notation, we could write that $g \circ f: x \rightarrow 2x^2$ and $f \circ g: x \rightarrow 4x^2$. Note that since $2x^2 \neq 4x^2$, $g \circ f \neq f \circ g$.