

Some Rules of Inference

Four rules for making logical inferences are symbolized below. A horizontal line separates the given information, or premises, from the conclusion. If you accept the given statement or statements as true, then you must accept as true the conclusions shown.

1. Modus Ponens

$$\begin{array}{l} p \rightarrow q \\ p \\ \hline \text{Therefore, } q \end{array}$$

2. Modus Tollens

$$\begin{array}{l} p \rightarrow q \\ \sim q \\ \hline \text{Therefore, } \sim p \end{array}$$

3. Simplification

$$\begin{array}{l} p \wedge q \\ \hline \text{Therefore, } p \end{array}$$

4. Disjunctive Syllogism

$$\begin{array}{l} p \vee q \\ \sim p \\ \hline \text{Therefore, } q \end{array}$$

You should convince yourself that these rules make good sense. For example, Rule 4 says that if you know that “ p or q ” is true and then you find out that p is not true, you must conclude that q is true.

Example 1 If today is Tuesday, then tomorrow is Wednesday.

Today is Tuesday.

Therefore, tomorrow is Wednesday. (Rule 1)

Example 2 If a figure is a triangle, then it is a polygon.

This figure is not a polygon.

Therefore, this figure is not a triangle. (Rule 2)

Example 3 It is Tuesday and it is April.

Therefore, it is Tuesday. (Rule 3)

Example 4 It is a square or it is a triangle.

It is not a square.

Therefore, it is a triangle. (Rule 4)

Example 5 Given: $p \rightarrow q$; $p \vee r$; $\sim q$

Prove: r

Proof:

Statements	Reasons
1. $p \rightarrow q$	1. Given
2. $\sim q$	2. Given
3. $\sim p$	3. Steps 1 and 2 and Modus Tollens
4. $p \vee r$	4. Given
5. r	5. Steps 3 and 4 and Disj. Syllogism