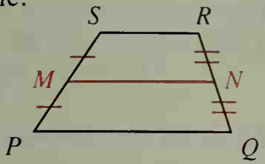
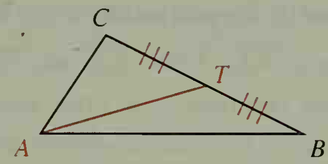
The **median** of a trapezoid is the segment that joins the midpoints of the legs. Note the difference between the median of a trapezoid and a median of a triangle.



$\overline{MN}$ is *the* median of trapezoid $PQRS$.



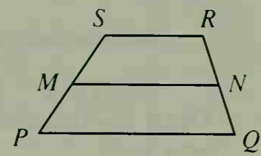
$\overline{AT}$ is *a* median of $\triangle ABC$.

Theorem 5-19

The median of a trapezoid

(1) is parallel to the bases;

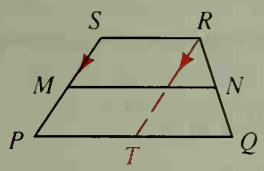
(2) has a length equal to the average of the base lengths.



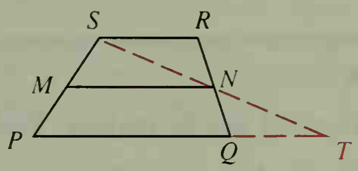
Given: Trapezoid $PQRS$ with median $\overline{MN}$

Prove: (1) $\overline{MN} \parallel \overline{PQ}$ and $\overline{MN} \parallel \overline{SR}$ (2) $MN = \frac{1}{2}(PQ + SR)$

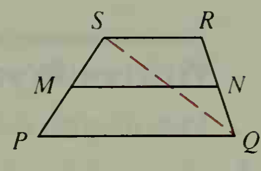
Plan for Proof: Again it is necessary to introduce auxiliary lines, and again there is more than one way to do this. Although any of the diagrams below could be used to prove the theorem, the proof below uses the second diagram.



Draw $\overline{RT} \parallel \overline{SP}$.



Draw $\overrightarrow{SN}$ intersecting $\overrightarrow{PQ}$ at T.

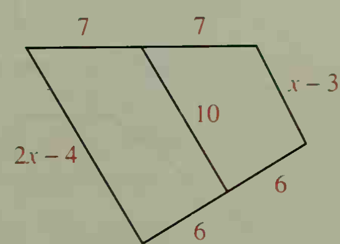


Draw $\overline{SQ}$.

Proof:

Extend $\overline{SN}$ to intersect $\overrightarrow{PQ}$ at T. $\triangle TNQ \cong \triangle SNR$ by ASA. Then $SN = NT$ and N is the midpoint of $\overline{ST}$. Using Theorem 5-11 and $\triangle PST$, (1) $\overline{MN} \parallel \overline{PQ}$ (and also $\overline{MN} \parallel \overline{SR}$), and (2) $MN = \frac{1}{2}PT = \frac{1}{2}(PQ + QT) = \frac{1}{2}(PQ + SR)$, since $\overline{QT} \cong \overline{SR}$.

Example A trapezoid and its median are shown. Find the value of x.



Solution

$$\begin{aligned} 10 &= \tfrac{1}{2}\,[(2x - 4) + (x - 3)] \\ 20 &= (2x - 4) + (x - 3) \\ 20 &= 3x - 7 \\ 27 &= 3x \\ 9 &= x \end{aligned}$$