Math Tutor CALCULATING WITH HALF-LIFE

The rate at which a sample of a radioactive nuclide decays is expressed in terms of half-life. This quantity is the time required for half of the atoms of a sample of a given nuclide to decay. For example, it takes 37.2 min for half of the nuclei of chlorine-38 to decay to argon-38. After 37.2 min, 0.50 g of a 1.0 g sample of chlorine-38 will remain and there will be 0.50 g of argon-38. After two half-lives (74.4 min), the fraction of chlorine-38 that remains will be $\frac{1}{2}$ of $\frac{1}{2}$, or $\frac{1}{4}$.

After n half-lives, the fraction of a radioactive nuclide that remains is $\left(\frac{1}{2}\right)^n$, or 2^{-n}.

If you know the amount of nuclide that was present initially and the amount of nuclide that remains, you can determine the number of half-lives that have passed.

Problem-Solving TIPS

- Familiarize yourself with the values of some common powers of two (2^n, $n = 1, 2, 3, 4, 5, 6$, etc). This will allow you to determine the number of half-lives quickly.

SAMPLE 1

The half-life of polonium-218 is 3.04 min. A sample of polonium contains 0.00558 g of $^{218}_{84}$Po. What mass of $^{218}_{84}$Po will remain after 18.24 min?

First, you must determine the number of half-lives that have passed in 18.24 min.

$$\text{number of half-lives} = \frac{\text{time elapsed}}{\text{half-life}} = \frac{18.24 \text{ min}}{3.04 \text{ min}} = 6.00 \text{ half-lives}$$

Then, to determine the mass of polonium-218 remaining, apply the following relationship.

$$\text{mass remaining} = \text{starting mass} \times \text{fraction remaining}$$

$$\text{mass } ^{218}_{84}\text{Po remaining} = 0.00558 \text{ g} \times \left(\frac{1}{2}\right)^6 = 0.00558 \text{ g} \times \frac{1}{64} = 8.72 \times 10^{-5} \text{ g } ^{218}_{84}\text{Po}$$

SAMPLE 2

The half-life of potassium-40 is 1.3×10^9 years. A volcanic rock contains $\frac{1}{8}$ of the amount of potassium-40 found in newly formed rocks. When was the rock formed?

First, determine the number of half-lives that have passed.

$$\text{fraction remaining} = \frac{1}{8} = \left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right)$$

Therefore, three half-lives have passed. The time since the rock was formed is 3 half-lives $\times \left(1.3 \times 10^9 \frac{\text{y}}{\text{half-life}}\right) = 3.9 \times 10^9$ y.

PRACTICE PROBLEMS

1. A sample of chromium contains 8.9×10^{-5} g of the radioactive nuclide chromium-51, which has a half-life of 28 days. What mass of chromium-51 will remain in the sample after 168 days?
2. The half-life of lead-202 is 53 000 years. A sample of lead contains only $\frac{1}{256}$ of the expected amount of lead-202. How old is the lead sample?