The area of the shaded region is first approximated by the sum of the areas of the five rectangles and then by the sum of the areas of the five trapezoids. Compare these approximations. (Calculus can be used to prove that the exact area is $\frac{7}{3}$. Note that $\frac{7}{3} \approx 2.33$.)

Area approximated by five rectangles:

$A \approx (1.2)^2(0.2) + (1.4)^2(0.2) + (1.6)^2(0.2) + (1.8)^2(0.2) + (2.0)^2(0.2)$

$A \approx 2.64$

Area approximated by five trapezoids:

$$A \approx \tfrac{1}{2}(0.2)[(1.0)^2 + (1.2)^2] + \tfrac{1}{2}(0.2)[(1.2)^2 + (1.4)^2] + \tfrac{1}{2}(0.2)[(1.4)^2 + (1.6)^2] + \tfrac{1}{2}(0.2)[(1.6)^2 + (1.8)^2] + \tfrac{1}{2}(0.2)[(1.8)^2 + (2.0)^2]$$

$A \approx 2.34$

The following computer program will compute and add the areas of the five trapezoids shown in the diagram on the preceding page.

```
10  LET X = 1
20  FOR N = 1 TO 5
30  LET B1 = X ↑ 2
40  LET B2 = (X + 0.2) ↑ 2
50  LET A = A + 0.5 * 0.2 * (B1 + B2)
60  LET X = X + 0.2
70  NEXT N
80  PRINT "AREA IS APPROXIMATELY ";A
90  END
```

Exercises

1. A better approximation can be found by using 100 smaller trapezoids with base vertices at 1.00, 1.01, 1.02, . . . , 1.99, 2.00. Change lines 20, 40, 50, and 60 as follows:

```
20  FOR N = 1 TO 100
40  LET B2 = (X + 0.01) ↑ 2
50  LET A = A + 0.5 * 0.01 * (B1 + B2)
60  LET X = X + 0.01
```

RUN the program to approximate the area of the shaded region.

2. Modify the given computer program so that it will use 1000 trapezoids with base vertices at 1.000, 1.001, 1.002, . . . , 2.000 to approximate the area of the shaded region. RUN the program.

3. Modify the given computer program so that it will use ten trapezoids to approximate the area of the region that is bounded by the graph of $y = x^2$, the x-axis, and *the vertical lines $x = 0$ and $x = 1$*. RUN the program. Compare your answer with that obtained on page 428, where ten rectangles were used. (*Note*: Calculus can be used to prove that the exact area is $\frac{1}{3}$.)