4. A chain of deductive reasoning is often used in geometric proofs. For example, if you are given three premises: p; $p \rightarrow q$; and $q \rightarrow r$, then you can conclude r. Prove the validity of this argument by filling in the truth table for $[p \wedge (p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow r$.

5. The following argument is not logically valid, because it is missing a premise.
"I have \$5.00 to spend for lunch. If the sandwich I want to buy costs \$3.50, then I'll have enough money left over to buy a beverage. If milk costs less than \$1.50, I'll buy it. The price of milk is \$1.00. Therefore, I'll buy milk with my sandwich."
   a. Add a premise that would make this argument complete and valid.
   b. Can you think of any reasons why I still might not buy milk?

6. The following argument is logically valid. But the conclusion that two equals one is nonsensical. Can you find the mistake in one of the conditionals below?
   1. Let $x = 1$. (Given.)
   2. If $x = 1$, then $x - 2 = -1$. (Subtract 2.)
   3. If $x - 2 = -1$, then $x^2 + x - 2 = x^2 - 1$. (Add x^2.)
   4. If $x^2 + x - 2 = x^2 - 1$, then $(x + 2)(x - 1) = (x + 1)(x - 1)$. (Factor.)
   5. If $(x + 2)(x - 1) = (x + 1)(x - 1)$, then $(x + 2) = (x + 1)$. (Divide by $x - 1$.)
   6. If $(x + 2) = (x + 1)$, then $2 = 1$. (Subtract x.)
   7. Therefore, if $x = 1$, then $2 = 1$.

Some Rules of Replacement

The symbol $\equiv$ means "is logically equivalent to." Thus Rule 5 below states that the conditional statement $p \rightarrow q$ is logically equivalent to its contrapositive, $\sim q \rightarrow \sim p$. Rules 6–10 give other logical equivalences. These can be verified by comparing the truth tables of the statements on both sides of the $\equiv$ sign.

5. Contrapositive Rule
$p \rightarrow q \equiv \sim q \rightarrow \sim p$

6. Double Negation
$\sim(\sim p) \equiv p$

7. Commutative Rules
$p \wedge q \equiv q \wedge p$
$p \vee q \equiv q \vee p$

8. Associative Rules
$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
$(p \vee q) \vee r \equiv p \vee (q \vee r)$

9. Distributive Rules
$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

10. DeMorgan's Rules
$\sim(p \wedge q) \equiv \sim p \vee \sim q$
$\sim(p \vee q) \equiv \sim p \wedge \sim q$

Any logically equivalent expressions can replace each other wherever they occur in a proof.