## Linear Equations

A linear equation has the following general form:

$$y = ax + b$$

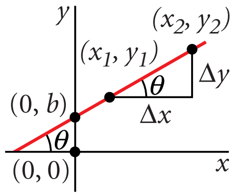


**Figure 1**

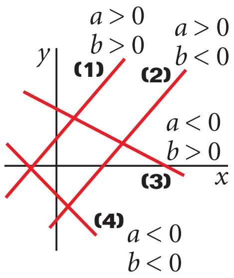


**Figure 2**

In this equation, $a$ and $b$ are constants. This equation is called linear because the graph of $y$ versus $x$ is a straight line, as shown in **Figure 1.** The constant $b$, called the *intercept,* represents the value of $y$ where the straight line intersects the $y$-axis. The constant $a$ is equal to the *slope* of the straight line and is also equal to the tangent of the angle that the line makes with the $x$-axis ($\theta$). If any two points on the straight line are specified by the coordinates $(x_1, y_1)$ and $(x_2, y_2)$, as in **Figure 1,** then the slope of the straight line can be expressed as follows:

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$$

For example, if the two points shown in **Figure 1** are (2, 4) and (6, 9), then the slope of the line is as follows:

$$\text{slope} = \frac{(9-4)}{(6-2)} = \frac{5}{4}$$

Note that $a$ and $b$ can have either positive or negative values. If $a > 0$, the straight line has a *positive* slope, as in **Figure 1.** If $a < 0$, the straight line has a *negative* slope. Furthermore, if $b > 0$, the $y$ intercept is positive (above the $x$-axis), while if $b < 0$, the $y$ intercept is negative (below the $x$-axis). **Figure 2** gives an example of each of these four possible cases, which are summarized in **Table 4.**

**Table 4** Linear Equations

| Constants | Slope | $y$ intercept |
|---|---|---|
| $a > 0, b > 0$ | positive slope | positive $y$ intercept |
| $a > 0, b < 0$ | positive slope | negative $y$ intercept |
| $a < 0, b > 0$ | negative slope | positive $y$ intercept |
| $a < 0, b < 0$ | negative slope | negative $y$ intercept |

## Solving Simultaneous Linear Equations

Consider the following equation:

$$3x + 5y = 15$$

This equation has two unknowns, $x$ and $y$. Such an equation does not have a unique solution. That is, $(x = 0, y = 3)$, $(x = 5, y = 0)$, and $\left(x = 2, y = \frac{9}{5}\right)$ are all solutions to this equation.

If a problem has two unknowns, a unique solution is possible only if there are two independent equations. In general, if a problem has $n$ unknowns, its solution requires $n$ independent equations. There are three basic methods that can be used to solve simultaneous equations. Each of these methods is discussed below, and an example is given for each.