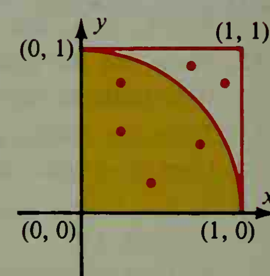
44. Find the coordinates of the point that is equidistant from $(-2, 5)$, $(8, 5)$, and $(6, 7)$.

45. Find the center and the radius of the circle $x^2 + 4x + y^2 - 8y = 16$. (*Hint*: Express the given equation in the form
$$(x - a)^2 + (y - b)^2 = r^2.)$$

♦ Computer Key-In

The graph shows a quarter-circle inscribed in a square with area 1. If points are picked at random inside the square, some of them will also be inside the quarter-circle. Let n be the number of points picked inside the square and let q be the number of these points that fall inside the quarter-circle. If many, many points are picked at random inside the square, the following ratios are approximately equal:

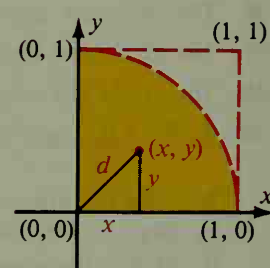


$$\frac{\text{Area of quarter-circle}}{\text{Area of square}} \approx \frac{q}{n}$$

$$\frac{\text{Area of quarter-circle}}{1} \approx \frac{q}{n}$$

$$\text{Area of whole circle} \approx 4 \times \frac{q}{n}$$

Any point (x, y) in the square region has coordinates such that $0 < x < 1$ and $0 < y < 1$. (Note that this restriction excludes points on the boundaries of the square.) A computer can pick a random point inside the unit square by choosing two random numbers x and y between 0 and 1. We let d be the distance from O to the point (x, y). By the Pythagorean Theorem, $d = \sqrt{x^2 + y^2}$. Do you see that if $d < 1$, the point lies inside the quarter-circle?



Exercises

1. Write a computer program to do all of the following:
   a. Choose n random points (x, y) inside the unit square.
   b. Using the distance formula test each point chosen to see whether it lies inside the quarter-circle.
   c. Count the number of points (q) which *do* lie inside the quarter-circle.
   d. Print out the value of $4 \times \frac{q}{n}$.

2. RUN your program for $n = 100$, $n = 500$, and $n = 1000$.

3. Calculate the area of the circle, using the formula given on page 446. Compare this result with your computer approximations.