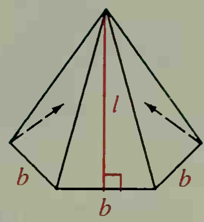
Theorem 12-3

The lateral area of a regular pyramid equals half the perimeter of the base times the slant height. (L.A. = $\frac{1}{2}pl$)

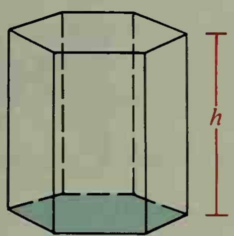
This formula is developed using Method 1 on the previous page. The area of one lateral face is $\frac{1}{2}bl$. Then:

$$\text{L.A.} = \left(\tfrac{1}{2}bl\right)n$$
$$= \tfrac{1}{2}(nb)l$$

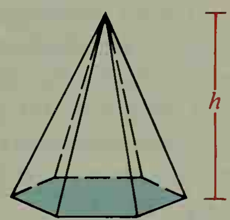
Since $nb = p$, $\quad \text{L.A.} = \frac{1}{2}pl$



The prism and pyramid below have congruent bases and equal heights. Since the volume of the prism is Bh, the volume of the pyramid must be less than Bh. In fact, it is exactly $\frac{1}{3}Bh$. This result is stated as Theorem 12-4. Although no proof will be given, Classroom Exercise 1 and the Computer Key-In on pages 488–489 help justify the formula.



$V = Bh$



$V = \frac{1}{3}Bh$

Theorem 12-4

The volume of a pyramid equals one third the area of the base times the height of the pyramid. ($V = \frac{1}{3}Bh$)

Example 3 Suppose the regular hexagonal pyramid shown at the right above Theorem 12-4 has base edges 6 and height 12. Find its volume.

Solution Find the area of the hexagonal base.

Divide the base into six equilateral triangles. Find the area of one triangle and multiply by 6.

Base area $= B = 6\left(\frac{1}{2} \cdot 6 \cdot 3\sqrt{3}\right) = 54\sqrt{3}$

Then $V = \frac{1}{3}Bh = \frac{1}{3} \cdot 54\sqrt{3} \cdot 12 = 216\sqrt{3}$