

Relating Algebra and Geometry

For products of numbers

1 is the identity.

$$a \cdot 1 = a \text{ and } 1 \cdot a = a$$

The inverse of a is written a^{-1} , or $\frac{1}{a}$.

$$a \cdot a^{-1} = 1 \text{ and } a^{-1} \cdot a = 1$$

For composites of mappings

I is the identity.

$$S \circ I = S \text{ and } I \circ S = S$$

The inverse of S is written S^{-1} .

$$S \circ S^{-1} = I \text{ and } S^{-1} \circ S = I$$

In general, the **inverse** of a transformation T is defined as the transformation S such that $S \circ T = I$. The inverses of some other transformations are illustrated below.

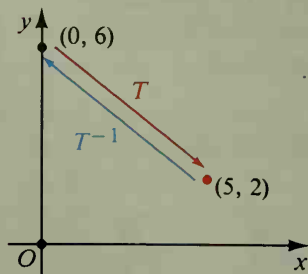
Example 1 Find the inverses of (a) translation $T: (x, y) \rightarrow (x + 5, y - 4)$,
(b) rotation $\mathcal{R}_{O, x}$, and (c) dilation $D_{O, 2}$.

Solution

a. $T^{-1}: (x, y) \rightarrow (x - 5, y + 4)$

$$T: (0, 6) \rightarrow (5, 2)$$

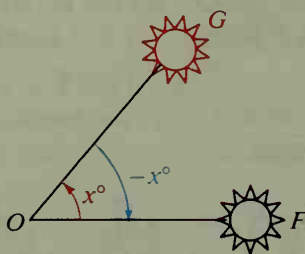
$$T^{-1}: (5, 2) \rightarrow (0, 6)$$



b. The inverse of $\mathcal{R}_{O, x}$
is $\mathcal{R}_{O, -x}$.

$$\mathcal{R}_{O, x}: F \rightarrow G$$

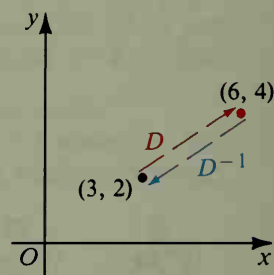
$$\mathcal{R}_{O, -x}: G \rightarrow F$$



c. The inverse of $D_{O, 2}$
is $D_{O, \frac{1}{2}}$.

$$D_{O, 2}: (3, 2) \rightarrow (6, 4)$$

$$D_{O, \frac{1}{2}}: (6, 4) \rightarrow (3, 2)$$



Example 2 What is the inverse of R_j ?
(Refer to the diagram at right.)

Solution Since $R_j \circ R_j = I$, the inverse of R_j is R_j itself.
In symbols, $R_j^{-1} = R_j$. Do you see that the
inverse of any reflection is that same reflection?

