Final velocity depends on initial velocity, acceleration, and displacement

So far, all of the equations for motion under uniform acceleration have required knowing the time interval. We can also obtain an expression that relates displacement, velocity, and acceleration without using the time interval. This method involves rearranging one equation to solve for Δt and substituting that expression in another equation, making it possible to find the final velocity of a uniformly accelerated object without knowing how long it has been accelerating. Start with the following equation for displacement:

$$\Delta x = \tfrac{1}{2}(v_i + v_f)\Delta t$$ Now, multiply both sides by 2.

$$2\Delta x = (v_i + v_f)\Delta t$$ Next, divide both sides by $(v_i + v_f)$ to solve for Δt.

$$\left(\frac{2\Delta x}{v_i + v_f}\right) = \Delta t$$

Now that we have an expression for Δt, we can substitute this expression into the equation for the final velocity.

$$v_f = v_i + a(\Delta t)$$

$$v_f = v_i + a\left(\frac{2\Delta x}{v_i + v_f}\right)$$

In its present form, this equation is not very helpful because v_f appears on both sides. To solve for v_f, first subtract v_i from both sides of the equation.

$$v_f - v_i = a\left(\frac{2\Delta x}{v_i + v_f}\right)$$

Next, multiply both sides by $(v_i + v_f)$ to get all the velocities on the same side of the equation.

$$(v_f - v_i)\,(v_f + v_i) = 2a\Delta x = v_f^{\,2} - v_i^{\,2}$$

Add $v_i^{\,2}$ to both sides to solve for $v_f^{\,2}$.

FINAL VELOCITY AFTER ANY DISPLACEMENT

$$v_f^{\,2} = v_i^{\,2} + 2a\Delta x$$

(final velocity)^2 = (initial velocity)^2 + 2(acceleration)(displacement)

When using this equation, you must take the square root of the right side of the equation to find the final velocity. Remember that the square root may be either positive or negative. If you have been consistent in your use of the sign convention, you will be able to determine which value is the right answer by reasoning based on the direction of the motion.

Did you know?

The word *physics* comes from the ancient Greek word for "nature." According to Aristotle, who assigned the name, physics is the study of natural events. Aristotle believed that the study of motion was the basis of physics. Galileo developed the foundations for the modern study of motion using mathematics. In 1632, Galileo published the first mathematical treatment of motion.

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