SAMPLE PROBLEM E

STRATEGY Projectiles Launched at an Angle

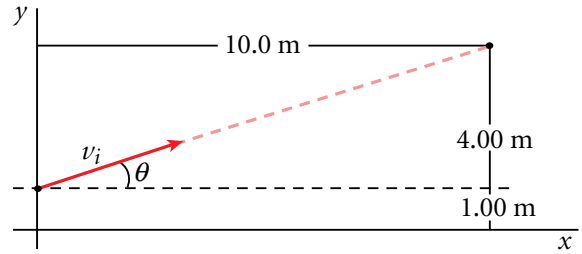
PROBLEM

A zookeeper finds an escaped monkey hanging from a light pole. Aiming her tranquilizer gun at the monkey, she kneels 10.0 m from the light pole, which is 5.00 m high. The tip of her gun is 1.00 m above the ground. At the same moment that the monkey drops a banana, the zookeeper shoots. If the dart travels at 50.0 m/s, will the dart hit the monkey, the banana, or neither one?

SOLUTION

1. Select a coordinate system.

The positive *y*-axis points up, and the positive *x*-axis points along the ground toward the pole. Because the dart leaves the gun at a height of 1.00 m, the vertical distance is 4.00 m.



2. Use the inverse tangent function to find the angle that the initial velocity makes with the *x*-axis.

$$\theta = \tan^{-1}\left(\frac{\Delta y}{\Delta x}\right) = \tan^{-1}\left(\frac{4.00\text{ m}}{10.0\text{ m}}\right) = 21.8°$$

3. Choose a kinematic equation to solve for time.

Rearrange the equation for motion along the *x*-axis to isolate the unknown, Δt, which is the time the dart takes to travel the horizontal distance.

$$\Delta x = (v_i \cos\theta)\Delta t$$

$$\Delta t = \frac{\Delta x}{v_i \cos\theta} = \frac{10.0\text{ m}}{(50.0\text{ m/s})(\cos 21.8°)} = 0.215\text{ s}$$

4. Find out how far each object will fall during this time.

Use the free-fall kinematic equation in both cases. For the banana, $v_i = 0$. Thus:

$$\Delta y_b = \tfrac{1}{2}a_y(\Delta t)^2 = \tfrac{1}{2}(-9.81\text{ m/s}^2)(0.215\text{ s})^2 = -0.227\text{ m}$$

The dart has an initial vertical component of velocity equal to $v_i \sin\theta$, so:

$$\Delta y_d = (v_i \sin\theta)\Delta t + \tfrac{1}{2}a_y(\Delta t)^2$$
$$\Delta y_d = (50.0\text{ m/s})(\sin 21.8°)(0.215\text{ s}) + \tfrac{1}{2}(-9.81\text{ m/s}^2)(0.215\text{ s})^2$$
$$\Delta y_d = 3.99\text{ m} - 0.227\text{ m} = 3.76\text{ m}$$

5. Analyze the results.

Find the final height of both the banana and the dart.

$$y_{banana,\,f} = y_{b,i} + \Delta y_b = 5.00\text{ m} + (-0.227\text{ m}) = \boxed{4.77\text{ m above the ground}}$$

$$y_{dart,\,f} = y_{d,i} + \Delta y_d = 1.00\text{ m} + 3.76\text{ m} = \boxed{4.76\text{ m above the ground}}$$

The dart hits the banana. The slight difference is due to rounding.