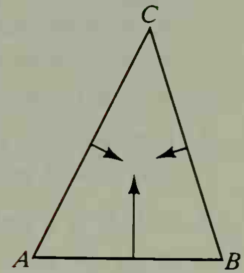
Theorem 10-2

The perpendicular bisectors of the sides of a triangle intersect in a point that is equidistant from the three vertices of the triangle.

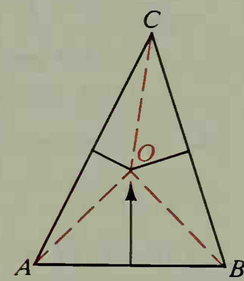


Given: $\triangle ABC$; the $\perp$ bisectors of $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$

Prove: The $\perp$ bisectors intersect in a point; that point is equidistant from A, B, and C.

Proof:

The perpendicular bisectors of $\overline{AC}$ and $\overline{BC}$ intersect at some point O. We will show that point O lies on the perpendicular bisector of $\overline{AB}$ and is equidistant from A, B, and C.



Draw $\overline{OA}$, $\overline{OB}$, and $\overline{OC}$. Since any point on the perpendicular bisector of a segment is equidistant from the endpoints of the segment (Theorem 4-5, page 153), $OA = OC$ and $OC = OB$. Thus $OA = OB$. Since any point equidistant from the endpoints of a segment lies on the perpendicular bisector of the segment (Theorem 4-6, page 153), O is on the perpendicular bisector of $\overline{AB}$. Since $OA = OB = OC$, point O is equidistant from A, B, and C.

The following theorems will be proved in Chapter 13.

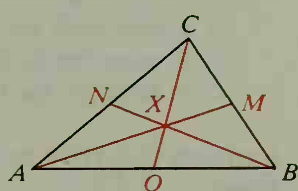
Theorem 10-3

The lines that contain the altitudes of a triangle intersect in a point.

Theorem 10-4

The medians of a triangle intersect in a point that is two thirds of the distance from each vertex to the midpoint of the opposite side.

According to Theorem 10-4, if $\overline{AM}$, $\overline{BN}$, and $\overline{CO}$ are medians of $\triangle ABC$, then:



$$AX = \tfrac{2}{3}AM$$
$$XN = \tfrac{1}{3}BN$$
$$CX:XO:CO = 2:1:3$$

The points of intersection described in the theorems in this section are sometimes called the *incenter* (point where the angle bisectors meet), *circumcenter* (point where the perpendicular bisectors meet), *orthocenter* (point where the altitudes meet), and *centroid* (point where the medians meet).