

Harmonics in a pipe closed at one end

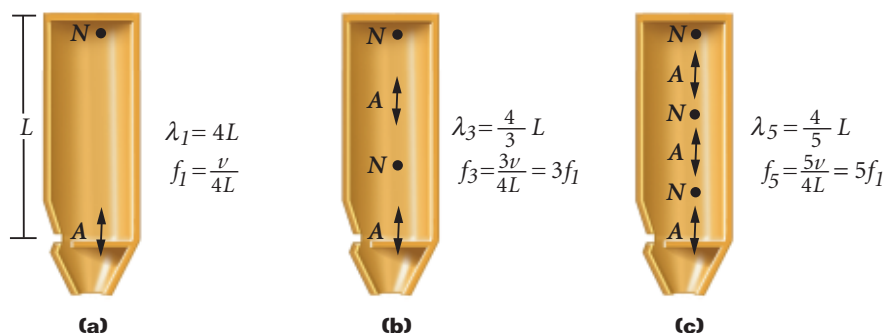


Figure 16

In a pipe closed at one end, the closed end is a node of displacement and the open end is an antinode of displacement. In this case, only the odd harmonics are present. The (a) first, (b) third, and (c) fifth harmonics are shown here.

If one end of a pipe is closed, only odd harmonics are present

When one end of an organ pipe is closed, as is illustrated in **Figure 16**, the movement of air molecules is restricted at this end, making this end a node. In this case, one end of the pipe is a node and the other is an antinode. As a result, a different set of standing waves can occur.

As shown in **Figure 16(a)**, the simplest possible standing wave that can exist in this pipe is one for which the length of the pipe is equal to one-fourth of a wavelength. Hence, the wavelength of this standing wave equals four times the length of the pipe. Thus, in this case, the fundamental frequency equals the velocity divided by four times the pipe length.

$$f_1 = \frac{v}{\lambda_1} = \frac{v}{4L}$$

For the case shown in **Figure 16(b)**, the length of the pipe is equal to three-fourths of a wavelength, so the wavelength is four-thirds the length of the pipe ($\lambda_3 = \frac{4}{3}L$). Substituting this value into the equation for frequency gives the frequency of this harmonic.

$$f_3 = \frac{v}{\lambda_3} = \frac{v}{\frac{4}{3}L} = \frac{3v}{4L} = 3f_1$$

The frequency of this harmonic is *three* times the fundamental frequency. Repeating this calculation for the case shown in **Figure 16(c)** gives a frequency equal to *five* times the fundamental frequency. Thus, only the odd-numbered harmonics vibrate in a pipe closed at one end. We can generalize the equation for the harmonic series of a pipe closed at one end as follows:

HARMONIC SERIES OF A PIPE CLOSED AT ONE END

$$f_n = n \frac{v}{4L} \quad n = 1, 3, 5, \dots$$

$$\text{frequency} = \text{harmonic number} \times \frac{(\text{speed of sound in the pipe})}{(4)(\text{length of vibrating air column})}$$

Quick Lab

A Pipe Closed at One End

MATERIALS LIST

- straw
- scissors

SAFETY



Always use caution when working with scissors.



Snip off the corners of one end of the straw so that the end tapers to a point, as shown above. Chew on this end to flatten it, and you create a double-reed instrument! Put your lips around the tapered end of the straw, press them together tightly, and blow through the straw. When you hear a steady tone, slowly snip off pieces of the straw at the other end. Be careful to keep about the same amount of pressure with your lips. How does the pitch change as the straw becomes shorter? How can you account for this change in pitch? You may be able to produce more than one tone for any given length of the straw. How is this possible?