11-6 Arc Lengths and Areas of Sectors

A pie chart is often used to analyze data or to help plan business strategy. The radii of a pie chart divide the interior of the circle into regions called sectors, whose areas represent the relative sizes of particular items. A sector of a circle is a region bounded by two radii and an arc of the circle. The shaded region of the diagram at the right below is called sector AOB. The unshaded region is also a sector.

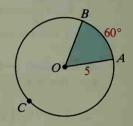


The length of \widehat{AB} in circle O is part of the circumference of the circle. Since $\widehat{mAB} = 60$ and $\frac{60}{360} = \frac{1}{6}$, the length of \widehat{AB} is $\frac{1}{6}$ of the circumference. Thus,

Length of
$$\widehat{AB} = \frac{1}{6}(2\pi \cdot 5) = \frac{5}{3}\pi$$
.

Similarly, the area of sector AOB is $\frac{1}{6}$ of the area of the circle. Thus,

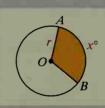
Area of sector
$$AOB = \frac{1}{6}(\pi \cdot 5^2) = \frac{25}{6}\pi$$
.



In general, if $\widehat{mAB} = x$:

Length of
$$\widehat{AB} = \frac{x}{360} \cdot 2\pi r$$

Area of sector $AOB = \frac{x}{360} \cdot \pi r^2$



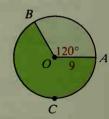
Example 1 In $\bigcirc O$ with radius 9, $m \angle AOB = 120$. Find the lengths of the arcs \widehat{AB} and \widehat{ACB} and the areas of the two sectors shown.

Solution $\widehat{mAB} = 120$, and $\widehat{mACB} = 240$.

Minor arc \widehat{AB} :

Length of
$$\widehat{AB} = \frac{120}{360} \cdot (2\pi \cdot 9) = \frac{1}{3}(18\pi) = 6\pi$$

Area of sector
$$AOB = \frac{120}{360} \cdot (\pi \cdot 9^2) = \frac{1}{3}(81\pi) = 27\pi$$



Major arc \widehat{ACB} :

Length of
$$\widehat{ACB} = \frac{240}{360} \cdot (2\pi \cdot 9) = \frac{2}{3}(18\pi) = 12\pi$$

Area of sector =
$$\frac{240}{360} \cdot (\pi \cdot 9^2) = \frac{2}{3}(81\pi) = 54\pi$$