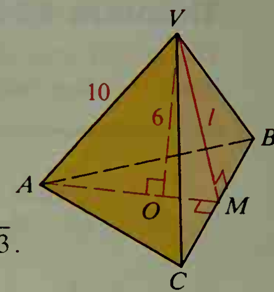
Example 4 A regular triangular pyramid has lateral edge 10 and height 6. Find the (a) lateral area and (b) volume.



Solution

a. In rt. $\triangle VOA$, $AO = \sqrt{10^2 - 6^2} = \sqrt{64} = 8$.

Since $AO = \frac{2}{3}AM$ (why?), $\frac{2}{3}AM = 8$, $AM = 12$, and $OM = 4$.

$l = \sqrt{6^2 + 4^2} = \sqrt{52} = 2\sqrt{13}$

In 30°-60°-90° $\triangle AMC$, $CM = \dfrac{12}{\sqrt{3}} = \dfrac{12\sqrt{3}}{3} = 4\sqrt{3}$.

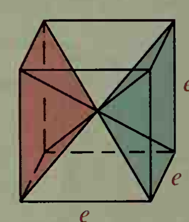
Base edge $= BC = 2 \cdot 4\sqrt{3} = 8\sqrt{3}$

L.A. $= \frac{1}{2}pl = \frac{1}{2}(3 \cdot 8\sqrt{3}) \cdot 2\sqrt{13} = 24\sqrt{39}$

b. Area of base $= B = \frac{1}{2} \cdot BC \cdot AM = \frac{1}{2} \cdot 8\sqrt{3} \cdot 12 = 48\sqrt{3}$

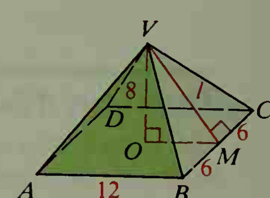
$V = \frac{1}{3}Bh = \frac{1}{3} \cdot 48\sqrt{3} \cdot 6 = 96\sqrt{3}$

Classroom Exercises



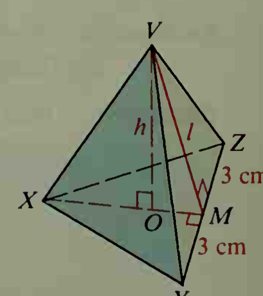
1. The diagonals of a cube intersect to divide the cube into six congruent pyramids as shown. The base of each pyramid is a face of the cube, and the height of each pyramid is $\frac{1}{2}e$.
   a. Use the formula for the volume of a cube to explain why the volume of each pyramid is $V = \frac{1}{6}e^3$.
   b. Use the formula in part (a) to show that $V = \frac{1}{3}Bh$. (*Note*: This exercise shows that $V = \frac{1}{3}Bh$ gives the correct result for *these* pyramids.)

V-$ABCD$ is a regular square pyramid. Find numerical answers.



2. $OM = \underline{\ ?\ }$
3. $l = \underline{\ ?\ }$
4. Area of $\triangle VBC = \underline{\ ?\ }$
5. L.A. $= \underline{\ ?\ }$
6. Volume $= \underline{\ ?\ }$
7. $VC = \underline{\ ?\ }$

Each edge of pyramid V-XYZ is 6 cm. Find numerical answers.



8. $XM = \underline{\ ?\ }$
9. $XO = \underline{\ ?\ }$
10. $h = \underline{\ ?\ }$
11. Base area $= \underline{\ ?\ }$
12. Volume $= \underline{\ ?\ }$
13. Slant height $= \underline{\ ?\ }$
14. L.A. $= \underline{\ ?\ }$
15. T.A. $= \underline{\ ?\ }$
16. Can the height of a regular pyramid be greater than the slant height? Explain.
17. Can the slant height of a regular pyramid be greater than the length of a lateral edge? Explain.
18. Can the area of the base of a regular pyramid be greater than the lateral area? Explain.