

Application*Steiner's Problem*

Four villages plan to build a system of roads of minimum length that will connect them all. Shown below are some plans for how to build the roads. Which plan shows the shortest road? Is there another way to connect the villages by an even shorter system of roads?

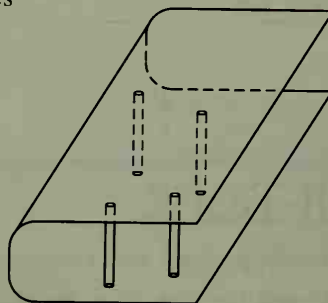


This problem was first investigated by the German mathematician Jacob Steiner (1796–1863), and carried his name, *Steiner's problem*.

Because a soap film automatically minimizes its surface area you can build a model that will help you solve Steiner's problem. You will need:

- a sheet of clear plastic
- 8 split-pin paper fasteners
- a drinking straw cut into four 3-cm long pieces

Bend the sheet of plastic without creasing it. Cut four small slits (to represent the location of the four villages) through both layers of the sheet. Insert the paper fasteners through all eight slits. Slip two fasteners through each of four straws, so that your model looks like the figure at the right. The halves of the plastic sheet should be parallel, and the straws perpendicular to them.



Dip the model in a soap solution and carefully lift it out. You should see a system of vertical soap films between the two sheets of plastic and joining the straws, revealing the solution to the problem. (Should any soap film adhere to the curved part of the plastic sheet, wet a drinking straw with the soap solution and push the straw through the soap films. You can suck air out through the straw to allow the films to form the minimum connection.)

Exercises

1. Gently place a protractor on top of the model and measure the angles where the soap films meet. What are the measures of these angles?

Make other models to find the shortest connection between the vertices of the following polygons. In each model, find the measures of the angles where the soap films meet.

2. Triangle
3. Square
4. Pentagon