♦ Computer Key-In

Suppose a, b, and c are positive integers such that $a^2 + b^2 = c^2$. Then the converse of the Pythagorean Theorem guarantees that a, b, and c are the lengths of the sides of a right triangle. Because of this, any such triple of integers is called a **Pythagorean triple.**

For example, 3, 4, 5 is a Pythagorean triple since $3^2 + 4^2 = 5^2$. Another triple is 6, 8, 10, since $6^2 + 8^2 = 10^2$. The triple 3, 4, 5 is called a *primitive* Pythagorean triple because no factor (other than 1) is common to all three integers. The triple 6, 8, 10 is *not* a primitive triple.

The following program in BASIC lists some Pythagorean triples.

```
10  FOR X = 2 TO 7
20    FOR Y = 1 TO X - 1
30      LET A = 2 * X * Y
40      LET B = X * X - Y * Y
50      LET C = X * X + Y * Y
60      PRINT A;",";B;",";C
70    NEXT Y
80  NEXT X
90  END
```

Exercises

1. Type and RUN the program. (If your computer uses a language other than BASIC, write and RUN a similar program.) What Pythagorean triples did it list? Which of these are primitive Pythagorean triples?

2. The program above uses a method for finding Pythagorean triples that was developed by Euclid around 320 B.C. His method can be stated as follows:

If x and y are positive integers with $y < x$, then $a = 2xy$, $b = x^2 - y^2$, and $c = x^2 + y^2$ is a Pythagorean triple.

To verify that Euclid's method is correct, show that the equation below is true.

$$(2xy)^2 + (x^2 - y^2)^2 = (x^2 + y^2)^2$$

3. Look at the primitive Pythagorean triples found in Exercise 1. List those triples that have an odd number as their lowest value. Do you notice a pattern in some of these triples?

Another method for finding Pythagorean triples begins with an odd number. If n is any positive integer, $2n + 1$ is an odd number. A triple is given by: $a = 2n + 1$, $b = 2n^2 + 2n$, $c = (2n^2 + 2n) + 1$.

For example, when $n = 3$, the triple is

$$2(3) + 1 = 7, \quad 2(3^2) + 2(3) = 24, \quad 24 + 1 = 25.$$

a. Use the formula to find another primitive triple with 33 as its lowest value. (*Hint*: $n = 16$)

b. Use the Pythagorean Theorem to verify the method described.