THE SIMPLE PENDULUM

As you have seen, the periodic motion of a mass-spring system is one example of simple harmonic motion. Now consider the trapeze acrobats shown in **Figure 3(a).** Like the vibrating mass-spring system, the swinging motion of a trapeze acrobat is a periodic vibration. Is a trapeze acrobat's motion an example of simple harmonic motion?

To answer this question, we will use a simple pendulum as a model of the acrobat's motion, which is a physical pendulum. A simple pendulum consists of a mass called a *bob,* which is attached to a fixed string, as shown in **Figure 3(b).** When working with a simple pendulum, we assume that the mass of the bob is concentrated at a point and that the mass of the string is negligible. Furthermore, we disregard the effects of friction and air resistance. For a physical pendulum, on the other hand, the distribution of the mass must be considered, and friction and air resistance also must be taken into account. To simplify our analysis, we will disregard these complications and use a simple pendulum to approximate a physical pendulum in all of our examples.

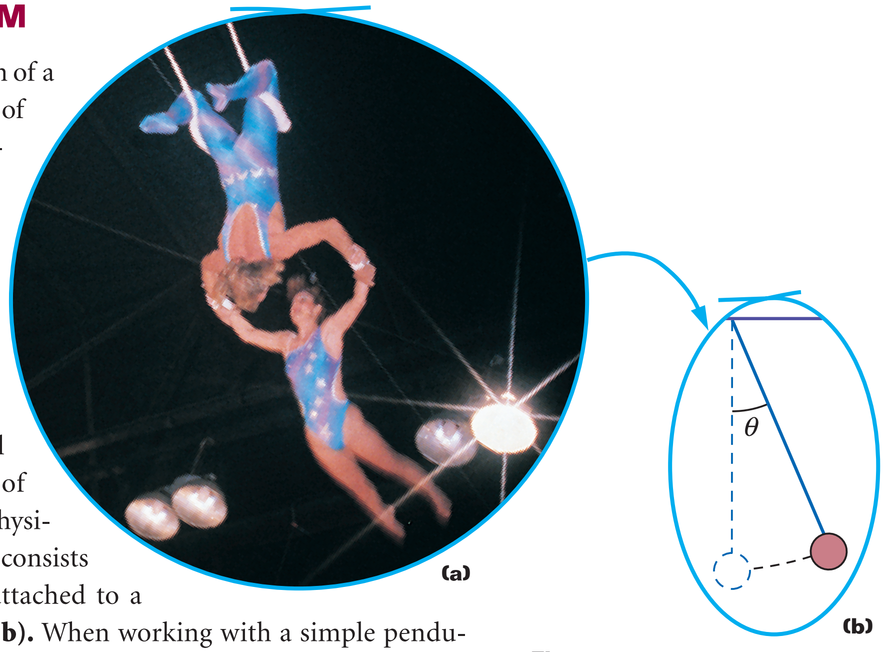

Figure 3
(a) The motion of these trapeze acrobats is modeled by **(b)** a simple pendulum.

The restoring force of a pendulum is a component of the bob's weight

To see whether the pendulum's motion is simple harmonic, we must first examine the forces exerted on the pendulum's bob to determine which force acts as the restoring force. If the restoring force is proportional to the displacement, then the pendulum's motion is simple harmonic. Let us select a coordinate system in which the *x*-axis is tangent to the direction of motion and the *y*-axis is perpendicular to the direction of motion. Because the bob is always changing its position, these axes will change at each point of the bob's motion.

The forces acting on the bob at any point include the force exerted by the string and the gravitational force. The force exerted by the string always acts along the *y*-axis, which is along the string. The gravitational force can be resolved into two components along the chosen axes, as shown in **Figure 4.** Because both the force exerted by the string and the *y* component of the gravitational force are perpendicular to the bob's motion, the *x* component of the gravitational force is the net force acting on the bob in the direction of its motion. In this case, the *x* component of the gravitational force always pulls the bob toward its equilibrium position and hence is the restoring force. Note that the restoring force ($F_{g,x} = F_g \sin\theta$) is zero at equilibrium because θ equals zero at this point.

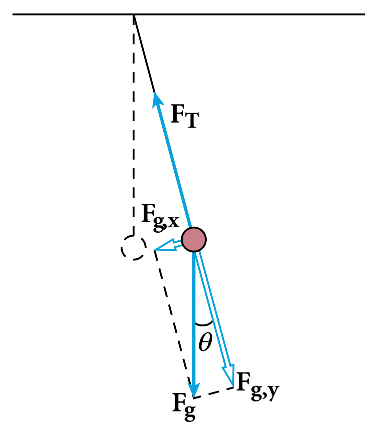

Figure 4
At any displacement from equilibrium, the weight of the bob **($\mathbf{F_g}$)** can be resolved into two components. The *x* component **($\mathbf{F_{g,x}}$)**, which is perpendicular to the string, is the only force acting on the bob in the direction of its motion.