

Logic

Statements and Truth Tables

In algebra, you have used letters to represent numbers. In logic, letters are used to represent statements that are either true or false. For example, p might represent the statement “Paris is the capital city of France,” and q might represent the statement “The moon is made of green cheese.” Deciding whether statements are true or false involves investigating the real world, not the “logic” of an argument.

Statements can be joined to form **compound statements**. Two important compound statements are defined below.

A **conjunction** is a compound statement composed of two statements joined by the word “and.” The symbol \wedge is used to represent the word “and.”

A **disjunction** is a compound statement composed of two statements joined by the word “or.” The symbol \vee is used to represent the word “or.”

Example 1 Statements: p Inez plays the flute.
 q Sue Yin plays the cello.
Conjunction: $p \wedge q$ Inez plays the flute and Sue Yin plays the cello.
Disjunction: $p \vee q$ Inez plays the flute or Sue Yin plays the cello.

The table at the left below is called a **truth table**. It tells you the conditions under which a conjunction is a true statement. “T” stands for “true” and “F” for “false.” The first row of the table shows that when statement p is true and statement q is true, the conjunction $p \wedge q$ is true. The other rows of the table show that $p \wedge q$ is false when either of its statements is false.

Truth table for
conjunction

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Truth table for
disjunction

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

A disjunction is true if either of its statements is true or both are true. This corresponds to what is called the *inclusive use* of “or.” (The *exclusive use* of “or” would imply that one of the statements is true, but not both. We deal only with the inclusive use of “or” in this course.) The first row of the truth table for disjunction shows that when both p and q are true, $p \vee q$ is true. The next two rows show that the compound statement $p \vee q$ is true when either of its statements is true. The last row shows that a disjunction is false when both of its statements are false.