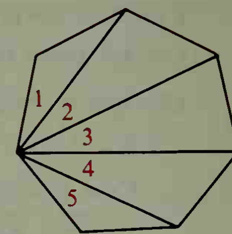
24. The diagram at the right shows a regular polygon with 7 sides.

a. Explain why the numbered angles are all congruent. (*Hint*: You may assume that a circle can be circumscribed about any regular polygon.)

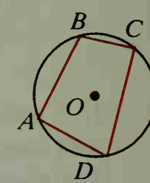
b. Will your reasoning apply to a regular polygon with any number of sides?



C **25.** Given: Vertices A, B, and C of quadrilateral $ABCD$ lie on $\odot O$; $m\angle A + m\angle C = 180$; $m\angle B + m\angle D = 180$.

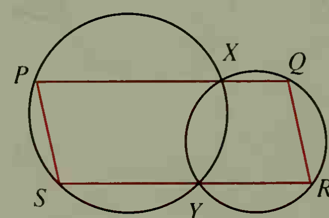
Prove: D lies on $\odot O$.

(*Hint*: Use an indirect proof. Assume temporarily that D is not on $\odot O$. You must then treat two cases: (1) D is inside $\odot O$, and (2) D is outside $\odot O$. In each case let X be the point where $\overrightarrow{AD}$ intersects $\odot O$ and draw $\overline{CX}$. Show that what you can conclude about $\angle AXC$ contradicts the given information.)



26. Given: $\overline{PQ} \parallel \overline{SR}$

Prove: $\overline{PS} \parallel \overline{QR}$



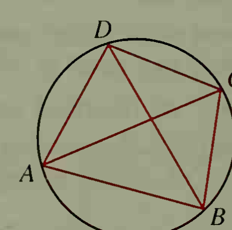
27. *Ptolemy's Theorem* states that in an inscribed quadrilateral, the sum of the products of its opposite sides is equal to the product of its diagonals. This means that for $ABCD$ shown,

$$AB \cdot CD + BC \cdot AD = AC \cdot BD.$$

Prove the theorem by choosing point Q on $\overline{AC}$ so that $\angle ADQ \cong \angle BDC$. Then show $\triangle ADQ \sim \triangle BDC$ and $\triangle ADB \sim \triangle QDC$. Use these similar triangles to show that

$$AQ = \frac{BC \cdot AD}{BD} \text{ and } QC = \frac{AB \cdot CD}{BD}.$$

Add these two equations and complete the proof.



28. Equilateral $\triangle ABC$ is inscribed in a circle. P is any point on $\overset{\frown}{BC}$. Prove $PA = PB + PC$. (*Hint*: Use Ptolemy's Theorem.)

★ **29.** Angle C of $\triangle ABC$ is a right angle. The sides of the triangle have the lengths shown. The smallest circle (not shown) through C that is tangent to $\overline{AB}$ intersects $\overline{AC}$ at J and $\overline{BC}$ at K. Express the distance JK in terms of a, b, and c.