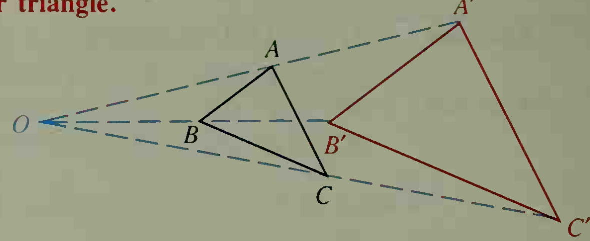
Theorem 14-5

A dilation maps any triangle to a similar triangle.

Given: $D_{O,k}: \triangle ABC \to \triangle A'B'C'$

Prove: $\triangle ABC \sim \triangle A'B'C'$



Key steps of proof:

1. $OA' = |k| \cdot OA$, $OB' = |k| \cdot OB$ (Definition of dilation)
2. $\triangle OAB \sim \triangle OA'B'$ (SAS Similarity Theorem)
3. $\dfrac{A'B'}{AB} = \dfrac{OA'}{OA} = |k|$ (Corr. sides of ~ ⚠ are in proportion.)
4. Similarly, $\dfrac{B'C'}{BC} = \dfrac{A'C'}{AC} = |k|$ (Repeat Steps 1–3 for $\triangle OBC$ and $\triangle OB'C'$ and for $\triangle OAC$ and $\triangle OA'C'$.)
5. $\triangle ABC \sim \triangle A'B'C'$ (SSS Similarity Theorem)

Corollary 1

A dilation maps an angle to a congruent angle.

Corollary 2

A dilation $D_{O,k}$ maps any segment to a parallel segment $|k|$ times as long.

Corollary 3

A dilation $D_{O,k}$ maps any polygon to a similar polygon whose area is k^2 times as large.

The diagram for the theorem above shows the case in which $k > 0$. You should draw the diagram for $k < 0$ and convince yourself that the proof is the same.

Theorem 14-5 can also be proved by using coordinates (see Exercise 28). To do this, you set the center of dilation at the origin, and describe $D_{O,k}$ in terms of coordinates by writing $D_{O,k}: (x, y) \to (kx, ky)$. You can see that this description satisfies the definition of a dilation because O, P, and P' are collinear (use slopes) and $OP' = |k| \cdot OP$ (use the distance formula).