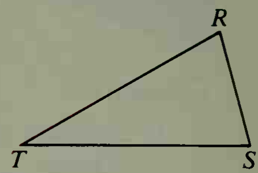
Theorem 6-3

If one angle of a triangle is larger than a second angle, then the side opposite the first angle is longer than the side opposite the second angle.



Given: $\triangle RST$; $m\angle S > m\angle T$

Prove: $RT > RS$

Proof:

Assume temporarily that $RT \not> RS$. Then either $RT = RS$ or $RT < RS$.

Case 1: If $RT = RS$, then $m\angle S = m\angle T$.

Case 2: If $RT < RS$, then $m\angle S < m\angle T$ by Theorem 6-2.

In either case there is a contradiction of the given fact that $m\angle S > m\angle T$. The assumption that $RT \not> RS$ must be false. It follows that $RT > RS$.

Corollary 1

The perpendicular segment from a point to a line is the shortest segment from the point to the line.

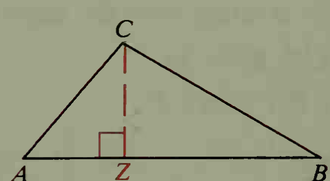
Corollary 2

The perpendicular segment from a point to a plane is the shortest segment from the point to the plane.

See Classroom Exercises 18 and 19 for proofs of the corollaries.

Theorem 6-4 *The Triangle Inequality*

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.



Given: $\triangle ABC$

Prove: (1) $AB + BC > AC$
(2) $AB + AC > BC$
(3) $AC + BC > AB$

Proof:

One of the sides, say $\overline{AB}$, is the longest side. (Or $\overline{AB}$ is at least as long as each of the other sides.) Then (1) and (2) are true. To prove (3), draw a line, $\overleftrightarrow{CZ}$, through C and perpendicular to $\overleftrightarrow{AB}$. (Through a point outside a line, there is exactly one line perpendicular to the given line.) By Corollary 1 of Theorem 6-3, $\overline{AZ}$ is the shortest segment from A to $\overline{CZ}$. Also, $\overline{BZ}$ is the shortest segment from B to $\overline{CZ}$. Therefore

$$AC > AZ \text{ and } BC > ZB.$$
$$AC + BC > AZ + ZB \text{ (Why?)}$$
$$AC + BC > AB \text{ (Why?)}$$