The proof of Theorem 8-1 is left as Exercise 40. The altitude to the hypotenuse divides the hypotenuse into two segments. Corollaries 1 and 2 of Theorem 8-1 deal with geometric means and the lengths of these segments.

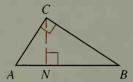
For simplicity in stating these corollaries, the words *segment*, *side*, *leg*, and *hypotenuse* are used to refer to the *length* of a segment rather than the segment itself. We will use this convention throughout the book when the context makes this meaning clear.

## **Corollary 1**

When the altitude is drawn to the hypotenuse of a right triangle, the length of the altitude is the geometric mean between the segments of the hypotenuse.

Given:  $\triangle ABC$  with rt.  $\angle ACB$ ; altitude  $\overline{CN}$ 

Prove: 
$$\frac{AN}{CN} = \frac{CN}{BN}$$



## Proof:

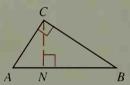
By Theorem 8-1,  $\triangle ANG \sim \triangle CNB$ . Because corresponding sides of similar triangles are in proportion,  $\frac{AN}{CN} = \frac{CN}{BN}$ .

## **Corollary 2**

When the altitude is drawn to the hypotenuse of a right triangle, each leg is the geometric mean between the hypotenuse and the segment of the hypotenuse that is adjacent to that leg.

Given:  $\triangle ABC$  with rt.  $\angle ACB$ ; altitude  $\overline{CN}$ 

Prove: (1) 
$$\frac{AB}{AC} = \frac{AC}{AN}$$
 and (2)  $\frac{AB}{BC} = \frac{BC}{BN}$ 



## Proof of (1):

By Theorem 8-1,  $\triangle ACB \sim \triangle ANC$ . Because corresponding sides of similar triangles are in proportion,  $\frac{AB}{AC} = \frac{AC}{AN}$ . The proof of (2) is very similar.

**Example 2** Use the diagram to find the values of h, a, and b.

First determine what parts of the "big" triangle are labeled h, a, and b:

h is the altitude to the hypotenuse,

a is a leg, and b is a leg. By Corollary 1,  $\frac{3}{h} = \frac{h}{7}$  and  $h = \sqrt{21}$ .

By Corollary 2, 
$$\frac{10}{a} = \frac{a}{3}$$
 and  $a = \sqrt{30}$ .

By Corollary 2, 
$$\frac{10}{b} = \frac{b}{7}$$
 and  $b = \sqrt{70}$ .

