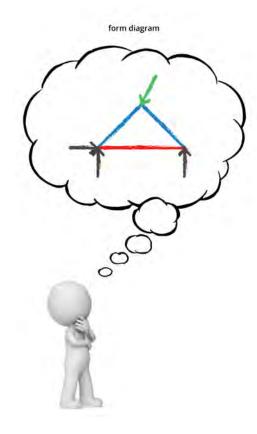
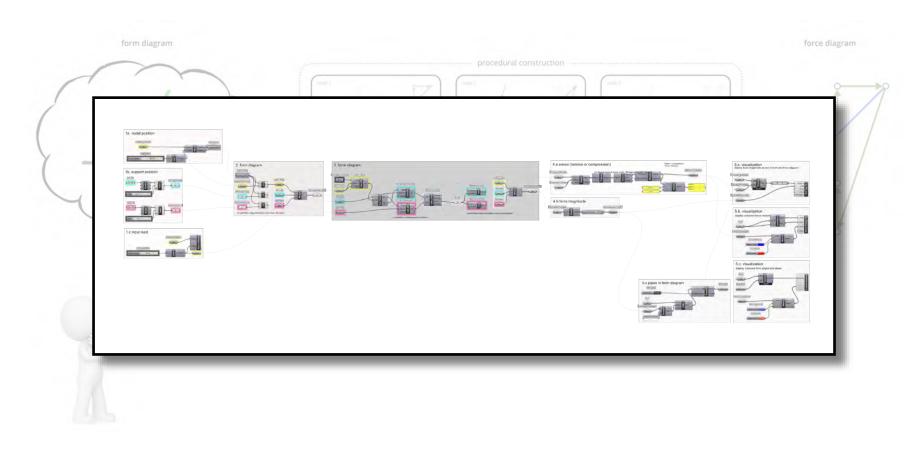
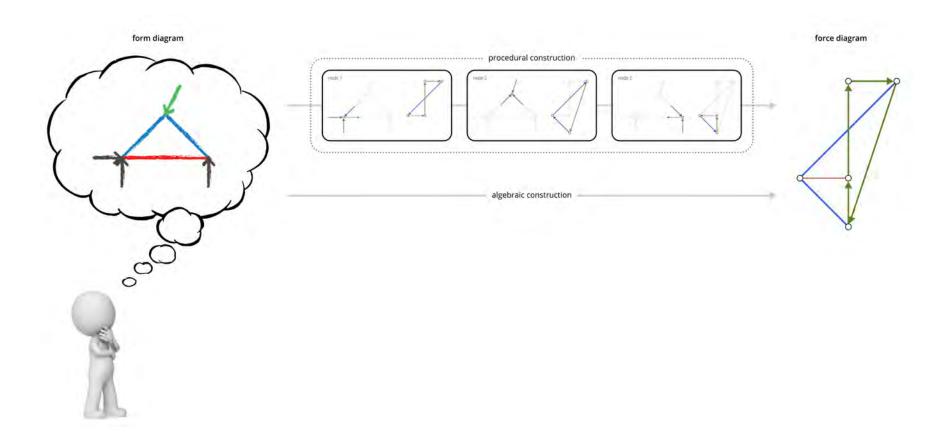


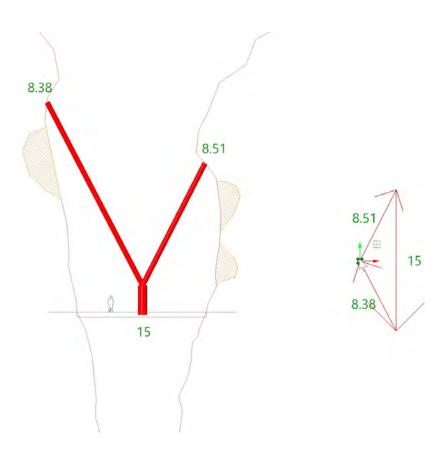
AGS ► Motivation

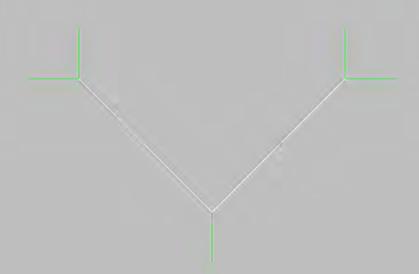


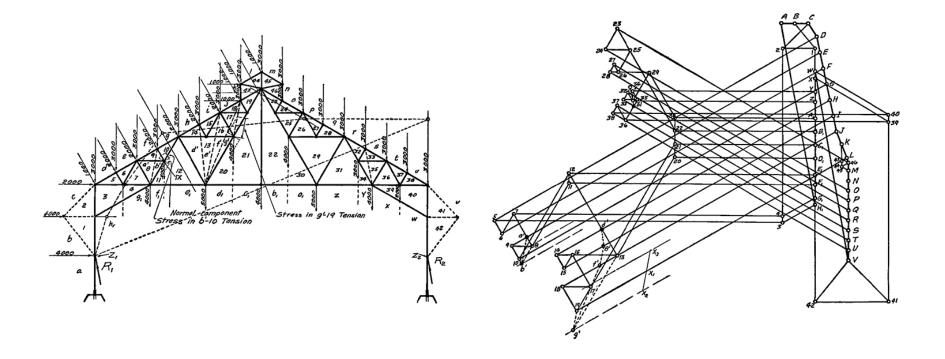
AGS ► Motivation

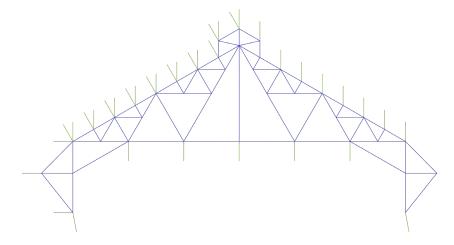


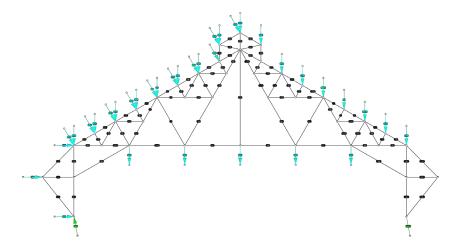


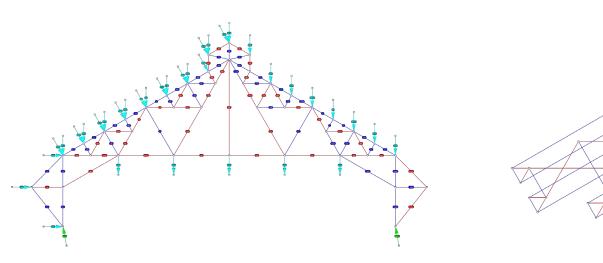


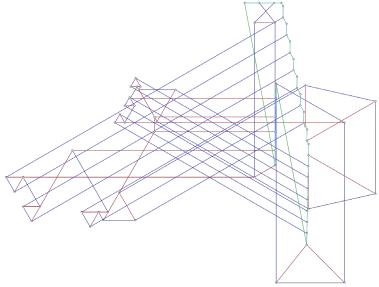


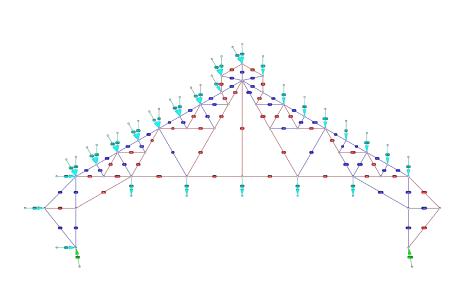


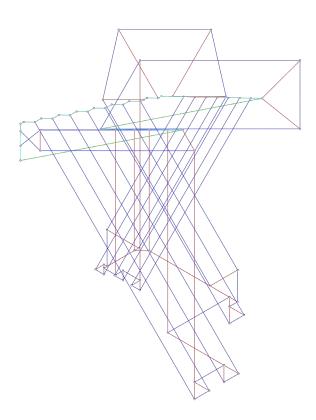


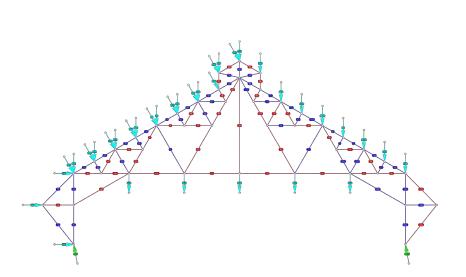


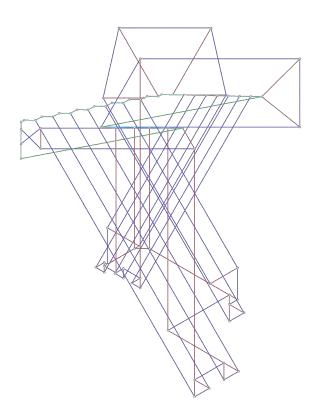


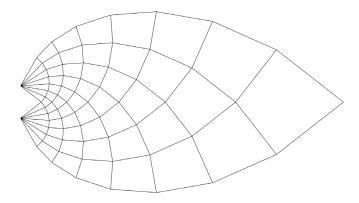


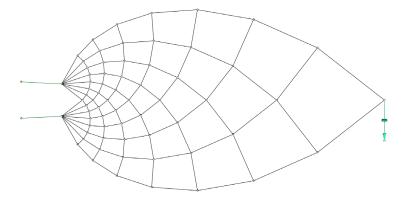


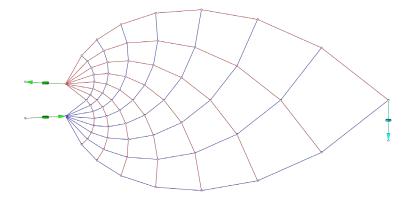


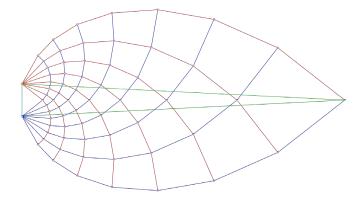


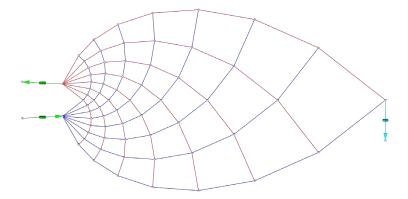


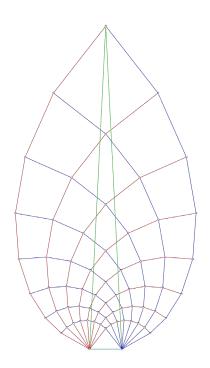


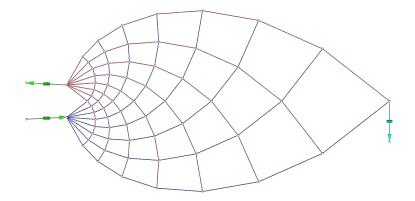


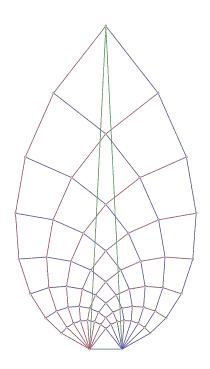


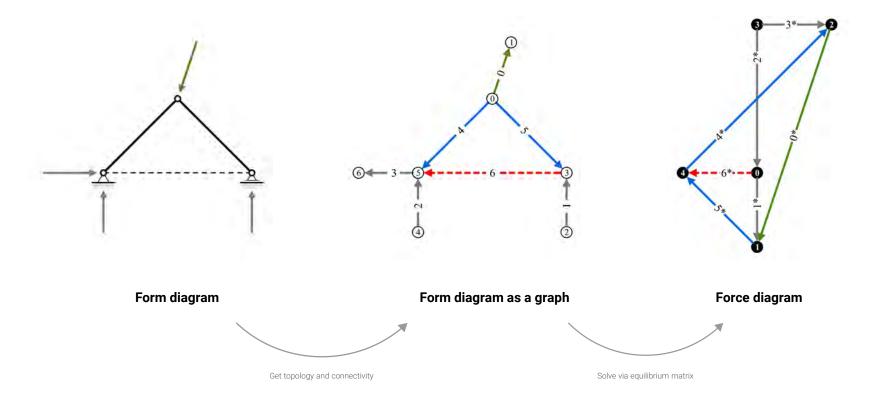












1. Form graph

Connectivity (edges & vertices)

Directed form graph

2. Force graph

Dual graph

- 3. Assign forces
- 4. Solve equilibrium matrix
- 5. Update force graph
- 6. Reciprocal form & force graphs

1. Form graph

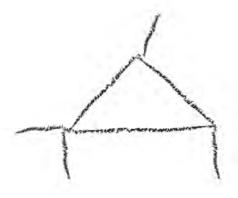
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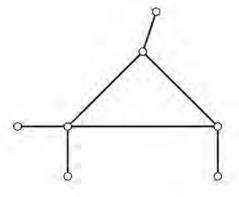
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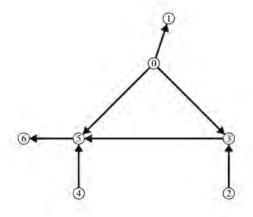
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30

1. Form graph

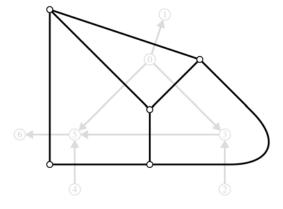
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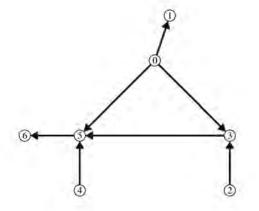
Connectivity (edges & vertices)

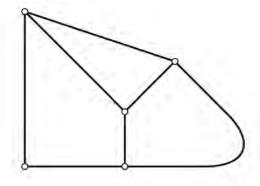
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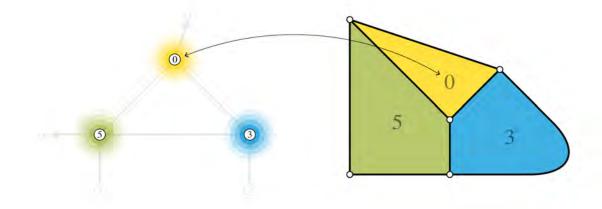
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32

1. Form graph

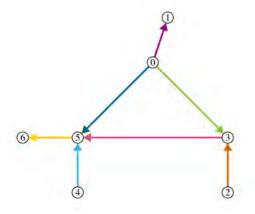
Connectivity (edges & vertices)

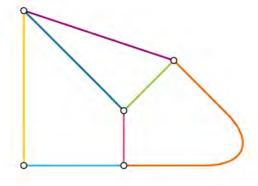
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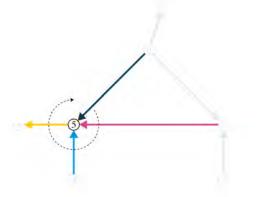
Connectivity (edges & vertices)

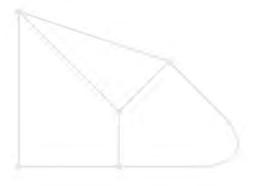
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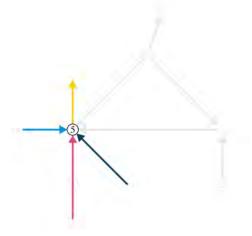
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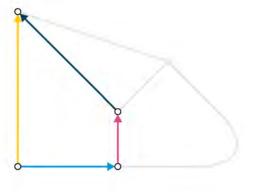
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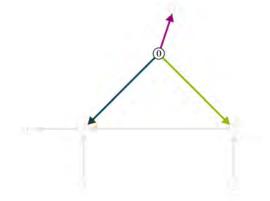
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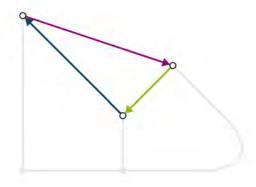
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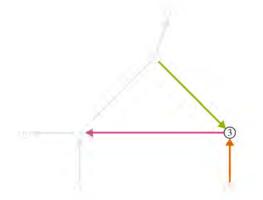
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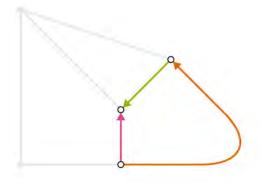
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37

1. Form graph

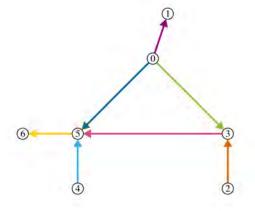
Connectivity (edges & vertices)

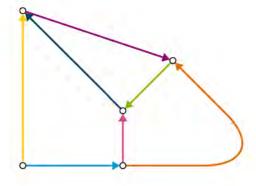
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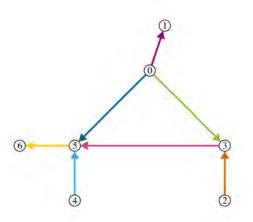
Connectivity (edges & vertices)

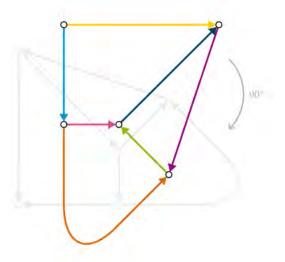
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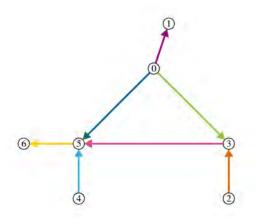
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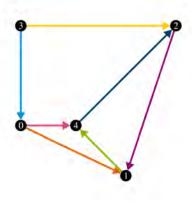
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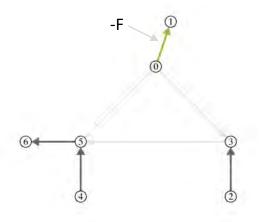
2. Force graph

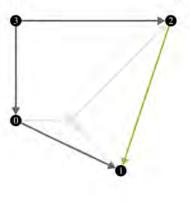
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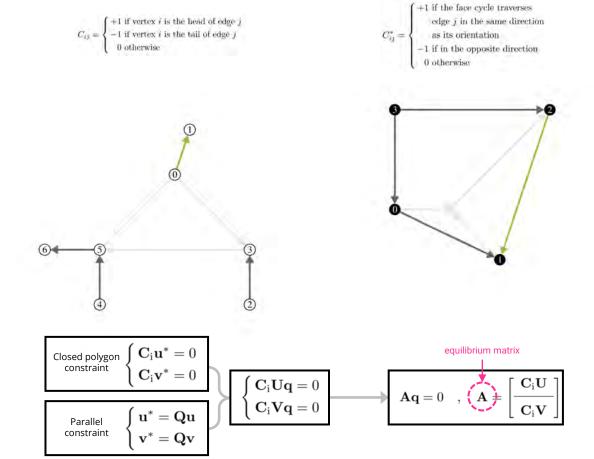
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2. Force graph

Dual graph

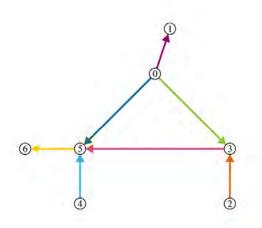
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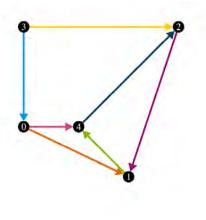
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$$\mathbf{A} = \left[\begin{array}{c|c} \mathbf{A}_{d} \mid \mathbf{A}_{id} \end{array}\right] \quad , \quad \mathbf{q} = \begin{bmatrix} \mathbf{q}_{d} \\ \hline \mathbf{q}_{id} \end{bmatrix} \qquad \mathbf{A}_{d} \mathbf{q}_{d} + \mathbf{A}_{id} \mathbf{q}_{id} = 0 \\ \mathbf{q}_{d} = -\mathbf{A}_{d}^{-1} \mathbf{A}_{id} \mathbf{q}_{id} \end{bmatrix} \qquad \mathbf{q}_{id} = \begin{bmatrix} -\mathbf{A}_{d}^{-1} \mathbf{A}_{id} \\ \hline \mathbf{I}_{k} \end{bmatrix} \mathbf{q}_{id} \qquad \mathbf{q}_{id} = \mathbf{Q}_{id} \mathbf{q}_{id}$$
 assigned loads/forces all force densities new force diagram coordinates

1. Form graph

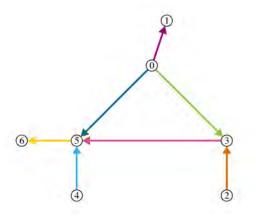
Connectivity (edges & vertices)

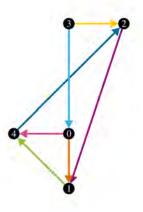
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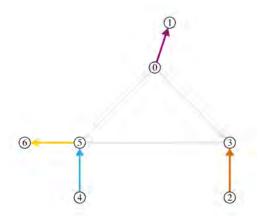
Connectivity (edges & vertices)

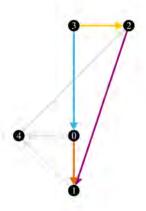
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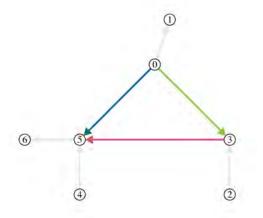
Connectivity (edges & vertices)

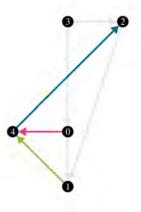
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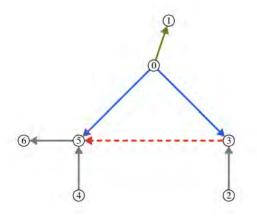
Connectivity (edges & vertices)

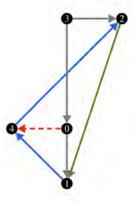
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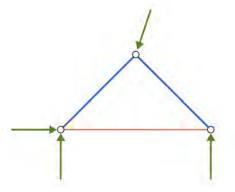
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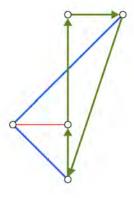
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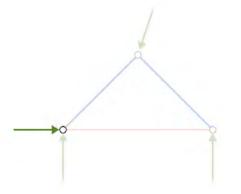
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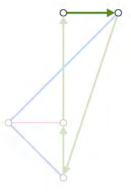
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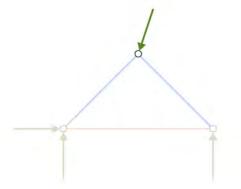
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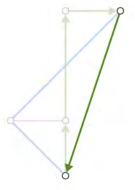
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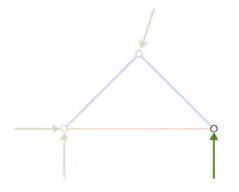
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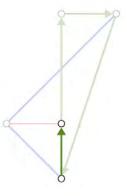
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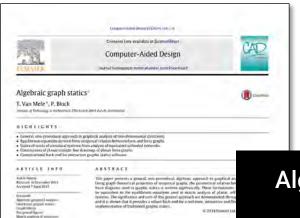
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1. Introduction



sponse to manipulations of the drawing by the user [3:7]. If ha seen demonstrated that such interactive implementations are not

stread feedback about the relation between for

expite its strengths compagnised (ingreactive) eraphic starits still has some drawbacks. The process of constructing drawines can easily become tedious and time-consuming and demands. a profound familianty with the specific prometric construction cinvolved (e.g. (4,11)). Furthermore, since the drawings produced by the CAD tools and interactive implementations are generated in a procedural manner according to the corresponding graphic statits "recipe", they tend to be designed for specific types of structures. Modifications to the Initial setup of the drawing (e.g. the number and/or connectivity of structural elements, enter of the logati.) Obercromore a complete redrair of the entire comprue-Him. All hough the process of making a graphic statics construction important for teaching and tearning, as it belos to got familiarmed with the specific geometric and structural relationships between

only extremely useful for educational purposes, but also for ad-

sanced research [8], in addition, several graphic statios tools have

also been developed as plug-ins for CAD environments (e.g. [f), (d)).

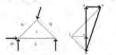
Company of the Architecture Ample of the Deservation and

reniem for research to practical projumes.

To fally explore the benefits of compareroed graphic trailes, a sensoral, non-procedural approach is reguled, which allows drawngs or he created without specific knowledge of the monument construction concedimes involved.

This paper presents a robust back-end for a graphics statics application. It allows the user to start from a connected two dimensional line drawing. The graph of this line drawing is suopportically constructed and analysed to assess the feasibility of the innur as a structural system or set of forces in equilibrium. If possible, a reciprocal force diagram is constructed, based on userdefined leading or self-stress soutitions. The two diagrams can then be manipulated interactively without breaking their tocolog scaland esemetrical relationship. As such the user can explore different states of equilibrium by explicit, researchic modify altern of the connected diagrams, or redistribution of forces werten given

1.2 Complications and cutting



this gray to construct the construction of the mentionally in the August.

two matrams are recovered: they consist of an equal number of lines, or that the corresponding lines in the IWA Carrains are parsitel (or perpondicities, or at any constant angle), and moresports ing lines which converge to a point in one diagram form a cassell.

The diagram to the left is the lives augrors and the one to the right the force diagram. How's notation is used to taket spaces in

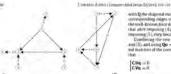


Fig. 2. These and from Assessed that the Security is directly people, in this colored, you distinct them, as the time, proph it and the property force graph to

mazon Carry

| of westex i in the tiesd of edge | if vertex i is the tail of ridge)

Note that these edge-directions may be chosen arbitrarily, but some sound for clarity. Here for example, all edges

th lower index to vertices with higher index. ence matrix & represents the tripology of raph G*. The recipies all graph has 1° vertices. of faces of the form graph, and the same numform yearth. The row vectors of C can be form graph by cycling in faces in a rmonter. 14). The entries of the filt new of Ct., corn-



ed C and C' the constitute difference oursels orn and furce others can be written as

H = Cx Y = Cy $\mathbf{u}' = \mathbf{C}'\mathbf{x}', \quad \mathbf{v}' = \mathbf{C}'\mathbf{v}'$ (3b) with x, y and x*, y* the x and y constitute vertex of G and G*; respectively. The reciprocal constraires, as described in Section 2.1. can be formulated in terms of the coordinate difference vectors (3a)

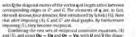
The first set of constraints, requiring that lines introducting a a node in the form diagram correspond to a closed polygon in the force diagram, can be imposed by expression that the sum of coordinate difference vectors of edges of the block graph connected to the same versex to the form graph should be acro [32].

with C compared of the story of C corresponding in the inner vertices of G.

The second set of constraints, requiring parallelity between corresponding edges to the form and force eraphy, can be imposed by writing the coordinate difference vectors u" and v" of the force scanhas a first tion of the corresponding vectors wand viol the form

$$u^* = Qu$$

 $v^* = Qv$



and materies of the coordinate difference vectors a and v. one finds

$$\begin{bmatrix} CAN_0 = 0 \\ CV_0 = 0 \end{bmatrix}$$
(5)

with A the
$$\|\Sigma_V = s\|$$
 equality and enough of G :
$$A = \begin{bmatrix} CU \\ C,V \end{bmatrix}.$$

Note that time all external forces are included in the firm graph. the right-hand side of (2) is a null vector. In the presented appersech, the equilibrium of a structural system is thus investigated. using the majes of self-nerses as an equivalent sudparted revenuels.

The marrier had independent states of self-stress of size antwork, consequently to the dimension of the nullipace of A | (b). The dimension of the nullspace can be calculated by SVD of A 1171 and in these serval to the resultier of adoes of which the force descrition can be chosen freely to explore different states of equilibrium of the represented structural system. We will call these the five in dependent edges. The structural interpretation of the states of salf-stress depends on the type of assembly or structural system represented by the form graph, as we will see in Section 2.6.

2.4. The firms graph

B can be seen from Fox (3h) and (5) that the encethmap vertices. x' and y' are related to the force demanies in the following way:

$$C'x' = Qx$$

$$C'y' = Qx.$$
(9)

This waters of equations cannot be select directly, once the imodence matrix of the force graph, Co is not source booked. The geometry of the force graph can be determined by sorring the

with L' the Laplacian matrix of the force graph. This system has an exact solution which can be calculated efficiently [18]; since L' in square and positive semi-definite (16).

Note that the vertex coordinates of the force graph are unuse up to a translation. Therefore, in order in obtain one specific columns, we solve the system of equations with the location of one potter chosen. In graphic statics, this is equivalent to cluming the first powe of the load line summediers on the drawing canyas to countries of the five distress.

L'v' = C'Ov

The approach consented in this paper is forced on the interpreation of the form and force diagrams of graphic statics as deal grapes, with the added constraint that corresponding edges about e parallel (or at any constant anale), as explained in Settion 2.2. If is essential that these graphs can be constructed automatically and

Algebraic graph(ic) statics (AGS)

$Form \rightarrow Force$

the reservance of the presented approach for three-dimensional

2. Theoretical featurement

in this section, we describe the theoretical framework for the graph-based, algebraic approach to graphic statics presented in ith paper, First, we briefly revisit traditional graphic statics, and describe the graph interpretation of form and force diagrams. Next, we formulate the recurrical constraints between these diagrams algebraically, and derive from them the typical equilibrium equalink the resmetry of the force graph can be readily derived from the solution of the equilibrium countings and the impolosical informaximum the form graph. Finally, we discuss the solution strategies ice different types of structural systems based on Sugaiter Value De-(SVD) of the equilibrium manys of their form graph and describe life interpretation of the obtained results in the contest of exaptor stories

3.1. Graphic reprise

Fig. 1 depicts a typicia anaphic statics drawing consistring of two discovery that movifies deposite the static envillagans of a nacmode structure and a set of applied loads and reaction forces. The

The Norm and force diagrams can be interpreted as directed graphs for which (directed) incidence or connectivity matrices can be constructed describing the topological relation between branches and nodes (Fig. 2). We define the graph of the form disgram as the form gright G, and the one of the force diagram as its rectimued force graph C. The force graph is the topological dual of the form graph with the added requirement that corresponding edges are parallel. The elements of G and G* are vertices, edges and Axoss. Elements of the force graph are superscripted with an untens. (*).

Due to the occurre of external forces in the form discress, the form graph contains leaf vertices. These are vertices of thegree une stocy trury have only one uscuretail inlge, which corresponds to an external force, in Section 2.5, we will use that it is a requirement of the presented approach that the leaf vertices and their edges can he drawn on the number of the month, in the nater or external space. The leaf vertices will threefore be referred to as other or extends vertices: the others as more or internal. Note that this requirement simply means that all external layers should be applied to vodes on the boundary of a structure. In Fig. 2, for example, vertices 1. 2. If and 6 are external vertices. They are, remectively, connected to educa it. 1, 2 and 3 representancitle external forces to the form diamain denicted in Fig. 1:

For a form wright E with enougher of edges and a number of vertices, the entries of the jtb column of the [1 a e) connectivity

been tolerand over the contract of the party of the contract o

Grantile statics is a well-known control for analysis and design

of two-dimensional structures based on Cremona's extensions o

Maxwell's theory of seciprocal figures [1,2], to graphic statics, the

relation between form and freces of a structural system is con-

tained in the reciprocal relation between two diagrams. A form

diagram describes the permetrical configuration of the (axial) to-

ternal and external forces of a two-dimensional structural vestero.

and a force diagram represents their equilibrium. The combina

tion of these two diagrams allows for an intuitive evaluation of

structural behaviour, performance and efficiency at a plance. The

example all nature of the method furthermore allows for a Vegal ver-

more transparent than anthmetic or numerical methods.

ification of both the evaluation process and results [3.4], making it.

Recent developments have demonstrated how the principles

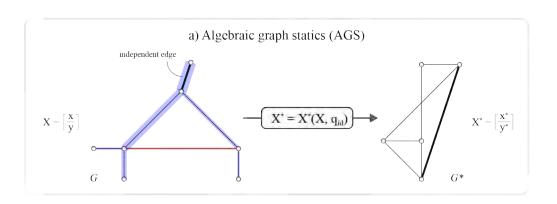
med the component by Charles C.L. Wangs

of graphic statics can be combined with madern commuter tech-

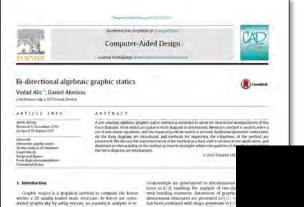
nolygies to create interactive drawings that provide real-time.

F. mail highways is seemed first (a.c.); T. Van Meter, principle Works, cit. 9. March 1

"Algebraic Graph Statics" | Van Mele and Block (2014)



AGS ► Theory



quired to calculate the intagritishe of the forces. The use of graphic ation of diverse structures in equilibrium, in a statics enow from the 19th and 29th conturies but lost popularity based approach to graphic statics is promoted due to the entergence of computational analysis methods. Howconstraints are employed to enline the result over, being an estudies method of visualizing form and forces, form and force diagrams. graphic staricy lists revently seen an increase in popularity. Digital implementations of graphic states h

Maxwell established that for axially haded structures, the graphical nature of the perhod line interactive eruble real time feedback of the relationship between from and

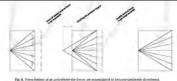
Several compages tools, that make one of graphs; statics have been developed [18-30]. However, urtil recently most of the toots and literature have only presented digital versions of graphs: was ics for some specific pre-defined structures. A peneral anarbonic version of graphic statics using a graph theoretic approach it preseried by Van Mese and Block in [37] that given a form diagram is able to penerate a force diagram directly. A graph througharguments for wilf-stressed networks is also presented in [17]. The equilibrium of the structural systems in [25] are investigated uning the states of self-stress of an equivalent unloaded perswork, making use of [7 i]. The algebraic approach of the graphic statics frame-work presented in [7] is well asked for computer implementation as a back and for a Company: Aided Design (CAD) suffware.

The ability to make changes in a general love diagram and subsequently compute a reciprocal form diagram has not been presented. The extension could be done in several different wires.

V. Rec D. Alexand / Computer Industriangs, 81 (2017) St. 30

"Bi-directional" AGS

Form ← Force



force diagram parameters since they must be perpendicular to the

Basis vector. From studying the basis vector for the left mult space, in

can be seen that, for instance, vertices 4 and 8 have to be moved by

the same amount horizontally, and by opposite amounts vertically

introducity further conditaints will introduce the rank and pumber of hous vectors for the left pull space. Finding an enactive method

revious example, and restricts the possible manapulations of the for presenting possible managerations of the force magram smooth be good in such cases.

4.2 Arch - Firm Indian

An extension of the previous examine is shown in No. II, which traditioner flow this control can be used to the design process of

Fig. 13. Manufaction of the best discussive and non-med familiar countries. The star disched lines represent the starting connective this example we have constrained tight budy motions, scaling and we arrive or the example shows in Fig. 10c, where off the edges of the bottom chard carry the same amount of force. Since the bottom Fig. 46a shows the starting position where the ton and bottom chord edges connect to the same verses in the force diamont marked with A in Vo. 10c', their opposite vertices are constrained to lie on a ritcle, shown in Fig. 10c. The diagonal elements inside

V 44, 0.4 -- /0 -- MARCH \$1(00) # #

sengths of free test edges.

chord edges do not carry the same amount of force in all offices. Edges in compression are shown in blue, edges in tension are shown in red in the form diagrams, and the thickness of the edgeis proportional to the force.

Two edges in the initial form in Fig. 104 in the form diagram do not carry any load, and their bottom vertices are marked with (F) A small manipulation of affacent vertices can leaf to large changes in the force diagram, as shown in (1), 10th. If the null space of the initial form is studied [182, 184], it is seen that it contains multi-modes where the zero force edges can rotate around their top vertices without affecting the force diagram. However, this is not

In official the form of a constant force wable trust is fairly straight-forward by manipulations in the force disgram of the true

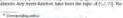
present in the shape in the 10th

of the tress carry a small amount of force compared to the other members of the structure. These could be triade smaller in another iteration, but have been kept in the figure for clarity.

An example of such a trust is seen in the Magazzini Generali warehouse at Chiaron Switzerland, designed by Robert Maillant Hill. The peacople of restraining chord forces to the same length can be used to find other similar forms as well, see for instance (20).

46. Just seasoned function structure

The T I shows our method armited to the design of an expensive.



nedes and polynops in the form disgram have reciprocal polynops.

and notes in the face diagram. The form diagram represents the structure, and the force diagram represents the static equilibcium [1] for that structure. Maxiveii's reciprocal diagrams were extended by Cremona [7], which provided a base for graphic statics. However, the first comprehensive presentation of graphic statics was by Culmann [1]. The method has historically been used to de-

sign structures by, for instance, Maurice Koechim (co-designer of

the (liflet Tower) and Robert Maillant [4], The reciprocity between

the form and force diagrams can provide the designer with insight

into the force distribution within a structure, and aid the intritive

masotry [5-7] and structural optimization [6]. Methods for using

graphic statics for the design of post-termioned limitedars has been

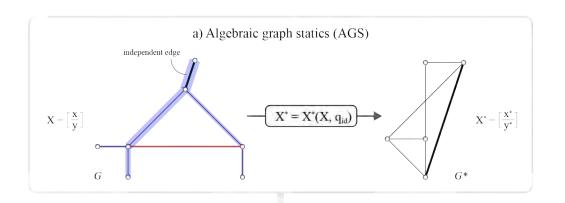
presented in [3]. The relationship between graphic statics and the

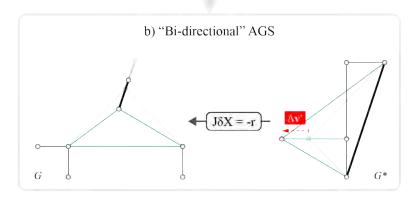
In recent literature it has been applied to the design of structural

understanding of the relationship between form and forces.

NAME AND POST OF PERSONS AND ADDRESS OF THE PARTY OF

"Bi-directional algebraic graphic statics" | Alic and Åkesson (2017)





AGS ► Theory



Concepted Decree of Sommer 201 Constitution of the Constitution of the Swimmfand, Superietter Lo-18, 2003

An interactive implementation of algebraic graphic statics for geometry-based teaching and design of structures

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This paper presents an interactive implementation of graphts, which can be integrated into a

CAD environment. Graphic statics is a well-known design and analysis method for rwo-dimensional discrete structures that relies an geometrical rather than analytical representation of the relation between the structure's accompany and the equilibrium of in internal forces. The method was formalised in the 19th century, but slowly disappeared from structural contracting practice over the 20th century. Recent developments have introduced Algebraic Graph Station (AGS), which formulates the geometric formulates the geometric formulates and a station of the station tionship between the graph representations of the recipiocal from and living diagrams in gr using linear alcubra. AGS and the extensions crubbs automatic construction of force of given from diagrams, and allow a few basic modifications of the force diagram from wh diagram is updated. This paper builds on the previous work of AGS by implementing a directional workflow allowing users to impose various constraints, and perform geometri tions in either the hour or fince diagram from which the other is automatically updates iterative accountric volver. The presented implementation of interactive. AGS provides a retaional back-end to hamest the advantages of teallifold graphic statics for garmeny-to

and design of structures introduction

Recent research has demonstrated how the principles of graphic statics can be combined tanional tools to create interactive drawings that provide visual feedback to the user in re-Such interactive implementations of graphic statics have not only introduced next and effect of teaching structural design [4], but this enabled advanced research [5, 6]. Despite its name elly, interactive graphic matics drawings will have some unjor drawbachs. The testimus and time-consuming process of constructing drawings in a procedural manner requires previous knowledge and experience with graphic station [7, 8]. More importantly, each drawing is representative of just one instance of a structure, meaning that topological changes to the design require a complete redraw of the form and force diagrams. Algebraic Graph Statics (AGS) introduced an algebraic method of formulating the reciprocal relationship between the form and force diagrams, which enables automatic saminestion of force diagrams from graph representations of form diagrams given by the user [9], "Di-discotional" AGS extended the medical, allowing prometric transformations of a freez diagram that result in an automatic reconfiguration of the corresponding form diagram [10]. Other methods for generating reciprocal diagrams have been presented using Airy stress functions [31] and projective geometry [12]. but these limit to self-stressed structures and add 3D polyhodral geometries into the workflow.

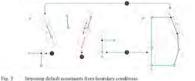
This paper presents a computational implementation and extension of previous research in AOS in an interactive design workflow. In order to create a fluid user experience while maintaining a robust back-end of solvers, various rules and constraints of graphic statics construction are explicitly defined. formulated and incorporated in an integrand computational pipeline using the CEMPAS framework (13). The examples presented in this paper damentrate how the proposed implententation can be used to maximum the inferent baseful of graphic-states-based amounted-design englacations in a amount and intuitive manner through controlled modifications; while minimizing the need for ground construction of form and force illagrams.

Conceptual Design of Structures 2011 e) Compute reciprocal force diagram G* and interit default constraints from form diagram. boundary conditions Interactively modify and add constraints to the form diagram. Interactively modify and add constraints to the force diagram Solve equilibrium by parallelisation

Interactive AGS

Form ←→ Force

the vertices in the form diagram with an externally applied load are constrained to remain on the line of action of the load. edges representing the reaction forces have their orientations fixed in both diagrams; and, edges representing the externally applied loads have their orientations fixed in both disgrams, and their lengths fixed in the force disgram.



Company Desgraf Streeters 2011

As introduced in Section 3.2, larger firecommunitodes can be imposed on the form diagram, which, in consequence, will be reflected as target lengths in the force diagram. Therefore, in order to assign equally distributed vertical leads to the arch, a target force -1.0 is applied to the leaded edges in the form diagram, or equivalently, target lengths of 1.0 to the loaded edges in the force diagram (Fig. 4b). With the default constraints from the boundary conditions, already imposed, the dual equilibrium is performed updating both form and force diagrams. The resulting form and force diagrams in Fig. 4r now shows the "correct," parabolic arch subjected to equally distributed vertical loads. Fig. 40 corresponds to one of the possible parabolic arch solutions, which depend on the magnitude of the unconstrained horizontal reactions, which in this case is equal 3.81 after the interactive parallelisation. Controlling this horizontal magnitude to after the such height will be discussed in the rext example.

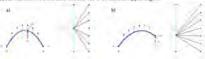
4.2 Form and force diagram modifications

The second example shows how the geometry of the arch from Fig. 4b can be modified through controlled translations of the vertices of both diagrams. After the transformations, the new diagrams are then parallelised (Section 3.3.2). Fig. 5 shows two possible manipulations on the ferce diagram. In Fig. Sa, the three vertices on the left side of the Torce diagram are dragged to decrease the magnitude of the internal forces, resulting in a taller arch. In Fig. 5b, the vertices are moved further to the right, such that the form diagram results in a geometry that corresponds to a function cable in tension.



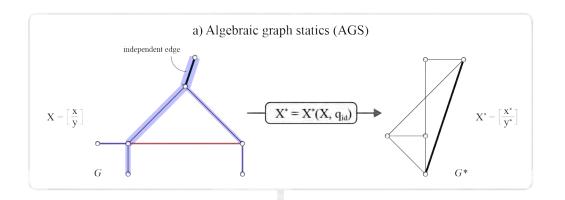
Moving the three vertices on the left side of the force diagram G* to the right, which sults in reduced internal forces and therefore a taller such in form diagram G, b) further sovement of three vertices until the forces flip from compression to tension, with form lagram (7 becoming a flancular cable

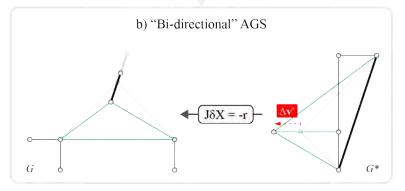
two examples of modifications in the vertices of the form diagram. In Fig. 6a. an internal such is moved up and its y exceedinate is fixed, which constrains the such to pass through his point. The target force magnitude constraints still apply, i.e., the loading case is equally distributed. After the translation of the internal vertex in the form diagram, both diagrams are updated (Fig. 84). Similarly, in Fig. 6b, the right support of the arch is moved up. After this modification, both diagrams are updated while respecting all imposed constraints, resulting in an irch with emform leads applied to it, but with uneven vertical force reactions due to the different support heights.

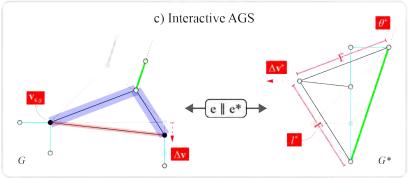


a) Update in form G and force G diagrams generated by moving, and constraining an internal vertex of the such, controlling its structural height, li) update in form U and force diagrams G* generated by moving one of the supports.

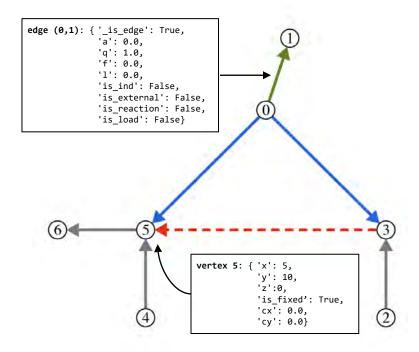




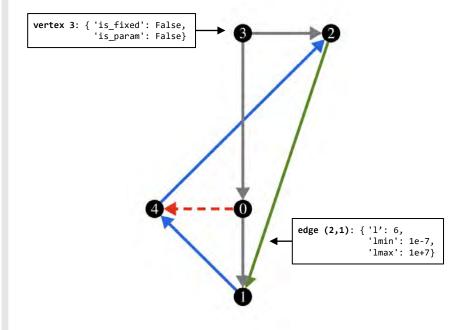




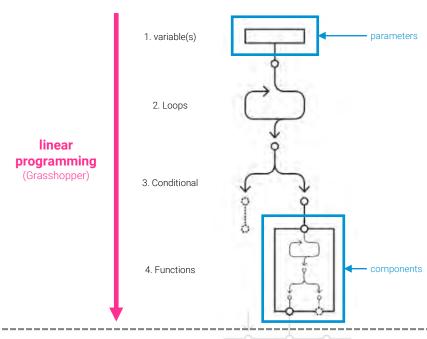
form diagram data



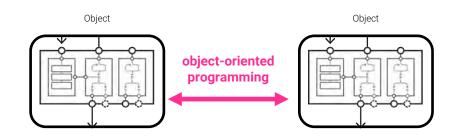
force diagram data



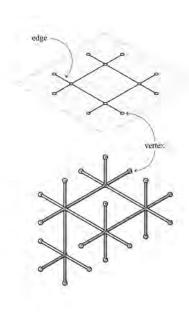
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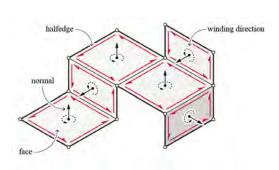


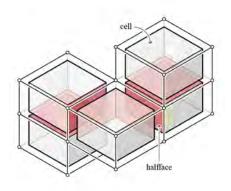
5. Object



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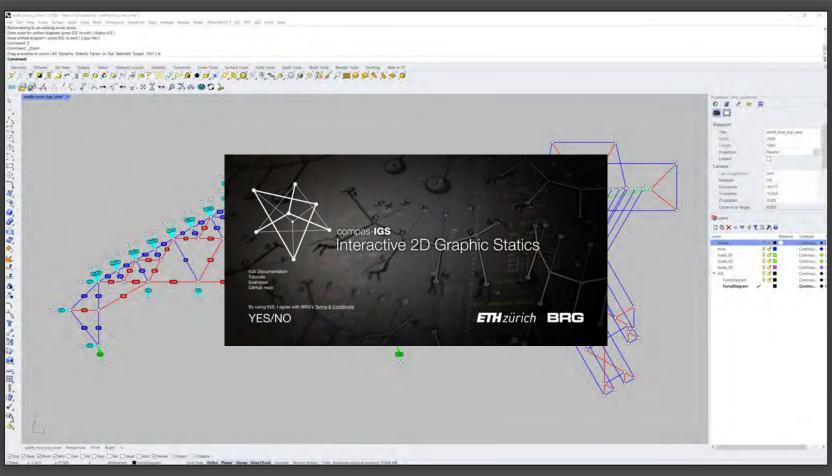


networknetwork of vertices

mesh network of faces

volmesh

network of cells

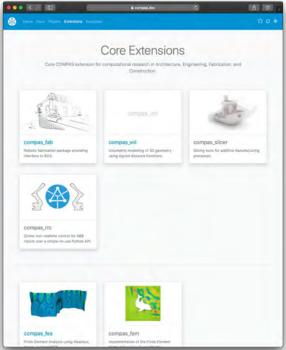


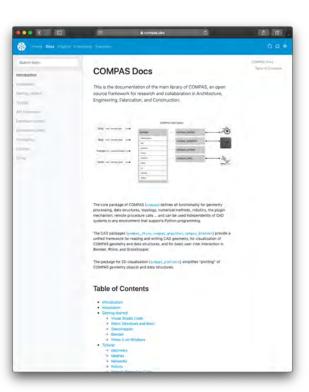


fixed

COMPAS ➤ Datastructures

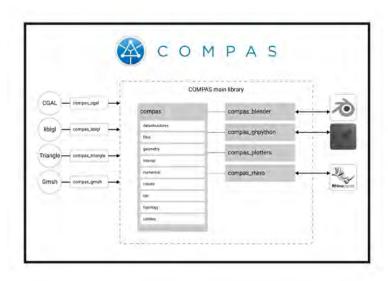


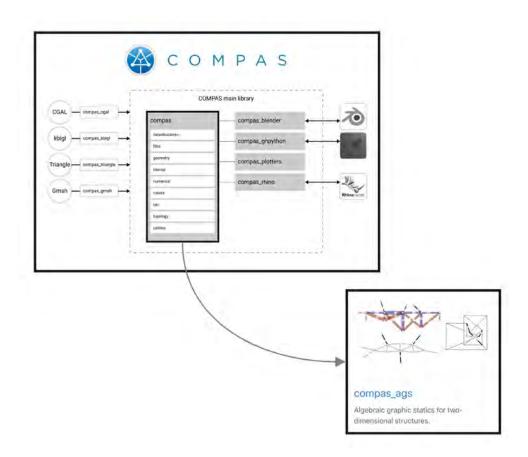




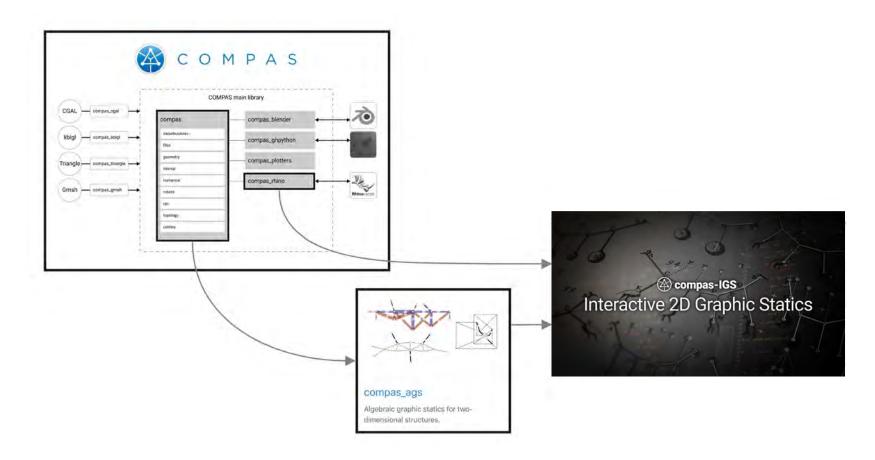
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IGS ► Overview 66



2D graphic statics

2.5D graphic statics

3D graphic statics

