063-0605-00L : Computational Structural Design 1

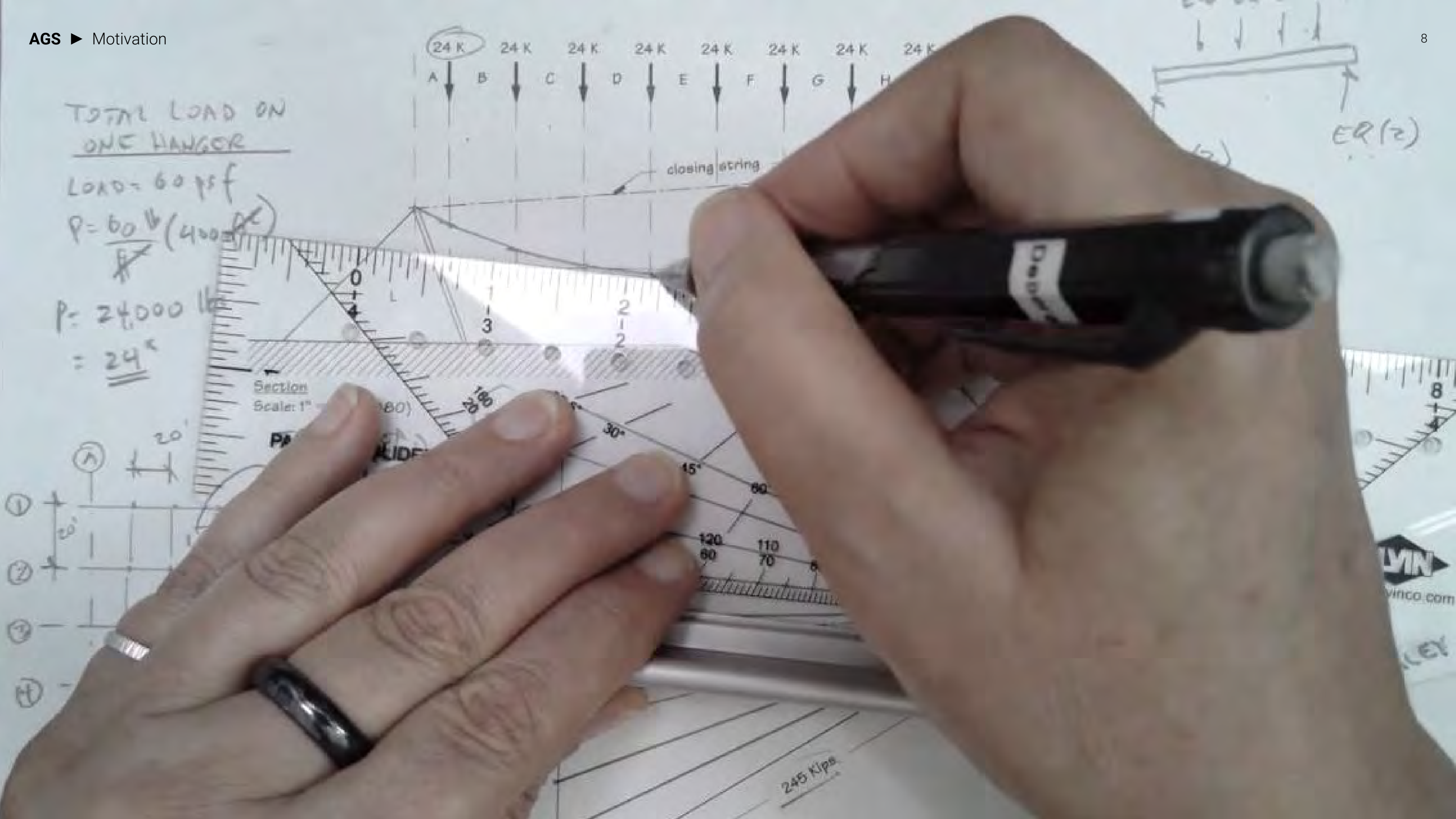
Computational Graphic Statics

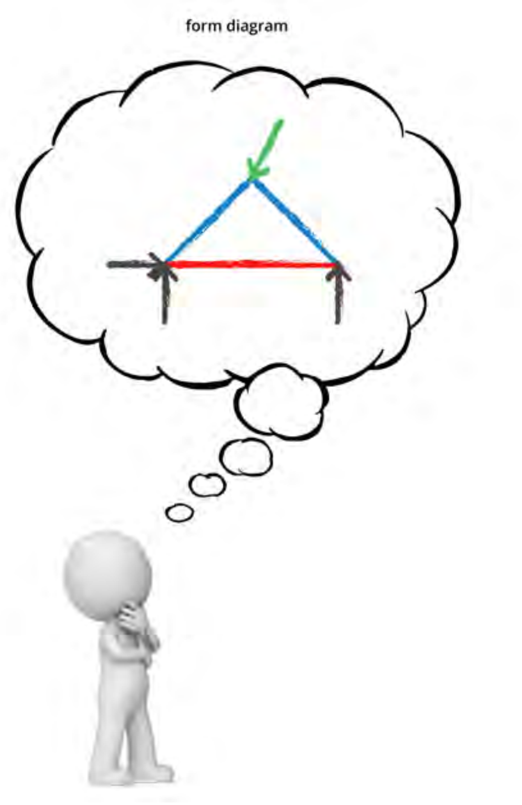
Lecture 4

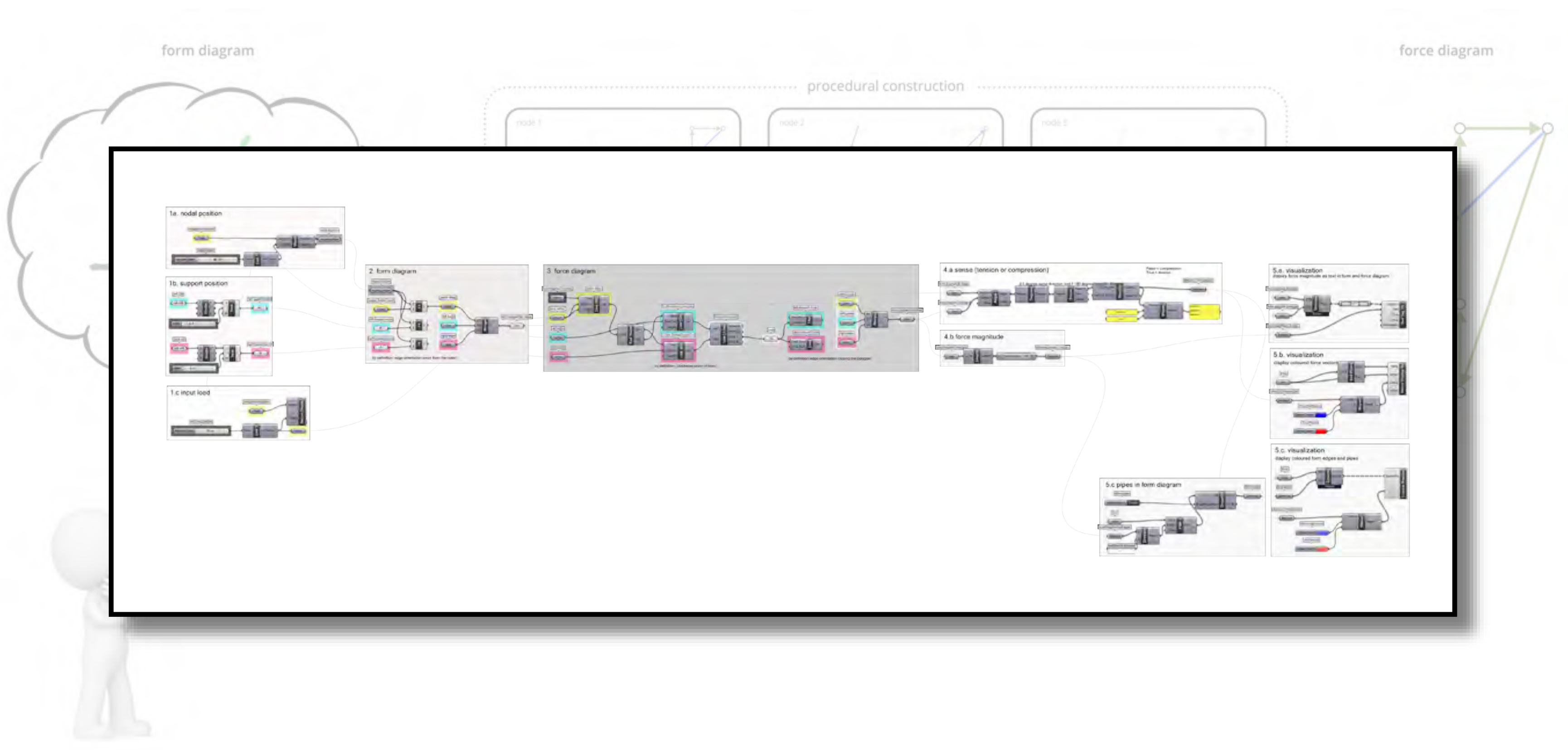
Algebraic Graphic Statics

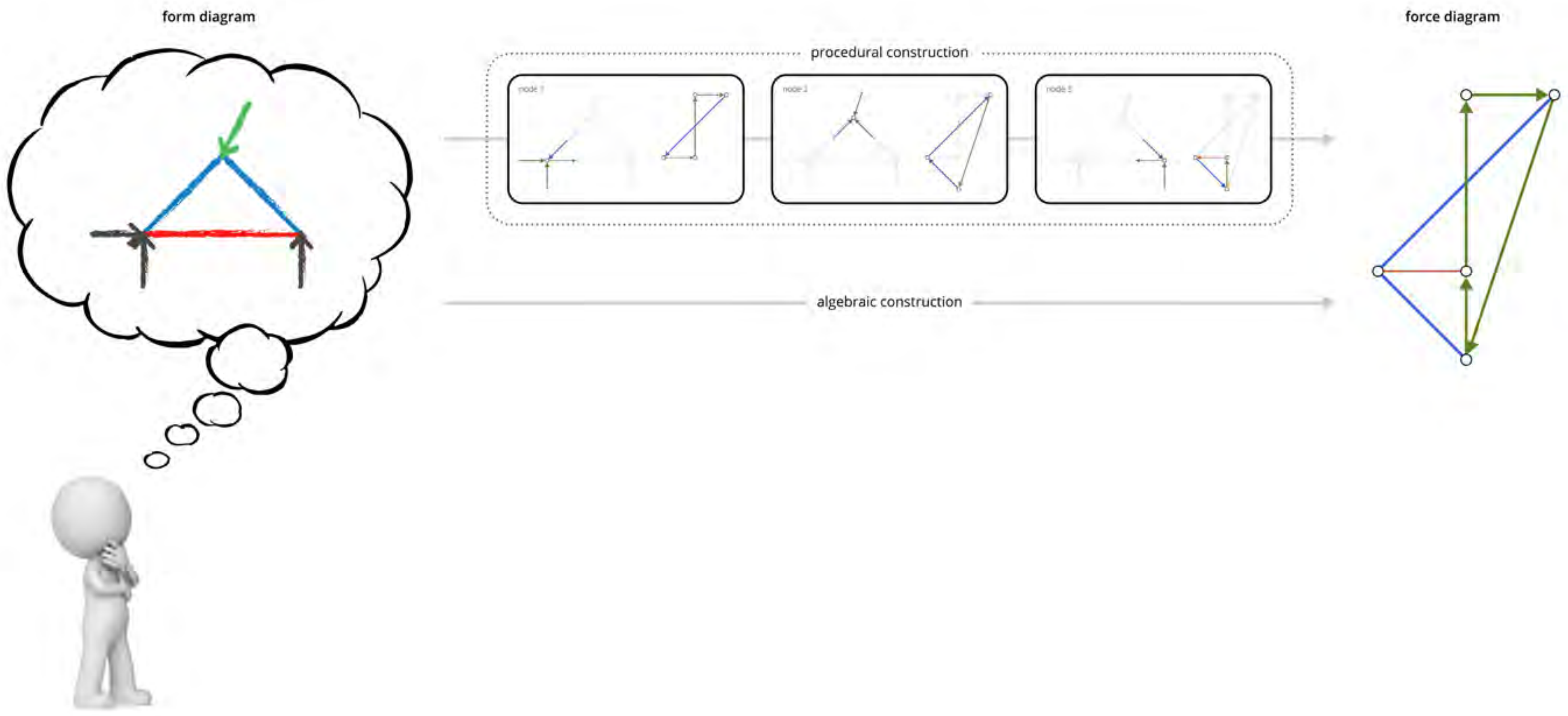
Friday, November 5th, 2021

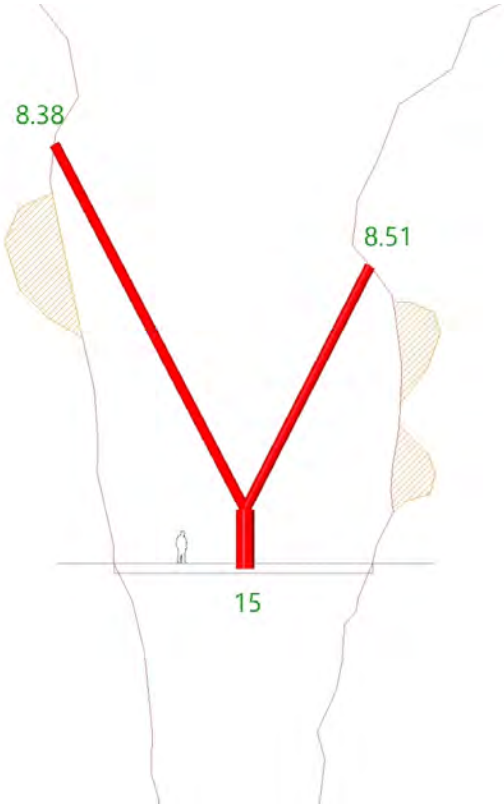
Dr. Juney Lee


24 K
A
B
C
D
E
F
G
H
closing string
TOTAL LOAD ON ONE HANGER
LOAD = 60 psf
P = 24,000 lb
= 24k
20'
Section
30°
45°
60
120
110
70
EQ (2)
245 Kips

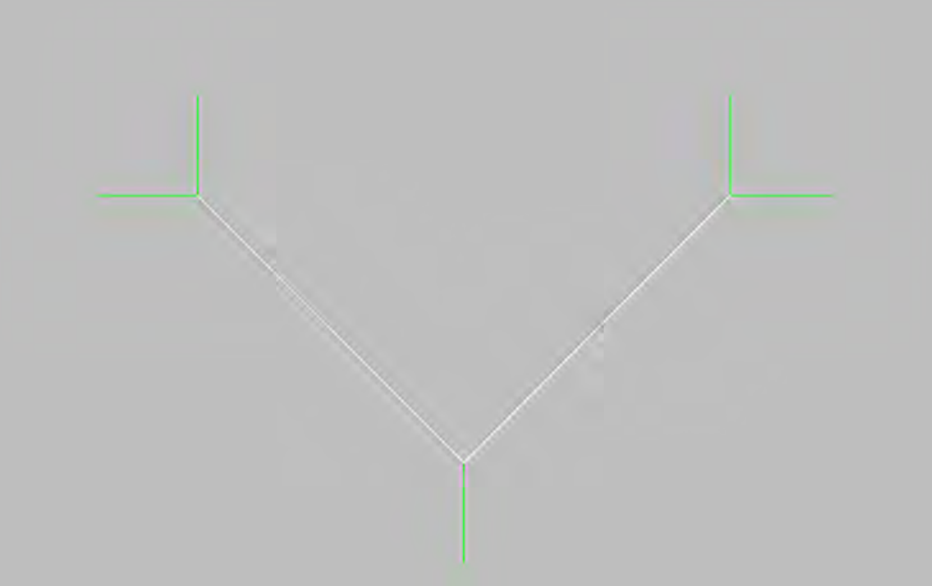

form diagram

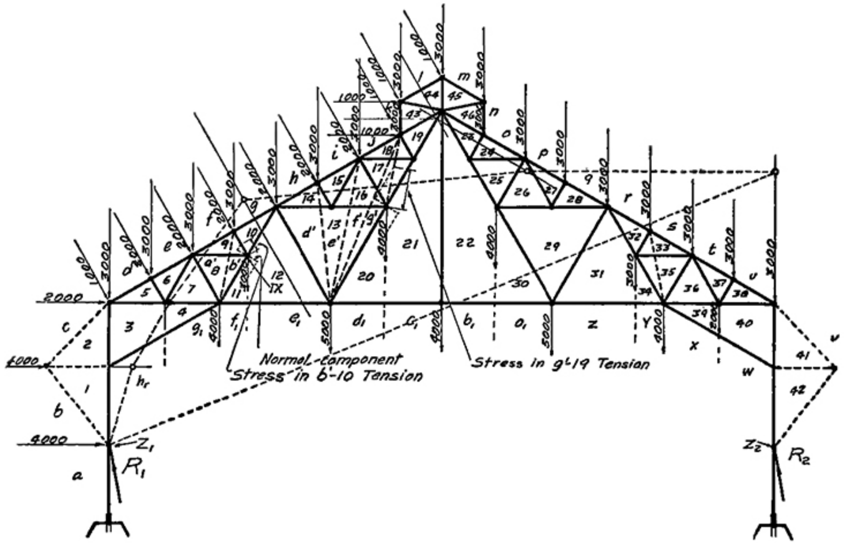

form diagram
procedural construction
force diagram
1a. nodal position
1b. support position
1.c input load
2. form diagram
3. force diagram
4.a sense (tension or compression)
4.b force magnitude
5.a. visualization
5.b. visualization
5.c. visualization
5.c pipes in form diagram

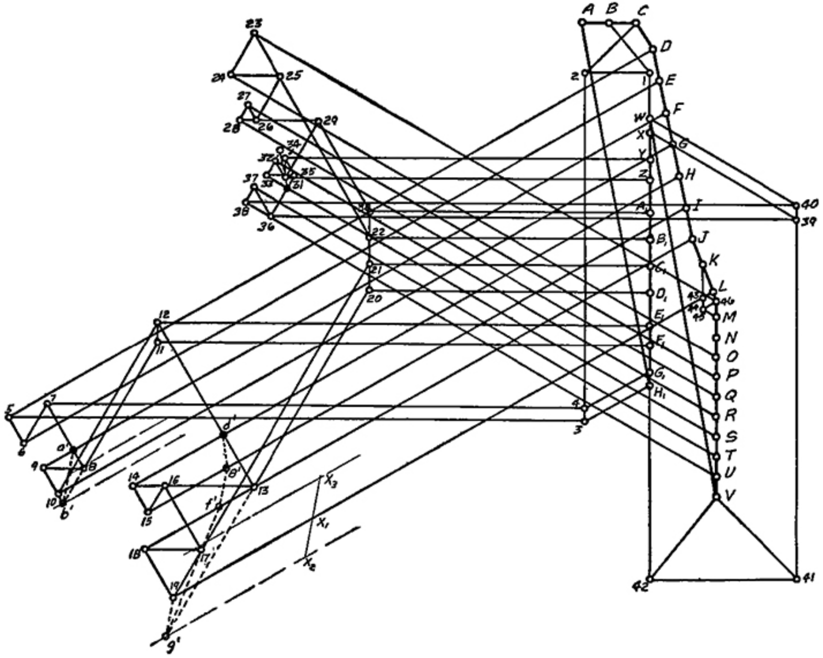

form diagram
procedural construction
algebraic construction
force diagram

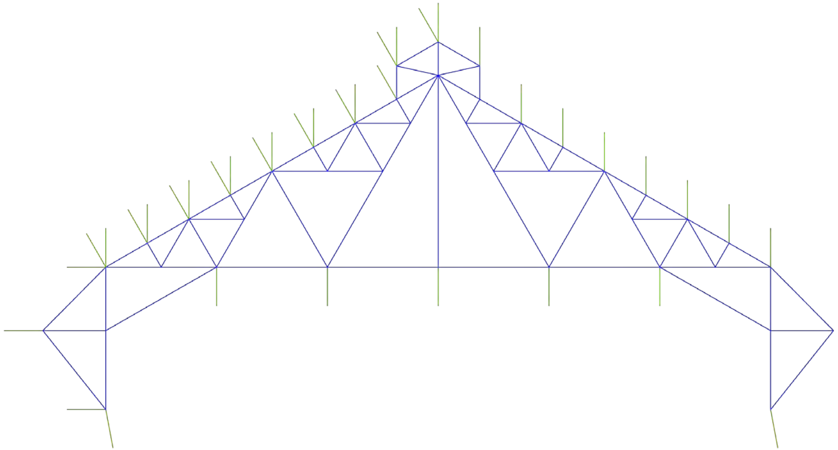

8.38
8.51
15

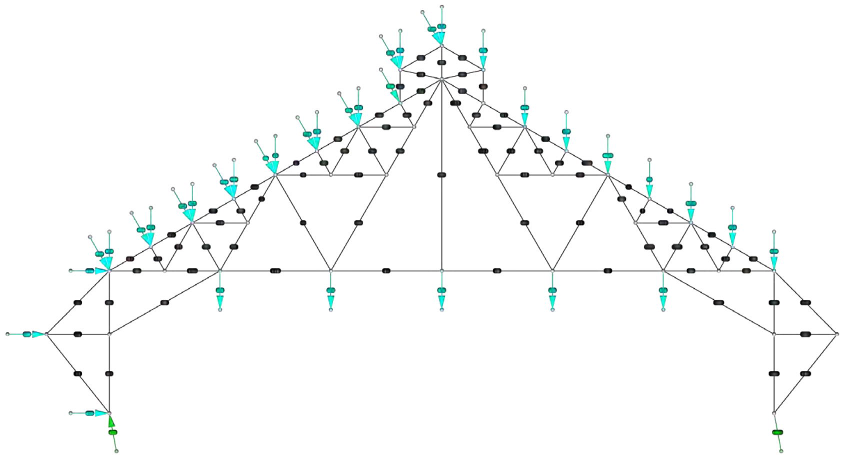

8.51
15
8.38

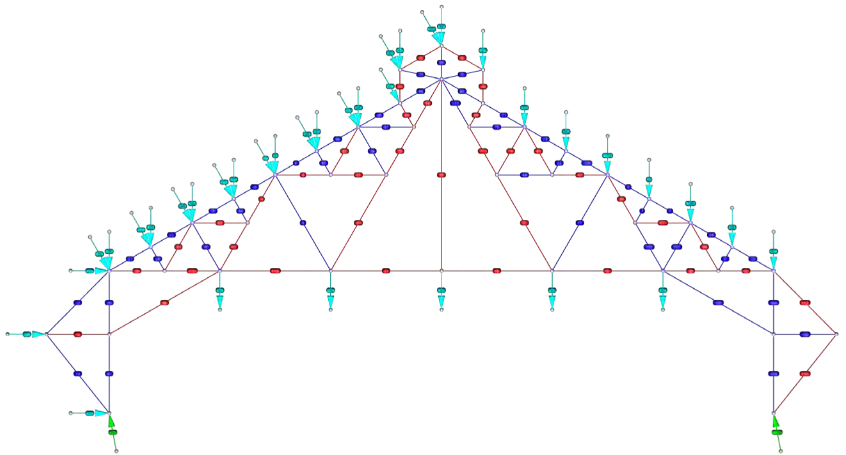


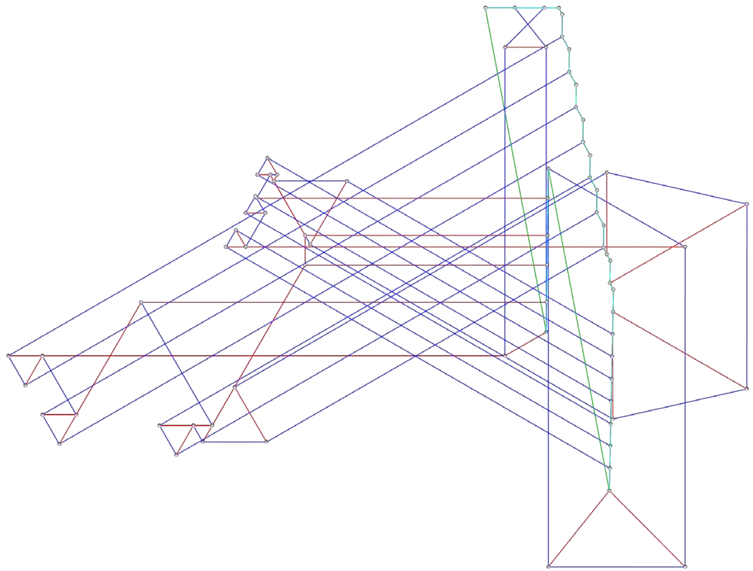

Normal Component
Stress in b-10 Tension
Stress in g'-19 Tension

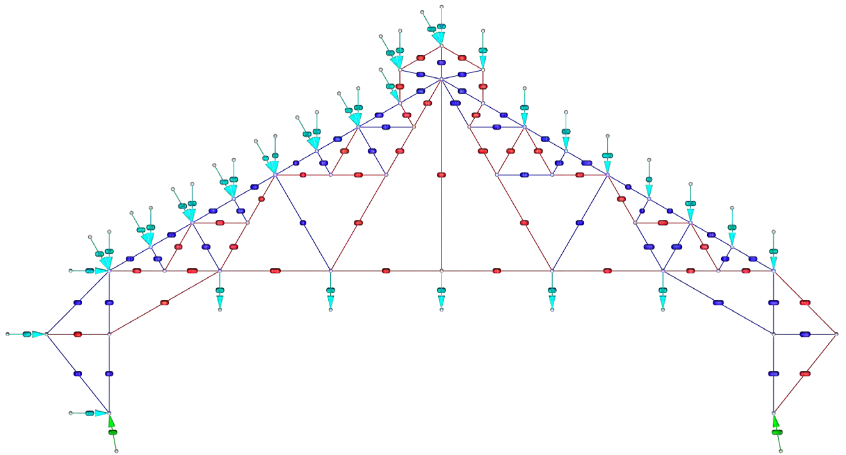


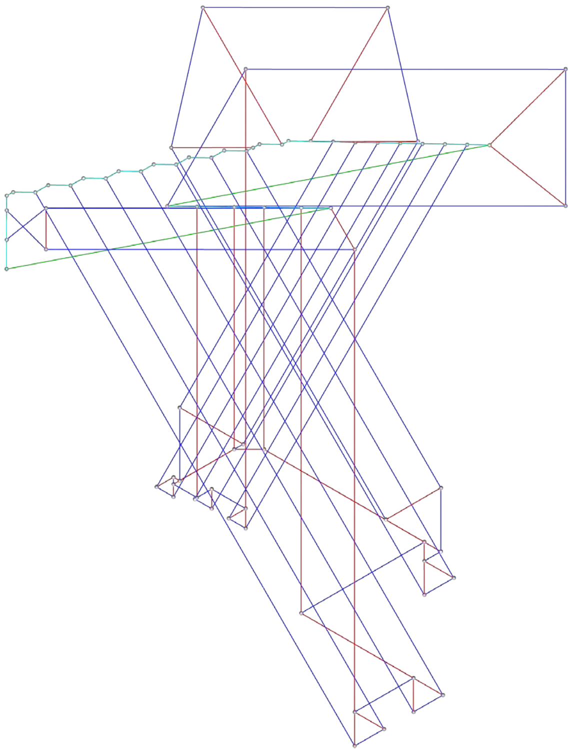


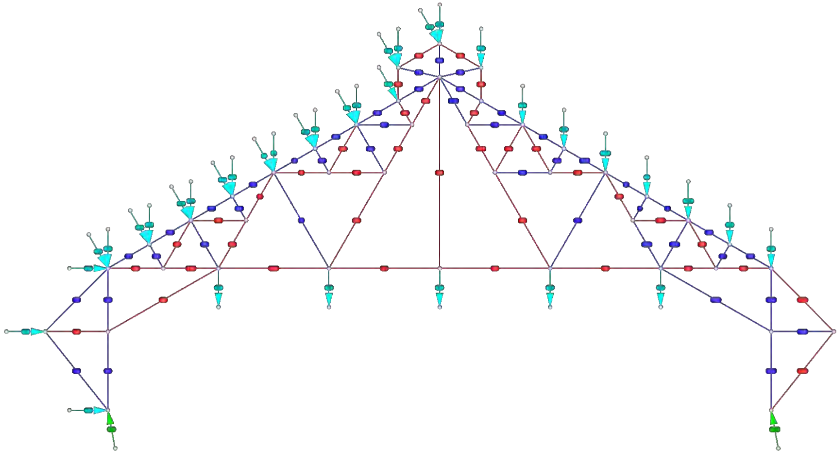


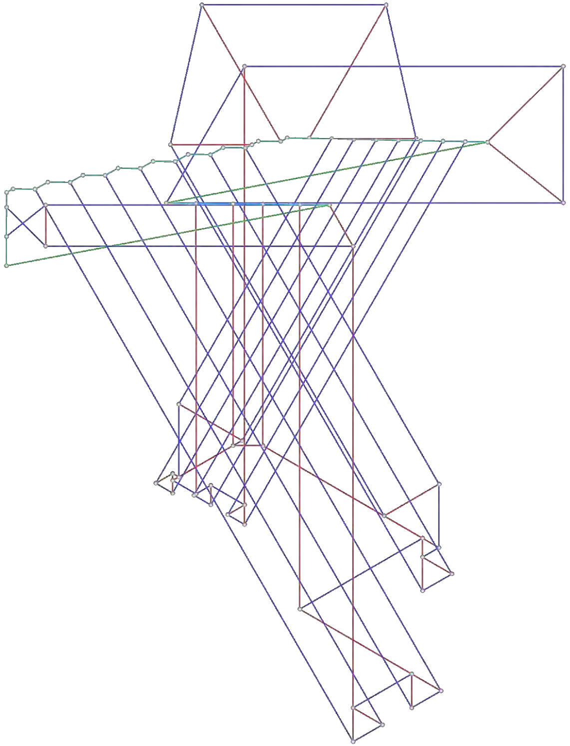


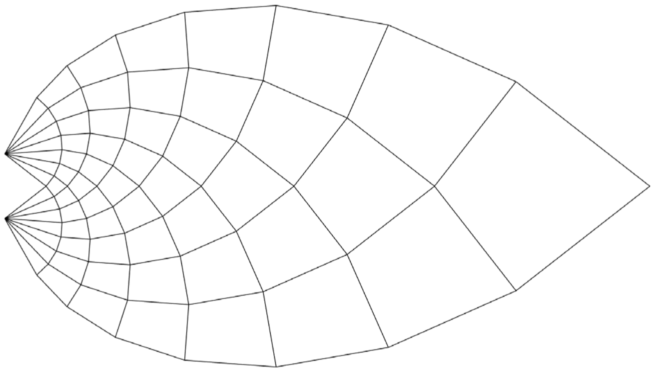


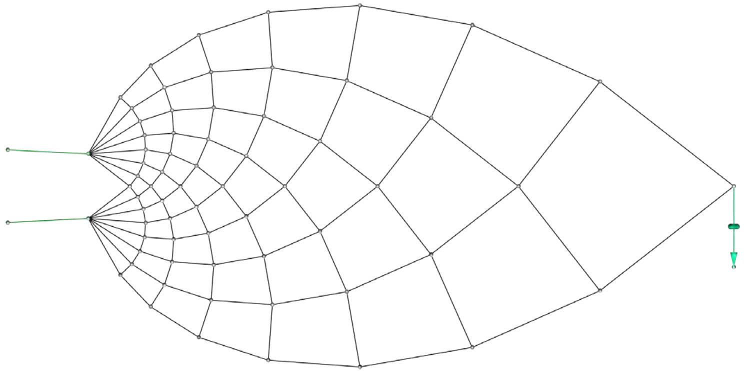


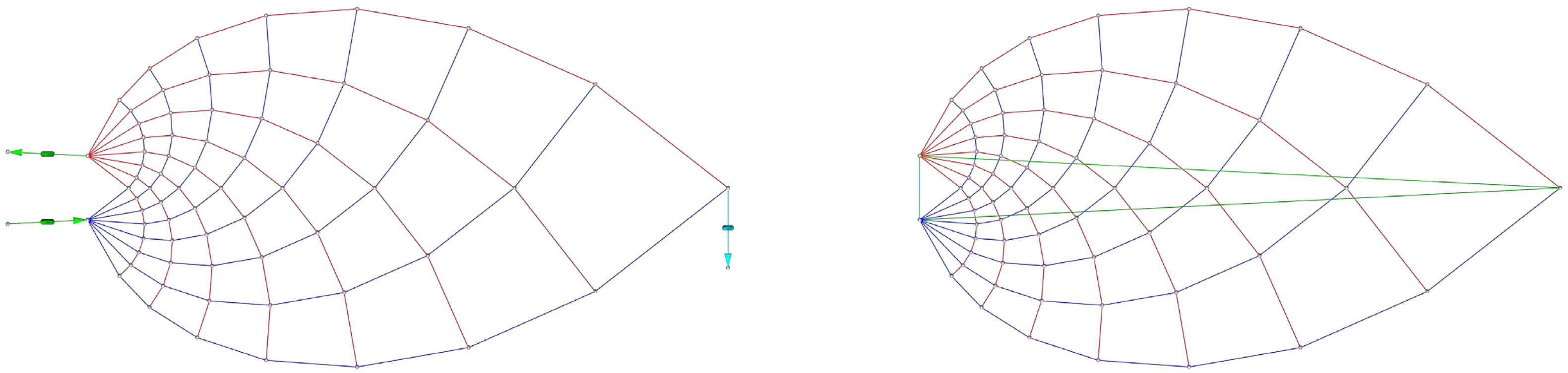


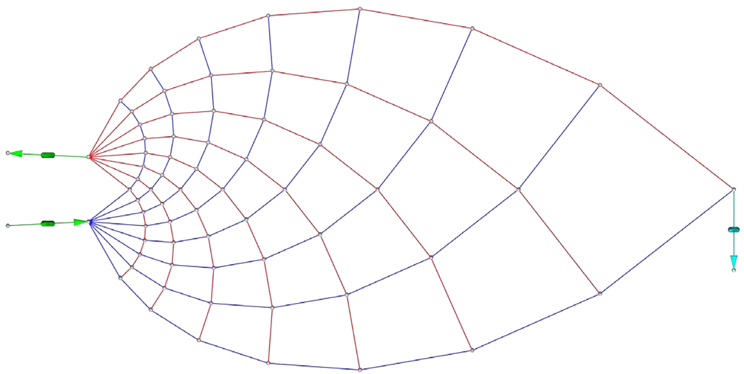


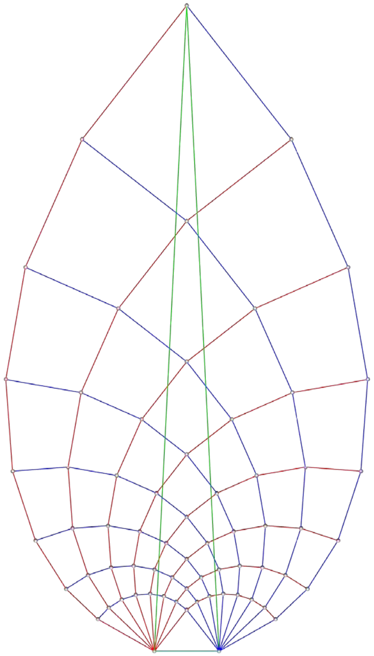


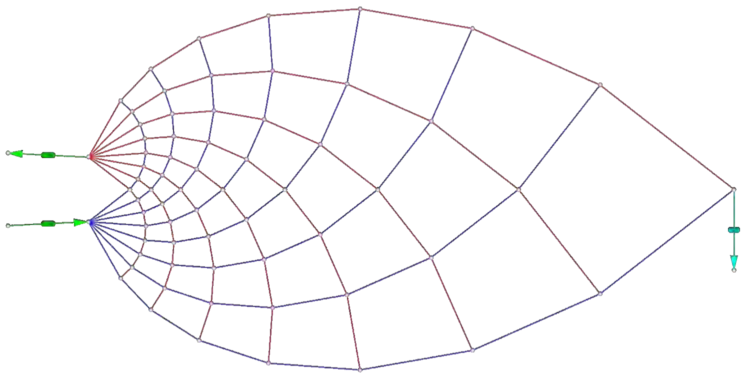


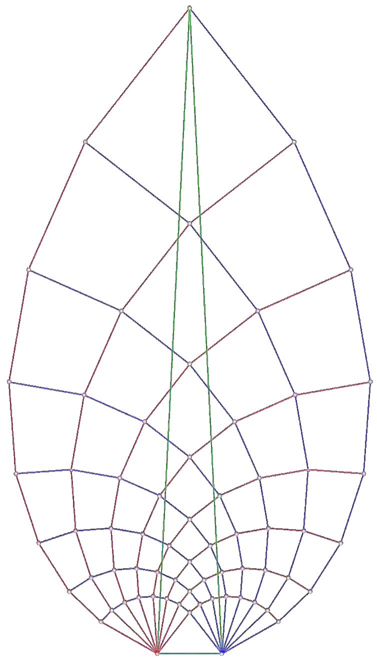


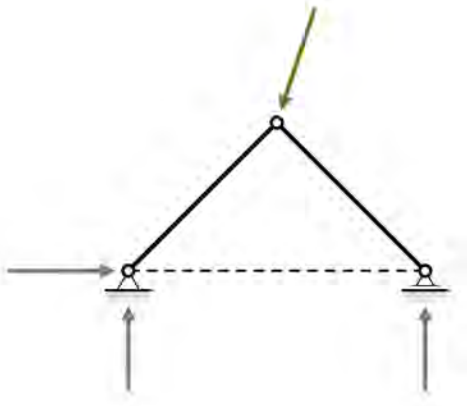




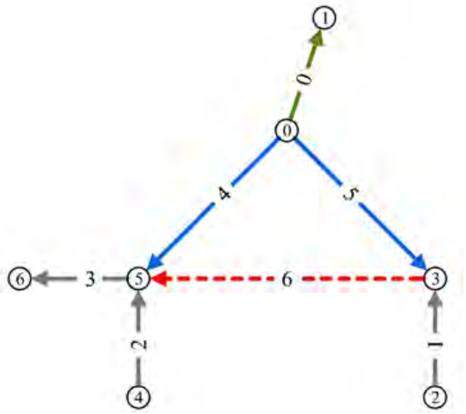




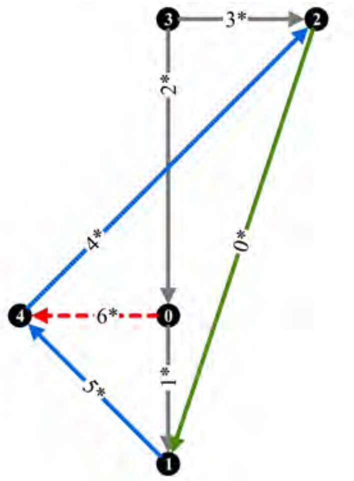




Form diagram



Form diagram as a graph



Force diagram

Get topology and connectivity

Solve via equilibrium matrix

1. Form graph

Connectivity (edges & vertices)

Directed form graph

2. Force graph

Dual graph

Directed force graph

3. Assign forces

4. Solve equilibrium matrix

5. Update force graph

6. Reciprocal form & force graphs

1. Form graph

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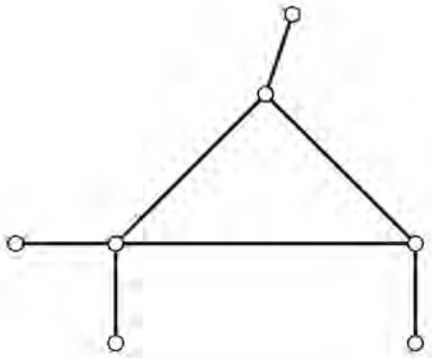

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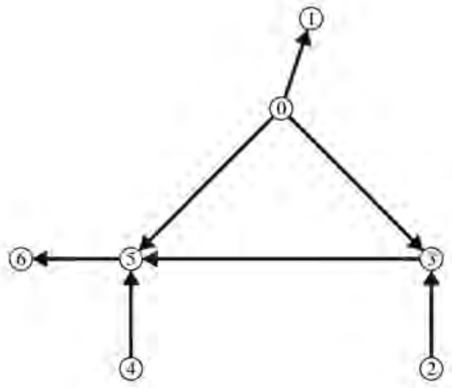

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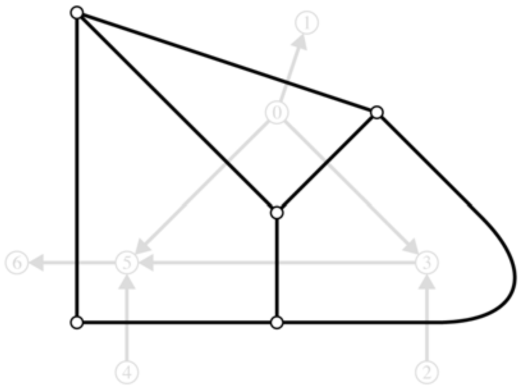

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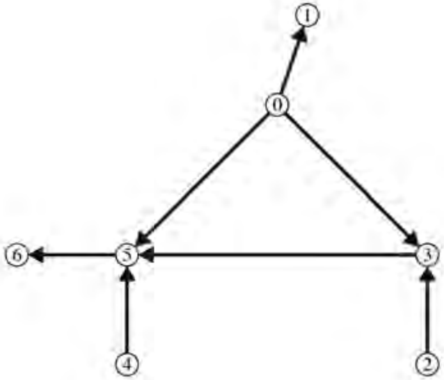

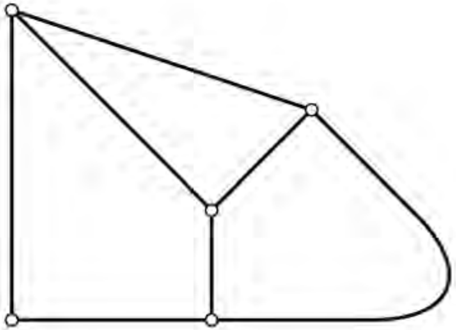

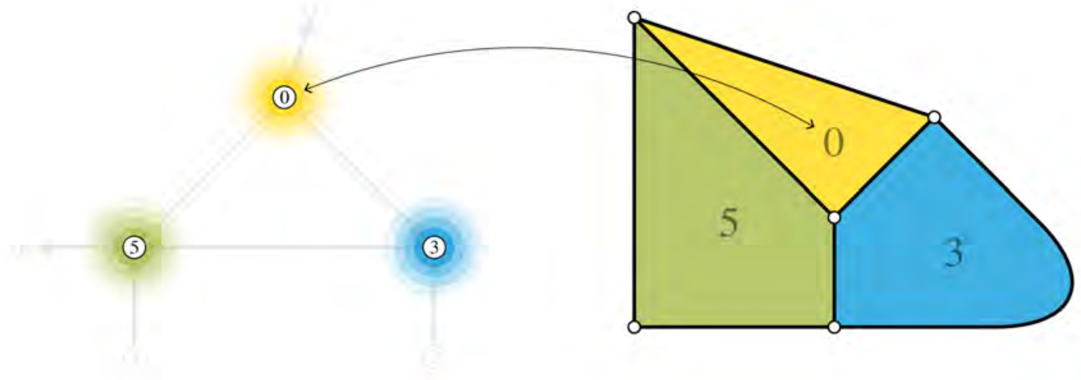
0
5
3
0
5
3

1. Form graph

Connectivity (edges & vertices)

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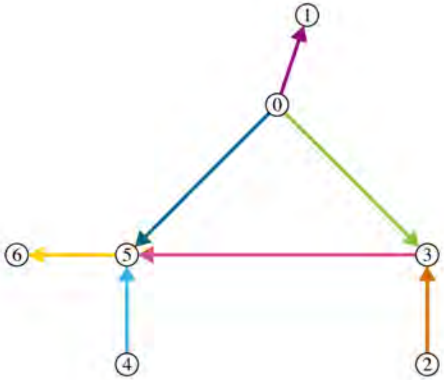

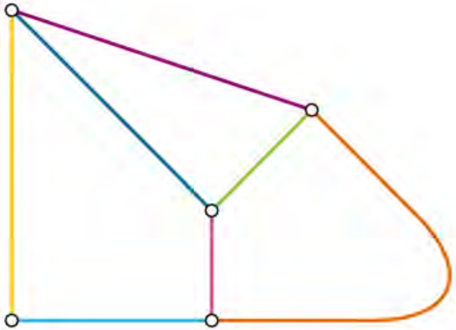

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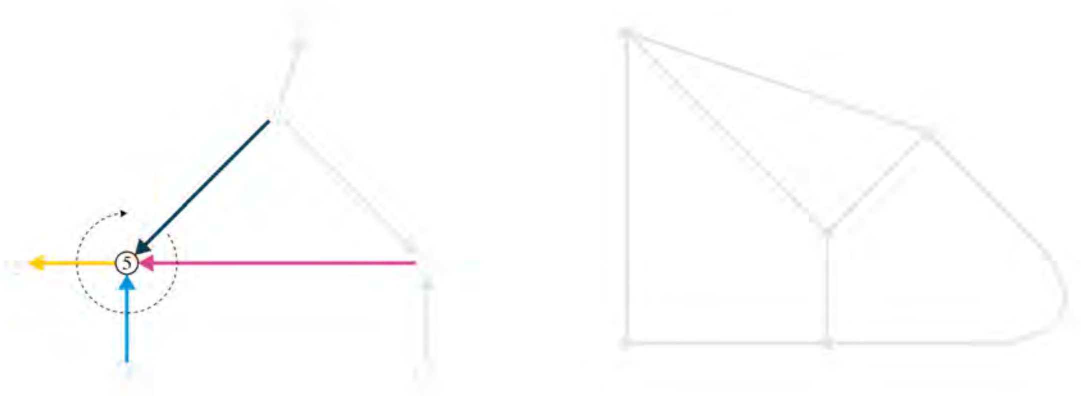

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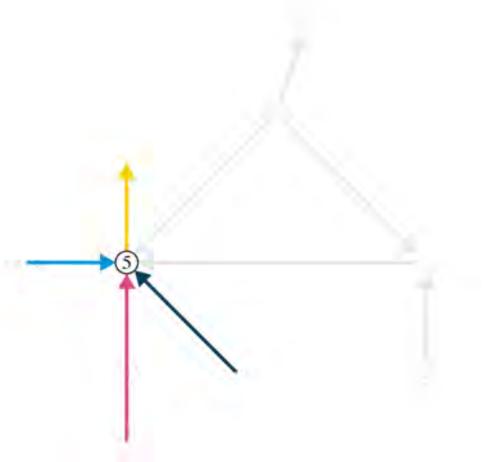

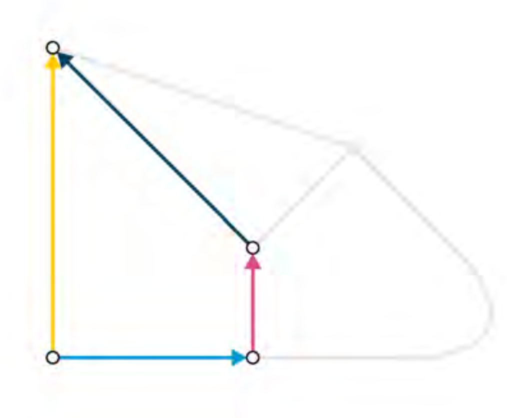

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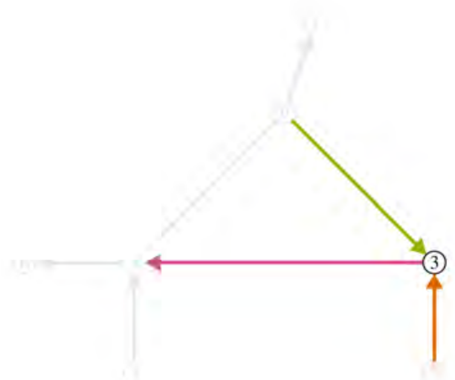
3

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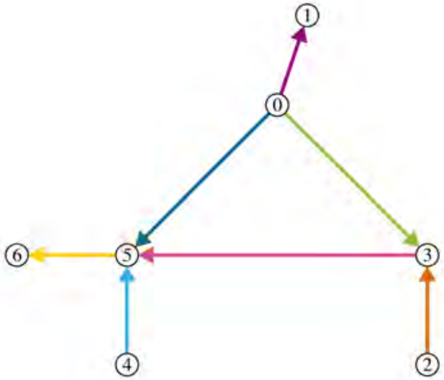

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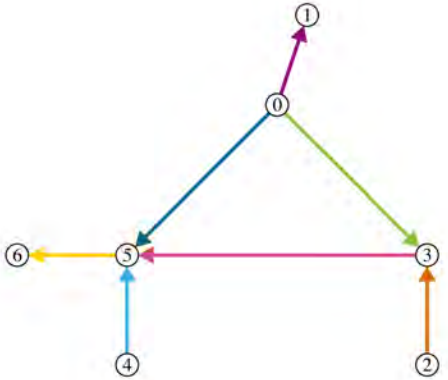

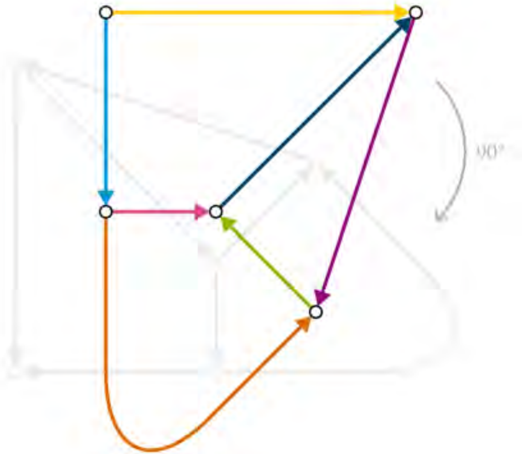

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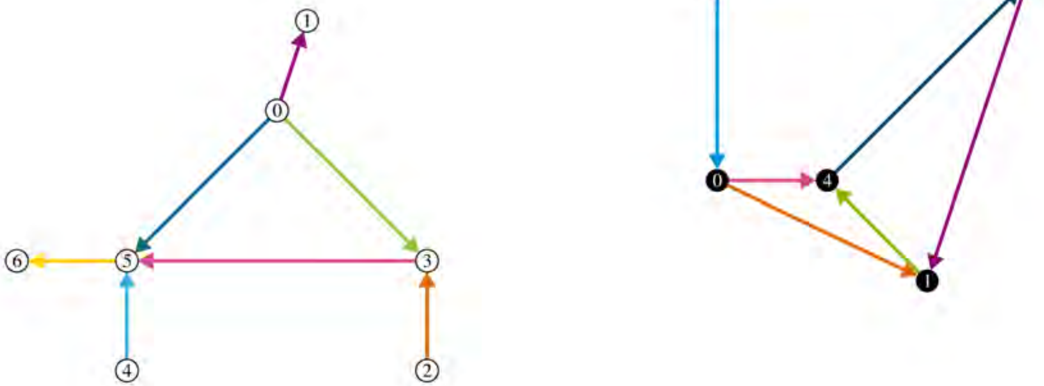

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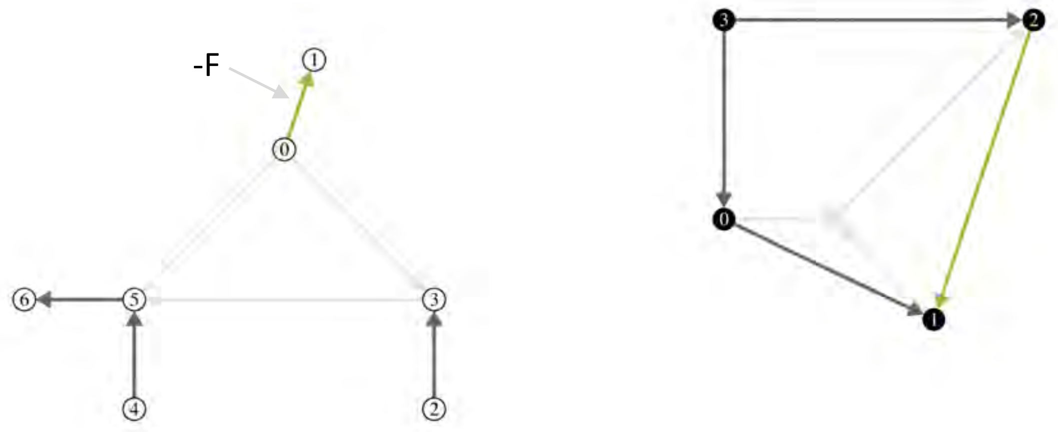
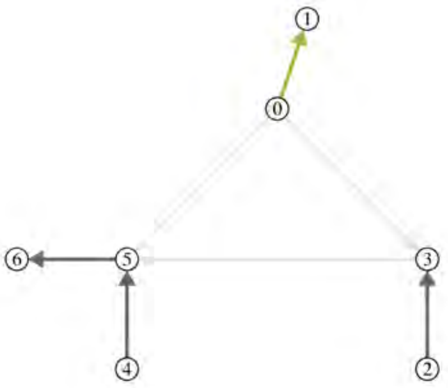
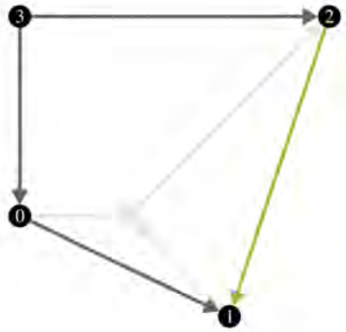


$$C_{ij} = \begin{cases} +1 \text{ if vertex } i \text{ is the head of edge } j \\ -1 \text{ if vertex } i \text{ is the tail of edge } j \\ 0 \text{ otherwise} \end{cases}$$

$$C^*_{ij} = \begin{cases} +1 \text{ if the face cycle traverses} \\ \quad \text{edge } j \text{ in the same direction} \\ \quad \text{as its orientation} \\ -1 \text{ if in the opposite direction} \\ 0 \text{ otherwise} \end{cases}$$





Closed polygon constraint $\begin{cases} \mathbf{C}_\text{i}\mathbf{u}^* = 0 \\ \mathbf{C}_\text{i}\mathbf{v}^* = 0 \end{cases}$

Parallel constraint $\begin{cases} \mathbf{u}^* = \mathbf{Q}\mathbf{u} \\ \mathbf{v}^* = \mathbf{Q}\mathbf{v} \end{cases}$

$$\begin{cases} \mathbf{C}_\text{i}\mathbf{U}\mathbf{q} = 0 \\ \mathbf{C}_\text{i}\mathbf{V}\mathbf{q} = 0 \end{cases}$$

equilibrium matrix

$$\mathbf{A}\mathbf{q} = 0 \quad , \quad \mathbf{A} = \begin{bmatrix} \mathbf{C}_\text{i}\mathbf{U} \\ \hline \mathbf{C}_\text{i}\mathbf{V} \end{bmatrix}$$

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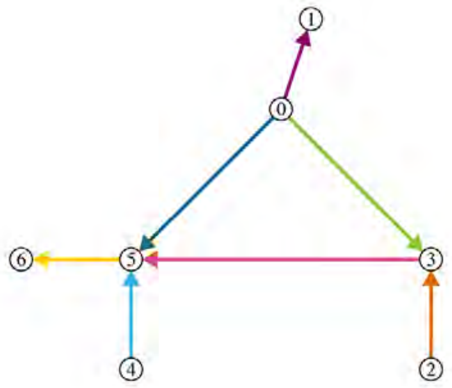

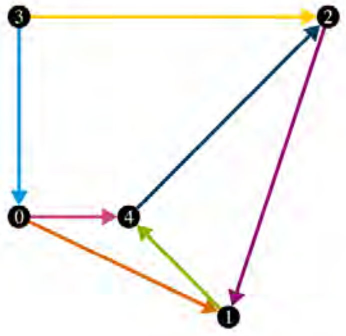

$$\mathbf{A} = \left[\ \mathbf{A}_{\mathrm{d}} \mid \mathbf{A}_{\mathrm{id}}\ \right] \quad , \quad \mathbf{q} = \left[\frac{\mathbf{q}_{\mathrm{d}}}{\mathbf{q}_{\mathrm{id}}} \right]$$

$$\mathbf{A}_{\mathrm{d}}\mathbf{q}_{\mathrm{d}} + \mathbf{A}_{\mathrm{id}}\mathbf{q}_{\mathrm{id}} = 0$$

$$\mathbf{q}_{\mathrm{d}} = -\mathbf{A}_{\mathrm{d}}^{-1}\mathbf{A}_{\mathrm{id}}\mathbf{q}_{\mathrm{id}}$$

$$\mathbf{q} = \left[\frac{-\mathbf{A}_{\mathrm{d}}^{-1}\mathbf{A}_{\mathrm{id}}}{\mathbf{I}_k} \right] \mathbf{q}_{\mathrm{id}}$$

$$\mathbf{x}^* = (\mathbf{C}^{*t})^{\dagger}\mathbf{Q}\mathbf{u}$$

$$\mathbf{y}^* = (\mathbf{C}^{*t})^{\dagger}\mathbf{Q}\mathbf{v}$$

assigned loads/forces

all force densities

new force diagram coordinates

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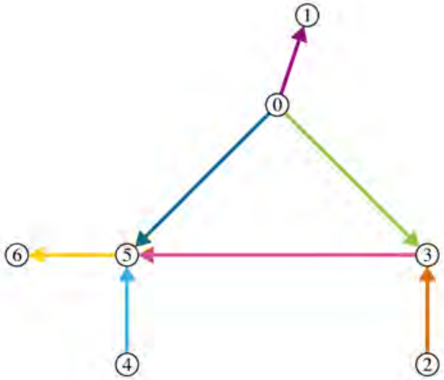

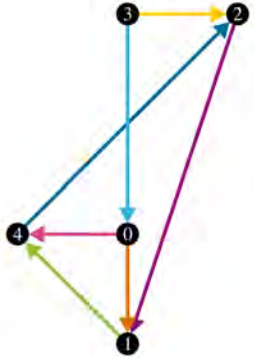

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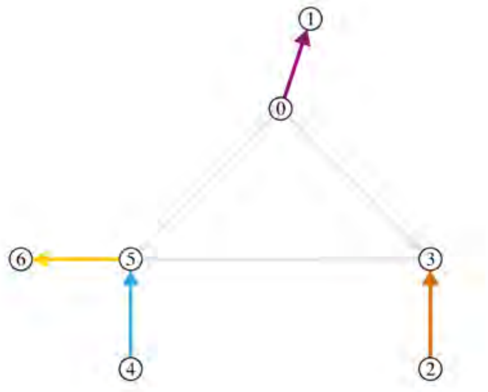

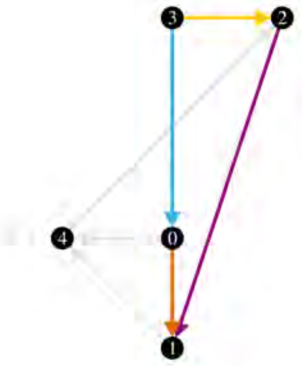

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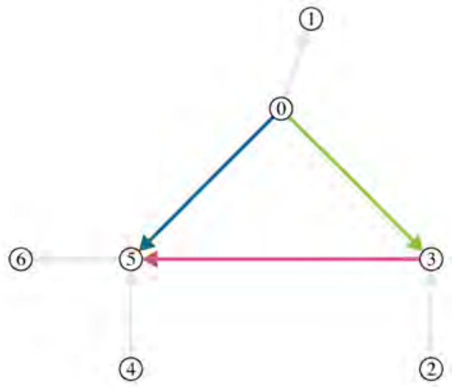

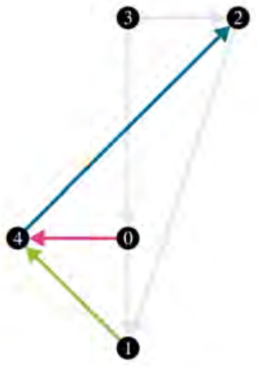

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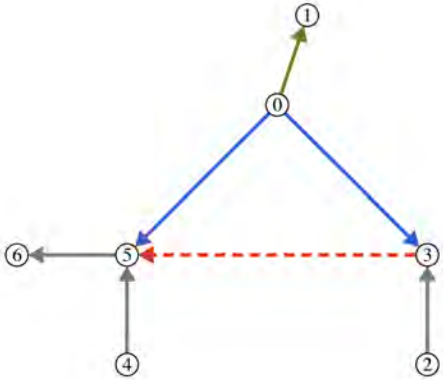

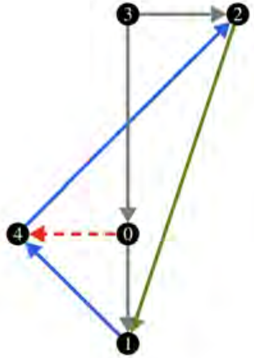

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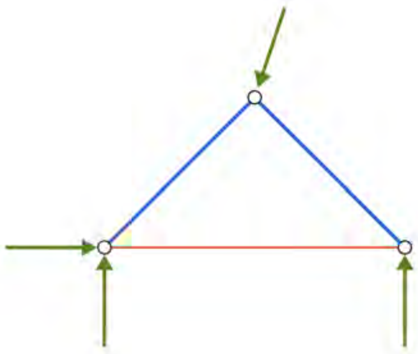
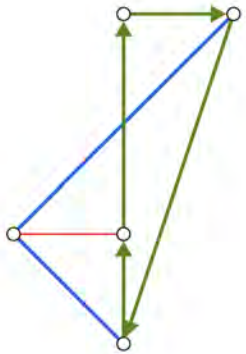
6. Reciprocal form & force graphs





1. Form graph

Connectivity (edges & vertices)

Directed form graph

2. Force graph

Dual graph

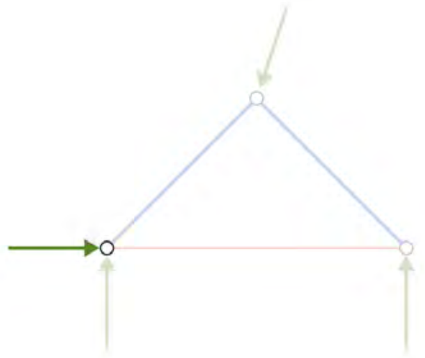
Directed force graph

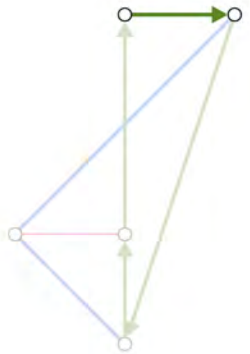
3. Assign forces

4. Solve equilibrium matrix

5. Update force graph

6. Reciprocal form & force graphs





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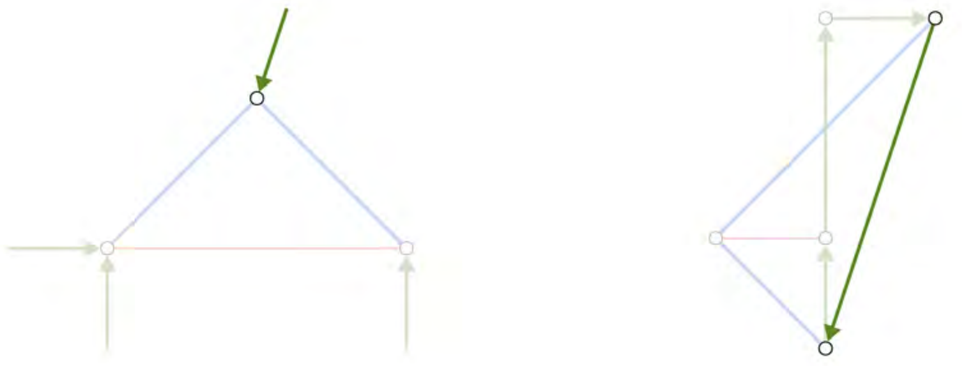

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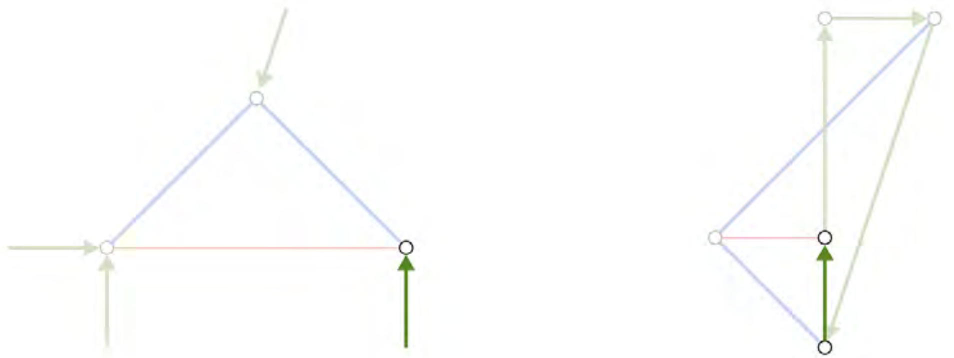



Computer-Aided Design

Algebraic graph statics

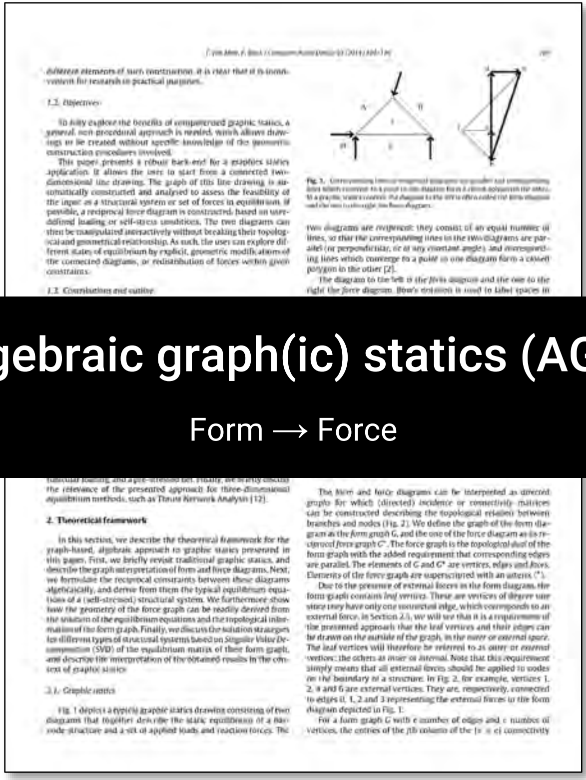
T. Van Mele, P. Block

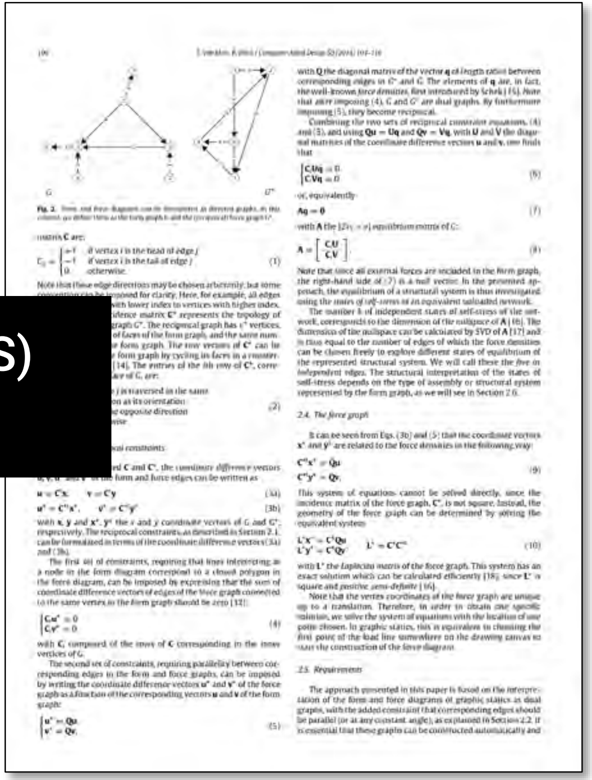
HIGHLIGHTS

ARTICLE INFO

ABSTRACT

1. Introduction


2. Theoretical framework

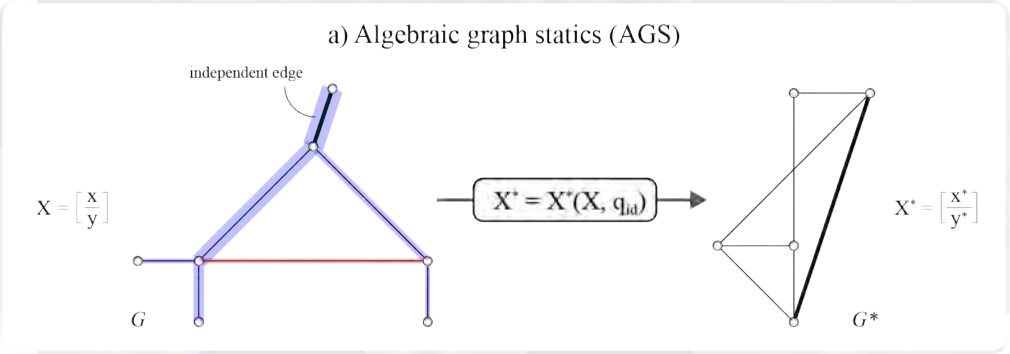

2.4. The force graph

2.5. Requirements

Algebraic graph(ic) statics (AGS)

Form ⟶ Force

“Algebraic Graph Statics” | Van Mele and Block (2014)

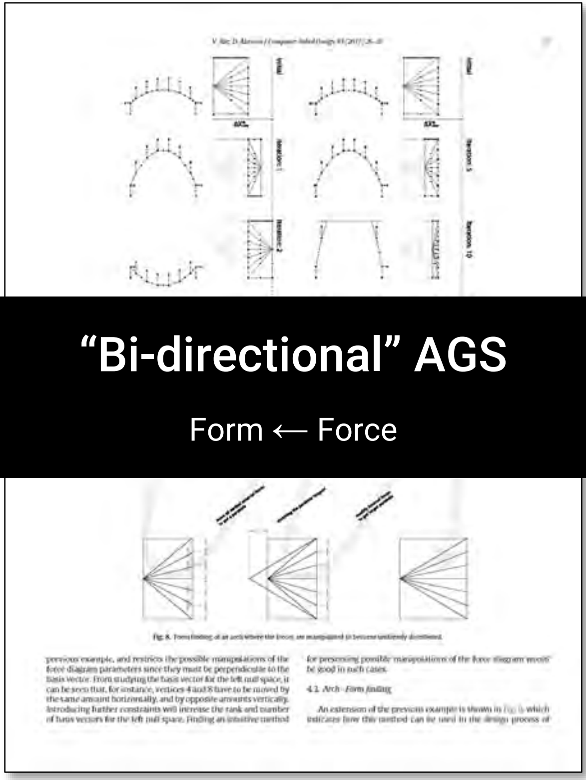

a) Algebraic graph statics (AGS)
independent edge
X = [x / y]
G
X* = X*(X, q_id)
G*
X* = [x* / y*]

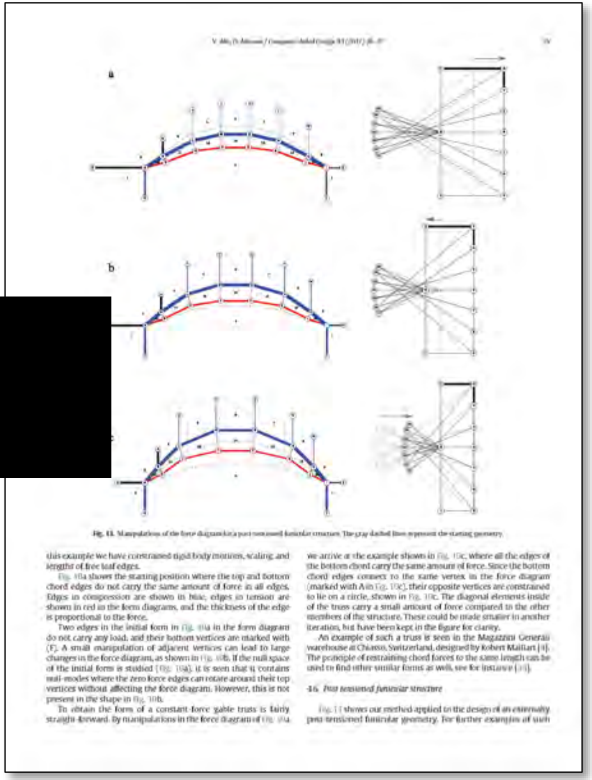
Computer-Aided Design

Bi-directional algebraic graphic statics

Vedad Alic *, Daniel Åkesson

1. Introduction

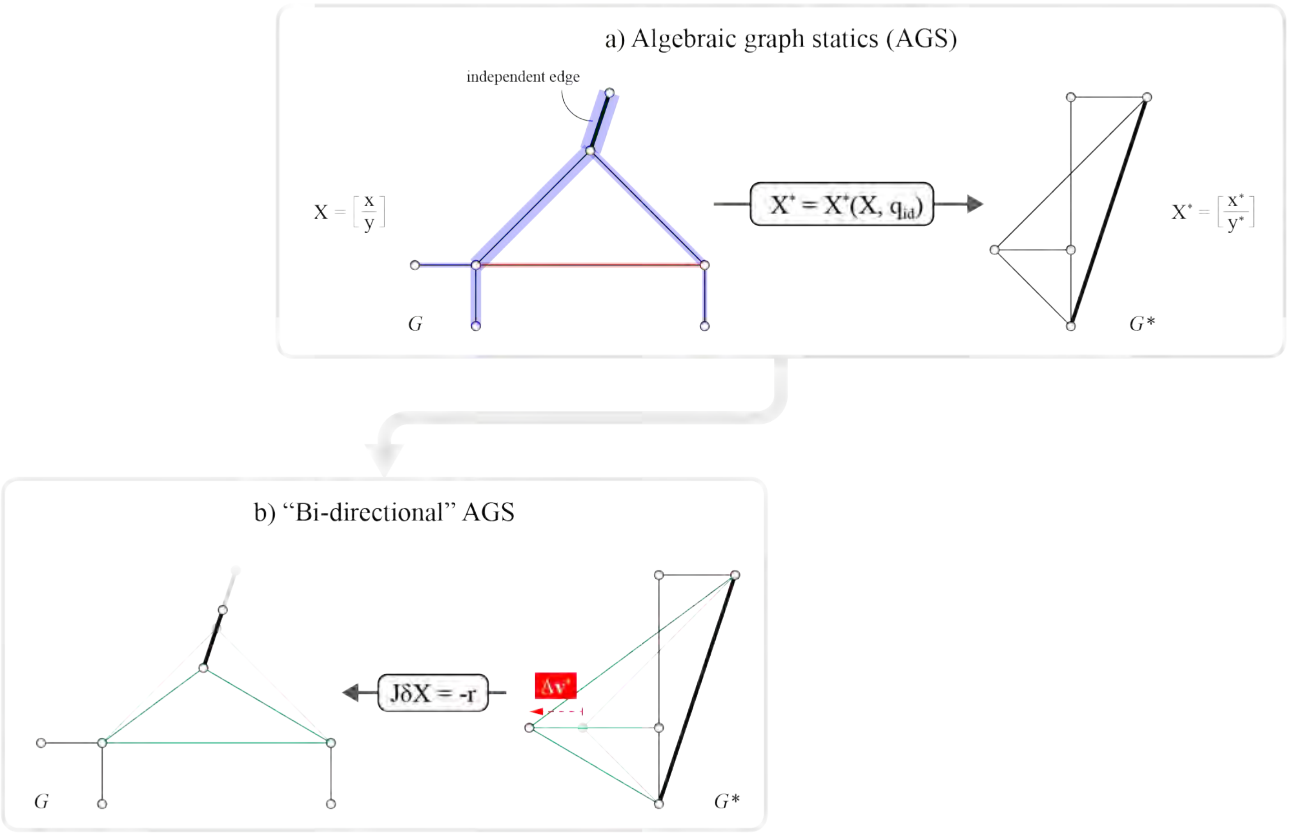


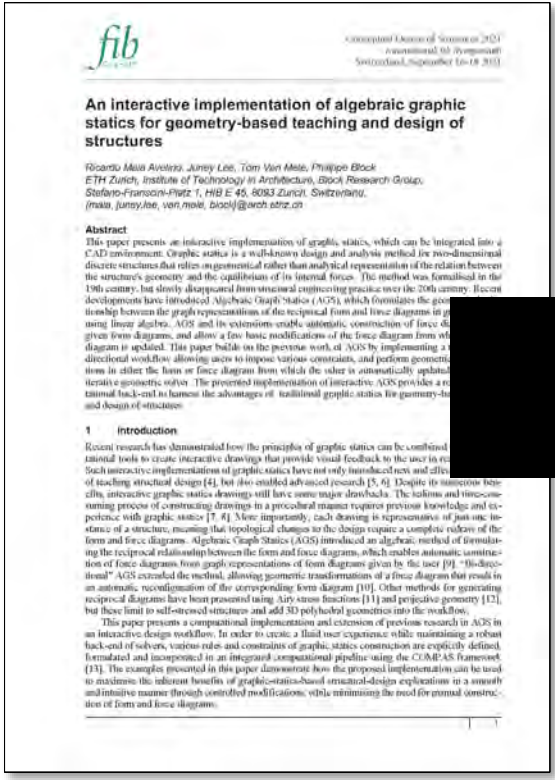


“Bi-directional” AGS

Form ⟵ Force

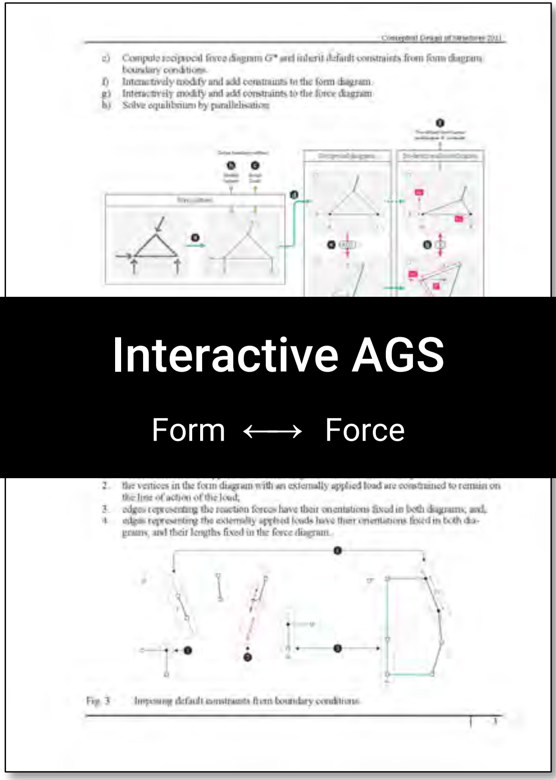
“Bi-directional algebraic graphic statics” | Alic and Åkesson (2017)

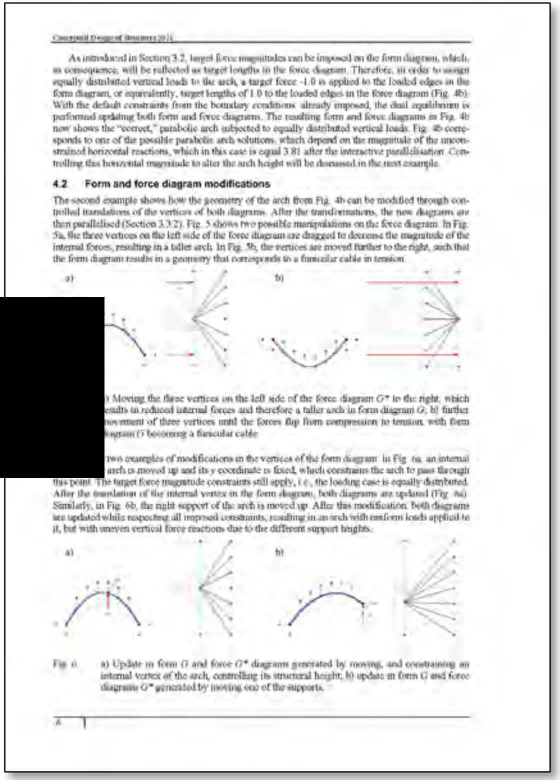

a) Algebraic graph statics (AGS)
independent edge
X = [x y]
X* = X*(X, q_id)
X* = [x* y*]
G
G*
b) “Bi-directional” AGS
JδX = -r
Δv*
G
G*



An interactive implementation of algebraic graphic statics for geometry-based teaching and design of structures

Ricardo Maia Avelino, Juney Lee, Tom Van Mele, Philippe Block
ETH Zurich, Institute of Technology in Architecture, Block Research Group,
Stefano-Franscini-Platz 1, HIB E 45, 8093 Zurich, Switzerland

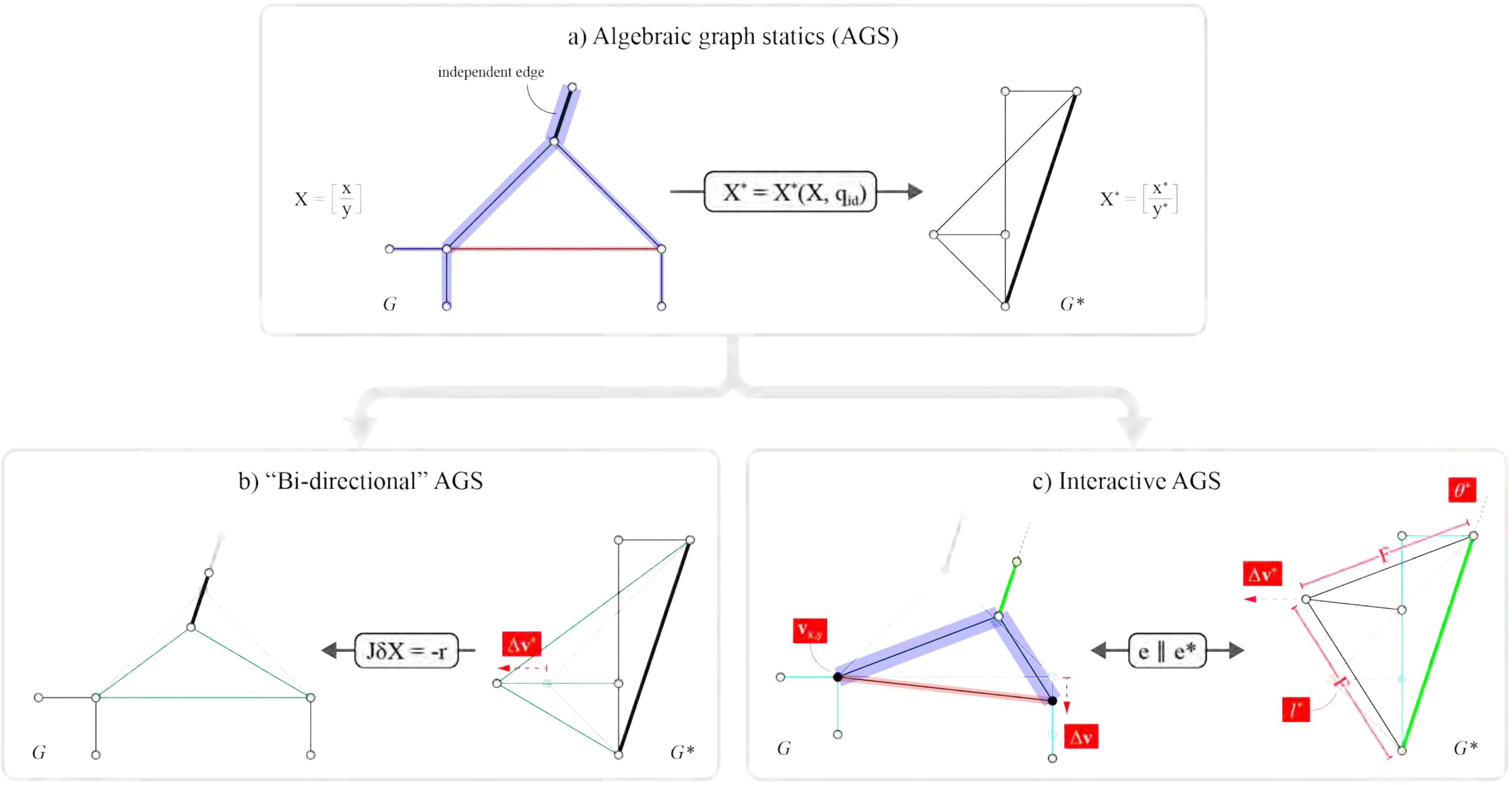




Interactive AGS

Form ⟷ Force

"An interactive implementation of algebraic graphic statics for geometry-based teaching and design of structures" | Avelino, Lee, Van Mele and Block (2021)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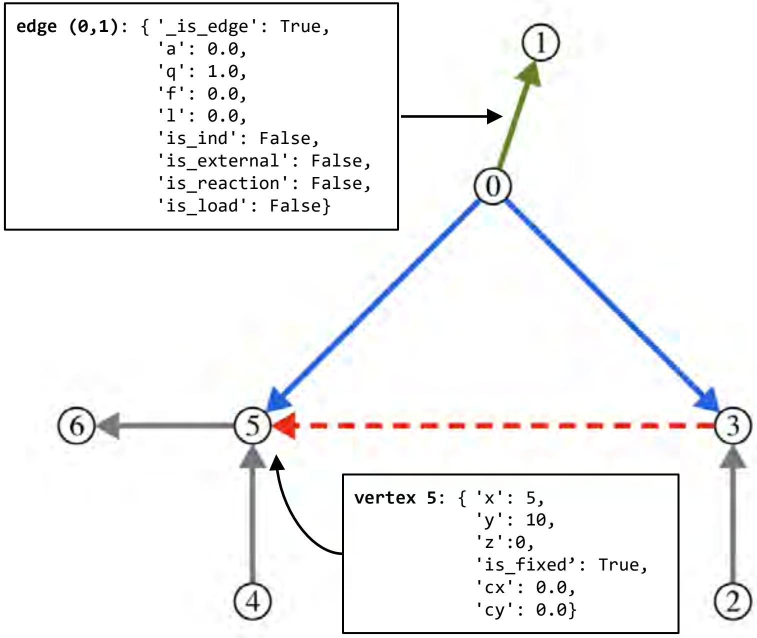
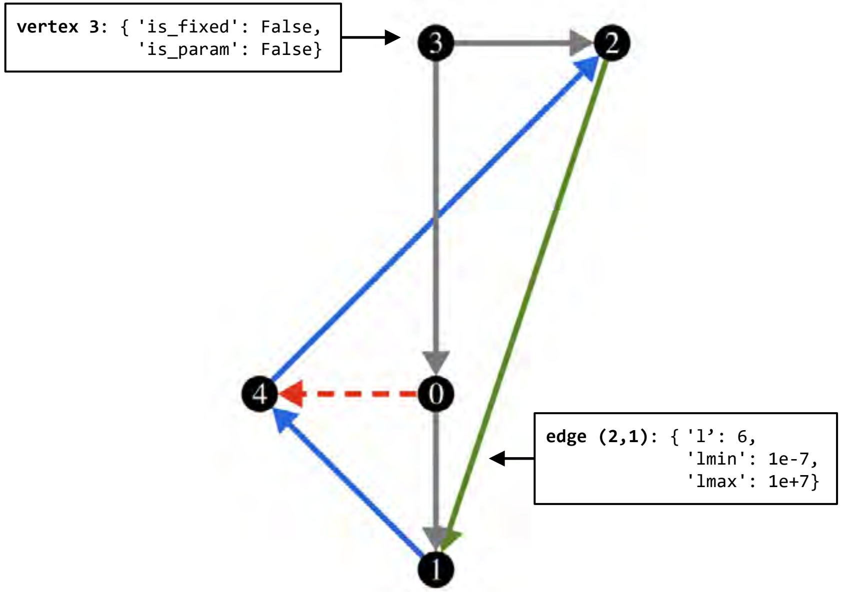
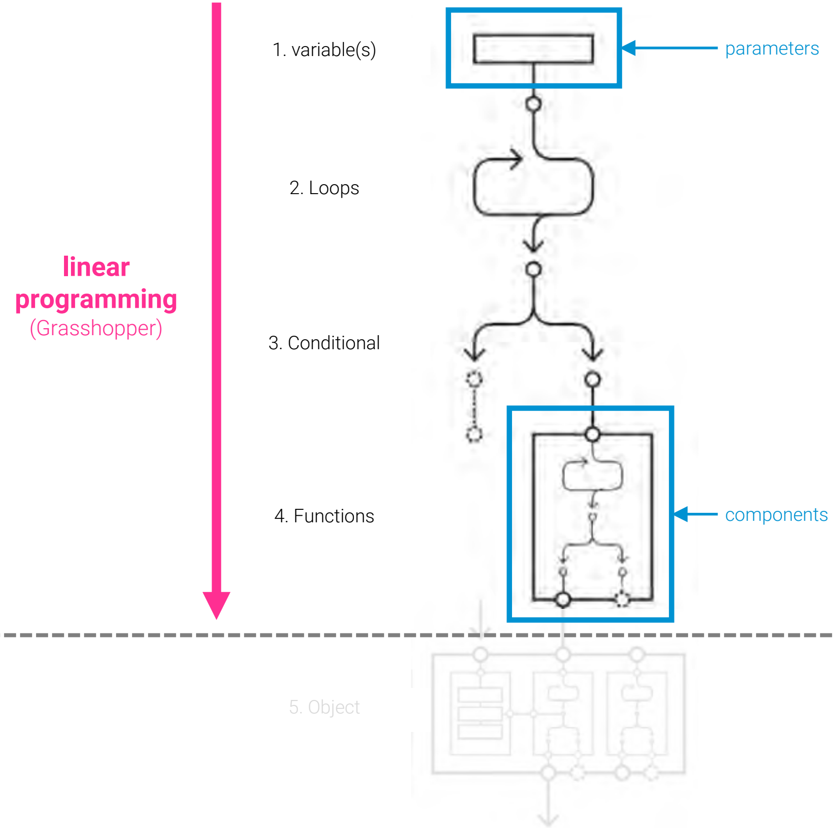
G*
b) "Bi-directional" AGS
JδX = -r
Δv*
G
G*
c) Interactive AGS
θ*
Δv*
V_x,y
e ∥ e*
Δv
l*
F
G
G*

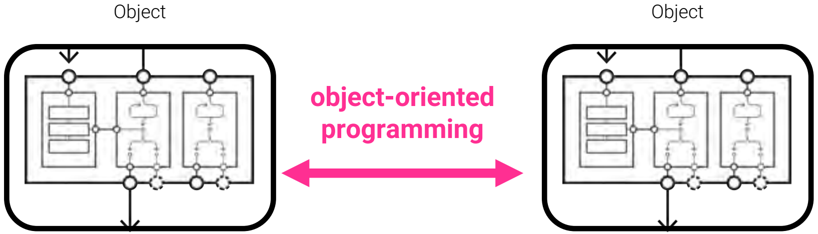
form diagram data

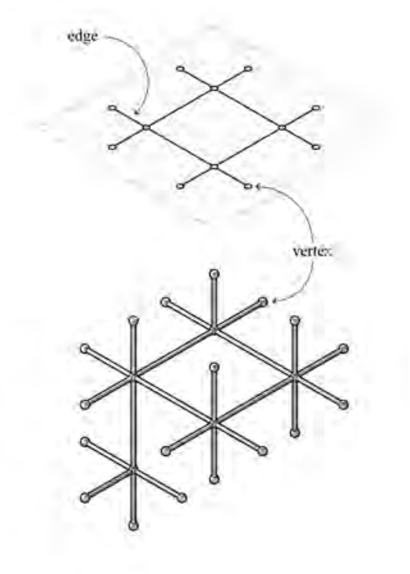


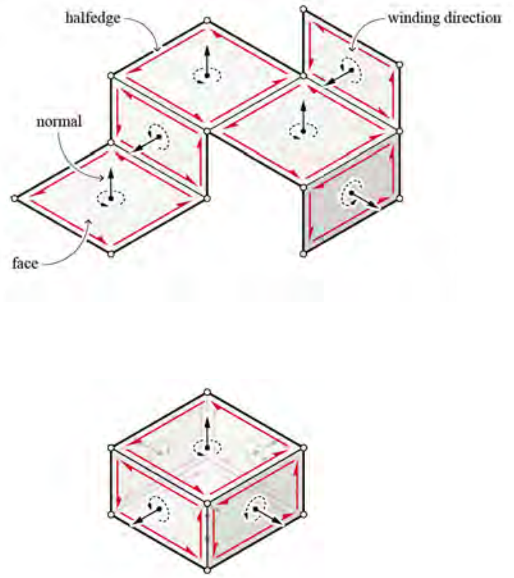
force diagram data

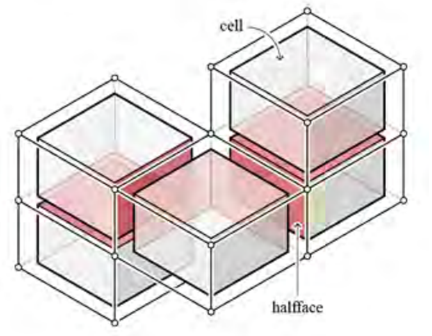



linear
programming
(Grasshopper)
1. variable(s)
parameters
2. Loops
3. Conditional
4. Functions
components
5. Object


Object
Object
object-oriented
programming







network

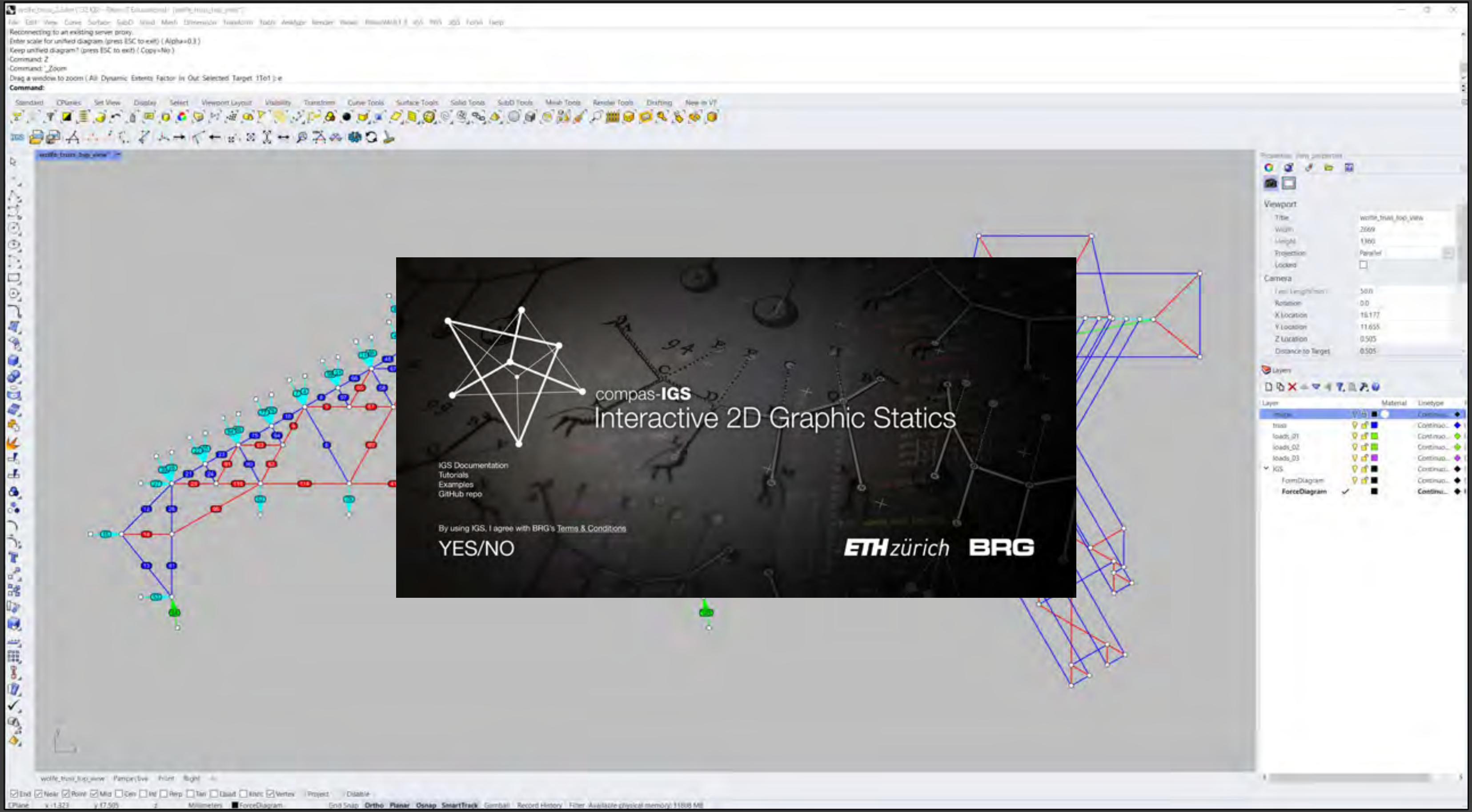
network of vertices

mesh

network of faces

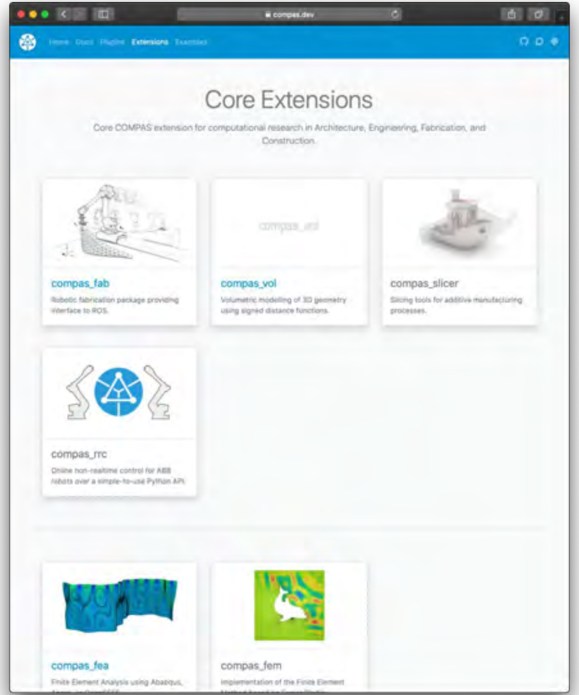
volmesh

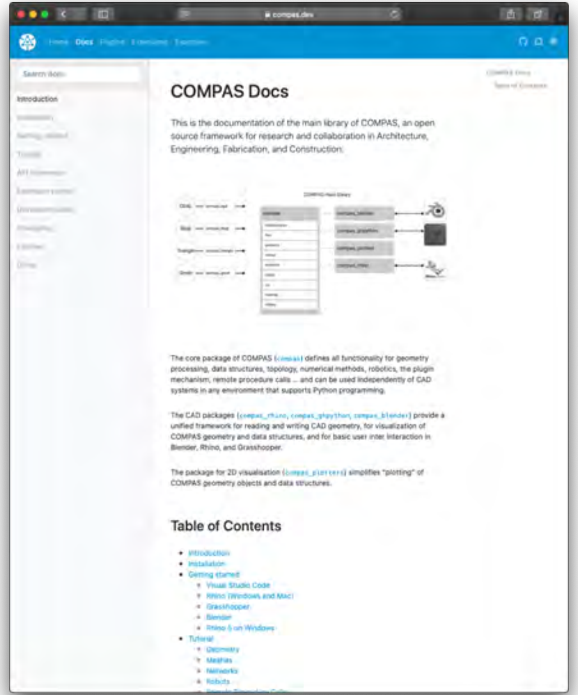
network of cells


compas-IGS
Interactive 2D Graphic Statics
IGS Documentation
Tutorials
Examples
GitHub repo
By using IGS, I agree with BRG's Terms & Conditions
YES/NO
ETH zürich
BRG

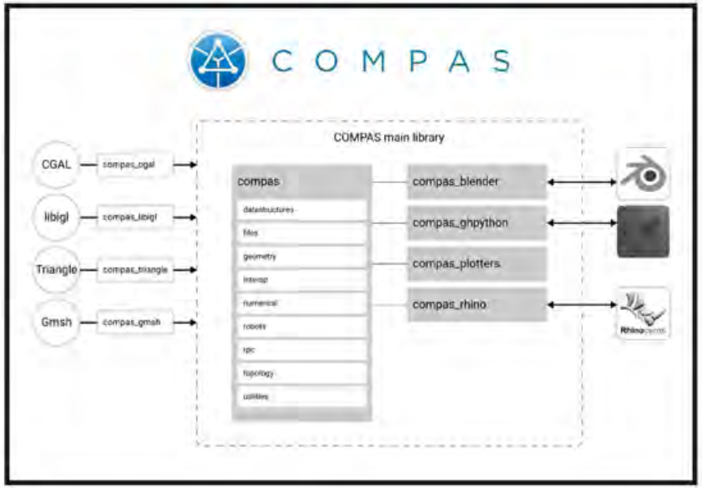
COMPAS

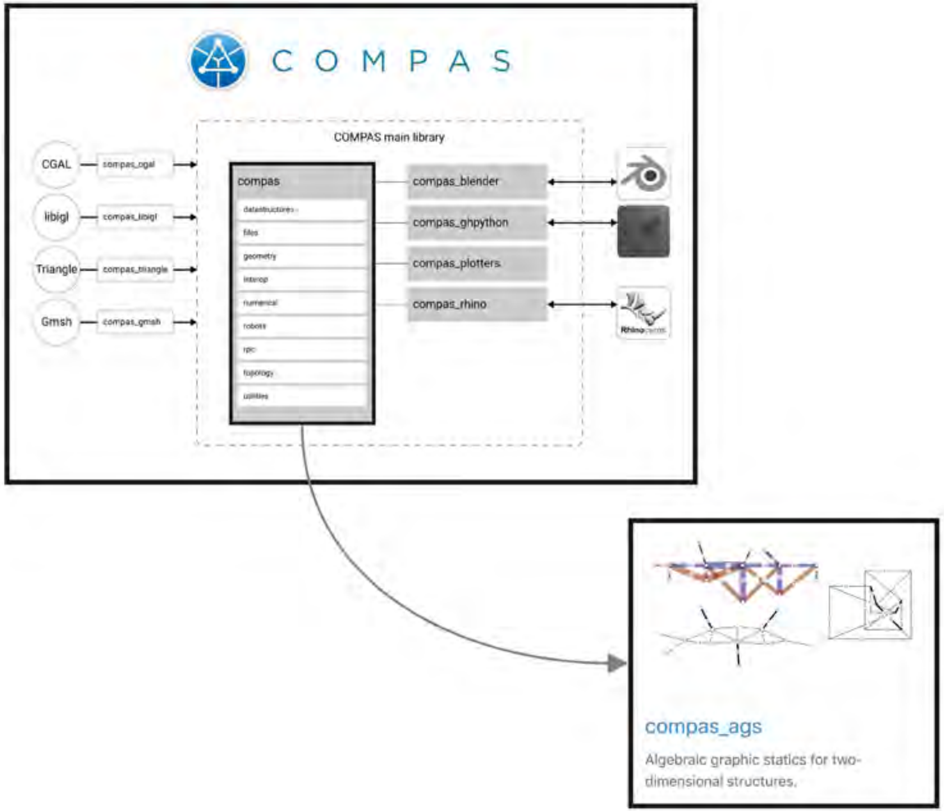
Open-source, Python-based framework for computational research and collaboration in architecture, engineering and digital fabrication





https://compas.dev


COMPAS
COMPAS main library
CGAL
compas_cgal
libigl
compas_libigl
Triangle
compas_triangle
Gmsh
compas_gmsh
compas
datastructures
files
geometry
interop
numerical
robots
rpc
topology
utilities
compas_blender
compas_ghpython
compas_plotters
compas_rhino
Rhino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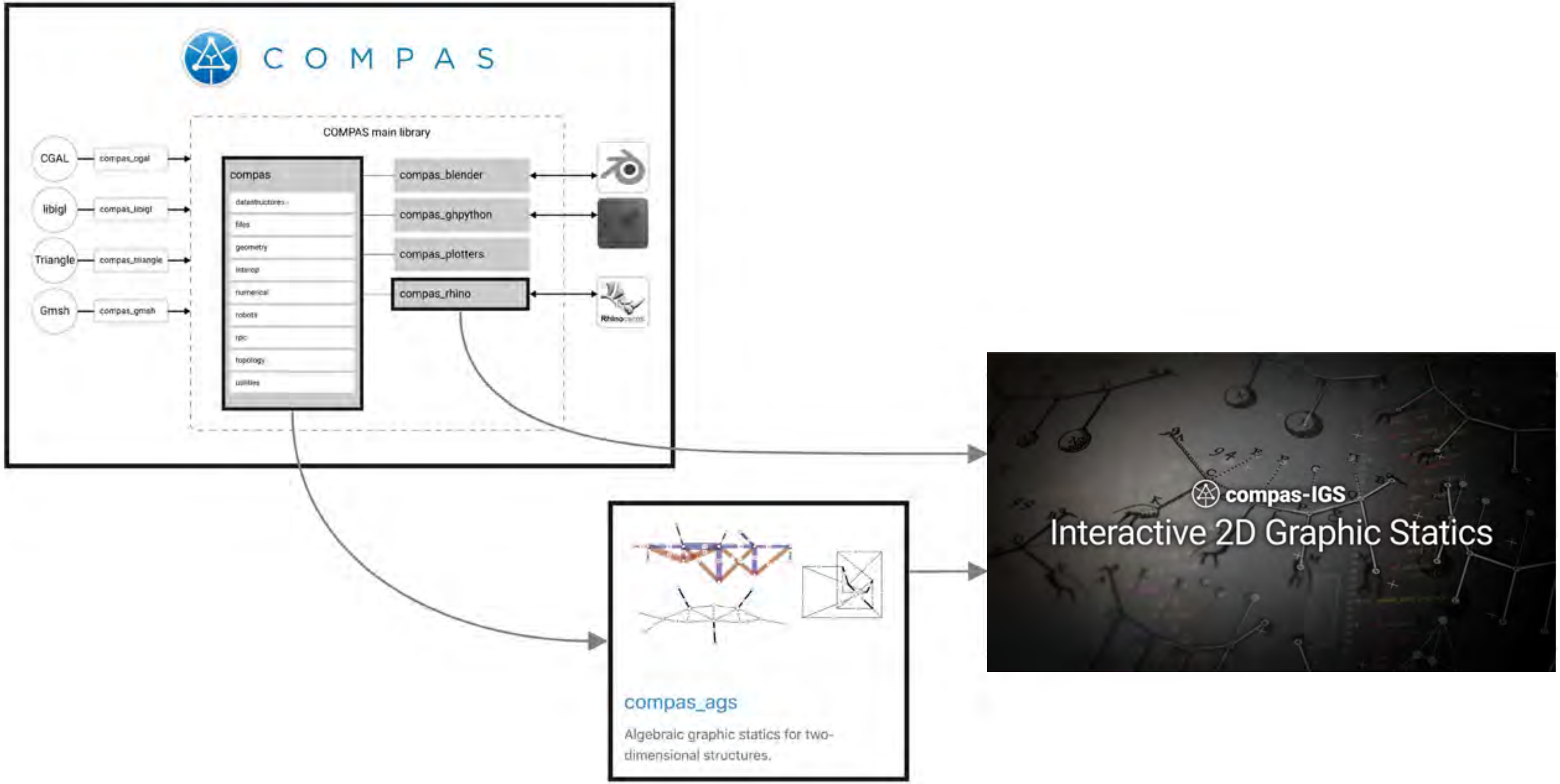
compas_rhino
compas_ags
Algebraic graphic statics for two-dimensional structures.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Triangle
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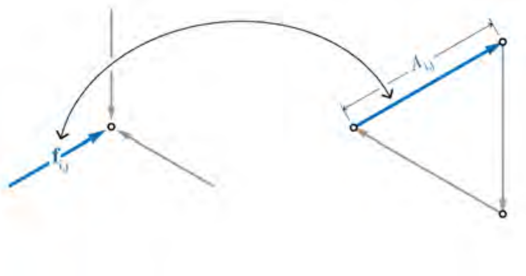
Algebraic graphic statics for two-dimensional structures.
compas-IGS
Interactive 2D Graphic Statics

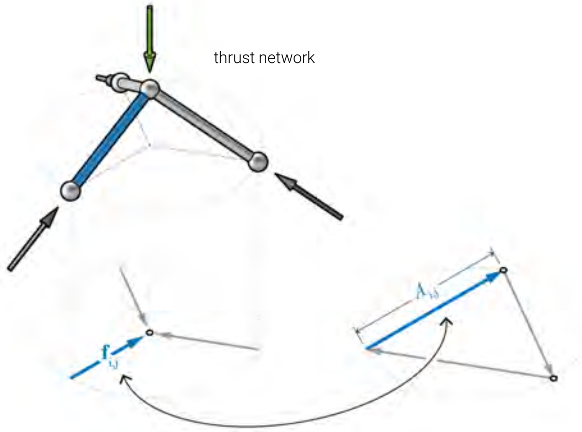
2D graphic statics



form diagram

force diagram

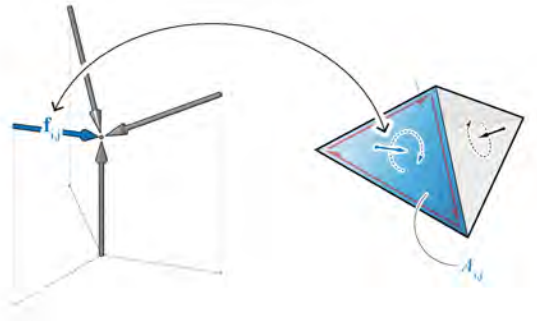
2.5D graphic statics



form diagram

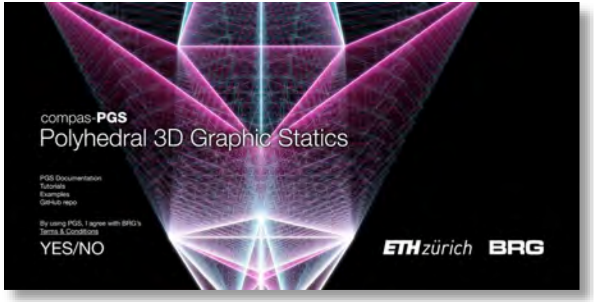
force diagram

3D graphic statics



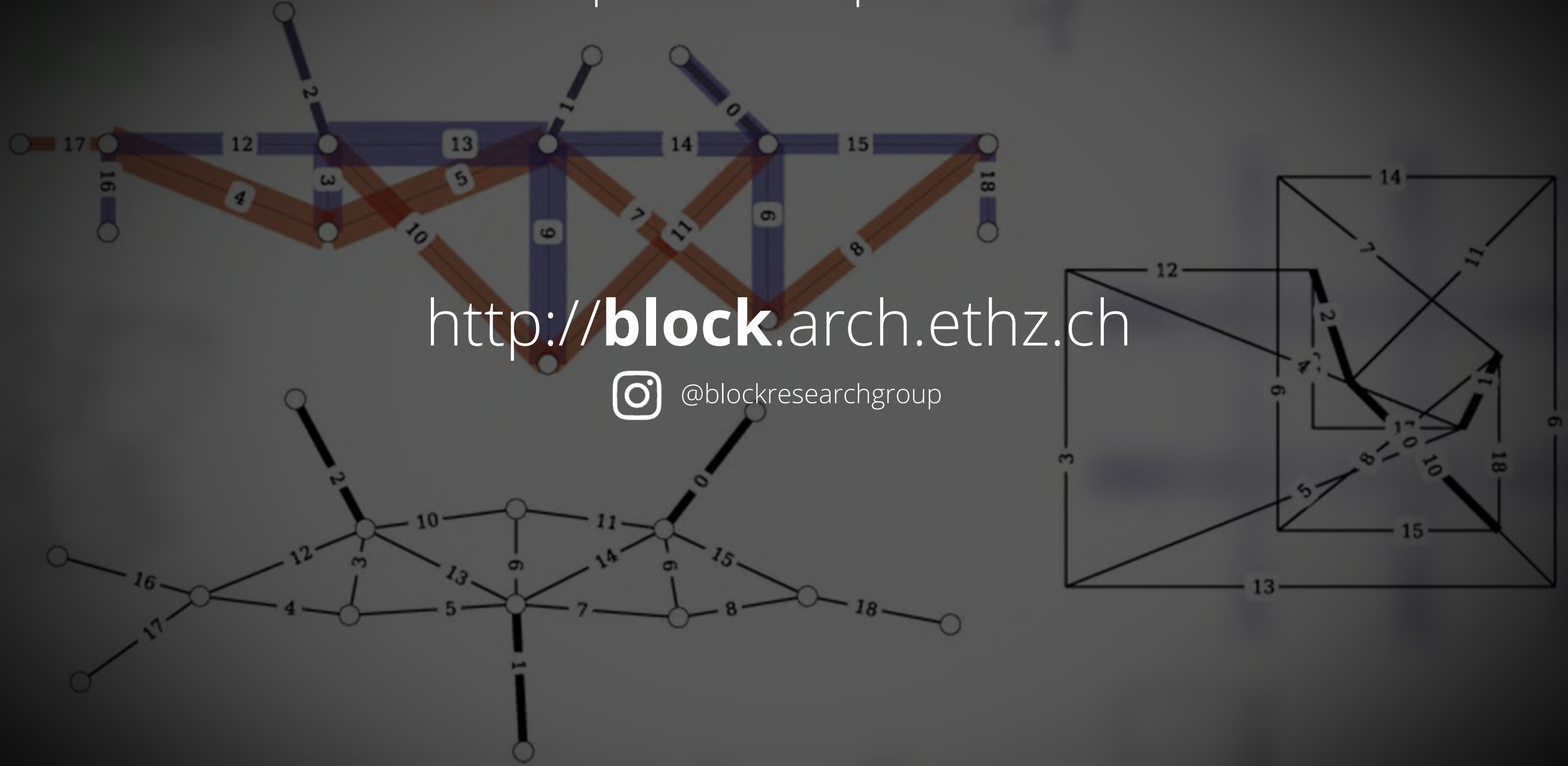
form diagram

force diagram



063-0605-00L : Computational Structural Design 1

Computational Graphic Statics



http://**block**.arch.ethz.ch

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