

063-0605-22L

Computational Structural Design I

Computational Graphic Statics

Dr. Lluís Enrique

Block Research Group (Prof. Dr. Philippe Block)

HS2022

Week 9 Friday, November 18th

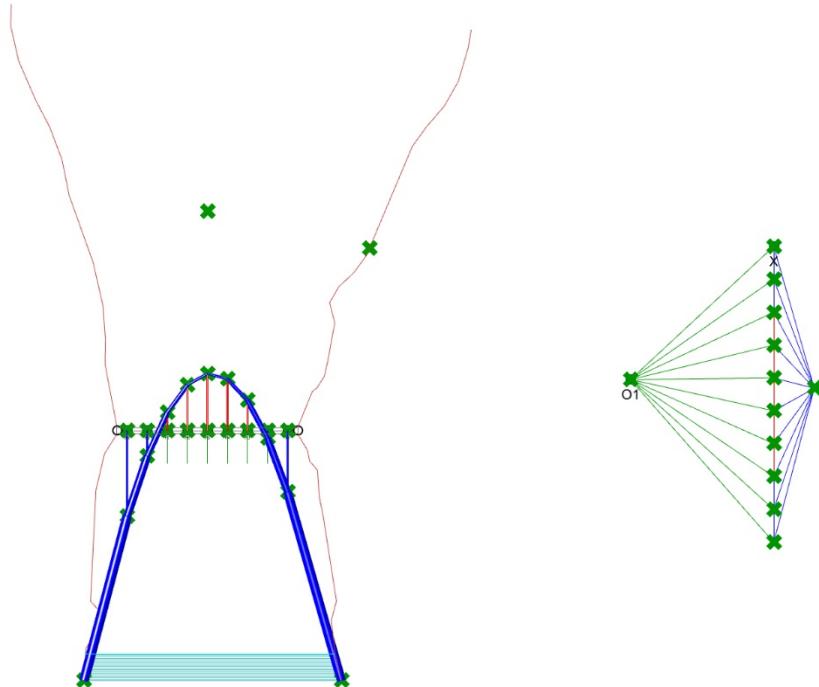
9:45 – 10:30 Recap of last week's exercise
Lecture: Algebraic Graphic statics

10:30 – 10:45 Break

10:45 – 11:30 **Tutorial: Interactive Graphic Statics**

11:30 – 11:45 Break

11:45 – 12:30 **Tutorial: Interactive Graphic Statics**



```

import rhinoscriptsyntax as rs

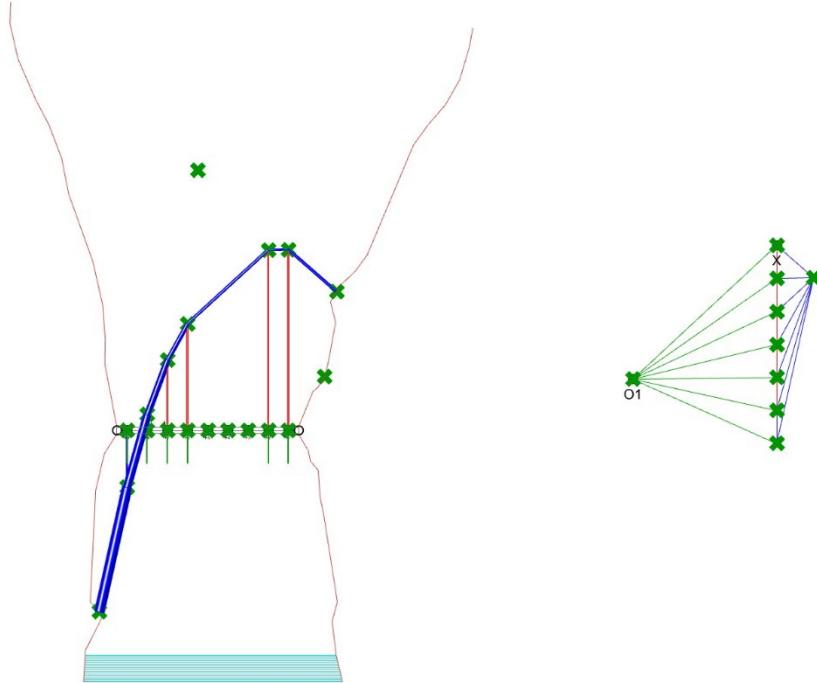
#1.a INPUT LOADS

#Task 1
#input
Lp_anc=[p1,p2,p3,p4,p5,p6,p7,p8,p9]
Ln_mag=[n1,n2,n3,n4,n5,n6,n7,n8,n9]

#vector loads
Lv_load=[]
for i in range (0,len(Lp_anc)):
    v=[0,Ln_mag[i],0]
    Lv_load.append(v)

#LOA (Line Of Action)
Ll_LOA=[]
for i in range (0,len(Lp_anc)):
    p=rs.CopyObject(Lp_anc[i])
    rs.MoveObject(p,[0,-10,0])
    LOA=rs.AddLine(Lp_anc[i],p)
    Ll_LOA.append(LOA)

```



```

import rhinoscriptsyntax as rs

#1.a INPUT LOADS

#input
Lp_anc_in=[p1,p2,p3,p4,p5,p6,p7,p8,p9]
Ln_mag_in=[n1,n2,n3,n4,n5,n6,n7,n8,n9]

#Task 2
Lp_anc=[]
Ln_mag=[]
for i in range (0,len(Lp_anc_in)):
    if Ln_mag_in[i] != 0:
        Lp_anc.append(Lp_anc_in[i])
        Ln_mag.append(Ln_mag_in[i])

#vector loads
Lv_load=[]
for i in range (0,len(Lp_anc)):
    v=[0,Ln_mag[i],0]
    Lv_load.append(v)

#LOA (Line Of Action)
Ll_LOA=[]
for i in range (0,len(Lp_anc)):
    p=rs.CopyObject(Lp_anc[i])
    rs.MoveObject(p,[0,-10,0])
    LOA=rs.AddLine(Lp_anc[i],p)
    Ll_LOA.append(LOA)

```

```
import rhinoscriptsyntax as rs

#1.a INPUT LOADS

n=[p1,p2,p3,p4,p5,p6,p7,p8,p9]
n=[n1,n2,n3,n4,n5,n6,n7,n8,n9]

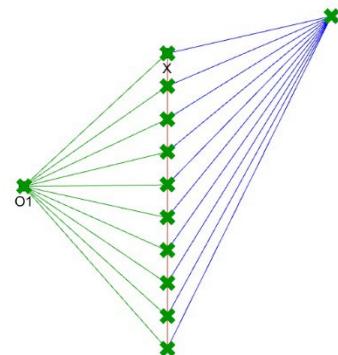
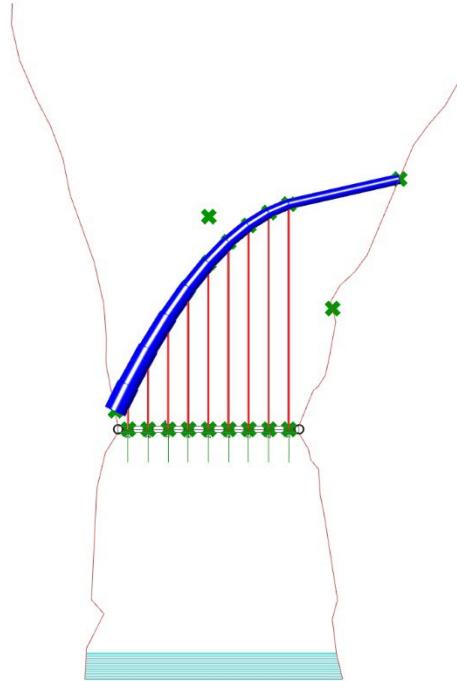
    and 3
]
]

for i in range (0,len(Lp_anc_in)):
    n_mag_in[i] != 0:
        if M == i:
            Lp_anc.append(Lp_anc_in[i])
            Ln_mag.append(Ln_mag_in[i]+Q)
        else:
            Lp_anc.append(Lp_anc_in[i])
            Ln_mag.append(Ln_mag_in[i])

loads
[]
for i in range (0,len(Lp_anc)):
    ,Ln_mag[i],0]
oad.append(v)

ne Of Action)
]
for i in range (0,len(Lp_anc)):
    .CopyObject(Lp_anc[i])
    ....rs.MoveObject(p,[0,-10,0])
    ....LOA=rs.AddLine(Lp_anc[i],p)
    ....L1_LOA.append(LOA)
```





```

import rhinoscriptsyntax as rs

#2. SUPPORTS AND REACTIONS

#creating lines of reaction forces in form diagram
l1=rs.AddLine(p_R,sp_left)
l2=rs.AddLine(p_R,sp_right)

#building reaction forces in force diagram
v1=rs.VectorCreate(Lp_loadline[-1],p_R)
l1_m=rs.MoveObject(l1,v1)

v2=rs.VectorCreate(Lp_loadline[0],p_R)
l2_m=rs.MoveObject(l2,v2)

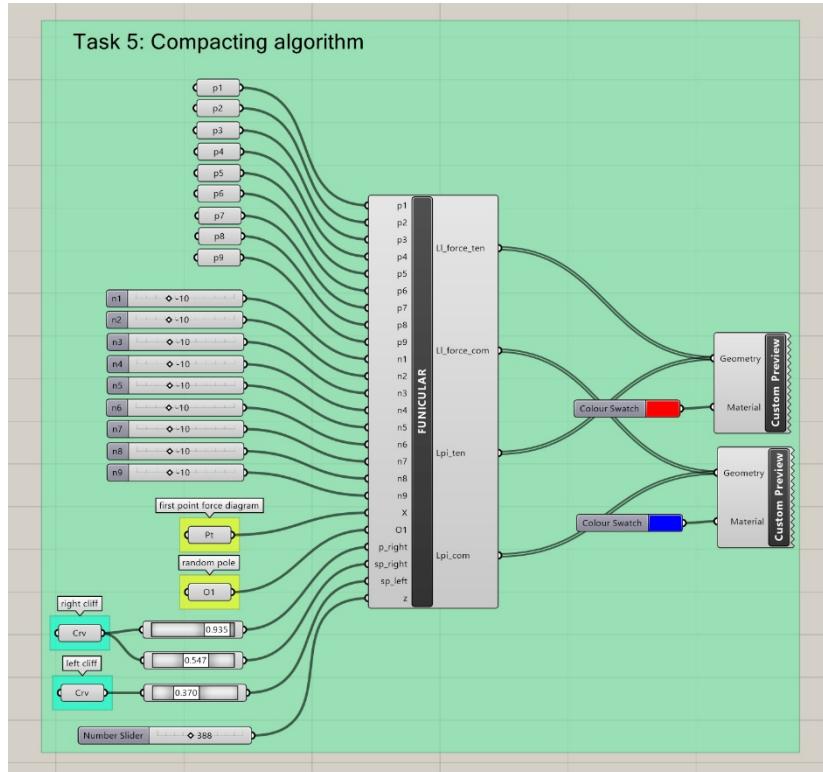
O2=rs.LineLineIntersection(l1_m,l2_m)[0]

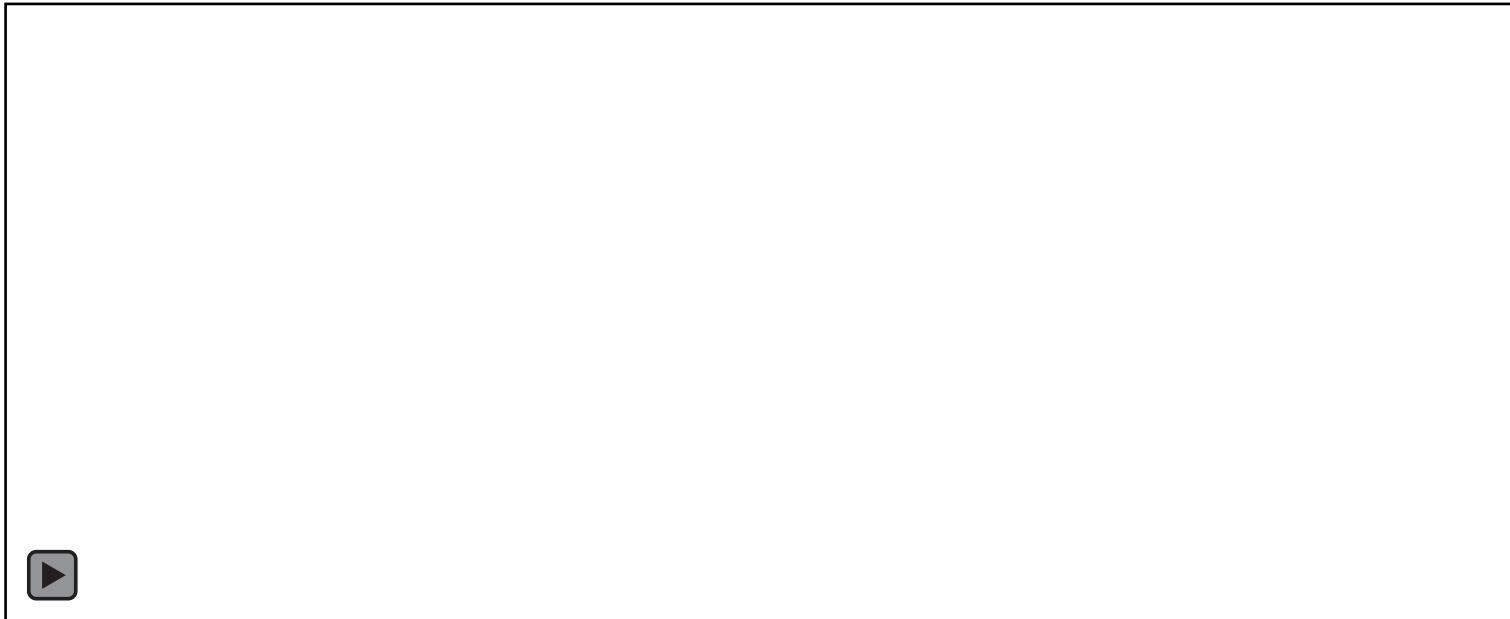
#Task 4
l_CS=rs.AddLine(sp_left,sp_right)
v=rs.VectorCreate(O2,sp_left)
l_CS_copy=rs.CopyObject(l_CS,v)

al=rs.AddLine(Lp_loadline[0],Lp_loadline[-1])
al_move=rs.MoveObject(al,[50,0,0])

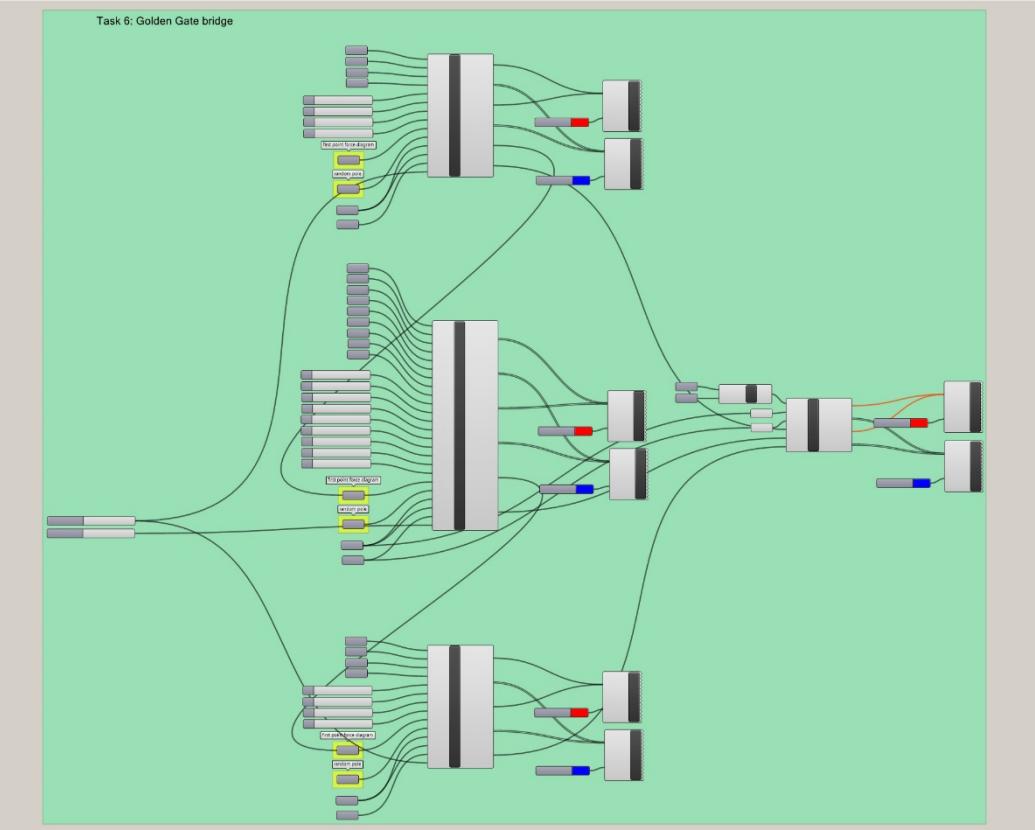
O2=rs.LineLineIntersection(l_CS_copy, al_move)[0]

```





Recap of exercise 3 – Task 6



```

import rhinoscriptsyntax as rs

l1_force=rs.AddLine(02_2,02_1)
l2_force=rs.AddLine(02_3,02_2)

v1=rs.VectorCreate(sp_right,02_1)
all_form=rs.CopyObject(l1_force,v1)
ip1=rs.LineLineIntersection(l1_terrain,all_form)[0]
l1_form=rs.AddLine(sp_right,ip1)

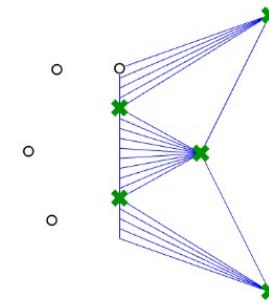
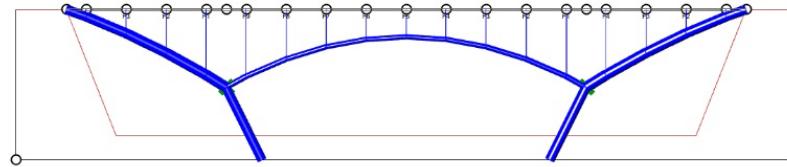
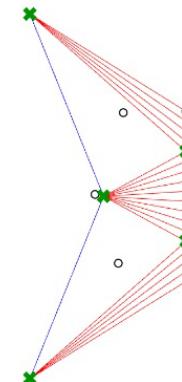
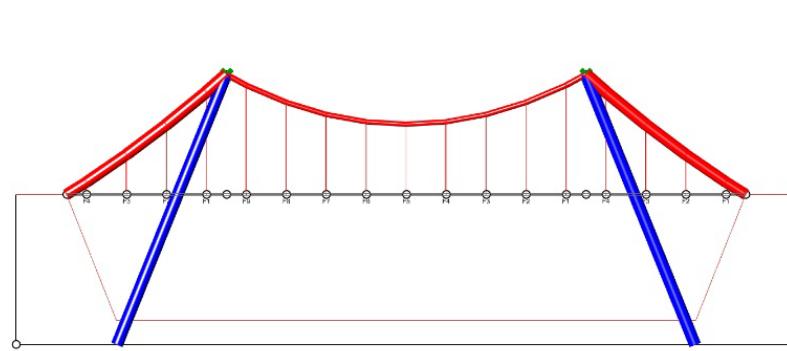
v2=rs.VectorCreate(sp_left,02_2)
al2_form=rs.CopyObject(l2_force,v2)
ip2=rs.LineLineIntersection(l1_terrain,al2_form)[0]
l2_form=rs.AddLine(sp_left,ip2)

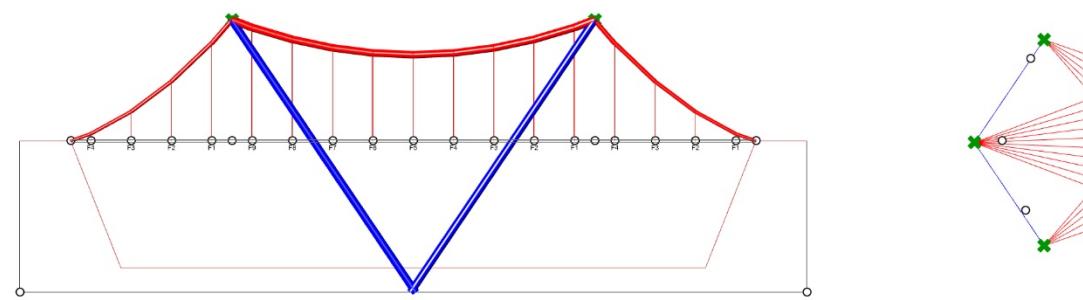
Ll_force=[l1_force,l2_force]
Ll_form=[l1_form,l2_form]

#check angle
Langle=[]
for i in range (0,len(Ll_form)):
    ....angle=rs.Angle2(Ll_form[i],Ll_force[i])
    ....Langle.append(round(angle[0]))

#colors in force diagram and pipes in form diagram
x=0.03
Ll_force_ten=[]
Ll_force_com=[]
Lpi_ten=[]
Lpi_com=[]
for i in range (0,len(Ll_form)):
    ....pi=rs.AddPipe(Ll_form[i],0,x*rs.CurveLength(Ll_force[i]),0,1)
    ....if Langle[i] ==0:
        ....Ll_force_ten.append(Ll_force[i])
        ....Lpi_ten.append(pi[0])
    ....else:
        ....Ll_force_com.append(Ll_force[i])
        ....Lpi_com.append(pi[0])

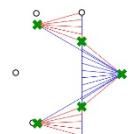
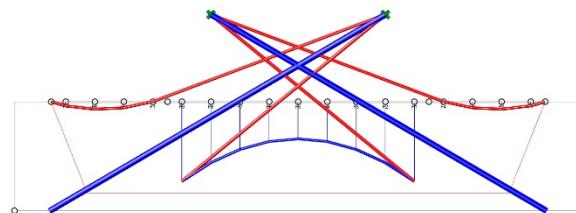
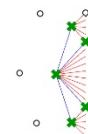
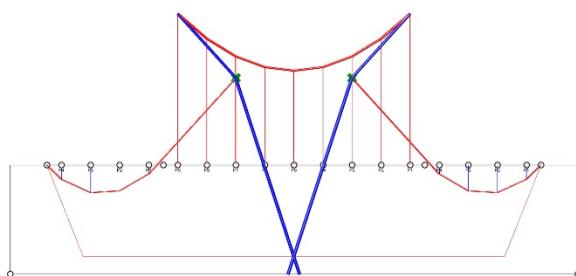
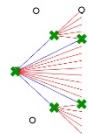
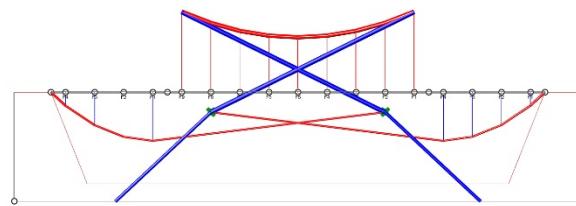
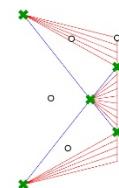
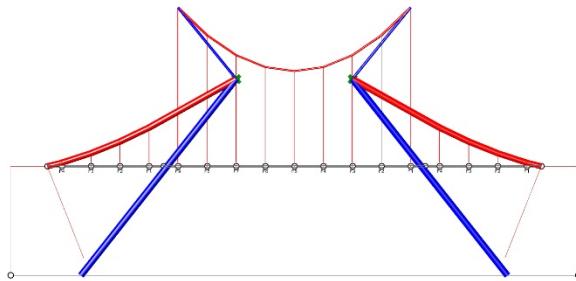
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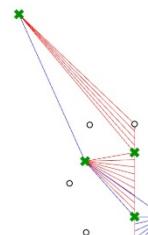
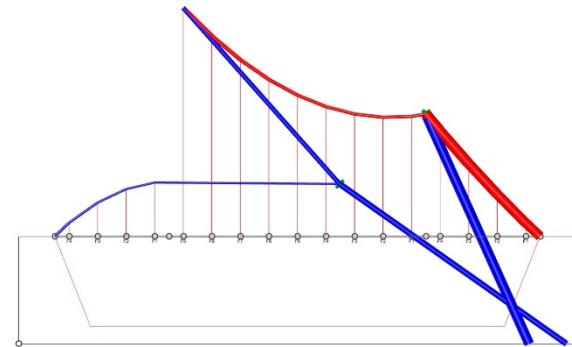
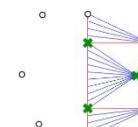
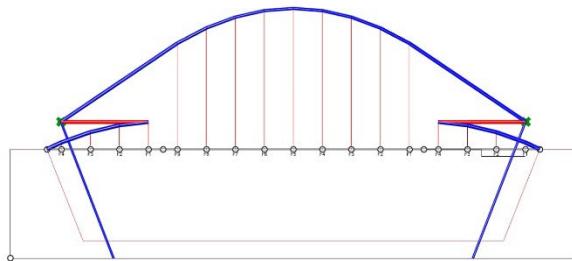
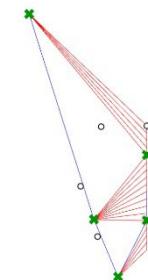
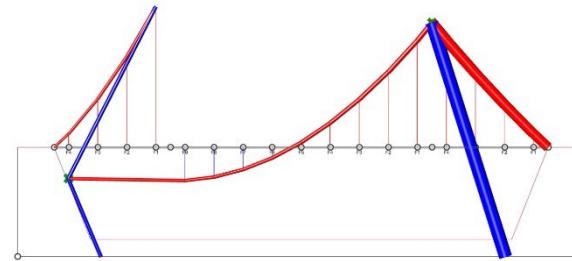
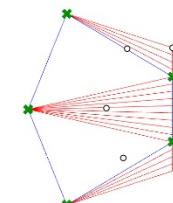
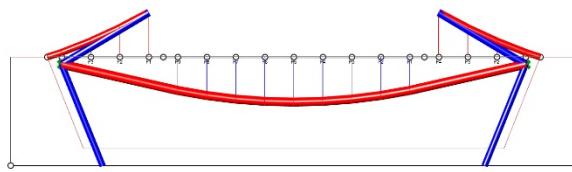
Recap of exercise 3 – Task 6

12

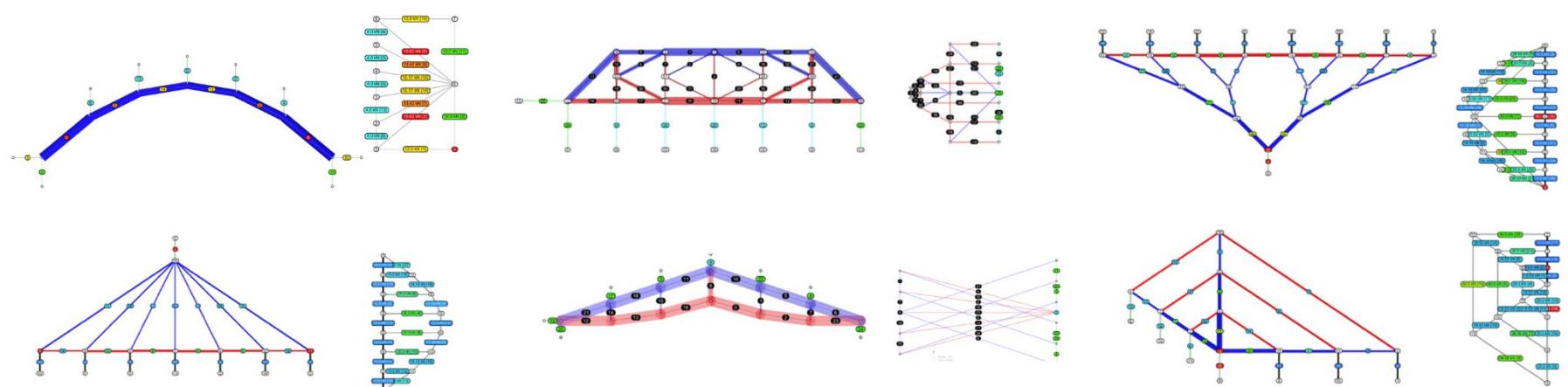
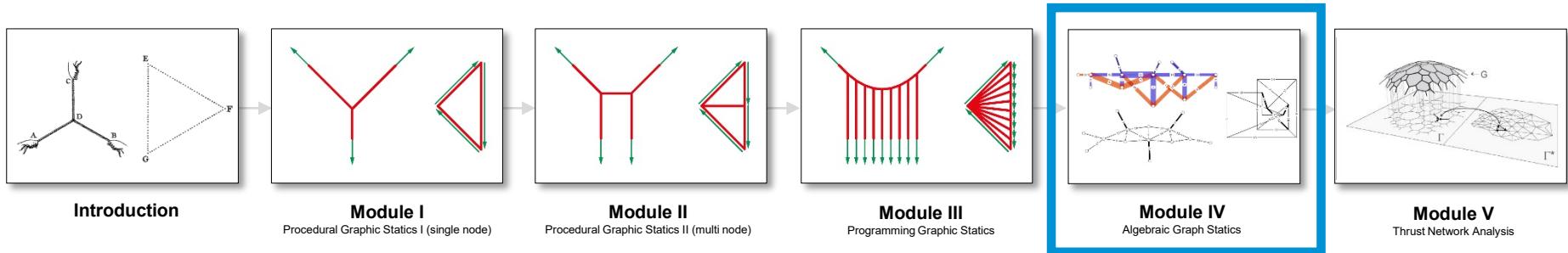


Recap of exercise 3 – Task 6

13



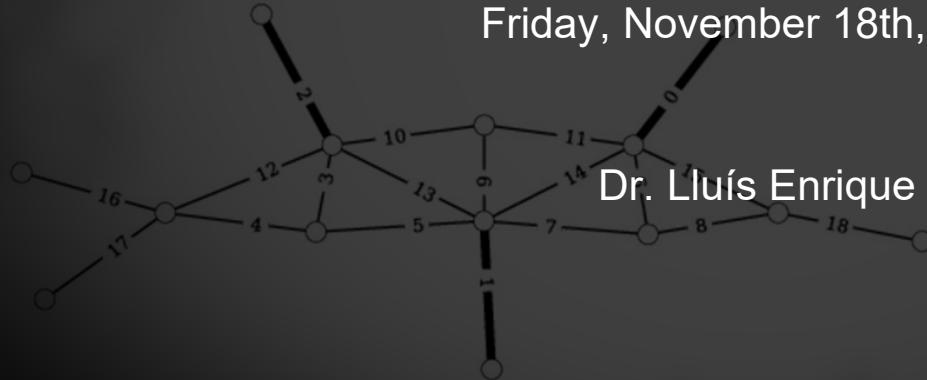
Typology	Topic	Module	Week		Topic	Lead
cables & arches	Introduction		Week 1 (23/9)	Lecture	Introduction to graphic statics	Dr. Lluis Enrique
				Tutorial	Introduction to Rhino and Grasshopper	
				Work session	graphic statics + Grasshopper	
			Week 2 (30/9)	Lecture	Quick recap of last week's exercise	Dr. Lluis Enrique
				Tutorial	Algorithmic design & thinking	
	Procedural GS (Grasshopper)			Work session	Single node bridge in Grasshopper	
		Week 3 (7/10)	Lecture	Quick recap of last week's exercise	Dr. Lluis Enrique	
			Tutorial	Computational graphic statics		
			Work session	Multi-node bridge in Grasshopper		
		Week 4 (14/10)	Lecture	Quick recap of last week's exercise	Dr. Lluis Enrique	
			Tutorial	Multi-node bridge in Grasshopper		
		Week 5 (21/10)	Work session	Multi-node bridge in Grasshopper		
Week 6: Seminar week						
cables & arches	Procedural GS (Python)		Week 7 (4/11)	Lecture	Programming	Dr. Lluis Enrique
				Tutorial	Multi-node bridge in Python	
			Week 8 (11/11)	Work session	Multi-node bridge in Python	
trusses	AGS		Week 9 (18/11)	Lecture	Quick recap of last week's exercise	Dr. Lluis Enrique
				Tutorial	Algebraic Graph Statics (AGS)	Dr. Lluis Enrique
			Week 10 (25/11)	Work session	Interactive Graphic Statics (IGS)	Chaoyu Du
					Truss analysis using IGS	
shells	TNA		Week 11 (02/12)	Lecture	Quick recap of last week's exercise	Dr. Lluis Enrique
				Tutorial	Thrust Network Analysis (TNA)	Dr. Lluis Enrique
			Week 12 (09/12)	Work session	RhinoVault 2 (rv2)	Selina Bitting
					Shell design using rv2	





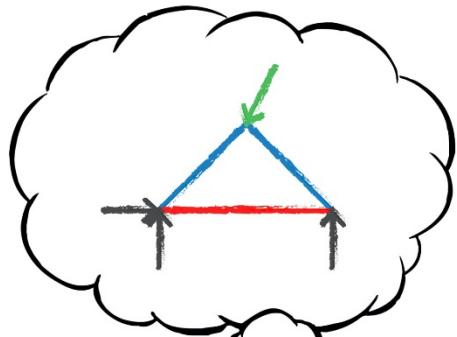
Algebraic Graph(ic) Statics

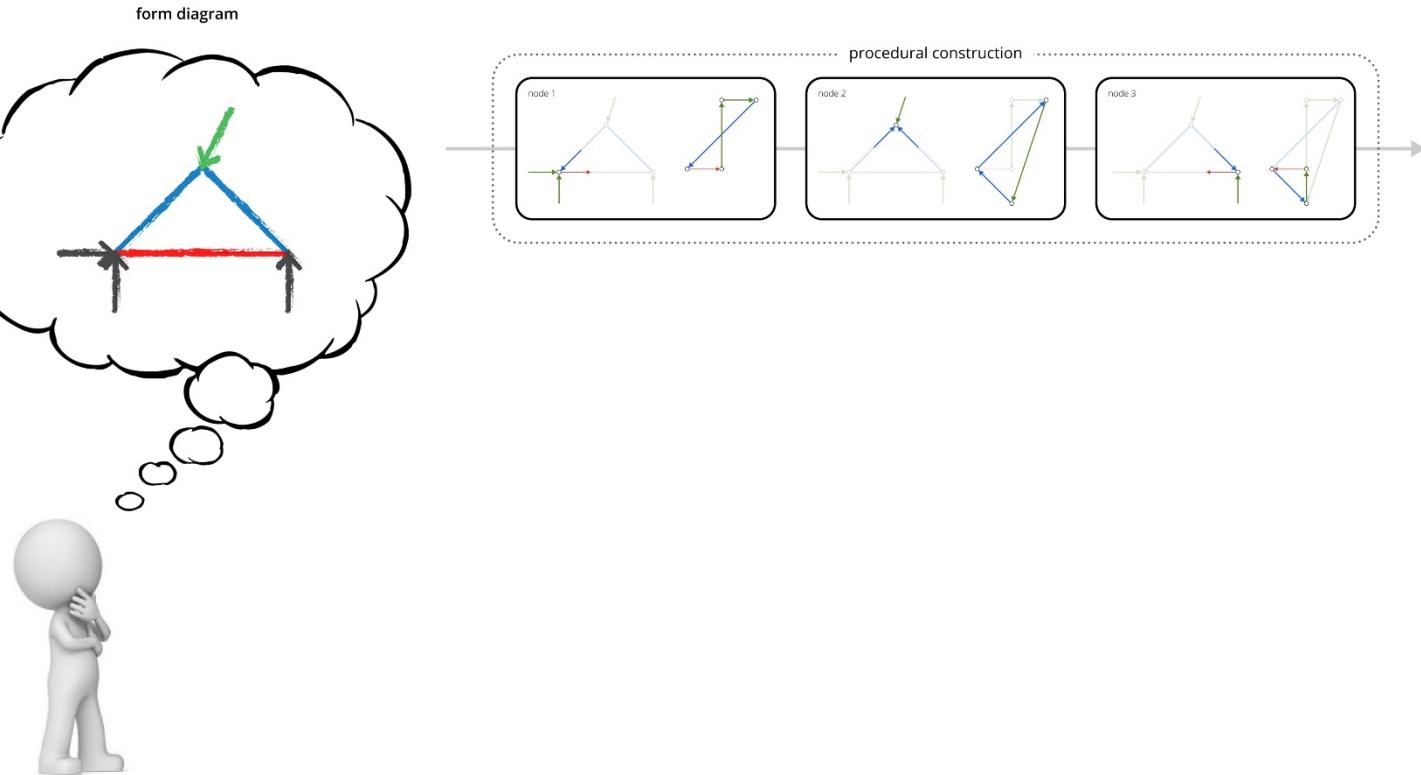
Friday, November 18th, 2022

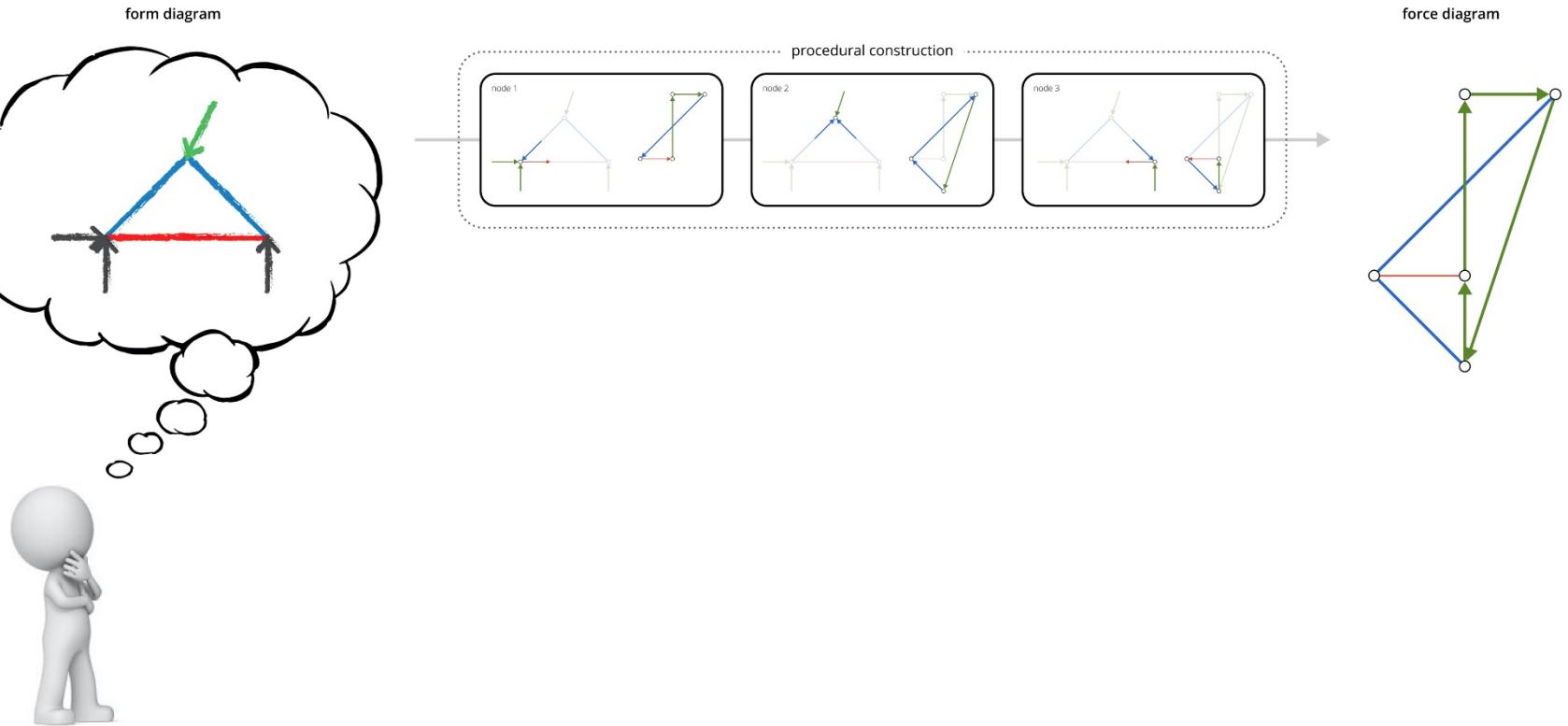


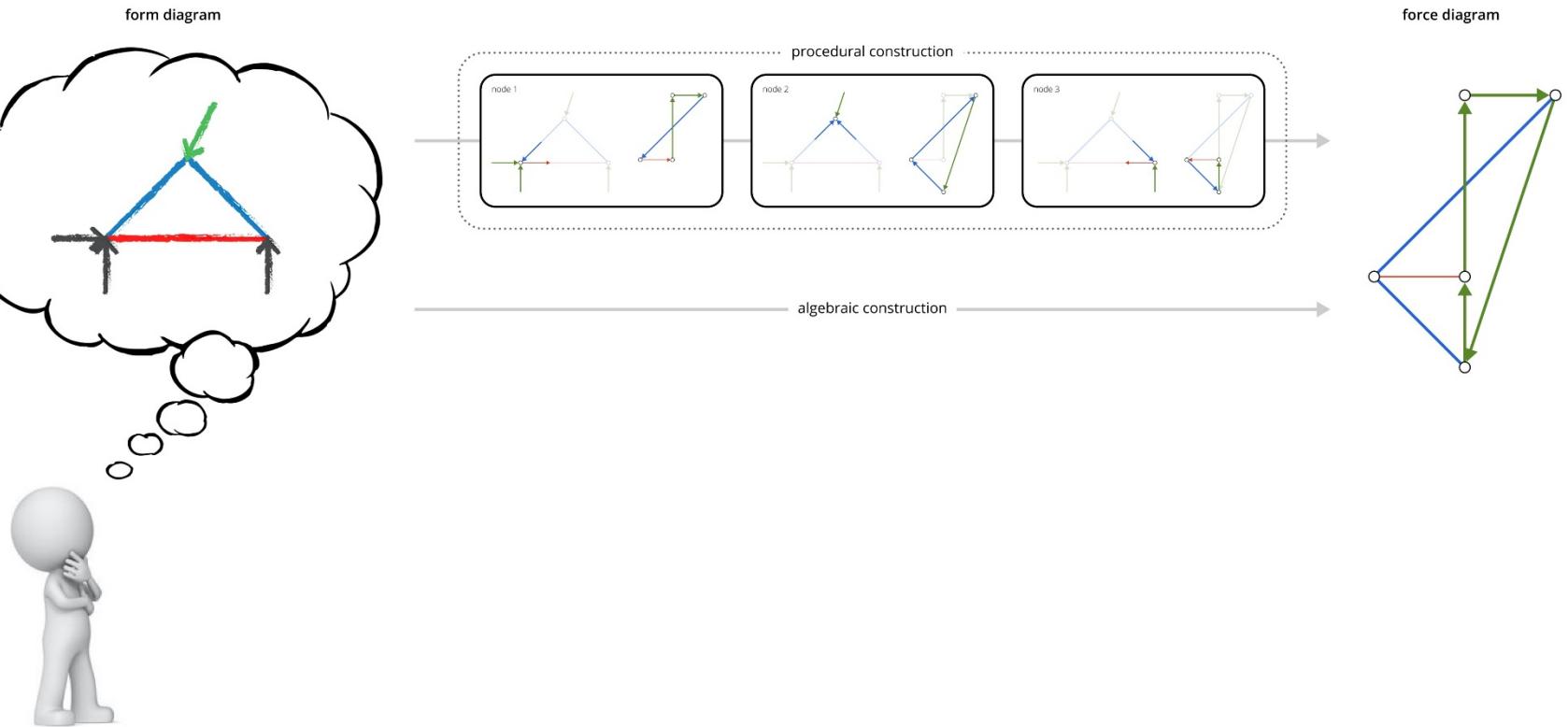
Dr. Lluís Enrique

form diagram



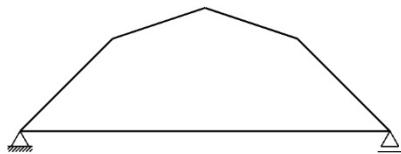
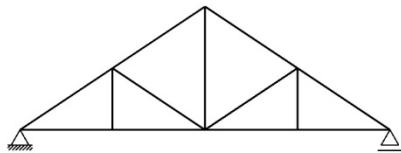




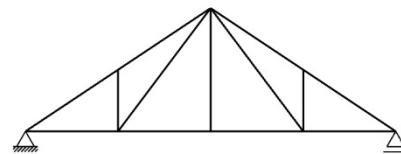
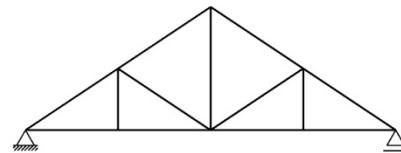


Differences between typology, topology and geometry

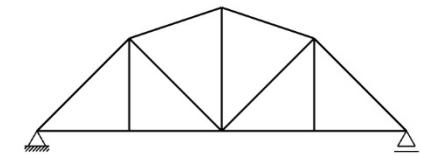
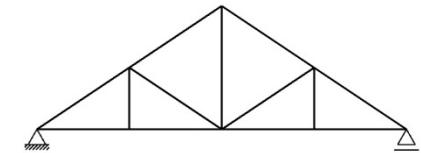
Typology



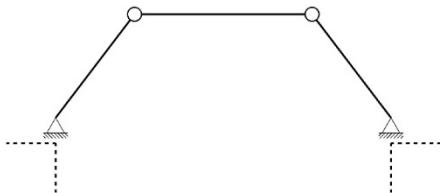
Topology



Geometry



Cables / Arches



b (bars): 3

r (reactions): 4

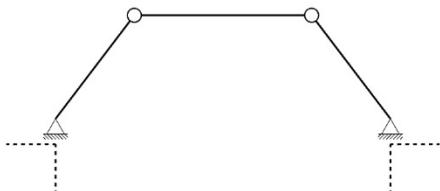
n (nodes): 4

$$\boxed{b + r < 2 \cdot n}$$

$$3 + 4 < 2 \cdot 4$$

Unstable!

Cables / Arches



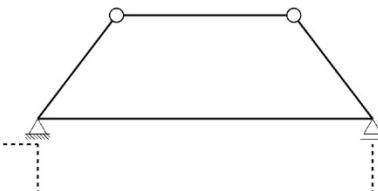
b (bars): 3
r (reactions): 4
n (nodes): 4

$$\boxed{\mathbf{b} + \mathbf{r} < 2^*\mathbf{n}}$$

$$3 + 4 < 2^*4$$

Unstable!

Arch-cables



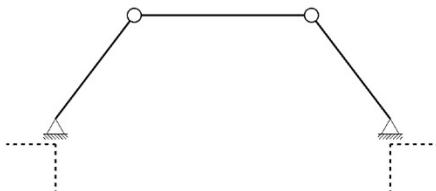
b (bars): 4
r (reactions): 3
n (nodes): 4

$$\boxed{\mathbf{b} + \mathbf{r} < 2^*\mathbf{n}}$$

$$4 + 3 < 2^*4$$

Unstable!

Cables / Arches



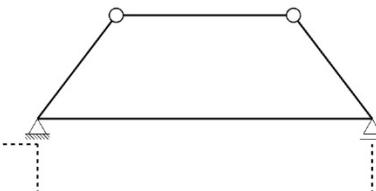
b (bars): 3
r (reactions): 4
n (nodes): 4

$$\boxed{b + r < 2*n}$$

$$3 + 4 < 2*4$$

Unstable!

Arch-cables



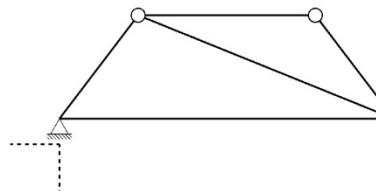
b (bars): 4
r (reactions): 3
n (nodes): 4

$$\boxed{b + r < 2*n}$$

$$4 + 3 < 2*4$$

Unstable!

Trusses



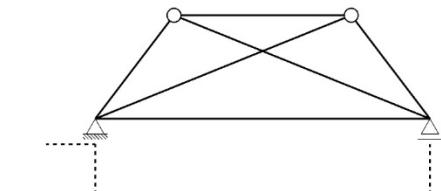
b (bars): 5
r (reactions): 3
n (nodes): 4

$$\boxed{b + r = 2*n}$$

$$5 + 3 = 2*4$$

Stable!

and
statically determinate!



b (bars): 6
r (reactions): 3
n (nodes): 4

$$\boxed{b + r > 2*n}$$

$$6 + 3 > 2*4$$

Stable!

but

statically indeterminate!

1. Form graph

Connectivity (edges & vertices)

Directed form graph

2. Force graph

Dual graph

Directed force graph

3. Assign forces

4. Solve equilibrium matrix

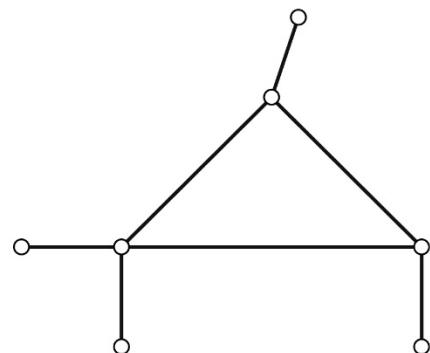
5. Update force graph

6. Reciprocal form & force graphs

1. Form graph

[Connectivity \(edges & vertices\)](#)

Directed form graph



2. Force graph

Dual graph

Directed force graph

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6. Reciprocal form & force graphs

1. Form graph

Connectivity (edges & vertices)

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2. Force graph

Dual graph

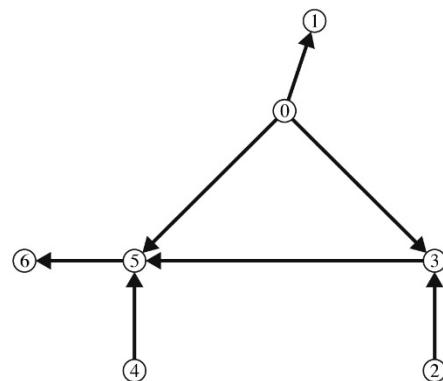
Directed force graph

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1. Form graph

Connectivity (edges & vertices)

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Dual graph

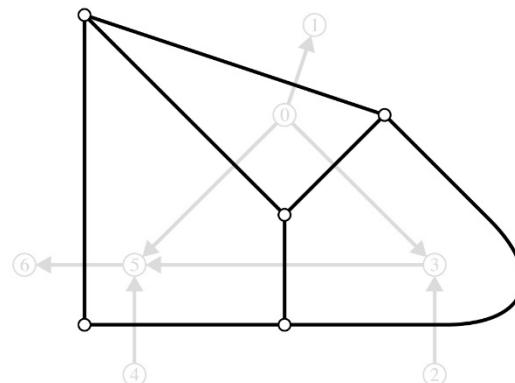
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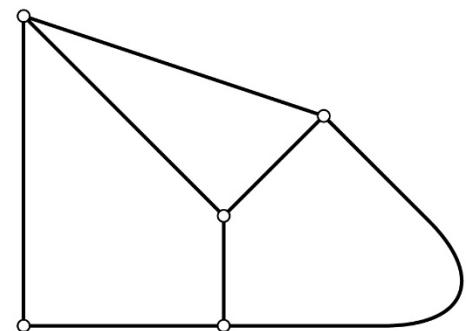
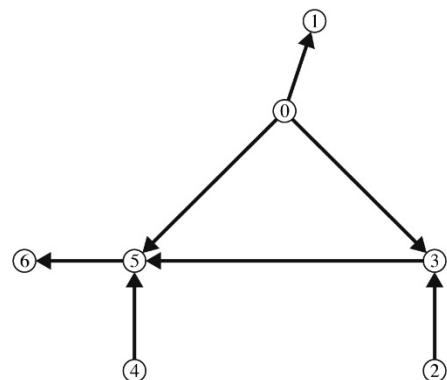
Directed force graph

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1. Form graph

Connectivity (edges & vertices)

Directed form graph

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Dual graph

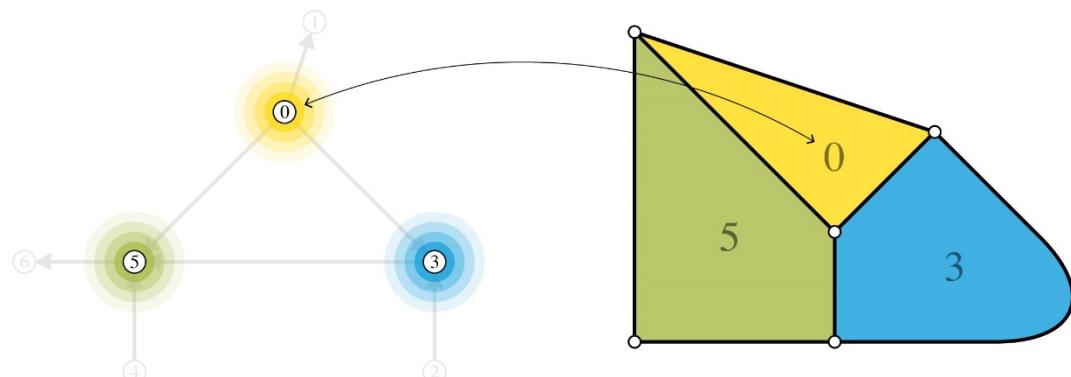
Directed force graph

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1. Form graph

Connectivity (edges & vertices)

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Dual graph

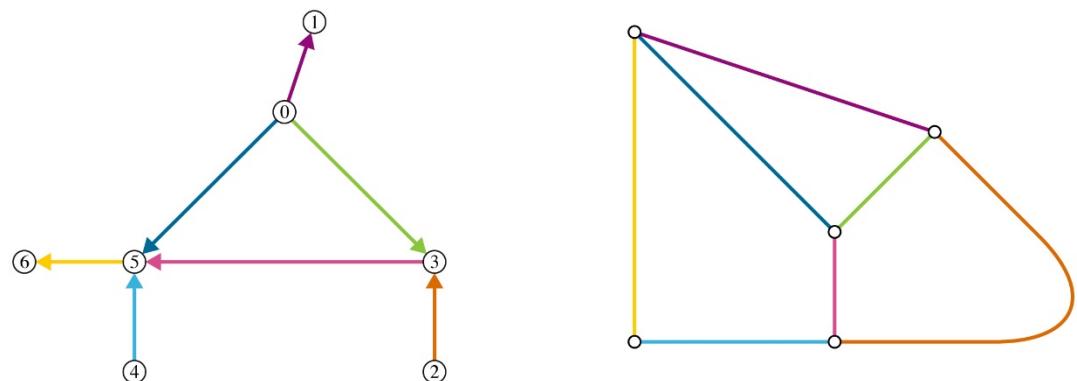
Directed force graph

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1. Form graph

Connectivity (edges & vertices)

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2. Force graph

Dual graph

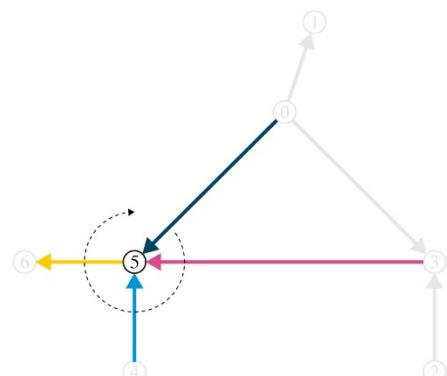
Directed force graph

3. Assign forces

4. Solve equilibrium matrix

5. Update force graph

6. Reciprocal form & force graphs



1. Form graph

Connectivity (edges & vertices)

Directed form graph

2. Force graph

Dual graph

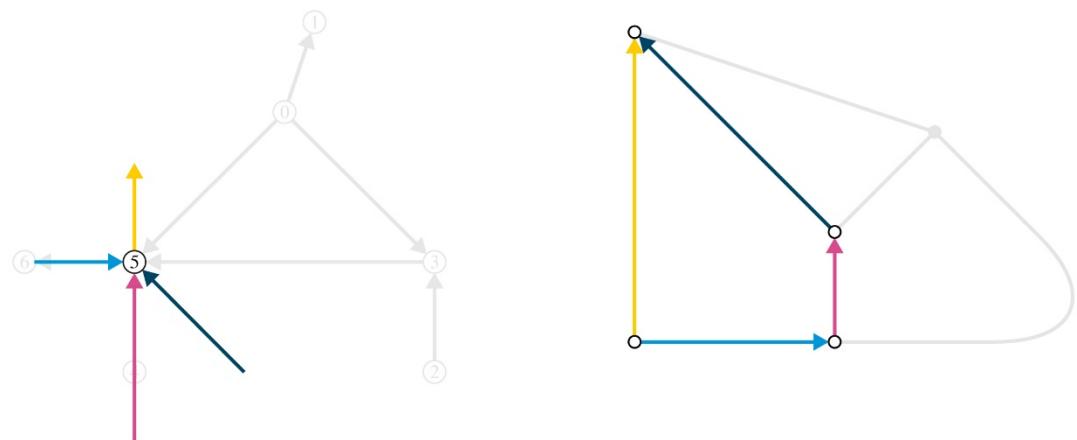
Directed force graph

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Dual graph

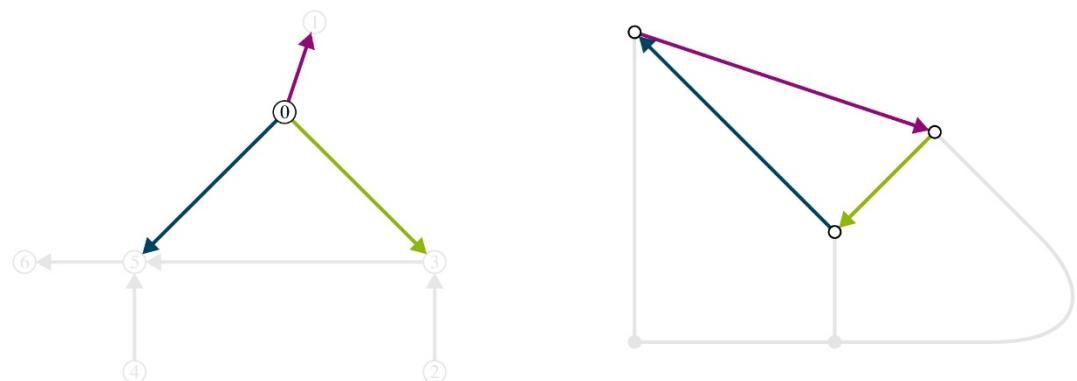
Directed force graph

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2. Force graph

Dual graph

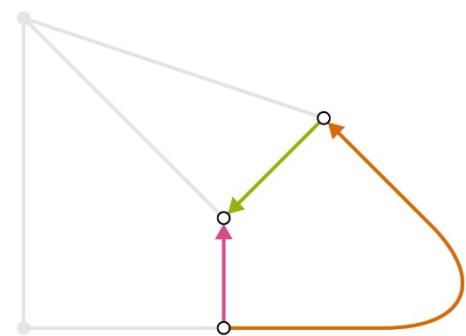
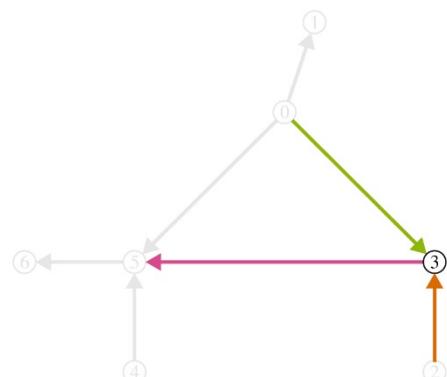
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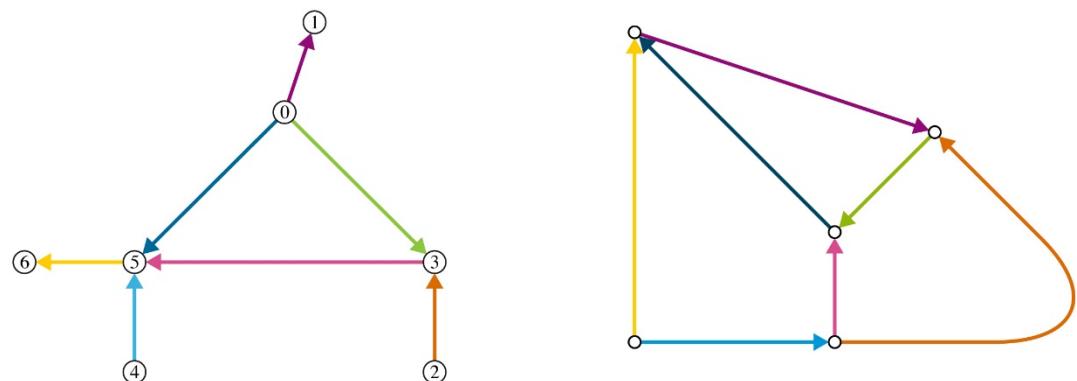
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Connectivity (edges & vertices)

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Dual graph

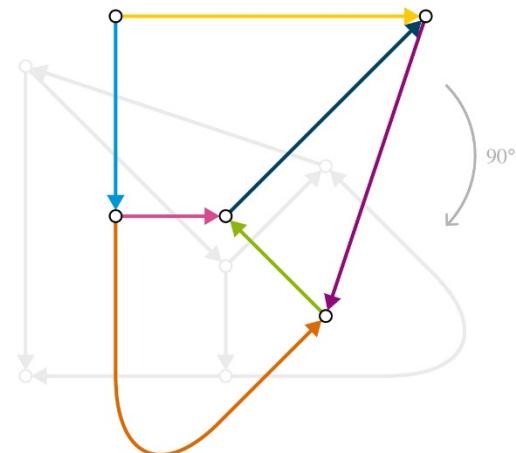
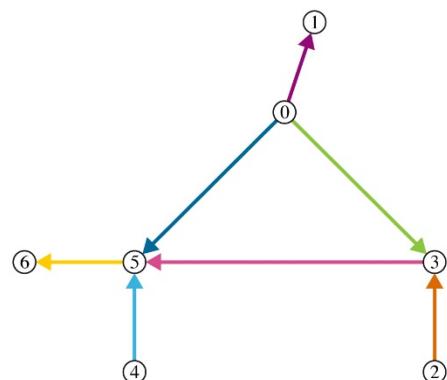
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Connectivity (edges & vertices)

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Dual graph

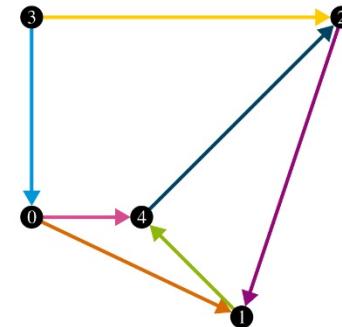
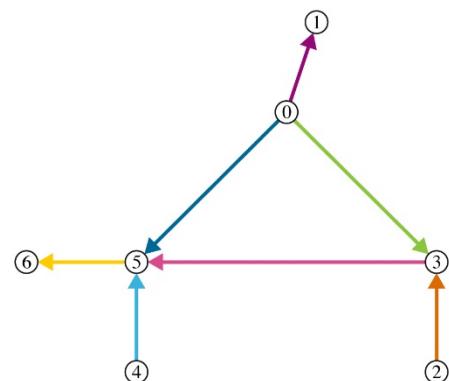
Directed force graph

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1. Form graph

Connectivity (edges & vertices)

Directed form graph

2. Force graph

Dual graph

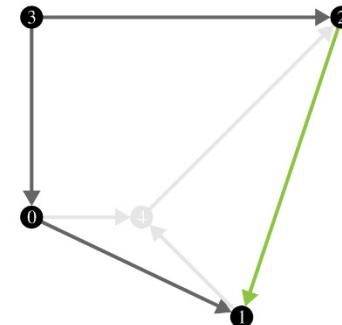
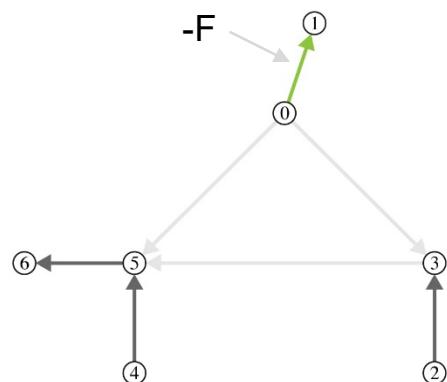
Directed force graph

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$$C_{ij} = \begin{cases} +1 & \text{if vertex } i \text{ is the head of edge } j \\ -1 & \text{if vertex } i \text{ is the tail of edge } j \\ 0 & \text{otherwise} \end{cases}$$

$$C_{ij}^* = \begin{cases} +1 & \text{if the face cycle traverses} \\ & \text{edge } j \text{ in the same direction} \\ & \text{as its orientation} \\ -1 & \text{if in the opposite direction} \\ 0 & \text{otherwise} \end{cases}$$

1. Form graph

Connectivity (edges & vertices)

Directed form graph

2. Force graph

Dual graph

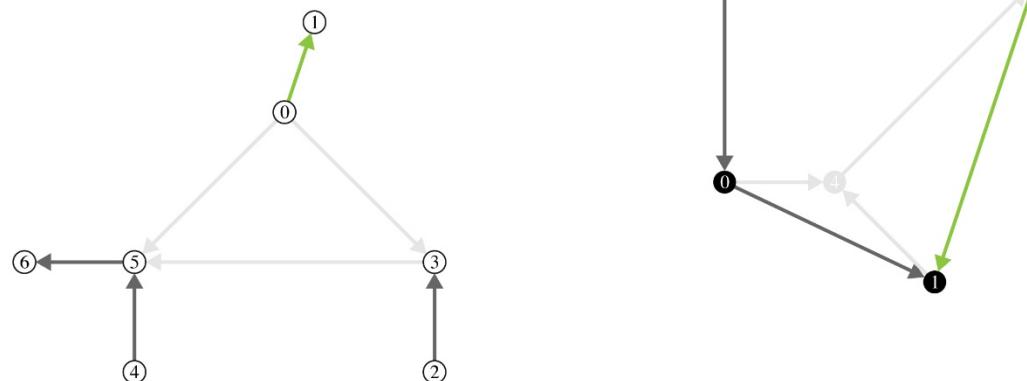
Directed force graph

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Closed polygon constraint

$$\begin{cases} C_i \mathbf{u}^* = 0 \\ C_i \mathbf{v}^* = 0 \end{cases}$$

Parallel constraint

$$\begin{cases} \mathbf{u}^* = Q\mathbf{u} \\ \mathbf{v}^* = Q\mathbf{v} \end{cases}$$

$$\begin{cases} C_i \mathbf{U}\mathbf{q} = 0 \\ C_i \mathbf{V}\mathbf{q} = 0 \end{cases}$$

equilibrium matrix

$$\mathbf{A}\mathbf{q} = 0 \quad , \quad \mathbf{A} = \left[\frac{\mathbf{C}_i \mathbf{U}}{\mathbf{C}_i \mathbf{V}} \right]$$

1. Form graph

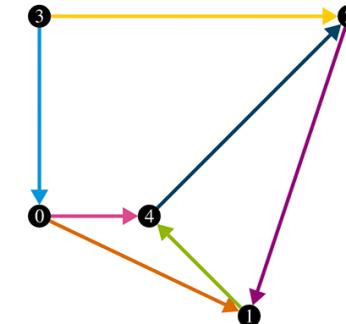
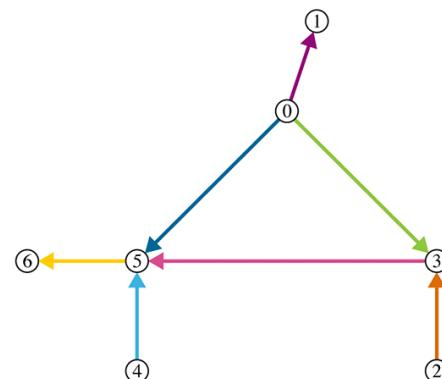
Connectivity (edges & vertices)

Directed form graph

2. Force graph

Dual graph

Directed force graph

3. Assign forces**4. Solve equilibrium matrix****5. Update force graph****6. Reciprocal form & force graphs**

$$\mathbf{A} = [\mathbf{A}_d \mid \mathbf{A}_{id}] , \quad \mathbf{q} = \begin{bmatrix} \mathbf{q}_d \\ \mathbf{q}_{id} \end{bmatrix}$$

$$\mathbf{A}_d \mathbf{q}_d + \mathbf{A}_{id} \mathbf{q}_{id} = 0$$

$$\mathbf{q}_d = -\mathbf{A}_d^{-1} \mathbf{A}_{id} \mathbf{q}_{id}$$

$$\mathbf{q} = \begin{bmatrix} -\mathbf{A}_d^{-1} \mathbf{A}_{id} \\ \mathbf{I}_k \end{bmatrix} \mathbf{q}_{id}$$

$$\mathbf{x}^* = (\mathbf{C}^{*t})^\dagger \mathbf{Q} \mathbf{u}$$

$$\mathbf{y}^* = (\mathbf{C}^{*t})^\dagger \mathbf{Q} \mathbf{v}$$

assigned loads/forces

all force densities

new force diagram coordinates

1. Form graph

Connectivity (edges & vertices)

Directed form graph

2. Force graph

Dual graph

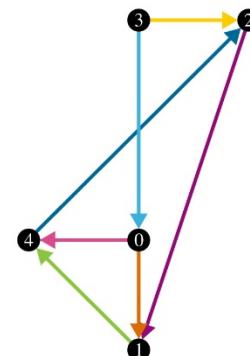
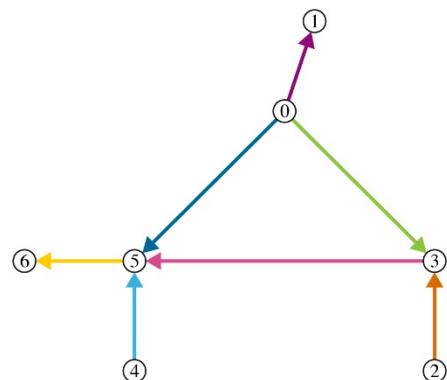
Directed force graph

3. Assign forces

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5. Update force graph

6. Reciprocal form & force graphs



1. Form graph

Connectivity (edges & vertices)

Directed form graph

2. Force graph

Dual graph

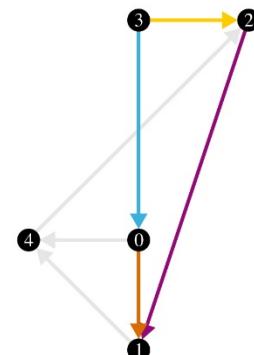
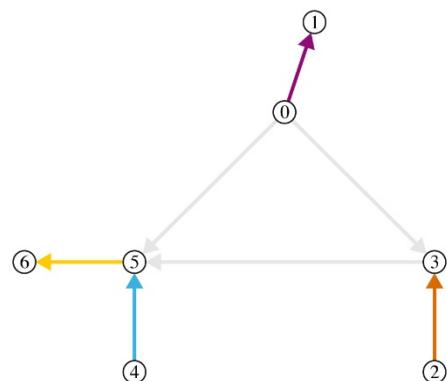
Directed force graph

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1. Form graph

Connectivity (edges & vertices)

Directed form graph

2. Force graph

Dual graph

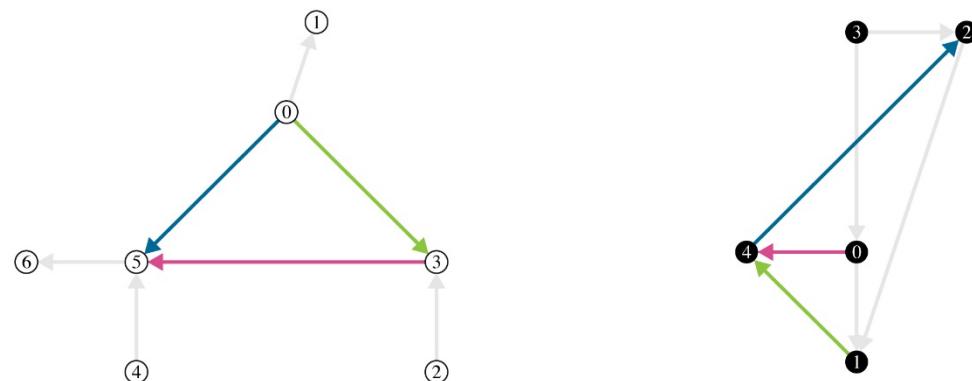
Directed force graph

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Connectivity (edges & vertices)

Directed form graph

2. Force graph

Dual graph

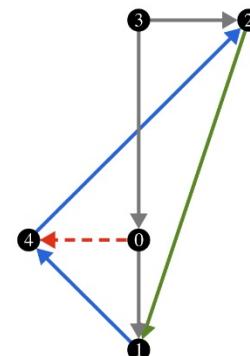
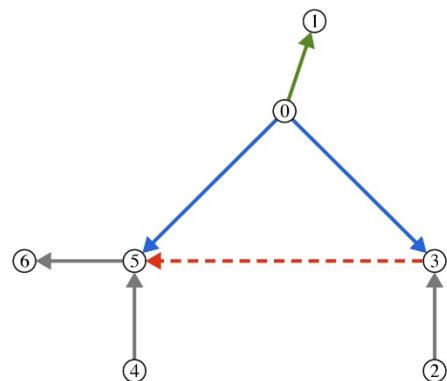
Directed force graph

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1. Form graph

Connectivity (edges & vertices)

Directed form graph

2. Force graph

Dual graph

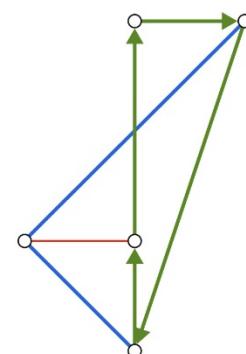
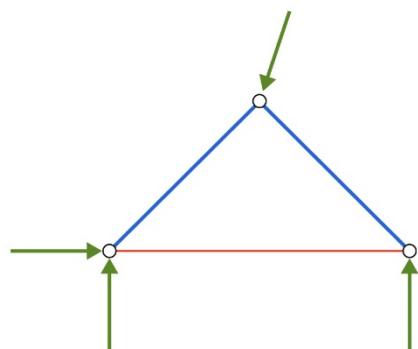
Directed force graph

3. Assign forces

4. Solve equilibrium matrix

5. Update force graph

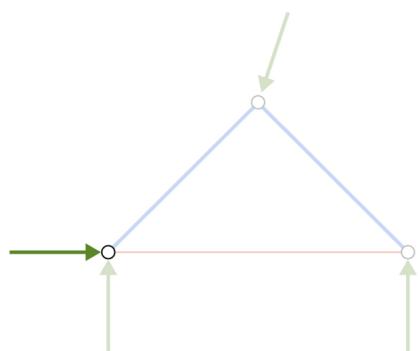
6. Reciprocal form & force graphs



1. Form graph

Connectivity (edges & vertices)

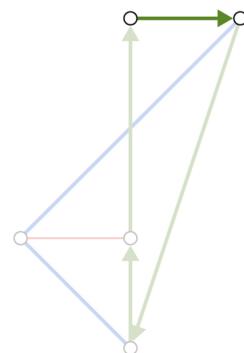
Directed form graph



2. Force graph

Dual graph

Directed force graph



3. Assign forces

4. Solve equilibrium matrix

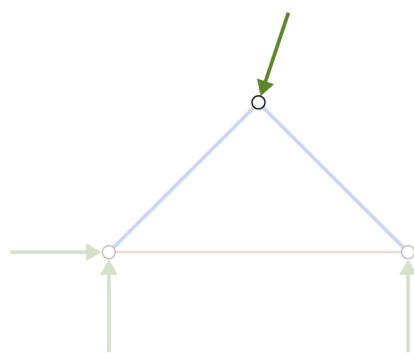
5. Update force graph

6. Reciprocal form & force graphs

1. Form graph

Connectivity (edges & vertices)

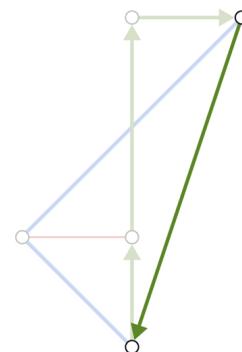
Directed form graph



2. Force graph

Dual graph

Directed force graph



3. Assign forces

4. Solve equilibrium matrix

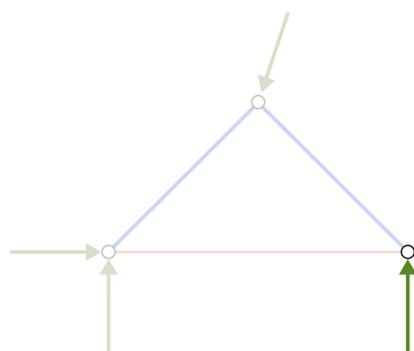
5. Update force graph

6. Reciprocal form & force graphs

1. Form graph

Connectivity (edges & vertices)

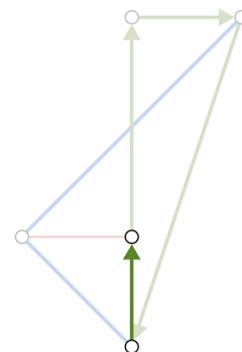
Directed form graph



2. Force graph

Dual graph

Directed force graph



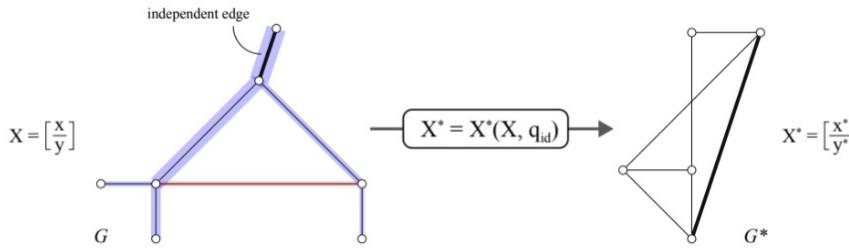
3. Assign forces

4. Solve equilibrium matrix

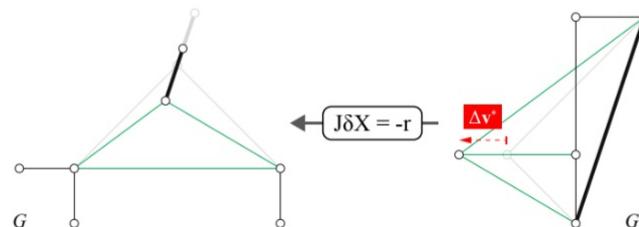
5. Update force graph

6. Reciprocal form & force graphs

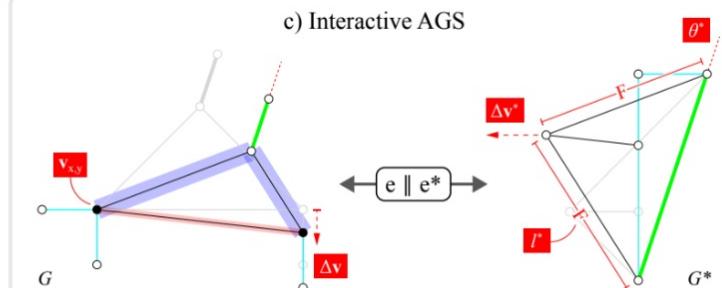
a) Algebraic graph statics (AGS)



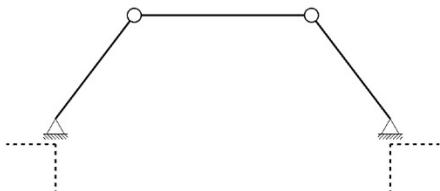
b) “Bi-directional” AGS



c) Interactive AGS



Cables / Arches

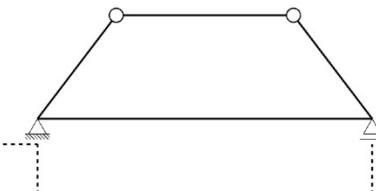


b (bars): 3
r (reactions): 4
n (nodes): 4

$$\boxed{b + r < 2*n}$$

$3 + 4 < 2*4$
 Unstable!

Arch-cables

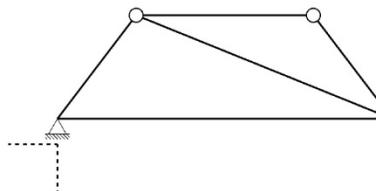


b (bars): 4
r (reactions): 3
n (nodes): 4

$$\boxed{b + r < 2*n}$$

$4 + 3 < 2*4$
 Unstable!

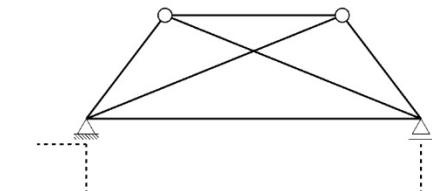
Trusses



b (bars): 5
r (reactions): 3
n (nodes): 4

$$\boxed{b + r = 2*n}$$

$5 + 3 = 2*4$
 Stable!
 and
 statically determinate!

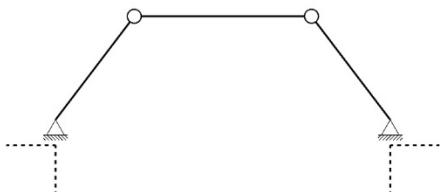


b (bars): 6
r (reactions): 3
n (nodes): 4

$$\boxed{b + r > 2*n}$$

$6 + 3 > 2*4$
 Stable!
 but
 statically indeterminate!

Cables / Arches

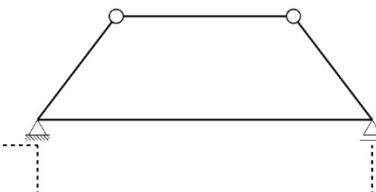


b (bars): 3
r (reactions): 4
n (nodes): 4

$$\boxed{b + r < 2*n}$$

$3 + 4 < 2*4$
 Unstable!

Arch-cables

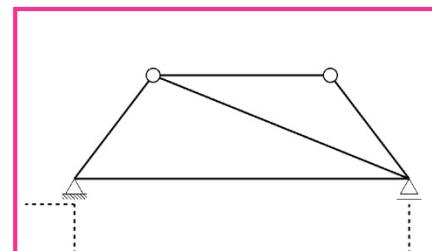


b (bars): 4
r (reactions): 3
n (nodes): 4

$$\boxed{b + r < 2*n}$$

$4 + 3 < 2*4$
 Unstable!

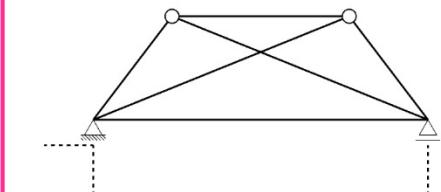
Trusses



b (bars): 5
r (reactions): 3
n (nodes): 4

$$\boxed{b + r = 2*n}$$

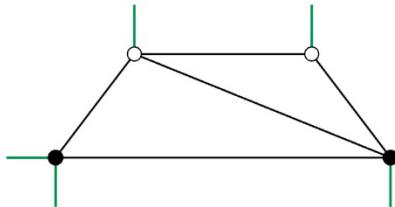
$5 + 3 = 2*4$
 Stable!
 and
 statically determinate!



b (bars): 6
r (reactions): 3
n (nodes): 4

$$\boxed{b + r > 2*n}$$

$6 + 3 > 2*4$
 Stable!
 but
 statically indeterminate!



b (bars): 5

e (external forces): 5

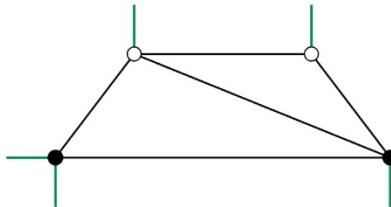
n (nodes): 4

$$\text{DOF} = b + e - (2 \cdot n)$$

$$5 + 5 - (2 \cdot 4) = 2$$

We must define the two external loads!

We have the freedom to define the two external loads!



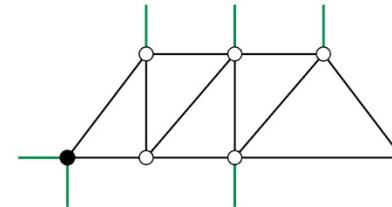
b (bars): 5
e (external forces): 5
n (nodes): 4

$$\text{DOF} = b + e - (2 \cdot n)$$

$$5 + 5 - (2 \cdot 4) = 2$$

We must define the two external loads!

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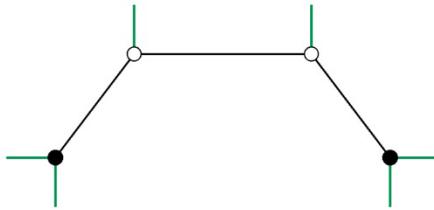
b (bars): 11
e (external forces): 7
n (nodes): 7

$$\text{DOF} = b + e - (2 \cdot n)$$

$$11 + 7 - (2 \cdot 7) = 4$$

We must define the four external loads!

We have the freedom to define the four external loads!



b (bars): 3

e (external forces): 6

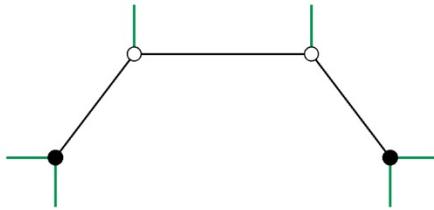
n (nodes): 4

$$\text{DOF} = b + e - (2 \cdot n)$$

$$3 + 6 - (2 \cdot 4) = 1$$

We must define one of
the external loads!

We have the freedom to
define one of the
external loads but not
the other one!



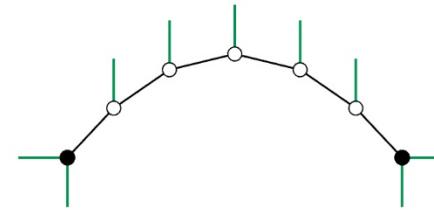
b (bars): 3
e (external forces): 6
n (nodes): 4

$$\boxed{\text{DOF} = b + e - (2 \cdot n)}$$

$$3 + 6 - (2 \cdot 4) = 1$$

We must define one of the external loads!

We have the freedom to define one of the external loads but not the other one!



b (bars): 6
e (external forces): 9
n (nodes): 7

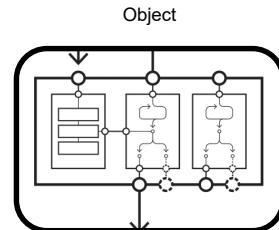
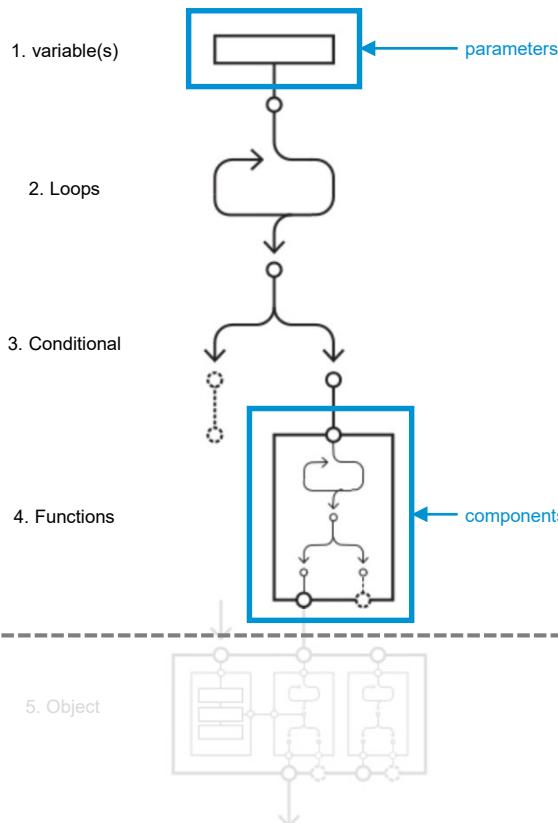
$$\boxed{\text{DOF} = b + e - (2 \cdot n)}$$

$$6 + 9 - (2 \cdot 7) = 1$$

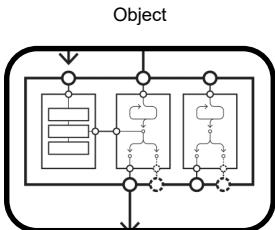
We must define one of the external loads!

We have the freedom to define only one of the external loads but not the other ones!

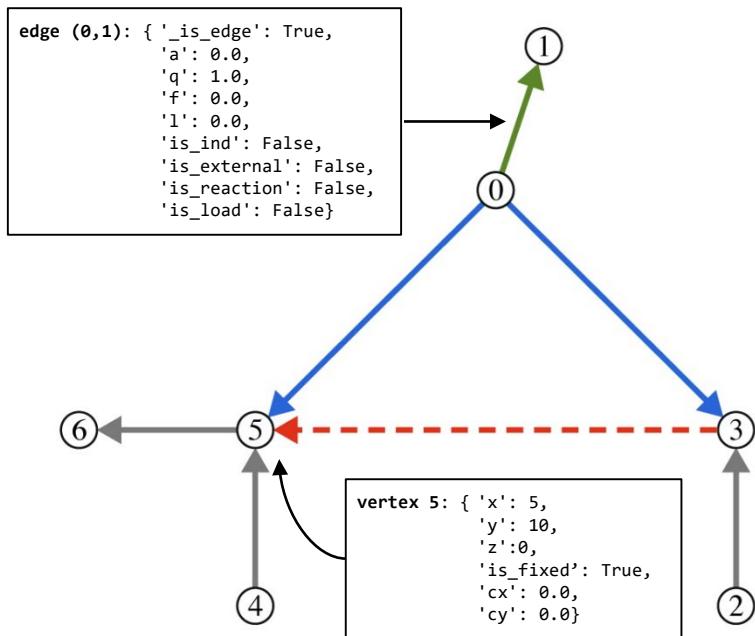
linear
programming
(Grasshopper)



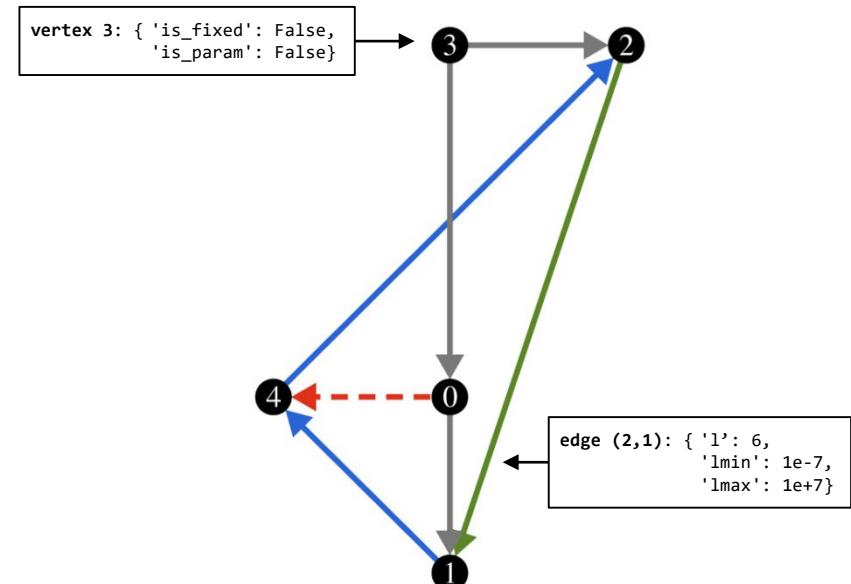
object-oriented
programming



form data



force data





COMPAS

Open-source, Python-based framework for computational research and collaboration
in architecture, engineering and digital fabrication

```
    ...+ vertex[key] - p[0])  
    attr['y'] += d * (c[1] - p[1])  
    attr['z'] += d * (c[2] - p[2])  
  
if callback:  
    callback(mesh, k, callback_args)  
  
smooth_mesh_length(mesh, lmin, lmax, fixed=None)  
  
if callback:  
    if not callable(callback):  
        raise Exception('Callback is not callable')  
  
fixed = fixed or []  
fixed = set(fixed)
```