Assignment No 10

Implement C++ Program for Expression Conversion as infix to postfix and its evaluation using stack based on given conditions :

1. Operands and Operator, both must be single Character

2. Input Postfix Expression must be in desired format

3. only +, -, *, / Operators are expected.

Theory:

When you write an arithmetic expression such as B * C, the form of the expression provides you with information so that you can interpret it correctly. In this case we know that the variable B is being multiplied by the variable C since the multiplication operator * appears between them in the expression. This type of notation is referred to as infix since the operator is in between the two operands that it is working on.

Consider another infix example, A + B * C. The operators + and * still appear between the operands, but there is a problem. Which operands do they work on? Does the + work on A and B or does the * take B and C? The expression seems ambiguous.

In fact, you have been reading and writing these types of expressions for a long time and they do not cause you any problem. The reason for this is that you know something about the operators + and *. Each operator has a precedence level. Operators of higher precedence are used before operators of lower precedence. The only thing that can change that order is the presence of parentheses. The precedence order for arithmetic operators places multiplication and division above addition and subtraction. If two operators of equal precedence appear, then a left-to-right ordering or associativity is used.

Let’s interpret the troublesome expression A + B * C using operator precedence. B and C are multiplied first, and A is then added to that result. (A + B) * C would force the addition of A and B to be done first before the multiplication. In expression A + B + C, by precedence (via associativity), the leftmost + would be done first.

Although all this may be obvious to you, remember that computers need to know exactly what operators to perform and in what order. One way to write an expression that guarantees there will be no confusion with respect to the order of operations is to create what is called a fully

parenthesized expression. This type of expression uses one pair of parentheses for each operator. The parentheses dictate the order of operations; there is no ambiguity. There is also no need to remember any precedence rules.

The expression A + B * C + D can be rewritten as ((A + (B * C)) + D) to show that the multiplication happens first, followed by the leftmost addition. A + B + C + D can be written as (((A + B) + C) + D) since the addition operations associate from left to right.

There are two other very important expression formats that may not seem obvious to you at first. Consider the infix expression A + B. What would happen if we moved the operator before the two operands? The resulting expression would be + A B. Likewise, we could move the operator to the end. We would get A B +. These look a bit strange.

These changes to the position of the operator with respect to the operands create two new expression formats, prefix and postfix. Prefix expression notation requires that all operators precede the two operands that they work on. Postfix, on the other hand, requires that its operators come after the corresponding operands. A few more examples should help to make this a bit clearer.

A + B * C would be written as + A * B C in prefix. The multiplication operator comes immediately before the operands B and C, denoting that * has precedence over +. The addition operator then appears before the A and the result of the multiplication.

In postfix, the expression would be A B C * +. Again, the order of operations is preserved since the * appears immediately after the B and the C, denoting that * has precedence, with + coming after. Although the operators moved and now appear either before or after their respective operands, the order of the operands stayed exactly the same relative to one another.

Table 2: Examples of Infix, Prefix, and Postfix

Infix Expression	Prefix Expression	Postfix Expression
A + B	+ A B	A B +
A + B * C	+ A * B C	A B C * +

Now consider the infix expression (A + B) * C. Recall that in this case, infix requires the parentheses to force the performance of the addition before the multiplication. However, when A + B was written in prefix, the addition operator was simply moved before the operands, + A B. The result of this operation becomes the first operand for the multiplication. The multiplication operator is moved in front of the entire expression, giving us * + A B C. Likewise, in postfix A B + forces the addition to happen first. The multiplication can be done to that result and the remaining operand C. The proper postfix expression is then A B + C *.

Consider these three expressions again (see Table 3). Something very important has happened. Where did the parentheses go? Why don't we need them in prefix and postfix? The answer is that the operators are no longer ambiguous with respect to the operands that they work on. Only infix notation requires the additional symbols. The order of operations within prefix and postfix expressions is completely determined by the position of the operator and nothing else. In many ways, this makes infix the least desirable notation to use.

Table 3: An Expression with Parentheses

Infix Expression	Prefix Expression	Postfix Expression
(A + B) * C	* + A B C	A B + C *

Table 4 shows some additional examples of infix expressions and the equivalent prefix and postfix expressions. Be sure that you understand how they are equivalent in terms of the order of the operations being performed.

Table 4: Additional Examples of Infix, Prefix, and Postfix

Infix Expression	Prefix Expression	Postfix Expression
A + B * C + D	+ + A * B C D	A B C * + D +
(A + B) * (C + D)	* + A B + C D	A B + C D + *
A * B + C * D	+ * A B * C D	A B * C D * +
A + B + C + D	+ + + A B C D	A B + C + D +

Infix Notation

We write expression in infix notation, e.g. a-b+c, where operators are used in-between operands. It is easy for us humans to read, write and speak in infix notation but the same does not go well with computing devices. An algorithm to process infix notation could be difficult and costly in terms of time and space consumption.

Prefix Notation

In this notation, operator is prefixed to operands, i.e. operator is written ahead of operands. For example +ab. This is equivalent to its infix notation a+b. Prefix notation is also known as Polish Notation.

Postfix Notation

This notation style is known as Reversed Polish Notation. In this notation style, operator is postfixed to the operands i.e., operator is written after the operands. For example ab+. This is equivalent to its infix notation a+b.

The below table briefly tries to show difference in all three notations −

S.n.	Infix Notation	Prefix Notation	Postfix Notation
1	a + b	+ a b	a b +
2	(a + b) * c	* + a b c	a b + c *
3	a * (b + c)	* a + b c	a b c + *
4	a / b + c / d	+ / a b / c d	a b / c d / +
5	(a + b) * (c + d)	* + a b + c d	a b + c d + *
6	((a + b) * c) - d	- * + a b c d	a b + c * d -

Parsing Expressions

As we have discussed, it is not very efficient way to design an algorithm or program to parse infix notations. Instead, these infix notations are first converted into either postfix or prefix notations and then computed.

To parse any arithmetic expression, we need to take care of operator precedence and associativity also.

Precedence

When an operand is in between two different operator, which operator will take the operand first, is decided by the precedence of an operator over others. For example −

As multiplication operation has precedence over addition, b * c will be evaluated firs. A table of operator precedence is provided later.

Associativity

Associativity describes the rule where operators with same precedence appear in an expression. For example, in expression a+b−c, both + and − has same precedence, then which part of expression will be evaluated first, is determined by associativity of those operators. Here, both + and − are left associative, so the expression will be evaluated as (a+b−c).

Precedence and associativity, determines the order of evaluation of an expression. An operator precedence and associativity table is given below (highest to lowest) −

S.n.	Operator	Precedence	Associativity
1	Esponentiation ^	Highest	Right Associative
2	Multiplication (*) & Division (/)	Second Highest	Left Associative
3	Addition (+) & Subtraction (−)	Lowest	Left Associative

The above table shows the default behavior of operators. At any point of time in expression evaluation, the order can be altered by using parenthesis. For example −

In a+b*c, the expression part b*c will be evaluated first, as multiplication as precedence over addition. We here use parenthesis to make a+b be evaluated first, like (a+b)*c.

Postfix Evaluation Algorithm

We shall now look at the algorithm on how to evaluate postfix notation −

Step 1 − scan the expression from left to right

Step 2 − if it is an operand push it to stack

Step 3 − if it is an operator pull operand from stack and perform operation

Step 4 − store the output of step 3, back to stack

Step 5 − scan the expression until all operands are consumed

Step 6 − pop the stack and perform operation

Conclusion: We have implement C++ program for expression conversion.