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Traffic-responsive signalling control through a modular/switching model represented via DTPN

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***Abstract*— Urban signalized traffic areas are considered in this paper, with the aim of minimizing congestion situations via a traffic-responsive signalling control procedure founded on a hierarchical Petri net (PN) representation of the system. The higher level of the PN representation consists of net modules, each one representing an intersection, a road, a signal staging, etc.; the description of each module in terms of deterministic-timed Petri nets (DTPN) is given at the lower level. Such a representation leads to a corresponding two-level control procedure. The high-level control system, which acts over the modular representation, switches among internal module structures so as to modify some parts of the model of the traffic system (e.g., signal plans, turning rates, etc.), depending on both state and time. The low-level control system, which acts over the DTPN representation, optimizes the performances of the traffic system, by solving a mathematical programming problem which minimizes the number of vehicles in the system. In the paper, the adopted model of the signalized urban area is briefly presented, and the two-level representation and the control system are described in details.**

I. INTRODUCTION

Traffic signalling control [1] of coordinated urban areas is often implemented in connection with two kinds of strategies. *Fixed-time strategies* consider a given time of a day and determine the optimal splits, the optimal cycle time, or even the optimal staging, based on historical values of traffic demand in the considered area. On the contrary, *traffic-responsive strategies* make use of real-time measurements to compute the suitable signal settings.

The control strategy here proposed belongs to this latter class, and is defined with reference to a hierarchical PN-based representation of the traffic area. In particular, two levels of representation are provided: the *high-level* representation consists of *net modules*, where each module represents an intersection, a road, a signal staging, etc.; the *low-level* representation is provided in terms of deterministic-timed Petri nets (DTPNs) [2]. A major advantage in using modules to build the DTPN representation of the system is the independence of the modules from their *internal representation*, that is, it is possible to obtain different low-level nets by simply switching the internal representation of a module from a certain structure (in terms of places, transitions, arcs, and tokens) to another one. The importance of this issue stands in the fact that the high-level representation maintains its validity throughout the day, whereas the low-level one switches in order to take into account different signal plans, or different turning rates at intersections.

Such a "time-invariance" property of the high-level representation results to be fruitful for the relevant control system, which again consists of two parts. A *high-level* control system switches among the internal module representations in order to modify the representation of the traffic system according to the system state, that is, the state of the DTPN, and/or based on some external variables; a *low-level* control system optimizes the performances of the traffic system by solving a mathematical programming problem which minimizes the number of vehicles in the considered area, that is, the number of tokens in the DTPN. The decision variables of the optimization problem are the *area stages*, that is, the time intervals during which a given combination of green and red lights does not change in a signalized area with several traffic signals. The optimization is accomplished over the low-level representation of the traffic area.

II. THE SIGNALIZED TRAFFIC AREA

The traffic area consists of n_I *intersections* I_i, $i = 1,\ldots,n_I$, and n_R *roads* R_j, $j = 1,\ldots,n_R$. Each road is characterized by its *number of lanes* s_j and its *capacity* c_j, that is, the maximum number of vehicles which can stay at once in the road itself. Let $R_{j(k)}$, $k = 1,\ldots,s_j$, be the k-th *lane* in road R_j. Moreover, eastbound and westbound directions, as well as southbound and northbound, are separately considered, that is, each direction is modelled by a road. An example of a traffic area which includes 2 intersections and 12 roads is in Fig. 1.

Let $IN(I_i)$ and $OUT(I_i)$ be the sets of *incoming roads* and *outgoing roads*, respectively, of intersection I_i. A *turning rate* value $\alpha_{j(k),l(h)}$, with $0 \le \alpha_{j(k),l(h)} \le 1$, is associated with each ordered pair $(R_{j(k)}, R_{l(h)})$, with $R_j \in IN(I_i)$, $k = 1,\ldots,s_j$, and $R_l \in OUT(I_i)$, $h = 1,\ldots,s_l$. Such a value expresses the percentage of vehicles coming from the k-th lane in road R_j, that is $R_{j(k)}$, and going to the h-th lane in road R_l, that is $R_{l(h)}$. A value $\alpha_{j(k),l(h)} = 0$ means that direction $R_{l(h)}$ is forbidden for vehicles coming from $R_{j(k)}$, whereas $\alpha_{j(k),l(h)} = 1$ represents a mandatory direction. To guarantee the flow conservation, the condition $\sum_{l:R_l \in OUT(I_i)} \sum_{h=1}^{s_l} \alpha_{j(k),l(h)} = 1$, $\forall R_j \in IN(I_i)$, $\forall k = 1,\ldots,s_j$, must always be verified for any signalized intersection I_i. In multi-lane roads, vehicles are allowed to change lane according to the relevant traffic rules. In this connection, let $\beta_{j(k)}$ be the percentages of vehicles, with $0 \le \beta_{j(k)} \le 1$, which exit from road R_j using lane $R_{j(k)}$, $k = 1,\ldots,s_j$. Obviously, it must be $\sum_{k=1}^{s_j} \beta_{j(k)} = 1$, $\forall R_j : s_j > 1$.

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In order to model the behaviour of vehicles when crossing an intersection, the whole intersection area is divided into a finite number of parts (*crossing sections*), so as to take into account the physical space that a vehicle crossing the intersection occupies [2]. In this connection, let $CS_{i,q}$, $q = 1,\ldots,Q_i$, be the q-th crossing section of signalized intersection I_i. The number of crossing sections within an intersection mainly depends on the geometry of the intersection itself (as an example, intersection I_1 in Fig. 1 is divided into 8 crossing sections whereas I_2 in only 4). The choice of using crossing sections allows an efficient management of macroscopic entities (turning rates $\alpha_{j(k),l(h)}$ and percentages $\beta_{j(k)}$) within a microscopic representation tool (the DTPN).

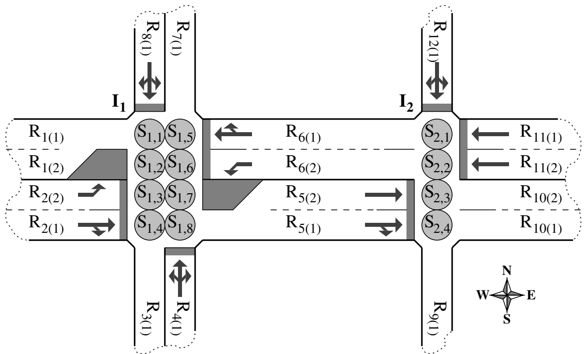

Fig. 1. An urban traffic area.

A multi-stage *traffic signal* can be associated with a signalized intersection. All traffic signals in the considered traffic area are here grouped in a finite number V of *signalized areas* SA_v, $v = 1,\ldots,V$. For each signalized area, one or more stage specifications can be defined (as usually is in real traffic systems). In this connection, let the h-th stage specification, $h = 1,\ldots,H_v^s$, of the v-th signalized area, $v = 1,\ldots,V$, consist of $F_{v,h}$ *"area" stages*, namely $\psi_{v,h,f}$, $f = 1,\ldots,F_{v,h}$. The duration of each area stage is not fixed, but constrained between a lower and an upper bound. Then, it can change in order to differently split the cycle, according to the traffic conditions. However, a default value $\psi^\star_{v,h,f}$ is also associated with stage f of stage specification h of the v-th signalized area, $v = 1,\ldots,V$, $h = 1,\ldots,H_v^s$, $f = 1,\ldots,F_{v,h}$. On the contrary, the *cycle time* of each signalized area, and of each stage specification, is assumed to be constant and equal to the fixed value C, that is, a global cycle time exists in the proposed model (different cycle times could be considered by assuming decentralized control systems, each one acting on a given signalized area). Then, the following constraint must hold for any possible area stage specification: $\sum_{f=1}^{F_{v,h}} \psi_{v,h,f} = C$, $v = 1,\ldots,V$, $h = 1,\ldots,H_v^s$. For a detailed description of the model, the reader can refer to [2].

III. THE MODEL REPRESENTATION

The signalized traffic area is microscopically represented by means of deterministic-timed Petri nets (DTPNs) [2]. The overall Petri net, namely, the DTPN^o, is obtained by merging, through a suitable procedure (not reported here for the sake of brevity), the three following distinct nets:

- the DTPN^a, representing signalized intersections and roads. Tokens within such a net model either vehicles in the urban area, or the availability of crossing sections within an intersection, or the available room of roads. Such a net includes both immediate and timed transitions: a timed transition represents the event of traversing a crossing section, or the event of travelling across a road; the firing time of timed transitions is assumed fixed and represents the average time necessary to traverse a crossing section or to travel across a road. Note that, the DTPN^a presents several conflicts necessary to model all the allowed vehicle drivers' behaviours in the traffic system;
- the DTPN^s, representing the staging of signalized areas. Such a net models all the area stages, and includes both immediate and timed transitions. A timed transition represents the event of being in a specific area stage, that is, the firing time of a timed transition coincides with the length of that area stage. The firing time of such timed transitions is not fixed and ranges in the interval $[d_j^{\min}, d_j^{\max}]$, where $d_j^{\min}$ and $d_j^{\max}$ are, respectively, the minimum and the maximum green time of the area stage represented by that timed transition. In this net, some places are connected with certain immediate transitions of the DTPN^a, so that vehicles entering the intersections act in accordance with green and red lights;
- the DTPN^c, representing macroscopic entities management. Such a net solves conflicts which are in the DTPN^a, according with turning rates α or percentages β. This is accomplished by connecting certain places of the DTPN^c with certain conflicting transitions of the DTPN^a, thus preventing the firing of one between two or more conflicting transitions. Such a net only includes immediate transitions.

A. High-level representation

The DTPN representing the traffic area can be considered as composed of a finite number of *DTPN-modules*, each one intended to the representation of a particular element of the model (an intersection, a road, a signalized area, and so on). A DTPN-module is a DTPN itself, which may be regarded as delimited by some of its immediate intersections, which are named *bound transitions*. In this connection, a DTPN-module interacts with the other modules in the overall representation via its bound transitions, that is, each bound transition of a certain module coincides with one (or more than one) bound transition of another module. A formal definition of DTPN-module is the following.

Definition 1: A DTPN-module (NM) is a couple $\{B, I, F\}$ where B is a finite set of p "bound" transitions, $p > 0$, I is the "internal representation" of the module, that is, a DTPN $\{P, T, Pre, Post, D, M_0\}$, and $F \subset (B \times I) \cup (I \times B)$ is a set of directed arcs between the bound transitions and the internal representation.

In the considered traffic model representation, five kinds of DTPN-modules are present, namely:

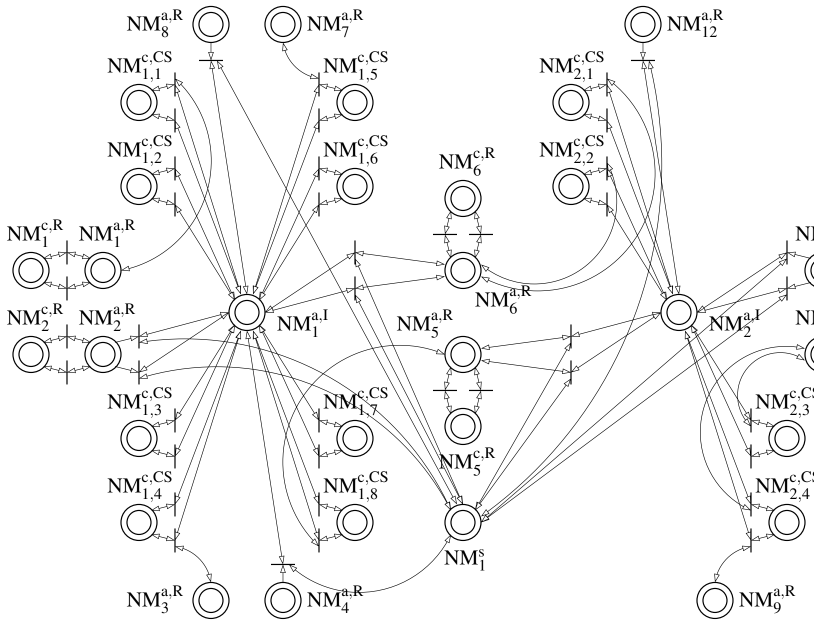

Fig. 2. The high-level representation of the signalized traffic area in Fig. 1.

- modules representing intersections: $NM_i^{a,I}$ is the DTPN-module representing I_i, $i = 1,\dots,n_I$;
- modules representing roads: $NM_j^{a,R}$ is the DTPN-module representing R_j, $j = 1,\dots,n_R$;
- modules representing signalized areas: NM_v^s is the DTPN-module representing SA_v, $v = 1,\dots,V$;
- modules representing the management of turning rate values associated with crossing sections: $NM_{i,q}^{c,CS}$ is the DTPN-module managing turning rate values in $CS_{i,q}$, $i = 1,\dots,n_I$, $q = 1,\dots,Q_i$;
- modules representing the management of exiting percentages of roads: $NM_j^{c,R}$ is the DTPN-module managing exiting percentages in R_j, $j : s_j > 1$, $j = 1,\dots,n_R$.

The $DTPN^a$, the $DTPN^s$, and the $DTPN^c$ can be determined by suitably merging the modules (the details about the merging procedure are here omitted for brevity). In particular, the $DTPN^a$ can be determined through a procedure f^a which suitably merges modules $NM_i^{a,I}$, $i = 1,\dots,n_I$, and $NM_j^{a,R}$, $j = 1,\dots,n_R$, that is

$$DTPN^a = f^a\left[NM_i^{a,I}, i = 1,\dots,n_I, NM_{j,h}^{a,R}, j = 1,\dots,n_R\right] \quad (1)$$

In the same way, the $DTPN^s$ and the $DTPN^c$ can be determined by respectively merging modules NM_v^s, $v = 1,\dots,V$, and modules $NM_{i,q}^{c,CS}$, $i = 1,\dots,n_I$, $q = 1,\dots,Q_i$, and $NM_j^{c,R}$, $j = 1,\dots,n_R$, that is

$$DTPN^s = f^s\left[NM_v^s, v = 1,\dots,V\right] \quad (2)$$

$$DTPN^c = f^c\big[NM_{i,q}^{c,CS}, i = 1,\dots,n_I, q = 1,\dots,Q_i, NM_j^{c,R}, j = 1,\dots,n_R\big] \quad (3)$$

Finally, let F^o a procedure which builds the overall DTPN representing the traffic and transportation system, starting from nets $DTPN^a$, $DTPN^s$, and $DTPN^c$, that is

$$DTPN^o = F^o\left[DTPN^a, DTPN^s, DTPN^c\right] \quad (4)$$

As an example, consider again the urban traffic area in Fig. 1, which consists of 2 intersections and 12 roads (6 single-lane and 6 two-lanes roads). Intersection I_1 has 8 crossing sections, whereas I_2 has 4. Moreover, only one signalized area controls the traffic flows in the system. Then, the overall system can be represented by 33 DTPN-modules. By applying the merging procedures, the resulting module representation of the overall system is that of Fig. 2.

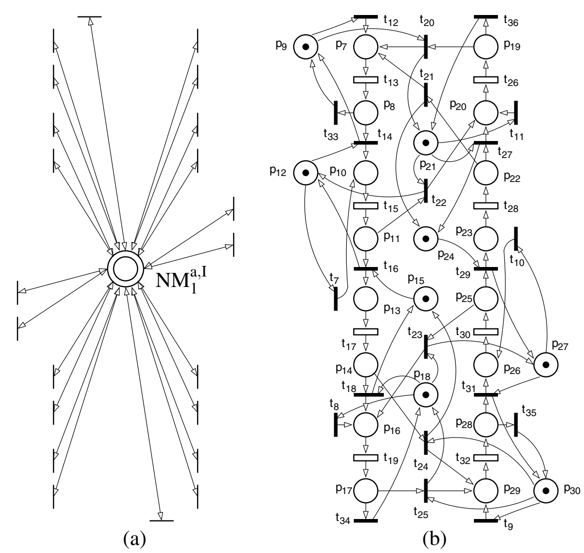

Fig. 3. (a) DTPN-module $NM_1^{a,I}$ and (b) its internal representation (bound transitions are depicted as thick lines).

The module representing intersection I_1, namely $NM_1^{a,I}$, has 22 bound transitions (see Fig. 3a), which interact with the bound transitions of: the 8 modules representing the management of turning rate values associated with its crossing sections ($NM_{1,q}^{c,CS}$, $q = 1,\dots,8$), the 4 modules representing its incoming roads ($NM_j^{a,R}$, $j = 2,4,6,8$), the 4 modules

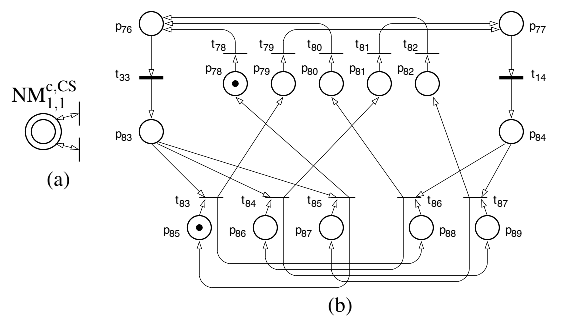

Fig. 4. (a) DTPN-module $\mathrm{NM}^{\mathrm{c,CS}}_{1,1}$ and (b) its internal representation.

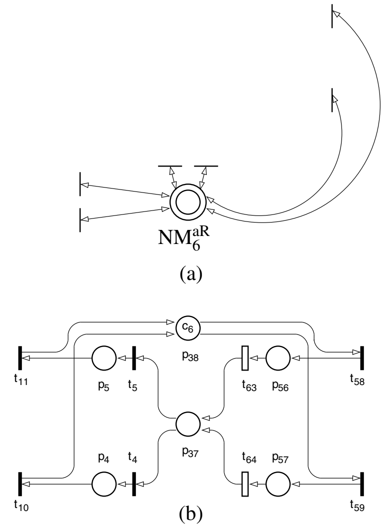

Fig. 5. (a) DTPN-module $\mathrm{NM}^{\mathrm{a,R}}_{6}$ and (b) its internal representation.

representing its outgoing roads ($\mathrm{NM}^{\mathrm{a,R}}_{j}$, $j = 1,3,5,7$), and the module representing the signalized area ($\mathrm{NM}^{\mathrm{s}}_{1}$). The adopted internal representation of $\mathrm{NM}^{\mathrm{a,I}}_{1}$ is in Fig. 3b. As an example of module representing the management of turning rate values, the reader can refer to the module of Fig. 4 which manages values associated with crossing section $CS_{1,1}$. In such a module, transitions t_{14} and t_{33} are alternately enabled in order to force the firing sequence $t_{33}t_{14}t_{33}t_{14}t_{33}$, so that transition t_{33} fires in the 60% of the times, and t_{14} fires in the remaining 40% (obviously, by considering time intervals long enough). An example of a module representing a road is in Fig. 5. Such a module ($\mathrm{NM}^{\mathrm{a,R}}_{6}$) represents the two-lane road R_6. It has 6 bound transitions, which interact with two bound transitions of the module representing I_1, that is, $\mathrm{NM}^{\mathrm{a,I}}_{1}$, with two bound transitions of $\mathrm{NM}^{\mathrm{a,I}}_{2}$, and with the two bound transitions of the module $\mathrm{NM}^{\mathrm{c,R}}_{6}$, which manages the exiting percentages β of such a road. Finally, the module in Fig. 6 represents the signalized area SA_1. In such a module, bound transitions coincide with some of those transitions representing I_1 and I_2. The stage specification defined through the internal representation of $\mathrm{NM}^{\mathrm{s}}_{1}$ (see Fig. 6b) consists of five area stages: in the first area stage, transitions t_8, t_{11}, and t_{50} are enabled, so that vehicles approaching I_1 from $R_{2(1)}$ and $R_{6(1)}$, and I_2 from $R_{12(1)}$, find green lights; in the second area stage, transitions t_8, t_{11}, t_{46}, t_{47}, t_{48}, and t_{49} are enabled, so that vehicles approaching I_1 from $R_{2(1)}$ and $R_{6(1)}$, and I_2 from $R_{5(1)}$, $R_{5(2)}$, $R_{11(1)}$, and $R_{11(2)}$, find green lights; and so on. The durations of the five area stages are represented through the firing time (variable) of timed transitions t_{68}, t_{70}, t_{72}, t_{74}, and t_{76}.

A major advantage in using modules to build the DTPN representing the overall system is the independence of the modules from their internal representation, that is, their bound transitions do not change whenever their internal representations are modified. That results in the possibility of providing, by means of the definition of different internal representations of a same DTPN-module, different $\mathrm{DTPN}^{\mathrm{o}}$ representing the overall system. In particular, in the proposed approach, it is assumed that different internal representations exist for modules representing signalized areas, the management of turning rate values associated with crossing sections, and the management of exiting percentages of roads, whereas only one internal representation is assumed for modules representing intersections and roads (however, different internal representations could be defined also for modules $\mathrm{NM}^{\mathrm{a,I}}_{i}$ and $\mathrm{NM}^{\mathrm{a,R}}_{j}$, so as to consider, for instance, different travel times across a road or a crossing section). In this connection, let H^{s}_{v}, $H^{\mathrm{c,CS}}_{i,q}$, and $H^{\mathrm{c,R}}_{j}$ be the number of internal representations provided for modules $\mathrm{NM}^{\mathrm{s}}_{v}$, $\mathrm{NM}^{\mathrm{c,CS}}_{i,q}$, and $\mathrm{NM}^{\mathrm{c,R}}_{j}$, respectively. Then, let

- $\mathrm{NM}^{\mathrm{s}}_{v,h}$, $v = 1,\ldots,V$, $h \in \{1,\ldots,H^{\mathrm{s}}_{v}\}$, be the DTPN-module representing SA_v through the h-th internal representation;
- $\mathrm{NM}^{\mathrm{c,CS}}_{i,q,h}$, $i = 1,\ldots,n_I$, $q = 1,\ldots,Q_i$, $h \in \{1,\ldots,H^{\mathrm{c,CS}}_{i,q}\}$, be the DTPN-module representing the management of turning rate values associated with $CS_{i,q}$ through the h-th internal representation;
- $\mathrm{NM}^{\mathrm{c,R}}_{j,h}$, $j = 1,\ldots,n_R$, $h \in \{1,\ldots,H^{\mathrm{c,R}}_{j}\}$, be the DTPN-module representing the management of exiting percentages of R_j, $j : s_j > 1$, through the h-th internal representation.

In turn, the equations which provide $\mathrm{DTPN}^{\mathrm{s}}$ and $\mathrm{DTPN}^{\mathrm{c}}$ are now functions of the internal representation h, that is, (2) and (3) become

$$\mathrm{DTPN}^{\mathrm{s}} = f^{\mathrm{s}}\left[\mathrm{NM}^{\mathrm{s}}_{v,h}, v = 1,\ldots,V, h \in \{1,\ldots,H^{\mathrm{s}}_{v}\}\right] \tag{5}$$

$$\begin{aligned}\mathrm{DTPN}^{\mathrm{c}} = f^{\mathrm{c}}\big[&\mathrm{NM}^{\mathrm{c,CS}}_{i,q,h}, i = 1,\ldots,n_I, q = 1,\ldots,Q_i,\\ &h \in \{1,\ldots,H^{\mathrm{c,CS}}_{i,q}\},\\ &\mathrm{NM}^{\mathrm{c,R}}_{j,h}, j = 1,\ldots,n_R, h \in \{1,\ldots,H^{\mathrm{c,R}}_{j}\}\big]\end{aligned} \tag{6}$$

A different internal representation of a module representing the management of turning rate values or exiting percentages may represent different rate values or percentages. As an example, consider the net in Fig. 7. In such an internal representation, transitions t_{14} and t_{33} are alternately enabled, so as to force the firing sequence $t_{33}t_{14}t_{33}t_{33}t_{33}t_{14}t_{33}t_{33}t_{14}t_{33}$, so that transition t_{33} fires in the 70% of the times, and t_{14} fires in the remaining 30%. Then, it is possible to adopt different internal representations in order to take into consideration different percentages to use, for instance, within particular day time intervals. Analogously, a different internal representation of a module representing a signalized area may be

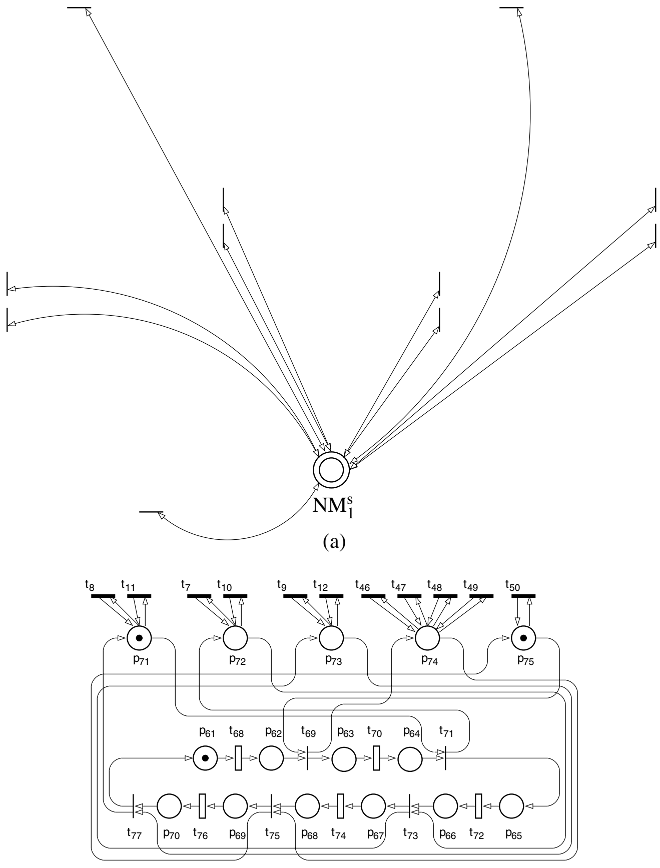

Fig. 6. (a) DTPN-module NM_1^s and (b) its internal representation.

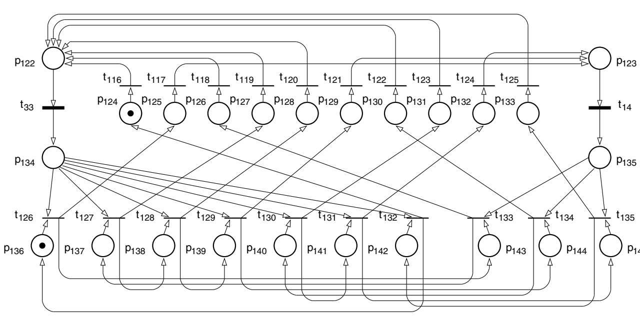

Fig. 7. Alternate internal representation for DTPN-module $NM_{1,1}^{c,CS}$.

used to adopt, within specific time intervals, a different area staging.

B. Low-level representation

The low-level representation of the signalized traffic area can be obtained from the high-level one by simply substituting the modules with their own internal representations. The resulting net is a DTPN.

The low-level net dynamics is expressed by its state equations. The DTPN state equations can be written assuming that the firing times of all timed transitions in the net are integer numbers with reference to a particular time unit δ (*sampling interval*). Such an assumption introduces a certain degree of approximation in the representation of a real traffic system. However, traffic signal plans are usually defined adopting a time unit equal to one second, and, moreover, the slow dynamics of urban traffic systems does not seem to require a shorter time unit. Then, a sampling interval equal to one second is adopted, even though a shorter interval, e.g., 1 millisecond, could be adopted, if necessary. The assumption of integrity allows the analytic representation of the DTPN system state evolution. The DTPN system state, at a given (integer) time instant, is represented by the joint information consisting of both the marking of the net and the residual firing times of transitions. The state evolution takes place by considering alternately *zero-length* time intervals, in which only immediate transitions fire, and *one-second-length* time intervals, in which only timed transitions fire or "burn" one time unit of their firing [3].

IV. THE TRAFFIC CONTROL

The characteristics of modularity proper to the proposed model allow to conceive a two-level traffic control structure. The higher level acts by changing the model of the traffic system whenever it moves to some specified condition, i.e., whenever the system state reaches a-priori fixed threshold values (the model may also change when specific time instants are reached). In fact, such an event enables a suitable switching condition, which, in turn, makes the internal structures of some modules change accordingly. On the other hand, the lower level optimizes the performances of the traffic system, represented by a DTPN-based model whose structure keeps fixed at this level, since it can only be changed by the higher level. The performance optimization is carried out by solving a mathematical programming problem which minimizes the number of vehicles in the system itself.

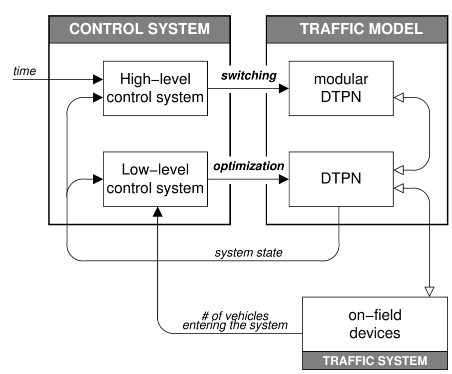

Fig. 8. The control system

The overall structure of the proposed traffic control system is depicted in Fig. 8, where the interactions of the control system itself with both the traffic model and the traffic system are put into evidence.

A. High-level control system

The high-level control system defines the structure of the Petri net representing the traffic model. In particular, the high-level control system provides the best internal representation of some Petri net modules on the basis of the current time instant and of the traffic system state. The meaning of such a choice is to allow the traffic system to switch from a set of signal stage specifications to another set, and to switch from a set of turning rates α and β to another set, then making the DTPN representation valid during the whole of the day (and not only during a specific time interval, e.g., the morning peak hour, where stages and turning rates are assumed fixed).

Let $\delta_v^{\mathrm{s}}(\tau_k)$, $v = 1, \ldots, V$, be the *switching variable*, at time instant τ_k, of the module representing the staging of the v-th signalized area. Then, $\delta_v^{\mathrm{s}}(\tau_k)$ assumes integer values 1 to H_v^{s}. Such a variable determines which internal representation, among the H_v^{s} available, has to be used during the $(k+1)$-th cycle. Moreover, let g_v^{s}, $v = 1, \ldots, V$, be the *switching function* to apply to the marking of the DTPN and to the time instant, in order to compute variable $\delta_v^{\mathrm{s}}(\tau_k)$, that is

$$\delta_v^{\mathrm{s}}(\tau_k) = g_v^{\mathrm{s}}\left[\underline{M}^{\mathrm{o}}(\tau_k), \tau_k\right] \qquad v = 1, \ldots, V \tag{7}$$

Analogously,

$$\delta_{i,q}^{\mathrm{c,CS}}(\tau_k) = g_{i,q}^{\mathrm{c,CS}}\left[\underline{M}^{\mathrm{o}}(\tau_k), \tau_k\right] \qquad i = 1, \ldots, n_I \tag{8}$$

$$\delta_j^{\mathrm{c,R}}(\tau_k) = g_j^{\mathrm{c,R}}\left[\underline{M}^{\mathrm{o}}(\tau_k), \tau_k\right] \qquad j = 1, \ldots, n_R \tag{9}$$

The structures of switching functions are not reported in this paper. However, the DTPN is now a time-dependent system (more specifically, a switching system). Then, equations (5), (6), and (4) are respectively replaced by the following

$$\mathrm{DTPN}^{\mathrm{s}}(\tau_k) = f^{\mathrm{s}}\left[\mathrm{NM}^{\mathrm{s}}_{v,\delta_v^{\mathrm{s}}(\tau_k)}, v = 1, \ldots, V\right] \tag{10}$$

$$\begin{aligned}\mathrm{DTPN}^{\mathrm{c}}(\tau_k) = f^{\mathrm{c}}\Big[&\mathrm{NM}^{\mathrm{c,CS}}_{i,q,\delta_{i,q}^{\mathrm{c,CS}}(\tau_k)}, i = 1, \ldots, n_I, q = 1, \ldots, Q_i, \\ &\mathrm{NM}^{\mathrm{c,R}}_{j,\delta_j^{\mathrm{c,R}}(\tau_k)}, j = 1, \ldots, n_R\Big]\end{aligned} \tag{11}$$

$$\mathrm{DTPN}^{\mathrm{o}}(\tau_k) = F^{\mathrm{o}}\left[\mathrm{DTPN}^{\mathrm{a}}, \mathrm{DTPN}^{\mathrm{s}}(\tau_k), \mathrm{DTPN}^{\mathrm{c}}(\tau_k)\right] \tag{12}$$

The net defined by (12) is the DTPN considered by the low-level control system, in the time interval $[\tau_k, \tau_{k+1})$, to optimize the lengths of stages.

B. Low-level control system

In the proposed model of an urban area, the duration of area stages may vary according with traffic conditions. This is modelled by deterministic-timed transitions of the DTPN$^{\mathrm{s}}$, whose firing times may range within a given interval. In this connection, the objective of the *low-level* control system is to solve an optimization problem which determines the optimal stages (that is, the optimal firing times in the DTPN$^{\mathrm{s}}$).

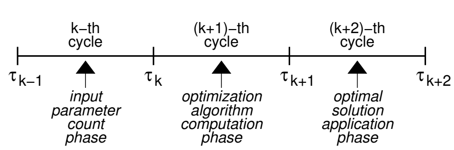

Fig. 9. Schematization of the optimization of area stages

Assume that the number of vehicles which enter the considered area, in the time interval $[\tau_{k-1}, \tau_k)$ (k-th cycle), can be counted (through, for instance, suitable counter devices such as electromagnetic induction loops and/or visual recognizers). This represents the *input parameter count phase* (Fig. 9). Moreover, assume that the traffic system dynamics is "slow" (such an hypothesis is usually fulfilled by urban traffic systems), then such a number can be considered to be an estimate of the number of the vehicles which will enter the considered area in the subsequent time interval, that is, in $[\tau_k, \tau_{k+1})$ ($(k+1)$-th cycle). Then, at τ_k, it is possible to set the firing time of "input" (or "source") transitions (in the proposed Petri net representation, an input transition is a timed transition having no incoming arcs), so that the number of tokens generated by their firings during the time interval $[\tau_k, \tau_{k+1})$ is equal to the number of counted vehicles during the interval $[\tau_{k-1}, \tau_k)$.

The optimal area stages $\psi^{\circ}_{k,v,h,f}$, $v = 1, \ldots, V$, $h = \delta_v^{\mathrm{s}}(\tau_k)$, $f = 1, \ldots, F$, are determined in the time interval $[\tau_k, \tau_{k+1})$ through the solution of an optimization problem whose objective function minimizes the number of tokens within the net (note that, as the numbers of tokens within the DTPN$^{\mathrm{s}}$ and the DTPN$^{\mathrm{c}}$ are constant, it is sufficient to minimize the number of tokens within the DTPN$^{\mathrm{a}}$), by optimizing the firing times of the timed transitions of the DTPN$^{\mathrm{s}}$. This corresponds to minimize the number of vehicles which are in the area at the end of the considered time interval, that is, at τ_{k+1}, by suitably setting the lengths of stages. The value of the objective function is constrained by both the cycle length, and the DTPN$^{\mathrm{o}}$ state equations (not reported here for the sake of brevity), initialized by the marking vector $\underline{M}^{\mathrm{o}}(\tau_k)$. This represents the *optimization algorithm computation phase*. Then, the computed optimal area stages are applied to the real system from time τ_{k+1}, that is, within the $(k+2)$-th semaphoric cycle, which is the *optimal solution application phase*. Note that, if the higher-level control system changes, at τ_{k+1}, one or more internal representations, then the computed optimal values can not be used, since they have been determined over a net which is no more the actual one; in this case, the default values for stages are adopted (see [4] for a detailed description of the control methodology).

V. CONCLUSIONS

In this paper, a two-level representation of urban signalized traffic areas has been introduced and discussed in details. The high modularity of the proposed PN-based model turns out to be a valuable feature, since it makes possible to use the same modular/switching system to rule the traffic flows through the considered signalized intersections during the whole day.

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