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ABSTRACT

Social influence network theory, which is both cognitive and structural, focuses on the contributions of networks of interpersonal influence to the formation of interpersonal agreements and group consensus. It entails a cognitive process when it deals with how actors integrate conflicting influential opinions to form revised opinions on an issue. It entails a social structure when it deals with an influence network that is defined by the pattern and strengths of the interpersonal influences among the members of a group. This article extends our work on social influence network theory, develops a set of formal implications of this theory, and assesses the theory in small groups of dyads, triads, and tetrads.

INTRODUCTION

The process of interpersonal influence that affects actors' attitudes and opinions is an important foundation of their socialization, identity, and decisions. It is also

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an important foundation of actors' efforts to shape their situations by modifying the attitudes and opinions of significant others with whom they interact. In groups, this influence process can produce shared understandings and agreements that define the culture of the group and frame the collective activities of its members. This process is also one of the key foundations of social control.

In classical sociological theory, social control refers to the occurrence and effectiveness of ongoing efforts in a group to formulate, agree upon, and implement collective courses of action (Janowitz 1975). The difficulty in arriving at a collective decision given actors with fixed discrepant preferences—as exemplified by Arrow's (1951) dilemma of social choice—is rendered moot when a social influence process produces consensus. The classical agenda on social control is not only to elucidate such social influence mechanisms, but also (taking the reduction of mechanisms based on coercive domination as an explicit value position) to discover conditions under which noncoercive mechanisms provide an effective basis of social control. Thus, the classical approach to the study of social control focuses on effects of noncoercive interpersonal influences arising from communication, social comparison, and bargaining.

Sociological theory on social influence mechanisms has addressed the origins of bargaining power (social exchange theory), the conditions of cooperation (rational choice theory), and the construction of role and status structures (expectation states theory). Surprisingly, however, there is not a well-developed formal sociological approach to the opinion formation process that may produce consensus in a group. The process of opinion formation can rarely be reduced to accepting or rejecting the consensus of others; typically, individuals form their opinions in a complex interpersonal environment in which influential opinions are in disagreement and liable to change. This article seeks to advance the classical sociological agenda on social influence and social control by showing how networks of interpersonal influence contribute to the formation of interpersonal agreements and consensus in such complex circumstances. Taking a formal approach, it extends the work of symbolic interactionists on the social construction of shared opinion and definitions of situations in dyads to address the problem of how N-actor agreements may develop.

Two interrelated theoretical problems must be solved to develop an account of how social influence networks affect attitudes and opinions and enter into the formation of N-actor agreements. The first problem is the development of a model of the *social process* that forms such agreements, that is, how actors modify their opinions taking into account their own circumstances and the influences of other actors. The second problem is the development of a model of the *social influence structure* that describes the configuration and strengths of interpersonal influences in a particular population, that is, the social structural context in which the influence process occurs. This article addresses both of these problems and builds upon previous work (Friedkin 1986, 1998; Friedkin and Cook 1990; Friedkin and Johnsen 1990). It extends the authors' work on social influence network theory, develops a set of formal implications of this theory, and assesses the theory in small groups of dyads, triads, and tetrads.

SOCIAL INFLUENCE NETWORK THEORY

The theory described in this article has been under development by social psychologists and mathematicians since the 1950s (DeGroot 1974; French 1956; Friedkin and Johnsen 1990; Harary 1959). This line of research began with French's (1956) formal theory of social power, which introduced a simple model of how a network of interpersonal influence enters into the process of opinion formation. Drawing on the algebra of a Markov Chain process, the theory was developed in a more general form by Harary (1959) and DeGroot (1974). These initial formulations described the formation of group consensus, but did not provide an adequate account of settled patterns of disagreement.[1] Friedkin and Johnsen's (1990) generalization of the theory addressed this limitation and integrated this line of social psychological work on opinion formation with another developing line of sociological work on network effects (Burt 1982; Doreian 1981; Erbring and Young 1979; Friedkin 1990; Marsden and Friedkin 1993). The distinguishing characteristic of this theoretical approach is that it attempts to model the flows of interpersonal influence that affect the opinions of actors; compare the work on social decision schemes (Stasser, Kerr, and Davis 1989) and social impact theory (Latane 1981).

Our theory postulates a simple recursive definition for the influence process in a group of N actors:

$$\mathbf{y}^{(t)} = \mathbf{AWy}^{(t-1)} + (\mathbf{I} - \mathbf{A})\mathbf{y}^{(1)} \tag{1}$$

for $t = 2,3,\ldots$, where $\mathbf{y}^{(1)}$ is an N x 1 vector of actors' initial opinions on an issue, $\mathbf{y}^{(t)}$ is an $N \times 1$ vector of actors' opinions at time t, $\mathbf{W} = [w_{ij}]$ is an $N \times N$ matrix of interpersonal influences ($0 \leq w_{ij} \leq 1$, $\Sigma_j^N w_{ij} = 1$), and $\mathbf{A} = \text{diag}(a_{11}, a_{22}, \ldots, a_{NN})$ is an $N \times N$ diagonal matrix of actors' susceptibilities to interpersonal influence on the issue ($0 \leq a_{ii} \leq 1$). Applying equation (1) iteratively, we obtain

$$\mathbf{y}^{(t)} = \mathbf{V}^{(t-1)}\mathbf{y}^{(1)} \tag{2}$$

where,

$$\mathbf{V}^{(t-1)} = (\mathbf{AW})^{(t-1)} + \left[\sum_{k=0}^{t-2} (\mathbf{AW})^k\right] (\mathbf{I} - \mathbf{A}) \tag{3}$$

for $t = 2.3\ldots.$

Six of this model's process assumptions are fundamental. (1) *Cognitive Weighted Averaging*: Actors are assumed to form their revised opinions through a weighted averaging of the influences on them. Flows of interpersonal influence are established by the *repeated* responses of actors to the (possibly changing) influential opinions on the issue. However, actors are not only influenced endogenously by the opinions of other actors, but also exogenously, at each point in the process, by the conditions that have formed their *initial* opinions. The relative weight of the endogenous and exogenous influences for each actor is determined by $\mathbf{A} = [a_{ii}]$, the coefficients of susceptibility to social influence.[2] (2) *Fixed Social Structure*: The social structure of the group of actors is assumed to be fixed during the entire process of opinion formation. This social structure consists of (a) the set of actors, (b) the direct influence network among them, represented by the matrix $\mathbf{W}$, (c) the susceptibilities of the actors to interpersonal influence, represented by the diagonal matrix $\mathbf{A}$, and (d) the actors' initial opinions, represented by the vector $\mathbf{Y}^{(1)}$. (3) *Determinism*: Given the direct influence matrix $\mathbf{W}$, the susceptibility matrix $\mathbf{A}$, and group members' initial opinion vector $\mathbf{Y}^{(1)}$, the subsequent opinion changes in the group are completely determined. (4) *Continuance*: The process of opinion formation in the group continues until all changes of opinion that may occur have played themselves out. (5) *Decomposability*: The opinion formation process is decomposable into time periods, defined by the times $t = 1,2,3,\ldots$, that may not be of the same length in real time. (6) *Simultaneity*: In each time period, simultaneous linear equations yield an accurate prediction of all the influence events that occur during that period.

We work with the following version of this general formulation which stipulates that

$$\mathbf{W} = \mathbf{AC} + \mathbf{I} - \mathbf{A} \tag{4}$$

where $\mathbf{C} = [c_{ij}]$ is an $N \times N$ matrix of relative interpersonal influences $(0 \le c_{ij} \le 1,\ c_{ii} = 0,\ \Sigma_j^N c_{ij} = 1)$[3]. That is, we have equated actors' lack of susceptibility to interpersonal influence with the weight that they place on their initial opinions

$$w_{ii} = 1 - a_{ii} \tag{5}$$

for all i and have distributed the cumulative weight of others ($a_{ii} = \Sigma_j w_{ij}$, $j \ne i$) according to the relative strength of the interpersonal influences, that is, for $i \ne j$

$$w_{ij} = a_{ii}c_{ij} \tag{6}$$

Assuming the process reaches an equilibrium, that is, $\lim_{t \to \infty} \mathbf{y}^{(t)} = \mathbf{y}^{(\infty)}$exists, equation (1) becomes

$$\mathbf{y}^{(\infty)} = \mathbf{AWy}^{(\infty)} + (\mathbf{I} - \mathbf{A})\mathbf{y}^{(1)} \tag{7}$$

and hence

$$(\mathbf{I} - \mathbf{AW})\mathbf{y}^{(\infty)} = (\mathbf{I} - \mathbf{A})\mathbf{y}^{(1)} \tag{8}$$

If, in addition, **I-AW** is nonsingular, then

$$\mathbf{y}^{(\infty)} = (\mathbf{I} - \mathbf{AW})^{-1}(\mathbf{I} - \mathbf{A})\mathbf{y}^{(1)} \tag{9}$$

whence actors' settled opinions are given by

$$\mathbf{y}^{(\infty)} = \mathbf{Vy}^{(1)} \tag{10}$$

where,

$$\mathbf{V} = (\mathbf{I} - \mathbf{AW})^{-1}(\mathbf{I} - \mathbf{A}) \tag{11}$$

More generally, by equation (2) we can obtain equation (10) if

$$\mathbf{V} = \lim_{t \to \infty} \mathbf{V}^{(t)} \tag{12}$$

exists. In either case, **V** is a matrix of reduced-form coefficients describing the total interpersonal effects that transform initial opinions into final opinions. The coefficients in $\mathbf{V} = [v_{\mathrm{ij}}]$ are nonnegative ($0 \leq v_{\mathrm{ij}} \leq 1$) and each row of **V** sums to unity ($\Sigma_j v_{ij} = 1$); hence, v_{ij} gives the *relative weight* of the initial opinion of actor j in determining the final opinion of actor i for all i and j. If $\mathbf{I} - \mathbf{AW}$ is nonsingular, then **V** can be derived directly from equation (11); otherwise, **V** can be estimated numerically from equation (3) for a sufficiently large t when $\lim_{t \to \infty} \mathbf{V}^{(t)}$exists.

Selected Special Cases

Our theory is developed for the *general case* in which there are individual differences in interpersonal influences and opinions, and it subsumes (as special cases) situations in which such individual differences are constrained. These special cases include certain classical situations:

1. A status order in which an individual is located in a stratified influence network,
2. A conformity situation in which a deviate is faced with a fixed consensus of others that the deviate either accepts (moves toward) or rejects,
3. A minority influence situation in which a deviate may change the majority consensual opinion, and
4. An intergroup conflict situation in which disagreement occurs between two factions.

In this section, we show how social influence network theory includes and integrates these special cases.

To see how various situations may be covered by the theory, consider the following example, which is taken from one of the tetrads involved in our experiments; we will describe these experiments later on in the article. In this group, each member (a) was presented with an issue on which opinions could range from 1 to 100, (b) independently formed an initial opinion on the issue, and (c) after a discussion of the issue, settled on a final opinion that may or may not have been in agreement with the settled opinions of certain other members of the group:

$$\mathbf{y}^{(1)} = \begin{bmatrix} 25 \\ 25 \\ 75 \\ 85 \end{bmatrix} \quad \mathbf{y}^{(\infty)} = \begin{bmatrix} 60 \\ 60 \\ 75 \\ 75 \end{bmatrix}$$

The influence network for this group

$$\mathbf{W} = \begin{bmatrix} .220 & .120 & .359 & .300 \\ .147 & .215 & .344 & .294 \\ 0 & 0 & 1 & 0 \\ .089 & .178 & .446 & .286 \end{bmatrix}$$

describes the distribution of relative interpersonal influences on the issue. The main diagonal of $\mathbf{W}$ are the actors' self-weights (w_{ii} = 1-a_{ii}) and, therefore, $\mathbf{A}$ = diag(.780,.785,0,.714) describes the actors' susceptibilities to interpersonal influence. The off-diagonal entries of $\mathbf{W}$ are interpersonal influences. For example, w_{13} = .359 indicates that the direct (unmediated) relative influence of actor 3 on actor 1 is .359. The total effects matrix

$$\mathbf{V} = \begin{bmatrix} .280 & .045 & .551 & .124 \\ .047 & .278 & .549 & .126 \\ 0 & 0 & 1 & 0 \\ .030 & .048 & .532 & .390 \end{bmatrix}$$

indicates the net influence of each actor on every other actor that arises from all of the flows of interpersonal influence (direct and indirect) among the members of the group. For example, $v_{13} = .551$ indicates that a little over 55 percent of the content of actor 1's final opinion is determined by actor 3. Comparing **W** and **V**, we can see that the influence of actor 3 on the other actors has been enhanced by the flows of influence in this network. It also is interesting to note that actors 1 and 2, who were in final agreement, have relatively slight net interpersonal effects on each other. The predicted final opinions are

$$\hat{\mathbf{y}}^{(\infty)} = \mathbf{V}\mathbf{y}^{(1)} = \begin{bmatrix} 60 \\ 60 \\ 75 \\ 75 \end{bmatrix}$$

which correspond exactly to the observed final opinions. The ability of the derived model to exactly reproduce the empirical data is not always assured, but it is not surprising in this case. The fit is always exact when equilibrium opinions are in disagreement (as in the present case) and $\Delta_i \geq 0$; see equation (A6) in the Appendix.

The social structure of the group is described by the distribution of initial opinions $\mathbf{y}^{(1)}$, and the influence network, **W**. The special cases described entail constraints on one or both of these components of the social structure of a group. These constraints can either be naturally occurring features of a group, as they were in the group described above, or they can be experimentally designed conditions under which subjects are placed in order to study a feature of social influence networks, process, or outcomes.

These special cases have an important property in common—they allow a simplification of equation (10),

$$y_i^{(\infty)} = \sum_{j=1}^{N} v_{ij} y_j^{(1)}$$

from which the total interpersonal effect of a subset of actors on another subset of actors can be described in terms of the initial and equilibrium opinions of other group members. This derivation is straightforward in a dyad:

$$y_i^{(\infty)} = \sum_{j=1}^{2} v_{ij} y_j^{(1)} = (1 - v_{ij}) y_i^{(1)} + v_{ij} y_j^{(1)} \tag{13}$$

and, hence,

$$v_{ij} = \frac{y_i^{(\infty)} - y_i^{(1)}}{y_j^{(1)} - y_i^{(1)}} \tag{14}$$

In larger groups a comparable formulation of interpersonal influence is

$$\tilde{v}_{ij} \equiv \sum_{j \neq 1} v_{ij} = \frac{y_i^{(\infty)} - y_i^{(1)}}{\bar{y}_i^{(1)} - y_i^{(1)}} \tag{15}$$

where $\bar{y}_i^{(1)} = \sum_{j \neq i} g_{ij} y_j^{(1)}$ and $g_{ij} = v_{ij}/(1 - v_{ii})$ for $v_{ii} \neq 1$ ($\tilde{v}_{ij} = 0$ for $v_{ii} = 1$). We will show how this general formulation simplifies in certain special cases.

One Actor Who Deviates from a Consensus

Our model captures, as a special case, the classical conformity situation in a group in which influence is exerted on an opinion of one member who deviates from the consensus of other group members. Consider equation (10) in scalar form

$$y_i^{(\infty)} = \Sigma_j v_{ij} y_j^{(1)}$$

with the *additional* stipulation of the existence of a consensus of initial opinions among all actors other than actor i. Let $\hat{y}_j^{(1)}$ equal the *consensus opinion* of all actors $j \neq i$. Hence, for, $j \neq i$,

$$y_i^{(\infty)} = v_{ii} y_i^{(1)} + \sum_{j=1}^{n} v_{ij} y_j^{(1)} = (1 - \tilde{v}_{ij}) y_i^{(1)} + \tilde{v}_{ij} \hat{y}_j^{(1)} \tag{16}$$

and equation (15) simplifies to

$$\tilde{v}_{ij} = \frac{y_i^{(\infty)} - y_i^{(1)}}{\hat{y}_j^{(1)} - y_i^{(1)}} \tag{17}$$

because $\bar{y}_i^{(1)} = \hat{y}_j^{(1)}$. The interpersonal influence is that of a *consensual collective other* on an actor. This formulation of the influence of a unanimous majority on a minority of one is identical to the measure proposed by Goldberg (1954), who referred to it as an "index of proportional conformity." While Goldberg proposed the index without theoretical justification, our approach provides a clear theoretical foundation.

Note that in this formulation, we have not assumed that the consensus is *fixed.* Hence, the initially deviant opinion may influence the opinions of other group members. The influence of the deviant member, actor i, on one of the other group members, actor j, determines the extent of that member's opinion change:

$$v_{ji} = \frac{y_j^{(\infty)} - \hat{y}_j^{(1)}}{y_i^{(1)} - \hat{y}_j^{(1)}} \tag{18}$$

Such influence is consistent with a situation in which the influence of a deviant breaks up the initial consensus of the other group members; it also is consistent with a situation in which the consensus of the other members is maintained, but changes as a result of the influence of the deviate. Such situations are the focus of a line of work on "minority influence" initiated by Moscovici (1985).

A further result is obtained by assuming so-called Asch (Asch 1951) conditions, in which the initial opinions of the other group members represent a *fixed* consensus, that is, where $w_{jj} = 1$ for all $j \neq$ i. for all. For convenience, letting $i = 1$, the influence network for this Asch-type situation is

$$\mathbf{W} = \begin{bmatrix} w_{11} & w_{12} & w_{13} & \cdots & w_{1n} \\ 0 & 1 & 0 & \ldots & 0 \\ 0 & 0 & 1 & \ldots & 0 \\ 0 & 0 & 0 & \ldots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & 0 & \ldots & 1 \end{bmatrix}$$

indicating that, while actor $i = 1$ may be influenced by each of the other group members, these other group members are not subject to any interpersonal influence. In an Asch-type experiment, these actors are confederates of the experimenter who instructs them not to alter their initial opinions. In this case, because the deviate cannot influence the other actors, the only issue is the extent to which the deviate will be influenced.

Moscovici (1976, 1985) has argued that the influence of a minority on a majority entails *compliance* (both public and private acceptance) and that the influence of a majority on a minority entails *conformity* (public but not necessarily private acceptance); that is, the character of the interpersonal influence differs. Although it is possible that an actor will conform to a consensus without changing his or her underlying opinion, it also is possible that such conformity can occur based on a response to one of the other members of the group; that is, the mere presence of a consensus does not necessarily imply that a group-level normative pressure exists. Indeed, it may be that what has been thought of as group-level pressure to conform to a consensus is nothing more than the aggregation of separate interpersonal influences that are exerted by each member of the consensus on the deviate.

Stratified Influence Networks

Many sociologists view the influence process as a source of power inequalities and dominance. In studies of natural groups, it has been shown that groups often quickly generate a consistent or "consensual" rank ordering of group members in terms of prestige and influence. Expectation states theory (Ridgeway and Walker 1995) suggests that these status orders arise because of shared beliefs about the value of certain status characteristics; for example, because actors value expertise similarly, they will tend to rank order the members of a task-oriented group in a similar way. Hence, a stratified influence structure will emerge

$$\mathbf{W} = \begin{bmatrix} w_{11} & w_{22} & \cdots & w_{NN} \\ w_{11} & w_{22} & \cdots & w_{NN} \\ \vdots & \vdots & \cdots & \vdots \\ w_{11} & w_{22} & \cdots & w_{NN} \end{bmatrix}$$

in which all actors accord the same influence to a particular actor and in which an actor's self-weight corresponds to his or her interpersonal influences.[4] Our interest focuses on the consequences of a maintained status order for the resolution of issues.

Status orders include as special cases an egalitarian situation ($w_{11} \approx w_{22} \approx \cdots \approx w_{NN}$) and domination ($w_{11} \approx 1$, $w_{22} \approx w_{33} \approx \cdots \approx w_{NN} \approx 0$). As an implication of the stochasticity of **W**, only *one* actor can be dominant in a status order (if, $w_{ii} = 1$, then $w_{jj} = 0$ for all $j \neq i$). In status orders with a dominant actor, a consensus will appear immediately at $t = 2$. Hence, status orders with a dominant actor are maximally efficient in producing consensus. This conclusion follows directly from equation (1) and the influence structure of a status order.

These implications have a particularly dramatic manifestation in a dyad. In a dyad, the formation of virtual agreement (regardless of $\mathbf{y}^{(1)}$) is consistent with two situations: (1) a status order in which one of the two actors is dominant or (2) an egalitarian influence structure in which $\mathbf{A} \approx \mathbf{I}$ (but $\mathbf{A} \neq \mathbf{I}$). In the former situation, the consensus will be the initial opinion of the dominant actor and will be formed immediately at $t = 2$. In the latter situation, the consensus will be the mean of the initial opinions of the two actors and will be formed very slowly. Indeed, in a dyad, for $\mathbf{A} = \mathbf{I}$, when both actors are maximally accommodating (place no weight on their own opinions) an equilibrium is unattainable.[5]

Influence Networks with Two Factions

Intergroup conflict often takes the form of a disagreement between *two* factions, each of which has a certain opinion (internal consensus) on an issue. In such a situation, the aggregate influence of the members of one faction on an actor in a different faction is

$$\tilde{v}_{i[j]} = \frac{y_i^{(\infty)} - \hat{y}_{[i]}^{(1)}}{\hat{y}_{[j]}^{(1)} - \hat{y}_{[i]}^{(1)}} \tag{19}$$

where $\hat{y}_{[i]}^{(1)}$ is the initial consensus value for every member of i's faction, and similarly for j, and $\tilde{v}_{i[j]}$ is the aggregate influence of j's faction on i. The aggregate influence of a faction on one of its own members is $1 - \tilde{v}_{i[j]}$ or

$$\tilde{v}_{i[i]} = \frac{y_i^{(\infty)} - \hat{y}_{[j]}^{(1)}}{\hat{y}_{[i]}^{(1)} - \hat{y}_{[j]}^{(1)}} \tag{20}$$

where $\hat{y}_{[j]}^{(1)}$ is the initial consensus value for every member of j's faction, and similarly for i, and $\tilde{v}_{i[i]}$ is the aggregate influence of i's faction (which includes i) on i.

MODEL VALIDATION

Our model of social influence captures various types of social influence situations and allows them to be viewed from a single theoretical perspective. Such theoretical integration is an important goal of a formal model. This section introduces the approach taken to examining the predictive accuracy of this model.

This assessment of the model will focus on three issues. First, we will analyze the fit between the observed and predicted equilibrium opinions of groups. This analysis is not only interested in the accuracy of these predictions, but also in an assessment of whether it would suffice to posit that group members' opinions converge to the mean of their initial opinions. The latter issue is not problematical to sociologists, who have long emphasized that social influences are stratified; however, it is an important issue to address in light of a continuing assertion among some psychologists (McGarty 1992; Turner and Oakes 1989) that the process of interpersonal influence entails a simple (as opposed to a weighted) averaging of group members' opinions. This assertion underlies a large body of work on group polarization (Isenberg 1986; Lamm and Myers 1978) in which it is assumed that the formation of a group consensus which is *not* the mean of group members' initial opinions is indicative of a process that is *different* from the process of social influence observed by Asch (1951) and Sherif (1936). Thus, our aim is to assess the importance of developing a structural (sociological) approach to opinion change, since if opinions do tend to converge to the mean of initial opinions, there is little call for the development of such an approach.

Then, we assess whether the parameter values that can be estimated from the pattern of initial and settled opinions of group members—the matrix **A** of individuals' susceptibilities to interpersonal influence—correspond to actors' subjective assessments of the extent to which their opinions have been influenced by the other members of the group. Such subjective reports are prone to systematic biases (e.g., actors' inflated estimates of their self-importance) and random errors; however, we do not require that subjective reports be unbiased to assess whether the derived values ($a_{11}, a_{22}, \ldots, a_{NN}$) correspond to the group members' subjective experiences. If we can show that there is a notable association between the estimated susceptibilities for actors and their subjective reports of influenceability, then the construct validity of the derived estimates will be importantly supported.

Finally, we will assess whether there is an association between the predicted efficiency with which an equilibrium is produced and the observed time taken by a group to reach a decision. The model stipulates that actors simultaneously revise their opinions and continue to do so until an equilibrium is reached. This simplifying process assumption is certainly flawed because interpersonal influences often occur in complex sequences. Moreover, group members may shortcircuit an influence process before equilibrium is attained and achieve a group decision that is based on a decision scheme (e.g., majority vote). Hence, there may not be any reliable association between the predicted and observed efficiency of the social influence process. It is important for us to ascertain whether the simplifying assumptions of the model are grossly misleading with respect to the actual efficiency with which groups attain consensus. An association between the observed and predicted efficiency of the social influence process would sup-

Table 1. Three Experiments

Experiments	*Groups*	*Issues*	*Trials*
Tetrads	50	5	250
Triads	32	3	96
Dyads	36	2	72

port the construct validity of the *function*, equation (1) with equation (5), that describes the *process* of interpersonal influence.

METHODS AND PROCEDURES

Three Experiments

Table 1 presents an overview of the experiments drawn upon in this article. The form of these experiments was the same: (1) Each member of a group privately recorded their initial opinion on an issue. (2) The members of the group discussed the issue over a telephone network in which all or only some pairs of members were allowed to communicate. (3) After some prespecified time or upon reaching group consensus or upon reaching a deadlock, the group members privately recorded their final opinions on the issue and provided estimates of the relative interpersonal influences of the other group members upon their final opinions.

These experiments provided data on three of the four theoretical constructs involved in the influence model described by equation (7): actors' initial opinions ($\mathbf{y}^{(1)}$), final opinions ($\mathbf{y}^{(\infty)}$), and relative interpersonal influences (**C**) on an issue. Given these data, the fourth construct, actors' susceptibilities to interpersonal influence (**A**), were estimated with the method described in the Appendix.

Tetrads Experiment

This experiment involved 50 groups of college students whose members were asked to attempt to resolve their initial differences of opinion on various issues they had been asked to consider. The subjects were randomly assigned to positions within one of five different networks of interpersonal communication.[6] During the experiment neither the structure of the communication network nor individuals' positions in the network were altered.

Each group member occupied a private room and was given an issue to consider in isolation from the other three group members. Each person was asked to form and record an initial opinion on the issue under consideration. The group members then discussed their opinions with one another by means of a telephone system; each of the telephones displayed the names of persons with whom direct communication was possible and had buttons that opened lines of communication to these persons. Hence, only dyadic communication was permitted and (depending on the network structure) only certain communication channels could be activated by each subject. For example, in the "slash" network, members B and D could not converse with each other, but both could converse with network members A and C. Group members were instructed that they might communicate with other members of the group as frequently as they liked but that they must communicate at least once with each person whose name was listed on their telephones.

Group members were given up to twenty minutes to discuss an issue. Each group was instructed that attaining consensus was desirable and that most groups we had encountered had been able to reach consensus:

> Your goal is to reach consensus. If it seems difficult to reach consensus, remember that most groups are able to come to some decision if those who disagree will restate their reasons and if the problem is reread carefully.

Upon reaching group consensus or upon reaching a deadlock, the group members were asked to record their final opinions on the issue.

Each group dealt with five discussion issues in sequence. To eliminate crossover effects, the order of the discussion issues was systematically varied among groups. We took three discussion issues from the "risky shift" literature: Sports, Surgery, and School (see below). Two other issues were developed that involved a judgment about appropriate monetary reward: Asbestos and Disaster (see below). We selected these issues because individuals' opinions could be represented by real numbers—subjective probabilities on the "risky shift" issues or dollars on the monetary issues.[7]

Triads Experiment

This experiment involved 32 groups of college students. Each group dealt with the three "risky shift" discussion issues (Sports, Surgery, and School) in the context of either a complete or chain communication network (see note 6). One half of the triads operated under a "high" pressure condition and one half operated under a "low" pressure condition. In the "high" pressure condition, subjects were instructed that attaining consensus was desirable and that most groups we had encountered had been able to reach consensus:

> We would like you to reach an agreement. If at the end of twenty minutes there are remaining differences that you believe might be reconciled, you may have an additional ten minutes for discussion. You may terminate the session at any time if you believe that the remaining differences of opinion cannot be reconciled. However, it has been our experience that most discussion groups are able to reach an agreement within the twenty (plus optional ten) minute time frame.

In the "low" pressure condition subjects were instructed that any outcome was alright:

When the buzzer sounds a second time it is the signal for you to begin telephone communication with the other person. Now is the time to reconsider your choice. Discuss the situation with the other person. The conversation that you will have may or may not lead you to alter your first opinion, and you may or may not come to an agreement. Any of these outcomes are OK with us. You will have twenty minutes in which to discuss the issue. You may have an additional ten minutes if you want them.

Dyads Experiment

This experiment involved 36 groups of college students. Dyads were given up to 30 minutes to discuss an issue. Each dyad dealt sequentially with two of the "risky shift" issues—Surgery and School—that were involved in the other experiments. Eighteen of the dyads were placed under the "high" pressure condition and 18 dyads were placed under the "low" pressure condition which were described above in the triads experiment.

Discussion Issues

We took three discussion issues from the "risky shift" literature (Cartwright 1971):

You are asked to choose one of two alternatives. One alternative involves greater risk than the other, while also offering a greater potential reward. Consider the alternatives. Then indicate what probability of success would be necessary for you to choose the alternative which is potentially more rewarding, but which also carries a greater degree of risk.

Sports

You are a captain of a college team. You are playing in the crucial contest against your team's traditional rival. The game has been an intense struggle and now, in the final seconds of the game, your team is slightly behind. Fortunately, you are in a good position to successfully complete a play that will almost certainly produce a tie score. You are also in a position to attempt a play that is much riskier. If successful, it would result in a victory for your team; if unsuccessful, your team's defeat.

School

You are a college senior planning to go on for a Ph.D. For the Ph.D., you may enter Quality University. Because of Quality's rigorous standards, only a fraction of the graduate students manage to receive the Ph.D. you desire. You may also make a different choice and, to enter O.K. University. O.K. has a much poorer reputation than Quality. At O.K. almost every student receives a Ph.D.

Surgery

You have just completed a visit to your family doctor and then to a cardiac specialist. You have been told that you have a severe heart ailment. Due to your heart disease, you must drastically curtail your customary way of life. There is an alternative. There is a medical operation available that has the potential to bring about a complete cure of your heart ailment. However, the operation could prove fatal.

The responses (opinions) of the subjects were restricted to one of twenty probability values: .05, .10, .15, ..., 1. Previous research indicated that subjects have heterogeneous initial opinions on these issues and take them seriously.

Two questions were developed concerning an issue of monetary reward.

Asbestos

The Elmwood Unified School District has some older buildings with asbestos ceiling tiles which must be removed. The job is dirty and tedious. Unskilled labor might be hired at the minimum wage of $5.50 per hour; however some members of the school board believe that the job calls for greater remuneration than $5.50 per hour in view of the potential hazards in dealing with asbestos. How much ought the personnel who are going to do this work be paid on an hourly basis?

Disaster

The 37th District Federal Court is hearing a case where plaintiffs in India have filed a class action suit against the Consolidated Chemical Company of Hunnicutt, Maryland. One hundred employees suffered irreversible lung damage as a consequence of an accident at company's plant in India and are no longer able to work at their former jobs at the plant. The average income for these workers was US $375 per year (in a country where the average per capita annual income in 1977 was US $150). Lawyers for the injured workers are seeking two million dollars per plaintiff for the lost wages they would have earned and for punitive damages. Lawyers for the company argue that if the company is forced to pay $200 million, it could not afford to maintain the plant, which has been marginally profitable. And while the 100 plaintiffs may gain in wealth, the remaining 3,500 workers who depend on the plant for their livelihood will find themselves out of a job. The lawyers for the company emphasize that while the company was not at fault in the accident, it is willing to work out some reasonable form of compensation. What is the dollar amount that would be just compensation for each of the plaintiffs?

Measure of Relative Interpersonal Influence

To operationalize the model, a measure is required of the relative interpersonal influence of each actor on another. This measure is employed in the matrix $\mathbf{C} = [c_{ij}]$ that enters into equation (4). This is an $N \times N$ matrix with zeros on the main diagonal in which $0 \leq c_{ij} \leq 1$ and $\Sigma_j^N \; c_{ij} = 1$. The construct c_{ij} is the relative direct influence of the opinion of actor j on actor i.

In other work (Friedkin 1998), an approach has been developed to $\mathbf{C}$ that derives a measure from features of structure of the communication network in which two actors are situated. Friedkin's approach assumes that the communication network structure has *evolved* to reflect actors' power bases (French and Raven 1959). In experimental settings this same approach could be taken if it were based on observational data on the frequency and nature of the communications that occur in a small group. However, we have adopted a cruder, but serviceable, approach that relies on subjects' assessments of the relative influence of other subjects in forming their final opinion on an issue (Hunter 1953; Laumann and Pappi 1976, chap. 5; Merton 1968, chap. 12; Tannenbaum 1974).

The present approach to the matrix of relative interpersonal influences (i.e., **C**) has strengths and weaknesses. A strength is that power bases can only translate into direct interpersonal influence if they are *perceived*. Thus, for example, French and Raven's (1959) exegesis of power bases emphasizes the *perceptual* mediation of interpersonal influence. However, there are obvious difficulties, involving bias and random error, with relying on subjective reports. A key concern is the occurrence of bias; for example, actors may underestimate the influence of other actors and inflate their own importance. We mitigate the effects of such bias by drawing only on the *relative* values of the subjective reports for a measure of the relative influence of other actors. Another concern is whether actors can accurately disentangle the relative influences of multiple actors on them, especially where there is not a dominant actor whose influence is evident.[8] To the extent that this task is prone to error, the measure of **C** will be flawed, and errors will be introduced into some of our predictions.

To solicit information on relative interpersonal influences, we asked our subjects (after recording their final opinions on an issue) to estimate the extent to which each other group member influenced their final opinion:

> You have been given a total of 20 poker chips. Each chip represents influence upon your final opinion. Divide the chips into two piles, *Pile A* and *Pile B*. *Pile A* will represent the extent to which the conversations you had with the other persons influenced your final opinion. *Pile B* will represent the extent to which the conversations you had with the other persons did not influence your final opinion.
>
> Now place *Pile B* to the side and focus only on *Pile A*. Consider the extent to which you feel each member of the group influenced your own final opinion. Distribute the chips in *Pile A* into piles for each of the other members of the group according to how much they influenced your final opinion.

Based on these data, the measure of relative interpersonal influence, c_{ij}, is the number of "chips" that actor i accords to actor j divided by the total number of "chips" that actor i accorded to all other members of the group (i.e., the number of chips in *Pile A*).

A potential weakness of this measurement of the influence network in a group is that it is based on information subjects provide *after* they have completed their discussion of an issue. However, the influence network that is formed in a group on an issue is significantly associated with the opinion outcomes of the group on the next issue resolution trial (these results are available upon request). These results are consistent with a degree of stability in the influence network of a group across issues and lend support to our measure of the influence network. We rely on the posterior subjective assessments of subjects for the specification of their interpersonal influences because such assessments are most likely to take into account the specific influences among them on each issue under discussion.

A Subjective Measure of Actor's Susceptibility to Interpersonal Influence

Note that as part of the task described above, subjects were asked to assess the extent to which the conversations they had with the other persons did not influence their final opinion (i.e., the relative size of *Pile B*). These data correspond to an actor's *self-weight* ($w_{ii} = 1 - a_{ii}$) but were not employed to operationalize the model. Thus, we are able to inquire whether the parameter values for actors' susceptibilities to interpersonal influence (i.e., the a_{ii} which have been derived analytically) correspond to the *perceived* extent to which an actor has been influenced by other actors.

This measure, a subject's report on extent with which other actors have *not* influenced his or her opinion, is likely to be heavily biased by subjects' tendency to inflate their own importance. Such bias will be reflected in the intercept of the regression of a_{ii} on s_{ii}

$$a_{ii} = b_0 + b_1 s_{ii} + e_i, \tag{21}$$

where s_{ii} is one minus the relative size of *Pile B*. Hence, we may rely on a test of the significance of b_1 (or more simply the correlation coefficient) to assess this association.

RESULTS

Table 2 reports the results of regressing the mean equilibrium opinion of the members of a group on the predicted mean opinion for the group. When a group has achieved consensus, the mean equilibrium opinion is simply that consensus. For each size group—dyad, triad, and tetrad—we report the results for groups that reached consensus (panel a) and groups that did not reach consensus (panel b). The model predicts group outcomes (in the case of consensus) and mean opinions (in the case of disagreement in a group) with a high degree of accuracy. The prediction is modestly less accurate for dyads than for triads and tetrads. The 90 percent confidence intervals are consistent with parameter values of zero (0) for the intercepts and one (1) for the slopes. In other analyses, which are not reported, this pattern of confidence intervals and high R-squares is maintained with two exceptions in samples that are divided according to issue (i.e., instead of the occurrence of consensus). One exception occurs in tetrads dealing with the disaster issue where the 95 percent confidence interval for the slope is [.673, .988], and the other exception occurs in dyads dealing with the surgery issue where the 95 percent

Table 2. Unstandardized OLS Coefficients from the Regression of Observed Mean Final Opinion of Group Members on Their Predicted Mean Final Opinion in Different Size Groups

(a) Groups that did not form a consensus

	Tetrads	Triads	Dyads
Predicted Group Mean	.997***	.999***	1.031***
	(.004)	(.014)	(.050)
Constant	−2.903	.400	−2.149
	(1.636)	(.926)	(3.346)
R-square	.999	.991	.940
Number of cases	40	45	29

(b) Groups that reached consensus

	Tetrads	Triads	Dyads
Predicted Group Mean	.999***	1.019***	.949***
	(.012)	(.021)	(.096)
Constant	−8.996	−1.379	3.993
	(5.684)	(1.282)	(6.224)
R-square	.986	.979	.703
Number of cases	210	51	43

Notes: * < .05 ** < .01 *** < .001.
Standard errors are in parentheses.

confidence intervals for the intercept and slope are [1.936, 22.321] and [.706, .988] respectively.

Social psychological investigations of interpersonal influence have been split between psychological investigations that have tended to assume that pressures toward accommodation result in a movement toward the mean of initial opinions and sociological investigations that have tended to assume that such pressures result in a structure of stratification or domination. Both compromise and domination (inequality) are evident in the outcomes of small groups and, hence, a general model of social influence should allow for both.

Table 3 describes a simple typology of group outcomes: (a) consensus versus disagreement outcomes, (b) in the case of consensus, whether the agreement is a new compromise position or one of the existing initial opinions of a group member, and (c) in the case of a consensus on one of the members' initial opinions, whether the initial opinion is one of the two most extreme opinions or not. If the

Table 3. A Typology of Group Outcomes

	Dyads	*Triads*	*Tetrads*
Consensus			
Consensus on an extreme initial opinion	17	18	31
Consensus on an internal initial opinion	0	13	73
Consensus on a new opinion	26	20	106
Disagreement	29	45	40
Total Number of Cases	72	96	250

Table 4. Pearson Product-Moment Correlations of Actors' Subjective Experience of Interpersonal Influence and the Derived Coefficient of Actor's Susceptibility to Such Influence

	Tetrads	*Triads*	*Dyads*
Surgery	.460***	.695***	.518***
	(200)	(96)	(71)
School	.389***	.769***	.609***
	(200)	(96)	(72)
Sports	.314***	.685***	
	(200)	(96)	
Asbestos	.439***		
	(200)		
Disaster	.562***		
	(200)		

Notes: ***$p < .001$ (1-tailed).
Number of cases in parentheses.

consensus involves a settlement on one of the two most extreme opinions in a group, the case is described as an instance of "consensus on an extreme initial opinion." If the consensus involves a settlement on an initial opinion that is *not* one of the two most extreme opinions in a group, the case is described as an instance of "consensus on an internal initial opinion." Finally, if the consensus involves a settlement upon an opinion that is not one of the group members' initial opinions, the case is described as an instance of "consensus on a new opinion."

The distribution of cases shows that group outcomes take a variety of forms and that no particular form is dominant. Obviously, it cannot be assumed that groups always reach consensus. Moreover, it cannot be assumed that groups strongly tend to form a consensus on compromise positions that are not held by any member or on one of the alternative existing initial opinions; they do both with substantial frequency. Finally, only a structure of unequal influence is consistent with

Table 5. Correlations of Time to Reach Settlement (Agreement or Stalemate) and the Predicted Inefficiency of the Influence Process

	Tetrads	*Triads*	*Dyads*
All Groups	.161**	.208*	.088
	(250)	(96)	(72)
Groups with no consensus			
Groups with no breach of initial opinions	.097	.049	–.065
	(39)	(44)	(24)
Groups with breach of initial opinions	—	—	.106
	(1)	(1)	(5)
Groups with consensus			
Groups with no breach of initial opinions	.259***	.339**	.310*
	(210)	(51)	(34)
Groups with breach of initial opinions	—	—	–.709*
	(0)	(0)	(9)

Note: Time and inefficiency measures are transformed into their natural logarithms.
* $p < .05$ ** $p < .01$ *** $p < .001$ (1-tailed).
Number of cases are in parentheses.

a settlement of opinion on an extreme opinion and such settlements occur with a substantial frequency.

Table 4 shows that the derived coefficient of an actor's susceptibility to interpersonal influence (a_{ii}) is correlated with the actor's subjective assessment of the extent to which the other actor(s) influenced his or her opinion on an issue. In triads and tetrads, the coefficient is equivalent to the aggregate relative weight of the interpersonal influences upon the actor. In a dyad, since there is only one other actor, the coefficient is equivalent to the relative weight of the other actor. All of the associations are statistically significant.

Table 5 reports the correlations between the observed and predicted rapidity with which a settled agreement or stalemate is reached in a group. The observed rapidity is reported as the natural logarithm of the time taken by a group to reach a conclusion. The predicted rapidity is reported as the natural logarithm of number of iterations for the process described by equation (1) to reach an equilibrium, where an equilibrium is defined as the attainment of $\max_i (|y_i^{(t)} - y_i^{(t-1)}|) < \varepsilon$, where $\varepsilon = 10^{-10}$, a small change in opinions from one time period to the next. Separate correlations are reported for groups that reached consensus and those that did not. The correlations are not significant in groups that did not reach consensus. Among the groups that reached consensus, the correlations are significant in tetrads and triads, but not in dyads. However, the expected association between observed and predicted efficiency appears in dyads upon further analysis.

In dyads there are a substantial number of groups (14 in 72 trials) in which the final opinions of one or more members are outside the range of the group members' initial opinions. Such cases are anomalous because the model predicts that all opinions must lie within the range of group members' initial opinions. These anomalies rarely occurred in triads and tetrads. In triads, a breach of the range of initial opinions occurred in one group in 96 trials and in tetrads such a breach occurred in one group in 250 trials. The association between the observed and predicted inefficiency of the social influence process is *negative* (i.e., inconsistent with our theoretical expectation) in the dyads with the anomaly and *positive* (i.e., consistent with our theoretical expectation) in groups without the anomaly.

Finally, we confirm the prediction for dyads concerning the efficiency of consensus formation: the production of consensus is more rapid in the 17 cases where it involves a settlement on an initial opinion (4.7 minutes) than in the 17 cases where it involves a settlement on a compromise opinion (8.4 minutes).[9] In three of the former cases, there was an initial consensus of opinion; when these three cases are excluded the difference is in the expected direction (5.0 minutes versus 8.4 minutes) but dampened ($t = 1.63$, d.f.=22, $p = .059$).

CONCLUSIONS

We have described the process of opinion change in a group as an interpersonal accommodation in which each member of a group of actors weighs his or her own and other members' opinions on an issue, and repetitively modifies his or her opinion until a settled opinion on the issue is formed. When an equilibrium is achieved, the result of this process is either consensus or disagreement.

The theory encompasses various classical situations in which interpersonal influence has been studied—conformity and minority influence situations, status orders, and factionalism. The distinguishing features of these situations are the structures of influence and distributions of initial opinion that they involve. We suggest that a single process of interpersonal influence is involved in each situation and different outcomes arise as a result of this process unfolding in a particular structural context.

This theoretical integration also brings together the psychologists' viewpoint on interpersonal influence as a strain toward the mean of actors' initial opinions and the sociologists' viewpoint on such influence as a source of inequality and domination. These outcomes—convergence on the mean of initial opinions and convergence on the initial opinion of a particular actor or subgroup—are special cases of our theory. Because the social structure of groups vary, the outcomes of groups can take a variety of forms. Opinions may settle on the mean of group members' initial opinions; they may settle on a compromise opinion that is different from the mean of initial opinions; they may settle on an initial opinion of a group member; and they may settle on more or less altered opinions that do not from a consensus. Because all of these types of outcomes are frequent, a general model of social influence must encompass them all. The applicability of the present theory to these situations is part of its appeal.

We also report evidence from three experiments on issue-resolution episodes in dyads, triads, and tetrads. We have shown that the model predicts the mean final opinion of group members with a high degree of accuracy. We have supported the construct validity of the model in two respects. First, we have shown that the analytically derived value for the parameter that indicates actors' susceptibility to interpersonal influence is associated with actors' subjective experiences of interpersonal influence in an issue-resolution episode. Second, we have shown that the predicted efficiency of settlements is associated with the observed time to reach a settlement.

The surprise in these data was the occurrence of a substantial number of final opinions in dyads that were outside the range of group members' initial opinions. The mechanism of social influence that we have postulated is not consistent with movements of opinion outside the range of a group's initial opinions. Because the settled opinions of actors are weighted averages of their group members' initial opinions, all of the actors' settled opinions are expected to be in the range of the group's initial opinions on the issue. For a dyad, this means that

$$y_1^{(1)} \leq \left\{ y_1^{(\infty)}, y_2^{(\infty)} \right\} \leq y_2^{(1)} \tag{22}$$

when $y_1^{(1)} \leq y_2^{(1)}$. Violations of this mechanism are infrequent, but they do occur and our theory does not explain them.

The research on group polarization comes closest to suggesting that there may be an internal strain in groups to breach the range of initial opinions. The strain, which stems from the interpersonal interactions of group members, takes the form of an escalation or reinforcement of an initial inclination of the group toward one pole of an opinion continuum. Clearly, given sufficient escalation, groups members' settled opinions might all lie on one or more positions that are more extreme than any of the positions originally entertained by the group members. Group polarization is properly viewed as a main effect on opinions that is theoretically distinct from the accommodation process.[10] However, group polarization does not provide an adequate explanation of the frequent occurrence of breaching opinions in dyads. If such polarization were at work, then there should have been more frequent breaches in the triads and tetrads, but there were not. Perhaps, polarization is inhibited by the presence of third parties (Simmel 1950).

Our current speculation is that in the intimate context of a dyad, members reported *initial* opinions that may not have been entirely fixed. It has been shown that subjects will modify a reported opinion, in the absence of any interpersonal influences, when they are asked to think about the initial opinion again (Tesser 1978; Tesser, Martin, and Mendolia 1995). We suspect that subjects involved in isolated dyads are more likely to reevaluate the grounds on which they formulated their initial opinion than are subjects in larger groups ($N \geq 3$), in which such reevaluation is inhibited by the more complex and pressing interpersonal environment in which the subject is placed. This is a testable hypothesis that predicts that the frequency of breaching opinions should be reduced in dyads if subjects are asked to report, rethink and rereport their initial opinion before group interaction.

We see two main tasks for future work. First, work on the measurement of influence structures should be pursued. The early work of Simon (1953) and March (1955, 1956) has not resulted in a program of research that attempts to develop quantitative measures of the direct (unmediated) interpersonal influence of one actor on another. Subjective data have been employed in the present study; elsewhere, a structural approach has been developed (Friedkin 1998). The task is to fill an $N \times N$ matrix with scores that accurately reflect the unmediated influence of actor j on actor i for all pairs of actors.

Second, the process implications of the theory must be explored. Simplifying process assumptions have been made to make our model mathematically tractable; hence, it is axiomatic that this model of social influence will be imperfect at some level. For instance, it is obvious that interpersonal influences do not occur in the simultaneous way that is assumed by the present model and that there are more or less complex sequences of interpersonal influences in a group. Moreover, it also is clear that there may be short-circuits in the process of interpersonal influence, that is, groups may jump to a resolution (through a social choice mechanism) before all possible opinion changes have been exhausted on an issue. Thus, the task is to assess the domain of behavior that the model may reliably address and to weigh the relative advantages of improving the detailed accuracy of the model against a potential decrease in its general applicability.

APPENDIX

There are various approaches for making the social influence theory described by equation (1) operational, which depend on the availability of measures for the theoretical constructs. We have developed the following approach for estimating actors' susceptibilities to interpersonal influence $\mathbf{A}$ when data is available on actors' initial opinions, final opinions, and relative interpersonal influences; that is, $\mathbf{y}^{(1)}$, $\mathbf{y}^{(\infty)}$ and $\mathbf{C}$ respectively.

Assuming equilibrium, the scalar equation of the reduced-form, equation (7), is

$$y_i^{(\infty)} = a_{ii}(1 - a_{ii})y_i^{(\infty)} + a_{ii}^2 \sum_{j \neq i} c_{ij} y_j^{(\infty)} - (1 - a_{ii})y_i^{(1)} \qquad \text{(A1)}$$

from which it follows that

$$y_i^{(\infty)} - y_i^{(1)} = a_{ii}(y_i^{(\infty)} - y_i^{(1)}) + a_{ii}^2(\bar{y}_i^{(\infty)} - y_i^{(\infty)}) \tag{A2}$$

and

$$(1 - a_{ii})(y_i^{(\infty)} - y_i^{(1)}) = a_{ii}^2(\bar{y}_i^{(\infty)} - y_i^{(\infty)}) \tag{A3}$$

where

$$\bar{y}_i^{(\infty)} = \sum_{j \neq i} c_{ij} y_j^{(\infty)} \tag{A4}$$

is a weighted average of the others' settled opinions.

Hence, for $\bar{y}_i^{(\infty)} - y_i^{(\infty)} \neq 0$

$$\frac{a_{ii}^2}{1 - a_{ii}} = \frac{y_i^{(\infty)} - y_i^{(1)}}{\bar{y}_i^{(\infty)} - y_i^{(\infty)}} \equiv \Delta_i \tag{A5}$$

and

$$a_{ii} = \frac{-\Delta_i \pm \sqrt{\Delta_i^2 + 4\Delta_i}}{2} \tag{A6}$$

The a_{ii} computed from equation (A6) is a complex number for $-4 < \Delta_i < 0$, greater than one for $\Delta_i \leq -4$, and less than one for $0 \leq \Delta_i$:

	$a_{ii} = \frac{-\Delta_i + \sqrt{\Delta_i^2 + 4\Delta_i}}{2}$	$a_{ii} = \frac{-\Delta_i - \sqrt{\Delta_i^2 + 4\Delta_i}}{2}$
$\Delta_i \leq -4$	$a_{ii} \geq 2$	$1 < a_{ii} \leq 2$
$-4 < \Delta_i < 0$	complex number	complex number
$\Delta_i \geq 0$	$0 \leq a_{ii} < 1$	$a_{ii} \leq 0$

But assumptions $w_{ii} = 1 - a_{ii}$ and $0 \leq w_{ii} \leq 1$ imply that $0 \leq a_{ii} \leq 1$. Hence, for each real valued Δ_i the estimate of a_{ii} is selected to be the real number in the legitimate range [0,1], which is numerically the closer (in the complex plane) to either of the a_{ii} computed from equation (A6). The resulting solution set for $\bar{y}_i^{(\infty)} - y_i^{(\infty)} \neq 0$ is:

$$\text{(a) } \Delta_i \leq -2 \Rightarrow a_{ii} = 1$$

$$\text{(b) } -2 < \Delta_i < 0 \Rightarrow a_{ii} = \frac{-\Delta_i}{2},$$

$$\text{(c) } \Delta_i \geq 0 \Rightarrow a_{ii} = \frac{-\Delta_i + \sqrt{\Delta_i^2 + 4\Delta_i}}{2}$$

Note that this solution assumes $\bar{y}_i^{(\infty)} - y_i^{(\infty)} \neq 0$, which will be violated when there is an equilibrium consensus or, in the absence of such consensus, when actor i has an equilibrium opinion that is the weighted average, $\bar{y}_i^{(\infty)}$, of the others' equilibrium opinions.

When $\bar{y}_i^{(\infty)} - y_i^{(\infty)} = 0$ and $y_i^{(\infty)} - y_i^{(1)} \neq 0$, then $a_{ii} = 1$; this implication follows from equation (A3). When $\bar{y}_i^{(\infty)} - y_i^{(\infty)} = 0$ and $y_i^{(\infty)} - y_i^{(1)} = 0$, then a_{ii} might be any value. Actor i's opinion has not changed either because actor i was not susceptible to interpersonal influence or because he or she was susceptible to such influence but remained in the same position as a result of exactly balancing cross-pressures. The former situation (which corresponds to $a_{ii} = 0$) is more likely than the latter. Therefore, it is assumed that $a_{ii} = 0$ with the understanding that this assumption is a potential source of error in the model.

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NOTES

1. Consensus production is a ubiquitous outcome of group interactions, but so are disagreements that cannot be reconciled except by social choice mechanisms. The theory focuses on the social process that leads either to consensus or to a disagreement that cannot be reduced by opinion changes (because the opinion change process has been played out). Horowitz (1962, p. 182) has commented that "any serious theory of agreements and decisions must at the same time be a theory of disagreements and the conditions under which decisions cannot be reached." This theory satisfies Horowitz's criterion; compre with Abelson (1964).

2. This weighted averaging mechanism is consistent with Festinger's (1953) viewpoint on interpersonal influence as a finite distributed quantity. He argued as follows: "When a person or a group attempts to influence someone, does that person or group produce a totally new force acting on the person, one which had not been present prior to the attempted influence? Our answer is No—an attempted influence does not produce any new motivation or force. Rather, what an influence attempt involves is the redirection of psychological forces which already exist" (p. 237). Extensive empirical support for the assumption of a weighted averaging mechanism appears in work related to Anderson's information integration theory (Anderson 1981, 1991, 1996).

3. An even distribution of relative interpersonal influences for actor i, that is, $c_{ij} = 1/(N-1)$ for all j, is assigned in the special case when actor i is not influenced by any other actor. For such an actor, $a_{ii} = 0$, $w_{ii} = 1$ and, therefore, the interpersonal influences on that actor are zero.

4. Expectation states theory assumes the same rank ordering of influences; hence, our definition of a status order is a stricter form of such consistency. A general formal definition of a stratified influence network represented by **W** is one in which the columns of **W** can be arranged such that $\mathrm{col}_{j_1}(\mathbf{W}) < \mathrm{col}_{j_2}(\mathbf{W}) \leq \cdots \leq \mathrm{col}_{j_N}(\mathbf{W})$, that is, entry-by-entry inequalities.

5. This extreme situation of equality of interpersonal influence may be rarely observed because of the greater influence of the opinion that is expressed first in a conversation and other effects that foster unequal interpersonal influences. However, even with such effects, two actors may be approximately equal in their influences on one another and slow to reach a consensus in dyads. See our later analysis of the data on dyads.

6. These networks are the star {A-C, B-C, D-C}, kite {A-B, A-C, B-C, C-D}, circle {A-B, B-C, C-D, D-A}, slash {A-B, B-C, C-D, D-A, A-C}, and complete {A-B, B-C, C-D, D-A, A-C, B-D} networks. Along with a chain {A-B-C-D}, these networks include all the nonisomorphic connected structures (connected graphs) that might occur in a group of four persons.

7. Further details about the design and experimental procedures (including the wording of the issues, instructions to subjects, and forms the subjects used to record their opinions) are available from the first author on request.

8. An actor may or may not perceive the direct influence of another actor. For instance, an actor may distort or forget the origins of his or her modified opinion on an issue. An actor's opinion may move toward the opinion of another actor who has a negligible total influence on this movement; conversely, his or her opinion may not move toward, and it may even more away from, the opinion of another actor who has had a substantial direct influence. Furthermore, the direct effect of one actor on another and the total effect of that actor may not correspond: the total effect of j on i may be large, so that i's opinion substantially reflects the initial opinion of j, and the direct effect of j on i may be slight; conversely, the total effect of j on i may be slight, and the direct effect of j on i may be large. The absence of congruence between direct effect and total effect is a potential source of instability in the structure and legitimacy of an influence network (Ridgeway and Walker 1995). We believe that the stability that is often achieved in small groups implies a congruence between these two effects. We hope to address this issue in future research.

9. These cases involve the 34 dyads in which there was a consensus and no breach of the range of initial opinions (see Table 5). The standard errors of these means are 1.9175 and .7111. Assuming independent samples and unequal variances, $t = 1.812$ (d.f. = 20) and $p = .042$ (1-tailed).

10. Hence, Friedkin and Johnsen (1990) modeled this effect by including an additional coefficient, δ, that is independent of the interpersonal influences, $\mathbf{y}^{(\infty)} = \delta \mathbf{V}\mathbf{y}^{(1)}$. Compare this with equation (10).

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