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Assignment – 6
Parameter Estimation

① Let random sample → $(x_1, x_2, x_3, \dots)$ ~~be a random sample~~ of ~~to~~ have size n taken from Normal Population with parameters mean $= \theta$, var $= \theta_2$.

find Maximum Likelihood Estimators of these 2 parameters.

→ $\theta_1 = \mu$; $\theta_2 = \sigma^2$

normal dist. $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}}$ ~~$= \frac{1}{\sqrt{2\pi\sigma^2}}$~~

$\therefore f(x, \theta_1, \theta_2) = \frac{1}{\sqrt{2\pi\theta_2}} e^{-\frac{1}{2}\frac{(x-\theta_1)^2}{\theta_2}}$

Now, $\theta_1 \in (-\infty, \infty)$
$\theta_2 \in [0, \infty)$

$\therefore L(\theta_1, \theta_2) = \prod_{i=1}^{n} f(x_i, \theta_1, \theta_2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\theta_2}} e^{-\frac{1}{2}\frac{(x_i-\theta_1)^2}{\theta_2}}$

$\Rightarrow \theta_2^{-n/2} \cdot (2\pi)^{-n/2} \cdot e^{-\frac{1}{2\theta_2}\sum_{i=1}^{n}(x_i-\theta_1)^2}$

$\therefore \log L(\theta_1, \theta_2) = -\frac{n}{2}\log\theta_2 - \frac{n}{2}\log(2\pi) - \frac{\sum(x_i-\theta_1)^2}{2\theta_2}$

Taking partial derivative (w.r.t θ_1) →

$\frac{d \log L(\theta_1, \theta_2)}{d\theta_1} = \frac{-2\sum(x_i-\theta_1)(-1)}{2\theta_2} = 0$

$\therefore \boxed{\hat{\theta}_1 = \mu = \frac{\sum x_i}{n} = \bar{x}}$

W.r.t. $\theta_2 \Rightarrow$

$$\frac{d \log L(\theta_1, \theta_2)}{d\theta_2} = \frac{-n}{2\theta_2} + \frac{\Sigma (x - \theta_1)^2}{2\theta_2^2} = 0$$

$$\therefore \hat{\theta_2} = \hat{\sigma^2} = \frac{\Sigma (x_i - \bar{x})^2}{n}$$

$$\therefore \hat{\mu} = \frac{\Sigma x_i}{n} \;\&\; \sigma^2 = \frac{\Sigma (x_i - \bar{x})^2}{n}$$

(2) Let X_1, X_2, ... X_n be a random sample from $B(m, \theta)$ distribution, where $\theta \in (0,1)$ is unknown & 'm' is known (+)ve integer. Compute value of θ using MLE.

$$\longrightarrow {}^{m}C_{x_i}\, \theta^{x_i} (1-\theta)^{m-x_i} = B(m, \theta)$$

$$L(\theta | x_1, x_2 \dots x_n) = \prod_{i=1}^{n} \binom{m}{x_i} \theta^{x_i} (1-\theta)^{m-x_i}$$

$$\therefore \log L(\theta | x_1, x_2 \dots x_n) = \sum_{i=1}^{n} \log \binom{m}{x_i} + x_i \log \theta + (m - x_i) \log(1-\theta)$$

$$\therefore \frac{dL}{d\theta} = \frac{\Sigma x_i}{\theta} + \frac{m - x_i}{1-\theta} = 0$$

$$\therefore \sum_{i=1}^{n} \frac{x_i - \theta m}{\theta(1-\theta)} = 0$$

$$\therefore \sum_{i=1}^{n} \frac{x_i}{\theta(1-\theta)} = \frac{m \cdot n}{1-\theta} \qquad \therefore \boxed{\theta = \frac{\sum_{i=1}^{n} x_i}{m \cdot n}}$$

$\therefore$ MLE $\longrightarrow$

$$\boxed{B(m, \theta) = \frac{\sum_{i=1}^{n} x_i}{m \cdot n}}$$