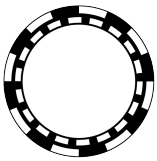
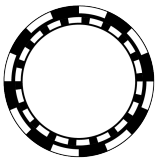


Mustang Math Tournament 2023

Risky Riding Colt Round



Basic Format

- This round contains 16 problems to be solved in 30 minutes.
- Every problem is multiple choice with exactly one correct answer.
- The problems are separated into four sets (Algebra x, Combinatorics $\{\}$, Geometry $\triangle$, and Number Theory $\equiv$) of 4 problems.
- Circling the correct answer to a problem on the answer sheet (backside) will grant you 2 points.
- *The poker chips are for grading purposes only, where graders will put 1s and 0s to mark correct and incorrect.*

Shooting The Moon

- Every problem has a "moonshine" answer, which is defined as the answer choice that is numerically furthest away from the correct answer.
  - For example, if the answer choices were $\{1, 2, 4, 8\}$ and 4 was the correct answer, 8 would be the moonshine answer as 4 is numerically furthest away from 8 than all other answer choices.
- For any given set, you may attempt to "shoot the moon" by circling the moonshine answer instead of the correct answer for all four problems.
- Successfully shooting the moon grants 12 points for the entire set. Unsuccessful attempts will be graded normally (2 points per correct answer, 0 points per incorrect answer).
- **Do not** circle multiple answers on a single problem, your answer will be invalidated.

Grader 1 Grader 2

A2 A3 A4 A5

C2 C3 C4 C5

G2 G3 G4 G5

N2 N3 N4 N5

2
x
332
528
x
2

3
x
$2\sqrt{6}$
5
$\sqrt{26}$
x
3

4
x
120 512
720 1024
x
4

5
x
32 42
52
62 65
x
5

2
{}
120
121
{}
2

3
{}
$\frac{1}{68}$
$\frac{1}{52}$
$\frac{1}{51}$
{}
3

4
{}
2 $\frac{9}{4}$
4 $\frac{9}{2}$
{}
4

5
{}
$\frac{1}{19}$ $\frac{1}{18}$
$\frac{3}{38}$
$\frac{2}{19}$ $\frac{1}{9}$
{}
5

2
△
20
$24\sqrt{3}$
▽
2

3
△
11
12
15
▽
3

4
△
120 150
$100\sqrt{3}$ 225
▽
4

5
△
40 60
80
120 140
▽
5

2
≡
7
8
≡
2

3
≡
$\frac{3379}{42}$
$\frac{247}{2}$
$\frac{1077}{7}$
≡
3

4
≡
3 4
6 7
≡
4

5
≡
302 407
526
563 599
≡
5

Algebra x

A2. 10 numbers, $a_1, a_2, \cdots, a_{10}$ are written in a row on a blackboard. For any integer s in the range $1 \leq s \leq 10$, the average of the first s numbers is equal to s^2. What is $a_5 + a_{10}$?

(A) 332 **(B)** 528

A3. Suppose x and y are distinct positive real numbers such that $x^3 - 20x = y^3 - 20y$ and $xy = 6$. Compute the value of $x + y$.

(A) $2\sqrt{6}$ **(B)** 5 **(C)** $\sqrt{26}$

A4. What is the value of $\dfrac{11 \times 12 \times 13 \times 14 \times \cdots \times 20}{1 \times 3 \times 5 \times 7 \times \cdots \times 19}$?

(A) 120 **(B)** 512 **(C)** 720 **(D)** 1024

A5. Let $7a + 2b = 54$ and let $4a + 7b = 43$. Evaluate $a + 12b$.

(A) 32 **(B)** 42 **(C)** 52 **(D)** 62 **(E)** 65

Combinatorics {}

C2. How many ways are there to arrange the letters in $BANANAS$ such that two A's never appear next to each other?

(A) 120 **(B)** 121

C3. A standard deck of 52 cards is shuffled into a random order. Given that the top card is a king, what is the probability that the bottom card is the king of diamonds?

(A) $\frac{1}{68}$ **(B)** $\frac{1}{52}$ **(C)** $\frac{1}{51}$

C4. Gerald rolls a standard six-sided die and lands on the number k. He then rerolls the dice k times. What is the expected number of primes that Gerald rolls including the initial roll?

(A) 2 **(B)** $\frac{9}{4}$ **(C)** 4 **(D)** $\frac{9}{2}$

C5. A box has 20 balls that are marked by the numbers 1 to 20. If 3 balls are randomly taken from the box of balls without replacement, what is the probability that one of them is the average of the other two?

(A) $\frac{1}{19}$ **(B)** $\frac{1}{18}$ **(C)** $\frac{3}{38}$ **(D)** $\frac{2}{19}$ **(E)** $\frac{1}{9}$

Geometry △

G2. Let Ω be a circle of radius 21. Circles ω_1 of radius 6 and ω_2 of radius 8 are internally tangent to Ω and externally tangent to each other. A chord of Ω with length L is tangent to both ω_1 and ω_2, which are on opposite sides of the chord. What is L?

(A) 20 **(B)** $24\sqrt{3}$

G3. Tristan constructs a shape by gluing together 18 equilateral triangles of side length 1 with no overlap. What is the smallest possible perimeter of Tristan's shape?

(A) 11 **(B)** 12 **(C)** 15

G4. Which of the following is not a possible area for a triangle with perimeter 60?

(A) 120 **(B)** 150 **(C)** $100\sqrt{3}$ **(D)** 225

G5. Equilateral triangle $\triangle ABC$ with side length 60 is cut into 3600 smaller equilateral triangles with side length 1. Point D is chosen on segment $\overline{AC}$ such that $AD = 20$. If a bug starts at point B and travels in a straight line path to point D, find the total number of triangles the bug passes through the interior of.

(A) 40 **(B)** 60 **(C)** 80 **(D)** 120 **(E)** 140

Number Theory ≡

N2. How many digits does the base-16 number 3421_{16} have in base-4?

(A) 7 **(B)** 8

N3. Let a and b be not necessarily distinct positive divisors of 42. What is the sum of the distinct possible values of $\frac{a}{b}$?

(A) $\frac{3379}{42}$ **(B)** $\frac{247}{2}$ **(C)** $\frac{1077}{7}$

N4. What digit O makes the 5-digit number $2O23O$ divisible by every answer choice except O?

(A) 3 **(B)** 4 **(C)** 6 **(D)** 7

N5. A *meaningful* number is a number whose prime factors sum to 42. What is the sum of the two smallest meaningful numbers?

(A) 302 **(B)** 407 **(C)** 526 **(D)** 563 **(E)** 599