Mustang Math Tournament 2023

Risky Riding Stallion Round

Basic Format

- This round contains 16 problems to be solved in 30 minutes.
- Every problem is multiple choice with exactly one correct answer.
- The problems are separated into four sets (Algebra x, Combinatorics $\{\}$, Geometry $\triangle$, and Number Theory $\equiv$) of 4 problems.
- Circling the correct answer to a problem on the answer sheet (backside) will grant you 2 points.
- *The poker chips are for grading purposes only, where graders will put 1s and 0s to mark correct and incorrect.*

Shooting The Moon

- Every problem has a "moonshine" answer, which is defined as the answer choice that is numerically furthest away from the correct answer.
  - For example, if the answer choices were $\{1, 2, 4, 8\}$ and 4 was the correct answer, 8 would be the moonshine answer as 4 is numerically furthest away from 8 than all other answer choices.
- For any given set, you may attempt to "shoot the moon" by circling the moonshine answer instead of the correct answer for all four problems.
- Successfully shooting the moon grants 12 points for the entire set. Unsuccessful attempts will be graded normally (2 points per correct answer, 0 points per incorrect answer).
- **Do not** circle multiple answers on a single problem, your answer will be invalidated.

Grader 1 Grader 2

2
x
332
528
x
2

3
x
35
37
40
x
3

4
x
120 512
720 1024
x
4

5
x
512 $512\sqrt{2}$
$32\sqrt{1023}$
1024 $1024\sqrt{2}$
x
5

A2
A3
A4
A5

2
{}
120
121
{}
2

3
{}
2
$\frac{9}{4}$
$\frac{9}{2}$
{}
3

4
{}
140 196
210 300
{}
4

5
{}
79 81
173
175 256
{}
5

C2
C3
C4
C5

2
$\triangle$
$\frac{1}{4}$
$\frac{3}{8}$
$\triangledown$
2

3
$\triangle$
20
$24\sqrt{3}$
$21\sqrt{5}$
$\triangledown$
3

4
$\triangle$
1 3
5 7
$\triangledown$
4

5
$\triangle$
$\frac{21}{4}$ 7
$\frac{21}{2}$
$\frac{63}{4}$ 21
$\triangledown$
5

G2
G3
G4
G5

2
$\equiv$
144
225
$\equiv$
2

3
$\equiv$
$\frac{3379}{42}$
$\frac{247}{2}$
$\frac{1077}{7}$
$\equiv$
3

4
$\equiv$
3 4
6 7
$\equiv$
4

5
$\equiv$
302 407
526
563 599
$\equiv$
5

N2
N3
N4
N5

Algebra x

A2. 10 numbers, $a_1, a_2, \cdots, a_{10}$ are written in a row on a blackboard. For any integer s in the range $1 \leq s \leq 10$, the average of the first s numbers is equal to s^2. What is $a_5 + a_{10}$?

(A) 332 **(B)** 528

A3. Cameron the Colt went galloping every day over the last 7 days. He galloped at least 3 miles every day and at most 10 miles over any 2 consecutive days. What is the maximum possible number of miles that Cameron the Colt could have galloped over all 7 days?

(A) 35 **(B)** 37 **(C)** 40

A4. What is the value of $\dfrac{11 \times 12 \times 13 \times 14 \times \cdots \times 20}{1 \times 3 \times 5 \times 7 \times \cdots \times 19}$?

(A) 120 **(B)** 512 **(C)** 720 **(D)** 1024

A5. A cricket starts at the point $(1, 1)$ on the coordinate plane. Every minute, if the cricket is on the point (x, y) and its distance to the origin $(0, 0)$ is r, it hops to the point $(x + y - r, x + y + r)$. How far is the cricket from the origin after 9 hops?

(A) 512 **(B)** $512\sqrt{2}$ **(C)** $32\sqrt{1023}$ **(D)** 1024 **(E)** $1024\sqrt{2}$

Combinatorics $\{\}$

C2. How many ways are there to arrange the letters in $BANANAS$ such that two A's never appear next to each other?

(A) 120 **(B)** 121

C3. Gerald rolls a standard six-sided die and lands on the number k. He then rerolls the dice k times. What is the expected number of primes that Gerald rolls, including his initial roll?

(A) 2 **(B)** $\frac{9}{4}$ **(C)** $\frac{9}{2}$

C4. At a party, $\frac{7}{17}$ of the people are wearing green jackets. An additional 300 people arrive on a bus. Now, $\frac{7}{12}$ of the people are wearing green jackets. What is the smallest possible number of people that were wearing green jackets on the bus?

(A) 140 **(B)** 196 **(C)** 210 **(D)** 300

C5. A 5 by 5 grid of square cells is initially empty. How many ways can four identical kings be placed on the grid such that every empty cell shares at least one corner with a cell occupied by a king? Rotations and reflections are considered distinct.

(A) 79 **(B)** 81 **(C)** 173 **(D)** 175 **(E)** 256

Geometry △

G2. In square $ABCD$, midpoints E and F are drawn on $\overline{BC}$ and $\overline{CD}$ respectively. Compute the ratio between the area of $\triangle AEF$ to $ABCD$.

(A) $\frac{1}{4}$ **(B)** $\frac{3}{8}$

G3. Let Ω be a circle of radius 21. Circles ω_1 of radius 6 and ω_2 of radius 8 are internally tangent to Ω and externally tangent to each other. A chord of Ω with length L is tangent to both ω_1 and ω_2, which are on opposite sides of the chord. What is L?

(A) 20 **(B)** $24\sqrt{3}$ **(C)** $21\sqrt{5}$

G4. Let $\triangle ABC$ be a triangle with $AB = 11$, $BC = 12$, and $CA = 13$. Let D be a point on side $\overline{BC}$, and let I_1 and I_2 be the incenters (centers of the circles inscribed in) triangles ADB and ADC. If $\overline{I_1I_2}$ is perpendicular to $\overline{AD}$, what is the length of $\overline{BD}$?

(A) 1 **(B)** 3 **(C)** 5 **(D)** 7

G5. Let $\triangle ABC$ have $AB = 13$, $BC = 14$, and $AC = 15$. Let M and N be the midpoints of $\overline{AB}$ and $\overline{AC}$. If a point P is uniformly randomly selected inside $\triangle ABC$, what is the expected area of $\triangle MNP$?

(A) $\frac{21}{4}$ **(B)** 7 **(C)** $\frac{21}{2}$ **(D)** $\frac{63}{4}$ **(E)** 21

Number Theory ≡

N2. The *importance* of a point (x, y) in the coordinate plane is $4x - 3y$. How many lattice points (a, b) with $1 \le a, b \le 100$ are there with importance values that are positive factors of 100?

(A) 144 **(B)** 225

N3. Let a and b be not necessarily distinct positive divisors of 42. What is the sum of the distinct possible values of $\frac{a}{b}$?

(A) $\frac{3379}{42}$ **(B)** $\frac{247}{2}$ **(C)** $\frac{1077}{7}$

N4. What digit O makes the 5-digit number $2O23O$ divisible by every answer choice except O?

(A) 3 **(B)** 4 **(C)** 6 **(D)** 7

N5. A *meaningful* number is a number whose prime factors sum to 42. What is the sum of the two smallest meaningful numbers?

(A) 302 **(B)** 407 **(C)** 526 **(D)** 563 **(E)** 599