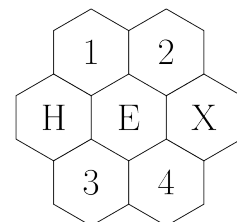


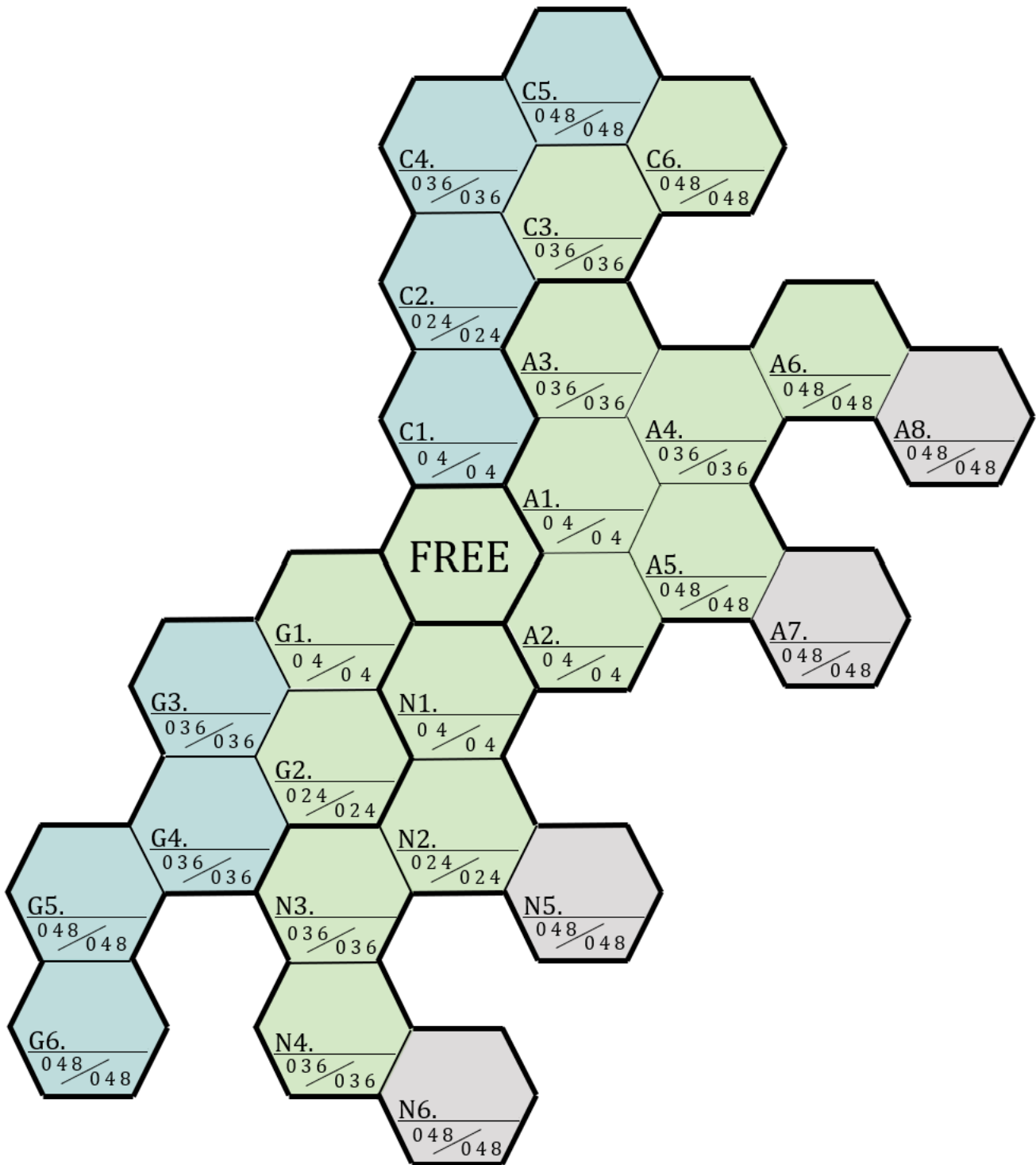
Mustang Math Tournament 2023

Herding Hexes Foal Round



Basic Format

- This round contains 26 problems to be solved in 45 minutes.
- Each problem corresponds to a hexagon on the answer grid (backside). Write the answer to each question inside its corresponding hexagon.
- The grid is separated by subject into 8 Algebra, 6 Combinatorics, 6 Geometry, 6 Number Theory, and 1 Free tile.
- Problems are identified as [Subject][Problem #] (e.g. Algebra Problem 5 is A5)
- A correct answer will grant 2 points for Problems 1 to 2, 3 points for Problems 3 to 4, and 4 points for all other problems in any given set.
- The score of a hexagonal tile is doubled if it can be connected back to the Free tile through other tiles (not necessarily of the same subject) that only contain correct answers.
- **Do not** write below the provided answer blank inside each hexagon (the space is for grading purposes).
- *Although this is a hex round, please write your answers in base-10 unless otherwise specified.*





Algebra

- A1. [2] The average of the six numbers $1, 3, 5, 7, 9, N$ is equal to the average of the five numbers $2, 4, 6, 8, 10$. What is N ?

Connects to FREE, A2, A3, A4, A5, C1

- A2. [2] Let $a \diamond b$ represent the value $ab + 100a$. What is the value of $18 \diamond 17 - 17 \diamond 18$?

Connects to FREE, A1, A5, N1

- A3. [3] The sequence $a_1, a_2, \dots, a_{2023}$ is a geometric sequence. Compute the common ratio if $a_{2023} = a_{2018} \cdot 243$.

Connects to A1, A4, C1, C2, C3

- A4. [3] Compute the value of m so that the points $(-2m + 1, -4)$ and $(-6m + 8, m - 6)$ lie on the same vertical line. Express your answer as a common fraction.

Connects to A1, A3, A5, A6

- A5. [4] Alon and Reese are superhumans running on an arbitrarily long, straight path. Alon runs consistently 10 miles per minute. Reese runs consistently at 35 miles per minute, however has very little stamina, so she needs to take a 6 minute break after every 2 minutes of running in order to rest (she immediately begins running again after every 6 minute break). How many times will they meet, not including the very beginning?

Connects to A1, A2, A4, A7

- A6. [4] If a, b , and c are the roots of the polynomial $x^3 - 4x^2 + 12x + 32$, compute $\frac{abc}{a+b+c}$.

Connects to A4, A8

- A7. [4] What positive real value x satisfies $x^x = (2x)^{2x}$? Express your answer as a common fraction.

Connects to A5

- A8. [4] Walking at constant rates, it takes Alice the same amount of time to walk from the park to the library as it takes Bob to walk from the school to the supermarket. Walking at the same constant rates, it takes Bob 44% longer to walk from the park to the library than it takes Alice to walk from the school to the supermarket. What is the ratio of Alice's walking speed to Bob's walking speed? Express your answer as a common fraction in simplest form.

Connects to A6



Combinatorics

- C1. [2] Bob uses a random number generator to pick 3 (not necessarily distinct) digits from 0 to 9. What is the probability that the sum of the digits is 27? Express your answer as a common fraction.

Connects to FREE, A1, A3, C2

- C2. [2] There are 10 lamps in a row from left to right, and all of them are initially off. One day, Mussy Mustang flips the switch of the first 4 lamps from the left. The next day, Mussy flips the switch of the first 5 lamps from the left. The next day, Mussy flips the switch of the first 6 lamps from the left, and so on. This process stops at the end of the day that Mussy flips the switch of all 10 lamps. At this point, how many lamps are on?

Connects to A3, C1, C3, C4

- C3. [3] Sami swaps 2 (not necessarily adjacent) letters in the word *HOOFS* to form a different string of letters. How many different strings of letters can result?

Connects to A3, C2, C4, C5, C6

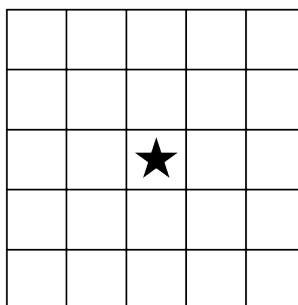
- C4. [3] Tristan flips a fair coin an infinite number of times and records the sequence of *H* (Heads) and *T* (Tails). What is the probability that the first instance of *TT* appears before the first instance of *HT*? Express your answer as a common fraction.

Connects to C2, C3, C5

- C5. [4] Angela is in charge of scheduling 5 meetings for her math club. The first meeting will be on Tuesday, September 3. The gap between adjacent meetings must be between 4 and 10 days, inclusive. Furthermore, meetings cannot be scheduled for Saturday nor Sunday. How many ways can Angela schedule the remaining 4 meetings? For example, this means that the second meeting must be on a weekday between September 7 and September 13.

Connects to C3, C4, C6

- C6. [4] In the 5 by 5 grid below, the center cell is marked with a star. Giorgio wants to color each of the 24 remaining cells red, blue, yellow, or green such that the cells of each color form a single rectangle. How many colorings are possible?



Connects to C3, C5



Geometry

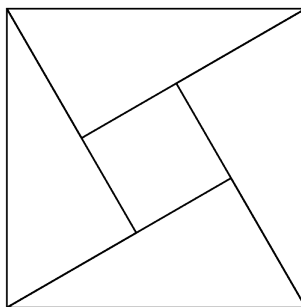
- G1. [2] If an isosceles triangle has vertex angle 12° larger than the other two, what is the degree measure of the vertex angle?

Connects to FREE, G2, G3, N1

- G2. [2] The vertices of a triangle are $(1, 1)$, $(4, 5)$, and $(9, 1)$ on a Cartesian coordinate plane. What is the area of the triangle formed by these three vertices?

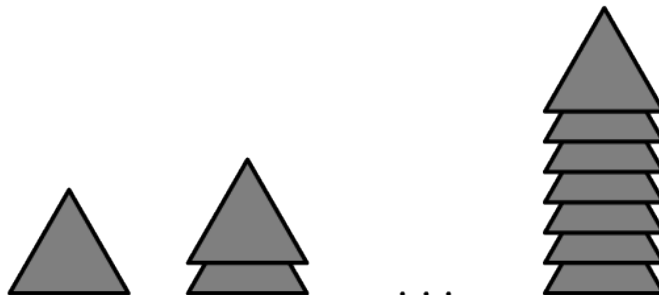
Connects to G1, G3, G4, N1, N2, N3

- G3. [3] In the diagram below, 4 congruent right-angled triangles and a small square are arranged to form a large square. If the area of the small square is 5 and the area of each triangle is 19, what is the side length of the large square?



Connects to G1, G2, G4

- G4. [3] Yuuki is making a Christmas tree in April using 7 equilateral triangles of length 2. He layers each triangle such that it covers exactly half the area of the triangle below it, as shown in the diagram below. What is the total area of Yuuki's tree?



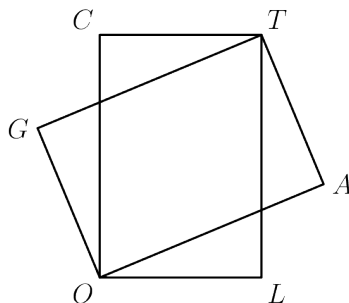
Connects to G2, G3, G5, N3

- G5. [4] Let $\triangle ABC$ be a triangle with $AB = 3$, $BC = 4$, and $CA = 5$. Let ω be a circle whose center lies on \overline{CA} . If ω passes through A and is tangent to \overline{BC} , what is the radius of ω ? Express your answer as a common fraction.

Connects to G4, G6



- G6. [4] Let $COLT$ and $GOAT$ be distinct congruent rectangles sharing the same diagonal \overline{OT} , as shown below. Suppose each rectangle has side lengths of 6 and 9. What is the total area covered by the two rectangles?



Connects to G5

Number Theory

- N1. [2] The 3-digit number \overline{ABC} is a perfect square and a multiple of 14. If $A+B+C \geq 18$, what is \overline{ABC} ?

Connects to FREE, A2, G1, G2, N2

- N2. [2] Albert wrote down a number N . Betty wrote down the number that equals the sum of the digits of N . Carol wrote down the sum of the digits of Betty's number. If Carol wrote the number 11, what is the smallest possible value N could have been?

Connects to G2, N1, N3, N5

- N3. [3] What fraction that lies strictly between $\frac{2}{3}$ and $\frac{3}{4}$ has the smallest positive integer denominator?

Connects to G2, G4, N2, N4

- N4. [3] 323323 is the product of n consecutive prime numbers. What is the value of n ?

Connects to N3, N6

- N5. [4] Positive integers x and y satisfy

$$\frac{\gcd(x, y)}{x} + \frac{1}{\text{lcm}(x, y)} = \gcd(x, y)$$

Compute $x + y$.

Connects to N2

- N6. [4] What is the smallest positive integer n such that n^n is divisible by 2023^{2023} but n is not divisible by 2023?

Connects to N4