MUSTANG MATH TOURNAMENT 2020

UNBRIDLED APPROXIMATION ROUND

Each question has an answer *X*, and you will submit an estimate *E*. Your score on question i will be $10 \times \left[\min\left(\frac{E}{X}, \frac{X}{E}\right)\right]^{P_i}$, on a scale from 0 to 10, based on how close your estimate is. The power P_i is unique to a problem and will be specified at the end of every question. A higher power simply indicates a steeper estimation score curve, so your estimate must be closer to the correct answer on these questions in order to receive a large portion of the 10 points.

ALL answers in this round must be integers or decimals, unless otherwise stated. Keep in mind that the exact answer may be irrational, so it is in your best interest to calculate as many decimal places as you can to maximize estimation points.

1. How many digits does the quantity 3^{87} have? ***P = 10***

2. You are given the first seven natural integers (1 to 7 inclusive), three multiplication operations, two division operations, and one addition operation. You are also permitted to use two sets of parentheses. If an operation must take place between every natural integer, then using all the given operations and symbols, the largest number that can be constructed is X. Determine X. ***P = 2***

   Note: You will also have to submit the expression (with numbers separated by standard operations and any necessary parentheses) you used to obtain your answer.

3. Using standard operations (addition, subtraction, multiplication, division), parentheses, and exponentiation, determine the minimum number of 3s needed to construct 2020. Remember that you can ONLY use 3s and you may NOT string together multiple 3s to form larger numbers like 33. ***P = 3***

   Note: You will also have to submit the expression (with 3s separated by operations and any necessary parentheses) you used to obtain your answer.

4. Estimate $\sqrt{1234_5}$ in base 10. ***P = 200***

5. You have a 99-by-99-unit grid with a total of 9,801 vertices. A red vertex follows two restrictions: it is a king's move apart from exactly one red vertex and a knight's move apart from exactly one red vertex. A "knight's move apart" is two vertices distanced $\sqrt{5}$ units apart. A "king's move apart" are two vertices either 1 or $\sqrt{2}$ units apart. What is the maximum number of red vertices that can be colored in the grid? ***P = 3***

6. Estimate the maximum number of whole spherical marbles of radius 10 cm that could be packed inside a $1\,m \times 2\,m \times 1\,m$ box. ***P = 2***

7. 2187 knights sit in a circle. The 1st knight kills the 3rd knight. The 4th knight kills the 6th knight. The 7th knight kills the 9th knight, and so on. After the 2187th knight, the killing wraps back around to the 1st knight. This process continues until only one knight is left. Determine the position of the last knight standing. ***P = 5***

8. Estimate the number of prime numbers between 2000 and 3000. ***P = 20***

9. You have an eight-by-eight chessboard. In how many ways can eight rooks be placed on this chessboard such that every column contains one rook and exactly four rows contain no rooks? ***P = 1***

10. Find $\left(\sqrt{17}+\sqrt{11}\right)^6$. ***P = 50***

11. Estimate the absolute value of the real part of $(3+4i)^{15}$. ***P = 0.5***

12. Estimate $\sin(10°)+\cos(10°)+\sin(20°)+\cos(20°)$ to the nearest hundredth. ***P = 50***