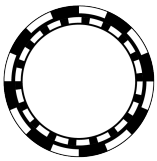
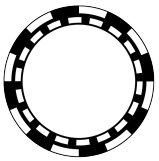


Mustang Math Tournament 2023

Risky Riding Foal Round



Basic Format

- This round contains 16 problems to be solved in 30 minutes.
- Every problem is multiple choice with exactly one correct answer.
- The problems are separated into four sets (Algebra x, Combinatorics $\{\}$, Geometry $\triangle$, and Number Theory $\equiv$) of 4 problems.
- Circling the correct answer to a problem on the answer sheet (backside) will grant you 2 points.
- *The poker chips are for grading purposes only, where graders will put 1s and 0s to mark correct and incorrect.*

Shooting The Moon

- Every problem has a "moonshine" answer, which is defined as the answer choice that is numerically furthest away from the correct answer.
  - For example, if the answer choices were $\{1, 2, 4, 8\}$ and 4 was the correct answer, 8 would be the moonshine answer as 4 is numerically furthest away from 8 than all other answer choices.
- For any given set, you may attempt to "shoot the moon" by circling the moonshine answer instead of the correct answer for all four problems.
- Successfully shooting the moon grants 12 points for the entire set. Unsuccessful attempts will be graded normally (2 points per correct answer, 0 points per incorrect answer).
- **Do not** circle multiple answers on a single problem, your answer will be invalidated.

Grader 1 Grader 2

2	3	4	5
x	x	x	x
332	10	0.4 1.5	32 42
	11		52
528	13	2.4 2.5	62 65
x	x	x	x
2	3	4	5

A2 A3 A4 A5

2	3	4	5
{}	{}	{}	{}
$\frac{2}{5}$	$\frac{1}{68}$	140 196	120 240
	$\frac{1}{52}$		360
$\frac{5}{9}$	$\frac{1}{51}$	210 300	720 1080
{}	{}	{}	{}
2	3	4	5

C2 C3 C4 C5

2	3	4	5
△	△	△	△
$\frac{1}{4}$	11	120 150	40 60
	12		80
$\frac{3}{8}$	15	$100\sqrt{3}$ 225	120 140
▽	▽	▽	▽
2	3	4	5

G2 G3 G4 G5

2	3	4	5
≡	≡	≡	≡
7	1	3 4	302 407
	3		526
8	4	6 7	563 599
≡	≡	≡	≡
2	3	4	5

N2 N3 N4 N5

Algebra x

A2. 10 numbers, $a_1, a_2, \cdots, a_{10}$ are written in a row on a blackboard. For any integer s in the range $1 \leq s \leq 10$, the average of the first s numbers is equal to s^2. What is $a_5 + a_{10}$?

(A) 332 **(B)** 528

A3. Find the smallest integer n such that $0.1 > \left(\frac{4}{5}\right)^n$.

(A) 10 **(B)** 11 **(C)** 13

A4. Tristan is trying to estimate $(1 + x)^2$, and ends up approximating it as $1 + 2x$. For what positive value of x is Tristan's approximation half the real value? Round x to the nearest tenth.

(A) 0.4 **(B)** 1.5 **(C)** 2.4 **(D)** 2.5

A5. Let $7a + 2b = 54$ and let $4a + 7b = 43$. Evaluate $a + 12b$.

(A) 32 **(B)** 42 **(C)** 52 **(D)** 62 **(E)** 65

Combinatorics {}

C2. A fair six-sided die has faces labelled $1, 1, 2, 3, 5$, and 8. Davy rolls the die twice and takes the sum of the numbers rolled. What is the probability that this sum is even?

(A) $\frac{2}{5}$ **(B)** $\frac{5}{9}$

C3. A standard deck of 52 cards is shuffled into a random order. Given that the top card is a king, what is the probability that the bottom card is the king of diamonds?

(A) $\frac{1}{68}$ **(B)** $\frac{1}{52}$ **(C)** $\frac{1}{51}$

C4. At a party, $\frac{7}{17}$ of the people are wearing green jackets. An additional 300 people arrive on a bus. Now, $\frac{7}{12}$ of the people are wearing green jackets. What is the smallest possible number of people that were wearing green jackets on the bus?

(A) 140 **(B)** 196 **(C)** 210 **(D)** 300

C5. How many ways are there to rearrange the letters in the word $MOONSHINE$ such that the word $MOON$ appears in that order continuously? ($SEMOONIHN$ is valid, but $MOOSNHINE$ and $NOOMNHSIE$ are not)

(A) 120 **(B)** 240 **(C)** 360 **(D)** 720 **(E)** 1080

Geometry △

G2. In square $ABCD$, midpoints E and F are drawn on $\overline{BC}$ and $\overline{CD}$ respectively. Compute the ratio between the area of $\triangle AEF$ to $ABCD$.

(A) $\frac{1}{4}$ **(B)** $\frac{3}{8}$

G3. Tristan constructs a shape by gluing together 18 equilateral triangles of side length 1 with no overlap. What is the smallest possible perimeter of Tristan's shape?

(A) 11 **(B)** 12 **(C)** 15

G4. Which of the following is not a possible area for a triangle with perimeter 60?

(A) 120 **(B)** 150 **(C)** $100\sqrt{3}$ **(D)** 225

G5. Equilateral triangle $\triangle ABC$ with side length 60 is cut into 3600 smaller equilateral triangles with side length 1. Point D is chosen on segment $\overline{AC}$ such that $AD = 20$. If a bug starts at point B and travels in a straight line path to point D, find the total number of triangles the bug passes through the interior of.

(A) 40 **(B)** 60 **(C)** 80 **(D)** 120 **(E)** 140

Number Theory ≡

N2. How many digits does the base-16 number 3421_{16} have in base-4?

(A) 7 **(B)** 8

N3. How many triples of positive integers (a, b, c) are there that satisfy $a \leq b \leq c$ and $abc = 2023$?

(A) 1 **(B)** 3 **(C)** 4

N4. What digit O makes the 5-digit number $2O23O$ divisible by every answer choice except O?

(A) 3 **(B)** 4 **(C)** 6 **(D)** 7

N5. A *meaningful* number is a number whose prime factors sum to 42. What is the sum of the two smallest meaningful numbers?

(A) 302 **(B)** 407 **(C)** 526 **(D)** 563 **(E)** 599