GALLOP ROUND 8

22. Carrie is sending out five invitations for her retirement party. Unfortunately, she hasn't gotten much sleep recently, so she randomly places each invitation in one of her five addressed envelopes. What is the expected number of invitees who will receive the correct invitation?

23. A positive integer N is the smallest possible value that has a remainder of 1 when divided by 3, a remainder of 2 when divided by 5, a remainder of 3 when divided by 7, and a remainder of 4 when divided by 11. A positive integer M is the smallest possible value that has a remainder of 1 when divided by 9, a remainder of 2 when divided by 8, a remainder of 3 when divided by 7, and a remainder of 4 when divided by 6. What is the product of the sum of the digits of N and the sum of the digits of M?

24. The A-excenter I_A of ΔABC is defined as the intersection of the external angle bisectors of $\angle B$ and $\angle C$ and the internal angle bisector of $\angle A$. An external angle bisector is the bisector of an exterior angle of the triangle. The incenter I of ΔABC is defined as the intersection of the three internal angle bisectors. If $\overline{AB} = 13$, $\overline{BC} = 14$ and $\overline{AC} = 15$, find $\overline{II_A}^2$.