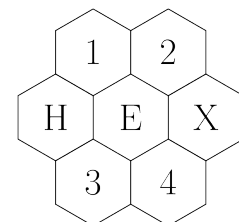


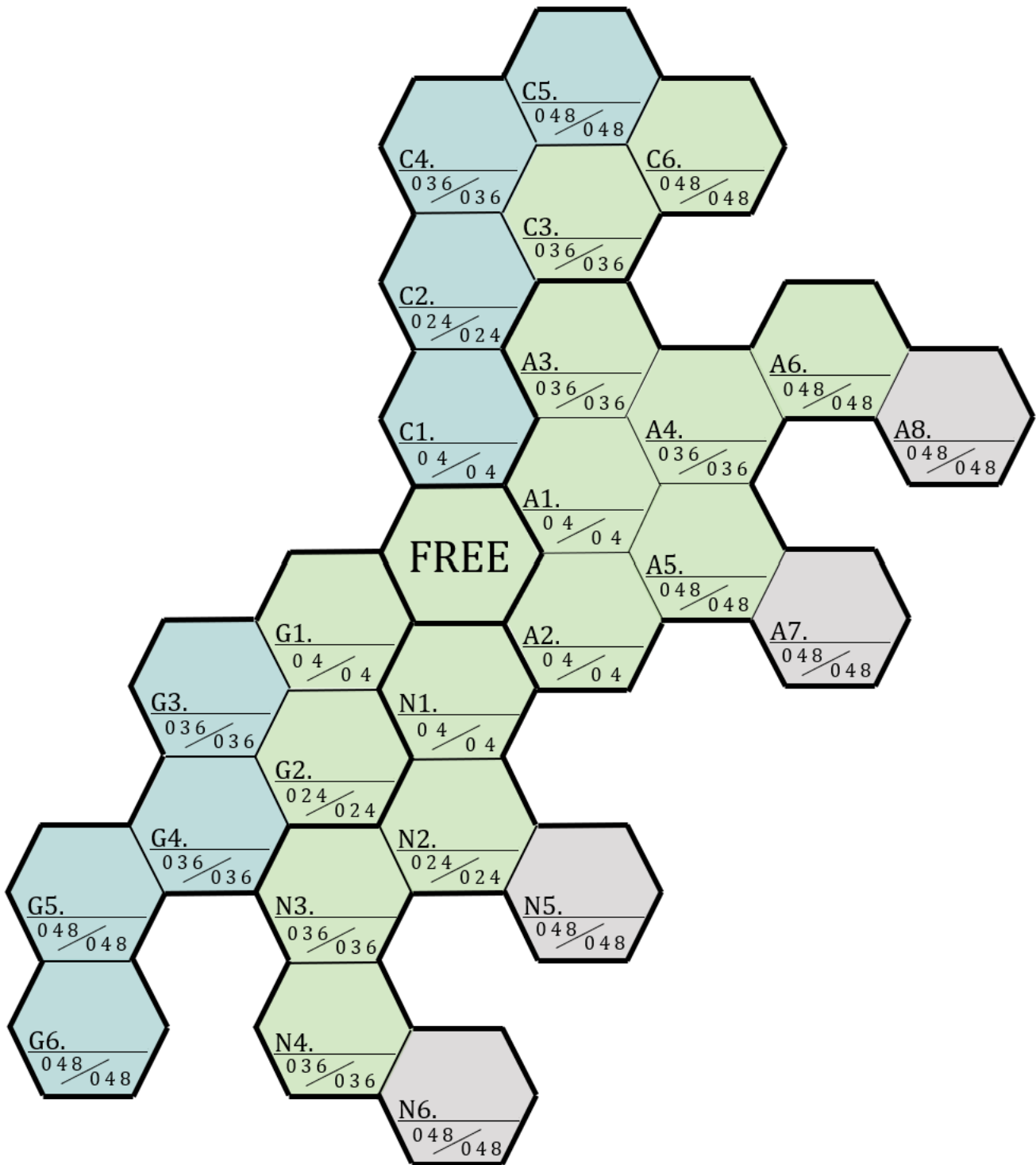
Mustang Math Tournament 2023

Herding Hexes Colt Round



Basic Format

- This round contains 26 problems to be solved in 45 minutes.
- Each problem corresponds to a hexagon on the answer grid (backside). Write the answer to each question inside its corresponding hexagon.
- The grid is separated by subject into 8 Algebra, 6 Combinatorics, 6 Geometry, 6 Number Theory, and 1 Free tile.
- Problems are identified as [Subject][Problem #] (e.g. Algebra Problem 5 is A5)
- A correct answer will grant 2 points for Problems 1 to 2, 3 points for Problems 3 to 4, and 4 points for all other problems in any given set.
- The score of a hexagonal tile is doubled if it can be connected back to the Free tile through other tiles (not necessarily of the same subject) that only contain correct answers.
- **Do not** write below the provided answer blank inside each hexagon (the space is for grading purposes).
- *Although this is a hex round, please write your answers in base-10 unless otherwise specified.*





Algebra

- A1. [2] The average of the six numbers $1, 3, 5, 7, 9, N$ is equal to the average of the five numbers $2, 4, 6, 8, 10$. What is N ?

Connects to FREE, A2, A3, A4, A5, C1

- A2. [2] Let $a \diamond b$ represent the value $ab + 100a$. What is the value of $18 \diamond 17 - 17 \diamond 18$?

Connects to FREE, A1, A5, N1

- A3. [3] The sequence $a_1, a_2, \dots, a_{2023}$ is a geometric sequence. Compute the common ratio if $a_{2023} = a_{2018} \cdot 243$.

Connects to A1, A4, C1, C2, C3

- A4. [3] Walking at constant rates, it takes Alice the same amount of time to walk from the park to the library as it takes Bob to walk from the school to the supermarket. Walking at the same constant rates, it takes Bob 44% longer to walk from the park to the library than it takes Alice to walk from the school to the supermarket. What is the ratio of Alice's walking speed to Bob's walking speed? Express your answer as a common fraction in simplest form.

Connects to A1, A3, A5, A6

- A5. [4] A photograph is 5 inches wide and 8 inches tall. It is mounted in a frame with a non-zero border x inches wide on all sides. If the border's width were doubled, the area of the frame would increase by 150%. What is the original width x ? Express your answer as a common fraction.

Connects to A1, A2, A4, A7

- A6. [4] What positive real value x satisfies $x^x = (2x)^{2x}$? Express your answer as a common fraction.

Connects to A4, A8

- A7. [4] Let a and b be real numbers, and define the function $f(x) = ax + b$. Given that $f(f(f(0))) = 2023$ and $f(f(f(1))) = 2031$, what is $f(0)$?

Connects to A5

- A8. [4] Let r_1, r_2 , and r_3 be the roots of $x^3 + x - 1$. What is value of the expression below?

$$\frac{1}{1 - r_1^3} + \frac{1}{1 - r_2^3} + \frac{1}{1 - r_3^3}$$

Connects to A6



Combinatorics

- C1. [2] There are 10 lamps in a row from left to right, and all of them are initially off. One day, Mussy Mustang flips the switch of the first 4 lamps from the left. The next day, Mussy flips the switch of the first 5 lamps from the left. The next day, Mussy flips the switch of the first 6 lamps from the left, and so on. This process stops at the end of the day that Mussy flips the switch of all 10 lamps. At this point, how many lamps are on?

Connects to FREE, A1, A3, C2

- C2. [2] Tristan flips a fair coin an infinite number of times and records the sequence of H (Heads) and T (Tails). What is the probability that the first instance of TT appears before the first instance of HT ? Express your answer as a common fraction.

Connects to A3, C1, C3, C4

- C3. [3] Bob uses a random number generator to pick 3 (not necessarily distinct) digits from 0 to 9. What is the probability that the sum of the digits is 27? Express your answer as a common fraction.

Connects to A3, C2, C4, C5, C6

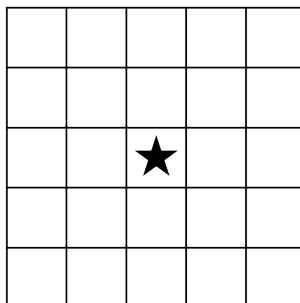
- C4. [3] Angela is in charge of scheduling 5 meetings for her math club. The first meeting will be on Tuesday, September 3. The gap between adjacent meetings must be between 4 and 10 days, inclusive. Furthermore, meetings cannot be scheduled for Saturday nor Sunday. How many ways can Angela schedule the remaining 4 meetings? For example, this means that the second meeting must be on a weekday between September 7 and September 13.

Connects to C2, C3, C5

- C5. [4] How many ways are there to arrange the letters in the word *MUSTANG* such that there is no string of three or more consecutive consonants?

Connects to C3, C4, C6

- C6. [4] In the 5 by 5 grid below, the center cell is marked with a star. Giorgio wants to color each of the 24 remaining cells red, blue, yellow, or green such that the cells of each color form a single rectangle. How many colorings are possible?



Connects to C3, C5

Geometry

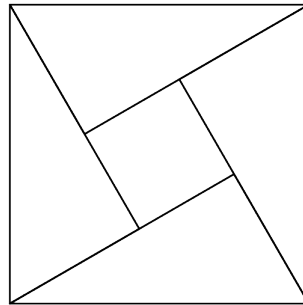
- G1. [2] If a regular hexagon is inscribed in equilateral triangle such that 2 points of the hexagon lie on each side of the triangle, what is the ratio of the area of hexagon to the area of the triangle? Express your answer as a common fraction.

Connects to FREE, G2, G3, N1

- G2. [2] If an isosceles triangle has vertex angle 12° larger than the other two, what is the degree measure of the vertex angle?

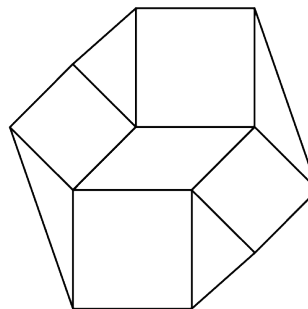
Connects to G1, G3, G4, N1, N2, N3

- G3. [3] In the diagram below, 4 congruent right-angled triangles and a small square are arranged to form a large square. If the area of the small square is 5 and the area of each triangle is 19, what is the side length of the large square?



Connects to G1, G2, G4

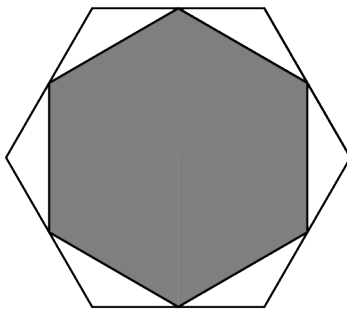
- G4. [3] In the diagram below, squares of length 6 and 8 have been constructed outside a parallelogram. The eight outer vertices are then connected to form an octagon with area of 302. What is the area of parallelogram?



Connects to G2, G3, G5, N3



- G5. [4] A regular hexagon has side length 4. Then, a second hexagon is formed by connecting the midpoints of the first hexagon. Compute the positive difference between the values of the perimeter and area of the second hexagon. Express your answer in simplest radical form.



Connects to G4, G6

- G6. [4] Let $\triangle ABC$ be an equilateral triangle and P be a point on the incircle (circle inscribed in) $\triangle ABC$. If the distances from P to \overline{AB} , \overline{AC} , and \overline{BC} are 2, 5, and a , what is the product of the possible values of a ?

Connects to G5

Number Theory

- N1. [2] The 3-digit number \overline{ABC} is a perfect square and a multiple of 14. If $A+B+C \geq 18$, what is \overline{ABC} ?

Connects to FREE, A2, G1, G2, N2

- N2. [2] Albert wrote down a number N . Betty wrote down the number that equals the sum of the digits of N . Carol wrote down the sum of the digits of Betty's number. If Carol wrote the number 11, what is the smallest possible value N could have been?

Connects to G2, N1, N3, N5

- N3. [3] Positive integers x and y satisfy

$$\frac{\gcd(x, y)}{x} + \frac{1}{\text{lcm}(x, y)} = \gcd(x, y)$$

Compute $x + y$.

Connects to G2, G4, N2, N4

- N4. [3] There is a pile of n coins, where n is an integer between 100 and 200 inclusive. Alice and Bob take turns playing a game, starting with Bob. Bob may remove either 3 or 5 coins on his turn, while Alice may remove either 2 or 4 coins on her turn. A player loses if, on their turn, they have no valid moves. For example, if it is Bob's turn and there are 2 coins on the stack, then Alice wins because there are not enough coins for Bob to remove either 3 or 5 of them. For how many values of n does Alice have a winning strategy, no matter how Bob plays?

Connects to N3, N6

- N5. [4] What is the greatest integer k such that 23^k evenly divides into $\gcd(2023! + 2025!, 2024! + 2026!)$?

Connects to N2

- N6. [4] What is the smallest positive integer n such that n^n is divisible by 2023^{2023} but n is not divisible by 2023?

Connects to N4