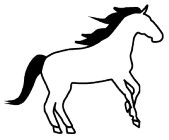
Sample Set (Gallop Rules)

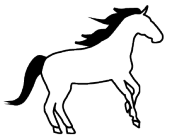


- This round contains 27 problems to be solved in 60 minutes.
- Problems are divided into 9 sets of 3 problems each.
- You must submit your current set before moving onto the next one.
- The 9^{th} and final set is an estimation round, where you will estimate the answers and points will be awarded by how close you are to the correct answer.
- *Be sure to dot your i's and dash your t's (i.e. carefully check your work) to submit each set with no remorse.*
- Point values for each set:

Round #	1	2	3	4	5	6	7	8	9	Total Pts.
Pts/Problem	10	11	12	13	14	16	18	21	25	420

MMT 2023 Gallop Colt Round April 29, 2023

Sample Set (Gallop Rules)



- This round contains 27 problems to be solved in 60 minutes.
- Problems are divided into 9 sets of 3 problems each.
- You must submit your current set before moving onto the next one.
- The 9^{th} and final set is an estimation round, where you will estimate the answers and points will be awarded by how close you are to the correct answer.
- *Be sure to dot your i's and dash your t's (i.e. carefully check your work) to submit each set with no remorse.*
- Point values for each set:

Round #	1	2	3	4	5	6	7	8	9	Total Pts.
Pts/Problem	10	11	12	13	14	16	18	21	25	420

Gallop Colt Round Set 1

1. [10] What is $16 \times 117 + 6 \times 8$?

2. [10] Let m and n be positive integers. Albert calculates the quantity $1 + 2 + 3 + \cdots + n$ and Betty calculates the quantity $5 + 6 + 7 + \cdots + m$. Given that Albert and Betty get the same sum, what is this sum?

3. [10] Let $ABCD$ be a square. If the largest circle that can fit inside $ABCD$ has area 2023π, what is the area of $ABCD$?

MMT 2023 Gallop Colt Round April 29, 2023

Gallop Colt Round Set 1

1. [10] What is $16 \times 117 + 6 \times 8$?

2. [10] Let m and n be positive integers. Albert calculates the quantity $1 + 2 + 3 + \cdots + n$ and Betty calculates the quantity $5 + 6 + 7 + \cdots + m$. Given that Albert and Betty get the same sum, what is this sum?

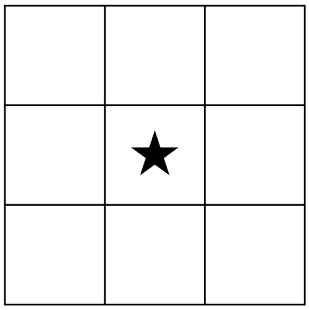
3. [10] Let $ABCD$ be a square. If the largest circle that can fit inside $ABCD$ has area 2023π, what is the area of $ABCD$?

Gallop Colt Round Set 2

4. [11] Let $\lfloor x \rfloor$ denote the greatest integer that is less than or equal to x. For example, $\lfloor 3.5 \rfloor = 3$ and $\lfloor -1.3 \rfloor = -2$. Evaluate the following expression:

$$\left\lfloor \frac{10}{10} \right\rfloor + \left\lfloor \frac{10}{9} \right\rfloor + \cdots + \left\lfloor \frac{10}{2} \right\rfloor + \left\lfloor \frac{10}{1} \right\rfloor + \left\lfloor \frac{10}{-1} \right\rfloor + \left\lfloor \frac{10}{-2} \right\rfloor + \cdots + \left\lfloor \frac{10}{-9} \right\rfloor + \left\lfloor \frac{10}{-10} \right\rfloor$$

5. [11] In the 3 by 3 grid below, the center cell is marked with a star. How many rectangles created by the gridlines contain the star?



6. [11] There exist pairs of primes (p, q) that satisfy the equation $20^2 + p = 23^2 - q$. Compute the sum of all distinct possible values of pq.

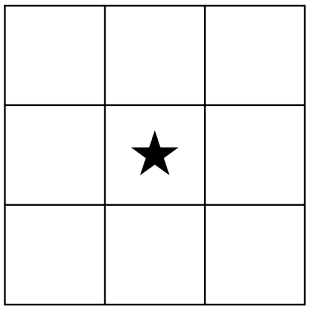
MMT 2023 Gallop Colt Round April 29, 2023

Gallop Colt Round Set 2

4. [11] Let $\lfloor x \rfloor$ denote the greatest integer that is less than or equal to x. For example, $\lfloor 3.5 \rfloor = 3$ and $\lfloor -1.3 \rfloor = -2$. Evaluate the following expression:

$$\left\lfloor \frac{10}{10} \right\rfloor + \left\lfloor \frac{10}{9} \right\rfloor + \cdots + \left\lfloor \frac{10}{2} \right\rfloor + \left\lfloor \frac{10}{1} \right\rfloor + \left\lfloor \frac{10}{-1} \right\rfloor + \left\lfloor \frac{10}{-2} \right\rfloor + \cdots + \left\lfloor \frac{10}{-9} \right\rfloor + \left\lfloor \frac{10}{-10} \right\rfloor$$

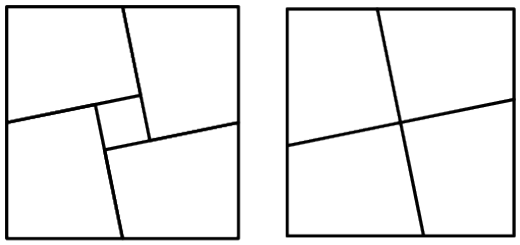
5. [11] In the 3 by 3 grid below, the center cell is marked with a star. How many rectangles created by the gridlines contain the star?



6. [11] There exist pairs of primes (p, q) that satisfy the equation $20^2 + p = 23^2 - q$. Compute the sum of all distinct possible values of pq.

Gallop Colt Round Set 3

7. [12] A square of side length 5 is cut into four identical quadrilaterals and a unit square, such that the four quadrilaterals can be moved and rotated to form a new square, as shown in the diagram below. What is the sum of the smallest and largest side lengths of one of the four quadrilaterals? Express your answer in simplest radical form.



Before After

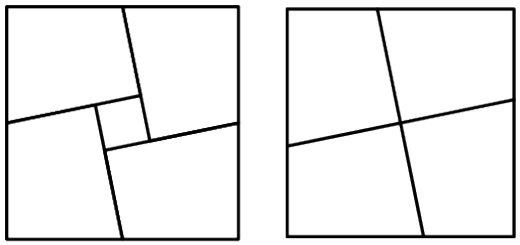
8. [12] Let O be the sum of all odd integers and E be the sum of all even integers between 0 and 9999, inclusive. Compute $\frac{O+E}{O-E}$.

9. [12] Let $\triangle ABC$ and $\triangle ACD$ be similar right triangles with $\angle ABC = \angle ACD = 90^\circ$ and $\angle BAC = \angle CAD$. If $AB = 20$ and $AD = 23$, what is the ratio of the area of $\triangle ABC$ to $\triangle ACD$? Express your answer as a common fraction.

MMT 2023 Gallop Colt Round April 29, 2023

Gallop Colt Round Set 3

7. [12] A square of side length 5 is cut into four identical quadrilaterals and a unit square, such that the four quadrilaterals can be moved and rotated to form a new square, as shown in the diagram below. What is the sum of the smallest and largest side lengths of one of the four quadrilaterals? Express your answer in simplest radical form.



Before After

8. [12] Let O be the sum of all odd integers and E be the sum of all even integers between 0 and 9999, inclusive. Compute $\frac{O+E}{O-E}$.

9. [12] Let $\triangle ABC$ and $\triangle ACD$ be similar right triangles with $\angle ABC = \angle ACD = 90^\circ$ and $\angle BAC = \angle CAD$. If $AB = 20$ and $AD = 23$, what is the ratio of the area of $\triangle ABC$ to $\triangle ACD$? Express your answer as a common fraction.

Gallop Colt Round Set 4

10. [13] 4 people each have a distinct number of coins from 1 to 4. If each person gives away all of their coins to someone else in the group at random, what is the probability no one ends up with the same number of coins they started with? Express your answer as a common fraction. Note that a person may receive coins from multiple people.

11. [13] Arpit needs to schedule a meeting with Anna. This week, both are available for 3 uniformly randomly selected days of the week (independent of each other). What is the probability that there is a day where both are available to meet?

12. [13] In right triangle $\triangle ABC$ with hypotenuse $\overline{AC}$, D lies on $\overline{AB}$ such that $\overline{DC}$ bisects angle C. Given that $DB = 1$ and $BC = 2$, compute AC. Express your answer as a common fraction.

MMT 2023 Gallop Colt Round April 29, 2023

Gallop Colt Round Set 4

10. [13] 4 people each have a distinct number of coins from 1 to 4. If each person gives away all of their coins to someone else in the group at random, what is the probability no one ends up with the same number of coins they started with? Express your answer as a common fraction. Note that a person may receive coins from multiple people.

11. [13] Arpit needs to schedule a meeting with Anna. This week, both are available for 3 uniformly randomly selected days of the week (independent of each other). What is the probability that there is a day where both are available to meet?

12. [13] In right triangle $\triangle ABC$ with hypotenuse $\overline{AC}$, D lies on $\overline{AB}$ such that $\overline{DC}$ bisects angle C. Given that $DB = 1$ and $BC = 2$, compute AC. Express your answer as a common fraction.

Gallop Colt Round Set 5

Welcome to the Cycle Round! In this round, the answer to a question will be used in question that follows, and so on until it loops back around. Good luck!

13. [14] Let r be the answer to the third question in this set. Compute the sum of the digits of r.

14. [14] Let u be the answer to the first question in this set. Compute the sum of the two solutions to the quadratic $x^2 + (x-1)^2 + (x-2)^2 + \cdots + (x-u)^2 = 2023^2$.

15. [14] Let n be the answer to the second question in this set. Arpit has a collection of n standard 6-sided dice. Compute the total number of dots on Arpit's dice.

MMT 2023 Gallop Colt Round April 29, 2023

Gallop Colt Round Set 5

Welcome to the Cycle Round! In this round, the answer to a question will be used in question that follows, and so on until it loops back around. Good luck!

13. [14] Let r be the answer to the third question in this set. Compute the sum of the digits of r.

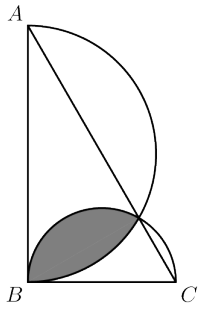
14. [14] Let u be the answer to the first question in this set. Compute the sum of the two solutions to the quadratic $x^2 + (x-1)^2 + (x-2)^2 + \cdots + (x-u)^2 = 2023^2$.

15. [14] Let n be the answer to the second question in this set. Arpit has a collection of n standard 6-sided dice. Compute the total number of dots on Arpit's dice.

Gallop Colt Round Set 6

16. [16] For a positive integer n, the function $f(n)$ switches the units digit and tens digit of n. For how many positive integers $n < 1000$ is $f(n)$ strictly greater than n? For example, $n = 156$ counts because $165 > 156$ but $n = 193$ does not because $139 < 193$. Note: If you have a single-digit number, assume the tens digit is 0. For example, 1 becomes 10 after switching digits.

17. [16] Right triangle ABC has $AB = 12\sqrt{3}$, $BC = 12$, and $AC = 24$. Let semicircles C_1 and C_2 have diameters $\overline{AB}$ and $\overline{BC}$, respectively. Compute the area of overlap between C_1 and C_2. Express your answer in simplest radical form in terms of π.



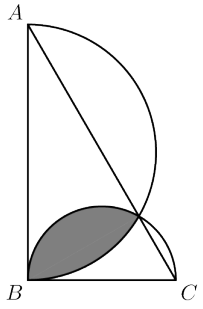
18. [16] Let n be a positive integer with exactly 16 positive divisors. If n^2 has exactly 63 positive divisors, how many positive divisors does n^3 have?

MMT 2023 Gallop Colt Round April 29, 2023

Gallop Colt Round Set 6

16. [16] For a positive integer n, the function $f(n)$ switches the units digit and tens digit of n. For how many positive integers $n < 1000$ is $f(n)$ strictly greater than n? For example, $n = 156$ counts because $165 > 156$ but $n = 193$ does not because $139 < 193$. Note: If you have a single-digit number, assume the tens digit is 0. For example, 1 becomes 10 after switching digits.

17. [16] Right triangle ABC has $AB = 12\sqrt{3}$, $BC = 12$, and $AC = 24$. Let semicircles C_1 and C_2 have diameters $\overline{AB}$ and $\overline{BC}$, respectively. Compute the area of overlap between C_1 and C_2. Express your answer in simplest radical form in terms of π.



18. [16] Let n be a positive integer with exactly 16 positive divisors. If n^2 has exactly 63 positive divisors, how many positive divisors does n^3 have?

Gallop Colt Round Set 7

19. [18] A convex polygon has distinct integer angle measure at each of its vertices. What is the maximum number of sides this polygon could have?

20. [18] Let $a_1, a_2, a_3, \cdots$ be a sequence of positive integers defined by $a_1 = 11$ and $a_{i+1} = a_i^2 - 2a_i + 2$ for all integers $i \geq 1$. What is the sum of the digits of the product $a_1 \times a_2 \times a_3 \times a_4 \times \ldots \times a_{10}$?

21. [18] The cubic $x^3 - 15x^2 - 41x + 119$ has roots a, b, and c. The cubic $4x^3 - 64x^2 - 85x + 289$ has roots c, d, and e. Given that the quintic $4x^5 - 56x^4 - 241x^3 + 567x^2 + 1173x - 2023$ has roots a, b, c, d, and e, compute c, the common root between all three polynomials.

MMT 2023 Gallop Colt Round April 29, 2023

Gallop Colt Round Set 7

19. [18] A convex polygon has distinct integer angle measure at each of its vertices. What is the maximum number of sides this polygon could have?

20. [18] Let $a_1, a_2, a_3, \cdots$ be a sequence of positive integers defined by $a_1 = 11$ and $a_{i+1} = a_i^2 - 2a_i + 2$ for all integers $i \geq 1$. What is the sum of the digits of the product $a_1 \times a_2 \times a_3 \times a_4 \times \ldots \times a_{10}$?

21. [18] The cubic $x^3 - 15x^2 - 41x + 119$ has roots a, b, and c. The cubic $4x^3 - 64x^2 - 85x + 289$ has roots c, d, and e. Given that the quintic $4x^5 - 56x^4 - 241x^3 + 567x^2 + 1173x - 2023$ has roots a, b, c, d, and e, compute c, the common root between all three polynomials.

Gallop Colt Round Set 8

22. [21] Alon picks two distinct prime numbers below 50 uniformly at random. What is the probability they add up to a multiple of 6? Express your answer as a common fraction.

23. [21] Six of the seven digits in the following set can be arranged to form a perfect square: $\{0, 3, 5, 6, 7, 8, 9\}$. Which digit is left out?

24. [21] In the empire of Oshenia, coins can be worth 5 different dollar values. Wynnston has two of each type of coin and discovered that he can combine some number of his coins to produce a sum of \$0, \$1, \$2, ..., and all integer dollar amounts up to $\$n$ (for example, if the values of the coins were $1, 2, 3, 4$, and 5, then Wynnston would be able to produce a combination of coins that sum to any integer between \$0 and \$30, inclusive). Find the maximum possible value of n.

MMT 2023 Gallop Colt Round April 29, 2023

Gallop Colt Round Set 8

22. [21] Alon picks two distinct prime numbers below 50 uniformly at random. What is the probability they add up to a multiple of 6? Express your answer as a common fraction.

23. [21] Six of the seven digits in the following set can be arranged to form a perfect square: $\{0, 3, 5, 6, 7, 8, 9\}$. Which digit is left out?

24. [21] In the empire of Oshenia, coins can be worth 5 different dollar values. Wynnston has two of each type of coin and discovered that he can combine some number of his coins to produce a sum of \$0, \$1, \$2, ..., and all integer dollar amounts up to $\$n$ (for example, if the values of the coins were $1, 2, 3, 4$, and 5, then Wynnston would be able to produce a combination of coins that sum to any integer between \$0 and \$30, inclusive). Find the maximum possible value of n.

Gallop Colt Round Set 9

25. [25] Estimate the expression below to the nearest integer.

$$\frac{3}{2} \times \frac{5}{4} \times \frac{7}{6} \times \cdots \times \frac{2023}{2022}$$

Submit a positive integer N. If the correct answer is A, you will receive $\max(25 - |A - N|, 0)$ points (if you do not submit a positive integer, you will receive 0 points).

26. [25] Anna the Anaconda initially has 1 gold coin. Every second, she has a 50% chance of doubling the number of coins she currently has. Else, she has a 50% chance of losing one coin. This process terminates whenever Anna loses all of her coins. The probability that Anna still has coins after 10^{2023} seconds is p. Estimate $\lfloor 100p \rfloor$, where $\lfloor x \rfloor$ denotes the largest integer less than or equal to x.

Submit a positive integer N. If the correct answer is A, you will receive $\max(25 - 5|A - N|, 0)$ points (if you do not submit a positive integer, you will receive 0 points).

27. [25] For nonnegative integers M, A, T, H, F, U, and N, let X be the number of possible quadruplets (M, A, T, H) and Y be the number of possible triplets (F, U, N) that satsfy the equations below. Estimate $\frac{X}{Y}$ to the nearest integer.

$$17M + 19A + 21T + 23H = 2023$$

$$9F + 10U + 11N = 1011$$

Submit a positive integer N. If the correct answer is A, you will receive $\max(25 - (A - N)^2, 0)$ points for this question (if you do not submit a positive integer, you will receive 0 points).

MMT 2023 Gallop Colt Round April 29, 2023

Gallop Colt Round Set 9

25. [25] Estimate the expression below to the nearest integer.

$$\frac{3}{2} \times \frac{5}{4} \times \frac{7}{6} \times \cdots \times \frac{2023}{2022}$$

Submit a positive integer N. If the correct answer is A, you will receive $\max(25 - |A - N|, 0)$ points (if you do not submit a positive integer, you will receive 0 points).

26. [25] Anna the Anaconda initially has 1 gold coin. Every second, she has a 50% chance of doubling the number of coins she currently has. Else, she has a 50% chance of losing one coin. This process terminates whenever Anna loses all of her coins. The probability that Anna still has coins after 10^{2023} seconds is p. Estimate $\lfloor 100p \rfloor$, where $\lfloor x \rfloor$ denotes the largest integer less than or equal to x.

Submit a positive integer N. If the correct answer is A, you will receive $\max(25 - 5|A - N|, 0)$ points (if you do not submit a positive integer, you will receive 0 points).

27. [25] For nonnegative integers M, A, T, H, F, U, and N, let X be the number of possible quadruplets (M, A, T, H) and Y be the number of possible triplets (F, U, N) that satsfy the equations below. Estimate $\frac{X}{Y}$ to the nearest integer.

$$17M + 19A + 21T + 23H = 2023$$

$$9F + 10U + 11N = 1011$$

Submit a positive integer N. If the correct answer is A, you will receive $\max(25 - (A - N)^2, 0)$ points for this question (if you do not submit a positive integer, you will receive 0 points).