# Model Comparison

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# 1 Case 1: Continuous Outcome and Continuous Mediator

## 1.1 Case 1-1: Continuous Outcome and Single Continuous Mediator Without Exposure-mediator Interaction

### 1.1.1 Data simulation

#### 1.1.1.1 Simulation Procedures

1. Simulate the exposure variable A from Binom(1,P(A=1)).
2. Simulate the covariate C from $N(\mu_C, \sigma_C^2)$.
3. Simulate the mediator M from $N((\beta_0 + \beta_1 * A + \beta_2 * C), \sigma_M^2)$.
4. Simulate the outcome Y from $N(\theta_0 + \theta_1 A + \theta_2 M + \theta_4 C, \sigma_Y^2)$.

#### 1.1.1.2 True Parameters

Table 1: True Model Parameters for Data Simulation

| Sample Size | $\theta_0$ | $\theta_1$ | $\theta_2$ | $\theta_4$ | $\beta_0$ | $\beta_1$ | $\beta_2$ | P(A=1) | $\mu_C$ | $\sigma_C$ | $\sigma_M$ | $\sigma_Y$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 10000 | -5 | 0.8 | 1.8 | 0.1 | -0.25 | 0.5 | 0.2 | 0.4 | 1 | 1 | 0.1 | 0.2 |

#### 1.1.1.3 True Models

True model for the mediator:

$$E[M|a,c] = \beta_0 + \beta_1 a + \beta_2 c$$

True model for the outcome:

$$E[Y|a,m^*,c] = \theta_0 + \theta_1 a + \theta_2 m^* + \theta_4 c$$

### 1.1.2 Causal Effects and Standard Errors Estimated By the Structural Equation Model with 3 Different Estimation Method

#### 1.1.2.1 Delta Method

```
causal_mediation(data = df_noint, model = "standard", est.method = "delta",
                 outcome = "contY_contM_noint", exposure = 'A', exposure.type = "binary",
                 mediator = 'M_cont', covariates = "C", EMint = FALSE,
                 yreg = "linear", mreg = "linear")
```

```
## $decomp3way
##         cde      cde_se        pnde     pnde_se        tnde     tnde_se
## 0.799536217 0.010662578 0.799536217 0.010662578 0.799536217 0.010662578
##        pnie     pnie_se        tnie     tnie_se          te       te_se
## 0.898282248 0.010530538 0.898282248 0.010530538 1.697818464 0.005504283
##          pm       pm_se
## 0.359693501 0.005574314
##
```

```
## $decomp4way
##            cde         cde_se         intref      intref_se         intmed
##    0.799536217    0.010662578    0.000000000    0.000000000    0.000000000
##      intmed_se            pie         pie_se             te          te_se
##    0.000000000    0.898282248    0.010530538    1.697818464    0.005504283
##       cde_prop    cde_prop_se    intref_prop intref_prop_se    intmed_prop
##    0.470919732    0.006030306    0.000000000    0.000000000    0.000000000
## intmed_prop_se       pie_prop    pie_prop_se     overall_pm  overall_pm_se
##    0.000000000    0.529080268    0.006030306    0.529080268    0.006030306
##    overall_int overall_int_se     overall_pe  overall_pe_se
##    0.000000000    0.000000000    0.529080268    0.006030306
```

#### 1.1.2.2 Bootstrapping

```r
causal_mediation(data = df_noint, model = "standard", est.method = "bootstrap",
                 outcome = "contY_contM_noint", exposure = 'A', exposure.type = "binary",
                 mediator = 'M_cont', covariates = "C", EMint = FALSE,
                 yreg = "linear", mreg = "linear", nboot = 500)
```

```
## $decomp3way
##         cde      cde_se        pnde     pnde_se        tnde     tnde_se
## 0.799536217 0.010070332 0.799536217 0.010070332 0.799536217 0.010070332
##        pnie     pnie_se        tnie     tnie_se          te       te_se
## 0.898282248 0.010512366 0.898282248 0.010512366 1.697818464 0.005398945
##          pm       pm_se
## 0.359693501 0.005400260
##
## $decomp4way
##            cde         cde_se         intref      intref_se         intmed
##    0.799536217    0.010070332    0.000000000    0.000000000    0.000000000
##      intmed_se            pie         pie_se             te          te_se
##    0.000000000    0.898282248    0.010512366    1.697818464    0.005398945
##       cde_prop    cde_prop_se    intref_prop intref_prop_se    intmed_prop
##    0.470919732    0.005843980    0.000000000    0.000000000    0.000000000
## intmed_prop_se       pie_prop    pie_prop_se     overall_pm  overall_pm_se
##    0.000000000    0.529080268    0.005843980    0.529080268    0.005843980
##    overall_int overall_int_se     overall_pe  overall_pe_se
##    0.000000000    0.000000000    0.529080268    0.005843980
```

#### 1.1.2.3 Simulation-based Approach

```r
causal_mediation(data = df_noint, model = "standard", est.method = "simulation",
                 outcome = "contY_contM_noint", exposure = 'A', exposure.type = "binary",
                 mediator = 'M_cont', covariates = "C", EMint = FALSE,
                 yreg = "linear", mreg = "linear",nsims = 500)[1:2]
```

```
## $decomp3way
##         cde      cde_se        pnde     pnde_se        tnde     tnde_se
## 0.799370598 0.010276748 0.799370598 0.010276748 0.799370598 0.010276748
##        pnie     pnie_se        tnie     tnie_se          te       te_se
## 0.898636303 0.009971016 0.898636303 0.009971016 1.698006901 0.005519370
##          pm       pm_se
## 0.359853960 0.005311127
##
```

```
## $decomp4way
##             cde         cde_se          intref      intref_se          intmed
##   7.993706e-01   1.027675e-02  -3.552714e-18   7.178826e-16  -3.552714e-18
##      intmed_se            pie          pie_se              te           te_se
##   6.398720e-16   8.986363e-01   9.971016e-03   1.698007e+00   5.519370e-03
##       cde_prop    cde_prop_se     intref_prop intref_prop_se     intmed_prop
##   4.707686e-01   5.743550e-03  -2.077740e-18   4.227304e-16  -2.112181e-18
## intmed_prop_se       pie_prop     pie_prop_se      overall_pm   overall_pm_se
##   3.768846e-16   5.292314e-01   5.743550e-03   5.292314e-01   5.743550e-03
##    overall_int overall_int_se      overall_pe  overall_pe_se
##  -4.189921e-18   3.724744e-16   5.292314e-01   5.743550e-03
```

#### 1.1.3 Causal Effects and Standard Errors Estimated By the Natural Effect Model

```r
causal_mediation(data = df_noint, model = "ne",
                 outcome = "contY_contM_noint", exposure = 'A', exposure.type = "binary",
                 mediator = 'M_cont', covariates = "C", EMint = FALSE,
                 yreg = "linear")
```

```
##                          Estimate  Std. Error
## natural direct effect   0.7995362 0.010517888
## natural indirect effect 0.8982822 0.010484385
## total effect            1.6978185 0.005509465
```

#### 1.1.4 Causal Effects and Standard Errors Estimated By the regression-based approach

##### 1.1.4.1 Bootstrapping

```r
causal_mediation(data = df_noint, model = "rb", est.method = "bootstrap",
                 outcome = "contY_contM_noint", exposure = 'A', exposure.type = "binary",
                 mediator = 'M_cont', covariates = "C", EMint = FALSE,
                 yreg = "linear", mreg = "linear", nboot = 500)
```

```
##         cde        pnde        tnde        pnie        tnie          te
## 0.799536217 0.799536217 0.799536217 0.898282248 0.898282248 1.697818464
##          pm      cde_se     pnde_se     tnde_se     pnie_se     tnie_se
## 0.359693501 0.010228245 0.010228245 0.010228245 0.010547126 0.010547126
##       te_se       pm_se
## 0.005630292 0.005442188
```

##### 1.1.4.2 Simulation-based Approach

```r
causal_mediation(data = df_noint, model = "rb", est.method = "simulation",
                 outcome = "contY_contM_noint", exposure = 'A', exposure.type = "binary",
                 mediator = 'M_cont', covariates = "C", EMint = FALSE,
                 yreg = "linear", mreg = "linear", nsims = 500)[1:2]
```

```
## $decomp3way
##         cde      cde_se        pnde     pnde_se        tnde     tnde_se
## 0.799054578 0.010378847 0.799054578 0.010378847 0.799054578 0.010378847
##        pnie     pnie_se        tnie     tnie_se          te       te_se
## 0.898585064 0.010796101 0.898585064 0.010796101 1.697639642 0.005279988
##          pm       pm_se
```

```
## 0.359931637 0.005580434
##
## $decomp4way
##             cde          cde_se         intref      intref_se         intmed
##   7.990546e-01    1.037885e-02  -1.598721e-17   6.757361e-16  -1.421085e-17
##      intmed_se             pie         pie_se             te          te_se
##   6.561924e-16    8.985851e-01   1.079610e-02   1.697640e+00   5.279988e-03
##       cde_prop     cde_prop_se    intref_prop intref_prop_se    intmed_prop
##   4.706869e-01    6.035808e-03  -9.421069e-18   3.981771e-16  -8.495286e-18
## intmed_prop_se        pie_prop    pie_prop_se     overall_pm  overall_pm_se
##   3.866775e-16    5.293131e-01   6.035808e-03   5.293131e-01   6.035808e-03
##    overall_int overall_int_se     overall_pe  overall_pe_se
##  -1.791636e-17    3.590311e-16   5.293131e-01   6.035808e-03
```

#### 1.1.5 Causal Effects and Standard Errors Estimated By the weighting-based approach

```r
causal_mediation(data = df_noint, model = "wb",
                 outcome = "contY_contM_noint", exposure = 'A', exposure.type = "binary",
                 mediator = 'M_cont', covariates = "C", EMint = FALSE,
                 yreg = "linear")
```

```
##         cde        pnde        tnde        pnie        tnie          te
## 0.799536217 0.799536217 0.799536217 0.898272607 0.898272607 1.697808824
##          pm      cde_se     pnde_se     tnde_se     pnie_se     tnie_se
## 0.359691029 0.010671001 0.010671001 0.010671001 0.010577364 0.010577364
##       te_se       pm_se
## 0.005773437 0.005568699
```

### 1.2 Case 1-2: Continuous Outcome and Single Continuous Mediator With Exposure-mediator Interaction

#### 1.2.1 Data simulation

##### 1.2.1.1 Simulation Procedures

1. Simulate the exposure variable A from Binom(1,P(A=1)).
2. Simulate the covariate C from $N(\mu_C, \sigma_C^2)$.
3. Simulate the mediator M from $N((\beta_0 + \beta_1 * A + \beta_2 * C), \sigma_M^2)$.
4. Simulate the outcome Y from $N(\theta_0 + \theta_1 A + \theta_2 M + \theta_3 AM + \theta_4 C, \sigma_Y^2)$.

##### 1.2.1.2 True Parameters

Table 2: True Model Parameters for Data Simulation

| Sample Size | $\theta_0$ | $\theta_1$ | $\theta_2$ | $\theta_3$ | $\theta_4$ | $\beta_0$ | $\beta_1$ | $\beta_2$ | P(A=1) | $\mu_C$ | $\sigma_C$ | $\sigma_M$ | $\sigma_Y$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 10000 | -5 | 0.8 | 1.8 | 0.2 | 0.1 | -0.25 | 0.5 | 0.2 | 0.4 | 1 | 1 | 0.1 | 0.2 |

##### 1.2.1.3 True Models

True model for the mediator:

$$E[M|a,c] = \beta_0 + \beta_1 a + \beta_2 c$$

True model for the outcome:

$$E[Y|a, m^*, c] = \theta_0 + \theta_1 a + \theta_2 m^* + \theta_3 a m^* + \theta_4 c$$

#### 1.2.2 Causal Effects and Standard Errors Estimated By the Structural Equation Model with 3 Different Estimation Method

##### 1.2.2.1 Delta Method

```
causal_mediation(data = df_int, model = "standard", est.method = "delta",
                 outcome = "contY_contM_int", exposure = 'A', exposure.type = "binary",
                 mediator = 'M_cont', covariates = "C", EMint = TRUE,
                 yreg = "linear", mreg = "linear")
```

```
## $decomp3way
##         cde      cde_se        pnde     pnde_se        tnde     tnde_se
## 0.805819272 0.011823428 0.796479690 0.012201966 0.893670431 0.011434780
##        pnie     pnie_se        tnie     tnie_se          te       te_se
## 0.895796887 0.011281648 0.992987628 0.012179303 1.789467318 0.005635842
##          pm       pm_se
## 0.383993804 0.006350551
##
## $decomp4way
##              cde          cde_se          intref       intref_se          intmed
##     0.8058192723    0.0118234281   -0.0093395822    0.0009151523    0.0971907406
##        intmed_se             pie          pie_se              te           te_se
##     0.0091672704    0.8957968870    0.0112816482    1.7894673177    0.0056358415
##         cde_prop     cde_prop_se     intref_prop intref_prop_se     intmed_prop
##     0.4503123719    0.0064001046   -0.0052191969    0.0005098454    0.0543126659
## intmed_prop_se        pie_prop     pie_prop_se      overall_pm   overall_pm_se
##     0.0051209162    0.5005941590    0.0061490183    0.5549068250    0.0066309098
##      overall_int overall_int_se     overall_pe   overall_pe_se
##     0.0490934691    0.0046290937    0.5496876281    0.0064001046
```

##### 1.2.2.2 Bootstrapping

```
causal_mediation(data = df_int, model = "standard", est.method = "bootstrap",
                 outcome = "contY_contM_int", exposure = 'A', exposure.type = "binary",
                 mediator = 'M_cont', covariates = "C", EMint = TRUE,
                 yreg = "linear", mreg = "linear", nboot = 500)
```

```
## $decomp3way
##         cde      cde_se        pnde     pnde_se        tnde     tnde_se
## 0.805819272 0.011599091 0.796479690 0.012025287 0.893670431 0.010954593
##        pnie     pnie_se        tnie     tnie_se          te       te_se
## 0.895796887 0.010613271 0.992987628 0.011773794 1.789467318 0.005753947
##          pm       pm_se
## 0.383993804 0.006189070
##
## $decomp4way
##              cde          cde_se          intref       intref_se          intmed
##     0.8058192723    0.0115990911   -0.0093395822    0.0009389173    0.0971907406
##        intmed_se             pie          pie_se              te           te_se
##     0.0094425843    0.8957968870    0.0106132710    1.7894673177    0.0057539473
```

```
##       cde_prop    cde_prop_se    intref_prop intref_prop_se    intmed_prop
##   0.4503123719   0.0062023729  -0.0052191969   0.0005236012   0.0543126659
## intmed_prop_se       pie_prop    pie_prop_se     overall_pm  overall_pm_se
##   0.0052768896   0.5005941590   0.0058074678   0.5549068250   0.0064612307
##    overall_int overall_int_se     overall_pe  overall_pe_se
##   0.0490934691   0.0047705577   0.5496876281   0.0062023729
```

#### 1.2.2.3 Simulation-based Approach

```r
causal_mediation(data = df_int, model = "standard", est.method = "simulation",
                 outcome = "contY_contM_int", exposure = 'A', exposure.type = "binary",
                 mediator = 'M_cont', covariates = "C", EMint = TRUE,
                 yreg = "linear", mreg = "linear",nsims = 500)[1:2]
```

```
## $decomp3way
##         cde      cde_se        pnde     pnde_se        tnde     tnde_se
## 0.805220958 0.012365853 0.795915711 0.012864006 0.892913127 0.010823825
##        pnie     pnie_se        tnie     tnie_se          te       te_se
## 0.896458641 0.010732336 0.993456058 0.012904129 1.789371769 0.005773639
##          pm       pm_se
## 0.384305959 0.006726705
##
## $decomp4way
##            cde         cde_se         intref      intref_se         intmed
##   0.8052209576   0.0123658526  -0.0093052464   0.0009614989   0.0969974162
##      intmed_se            pie         pie_se             te          te_se
##   0.0095726274   0.8964586415   0.0107323358   1.7893717689   0.0057736389
##       cde_prop    cde_prop_se    intref_prop intref_prop_se    intmed_prop
##   0.4500014756   0.0067177969  -0.0052001332   0.0005352978   0.0542076261
## intmed_prop_se       pie_prop    pie_prop_se     overall_pm  overall_pm_se
##   0.0053463791   0.5009910316   0.0058024112   0.5551986576   0.0070173406
##    overall_int overall_int_se     overall_pe  overall_pe_se
##   0.0490074928   0.0048297879   0.5499985244   0.0067177969
```

### 1.2.3 Causal Effects and Standard Errors Estimated By the Natural Effect Model

```r
causal_mediation(data = df_int, model = "ne",
                 outcome = "contY_contM_int", exposure = 'A', exposure.type = "binary",
                 mediator = 'M_cont', covariates = "C", EMint = TRUE,
                 yreg = "linear")
```

```
##                        Estimate  Std. Error
## pure direct effect    0.7963237 0.012412380
## total direct effect   0.8939014 0.011335486
## pure indirect effect  0.8956434 0.011082262
## total indirect effect 0.9932211 0.012280539
## total effect          1.7895448 0.005660758
```

### 1.2.4 Causal Effects and Standard Errors Estimated By the regression-based approach

#### 1.2.4.1 Bootstrapping

```r
causal_mediation(data = df_int, model = "rb", est.method = "bootstrap",
                 outcome = "contY_contM_int", exposure = 'A', exposure.type = "binary",
                 mediator = 'M_cont', covariates = "C", EMint = TRUE,
                 yreg = "linear", mreg = "linear", nboot = 500)
```

```
##         cde        pnde        tnde        pnie        tnie          te 
## 0.805819272 0.796479690 0.893670431 0.895796887 0.992987628 1.789467318 
##          pm      cde_se     pnde_se     tnde_se     pnie_se     tnie_se 
## 0.383993804 0.012250781 0.012685208 0.011473190 0.010757211 0.011977640 
##       te_se       pm_se 
## 0.005474560 0.006457796
```

#### 1.2.4.2 Simulation-based Approach

```r
causal_mediation(data = df_int, model = "rb", est.method = "simulation",
                 outcome = "contY_contM_int", exposure = 'A', exposure.type = "binary",
                 mediator = 'M_cont', covariates = "C", EMint = TRUE,
                 yreg = "linear", mreg = "linear", nsims = 500)[1:2]
```

```
## $decomp3way
##         cde      cde_se        pnde     pnde_se        tnde     tnde_se 
## 0.806404804 0.011873030 0.797135538 0.012361292 0.893816984 0.010846555 
##        pnie     pnie_se        tnie     tnie_se          te       te_se 
## 0.895504604 0.010973390 0.992186050 0.012640065 1.789321588 0.005310391 
##          pm       pm_se 
## 0.383638326 0.006529716 
## 
## $decomp4way
##            cde          cde_se          intref       intref_se          intmed 
##   0.8064048035    0.0118730300   -0.0092692652    0.0009555224    0.0966814459 
##      intmed_se             pie          pie_se              te           te_se 
##   0.0095191880    0.8955046038    0.0109733898    1.7893215880    0.0053103906 
##       cde_prop     cde_prop_se     intref_prop intref_prop_se     intmed_prop 
##   0.4506767539    0.0065316715   -0.0051801942    0.0005324121    0.0540322849 
## intmed_prop_se        pie_prop     pie_prop_se      overall_pm   overall_pm_se 
##   0.0053148506    0.5004711554    0.0059139804    0.5545034403    0.0068197119 
##    overall_int overall_int_se      overall_pe   overall_pe_se 
##   0.0488520907    0.0047996400    0.5493232461    0.0065316715
```

### 1.2.5 Causal Effects and Standard Errors Estimated By the weighting-based approach

```r
causal_mediation(data = df_int, model = "wb",
                 outcome = "contY_contM_int", exposure = 'A', exposure.type = "binary",
                 mediator = 'M_cont', covariates = "C", EMint = TRUE,
                 yreg = "linear", nboot = 500)
```

```
##         cde        pnde        tnde        pnie        tnie          te 
## 0.805819272 0.796478000 0.893666784 0.895771537 0.992960322 1.789438321 
##          pm      cde_se     pnde_se     tnde_se     pnie_se     tnie_se 
## 0.383987801 0.011841361 0.012289951 0.011218262 0.011007469 0.012180542 
##       te_se       pm_se 
## 0.005671137 0.006380443
```

## 1.3 Case 1-3: Continuous Outcome and Multiple Continuous Mediators Without Interaction

### 1.3.1 Data simulation

#### 1.3.1.1 Simulation Procedures

1. Simulate the exposure variable A from Binom(1,P(A=1)).
2. Simulate the covariate C from $N(\mu_C, \sigma_C^2)$.
3. Simulate the first mediator M1 from $N((\beta_{01} + \beta_{11} * A + \beta_{21} * C), \sigma_{M1}^2)$ and the second mediator M2 from $N((\beta_{02} + \beta_{12} * A + \beta_{22} * C), \sigma_{M2}^2)$.
4. Simulate the outcome Y from $N(\theta_0 + \theta_1 A + \theta_2 M1 + \theta_3 M2 + \theta_4 C, \sigma_Y^2)$.

#### 1.3.1.2 True Parameters

Table 3: True Model Parameters for Data Simulation

| Sample Size | $\theta_0$ | $\theta_1$ | $\theta_2$ | $\theta_3$ | $\theta_4$ | $\beta_{01}$ | $\beta_{11}$ | $\beta_{21}$ |
|---|---|---|---|---|---|---|---|---|
| 10000 | -5 | 0.8 | 1.8 | 1.2 | 0.1 | -0.25 | 0.5 | 0.2 |
| $\beta_{02}$ | $\beta_{12}$ | $\beta_{22}$ | P(A=1) | $\mu_C$ | $\sigma_C$ | $\sigma_{M1}$ | $\sigma_{M2}$ | $\sigma_Y$ |
| -0.3 | 0.4 | 0.3 | 0.4 | 1 | 1 | 0.1 | 0.1 | 0.2 |

#### 1.3.1.3 True Models

True model for the first mediator:

$$E[M1|a, c] = \beta_{01} + \beta_{11} a + \beta_{21} c$$

True model for the second mediator:

$$E[M2|a, c] = \beta_{02} + \beta_{12} a + \beta_{22} c$$

True model for the outcome:

$$E[Y|a, m^*, c] = \theta_0 + \theta_1 a + \theta_2 m1^* + \theta_3 m2^* + \theta_4 c$$

### 1.3.2 Causal Effects and Standard Errors Estimated By the Natural Effect Model

```
causal_mediation(data = df_mulipleM_noint, model = "ne",
                 outcome = "contY_2contM_noint", exposure = 'A', exposure.type = "binary",
                 mediator = c('M_cont1','M_cont2'), covariates = "C",
                 yreg = "linear")
```

```
##                            Estimate  Std. Error
## natural direct effect    0.8140153 0.013428091
## natural indirect effect 1.3690681 0.013488512
## total effect             2.1830834 0.005959846
```

### 1.3.3 Causal Effects and Standard Errors Estimated By the regression-based approach

#### 1.3.3.1 Bootstrapping

```r
causal_mediation(data = df_mulipleM_noint, model = "rb", est.method = "bootstrap",
                 outcome = "contY_2contM_noint", exposure = 'A', exposure.type = "binary",
                 mediator = c('M_cont1','M_cont2'), covariates = "C",
                 yreg = "linear", m_star = c(0,0), mreg = c("linear","linear"), nboot = 500)
```

```
##         cde        pnde        tnde        pnie        tnie          te
## 0.814015300 0.814015300 0.814015300 1.369068149 1.369068149 2.183083448
##          pm      cde_se     pnde_se     tnde_se     pnie_se     tnie_se
## 0.456797811 0.012980586 0.012980586 0.012980586 0.012935898 0.012935898
##       te_se       pm_se
## 0.005770399 0.006157335
```

#### 1.3.3.2 Simulation-based Approach

```r
causal_mediation(data = df_mulipleM_noint, model = "rb", est.method = "simulation",
                 outcome = "contY_2contM_noint", exposure = 'A', exposure.type = "binary",
                 mediator = c('M_cont1','M_cont2'), covariates = "C",
                 yreg = "linear", m_star = c(0,0), mreg = c("linear","linear"), nsims = 500)[1:2]
```

```
## $decomp3way
##         cde      cde_se        pnde     pnde_se        tnde     tnde_se
## 0.814458976 0.013108561 0.814458976 0.013108561 0.814458976 0.013108561
##        pnie     pnie_se        tnie     tnie_se          te       te_se
## 1.369100401 0.013279630 1.369100401 0.013279630 2.183559377 0.005980597
##          pm       pm_se
## 0.456695675 0.006246513
##
## $decomp4way
##            cde          cde_se          intref       intref_se          intmed
##   8.144590e-01    1.310856e-02   -7.105427e-18    7.045189e-16   -1.953993e-17
##      intmed_se             pie          pie_se              te           te_se
##   5.626585e-16    1.369100e+00    1.327963e-02    2.183559e+00    5.980597e-03
##       cde_prop     cde_prop_se     intref_prop intref_prop_se     intmed_prop
##   3.729956e-01    5.884243e-03   -3.197033e-18    3.226757e-16   -9.028902e-18
## intmed_prop_se        pie_prop     pie_prop_se      overall_pm   overall_pm_se
##   2.576553e-16    6.270044e-01    5.884243e-03    6.270044e-01    5.884243e-03
##    overall_int overall_int_se      overall_pe   overall_pe_se
##  -1.222593e-17    2.706935e-16    6.270044e-01    5.884243e-03
```

### 1.3.4 Causal Effects and Standard Errors Estimated By the weighting-based approach

```r
causal_mediation(data = df_mulipleM_noint, model = "wb",
                 outcome = "contY_2contM_noint", exposure = 'A', exposure.type = "binary",
                 mediator = c('M_cont1','M_cont2'), covariates = "C",
                 yreg = "linear", m_star = c(0,0), nboot = 500)
```

```
##         cde        pnde        tnde        pnie        tnie          te
## 0.814015300 0.814015300 0.814015300 1.369076890 1.369076890 2.183092190
##          pm      cde_se     pnde_se     tnde_se     pnie_se     tnie_se
## 0.456799396 0.012976066 0.012976066 0.012976066 0.012963488 0.012963488
##       te_se       pm_se
## 0.006146467 0.006135309
```

## 1.4 Case 1-4: Continuous Outcome and Multiple Continuous Mediators With Exposure-mediator Interaction

### 1.4.1 Data simulation

#### 1.4.1.1 Simulation Procedures

1. Simulate the exposure variable A from Binom(1,P(A=1)).
2. Simulate the covariate C from $N(\mu_C, \sigma_C^2)$.
3. Simulate the first mediator M1 from $N((\beta_{01} + \beta_{11} * A + \beta_{21} * C), \sigma_{M1}^2)$ and the second mediator M2 from $N((\beta_{02} + \beta_{12} * A + \beta_{22} * C), \sigma_{M2}^2)$.
4. Simulate the outcome Y from $N(\theta_0 + \theta_1 A + \theta_2 M1 + \theta_3 M2 + \theta_4 AM1 + \theta_5 AM2 + \theta_6 C, \sigma_Y^2)$.

#### 1.4.1.2 True Parameters

Table 4: True Model Parameters for Data Simulation

| Sample Size | $\theta_0$ | $\theta_1$ | $\theta_2$ | $\theta_3$ | $\theta_4$ | $\theta_5$ | $\theta_6$ | $\beta_{01}$ | $\beta_{11}$ |
|---|---|---|---|---|---|---|---|---|---|
| 10000 | -5 | 0.8 | 1.8 | 1.2 | 0.6 | 0.4 | 0.1 | -0.25 | 0.5 |
| $\beta_{21}$ | $\beta_{02}$ | $\beta_{12}$ | $\beta_{22}$ | P(A=1) | $\mu_C$ | $\sigma_C$ | $\sigma_{M1}$ | $\sigma_{M2}$ | $\sigma_Y$ |
| 0.2 | -0.3 | 0.4 | 0.3 | 0.4 | 1 | 1 | 0.1 | 0.1 | 0.2 |

#### 1.4.1.3 True Models

True model for the first mediator:

$$E[M1|a, c] = \beta_{01} + \beta_{11} a + \beta_{21} c$$

True model for the second mediator:

$$E[M2|a, c] = \beta_{02} + \beta_{12} a + \beta_{22} c$$

True model for the outcome:

$$E[Y|a, m^*, c] = \theta_0 + \theta_1 a + \theta_2 m1^* + \theta_3 m2^* + \theta_4 a m1^* + \theta_5 a m2^* + \theta_6 c$$

### 1.4.2 Causal Effects and Standard Errors Estimated By the Natural Effect Model

```
causal_mediation(data = df_mulipleM_EMint, model = "ne",
                 outcome = "contY_2contM_EMint", exposure = 'A', exposure.type = "binary",
                 mediator = c('M_cont1','M_cont2'), EMint = TRUE, EMint.terms = c("A*M_cont1", "A*M_con
                 covariates = "C", yreg = "linear")
```

```
##                        Estimate  Std. Error
## pure direct effect    0.7853913 0.015009406
## total direct effect   1.2405390 0.014570541
## pure indirect effect  1.3710444 0.014236126
## total indirect effect 1.8261921 0.015479281
## total effect          2.6115834 0.007146466
```

### 1.4.3 Causal Effects and Standard Errors Estimated By the regression-based approach

#### 1.4.3.1 Bootstrapping

```
causal_mediation(data = df_mulipleM_EMint, model = "rb", est.method = "bootstrap",
                 outcome = "contY_2contM_EMint", exposure = 'A', exposure.type = "binary",
                 mediator = c('M_cont1','M_cont2'), covariates = "C",
                 EMint = TRUE, EMint.terms = c("A*M_cont1", "A*M_cont2"),
                 yreg = "linear", m_star = c(0,0), mreg = c("linear","linear"), nboot = 500)
```

```
##         cde        pnde        tnde        pnie        tnie          te
## 0.817875337 0.787094008 1.237983925 1.372749512 1.823639429 2.610733437
##          pm      cde_se     pnde_se     tnde_se     pnie_se     tnie_se
## 0.536707487 0.013968449 0.014510503 0.014488742 0.014586310 0.014991481
##       te_se       pm_se
## 0.007030555 0.006468908
```

#### 1.4.3.2 Simulation-based Approach

```
causal_mediation(data = df_mulipleM_EMint, model = "rb", est.method = "simulation",
                 outcome = "contY_2contM_EMint", exposure = 'A', exposure.type = "binary",
                 mediator = c('M_cont1','M_cont2'), covariates = "C",
                 EMint = TRUE, EMint.terms = c("A*M_cont1", "A*M_cont2"),
                 yreg = "linear", m_star = c(0,0), mreg = c("linear","linear"), nsims = 500)[1:2]
```

```
## $decomp3way
##         cde      cde_se        pnde     pnde_se        tnde     tnde_se
## 0.818493508 0.015393511 0.787580788 0.015949537 1.238832985 0.014821854
##        pnie     pnie_se        tnie     tnie_se          te       te_se
## 1.371924443 0.014652644 1.823176640 0.016307122 2.610757428 0.006559533
##          pm       pm_se
## 0.536524122 0.007127185
##
## $decomp4way
##            cde         cde_se         intref      intref_se         intmed
##   0.8184935075   0.0153935112  -0.0309127197   0.0019013933   0.4512521969
##      intmed_se            pie         pie_se             te          te_se
##   0.0102513684   1.3719244430   0.0146526436   2.6107574277   0.0065595332
##       cde_prop    cde_prop_se    intref_prop intref_prop_se    intmed_prop
##   0.3135073849   0.0058051156  -0.0118402405   0.0007233659   0.1728432800
## intmed_prop_se       pie_prop    pie_prop_se     overall_pm  overall_pm_se
##   0.0038979332   0.5254895757   0.0055033783   0.6983328557   0.0060406890
##    overall_int overall_int_se     overall_pe  overall_pe_se
##   0.1610030394   0.0033506214   0.6864926151   0.0058051156
```

### 1.4.4 Causal Effects and Standard Errors Estimated By the weighting-based approach

```
causal_mediation(data = df_mulipleM_EMint, model = "wb",
                 outcome = "contY_2contM_EMint", exposure = 'A', exposure.type = "binary",
                 mediator = c('M_cont1','M_cont2'), covariates = "C",
                 EMint = TRUE, EMint.terms = c("A*M_cont1", "A*M_cont2"),
                 yreg = "linear", m_star = c(0,0), nboot = 500)
```

```
##         cde        pnde        tnde        pnie        tnie          te
```

```
## 0.817875337 0.787095371 1.237988061 1.372758268 1.823650958 2.610746329
##          pm      cde_se     pnde_se     tnde_se     pnie_se     tnie_se
## 0.536708628 0.013648504 0.014077320 0.014294654 0.014508262 0.015044391
##       te_se       pm_se
## 0.006906550 0.006337236
```

## 1.5 Case 1-5: Continuous Outcome and Multiple Continuous Mediators With Mediator-mediator Interaction

### 1.5.1 Data simulation

#### 1.5.1.1 Simulation Procedures

1. Simulate the exposure variable A from Binom(1,P(A=1)).
2. Simulate the covariate C from $N(\mu_C, \sigma_C^2)$.
3. Simulate the first mediator M1 from $N((\beta_{01} + \beta_{11} * A + \beta_{21} * C), \sigma_{M1}^2)$ and the second mediator M2 from $N((\beta_{02} + \beta_{12} * A + \beta_{22} * C), \sigma_{M2}^2)$.
4. Simulate the outcome Y from $N(\theta_0 + \theta_1 A + \theta_2 M1 + \theta_3 M2 + \theta_4 M1M2 + \theta_5 C, \sigma_Y^2)$.

#### 1.5.1.2 True Parameters

Table 5: True Model Parameters for Data Simulation

| Sample Size | $\theta_0$ | $\theta_1$ | $\theta_2$ | $\theta_3$ | $\theta_4$ | $\theta_5$ | $\beta_{01}$ | $\beta_{11}$ | $\beta_{21}$ |
|---|---|---|---|---|---|---|---|---|---|
| 10000 | -5 | 0.8 | 1.8 | 1.2 | 0.6 | 0.1 | -0.25 | 0.5 | 0.2 |
| $\beta_{02}$ | $\beta_{12}$ | $\beta_{22}$ | P(A=1) | $\mu_C$ | $\sigma_C$ | $\sigma_{M1}$ | $\sigma_{M2}$ | $\sigma_Y$ | |
| -0.3 | 0.4 | 0.3 | 0.4 | 1 | 1 | 0.1 | 0.1 | 0.2 | |

#### 1.5.1.3 True Models

True model for the first mediator:

$$E[M1|a, c] = \beta_{01} + \beta_{11} a + \beta_{21} c$$

True model for the second mediator:

$$E[M2|a, c] = \beta_{02} + \beta_{12} a + \beta_{22} c$$

True model for the outcome:

$$E[Y|a, m^*, c] = \theta_0 + \theta_1 a + \theta_2 m1^* + \theta_3 m2^* + \theta_4 m1^* m2^* + \theta_5 c$$

### 1.5.2 Causal Effects and Standard Errors Estimated By the Natural Effect Model

```
causal_mediation(data = df_mulipleM_MMint, model = "ne",
                 outcome = "contY_2contM_MMint", exposure = 'A', exposure.type = "binary",
                 mediator = c('M_cont1','M_cont2'), MMint = TRUE, MMint.terms = c("M_cont1*M_cont2"),
                 covariates = "C", yreg = "linear")
```

```
##                          Estimate  Std. Error
## natural direct effect   0.8142043 0.013432179
## natural indirect effect 1.4759987 0.013751029
## total effect            2.2902030 0.006556647
```

### 1.5.3 Causal Effects and Standard Errors Estimated By the weighting-based approach

```r
causal_mediation(data = df_mulipleM_MMint, model = "wb",
                 outcome = "contY_2contM_MMint", exposure = 'A', exposure.type = "binary",
                 mediator = c('M_cont1','M_cont2'), covariates = "C",
                 MMint = TRUE, MMint.terms = c("M_cont1*M_cont2"),
                 yreg = "linear", m_star = c(0,0), nboot = 500)
```

```
##         cde        pnde        tnde        pnie        tnie          te 
## 0.814204288 0.814204288 0.814204288 1.475509099 1.475509099 2.289713387 
##          pm      cde_se     pnde_se     tnde_se     pnie_se     tnie_se 
## 0.475369921 0.014100144 0.014100144 0.014100144 0.013932282 0.013932282 
##       te_se       pm_se 
## 0.006896718 0.006485043
```

## 1.6 Case 1-6: Continuous Outcome and Multiple Continuous Mediators With Exposure-mediator-mediator Interaction

### 1.6.1 Data simulation

#### 1.6.1.1 Simulation Procedures

1. Simulate the exposure variable A from Binom(1,P(A=1)).
2. Simulate the covariate C from $N(\mu_C, \sigma_C^2)$.
3. Simulate the first mediator M1 from $N((\beta_{01} + \beta_{11} * A + \beta_{21} * C), \sigma_{M1}^2)$ and the second mediator M2 from $N((\beta_{02} + \beta_{12} * A + \beta_{22} * C), \sigma_{M2}^2)$.
4. Simulate the outcome Y from $N(\theta_0 + \theta_1 A + \theta_2 M1 + \theta_3 M2 + \theta_4 AM1 + \theta_5 AM2 + \theta_6 M1M2 + \theta_7 AM1M2 + \theta_8 C, \sigma_Y^2)$.

#### 1.6.1.2 True Parameters

Table 6: True Model Parameters for Data Simulation

| Sample Size | $\theta_0$ | $\theta_1$ | $\theta_2$ | $\theta_3$ | $\theta_4$ | $\theta_5$ | $\theta_6$ | $\theta_7$ | $\theta_8$ | $\beta_{01}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 10000 | -5 | 0.8 | 1.8 | 1.2 | 0.6 | 0.4 | 0.3 | 0.5 | 0.1 | -0.25 |
| $\beta_{11}$ | $\beta_{21}$ | $\beta_{02}$ | $\beta_{12}$ | $\beta_{22}$ | P(A=1) | $\mu_C$ | $\sigma_C$ | $\sigma_{M1}$ | $\sigma_{M2}$ | $\sigma_Y$ |
| 0.5 | 0.2 | -0.3 | 0.4 | 0.3 | 0.4 | 1 | 1 | 0.1 | 0.1 | 0.2 |

#### 1.6.1.3 True Models

True model for the first mediator:

$$E[M1|a,c] = \beta_{01} + \beta_{11}a + \beta_{21}c$$

True model for the second mediator:

$$E[M2|a,c] = \beta_{02} + \beta_{12}a + \beta_{22}c$$

True model for the outcome:

$$E[Y|a,m^*,c] = \theta_0 + \theta_1 a + \theta_2 m1^* + \theta_3 m2^* + \theta_4 am1^* + \theta_5 am2^* + \theta_6 m1^* m2^* + \theta_7 am1^* m2^* + \theta_8 c$$

#### 1.6.2 Causal Effects and Standard Errors Estimated By the Natural Effect Model

```
causal_mediation(data = df_mulipleM_mint, model = "ne",
                 outcome = "contY_2contM_mint", exposure = 'A', exposure.type = "binary",
                 mediator = c('M_cont1','M_cont2'),
                 EMMint = TRUE, EMMint.terms = c("A*M_cont1*M_cont2"),
                 covariates = "C", yreg = "linear")
```

```
##                         Estimate  Std. Error
## pure direct effect     0.813262 0.016339811
## total direct effect    1.361424 0.016085313
## pure indirect effect   1.424536 0.015276143
## total indirect effect 1.972698 0.017431199
## total effect           2.785960 0.008701698
```

#### 1.6.3 Causal Effects and Standard Errors Estimated By the weighting-based approach

```
causal_mediation(data = df_mulipleM_mint, model = "wb",
                 outcome = "contY_2contM_mint", exposure = 'A', exposure.type = "binary",
                 mediator = c('M_cont1','M_cont2'), covariates = "C",
                 EMMint = TRUE, EMMint.terms = c("A*M_cont1*M_cont2"),
                 yreg = "linear", m_star = c(0,0), nboot = 500)
```

```
##         cde        pnde        tnde        pnie        tnie          te
## 0.816958390 0.815601331 1.356697305 1.426468345 1.967564319 2.783165650
##          pm      cde_se     pnde_se     tnde_se     pnie_se     tnie_se
## 0.546732903 0.015704114 0.016968880 0.016329629 0.016318999 0.017720944
##       te_se       pm_se
## 0.008476629 0.007200665
```

# 2 Case 2: Continuous Outcome and Binary Mediator

## 2.1 Case 2-1: Continuous Outcome and Single Binary Mediator Without Exposure-mediator Interaction

### 2.1.1 Data simulation

#### 2.1.1.1 Simulation Procedures

1. Simulate the exposure variable A from Binom(1,P(A=1)).
2. Simulate the covariate C from $N(\mu_C, \sigma_C^2)$.
3. Simulate the mediator M from $Bernoulli(expit(\beta_0 + \beta_1 * A + \beta_2 * C))$.
4. Simulate the outcome Y from $N(\theta_0 + \theta_1 A + \theta_2 M + \theta_4 C, \sigma_Y^2)$.

#### 2.1.1.2 True Parameters

Table 7: True Model Parameters for Data Simulation

| Sample Size | $\theta_0$ | $\theta_1$ | $\theta_2$ | $\theta_4$ | $\beta_0$ | $\beta_1$ | $\beta_2$ | P(A=1) | $\mu_C$ | $\sigma_C$ | $\sigma_Y$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 10000 | -5 | 0.8 | 1.8 | 0.1 | -0.25 | 0.5 | 0.2 | 0.4 | 1 | 1 | 0.2 |

#### 2.1.1.3 True Models

True model for the mediator:

$$logitE[M|a,c] = \beta_0 + \beta_1 a + \beta_2 c$$

True model for the outcome:

$$E[Y|a,m^*,c] = \theta_0 + \theta_1 a + \theta_2 m^* + \theta_4 c$$

### 2.1.2 Causal Effects and Standard Errors Estimated By the Structural Equation Model with 3 Different Estimation Method

#### 2.1.2.1 Delta Method

```
causal_mediation(data = df_noint, model = "standard", est.method = "delta",
                 outcome = "contY_binM_noint", exposure = 'A', exposure.type = "binary",
                 mediator = 'M_bin', covariates = "C", EMint = FALSE,
                 yreg = "linear", mreg = "logistic")
```

```
## $decomp3way
##         cde      cde_se        pnde     pnde_se        tnde     tnde_se
## 0.791521270 0.004108150 0.791521270 0.004108150 0.791521270 0.004108150
##        pnie     pnie_se        tnie     tnie_se          te       te_se
## 0.217342565 0.018260492 0.217342565 0.018260492 1.008863835 0.018704267
##          pm       pm_se
## 0.120720042 0.008936873
##
## $decomp4way
##            cde          cde_se          intref       intref_se          intmed
##      0.79152127      0.00410815      0.00000000      0.00000000      0.00000000
##      intmed_se             pie          pie_se              te           te_se
##      0.00000000      0.21734256      0.01826049      1.00886383      0.01870427
##       cde_prop     cde_prop_se     intref_prop  intref_prop_se     intmed_prop
##     0.78456700      0.01423054      0.00000000      0.00000000      0.00000000
## intmed_prop_se        pie_prop     pie_prop_se      overall_pm   overall_pm_se
##      0.00000000      0.21543300      0.01423054      0.21543300      0.01423054
##    overall_int  overall_int_se      overall_pe   overall_pe_se
##      0.00000000      0.00000000      0.21543300      0.01423054
```

#### 2.1.2.2 Bootstrapping

```
causal_mediation(data = df_noint, model = "standard", est.method = "bootstrap",
                 outcome = "contY_binM_noint", exposure = 'A', exposure.type = "binary",
                 mediator = 'M_bin', covariates = "C", EMint = FALSE,
                 yreg = "linear", mreg = "logistic", nboot = 500)
```

```
## $decomp3way
##         cde      cde_se        pnde     pnde_se        tnde     tnde_se
## 0.791521270 0.004088300 0.791521270 0.004088300 0.791521270 0.004088300
##        pnie     pnie_se        tnie     tnie_se          te       te_se
## 0.217342565 0.019551837 0.217342565 0.019551837 1.008863835 0.020226066
##          pm       pm_se
## 0.120720042 0.009538419
##
## $decomp4way
##            cde          cde_se          intref       intref_se          intmed
```

```
##      0.79152127      0.00408830      0.00000000      0.00000000      0.00000000
##       intmed_se             pie          pie_se              te           te_se
##      0.00000000      0.21734256      0.01955184      1.00886383      0.02022607
##        cde_prop     cde_prop_se     intref_prop  intref_prop_se     intmed_prop
##      0.78456700      0.01520693      0.00000000      0.00000000      0.00000000
## intmed_prop_se        pie_prop     pie_prop_se      overall_pm   overall_pm_se
##      0.00000000      0.21543300      0.01520693      0.21543300      0.01520693
##     overall_int overall_int_se      overall_pe   overall_pe_se
##      0.00000000      0.00000000      0.21543300      0.01520693
```

#### 2.1.2.3 Simulation-based Approach

```r
causal_mediation(data = df_noint, model = "standard", est.method = "simulation",
                 outcome = "contY_binM_noint", exposure = 'A', exposure.type = "binary",
                 mediator = 'M_bin', covariates = "C", EMint = FALSE,
                 yreg = "linear", mreg = "logistic",nsims = 500)[1:2]
```

```
## $decomp3way
##         cde      cde_se        pnde     pnde_se        tnde     tnde_se
## 0.791163589 0.003927357 0.791163589 0.003927357 0.791163589 0.003927357
##        pnie     pnie_se        tnie     tnie_se          te       te_se
## 0.215158211 0.017892328 0.215158211 0.017892328 1.006321800 0.018116754
##          pm       pm_se
## 0.119616496 0.008808862
##
## $decomp4way
##             cde          cde_se         intref       intref_se          intmed
##    7.911636e-01    3.927357e-03   -3.730349e-17    5.673591e-16    2.842171e-17
##       intmed_se             pie          pie_se              te           te_se
##    4.498157e-16    2.151582e-01    1.789233e-02    1.006322e+00    1.811675e-02
##        cde_prop     cde_prop_se     intref_prop  intref_prop_se     intmed_prop
##    7.864364e-01    1.406498e-02   -3.710782e-17    5.644115e-16    2.785925e-17
## intmed_prop_se        pie_prop     pie_prop_se      overall_pm   overall_pm_se
##    4.470223e-16    2.135636e-01    1.406498e-02    2.135636e-01    1.406498e-02
##     overall_int overall_int_se      overall_pe   overall_pe_se
##   -9.248568e-18    5.518411e-16    2.135636e-01    1.406498e-02
```

### 2.1.3 Causal Effects and Standard Errors Estimated By the Natural Effect Model

```r
causal_mediation(data = df_noint, model = "ne",
                 outcome = "contY_binM_noint", exposure = 'A', exposure.type = "binary",
                 mediator = 'M_bin', covariates = "C", EMint = FALSE,
                 yreg = "linear")
```

```
##                          Estimate  Std. Error
## natural direct effect   0.7915213 0.004107422
## natural indirect effect 0.2146027 0.018045308
## total effect            1.0061240 0.018516073
```

### 2.1.4 Causal Effects and Standard Errors Estimated By the regression-based approach

#### 2.1.4.1 Bootstrapping

```r
causal_mediation(data = df_noint, model = "rb", est.method = "bootstrap",
                 outcome = "contY_binM_noint", exposure = 'A', exposure.type = "binary",
                 mediator = 'M_bin', covariates = "C", EMint = FALSE,
                 yreg = "linear", mreg = "logistic", nboot = 500)
```

```
##         cde        pnde        tnde        pnie        tnie          te
## 0.791521270 0.791521270 0.791521270 0.217342565 0.217342565 1.008863835
##          pm      cde_se     pnde_se     tnde_se     pnie_se     tnie_se
## 0.120720042 0.004189819 0.004189819 0.004189819 0.017781854 0.017781854
##       te_se       pm_se
## 0.018352508 0.008721204
```

#### 2.1.4.2 Simulation-based Approach

```r
causal_mediation(data = df_noint, model = "rb", est.method = "simulation",
                 outcome = "contY_binM_noint", exposure = 'A', exposure.type = "binary",
                 mediator = 'M_bin', covariates = "C", EMint = FALSE,
                 yreg = "linear", mreg = "logistic", nsims = 500)[1:2]
```

```
## $decomp3way
##         cde      cde_se        pnde     pnde_se        tnde     tnde_se
## 0.791588632 0.004056042 0.791588632 0.004056042 0.791588632 0.004056042
##        pnie     pnie_se        tnie     tnie_se          te       te_se
## 0.214318009 0.016497055 0.214318009 0.016497055 1.005906640 0.017064344
##          pm       pm_se
## 0.119159127 0.008099400
##
## $decomp4way
##             cde          cde_se          intref       intref_se          intmed
##   7.915886e-01    4.056042e-03    2.486900e-17    5.952215e-16   -1.154632e-17
##       intmed_se             pie          pie_se              te           te_se
##   4.544964e-16    2.143180e-01    1.649705e-02    1.005907e+00    1.706434e-02
##        cde_prop     cde_prop_se     intref_prop  intref_prop_se     intmed_prop
##   7.871495e-01    1.294405e-02    2.431927e-17    5.919570e-16   -1.086782e-17
## intmed_prop_se        pie_prop     pie_prop_se      overall_pm   overall_pm_se
##   4.514423e-16    2.128505e-01    1.294405e-02    2.128505e-01    1.294405e-02
##     overall_int overall_int_se      overall_pe   overall_pe_se
##   1.345145e-17    5.616998e-16    2.128505e-01    1.294405e-02
```

### 2.1.5 Causal Effects and Standard Errors Estimated By the weighting-based approach

```r
causal_mediation(data = df_noint, model = "wb",
                 outcome = "contY_binM_noint", exposure = 'A', exposure.type = "binary",
                 mediator = 'M_bin', covariates = "C", EMint = FALSE,
                 yreg = "linear")
```

```
##         cde        pnde        tnde        pnie        tnie          te
## 0.791521270 0.791521270 0.791521270 0.214687261 0.214687261 1.006208531
##          pm      cde_se     pnde_se     tnde_se     pnie_se     tnie_se
## 0.119421317 0.004035054 0.004035054 0.004035054 0.017912733 0.017912733
##       te_se       pm_se
## 0.018440408 0.008782299
```

## 2.2 Case 2-2: Continuous Outcome and Single Binary Mediator With Exposure-mediator Interaction

### 2.2.1 Data simulation

#### 2.2.1.1 Simulation Procedures

1. Simulate the exposure variable A from Binom(1,P(A=1)).
2. Simulate the covariate C from $N(\mu_C, \sigma_C^2)$.
3. Simulate the mediator M from $Bernoulli(expit(\beta_0 + \beta_1 * A + \beta_2 * C))$.
4. Simulate the outcome Y from $N(\theta_0 + \theta_1 A + \theta_2 M + \theta_3 AM + \theta_4 C, \sigma_Y^2)$.

#### 2.2.1.2 True Parameters

Table 8: True Model Parameters for Data Simulation

| Sample Size | $\theta_0$ | $\theta_1$ | $\theta_2$ | $\theta_3$ | $\theta_4$ | $\beta_0$ | $\beta_1$ | $\beta_2$ | P(A=1) | $\mu_C$ | $\sigma_C$ | $\sigma_Y$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 10000 | -5 | 0.8 | 1.8 | 0.2 | 0.1 | -0.25 | 0.5 | 0.2 | 0.4 | 1 | 1 | 0.2 |

#### 2.2.1.3 True Models

True model for the mediator:

$$logitE[M|a,c] = \beta_0 + \beta_1 a + \beta_2 c$$

True model for the outcome:

$$E[Y|a,m^*,c] = \theta_0 + \theta_1 a + \theta_2 m^* + \theta_3 am^* + \theta_4 c$$

### 2.2.2 Causal Effects and Standard Errors Estimated By the Structural Equation Model with 3 Different Estimation Method

#### 2.2.2.1 Delta Method

```
causal_mediation(data = df_int, model = "standard", est.method = "delta",
                 outcome = "contY_binM_int", exposure = 'A', exposure.type = "binary",
                 mediator = 'M_bin', covariates = "C", EMint = TRUE,
                 yreg = "linear", mreg = "logistic")
```

```
## $decomp3way
##         cde      cde_se        pnde     pnde_se        tnde     tnde_se
## 0.799138924 0.006195933 0.900955189 0.004334708 0.924746446 0.004396184
##        pnie     pnie_se        tnie     tnie_se          te       te_se
## 0.209149773 0.018229626 0.232941029 0.020306035 1.133896218 0.019876422
##          pm       pm_se
## 0.114475695 0.008949382
##
## $decomp4way
##            cde          cde_se         intref      intref_se          intmed
##    0.799138924     0.006195933    0.101816265    0.004296866     0.023791257
##      intmed_se             pie         pie_se             te           te_se
##    0.002281682     0.209149773    0.018229626    1.133896218     0.019876422
##       cde_prop     cde_prop_se    intref_prop intref_prop_se     intmed_prop
##    0.704772546     0.012824459    0.089793284    0.004361941     0.020981864
## intmed_prop_se        pie_prop    pie_prop_se     overall_pm   overall_pm_se
```

```
##    0.001694070    0.184452306    0.012936115    0.205434171    0.014410592
##    overall_int overall_int_se     overall_pe  overall_pe_se
##    0.110775148    0.004607163    0.295227454    0.012824459
```

#### 2.2.2.2 Bootstrapping

```r
causal_mediation(data = df_int, model = "standard", est.method = "bootstrap",
                 outcome = "contY_binM_int", exposure = 'A', exposure.type = "binary",
                 mediator = 'M_bin', covariates = "C", EMint = TRUE,
                 yreg = "linear", mreg = "logistic", nboot = 500)
```

```
## $decomp3way
##         cde      cde_se        pnde     pnde_se        tnde     tnde_se
## 0.799138924 0.006470607 0.900955189 0.004383776 0.924746446 0.004352664
##        pnie     pnie_se        tnie     tnie_se          te       te_se
## 0.209149773 0.017422704 0.232941029 0.019463820 1.133896218 0.019029646
##          pm       pm_se
## 0.114475695 0.008595866
##
## $decomp4way
##            cde         cde_se         intref      intref_se         intmed
##    0.799138924    0.006470607    0.101816265    0.004304194    0.023791257
##      intmed_se            pie         pie_se             te          te_se
##    0.002254919    0.209149773    0.017422704    1.133896218    0.019029646
##       cde_prop    cde_prop_se    intref_prop intref_prop_se    intmed_prop
##    0.704772546    0.012625300    0.089793284    0.004282019    0.020981864
## intmed_prop_se       pie_prop    pie_prop_se     overall_pm  overall_pm_se
##    0.001689686    0.184452306    0.012392175    0.205434171    0.013855725
##    overall_int overall_int_se     overall_pe  overall_pe_se
##    0.110775148    0.004655821    0.295227454    0.012625300
```

#### 2.2.2.3 Simulation-based Approach

```r
causal_mediation(data = df_int, model = "standard", est.method = "simulation",
                 outcome = "contY_binM_int", exposure = 'A', exposure.type = "binary",
                 mediator = 'M_bin', covariates = "C", EMint = TRUE,
                 yreg = "linear", mreg = "logistic",nsims = 500)[1:2]
```

```
## $decomp3way
##         cde      cde_se        pnde     pnde_se        tnde     tnde_se
## 0.799298802 0.006600643 0.901225732 0.004574672 0.924774790 0.004500683
##        pnie     pnie_se        tnie     tnie_se          te       te_se
## 0.206862198 0.018053245 0.230411256 0.020133413 1.131636988 0.019581593
##          pm       pm_se
## 0.113266767 0.008907986
##
## $decomp4way
##            cde         cde_se         intref      intref_se         intmed
##    0.799298802    0.006600643    0.101926930    0.004366056    0.023549058
##      intmed_se            pie         pie_se             te          te_se
##    0.002300726    0.206862198    0.018053245    1.131636988    0.019581593
##       cde_prop    cde_prop_se    intref_prop intref_prop_se    intmed_prop
##    0.706520562    0.012931721    0.090108760    0.004406463    0.020785297
## intmed_prop_se       pie_prop    pie_prop_se     overall_pm  overall_pm_se
```

```
##    0.001723145    0.182585381    0.012877070    0.203370678    0.014366883
##    overall_int overall_int_se     overall_pe  overall_pe_se
##    0.110894057    0.004731674    0.293479438    0.012931721
```

#### 2.2.3 Causal Effects and Standard Errors Estimated By the Natural Effect Model

```r
causal_mediation(data = df_int, model = "ne",
                 outcome = "contY_binM_int", exposure = 'A', exposure.type = "binary",
                 mediator = 'M_bin', covariates = "C", EMint = TRUE,
                 yreg = "linear")
```

```
##                         Estimate  Std. Error
## pure direct effect    0.9009865 0.004345408
## total direct effect   0.9244763 0.004388925
## pure indirect effect  0.2070426 0.018060639
## total indirect effect 0.2305324 0.020113014
## total effect          1.1315189 0.019699705
```

#### 2.2.4 Causal Effects and Standard Errors Estimated By the regression-based approach

##### 2.2.4.1 Bootstrapping

```r
causal_mediation(data = df_int, model = "rb", est.method = "bootstrap",
                 outcome = "contY_binM_int", exposure = 'A', exposure.type = "binary",
                 mediator = 'M_bin', covariates = "C", EMint = TRUE,
                 yreg = "linear", mreg = "logistic", nboot = 500)
```

```
##         cde        pnde        tnde        pnie        tnie          te
## 0.799138924 0.900955189 0.924746446 0.209149773 0.232941029 1.133896218
##          pm      cde_se     pnde_se     tnde_se     pnie_se     tnie_se
## 0.114475695 0.006277942 0.004367642 0.004636581 0.018483613 0.020648793
##       te_se       pm_se
## 0.020605417 0.009070593
```

##### 2.2.4.2 Simulation-based Approach

```r
causal_mediation(data = df_int, model = "rb", est.method = "simulation",
                 outcome = "contY_binM_int", exposure = 'A', exposure.type = "binary",
                 mediator = 'M_bin', covariates = "C", EMint = TRUE,
                 yreg = "linear", mreg = "logistic", nsims = 500)[1:2]
```

```
## $decomp3way
##         cde      cde_se        pnde     pnde_se        tnde     tnde_se
## 0.799250929 0.006114855 0.901059039 0.004285467 0.924575377 0.004424837
##        pnie     pnie_se        tnie     tnie_se          te       te_se
## 0.206913301 0.018232183 0.230429639 0.020269672 1.131488678 0.019886333
##          pm       pm_se
## 0.113290131 0.008951479
##
## $decomp4way
##            cde         cde_se        intref      intref_se         intmed
##    0.799250929    0.006114855    0.101808110    0.004512823    0.023516338
##      intmed_se            pie         pie_se             te          te_se
```

```
##    0.002261827    0.206913301    0.018232183    1.131488678    0.019886333
##       cde_prop    cde_prop_se    intref_prop intref_prop_se    intmed_prop
##    0.706576150    0.012864579    0.090016753    0.004538845    0.020758852
## intmed_prop_se       pie_prop    pie_prop_se     overall_pm  overall_pm_se
##    0.001686638    0.182648245    0.013010453    0.203407097    0.014457438
##    overall_int overall_int_se     overall_pe  overall_pe_se
##    0.110775605    0.004818411    0.293423850    0.012864579
```

#### 2.2.5 Causal Effects and Standard Errors Estimated By the weighting-based approach

```
causal_mediation(data = df_int, model = "wb",
                 outcome = "contY_binM_int", exposure = 'A', exposure.type = "binary",
                 mediator = 'M_bin', covariates = "C", EMint = TRUE,
                 yreg = "linear", nboot = 500)
```

```
##         cde        pnde        tnde        pnie        tnie          te
## 0.799138924 0.900965535 0.924518725 0.207049668 0.230602858 1.131568393
##          pm      cde_se     pnde_se     tnde_se     pnie_se     tnie_se
## 0.113455847 0.006131699 0.004373181 0.004382605 0.018720211 0.020892364
##       te_se       pm_se
## 0.020302233 0.009256227
```

### 2.3 Case 2-3: Continuous Outcome and Multiple Binary Mediators Without Interaction

#### 2.3.1 Data simulation

##### 2.3.1.1 Simulation Procedures

1. Simulate the exposure variable A from Binom(1,P(A=1)).
2. Simulate the covariate C from $N(\mu_C, \sigma_C^2)$.
3. Simulate the first mediator M1 from $Bernoulli(expit(\beta_{01} + \beta_{11} * A + \beta_{21} * C))$ and the second mediator M2 from $Bernoulli(expit(\beta_{02} + \beta_{12} * A + \beta_{22} * C))$.
4. Simulate the outcome Y from $N(\theta_0 + \theta_1 A + \theta_2 M1 + \theta_3 M2 + \theta_4 C, \sigma_Y^2)$.

##### 2.3.1.2 True Parameters

Table 9: True Model Parameters for Data Simulation

| Sample Size | $\theta_0$ | $\theta_1$ | $\theta_2$ | $\theta_3$ | $\theta_4$ | $\beta_{01}$ | $\beta_{11}$ | $\beta_{21}$ |
|---|---|---|---|---|---|---|---|---|
| 10000 | -5 | 0.8 | 1.8 | 1.2 | 0.1 | -0.25 | 0.5 | 0.2 |
| $\beta_{02}$ | $\beta_{12}$ | $\beta_{22}$ | P(A=1) | $\mu_C$ | $\sigma_C$ | $\sigma_{M1}$ | $\sigma_{M2}$ | $\sigma_Y$ |
| -0.3 | 0.4 | 0.3 | 0.4 | 1 | 1 | 0.1 | 0.1 | 0.2 |

##### 2.3.1.3 True Models

True model for the first mediator:

$$logitE[M1|a,c] = \beta_{01} + \beta_{11}a + \beta_{21}c$$

True model for the second mediator:

$$logitE[M2|a,c] = \beta_{02} + \beta_{12}a + \beta_{22}c$$

True model for the outcome:

$$E[Y|a,m^*,c] = \theta_0 + \theta_1 a + \theta_2 m1^* + \theta_3 m2^* + \theta_4 c$$

### 2.3.2 Causal Effects and Standard Errors Estimated By the Natural Effect Model

```
causal_mediation(data = df_mulipleM_noint, model = "ne",
                 outcome = "contY_2binM_noint", exposure = 'A', exposure.type = "binary",
                 mediator = c('M_bin1','M_bin2'), covariates = "C",
                 yreg = "linear")
```

```
##                          Estimate  Std. Error
## natural direct effect   0.8023417 0.004155421
## natural indirect effect 0.3399026 0.021546199
## total effect            1.1422444 0.021903684
```

### 2.3.3 Causal Effects and Standard Errors Estimated By the regression-based approach

#### 2.3.3.1 Bootstrapping

```
causal_mediation(data = df_mulipleM_noint, model = "rb", est.method = "bootstrap",
                 outcome = "contY_2binM_noint", exposure = 'A', exposure.type = "binary",
                 mediator = c('M_bin1','M_bin2'), covariates = "C",
                 yreg = "linear", m_star = c(0,0), mreg = c("logistic","logistic"), nboot = 500)
```

```
##         cde        pnde        tnde        pnie        tnie          te
## 0.802341726 0.802341726 0.802341726 0.345503379 0.345503379 1.147845104
##          pm      cde_se     pnde_se     tnde_se     pnie_se     tnie_se
## 0.177164246 0.004190714 0.004190714 0.004190714 0.022329989 0.022329989
##       te_se       pm_se
## 0.022734001 0.009448640
```

#### 2.3.3.2 Simulation-based Approach

```
causal_mediation(data = df_mulipleM_noint, model = "rb", est.method = "simulation",
                 outcome = "contY_2binM_noint", exposure = 'A', exposure.type = "binary",
                 mediator = c('M_bin1','M_bin2'), covariates = "C",
                 yreg = "linear", m_star = c(0,0), mreg = c("logistic","logistic"), nsims = 500)[1:2]
```

```
## $decomp3way
##         cde      cde_se        pnde     pnde_se        tnde     tnde_se
## 0.802770488 0.003827756 0.802770488 0.003827756 0.802770488 0.003827756
##        pnie     pnie_se        tnie     tnie_se          te       te_se
## 0.341200384 0.021875779 0.341200384 0.021875779 1.143970872 0.022270446
##          pm       pm_se
## 0.175165984 0.009262060
##
## $decomp4way
##            cde          cde_se           intref       intref_se          intmed
```

```
##    8.027705e-01   3.827756e-03  -2.220446e-17   5.622081e-16   2.220446e-17
##       intmed_se            pie         pie_se             te          te_se
##    3.599078e-16   3.412004e-01   2.187578e-02   1.143971e+00   2.227045e-02
##        cde_prop    cde_prop_se    intref_prop intref_prop_se    intmed_prop
##    7.019926e-01   1.339908e-02  -1.906931e-17   4.912966e-16   1.903526e-17
## intmed_prop_se       pie_prop    pie_prop_se     overall_pm overall_pm_se
##    3.142368e-16   2.980074e-01   1.339908e-02   2.980074e-01   1.339908e-02
##     overall_int overall_int_se     overall_pe overall_pe_se
##   -3.405505e-20   4.586716e-16   2.980074e-01   1.339908e-02
```

#### 2.3.4 Causal Effects and Standard Errors Estimated By the weighting-based approach

```
causal_mediation(data = df_mulipleM_noint, model = "wb",
                 outcome = "contY_2binM_noint", exposure = 'A', exposure.type = "binary",
                 mediator = c('M_bin1','M_bin2'), covariates = "C",
                 yreg = "linear", m_star = c(0,0), nboot = 500)
```

```
##         cde        pnde        tnde        pnie        tnie          te
## 0.802341726 0.802341726 0.802341726 0.340016151 0.340016151 1.142357876
##          pm      cde_se     pnde_se     tnde_se     pnie_se     tnie_se
## 0.174842505 0.004293909 0.004293909 0.004293909 0.022793234 0.022793234
##       te_se       pm_se
## 0.023078577 0.009718155
```

### 2.4 Case 2-4: Continuous Outcome and Multiple Binary Mediators With Exposure-mediator Interaction

#### 2.4.1 Data simulation

##### 2.4.1.1 Simulation Procedures

1. Simulate the exposure variable A from Binom(1,P(A=1)).
2. Simulate the covariate C from $N(\mu_C, \sigma_C^2)$.
3. Simulate the first mediator M1 from $Bernoulli(expit(\beta_{01} + \beta_{11} * A + \beta_{21} * C))$ and the second mediator M2 from $Bernoulli(expit(\beta_{02} + \beta_{12} * A + \beta_{22} * C))$.
4. Simulate the outcome Y from $N(\theta_0 + \theta_1 A + \theta_2 M1 + \theta_3 M2 + \theta_4 AM1 + \theta_5 AM2 + \theta_6 C, \sigma_Y^2)$.

##### 2.4.1.2 True Parameters

Table 10: True Model Parameters for Data Simulation

| Sample Size | $\theta_0$ | $\theta_1$ | $\theta_2$ | $\theta_3$ | $\theta_4$ | $\theta_5$ | $\theta_6$ | $\beta_{01}$ | $\beta_{11}$ |
|---|---|---|---|---|---|---|---|---|---|
| 10000 | -5 | 0.8 | 1.8 | 1.2 | 0.6 | 0.4 | 0.1 | -0.25 | 0.5 |
| $\beta_{21}$ | $\beta_{02}$ | $\beta_{12}$ | $\beta_{22}$ | P(A=1) | $\mu_C$ | $\sigma_C$ | $\sigma_{M1}$ | $\sigma_{M2}$ | $\sigma_Y$ |
| 0.2 | -0.3 | 0.4 | 0.3 | 0.4 | 1 | 1 | 0.1 | 0.1 | 0.2 |

##### 2.4.1.3 True Models

True model for the first mediator:

$$logitE[M1|a, c] = \beta_{01} + \beta_{11}a + \beta_{21}c$$

True model for the second mediator:

$$logitE[M2|a,c] = \beta_{02} + \beta_{12}a + \beta_{22}c$$

True model for the outcome:

$$E[Y|a,m^*,c] = \theta_0 + \theta_1 a + \theta_2 m1^* + \theta_3 m2^* + \theta_4 am1^* + \theta_5 am2^* + \theta_6 c$$

### 2.4.2 Causal Effects and Standard Errors Estimated By the Natural Effect Model

```
causal_mediation(data = df_mulipleM_EMint, model = "ne",
                 outcome = "contY_2binM_EMint", exposure = 'A', exposure.type = "binary",
                 mediator = c('M_bin1','M_bin2'), EMint = TRUE, EMint.terms = c("A*M_bin1", "A*M_bin2")
                 covariates = "C", yreg = "linear")
```

```
##                          Estimate  Std. Error
## pure direct effect     1.2998982 0.006264900
## total direct effect    1.4132225 0.006961218
## pure indirect effect   0.3398707 0.021579701
## total indirect effect 0.4531950 0.028708672
## total effect           1.7530932 0.026336476
```

### 2.4.3 Causal Effects and Standard Errors Estimated By the regression-based approach

#### 2.4.3.1 Bootstrapping

```
causal_mediation(data = df_mulipleM_EMint, model = "rb", est.method = "bootstrap",
                 outcome = "contY_2binM_EMint", exposure = 'A', exposure.type = "binary",
                 mediator = c('M_bin1','M_bin2'), covariates = "C",
                 EMint = TRUE, EMint.terms = c("A*M_bin1", "A*M_bin2"),
                 yreg = "linear", m_star = c(0,0), mreg = c("logistic","logistic"), nboot = 500)
```

```
##         cde        pnde        tnde        pnie        tnie          te
## 0.805229822 1.300123133 1.414707690 0.345759609 0.460344166 1.760467299
##          pm      cde_se     pnde_se     tnde_se     pnie_se     tnie_se
## 0.150410248 0.008146893 0.006295196 0.006853530 0.021943013 0.029249550
##       te_se       pm_se
## 0.026671367 0.008443445
```

#### 2.4.3.2 Simulation-based Approach

```
causal_mediation(data = df_mulipleM_EMint, model = "rb", est.method = "simulation",
                 outcome = "contY_2binM_EMint", exposure = 'A', exposure.type = "binary",
                 mediator = c('M_bin1','M_bin2'), covariates = "C",
                 EMint = TRUE, EMint.terms = c("A*M_bin1", "A*M_bin2"),
                 yreg = "linear", m_star = c(0,0), mreg = c("logistic","logistic"), nsims = 500)[1:2]
```

```
## $decomp3way
##         cde      cde_se        pnde     pnde_se        tnde     tnde_se
## 0.805071891 0.007713810 1.300000123 0.006027672 1.412817950 0.007075601
##        pnie     pnie_se        tnie     tnie_se          te       te_se
## 0.340377516 0.022450321 0.453195343 0.029873141 1.753195466 0.027613242
##          pm       pm_se
```

```
## 0.148366519 0.008623921
##
## $decomp4way
##             cde          cde_se         intref      intref_se         intmed
##     0.805071891     0.007713810    0.494928232    0.007196913    0.112817827
##       intmed_se             pie         pie_se             te          te_se
##     0.007536840     0.340377516    0.022450321    1.753195466    0.027613242
##        cde_prop     cde_prop_se    intref_prop intref_prop_se    intmed_prop
##     0.459310114     0.008123992    0.282392205    0.006995772    0.064300399
## intmed_prop_se        pie_prop    pie_prop_se     overall_pm  overall_pm_se
##     0.003334220     0.193997282    0.009837004    0.258297681    0.013087135
##     overall_int overall_int_se     overall_pe  overall_pe_se
##     0.346692604     0.005080263    0.540689886    0.008123992
```

### 2.4.4 Causal Effects and Standard Errors Estimated By the weighting-based approach

```
causal_mediation(data = df_mulipleM_EMint, model = "wb",
                 outcome = "contY_2binM_EMint", exposure = 'A', exposure.type = "binary",
                 mediator = c('M_bin1','M_bin2'), covariates = "C",
                 EMint = TRUE, EMint.terms = c("A*M_bin1", "A*M_bin2"),
                 yreg = "linear", m_star = c(0,0), nboot = 500)
```

```
##         cde        pnde        tnde        pnie        tnie          te
## 0.805229822 1.300169158 1.412928923 0.340272006 0.453031771 1.753200929
##          pm      cde_se     pnde_se     tnde_se     pnie_se     tnie_se
## 0.148371065 0.007691073 0.006137175 0.007022717 0.021459485 0.028527018
##       te_se       pm_se
## 0.026343641 0.008245805
```

## 2.5 Case 2-5: Continuous Outcome and Multiple Binary Mediators With Mediator-mediator Interaction

### 2.5.1 Data simulation

#### 2.5.1.1 Simulation Procedures

1. Simulate the exposure variable A from Binom(1,P(A=1)).
2. Simulate the covariate C from $N(\mu_C, \sigma_C^2)$.
3. Simulate the first mediator M1 from $Bernoulli(expit(\beta_{01} + \beta_{11} * A + \beta_{21} * C))$ and the second mediator M2 from $Bernoulli(expit(\beta_{02} + \beta_{12} * A + \beta_{22} * C))$.
4. Simulate the outcome Y from $N(\theta_0 + \theta_1 A + \theta_2 M1 + \theta_3 M2 + \theta_4 M1M2 + \theta_5 C, \sigma_Y^2)$.

#### 2.5.1.2 True Parameters

Table 11: True Model Parameters for Data Simulation

| Sample Size | $\theta_0$ | $\theta_1$ | $\theta_2$ | $\theta_3$ | $\theta_4$ | $\theta_5$ | $\beta_{01}$ | $\beta_{11}$ | $\beta_{21}$ |
|---|---|---|---|---|---|---|---|---|---|
| 10000 | -5 | 0.8 | 1.8 | 1.2 | 0.6 | 0.1 | -0.25 | 0.5 | 0.2 |
| $\beta_{02}$ | $\beta_{12}$ | $\beta_{22}$ | P(A=1) | $\mu_C$ | $\sigma_C$ | $\sigma_{M1}$ | $\sigma_{M2}$ | $\sigma_Y$ | |
| -0.3 | 0.4 | 0.3 | 0.4 | 1 | 1 | 0.1 | 0.1 | 0.2 | |

#### 2.5.1.3 True Models

True model for the first mediator:

$$logitE[M1|a,c] = \beta_{01} + \beta_{11}a + \beta_{21}c$$

True model for the second mediator:

$$logitE[M2|a,c] = \beta_{02} + \beta_{12}a + \beta_{22}c$$

True model for the outcome:

$$E[Y|a,m^*,c] = \theta_0 + \theta_1 a + \theta_2 m1^* + \theta_3 m2^* + \theta_4 m1^* m2^* + \theta_5 c$$

### 2.5.2 Causal Effects and Standard Errors Estimated By the Natural Effect Model

```
causal_mediation(data = df_mulipleM_MMint, model = "ne",
                 outcome = "contY_2binM_MMint", exposure = 'A', exposure.type = "binary",
                 mediator = c('M_bin1','M_bin2'), MMint = TRUE, MMint.terms = c("M_bin1*M_bin2"),
                 covariates = "C", yreg = "linear")
```

```
##                          Estimate  Std. Error
## natural direct effect   0.8022898 0.004155326
## natural indirect effect 0.4174735 0.026409800
## total effect            1.2197633 0.026700816
```

### 2.5.3 Causal Effects and Standard Errors Estimated By the weighting-based approach

```
causal_mediation(data = df_mulipleM_MMint, model = "wb",
                 outcome = "contY_2binM_MMint", exposure = 'A', exposure.type = "binary",
                 mediator = c('M_bin1','M_bin2'), covariates = "C",
                 MMint = TRUE, MMint.terms = c("M_bin1*M_bin2"),
                 yreg = "linear", m_star = c(0,0), nboot = 500)
```

```
##         cde        pnde        tnde        pnie        tnie          te
## 0.802289803 0.802289803 0.802289803 0.417589527 0.417589527 1.219879330
##          pm      cde_se     pnde_se     tnde_se     pnie_se     tnie_se
## 0.206505737 0.004327272 0.004327272 0.004327272 0.024747743 0.024747743
##       te_se       pm_se
## 0.025179325 0.009780241
```

## 2.6 Case 2-6: Continuous Outcome and Multiple Binary Mediators With Exposure-mediator-mediator Interaction

### 2.6.1 Data simulation

#### 2.6.1.1 Simulation Procedures

1. Simulate the exposure variable A from Binom(1,P(A=1)).
2. Simulate the covariate C from $N(\mu_C, \sigma_C^2)$.
3. Simulate the first mediator M1 from $Bernoulli(expit(\beta_{01} + \beta_{11} * A + \beta_{21} * C))$ and the second mediator M2 from $Bernoulli(expit(\beta_{02} + \beta_{12} * A + \beta_{22} * C))$.

4. Simulate the outcome Y from $N(\theta_0+\theta_1 A+\theta_2 M1+\theta_3 M2+\theta_4 AM1+\theta_5 AM2+\theta_6 M1M2+\theta_7 AM1M2+\theta_8 C, \sigma_Y^2)$.

#### 2.6.1.2 True Parameters

Table 12: True Model Parameters for Data Simulation

| Sample Size | $\theta_0$ | $\theta_1$ | $\theta_2$ | $\theta_3$ | $\theta_4$ | $\theta_5$ | $\theta_6$ | $\theta_7$ | $\theta_8$ | $\beta_{01}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 10000 | -5 | 0.8 | 1.8 | 1.2 | 0.6 | 0.4 | 0.3 | 0.5 | 0.1 | -0.25 |
| $\beta_{11}$ | $\beta_{21}$ | $\beta_{02}$ | $\beta_{12}$ | $\beta_{22}$ | P(A=1) | $\mu_C$ | $\sigma_C$ | $\sigma_{M1}$ | $\sigma_{M2}$ | $\sigma_Y$ |
| 0.5 | 0.2 | -0.3 | 0.4 | 0.3 | 0.4 | 1 | 1 | 0.1 | 0.1 | 0.2 |

#### 2.6.1.3 True Models

True model for the first mediator:

$$logitE[M1|a,c] = \beta_{01} + \beta_{11}a + \beta_{21}c$$

True model for the second mediator:

$$logitE[M2|a,c] = \beta_{02} + \beta_{12}a + \beta_{22}c$$

True model for the outcome:

$$E[Y|a,m^*,c] = \theta_0 + \theta_1 a + \theta_2 m1^* + \theta_3 m2^* + \theta_4 am1^* + \theta_5 am2^* + \theta_6 m1^* m2^* + \theta_7 am1^* m2^* + \theta_8 c$$

### 2.6.2 Causal Effects and Standard Errors Estimated By the Natural Effect Model

```
causal_mediation(data = df_mulipleM_mint, model = "ne",
                 outcome = "contY_2binM_mint", exposure = 'A', exposure.type = "binary",
                 mediator = c('M_bin1','M_bin2'),
                 EMMint = TRUE, EMMint.terms = c("A*M_bin1*M_bin2"),
                 covariates = "C", yreg = "linear")
```

```
##                        Estimate  Std. Error
## pure direct effect    1.4240397 0.008230877
## total direct effect   1.6024202 0.010010530
## pure indirect effect  0.3783645 0.023969710
## total indirect effect 0.5567450 0.035184539
## total effect          1.9807847 0.031613960
```

### 2.6.3 Causal Effects and Standard Errors Estimated By the weighting-based approach

```
causal_mediation(data = df_mulipleM_mint, model = "wb",
                 outcome = "contY_2binM_mint", exposure = 'A', exposure.type = "binary",
                 mediator = c('M_bin1','M_bin2'), covariates = "C",
                 EMMint = TRUE, EMMint.terms = c("A*M_bin1*M_bin2"),
                 yreg = "linear", m_star = c(0,0), nboot = 500)
```

```
##         cde        pnde        tnde        pnie        tnie          te
## 0.813729875 1.424543847 1.601781676 0.378999283 0.556237113 1.980780959
##          pm      cde_se     pnde_se     tnde_se     pnie_se     tnie_se
## 0.163343336 0.009418434 0.008085553 0.010214080 0.023680251 0.034778750
##       te_se       pm_se
## 0.031575774 0.008950341
```

# 3 Case 3: Binary Outcome and Continuous Mediator

## 3.1 Case 3-1: Binary Outcome and Single Continuous Mediator Without Exposure-mediator Interaction

### 3.1.1 Data simulation

#### 3.1.1.1 Simulation Procedures

1. Simulate the exposure variable A from Binom(1,P(A=1)).
2. Simulate the covariate C from $N(\mu_C, \sigma_C^2)$.
3. Simulate the mediator M from $N((\beta_0 + \beta_1 * A + \beta_2 * C), \sigma_M^2)$.
4. Simulate the outcome Y from $Bernoulli(\theta_0 + \theta_1 A + \theta_2 M + \theta_4 C)$.

#### 3.1.1.2 True Parameters

Table 13: True Model Parameters for Data Simulation

| Sample Size | $\theta_0$ | $\theta_1$ | $\theta_2$ | $\theta_4$ | $\beta_0$ | $\beta_1$ | $\beta_2$ | P(A=1) | $\mu_C$ | $\sigma_C$ | $\sigma_M$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 10000 | -5 | 0.8 | 1.8 | 0.1 | -0.25 | 0.5 | 0.2 | 0.4 | 1 | 1 | 0.1 |

#### 3.1.1.3 True Models

True model for the mediator:

$$E[M|a, c] = \beta_0 + \beta_1 a + \beta_2 c$$

True model for the outcome:

$$logitE[Y|a, m^*, c] = \theta_0 + \theta_1 a + \theta_2 m^* + \theta_4 c$$

### 3.1.2 Causal Effects and Standard Errors Estimated By the Structural Equation Model with 3 Different Estimation Method

#### 3.1.2.1 Delta Method

```
causal_mediation(data = df_noint, model = "standard", est.method = "delta",
                 outcome = "binY_contM_noint", exposure = 'A', exposure.type = "binary",
                 mediator = 'M_cont', covariates = "C", EMint = FALSE,
                 yreg = "logistic", mreg = "linear")
```

```
## $decomp3way
##     cde_rr   cde_rr_se    pnde_rr pnde_rr_se    tnde_rr tnde_rr_se
##  2.9971114  1.1233382  2.9971114  1.1233382  2.9971114  1.1233382
##    pnie_rr pnie_rr_se    tnie_rr tnie_rr_se      te_rr    te_rr_se
##  1.9649169  0.6598507  1.9649169  0.6598507  5.8890748  0.9752616
##         pm       pm_se
##  0.5915155  0.2068861
##
## $decomp4way
##            cde_err         cde_err_se         intref_err
##        2.121910290        1.145252377       -0.124798865
##      intref_err_se         intmed_err      intmed_err_se
##        0.032471577        1.927046533        0.590487171
```

```
##            pie_err         pie_err_se          total_err
##        0.964916884        0.659850652        4.889074842
##       total_err_se       cde_err_prop    cde_err_prop_se
##        0.975261567        0.434010597        0.208801011
##    intref_err_prop intref_err_prop_se    intmed_err_prop
##       -0.025526070        0.002306524        0.394153617
## intmed_err_prop_se       pie_err_prop    pie_err_prop_se
##        0.073439665        0.197361856        0.140685186
##         overall_pm      overall_pm_se        overall_int
##        0.591515473        0.206886122        0.368627547
##     overall_int_se         overall_pe      overall_pe_se
##        0.074864762        0.565989403        0.208801011
```

#### 3.1.2.2 Bootstrapping

```
causal_mediation(data = df_noint, model = "standard", est.method = "bootstrap",
                 outcome = "binY_contM_noint", exposure = 'A', exposure.type = "binary",
                 mediator = 'M_cont', covariates = "C", EMint = FALSE,
                 yreg = "logistic", mreg = "linear", nboot = 500)
```

```
## $decomp3way
##     cde_rr  cde_rr_se    pnde_rr pnde_rr_se    tnde_rr tnde_rr_se
##  2.9971114  1.2378319  2.9971114  1.2378319  2.9971114  1.2378319
##    pnie_rr pnie_rr_se    tnie_rr tnie_rr_se      te_rr   te_rr_se
##  1.9649169  0.7478846  1.9649169  0.7478846  5.8890748  1.0022577
##         pm      pm_se
##  0.5915155  0.2261877
##
## $decomp4way
##            cde_err         cde_err_se         intref_err
##        2.121910290        1.235292177       -0.124798865
##      intref_err_se         intmed_err      intmed_err_se
##        0.048987014        1.927046533        0.745217255
##            pie_err         pie_err_se          total_err
##        0.964916884        0.747884609        4.889074842
##       total_err_se       cde_err_prop    cde_err_prop_se
##        1.002257719        0.434010597        0.223078014
##    intref_err_prop intref_err_prop_se    intmed_err_prop
##       -0.025526070        0.008084617        0.394153617
## intmed_err_prop_se       pie_err_prop    pie_err_prop_se
##        0.116381641        0.197361856        0.155937437
##         overall_pm      overall_pm_se        overall_int
##        0.591515473        0.226187652        0.368627547
##     overall_int_se         overall_pe      overall_pe_se
##        0.109666690        0.565989403        0.223078014
```

#### 3.1.2.3 Simulation-based Approach

```
causal_mediation(data = df_noint, model = "standard", est.method = "simulation",
                 outcome = "binY_contM_noint", exposure = 'A', exposure.type = "binary",
                 mediator = 'M_cont', covariates = "C", EMint = FALSE,
                 yreg = "logistic", mreg = "linear",nsims = 500)[1:2]
```

```
## $decomp3way
```

```
##     cde_rr  cde_rr_se    pnde_rr pnde_rr_se    tnde_rr tnde_rr_se
##  3.2224956  1.3162520  3.2165261  1.3206891  3.1767594  1.3305863
##    pnie_rr pnie_rr_se    tnie_rr tnie_rr_se      te_rr   te_rr_se
##  2.0510164  0.7330560  2.0206273  0.7224163  5.7590512  0.9512526
##         pm      pm_se
##  0.5389860  0.2477529
##
## $decomp4way
##            cde_err         cde_err_se         intref_err
##       2.2138943781       1.3221853054       0.0026317027
##      intref_err_se         intmed_err      intmed_err_se
##       0.0413015763       1.4915086786       0.7991702066
##            pie_err         pie_err_se          total_err
##       1.0510164249       0.7330559999       4.7590511843
##       total_err_se       cde_err_prop    cde_err_prop_se
##       0.9512526291       0.4602561867       0.2476777854
##    intref_err_prop intref_err_prop_se    intmed_err_prop
##       0.0007578469       0.0081592609       0.3090831834
## intmed_err_prop_se       pie_err_prop    pie_err_prop_se
##       0.1372403979       0.2299027830       0.1698308374
##         overall_pm      overall_pm_se        overall_int
##       0.5389859664       0.2477529392       0.3098410303
##     overall_int_se         overall_pe      overall_pe_se
##       0.1389793282       0.5397438133       0.2476777854
```

### 3.1.3 Causal Effects and Standard Errors Estimated By the Natural Effect Model

```r
causal_mediation(data = df_noint, model = "ne",
                 outcome = "binY_contM_noint", exposure = 'A', exposure.type = "binary",
                 mediator = 'M_cont', covariates = "C", EMint = FALSE,
                 yreg = "logistic")
```

```
##                         Estimate Std. Error
## natural direct effect   2.995350  12.361092
## natural indirect effect 1.968323   4.193891
## total effect            5.895817 147.847844
```

### 3.1.4 Causal Effects and Standard Errors Estimated By the regression-based approach

#### 3.1.4.1 Bootstrapping

```r
causal_mediation(data = df_noint, model = "rb", est.method = "bootstrap",
                 outcome = "binY_contM_noint", exposure = 'A', exposure.type = "binary",
                 mediator = 'M_cont', covariates = "C", EMint = FALSE,
                 yreg = "logistic", mreg = "linear", nboot = 500)
```

```
##     RRcde    RRpnde    RRtnde    RRpnie    RRtnie      RRte        pm
## 2.9971114 2.9971114 2.9971114 1.9649169 1.9649169 5.8890748 0.5915155
##  RRcde_se RRpnde_se RRtnde_se RRpnie_se RRtnie_se   RRte_se     pm_se
## 1.1637934 1.1637934 1.1637934 0.7667883 0.7667883 1.0396619 0.2126101
```

#### 3.1.4.2 Simulation-based Approach

```r
causal_mediation(data = df_noint, model = "rb", est.method = "simulation",
                 outcome = "binY_contM_noint", exposure = 'A', exposure.type = "binary",
                 mediator = 'M_cont', covariates = "C", EMint = FALSE,
                 yreg = "logistic", mreg = "linear", nsims = 500)[1:2]
```

```
## $decomp3way
##     cde_rr  cde_rr_se    pnde_rr pnde_rr_se    tnde_rr tnde_rr_se
##  3.1580450  1.1722206  3.1506447  1.1706454  3.1085873  1.1687789
##    pnie_rr pnie_rr_se    tnie_rr tnie_rr_se      te_rr   te_rr_se
##  2.0669159  0.7613704  2.0365257  0.7517779  5.7236318  0.9357483
##         pm      pm_se
##  0.5479886  0.2232655
##
## $decomp4way
##            cde_err         cde_err_se         intref_err
##        2.143868468        1.161578680        0.006776275
##      intref_err_se         intmed_err      intmed_err_se
##        0.034857014        1.506071097        0.683165482
##            pie_err         pie_err_se          total_err
##        1.066915925        0.761370420        4.723631766
##       total_err_se       cde_err_prop    cde_err_prop_se
##        0.935748274        0.450424462        0.221163663
##    intref_err_prop intref_err_prop_se    intmed_err_prop
##        0.001586922        0.006990577        0.313866116
## intmed_err_prop_se       pie_err_prop    pie_err_prop_se
##        0.111490735        0.234122500        0.176242289
##         overall_pm      overall_pm_se        overall_int
##        0.547988616        0.223265462        0.315453038
##     overall_int_se         overall_pe      overall_pe_se
##        0.110992169        0.549575538        0.221163663
```

#### 3.1.5 Causal Effects and Standard Errors Estimated By the weighting-based approach

```r
causal_mediation(data = df_noint, model = "wb",
                 outcome = "binY_contM_noint", exposure = 'A', exposure.type = "binary",
                 mediator = 'M_cont', covariates = "C", EMint = FALSE,
                 yreg = "logistic")
```

```
##     RRcde    RRpnde    RRtnde    RRpnie    RRtnie      RRte        pm
## 2.9488546 2.9410239 2.8891481 1.9522780 1.9178424 5.6404203 0.5817138
##  RRcde_se RRpnde_se RRtnde_se RRpnie_se RRtnie_se   RRte_se     pm_se
## 1.2578324 1.2444119 1.2441779 0.7286536 0.7149112 0.9679001 0.2224177
```

### 3.2 Case 3-2: Binary Outcome and Single Continuous Mediator With Exposure-mediator Interaction

#### 3.2.1 Data simulation

##### 3.2.1.1 Simulation Procedures

1. Simulate the exposure variable A from Binom(1,P(A=1)).
2. Simulate the covariate C from $N(\mu_C, \sigma_C^2)$.

3. Simulate the mediator M from $N((\beta_0 + \beta_1 * A + \beta_2 * C), \sigma_M^2)$.
4. Simulate the outcome Y from $Bernoulli(\theta_0 + \theta_1 A + \theta_2 M + \theta_3 AM + \theta_4 C)$.

#### 3.2.1.2 True Parameters

Table 14: True Model Parameters for Data Simulation

| Sample Size | $\theta_0$ | $\theta_1$ | $\theta_2$ | $\theta_3$ | $\theta_4$ | $\beta_0$ | $\beta_1$ | $\beta_2$ | P(A=1) | $\mu_C$ | $\sigma_C$ | $\sigma_M$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 10000 | -5 | 0.8 | 1.8 | 0.2 | 0.1 | -0.25 | 0.5 | 0.2 | 0.4 | 1 | 1 | 0.1 |

#### 3.2.1.3 True Models

True model for the mediator:

$$E[M|a,c] = \beta_0 + \beta_1 a + \beta_2 c$$

True model for the outcome:

$$logitE[Y|a,m^*,c] = \theta_0 + \theta_1 a + \theta_2 m^* + \theta_3 am^* + \theta_4 c$$

### 3.2.2 Causal Effects and Standard Errors Estimated By the Structural Equation Model with 3 Different Estimation Method

#### 3.2.2.1 Delta Method

```
causal_mediation(data = df_int, model = "standard", est.method = "delta",
                 outcome = "binY_contM_int", exposure = 'A', exposure.type = "binary",
                 mediator = 'M_cont', covariates = "C", EMint = TRUE,
                 yreg = "logistic", mreg = "linear")
```

```
## $decomp3way
##     cde_rr   cde_rr_se    pnde_rr pnde_rr_se    tnde_rr tnde_rr_se
## 0.70056870 0.29270734 0.69548658 0.29075660 0.94244120 0.40991157
##    pnie_rr pnie_rr_se    tnie_rr tnie_rr_se      te_rr   te_rr_se
## 5.27439178 2.34350964 7.14723229 2.60920119 4.97080412 0.87736506
##         pm      pm_se
## 1.07668810 0.08288859
##
## $decomp4way
##            cde_err         cde_err_se        intref_err
##      -0.3325281066       0.3270342999       0.0280146839
##      intref_err_se         intmed_err      intmed_err_se
##       0.0381088765       0.0009257599       2.1201493762
##            pie_err         pie_err_se          total_err
##       4.2743917846       2.3435096391       3.9708041218
##       total_err_se       cde_err_prop    cde_err_prop_se
##       0.8773650557      -0.0837432662       0.0926244309
##    intref_err_prop intref_err_prop_se    intmed_err_prop
##       0.0070551664       0.0102284422       0.0002331417
## intmed_err_prop_se       pie_err_prop    pie_err_prop_se
##       0.5339309801       1.0764549581       0.5782534579
##         overall_pm      overall_pm_se        overall_int
##       1.0766880998       0.0828885897       0.0072883081
##     overall_int_se         overall_pe      overall_pe_se
##       0.5265047447       1.0837432662       0.0926244309
```

#### 3.2.2.2 Bootstrapping

```r
causal_mediation(data = df_int, model = "standard", est.method = "bootstrap",
                 outcome = "binY_contM_int", exposure = 'A', exposure.type = "binary",
                 mediator = 'M_cont', covariates = "C", EMint = TRUE,
                 yreg = "logistic", mreg = "linear", nboot = 500)
```

```
## $decomp3way
##     cde_rr  cde_rr_se    pnde_rr pnde_rr_se    tnde_rr tnde_rr_se
## 0.70056870 0.26799335 0.69548658 0.26422988 0.94244120 0.52027472
##    pnie_rr pnie_rr_se    tnie_rr tnie_rr_se      te_rr   te_rr_se
## 5.27439178 3.16301683 7.14723229 2.45730830 4.97080412 0.92819474
##         pm      pm_se
## 1.07668810 0.07516814
##
## $decomp4way
##            cde_err         cde_err_se         intref_err
##      -0.3325281066       0.2963286707       0.0280146839
##      intref_err_se         intmed_err      intmed_err_se
##       0.0351591811       0.0009257599       2.8738298290
##            pie_err         pie_err_se          total_err
##       4.2743917846       3.1630168296       3.9708041218
##       total_err_se       cde_err_prop    cde_err_prop_se
##       0.9281947368      -0.0837432662       0.0835866197
##    intref_err_prop intref_err_prop_se    intmed_err_prop
##       0.0070551664       0.0094802187       0.0002331417
## intmed_err_prop_se       pie_err_prop    pie_err_prop_se
##       0.6590216192       1.0764549581       0.6913528880
##         overall_pm      overall_pm_se        overall_int
##       1.0766880998       0.0751681410       0.0072883081
##     overall_int_se         overall_pe      overall_pe_se
##       0.6528310554       1.0837432662       0.0835866197
```

#### 3.2.2.3 Simulation-based Approach

```r
causal_mediation(data = df_int, model = "standard", est.method = "simulation",
                 outcome = "binY_contM_int", exposure = 'A', exposure.type = "binary",
                 mediator = 'M_cont', covariates = "C", EMint = TRUE,
                 yreg = "logistic", mreg = "linear",nsims = 500)[1:2]
```

```
## $decomp3way
##     cde_rr  cde_rr_se    pnde_rr pnde_rr_se    tnde_rr tnde_rr_se
## 0.77689479 0.29939506 0.79657360 0.30569964 1.11189545 0.58950688
##    pnie_rr pnie_rr_se    tnie_rr tnie_rr_se      te_rr   te_rr_se
## 5.57099041 2.57220457 7.03245646 2.18080800 5.06502574 0.83155631
##         pm      pm_se
## 1.05946535 0.08150221
##
## $decomp4way
##            cde_err         cde_err_se         intref_err
##        -0.25266067         0.32553427         0.04923427
##      intref_err_se         intmed_err      intmed_err_se
##         0.04839212        -0.30253827         2.44967354
##            pie_err         pie_err_se          total_err
##         4.57099041         2.57220457         4.06502574
```

```
##       total_err_se       cde_err_prop    cde_err_prop_se
##         0.83155631        -0.07264172         0.08911069
##    intref_err_prop intref_err_prop_se     intmed_err_prop
##         0.01317637         0.01350499        -0.09852101
## intmed_err_prop_se       pie_err_prop     pie_err_prop_se
##         0.62423521         1.15798636         0.66990637
##         overall_pm      overall_pm_se         overall_int
##         1.05946535         0.08150221        -0.08534464
##     overall_int_se         overall_pe      overall_pe_se
##         0.62473172         1.07264172         0.08911069
```

#### 3.2.3 Causal Effects and Standard Errors Estimated By the Natural Effect Model

```
causal_mediation(data = df_int, model = "ne",
                 outcome = "binY_contM_int", exposure = 'A', exposure.type = "binary",
                 mediator = 'M_cont', covariates = "C", EMint = TRUE,
                 yreg = "logistic")
```

```
##                        Estimate Std. Error
## pure direct effect    0.7376720   1.255747
## total direct effect   0.9939177   1.917615
## pure indirect effect  5.2318768 128.909645
## total indirect effect 7.0492778 644.236537
## total effect          5.2000547  73.252438
```

#### 3.2.4 Causal Effects and Standard Errors Estimated By the regression-based approach

##### 3.2.4.1 Bootstrapping

```
causal_mediation(data = df_int, model = "rb", est.method = "bootstrap",
                 outcome = "binY_contM_int", exposure = 'A', exposure.type = "binary",
                 mediator = 'M_cont', covariates = "C", EMint = TRUE,
                 yreg = "logistic", mreg = "linear", nboot = 500)
```

```
##      RRcde     RRpnde     RRtnde     RRpnie     RRtnie       RRte
## 0.70056870 0.70959593 0.96156053 5.27439178 7.14723229 5.07164697
##         pm   RRcde_se  RRpnde_se  RRtnde_se  RRpnie_se  RRtnie_se
## 1.07132349 0.31635050 0.31102224 0.56762158 2.92287539 2.41997296
##    RRte_se      pm_se
## 0.94164138 0.08126727
```

##### 3.2.4.2 Simulation-based Approach

```
causal_mediation(data = df_int, model = "rb", est.method = "simulation",
                 outcome = "binY_contM_int", exposure = 'A', exposure.type = "binary",
                 mediator = 'M_cont', covariates = "C", EMint = TRUE,
                 yreg = "logistic", mreg = "linear", nsims = 500)[1:2]
```

```
## $decomp3way
##     cde_rr   cde_rr_se    pnde_rr pnde_rr_se    tnde_rr tnde_rr_se
## 0.75984936 0.29796200 0.77841292 0.29870894 1.11075571 0.59859736
##    pnie_rr pnie_rr_se    tnie_rr tnie_rr_se      te_rr   te_rr_se
## 5.72516348 2.73743720 7.35975348 2.54754751 5.14079526 0.89890709
```

```
##         pm      pm_se
## 1.06371820 0.07894126
##
## $decomp4way
##            cde_err         cde_err_se         intref_err
##        -0.27569588         0.33005910         0.05410880
##      intref_err_se         intmed_err      intmed_err_se
##         0.05775866        -0.36278114         2.62912697
##            pie_err         pie_err_se          total_err
##         4.72516348         2.73743720         4.14079526
##       total_err_se      cde_err_prop    cde_err_prop_se
##         0.89890709        -0.07804537         0.08904818
##    intref_err_prop intref_err_prop_se    intmed_err_prop
##         0.01432717         0.01534747        -0.11775494
## intmed_err_prop_se       pie_err_prop    pie_err_prop_se
##         0.66852844         1.18147314         0.71357670
##         overall_pm      overall_pm_se        overall_int
##         1.06371820         0.07894126        -0.10342778
##     overall_int_se         overall_pe      overall_pe_se
##         0.66802669         1.07804537         0.08904818
```

### 3.2.5 Causal Effects and Standard Errors Estimated By the weighting-based approach

```
causal_mediation(data = df_int, model = "wb",
                 outcome = "binY_contM_int", exposure = 'A', exposure.type = "binary",
                 mediator = 'M_cont', covariates = "C", EMint = TRUE,
                 yreg = "logistic", nboot = 500)
```

```
##      RRcde     RRpnde     RRtnde     RRpnie     RRtnie       RRte
## 0.70201835 0.74209805 0.99495708 5.03177749 6.74628188 5.00640263
##         pm   RRcde_se  RRpnde_se  RRtnde_se  RRpnie_se  RRtnie_se
## 1.06437245 0.31216183 0.30506132 0.56324980 2.28289278 2.19711693
##    RRte_se      pm_se
## 0.91046555 0.07835301
```

## 3.3 Case 3-3: Binary Outcome and Multiple Continuous Mediators Without Interaction

### 3.3.1 Data simulation

#### 3.3.1.1 Simulation Procedures

1. Simulate the exposure variable A from Binom(1,P(A=1)).
2. Simulate the covariate C from $N(\mu_C, \sigma_C^2)$.
3. Simulate the first mediator M1 from $N((\beta_{01} + \beta_{11} * A + \beta_{21} * C), \sigma_{M1}^2)$ and the second mediator M2 from $N((\beta_{02} + \beta_{12} * A + \beta_{22} * C), \sigma_{M2}^2)$.
4. Simulate the outcome Y from $Bernoulli(\theta_0 + \theta_1 A + \theta_2 M1 + \theta_3 M2 + \theta_4 C)$.

#### 3.3.1.2 True Parameters

Table 15: True Model Parameters for Data Simulation

| Sample Size | $\theta_0$ | $\theta_1$ | $\theta_2$ | $\theta_3$ | $\theta_4$ | $\beta_{01}$ | $\beta_{11}$ | $\beta_{21}$ |
|---|---|---|---|---|---|---|---|---|
| 10000 | -5 | 0.8 | 1.8 | 1.2 | 0.1 | -0.25 | 0.5 | 0.2 |
| $\beta_{02}$ | $\beta_{12}$ | $\beta_{22}$ | P(A=1) | $\mu_C$ | $\sigma_C$ | $\sigma_{M1}$ | $\sigma_{M2}$ | |
| -0.3 | 0.4 | 0.3 | 0.4 | 1 | 1 | 0.1 | 0.1 | |

#### 3.3.1.3 True Models

True model for the first mediator:

$$E[M1|a, c] = \beta_{01} + \beta_{11}a + \beta_{21}c$$

True model for the second mediator:

$$E[M2|a, c] = \beta_{02} + \beta_{12}a + \beta_{22}c$$

True model for the outcome:

$$logitE[Y|a, m^*, c] = \theta_0 + \theta_1 a + \theta_2 m1^* + \theta_3 m2^* + \theta_4 c$$

### 3.3.2 Causal Effects and Standard Errors Estimated By the Natural Effect Model

```
causal_mediation(data = df_mulipleM_noint, model = "ne",
                 outcome = "binY_2contM_noint", exposure = 'A', exposure.type = "binary",
                 mediator = c('M_cont1','M_cont2'), covariates = "C",
                 yreg = "logistic")
```

```
##                         Estimate Std. Error
## natural direct effect   3.290425  16.785644
## natural indirect effect 2.277700   5.904462
## total effect            7.494602 680.988519
```

### 3.3.3 Causal Effects and Standard Errors Estimated By the regression-based approach

#### 3.3.3.1 Bootstrapping

```
causal_mediation(data = df_mulipleM_noint, model = "rb", est.method = "bootstrap",
                 outcome = "binY_2contM_noint", exposure = 'A', exposure.type = "binary",
                 mediator = c('M_cont1','M_cont2'), covariates = "C",
                 yreg = "logistic", m_star = c(0,0), mreg = c("linear","linear"), nboot = 500)
```

```
##     RRcde    RRpnde    RRtnde    RRpnie    RRtnie      RRte        pm
## 3.2935851 3.2935851 3.2935851 2.2814924 2.2814924 7.5142894 0.6479148
##  RRcde_se RRpnde_se RRtnde_se RRpnie_se RRtnie_se   RRte_se     pm_se
## 1.4351432 1.4351432 1.4351432 0.9868772 0.9868772 1.1794101 0.2084093
```

#### 3.3.3.2 Simulation-based Approach

```
causal_mediation(data = df_mulipleM_noint, model = "rb", est.method = "simulation",
                 outcome = "binY_2contM_noint", exposure = 'A', exposure.type = "binary",
                 mediator = c('M_cont1','M_cont2'), covariates = "C",
                 yreg = "logistic", m_star = c(0,0), mreg = c("linear","linear"), nsims = 500)[1:2]
```

```
## $decomp3way
##     cde_rr  cde_rr_se    pnde_rr pnde_rr_se    tnde_rr tnde_rr_se
##  3.4896155  1.2699968  3.4486659  1.2679719  3.3350603  1.2613588
##    pnie_rr pnie_rr_se    tnie_rr tnie_rr_se      te_rr   te_rr_se
##  2.3564420  0.8310737  2.2711578  0.8033489  7.0023954  0.9643732
##         pm      pm_se
##  0.5927106  0.2014051
##
## $decomp4way
##            cde_err         cde_err_se         intref_err
##         2.26095872         1.29907391         0.18770717
##      intref_err_se         intmed_err      intmed_err_se
##         0.14454492         2.19728750         0.79774079
##            pie_err         pie_err_se          total_err
##         1.35644199         0.83107367         6.00239538
##       total_err_se       cde_err_prop    cde_err_prop_se
##         0.96437323         0.37593623         0.20779387
##    intref_err_prop intref_err_prop_se    intmed_err_prop
##         0.03135315         0.02299733         0.36184029
## intmed_err_prop_se       pie_err_prop    pie_err_prop_se
##         0.10388800         0.23087032         0.14774934
##         overall_pm      overall_pm_se        overall_int
##         0.59271061         0.20140514         0.39319344
##     overall_int_se         overall_pe      overall_pe_se
##         0.11686889         0.62406377         0.20779387
```

### 3.3.4 Causal Effects and Standard Errors Estimated By the weighting-based approach

```
causal_mediation(data = df_mulipleM_noint, model = "wb",
                 outcome = "binY_2contM_noint", exposure = 'A', exposure.type = "binary",
                 mediator = c('M_cont1','M_cont2'), covariates = "C",
                 yreg = "logistic", m_star = c(0,0), nboot = 500)
```

```
##     RRcde    RRpnde    RRtnde    RRpnie    RRtnie      RRte        pm
## 3.2226564 3.1725837 3.0427245 2.2367381 2.1451846 6.8057777 0.6257894
##  RRcde_se RRpnde_se RRtnde_se RRpnie_se RRtnie_se   RRte_se     pm_se
## 1.4300168 1.3694863 1.3654104 0.8394656 0.8078362 0.9825347 0.2037975
```

## 3.4 Case 3-4: Binary Outcome and Multiple Continuous Mediators With Exposure-mediator Interaction

### 3.4.1 Data simulation

#### 3.4.1.1 Simulation Procedures

1. Simulate the exposure variable A from Binom(1,P(A=1)).
2. Simulate the covariate C from $N(\mu_C, \sigma_C^2)$.
3. Simulate the first mediator M1 from $N((\beta_{01} + \beta_{11} * A + \beta_{21} * C), \sigma_{M1}^2)$ and the second mediator M2 from $N((\beta_{02} + \beta_{12} * A + \beta_{22} * C), \sigma_{M2}^2)$.
4. Simulate the outcome Y from $Bernoulli(\theta_0 + \theta_1 A + \theta_2 M1 + \theta_3 M2 + \theta_4 AM1 + \theta_5 AM2 + \theta_6 C)$.

#### 3.4.1.2 True Parameters

Table 16: True Model Parameters for Data Simulation

| Sample Size | $\theta_0$ | $\theta_1$ | $\theta_2$ | $\theta_3$ | $\theta_4$ | $\theta_5$ | $\theta_6$ | $\beta_{01}$ | $\beta_{11}$ |
|---|---|---|---|---|---|---|---|---|---|
| 10000 | -5 | 0.8 | 1.8 | 1.2 | 0.6 | 0.4 | 0.1 | -0.25 | 0.5 |
| $\beta_{21}$ | $\beta_{02}$ | $\beta_{12}$ | $\beta_{22}$ | P(A=1) | $\mu_C$ | $\sigma_C$ | $\sigma_{M1}$ | $\sigma_{M2}$ | |
| 0.2 | -0.3 | 0.4 | 0.3 | 0.4 | 1 | 1 | 0.1 | 0.1 | |

#### 3.4.1.3 True Models

True model for the first mediator:

$$E[M1|a,c] = \beta_{01} + \beta_{11}a + \beta_{21}c$$

True model for the second mediator:

$$E[M2|a,c] = \beta_{02} + \beta_{12}a + \beta_{22}c$$

True model for the outcome:

$$logitE[Y|a,m^*,c] = \theta_0 + \theta_1 a + \theta_2 m1^* + \theta_3 m2^* + \theta_4 am1^* + \theta_5 am2^* + \theta_6 c$$

### 3.4.2 Causal Effects and Standard Errors Estimated By the Natural Effect Model

```
causal_mediation(data = df_mulipleM_EMint, model = "ne",
                 outcome = "binY_2contM_EMint", exposure = 'A', exposure.type = "binary",
                 mediator = c('M_cont1','M_cont2'), EMint = TRUE, EMint.terms = c("A*M_cont1", "A*M_con
                 covariates = "C", yreg = "logistic")
```

```
##                        Estimate   Std. Error
## pure direct effect     3.475523 1.904048e+01
## total direct effect   11.573448 7.311215e+04
## pure indirect effect   1.254642 2.349678e+00
## total indirect effect  4.177942 3.611974e+01
## total effect          14.520533 7.683104e+05
```

### 3.4.3 Causal Effects and Standard Errors Estimated By the regression-based approach

#### 3.4.3.1 Bootstrapping

```
causal_mediation(data = df_mulipleM_EMint, model = "rb", est.method = "bootstrap",
                 outcome = "binY_2contM_EMint", exposure = 'A', exposure.type = "binary",
                 mediator = c('M_cont1','M_cont2'), covariates = "C",
                 EMint = TRUE, EMint.terms = c("A*M_cont1", "A*M_cont2"),
                 yreg = "logistic", m_star = c(0,0), mreg = c("linear","linear"), nboot = 500)
```

```
##       RRcde      RRpnde      RRtnde      RRpnie      RRtnie        RRte
##  3.91168674  2.76865749  9.28718160  1.27869047  4.28923789 11.87543062
##          pm    RRcde_se   RRpnde_se   RRtnde_se   RRpnie_se   RRtnie_se
##  0.83737127  1.69512764  1.07514713  4.09921450  0.69628568  1.42602779
##     RRte_se       pm_se
##  2.39860154  0.08713069
```

#### 3.4.3.2 Simulation-based Approach

```
causal_mediation(data = df_mulipleM_EMint, model = "rb", est.method = "simulation",
                 outcome = "binY_2contM_EMint", exposure = 'A', exposure.type = "binary",
                 mediator = c('M_cont1','M_cont2'), covariates = "C",
                 EMint = TRUE, EMint.terms = c("A*M_cont1", "A*M_cont2"),
                 yreg = "logistic", m_star = c(0,0), mreg = c("linear","linear"), nsims = 500)[1:2]
```

```
## $decomp3way
##      cde_rr   cde_rr_se     pnde_rr  pnde_rr_se     tnde_rr  tnde_rr_se
##  4.13871316  1.50038587  3.63980359  1.20016822 10.54110644  5.62949285
##     pnie_rr  pnie_rr_se     tnie_rr  tnie_rr_se       te_rr    te_rr_se
##  1.42490150  0.64467094  3.71319725  1.13846865 12.41953700  1.80282030
##          pm       pm_se
##  0.76941889  0.09643403
##
## $decomp4way
##            cde_err         cde_err_se         intref_err
##         2.15891859         1.07771736         0.48088500
##      intref_err_se         intmed_err      intmed_err_se
##         0.21953310         8.35483191         1.66964459
##            pie_err         pie_err_se          total_err
##         0.42490150         0.64467094        11.41953700
##       total_err_se       cde_err_prop    cde_err_prop_se
##         1.80282030         0.18877283         0.08779092
##    intref_err_prop intref_err_prop_se    intmed_err_prop
##         0.04180828         0.01756805         0.73100147
## intmed_err_prop_se       pie_err_prop    pie_err_prop_se
##         0.08126828         0.03841742         0.05815714
##         overall_pm      overall_pm_se        overall_int
##         0.76941889         0.09643403         0.77280976
##     overall_int_se         overall_pe      overall_pe_se
##         0.07783217         0.81122717         0.08779092
```

### 3.4.4 Causal Effects and Standard Errors Estimated By the weighting-based approach

```
causal_mediation(data = df_mulipleM_EMint, model = "wb",
                 outcome = "binY_2contM_EMint", exposure = 'A', exposure.type = "binary",
                 mediator = c('M_cont1','M_cont2'), covariates = "C",
                 EMint = TRUE, EMint.terms = c("A*M_cont1", "A*M_cont2"),
                 yreg = "logistic", m_star = c(0,0), nboot = 500)
```

```
##       RRcde      RRpnde      RRtnde      RRpnie      RRtnie        RRte
##  3.83259958  3.26561075  9.09959654  1.26306610  3.51952291 11.49339187
##          pm    RRcde_se   RRpnde_se   RRtnde_se   RRpnie_se   RRtnie_se
##  0.78409167  1.55275925  1.10667241  4.97108645  0.58936706  1.03419222
##     RRte_se       pm_se
##  1.51210094  0.09309684
```

### 3.5 Case 3-5: Binary Outcome and Multiple Continuous Mediators With Mediator-mediator Interaction

#### 3.5.1 Data simulation

##### 3.5.1.1 Simulation Procedures

1. Simulate the exposure variable A from Binom(1,P(A=1)).
2. Simulate the covariate C from $N(\mu_C, \sigma_C^2)$.
3. Simulate the first mediator M1 from $N((\beta_{01} + \beta_{11} * A + \beta_{21} * C), \sigma_{M1}^2)$ and the second mediator M2 from $N((\beta_{02} + \beta_{12} * A + \beta_{22} * C), \sigma_{M2}^2)$.
4. Simulate the outcome Y from $Bernoulli(\theta_0 + \theta_1 A + \theta_2 M1 + \theta_3 M2 + \theta_4 M1M2 + \theta_5 C)$.

##### 3.5.1.2 True Parameters

Table 17: True Model Parameters for Data Simulation

| Sample Size | $\theta_0$ | $\theta_1$ | $\theta_2$ | $\theta_3$ | $\theta_4$ | $\theta_5$ | $\beta_{01}$ | $\beta_{11}$ | $\beta_{21}$ |
|---|---|---|---|---|---|---|---|---|---|
| 10000 | -5 | 0.8 | 1.8 | 1.2 | 0.6 | 0.1 | -0.25 | 0.5 | 0.2 |
| $\beta_{02}$ | $\beta_{12}$ | $\beta_{22}$ | P(A=1) | $\mu_C$ | $\sigma_C$ | $\sigma_{M1}$ | $\sigma_{M2}$ | | |
| -0.3 | 0.4 | 0.3 | 0.4 | 1 | 1 | 0.1 | 0.1 | | |

##### 3.5.1.3 True Models

True model for the first mediator:

$$E[M1|a,c] = \beta_{01} + \beta_{11}a + \beta_{21}c$$

True model for the second mediator:

$$E[M2|a,c] = \beta_{02} + \beta_{12}a + \beta_{22}c$$

True model for the outcome:

$$logitE[Y|a,m^*,c] = \theta_0 + \theta_1 a + \theta_2 m1^* + \theta_3 m2^* + \theta_4 m1^* m2^* + \theta_5 c$$

#### 3.5.2 Causal Effects and Standard Errors Estimated By the Natural Effect Model

```
causal_mediation(data = df_mulipleM_MMint, model = "ne",
                 outcome = "binY_2contM_MMint", exposure = 'A', exposure.type = "binary",
                 mediator = c('M_cont1','M_cont2'), MMint = TRUE, MMint.terms = c("M_cont1*M_cont2"),
                 covariates = "C", yreg = "logistic")
```

```
##                          Estimate  Std. Error
## natural direct effect    2.322155    6.175297
## natural indirect effect 4.187911   38.872196
## total effect             9.724981 6209.261092
```

#### 3.5.3 Causal Effects and Standard Errors Estimated By the weighting-based approach

```r
causal_mediation(data = df_mulipleM_MMint, model = "wb",
                 outcome = "binY_2contM_MMint", exposure = 'A', exposure.type = "binary",
                 mediator = c('M_cont1','M_cont2'), covariates = "C",
                 MMint = TRUE, MMint.terms = c("M_cont1*M_cont2"),
                 yreg = "logistic", m_star = c(0,0), nboot = 500)
```

```
##     RRcde    RRpnde    RRtnde    RRpnie    RRtnie      RRte        pm
## 2.3136031 2.2753551 2.1003353 3.9620695 3.6573079 8.3216744 0.8258110
##  RRcde_se RRpnde_se RRtnde_se RRpnie_se RRtnie_se   RRte_se     pm_se
## 0.9193629 0.8739691 0.7757852 1.3091830 1.3189521 1.1472964 0.1095918
```

## 3.6 Case 3-6: Binary Outcome and Multiple Continuous Mediators With Exposure-mediator-mediator Interaction

### 3.6.1 Data simulation

#### 3.6.1.1 Simulation Procedures

1. Simulate the exposure variable A from Binom(1,P(A=1)).
2. Simulate the covariate C from $N(\mu_C, \sigma_C^2)$.
3. Simulate the first mediator M1 from $N((\beta_{01} + \beta_{11} * A + \beta_{21} * C), \sigma_{M1}^2)$ and the second mediator M2 from $N((\beta_{02} + \beta_{12} * A + \beta_{22} * C), \sigma_{M2}^2)$.
4. Simulate the outcome Y from $Bernoulli(\theta_0 + \theta_1 A + \theta_2 M1 + \theta_3 M2 + \theta_4 AM1 + \theta_5 AM2 + \theta_6 M1M2 + \theta_7 AM1M2 + \theta_8 C)$.

#### 3.6.1.2 True Parameters

Table 18: True Model Parameters for Data Simulation

| Sample Size | $\theta_0$ | $\theta_1$ | $\theta_2$ | $\theta_3$ | $\theta_4$ | $\theta_5$ | $\theta_6$ | $\theta_7$ | $\theta_8$ | $\beta_{01}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 10000 | -5 | 0.8 | 1.8 | 1.2 | 0.6 | 0.4 | 0.3 | 0.5 | 0.1 | -0.25 |
| $\beta_{11}$ | $\beta_{21}$ | $\beta_{02}$ | $\beta_{12}$ | $\beta_{22}$ | P(A=1) | $\mu_C$ | $\sigma_C$ | $\sigma_{M1}$ | $\sigma_{M2}$ | |
| 0.5 | 0.2 | -0.3 | 0.4 | 0.3 | 0.4 | 1 | 1 | 0.1 | 0.1 | |

#### 3.6.1.3 True Models

True model for the first mediator:

$$E[M1|a,c] = \beta_{01} + \beta_{11}a + \beta_{21}c$$

True model for the second mediator:

$$E[M2|a,c] = \beta_{02} + \beta_{12}a + \beta_{22}c$$

True model for the outcome:

$$logitE[Y|a,m^*,c] = \theta_0 + \theta_1 a + \theta_2 m1^* + \theta_3 m2^* + \theta_4 am1^* + \theta_5 am2^* + \theta_6 m1^* m2^* + \theta_7 am1^* m2^* + \theta_8 c$$

### 3.6.2 Causal Effects and Standard Errors Estimated By the Natural Effect Model

```r
causal_mediation(data = df_mulipleM_mint, model = "ne",
                 outcome = "binY_2contM_mint", exposure = 'A', exposure.type = "binary",
                 mediator = c('M_cont1','M_cont2'),
                 EMMint = TRUE, EMMint.terms = c("A*M_cont1*M_cont2"),
                 covariates = "C", yreg = "logistic")
```

```
##                          Estimate   Std. Error
## pure direct effect      4.142893 3.795326e+01
## total direct effect     9.448795 8.971229e+03
## pure indirect effect    2.213898 6.548497e+00
## total indirect effect   5.049289 9.171471e+01
## total effect           20.918664 4.673947e+08
```

#### 3.6.3 Causal Effects and Standard Errors Estimated By the weighting-based approach

```r
causal_mediation(data = df_mulipleM_mint, model = "wb",
                 outcome = "binY_2contM_mint", exposure = 'A', exposure.type = "binary",
                 mediator = c('M_cont1','M_cont2'), covariates = "C",
                 EMMint = TRUE, EMMint.terms = c("A*M_cont1*M_cont2"),
                 yreg = "logistic", m_star = c(0,0), nboot = 500)
```

```
##       RRcde      RRpnde      RRtnde      RRpnie      RRtnie        RRte
##  4.84117129  3.76798669  6.85568816  2.11654518  3.85096204 14.51037371
##          pm    RRcde_se   RRpnde_se   RRtnde_se   RRpnie_se   RRtnie_se
##  0.79512138  2.06819109  1.27121745  4.59241813  1.09089275  1.24458649
##     RRte_se       pm_se
##  2.00518463  0.08475051
```

# 4 Case 4: Binary Outcome and Binary Mediator

## 4.1 Case 4-1: Binary Outcome and Single Binary Mediator Without Exposure-mediator Interaction

### 4.1.1 Data simulation

#### 4.1.1.1 Simulation Procedures

1. Simulate the exposure variable A from Binom(1,P(A=1)).
2. Simulate the covariate C from $N(\mu_C, \sigma_C^2)$.
3. Simulate the mediator M from $Bernoulli(\beta_0 + \beta_1 * A + \beta_2 * C)$.
4. Simulate the outcome Y from $Bernoulli(\theta_0 + \theta_1 A + \theta_2 M + \theta_4 C)$.

#### 4.1.1.2 True Parameters

Table 19: True Model Parameters for Data Simulation

| Sample Size | $\theta_0$ | $\theta_1$ | $\theta_2$ | $\theta_4$ | $\beta_0$ | $\beta_1$ | $\beta_2$ | P(A=1) | $\mu_C$ | $\sigma_C$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 10000 | -5 | 0.8 | 1.8 | 0.1 | -0.25 | 0.5 | 0.2 | 0.4 | 1 | 1 |

#### 4.1.1.3 True Models

True model for the mediator:

$$logitE[M|a,c] = \beta_0 + \beta_1 a + \beta_2 c$$

True model for the outcome:

$$logitE[Y|a,m^*,c] = \theta_0 + \theta_1 a + \theta_2 m^* + \theta_4 c$$

### 4.1.2 Causal Effects and Standard Errors Estimated By the Structural Equation Model with 3 Different Estimation Method

#### 4.1.2.1 Delta Method

```
causal_mediation(data = df_noint, model = "standard", est.method = "delta",
                 outcome = "binY_binM_noint", exposure = 'A', exposure.type = "binary",
                 mediator = 'M_bin', covariates = "C", EMint = FALSE,
                 yreg = "logistic", mreg = "logistic")
```

```
## $decomp3way
##     cde_rr  cde_rr_se    pnde_rr pnde_rr_se    tnde_rr tnde_rr_se
## 2.53081815 0.27175459 2.53081815 0.27175459 2.53081815 0.27175459
##    pnie_rr pnie_rr_se    tnie_rr tnie_rr_se      te_rr   te_rr_se
## 1.16532353 0.01788651 1.16532353 0.01788651 2.94922193 0.31823782
##         pm      pm_se
## 0.21465169 0.02210045
##
## $decomp4way
##            cde_err         cde_err_se         intref_err
##         0.49669140         0.11128769         1.03412675
##      intref_err_se         intmed_err      intmed_err_se
##         0.28972841         0.25308026         0.05174086
##            pie_err         pie_err_se          total_err
##         0.16532353         0.01788651         1.94922193
##       total_err_se       cde_err_prop    cde_err_prop_se
##         0.31823782         0.25481521         0.03581582
##    intref_err_prop intref_err_prop_se    intmed_err_prop
##         0.53053310         0.13842108         0.12983655
## intmed_err_prop_se       pie_err_prop    pie_err_prop_se
##         0.01113228         0.08481514         0.01578488
##         overall_pm      overall_pm_se        overall_int
##         0.21465169         0.02210045         0.66036965
##     overall_int_se         overall_pe      overall_pe_se
##         0.14272277         0.74518479         0.15430278
```

#### 4.1.2.2 Bootstrapping

```
causal_mediation(data = df_noint, model = "standard", est.method = "bootstrap",
                 outcome = "binY_binM_noint", exposure = 'A', exposure.type = "binary",
                 mediator = 'M_bin', covariates = "C", EMint = FALSE,
                 yreg = "logistic", mreg = "logistic", nboot = 500)
```

```
## $decomp3way
##     cde_rr  cde_rr_se    pnde_rr pnde_rr_se    tnde_rr tnde_rr_se
## 2.53081815 0.27303340 2.53081815 0.27303340 2.53081815 0.27303340
```

```
##    pnie_rr pnie_rr_se    tnie_rr tnie_rr_se      te_rr   te_rr_se
## 1.16532353 0.01802655 1.16532353 0.01802655 2.94922193 0.31924137
##         pm      pm_se
## 0.21465169 0.02234182
##
## $decomp4way
##            cde_err          cde_err_se         intref_err
##         0.49669140          0.11426657         1.03412675
##      intref_err_se          intmed_err      intmed_err_se
##         0.19074957          0.25308026         0.05191642
##            pie_err          pie_err_se          total_err
##         0.16532353          0.01802655         1.94922193
##       total_err_se        cde_err_prop     cde_err_prop_se
##         0.31924137          0.25481521         0.03685228
##    intref_err_prop intref_err_prop_se    intmed_err_prop
##         0.53053310          0.02813902         0.12983655
## intmed_err_prop_se       pie_err_prop     pie_err_prop_se
##         0.01117655          0.08481514         0.01618042
##         overall_pm      overall_pm_se        overall_int
##         0.21465169          0.02234182         0.66036965
##     overall_int_se          overall_pe      overall_pe_se
##         0.03279431          0.74518479         0.03685228
```

#### 4.1.2.3 Simulation-based Approach

```r
causal_mediation(data = df_noint, model = "standard", est.method = "simulation",
                 outcome = "binY_binM_noint", exposure = 'A', exposure.type = "binary",
                 mediator = 'M_bin', covariates = "C", EMint = FALSE,
                 yreg = "logistic", mreg = "logistic",nsims = 500)[1:2]
```

```
## $decomp3way
##     cde_rr  cde_rr_se    pnde_rr pnde_rr_se    tnde_rr tnde_rr_se
## 2.51667509 0.25947896 2.47522699 0.25094298 2.46057412 0.24776756
##    pnie_rr pnie_rr_se    tnie_rr tnie_rr_se      te_rr   te_rr_se
## 1.21486494 0.02897696 1.20773755 0.02809938 2.99007234 0.31795838
##         pm      pm_se
## 0.25951873 0.02845949
##
## $decomp4way
##            cde_err          cde_err_se         intref_err
##         0.66109562          0.12184930         0.81413137
##      intref_err_se          intmed_err      intmed_err_se
##         0.15122054          0.29998041         0.06921453
##            pie_err          pie_err_se          total_err
##         0.21486494          0.02897696         1.99007234
##       total_err_se        cde_err_prop     cde_err_prop_se
##         0.31795838          0.33268643         0.03583613
##    intref_err_prop intref_err_prop_se    intmed_err_prop
##         0.40779484          0.02077439         0.14953440
## intmed_err_prop_se       pie_err_prop     pie_err_prop_se
##         0.01557307          0.10998434         0.01945207
##         overall_pm      overall_pm_se        overall_int
##         0.25951873          0.02845949         0.55732923
##     overall_int_se          overall_pe      overall_pe_se
```

```
##         0.02956083         0.66731357         0.03583613
```

### 4.1.3 Causal Effects and Standard Errors Estimated By the Natural Effect Model

```
causal_mediation(data = df_noint, model = "ne",
                 outcome = "binY_binM_noint", exposure = 'A', exposure.type = "binary",
                 mediator = 'M_bin', covariates = "C", EMint = FALSE,
                 yreg = "logistic")
```

```
##                         Estimate Std. Error
## natural direct effect   2.491130  3.9217162
## natural indirect effect 1.163086  0.4003889
## total effect            2.897398  5.9039492
```

### 4.1.4 Causal Effects and Standard Errors Estimated By the weighting-based approach

```
causal_mediation(data = df_noint, model = "wb",
                 outcome = "binY_binM_noint", exposure = 'A', exposure.type = "binary",
                 mediator = 'M_bin', covariates = "C", EMint = FALSE,
                 yreg = "logistic")
```

```
##      RRcde     RRpnde     RRtnde     RRpnie     RRtnie       RRte
## 2.49912677 2.40008795 2.39427658 1.15615980 1.15336037 2.76816633
##         pm   RRcde_se  RRpnde_se  RRtnde_se  RRpnie_se  RRtnie_se
## 0.20816954 0.27081087 0.24959815 0.24828800 0.01721517 0.01704678
##    RRte_se      pm_se
## 0.28882096 0.02279017
```

## 4.2 Case 3-2: Binary Outcome and Single Continuous Mediator With Exposure-mediator Interaction

### 4.2.1 Data simulation

#### 4.2.1.1 Simulation Procedures

1. Simulate the exposure variable A from Binom(1,P(A=1)).
2. Simulate the covariate C from $N(\mu_C, \sigma_C^2)$.
3. Simulate the mediator M from $Bernoulli(\beta_0 + \beta_1 * A + \beta_2 * C)$.
4. Simulate the outcome Y from $Bernoulli(\theta_0 + \theta_1 A + \theta_2 M + \theta_3 AM + \theta_4 C)$.

#### 4.2.1.2 True Parameters

Table 20: True Model Parameters for Data Simulation

| Sample Size | $\theta_0$ | $\theta_1$ | $\theta_2$ | $\theta_3$ | $\theta_4$ | $\beta_0$ | $\beta_1$ | $\beta_2$ | P(A=1) | $\mu_C$ | $\sigma_C$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 10000 | -5 | 0.8 | 1.8 | 0.2 | 0.1 | -0.25 | 0.5 | 0.2 | 0.4 | 1 | 1 |

#### 4.2.1.3 True Models

True model for the mediator:

$$logitE[M|a,c] = \beta_0 + \beta_1 a + \beta_2 c$$

True model for the outcome:

$$logitE[Y|a,m^*,c] = \theta_0 + \theta_1 a + \theta_2 m^* + \theta_3 am^* + \theta_4 c$$

### 4.2.2 Causal Effects and Standard Errors Estimated By the Structural Equation Model with 3 Different Estimation Method

#### 4.2.2.1 Delta Method

```r
causal_mediation(data = df_int, model = "standard", est.method = "delta",
                 outcome = "binY_binM_int", exposure = 'A', exposure.type = "binary",
                 mediator = 'M_bin', covariates = "C", EMint = TRUE,
                 yreg = "logistic", mreg = "logistic")
```

```
## $decomp3way
##     cde_rr   cde_rr_se    pnde_rr pnde_rr_se    tnde_rr tnde_rr_se
## 1.28190994 0.44410143 2.49033078 0.25717803 2.55434974 0.26555403
##    pnie_rr pnie_rr_se    tnie_rr tnie_rr_se      te_rr   te_rr_se
## 1.17160022 0.02048550 1.20171856 0.02081452 2.99267672 0.31200076
##         pm      pm_se
## 0.25209606 0.02372341
##
## $decomp4way
##            cde_err          cde_err_se         intref_err
##         0.07488245          0.10969912         1.41544833
##      intref_err_se          intmed_err      intmed_err_se
##         0.31086005          0.33074572         0.06452986
##            pie_err          pie_err_se          total_err
##         0.17160022          0.02048550         1.99267672
##       total_err_se        cde_err_prop     cde_err_prop_se
##         0.31200076          0.03757883         0.05364763
##    intref_err_prop intref_err_prop_se     intmed_err_prop
##         0.71032512          0.13092805         0.16598062
## intmed_err_prop_se        pie_err_prop     pie_err_prop_se
##         0.01595921          0.08611543         0.01611205
##         overall_pm       overall_pm_se        overall_int
##         0.25209606          0.02372341         0.87630574
##     overall_int_se          overall_pe      overall_pe_se
##         0.14044748          0.96242117         0.13896609
```

#### 4.2.2.2 Bootstrapping

```r
causal_mediation(data = df_int, model = "standard", est.method = "bootstrap",
                 outcome = "binY_binM_int", exposure = 'A', exposure.type = "binary",
                 mediator = 'M_bin', covariates = "C", EMint = TRUE,
                 yreg = "logistic", mreg = "logistic", nboot = 500)
```

```
## $decomp3way
##     cde_rr   cde_rr_se    pnde_rr pnde_rr_se    tnde_rr tnde_rr_se
## 1.28190994 0.51750781 2.49033078 0.25143388 2.55434974 0.25682983
```

```
##    pnie_rr pnie_rr_se    tnie_rr tnie_rr_se      te_rr   te_rr_se
## 1.17160022 0.01989578 1.20171856 0.02045967 2.99267672 0.30237743
##         pm      pm_se
## 0.25209606 0.02365800
##
## $decomp4way
##            cde_err          cde_err_se         intref_err
##         0.07488245          0.11243051         1.41544833
##      intref_err_se          intmed_err      intmed_err_se
##         0.23173500          0.33074572         0.06150066
##            pie_err          pie_err_se          total_err
##         0.17160022          0.01989578         1.99267672
##       total_err_se        cde_err_prop    cde_err_prop_se
##         0.30237743          0.03757883         0.05471412
##    intref_err_prop intref_err_prop_se    intmed_err_prop
##         0.71032512          0.04915777         0.16598062
## intmed_err_prop_se        pie_err_prop    pie_err_prop_se
##         0.01601837          0.08611543         0.01543082
##         overall_pm       overall_pm_se        overall_int
##         0.25209606          0.02365800         0.87630574
##     overall_int_se          overall_pe      overall_pe_se
##         0.05695962          0.96242117         0.05471412
```

#### 4.2.2.3 Simulation-based Approach

```
causal_mediation(data = df_int, model = "standard", est.method = "simulation",
                 outcome = "binY_binM_int", exposure = 'A', exposure.type = "binary",
                 mediator = 'M_bin', covariates = "C", EMint = TRUE,
                 yreg = "logistic", mreg = "logistic",nsims = 500)[1:2]
```

```
## $decomp3way
##     cde_rr  cde_rr_se    pnde_rr pnde_rr_se    tnde_rr tnde_rr_se
## 1.32205907 0.52880617 1.82828223 0.35753860 1.96974428 0.31089258
##    pnie_rr pnie_rr_se    tnie_rr tnie_rr_se      te_rr   te_rr_se
## 1.23912702 0.04057785 1.34396157 0.05556037 2.44605232 0.43197268
##         pm      pm_se
## 0.45182482 0.11048440
##
## $decomp4way
##            cde_err          cde_err_se         intref_err
##         0.11022994          0.17706346         0.71805229
##      intref_err_se          intmed_err      intmed_err_se
##         0.19438300          0.37864307         0.07731425
##            pie_err          pie_err_se          total_err
##         0.23912702          0.04057785         1.44605232
##       total_err_se        cde_err_prop    cde_err_prop_se
##         0.43197268          0.04569298         0.13039868
##    intref_err_prop intref_err_prop_se    intmed_err_prop
##         0.50248220          0.04357545         0.27514265
## intmed_err_prop_se        pie_err_prop    pie_err_prop_se
##         0.06560168          0.17668217         0.05246814
##         overall_pm       overall_pm_se        overall_int
##         0.45182482          0.11048440         0.77762485
##     overall_int_se          overall_pe      overall_pe_se
```

```
##          0.09398971          0.95430702          0.13039868
```

#### 4.2.3 Causal Effects and Standard Errors Estimated By the Natural Effect Model

```
causal_mediation(data = df_int, model = "ne",
                 outcome = "binY_binM_int", exposure = 'A', exposure.type = "binary",
                 mediator = 'M_bin', covariates = "C", EMint = TRUE,
                 yreg = "logistic")
```

```
##                        Estimate Std. Error
## pure direct effect     2.403750  3.5079031
## total direct effect    2.483342  3.8107223
## pure indirect effect   1.170043  0.4313725
## total indirect effect 1.208785  0.4530739
## total effect           2.905618  5.8282474
```

#### 4.2.4 Causal Effects and Standard Errors Estimated By the weighting-based approach

```
causal_mediation(data = df_int, model = "wb",
                 outcome = "binY_binM_int", exposure = 'A', exposure.type = "binary",
                 mediator = 'M_bin', covariates = "C", EMint = TRUE,
                 yreg = "logistic", nboot = 500)
```

```
##      RRcde     RRpnde     RRtnde     RRpnie     RRtnie       RRte
## 1.27913959 2.32352057 2.37952107 1.16585507 1.19395401 2.77417671
##         pm   RRcde_se  RRpnde_se  RRtnde_se  RRpnie_se  RRtnie_se
## 0.25400860 0.52741074 0.22314522 0.22866928 0.01980619 0.02044251
##    RRte_se      pm_se
## 0.27052997 0.02471018
```

### 4.3 Case 4-3: Binary Outcome and Multiple Binary Mediators Without Interaction

#### 4.3.1 Data simulation

##### 4.3.1.1 Simulation Procedures

1. Simulate the exposure variable A from Binom(1,P(A=1)).
2. Simulate the covariate C from $N(\mu_C, \sigma_C^2)$.
3. Simulate the first mediator M1 from $Bernoulli(\beta_{01} + \beta_{11} * A + \beta_{21} * C)$ and the second mediator M2 from $Bernoulli(\beta_{02} + \beta_{12} * A + \beta_{22} * C)$.
4. Simulate the outcome Y from $Bernoulli(\theta_0 + \theta_1 A + \theta_2 M1 + \theta_3 M2 + \theta_4 C)$.

##### 4.3.1.2 True Parameters

Table 21: True Model Parameters for Data Simulation

| Sample Size | $\theta_0$ | $\theta_1$ | $\theta_2$ | $\theta_3$ | $\theta_4$ | $\beta_{01}$ | $\beta_{11}$ | $\beta_{21}$ |
|---|---|---|---|---|---|---|---|---|
| 10000 | -5 | 0.8 | 1.8 | 1.2 | 0.1 | -0.25 | 0.5 | 0.2 |
| $\beta_{02}$ | $\beta_{12}$ | $\beta_{22}$ | P(A=1) | $\mu_C$ | $\sigma_C$ | | | |
| -0.3 | 0.4 | 0.3 | 0.4 | 1 | 1 | | | |

#### 4.3.1.3 True Models

True model for the first mediator:

$$logitE[M1|a,c] = \beta_{01} + \beta_{11}a + \beta_{21}c$$

True model for the second mediator:

$$logitE[M2|a,c] = \beta_{02} + \beta_{12}a + \beta_{22}c$$

True model for the outcome:

$$logitE[Y|a,m^*,c] = \theta_0 + \theta_1 a + \theta_2 m1^* + \theta_3 m2^* + \theta_4 c$$

### 4.3.2 Causal Effects and Standard Errors Estimated By the Natural Effect Model

```
causal_mediation(data = df_mulipleM_noint, model = "ne",
                 outcome = "binY_2binM_noint", exposure = 'A', exposure.type = "binary",
                 mediator = c('M_bin1','M_bin2'), covariates = "C",
                 yreg = "logistic")
```

```
##                         Estimate Std. Error
## natural direct effect   2.214931  2.4530107
## natural indirect effect 1.326184  0.5412259
## total effect            2.937407  5.0934849
```

### 4.3.3 Causal Effects and Standard Errors Estimated By the weighting-based approach

```
causal_mediation(data = df_mulipleM_noint, model = "wb",
                 outcome = "binY_2binM_noint", exposure = 'A', exposure.type = "binary",
                 mediator = c('M_bin1','M_bin2'), covariates = "C",
                 yreg = "logistic", m_star = c(0,0), nboot = 500)
```

```
##      RRcde     RRpnde     RRtnde     RRpnie     RRtnie       RRte
## 2.32460771 2.07261588 2.04345828 1.29938222 1.28110248 2.65523335
##         pm   RRcde_se  RRpnde_se  RRtnde_se  RRpnie_se  RRtnie_se
## 0.35198510 0.17593191 0.13850050 0.13425351 0.02668605 0.02507954
##    RRte_se      pm_se
## 0.17796262 0.02671780
```

## 4.4 Case 4-4: Binary Outcome and Multiple Binary Mediators With Exposure-mediator Interaction

### 4.4.1 Data simulation

#### 4.4.1.1 Simulation Procedures

1. Simulate the exposure variable A from Binom(1,P(A=1)).
2. Simulate the covariate C from $N(\mu_C, \sigma_C^2)$.
3. Simulate the first mediator M1 from $Bernoulli(\beta_{01} + \beta_{11} * A + \beta_{21} * C)$ and the second mediator M2 from $Bernoulli(\beta_{02} + \beta_{12} * A + \beta_{22} * C)$.
4. Simulate the outcome Y from $Bernoulli(\theta_0 + \theta_1 A + \theta_2 M1 + \theta_3 M2 + \theta_4 AM1 + \theta_5 AM2 + \theta_6 C)$.

#### 4.4.1.2 True Parameters

Table 22: True Model Parameters for Data Simulation

| Sample Size | $\theta_0$ | $\theta_1$ | $\theta_2$ | $\theta_3$ | $\theta_4$ | $\theta_5$ | $\theta_6$ | $\beta_{01}$ | $\beta_{11}$ |
|---|---|---|---|---|---|---|---|---|---|
| 10000 | -5 | 0.8 | 1.8 | 1.2 | 0.6 | 0.4 | 0.1 | -0.25 | 0.5 |
| $\beta_{21}$ | $\beta_{02}$ | $\beta_{12}$ | $\beta_{22}$ | P(A=1) | $\mu_C$ | $\sigma_C$ | | | |
| 0.2 | -0.3 | 0.4 | 0.3 | 0.4 | 1 | 1 | | | |

#### 4.4.1.3 True Models

True model for the first mediator:

$$logitE[M1|a,c] = \beta_{01} + \beta_{11}a + \beta_{21}c$$

True model for the second mediator:

$$logitE[M2|a,c] = \beta_{02} + \beta_{12}a + \beta_{22}c$$

True model for the outcome:

$$logitE[Y|a,m^*,c] = \theta_0 + \theta_1 a + \theta_2 m1^* + \theta_3 m2^* + \theta_4 am1^* + \theta_5 am2^* + \theta_6 c$$

### 4.4.2 Causal Effects and Standard Errors Estimated By the Natural Effect Model

```
causal_mediation(data = df_mulipleM_EMint, model = "ne",
                 outcome = "binY_2binM_EMint", exposure = 'A', exposure.type = "binary",
                 mediator = c('M_bin1','M_bin2'), EMint = TRUE, EMint.terms = c("A*M_bin1", "A*M_bin2")
                 covariates = "C", yreg = "logistic")
```

```
##                         Estimate Std. Error
## pure direct effect      3.719627  9.3209370
## total direct effect     3.957302 11.9326205
## pure indirect effect    1.335317  0.5809628
## total indirect effect   1.420641  0.6511138
## total effect            5.284255 46.3684825
```

### 4.4.3 Causal Effects and Standard Errors Estimated By the weighting-based approach

```
causal_mediation(data = df_mulipleM_EMint, model = "wb",
                 outcome = "binY_2binM_EMint", exposure = 'A', exposure.type = "binary",
                 mediator = c('M_bin1','M_bin2'), covariates = "C",
                 EMint = TRUE, EMint.terms = c("A*M_bin1", "A*M_bin2"),
                 yreg = "logistic", m_star = c(0,0), nboot = 500)
```

```
##      RRcde     RRpnde     RRtnde     RRpnie     RRtnie       RRte
## 3.30127398 2.80612102 2.74459169 1.27995454 1.25188920 3.51295260
##         pm   RRcde_se  RRpnde_se  RRtnde_se  RRpnie_se  RRtnie_se
## 0.28127533 0.65176608 0.11945995 0.11363751 0.02507274 0.01978510
##    RRte_se      pm_se
## 0.15269764 0.01713566
```

## 4.5 Case 4-5: Binary Outcome and Multiple Binary Mediators With Mediator-mediator Interaction

### 4.5.1 Data simulation

#### 4.5.1.1 Simulation Procedures

1. Simulate the exposure variable A from Binom(1,P(A=1)).
2. Simulate the covariate C from $N(\mu_C, \sigma_C^2)$.
3. Simulate the first mediator M1 from $Bernoulli(\beta_{01} + \beta_{11} * A + \beta_{21} * C)$ and the second mediator M2 from $Bernoulli(\beta_{02} + \beta_{12} * A + \beta_{22} * C)$.
4. Simulate the outcome Y from $Bernoulli(\theta_0 + \theta_1 A + \theta_2 M1 + \theta_3 M2 + \theta_4 M1M2 + \theta_5 C)$.

#### 4.5.1.2 True Parameters

Table 23: True Model Parameters for Data Simulation

| Sample Size | $\theta_0$ | $\theta_1$ | $\theta_2$ | $\theta_3$ | $\theta_4$ | $\theta_5$ | $\beta_{01}$ | $\beta_{11}$ | $\beta_{21}$ |
|---|---|---|---|---|---|---|---|---|---|
| 10000 | -5 | 0.8 | 1.8 | 1.2 | 0.6 | 0.1 | -0.25 | 0.5 | 0.2 |
| $\beta_{02}$ | $\beta_{12}$ | $\beta_{22}$ | P(A=1) | $\mu_C$ | $\sigma_C$ | | | | |
| -0.3 | 0.4 | 0.3 | 0.4 | 1 | 1 | | | | |

#### 4.5.1.3 True Models

True model for the first mediator:

$$logitE[M1|a, c] = \beta_{01} + \beta_{11}a + \beta_{21}c$$

True model for the second mediator:

$$logitE[M2|a, c] = \beta_{02} + \beta_{12}a + \beta_{22}c$$

True model for the outcome:

$$logitE[Y|a, m^*, c] = \theta_0 + \theta_1 a + \theta_2 m1^* + \theta_3 m2^* + \theta_4 m1^* m2^* + \theta_5 c$$

### 4.5.2 Causal Effects and Standard Errors Estimated By the Natural Effect Model

```
causal_mediation(data = df_mulipleM_MMint, model = "ne",
                 outcome = "binY_2binM_MMint", exposure = 'A', exposure.type = "binary",
                 mediator = c('M_bin1','M_bin2'), MMint = TRUE, MMint.terms = c("M_bin1*M_bin2"),
                 covariates = "C", yreg = "logistic")
```

```
##                          Estimate Std. Error
## natural direct effect   2.006796  1.8168242
## natural indirect effect 1.411036  0.6382357
## total effect            2.831662  4.2684428
```

### 4.5.3 Causal Effects and Standard Errors Estimated By the weighting-based approach

```r
causal_mediation(data = df_mulipleM_MMint, model = "wb",
                 outcome = "binY_2binM_MMint", exposure = 'A', exposure.type = "binary",
                 mediator = c('M_bin1','M_bin2'), covariates = "C",
                 MMint = TRUE, MMint.terms = c("M_bin1*M_bin2"),
                 yreg = "logistic", m_star = c(0,0), nboot = 500)
```

```
##      RRcde     RRpnde     RRtnde     RRpnie     RRtnie       RRte
## 2.22700334 1.85121805 1.81743465 1.36077307 1.33593994 2.47311613
##         pm   RRcde_se  RRpnde_se  RRtnde_se  RRpnie_se  RRtnie_se
## 0.42216501 0.15055883 0.09872063 0.09418609 0.02893633 0.02683497
##    RRte_se      pm_se
## 0.13497029 0.02665281
```

## 4.6 Case 4-6: Binary Outcome and Multiple Binary Mediators With Exposure-mediator-mediator Interaction

### 4.6.1 Data simulation

#### 4.6.1.1 Simulation Procedures

1. Simulate the exposure variable A from Binom(1,P(A=1)).
2. Simulate the covariate C from $N(\mu_C, \sigma_C^2)$.
3. Simulate the first mediator M1 from $Bernoulli(\beta_{01} + \beta_{11} * A + \beta_{21} * C)$ and the second mediator M2 from $Bernoulli(\beta_{02} + \beta_{12} * A + \beta_{22} * C)$.
4. Simulate the outcome Y from $Bernoulli(\theta_0 + \theta_1 A + \theta_2 M1 + \theta_3 M2 + \theta_4 AM1 + \theta_5 AM2 + \theta_6 M1M2 + \theta_7 AM1M2 + \theta_8 C)$.

#### 4.6.1.2 True Parameters

Table 24: True Model Parameters for Data Simulation

| Sample Size | $\theta_0$ | $\theta_1$ | $\theta_2$ | $\theta_3$ | $\theta_4$ | $\theta_5$ | $\theta_6$ | $\theta_7$ | $\theta_8$ | $\beta_{01}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 10000 | -5 | 0.8 | 1.8 | 1.2 | 0.6 | 0.4 | 0.3 | 0.5 | 0.1 | -0.25 |
| $\beta_{11}$ | $\beta_{21}$ | $\beta_{02}$ | $\beta_{12}$ | $\beta_{22}$ | P(A=1) | $\mu_C$ | $\sigma_C$ | $\sigma_{M1}$ | $\sigma_{M2}$ | |
| 0.5 | 0.2 | -0.3 | 0.4 | 0.3 | 0.4 | 1 | 1 | 0.1 | 0.1 | |

#### 4.6.1.3 True Models

True model for the first mediator:

$$logitE[M1|a, c] = \beta_{01} + \beta_{11} a + \beta_{21} c$$

True model for the second mediator:

$$logitE[M2|a, c] = \beta_{02} + \beta_{12} a + \beta_{22} c$$

True model for the outcome:

$$logitE[Y|a, m^*, c] = \theta_0 + \theta_1 a + \theta_2 m1^* + \theta_3 m2^* + \theta_4 am1^* + \theta_5 am2^* + \theta_6 m1^* m2^* + \theta_7 am1^* m2^* + \theta_8 c$$

### 4.6.2 Causal Effects and Standard Errors Estimated By the Natural Effect Model

```
causal_mediation(data = df_mulipleM_mint, model = "ne",
                 outcome = "binY_2binM_mint", exposure = 'A', exposure.type = "binary",
                 mediator = c('M_bin1','M_bin2'),
                 EMMint = TRUE, EMMint.terms = c("A*M_bin1*M_bin2"),
                 covariates = "C", yreg = "logistic")
```

```
##                          Estimate  Std. Error
## pure direct effect     4.411091  19.0469420
## total direct effect    4.753790  26.6536534
## pure indirect effect   1.389864   0.6289652
## total indirect effect  1.497844   0.7506387
## total effect           6.607124 178.3105685
```

### 4.6.3 Causal Effects and Standard Errors Estimated By the weighting-based approach

```
causal_mediation(data = df_mulipleM_mint, model = "wb",
                 outcome = "binY_2binM_mint", exposure = 'A', exposure.type = "binary",
                 mediator = c('M_bin1','M_bin2'), covariates = "C",
                 EMMint = TRUE, EMMint.terms = c("A*M_bin1*M_bin2"),
                 yreg = "logistic", m_star = c(0,0), nboot = 500)
```

```
##        RRcde       RRpnde       RRtnde       RRpnie       RRtnie
## 8.510133e+00 3.748803e+00 3.699299e+00 1.353940e+00 1.336061e+00
##         RRte           pm     RRcde_se    RRpnde_se    RRtnde_se
## 5.008630e+00 3.142788e-01 4.587229e+05 1.960546e-01 1.920155e-01
##    RRpnie_se    RRtnie_se      RRte_se        pm_se
## 3.067589e-02 2.929784e-02 2.780990e-01 1.951116e-02
```