BT209

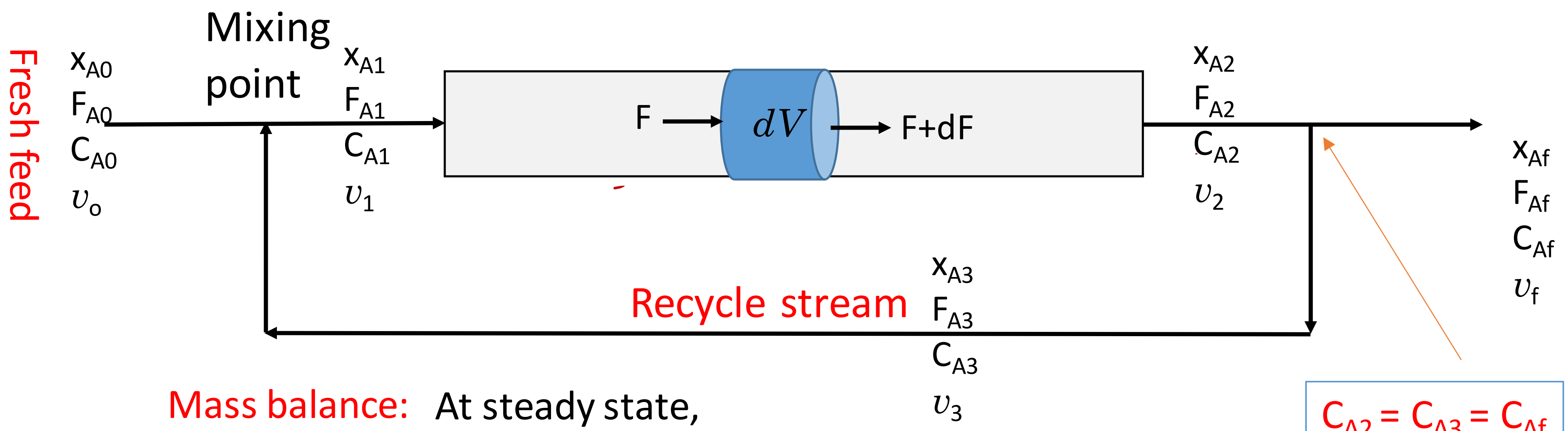
Bioreaction Engineering

20/03/2023

Recycle reactor

- In certain situations it is found to be advantageous to divide the product stream from a plug flow reactor and return a portion of it to the entrance of the reactor.
- Let the recycle ratio R be defined as

$$\boldsymbol{R} = \frac{\text{volume of fluid returned to the reactor entrance}}{\text{volume leaving the system}} = \upsilon_3/\upsilon_f$$



$C_{A2} = C_{A3} = C_{Af}$
$X_{A2} = x_{A3} = x_{Af}$

Mass balance: At steady state,

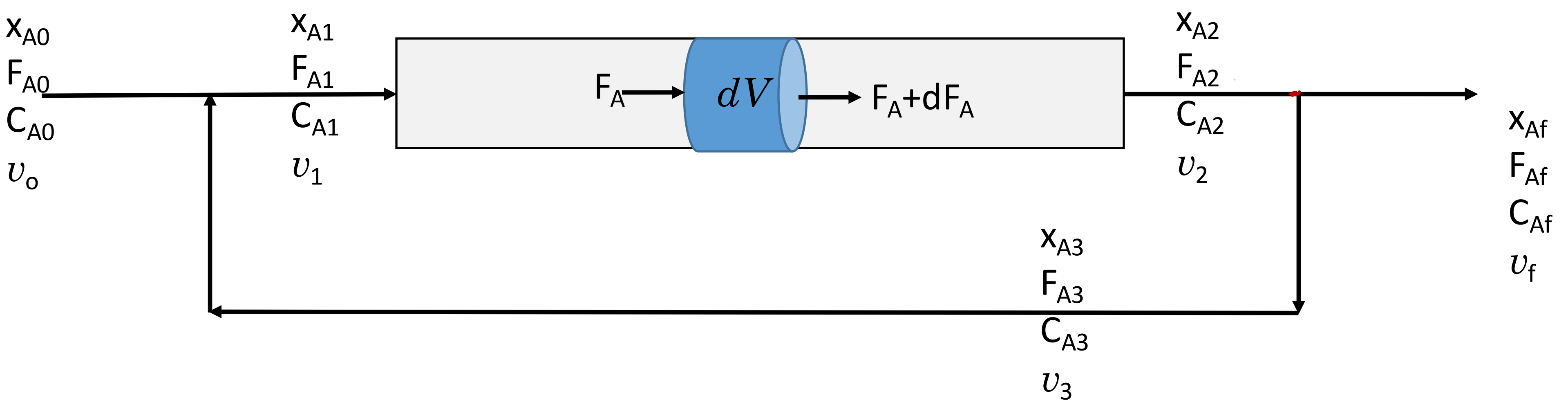
$F_A=(F_A+dF_A)+(-r_A)dV$

$-dF_A = (-r_A)dV$

Recycle ratio: $R=\upsilon_3/\upsilon_f$

$R= C_{Af}\,\upsilon_3/ C_{Af}\,\upsilon_f = C_{A3}\,\upsilon_3/ C_{Af}\,\upsilon_f = F_{A3}/ F_{Af}$

Cont..

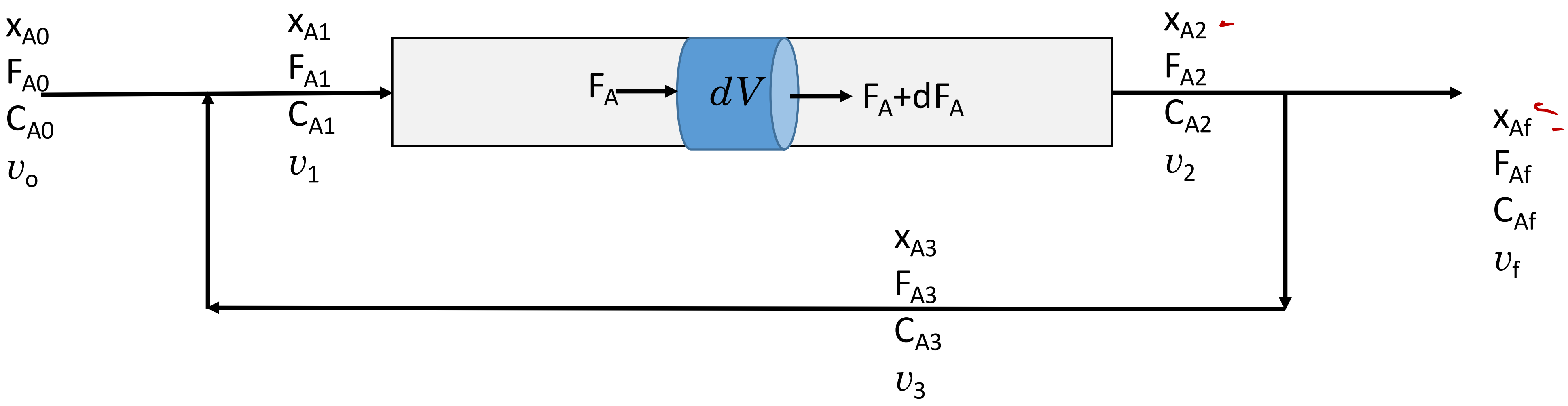


$$C_{Af} = C_{A0}\ (1-x_{Af})\ /(1+\varepsilon_A x_{Af})$$
$$\upsilon_f = \upsilon_o(1+\varepsilon_A x_{Af})$$
$$F_{Af} = F_{A0}\ (1-x_{Af})$$

$$F_{A2}=F_{A3}+F_{Af}$$
$$F_{A2}=RF_{Af} + F_{Af} = F_{Af}(R+1)$$

$$F_{A2} = F_{A0}\,(R+1)\ (1-x_{Af})$$
$$F_{A2} = F_{A0}\,(R+1)\ (1-x_{A2})$$

Cont..



$C_{A2} = C_{A3} = C_{Af}$

$X_{A2} = x_{A3} = x_{Af}$

$F_{A2} = F_{A0}\,(R+1)\,(1-x_{A2})$

At any position in the reactor, $F_A = F_{A0}\,(R+1)\,(1-x_A)$

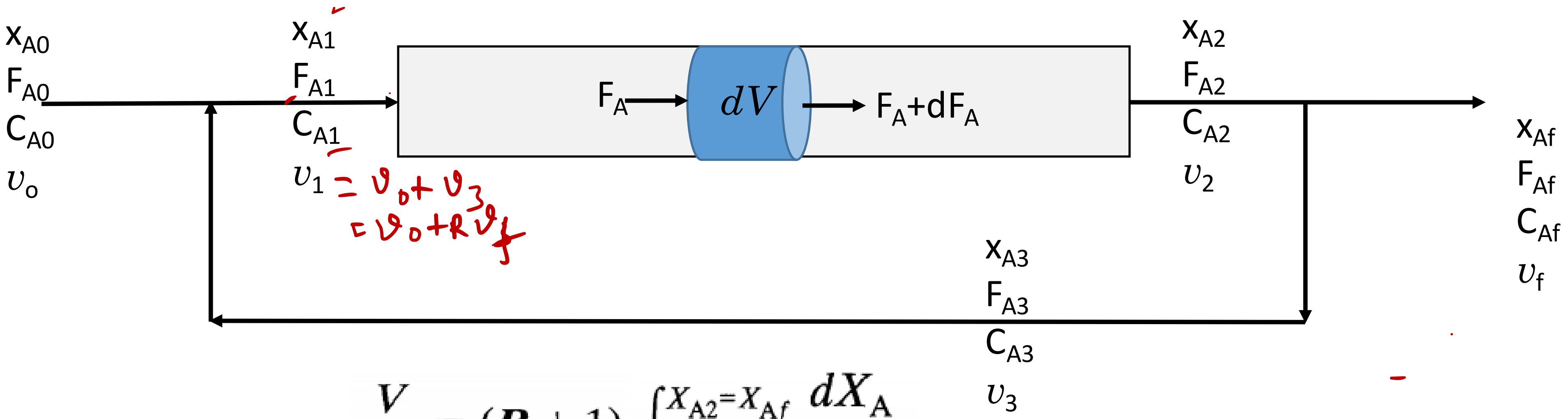
At constant R, $dF_A = -F_{A0}\,(R+1)\,dx_A$

Mass balance: $-dF_A = (-r_A)dV$

$(-r_A)dV = F_{A0}\,(R+1)\,dx_A$

$$\frac{V}{F_{A0}} = (\boldsymbol{R}+1)\int_{X_{A1}}^{X_{A2}=X_{Af}} \frac{dX_A}{-r_A}$$

Cont..

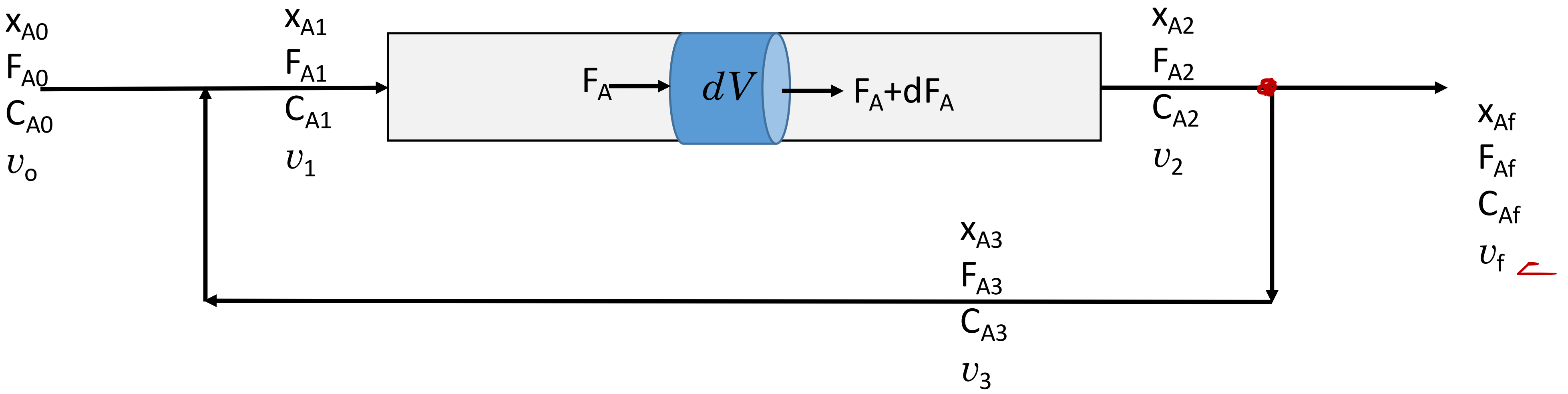


$$\frac{V}{F_{A0}} = (\boldsymbol{R}+1)\int_{X_{A1}}^{X_{A2}=X_{Af}} \frac{dX_A}{-r_A}$$

$$C_{A1} = \frac{F_{A1}}{v_1} = \frac{F_{A0}+F_{A3}}{v_0+\boldsymbol{R}v_f} = \frac{F_{A0}+\boldsymbol{R}F_{A0}(1-X_{Af})}{v_0+\boldsymbol{R}v_0(1+\varepsilon_A X_{Af})} = C_{A0}\left(\frac{1+\boldsymbol{R}-\boldsymbol{R}X_{Af}}{1+\boldsymbol{R}+\boldsymbol{R}\varepsilon_A X_{Af}}\right)$$

$$X_{A1} = \frac{1-C_{A1}/C_{A0}}{1+\varepsilon_A C_{A1}/C_{A0}} \quad \Rightarrow \quad X_{A1} = \left(\frac{\boldsymbol{R}}{\boldsymbol{R}+1}\right)X_{Af}$$

Cont..



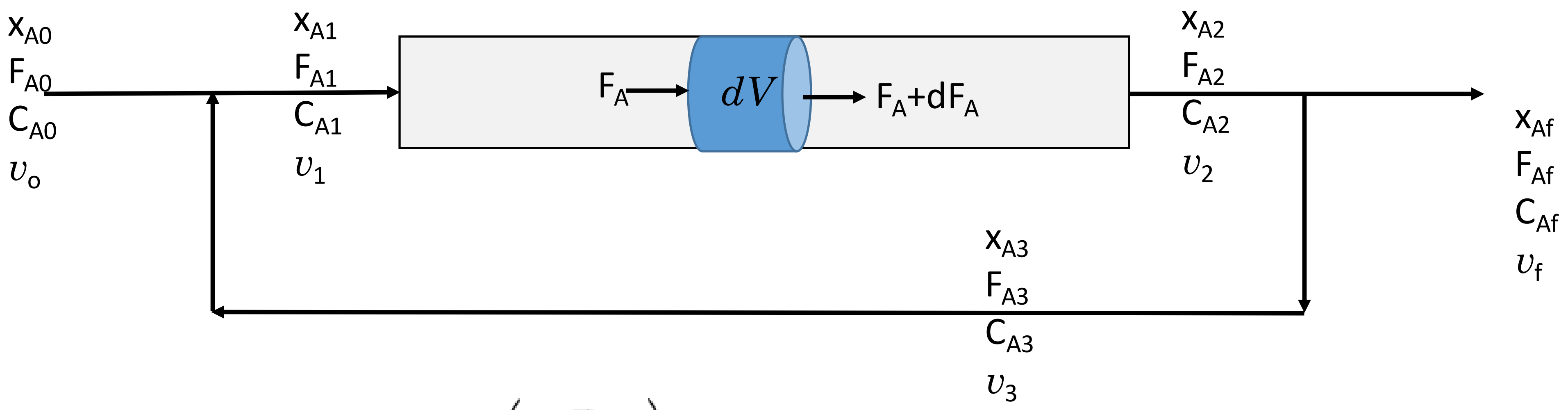
$$\frac{V}{F_{A0}} = (\boldsymbol{R}+1)\int_{X_{A1}}^{X_{A2}=X_{Af}} \frac{dX_A}{-r_A}$$

$$X_{A1} = \left(\frac{\boldsymbol{R}}{\boldsymbol{R}+1}\right) X_{Af}$$

$$\frac{V}{F_{A0}} = (\boldsymbol{R}+1)\int_{\left(\frac{\boldsymbol{R}}{\boldsymbol{R}+1}\right) X_{Af}}^{X_{Af}} \frac{dX_A}{-r_A}$$

Cont..

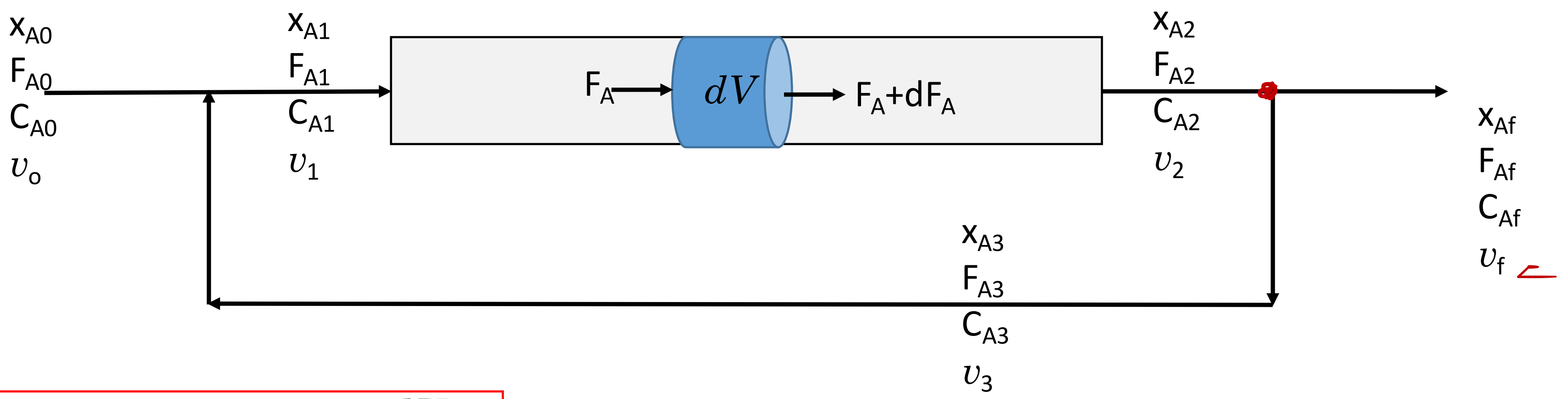
For the special case where density changes are negligible we may write this equation in terms of concentrations, or



$$X_{A1} = \left(\frac{\boldsymbol{R}}{\boldsymbol{R}+1}\right) X_{Af}$$

$$\boxed{\tau = \frac{C_{A0}V}{F_{A0}} = -(\boldsymbol{R}+1)\int_{\frac{C_{A0}+\boldsymbol{R}C_{Af}}{\boldsymbol{R}+1}}^{C_{Af}} \frac{dC_A}{-r_A} \quad \ldots \varepsilon_A = 0}$$

$R = \infty$



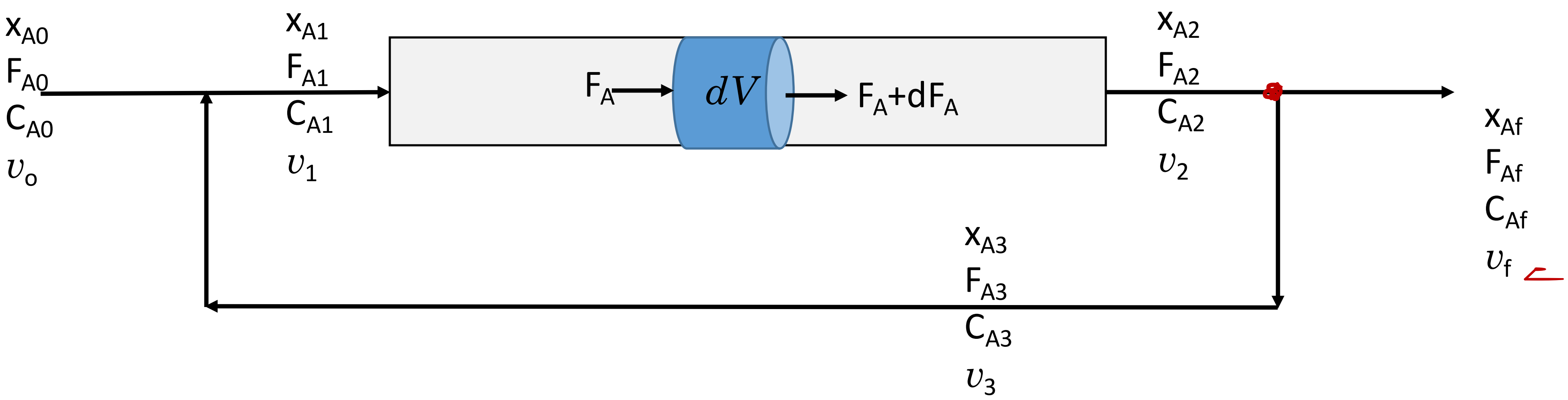
$$\boxed{\begin{aligned} \frac{V}{F_{A0}} &= (\boldsymbol{R}+1)\int_{X_{A1}}^{X_{A2}=X_{Af}} \frac{dX_A}{-r_A} \\ \frac{V}{F_{A0}} &= (\boldsymbol{R}+1)\int_{\left(\frac{\boldsymbol{R}}{\boldsymbol{R}+1}\right)X_{Af}}^{X_{Af}} \frac{dX_A}{-r_A} \end{aligned}}$$

$$X_{A1} = \left(\frac{\boldsymbol{R}}{\boldsymbol{R}+1}\right)X_{Af}$$

$$R+1 = \frac{X_{Af}}{X_{Af}-X_{A1}}$$

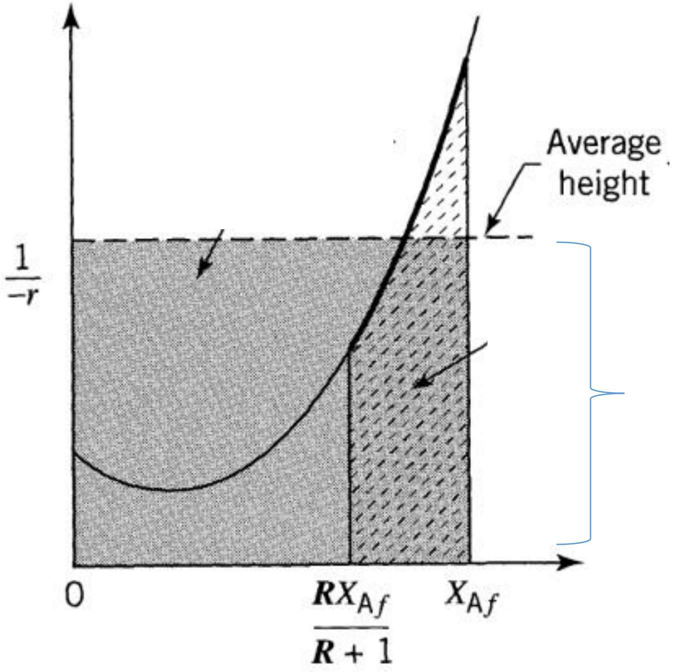
$$\frac{V}{F_{A0}} = \frac{X_{Af}\displaystyle\int_{X_{A1}}^{X_{A2}=X_{Af}} \frac{dX_A}{-r_A}}{X_{Af}-X_{A1}}$$

$R \doteq \infty$



$$\frac{V}{F_{A0}} = \frac{X_{Af}\int_{X_{A1}}^{X_{A2}=X_{Af}} \frac{dX_A}{-r_A}}{X_{Af}-X_{A1}}$$

Average value of function



$$R \doteq \infty$$

$$X_{A1} = \left(\frac{R}{R+1}\right) X_{Af}$$

X_{A1} *will close to* X_{Af} *at* $R \rightarrow \infty$

$$\frac{V}{F_{A0}} = \frac{X_{Af}}{-r_{Af}}$$

Cont..

For the extremes of negligible and infinite recycle the system approaches plug flow and mixed flow

$$\frac{V}{F_{A0}} = (\boldsymbol{R}+1)\int_{\frac{\boldsymbol{R}}{\boldsymbol{R}+1}X_{Af}}^{X_{Af}} \frac{dX_A}{-r_A}$$

$\boldsymbol{R} = 0$ $\downarrow$

$$\frac{V}{F_{A0}} = \int_{A}^{X_{Af}} \frac{dX_A}{-r_A}$$

plug flow

$\boldsymbol{R} = \infty$ $\downarrow$

$$\frac{V}{F_{A0}} = \frac{X_{Af}}{-r_{Af}}$$

mixed flow

Cont..

Integration of the recycle equation gives, for *first-order reaction*, $\varepsilon_A = 0$

$$\boxed{\frac{k\tau}{\boldsymbol{R}+1} = \ln\left[\frac{C_{A0} + \boldsymbol{R}C_{Af}}{(\boldsymbol{R}+1)C_{Af}}\right]}$$

and for *second-order reaction*, $2A \rightarrow$ products, $-r_A = kC_A^2$, $\varepsilon_A = 0$,

$$\boxed{\frac{kC_{A0}\tau}{\boldsymbol{R}+1} = \frac{C_{A0}(C_{A0} - C_{Af})}{C_{Af}(C_{A0} + \boldsymbol{R}C_{Af})}}$$