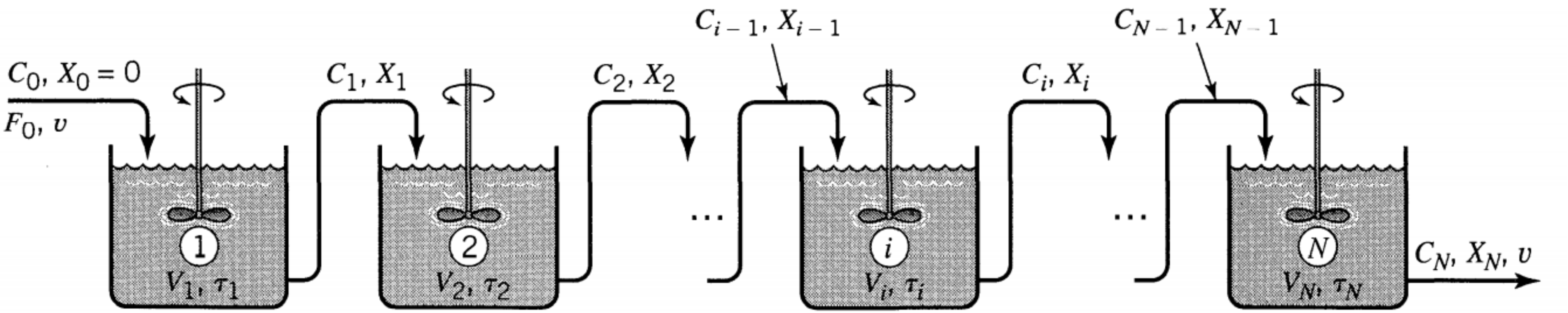
BT209

Bioreaction Engineering

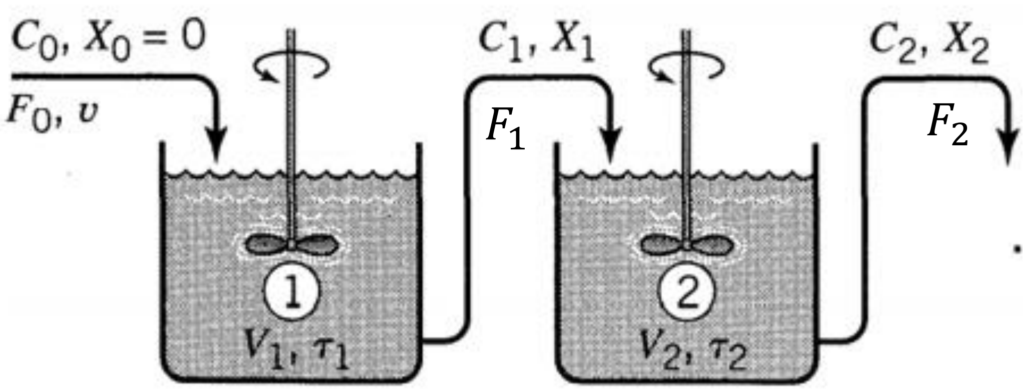
13/03/2023

Mixed Flow Reactors in Series

- Quantitatively evaluate the behavior of a series of N mixed flow reactors.
- Density changes will be assumed to be negligible; hence ε = 0



➢Density changes will be assumed to be negligible; hence ε = 0



$$\tau_1 = \frac{V_1}{\vartheta} = \frac{C_0 V_1}{F_0}$$

$$\tau_2 = \frac{V_2}{\vartheta} = \frac{C_0 V_1}{F_0}$$

$$\text{input} = \text{output} + \text{disappearance by reaction} + \cancelto{=0}{\text{accumulation}}$$

Reactor 1:

$$F_0 = F_1 + (-r_{A\ at\ C1})V_1$$

$$F_0 = F_0(1 - X_1) + (-r_{A\ at\ C1})V_1$$

$$\frac{V_1}{F_0} = \frac{X_1}{-r_{A\ at\ C1}}$$

$$\boxed{\frac{V_1}{F_0} = \frac{\tau_1}{C_0} = \frac{X_1}{-r_{A\ at\ C1}}}$$

Reactor 2:

$$F_1 = F_2 + (-r_{A\ \mathrm{at}\ C2})V_2$$

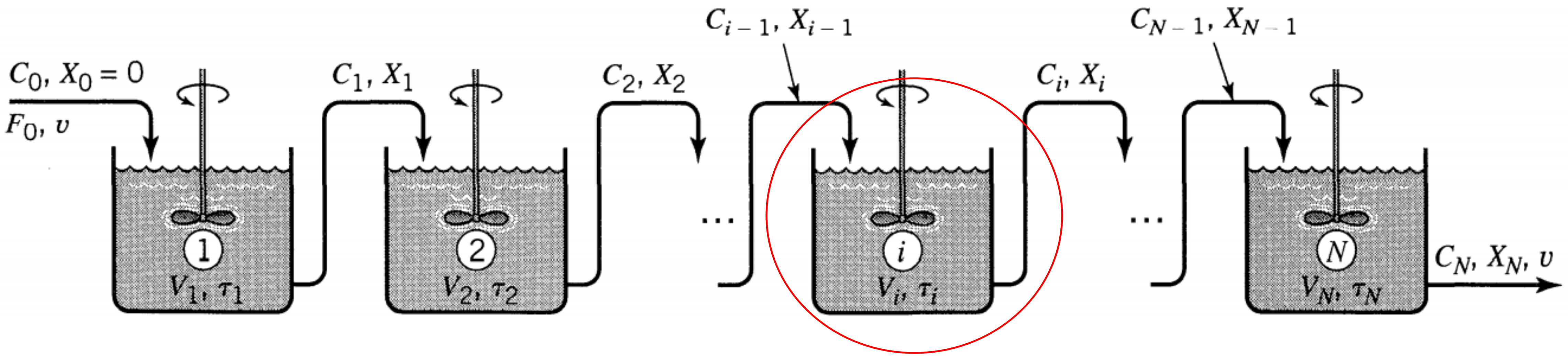
$$F_0(1 - X_1) = F_0(1 - X_2) + (-r_{A\ at\ C1})V_2$$

$$\frac{V_2}{F_0} = \frac{X_2 - X_1}{-r_{A\ at\ C2}}$$

$$\boxed{\frac{V_2}{F_0} = \frac{\tau_2}{C_0} = \frac{X_2 - X_1}{-r_{A\ at\ C2}}}$$

Mixed Flow Reactors in Series

- Density changes will be assumed to be negligible; hence ε = 0

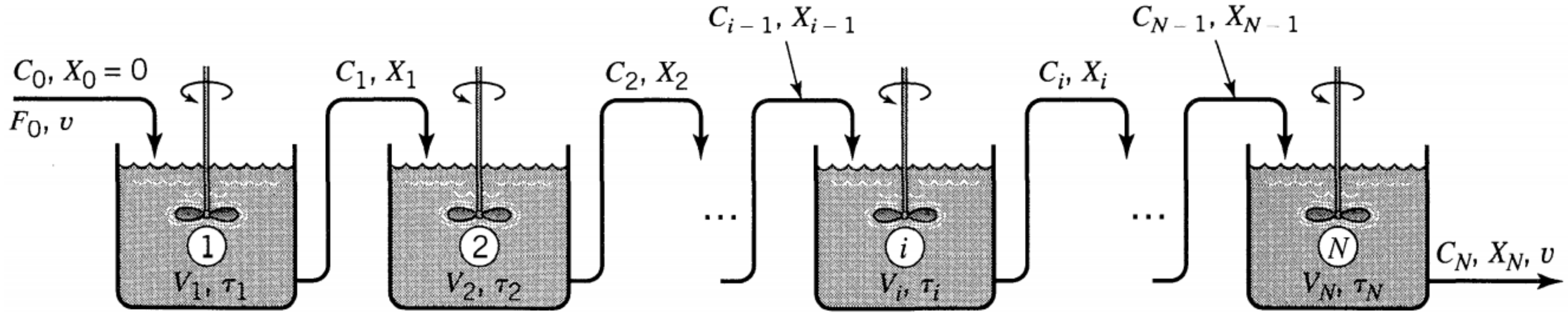


Material balance for *i*th reactor:

$$\frac{V_i}{F_0} = \frac{\tau_i}{C_0} = \frac{X_i - X_{i-1}}{-r_{A\ at\ Ci}}$$

$$\tau_i = \frac{C_0 V_i}{F_0} = \frac{V_i}{v} = \frac{C_0(X_i - X_{i-1})}{-r_{Ai}}$$

Mixed Flow Reactors in Series with 1st order reaction



$$\tau_i = \frac{C_0 V_i}{F_0} = \frac{V_i}{v} = \frac{C_0(X_i - X_{i-1})}{-r_{Ai}}$$

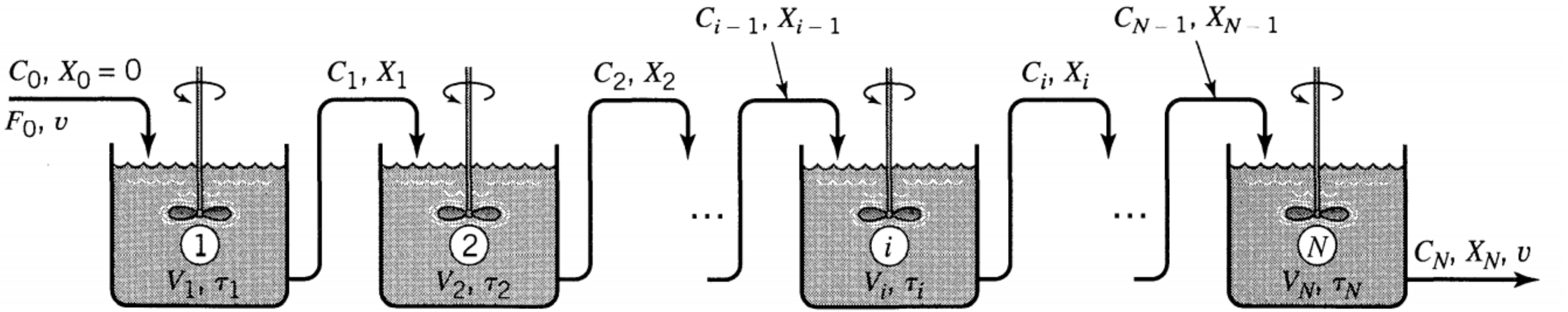
Because $\varepsilon = 0$ this may be written in terms of concentrations. Hence

$$\tau_i = \frac{C_0[(1 - C_i/C_0) - (1 - C_{i-1}/C_0)]}{kC_i} = \frac{C_{i-1} - C_i}{kC_i}$$

$$\boxed{\frac{C_{i-1}}{C_i} = 1 + k\tau_i}$$

Equal-size Mixed Flow Reactors in Series with 1st order reaction

- Density changes assumed to be negligible; hence **ε = 0**



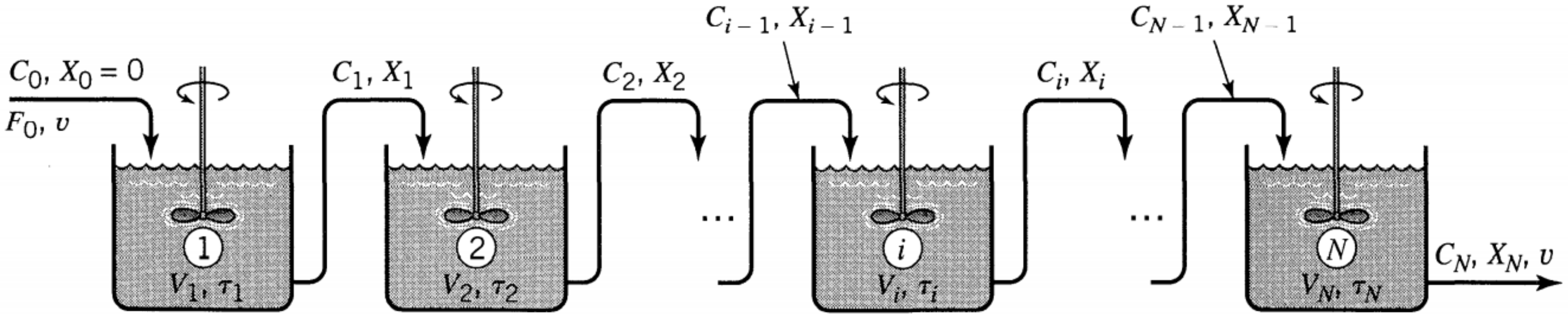
$$\boxed{\frac{C_{i-1}}{C_i} = 1 + k\tau_i}$$

Now the space-time τ (or mean residence time t) is the same in all the equal-size reactors of volume V_i. Therefore,

$$\frac{C_0}{C_N} = \frac{C_0}{C_1}\frac{C_1}{C_2} \cdots \frac{C_{N-1}}{C_N} = (1 + k\tau_i)^N$$

$$\frac{C_0}{C_N} = \frac{1}{1 - X_N}$$

Cont..



$$\boxed{\frac{C_0}{C_N} = (1 + k\tau_i)^N}$$

Total space time of the system: $\tau_{N \text{ reactors}} = N\tau_i$

$$\tau_{N \text{ reactors}} = N\tau_i = \frac{N}{k}\left[\left(\frac{C_0}{C_N}\right)^{1/N} - 1\right]$$

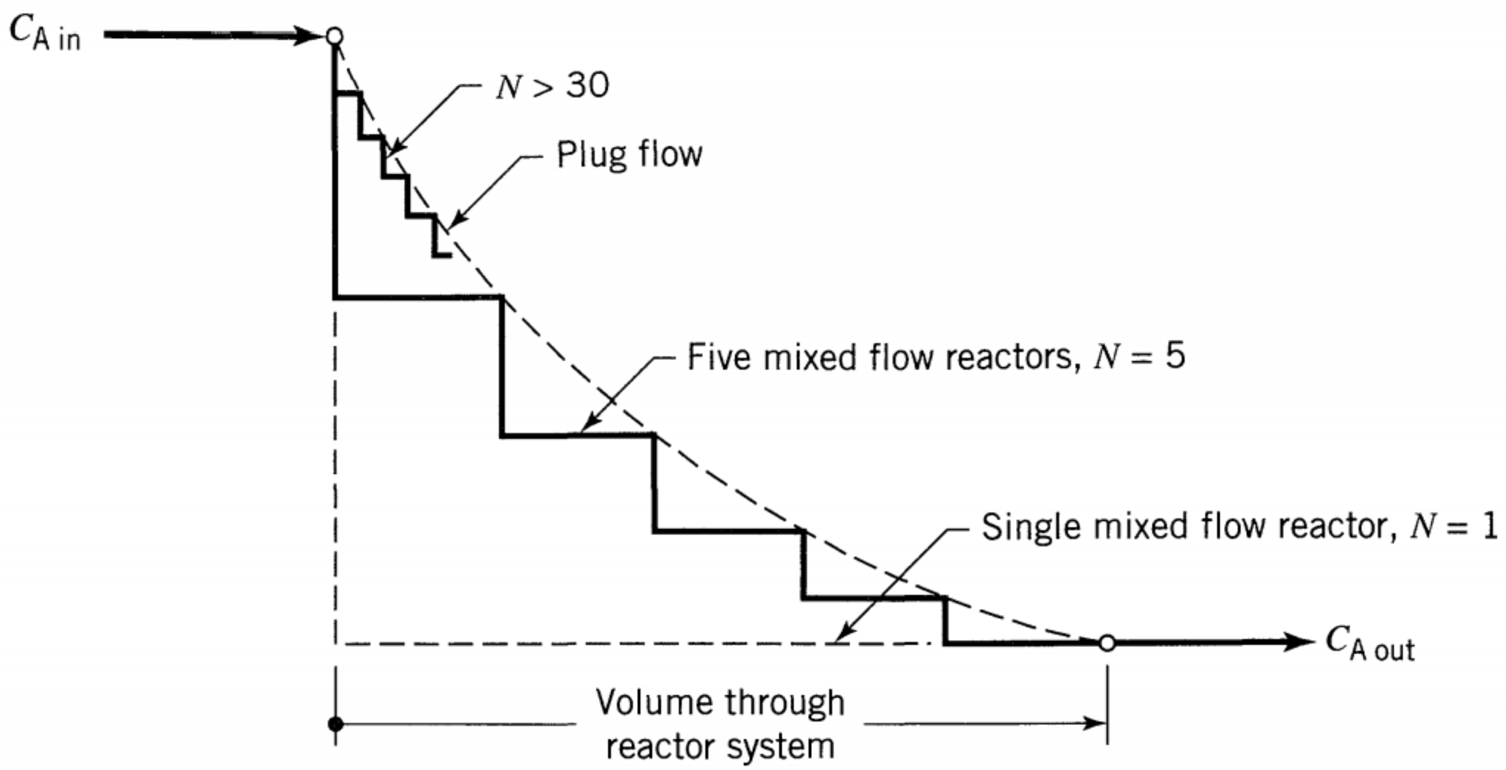
In the limit, for $N \to \infty$, this equation reduces to the plug flow equation

$$\tau_p = \frac{1}{k}\ln\frac{C_0}{C}$$

Graphical representation (for any order of reaction)

- In plug flow, the concentration of reactant decreases progressively through the system; in mixed flow, the concentration drops immediately to a low value.

- Because of this fact, a plug flow reactor is more efficient than a mixed flow reactor for reactions whose rates increase with reactant concentration, such as nth-order irreversible reactions, $n > 0$.

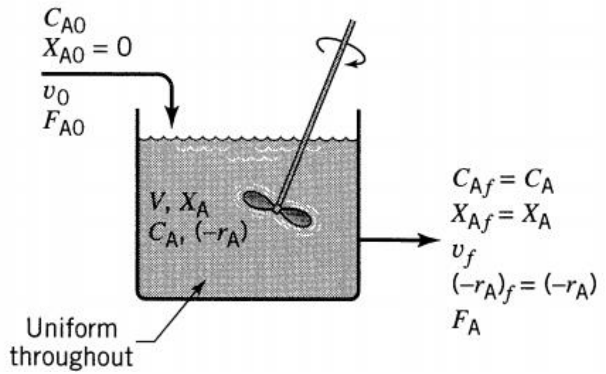


- Consider a system of N mixed flow reactors connected in series. A change in concentration as fluid moves from reactor to reactor.

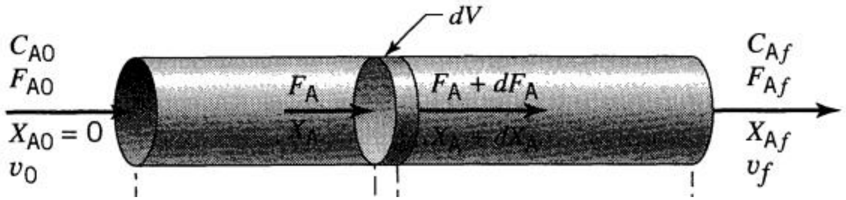
- This stepwise drop in concentration, illustrated in Fig, suggests that the larger the number of units in series, the closer should the behavior of the system approach plug flow.

Holding Time and Space Time for Flow Reactors

Space time $$\tau = \begin{pmatrix} \text{time needed to} \\ \text{treat one reactor} \\ \text{volume of feed} \end{pmatrix} = \frac{V}{v_0} = \frac{C_{A0}V}{F_{A0}}, \quad \text{[hr]}$$



Holding time $$\bar{t} = \begin{pmatrix} \text{mean residence time} \\ \text{of flowing material} \\ \text{in the reactor} \end{pmatrix}$$



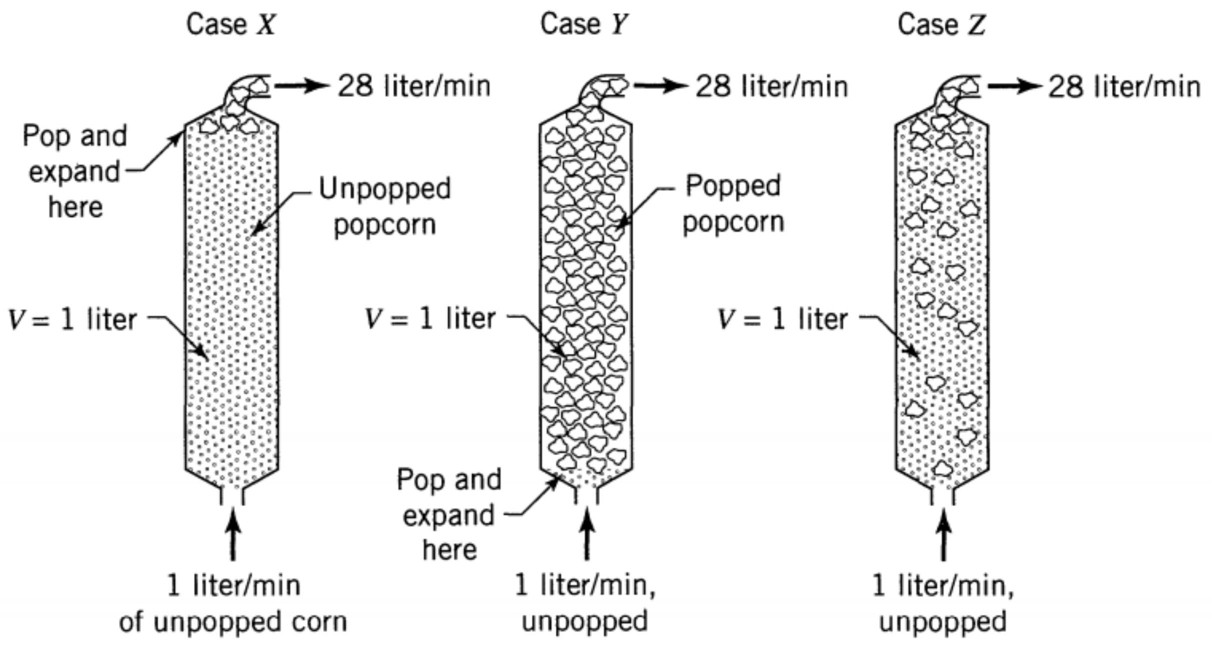
For constant density systems (all liquids and constant density gases)

$$\tau = \bar{t} = \frac{V}{v}$$

For changing density systems $\bar{t} \neq \tau$ and $\bar{t} \neq V/v_0$

Cont.

Consider three cases of the steady-flow popcorn popper which takes in 1 liter/min of raw corn and produces 28 liters/min of product popcorn



For the same τ value the $\bar{t}$ values differ in these three cases

In case X: all the popping occurs at the back end of the reactor.
In case Y: all the popping occurs at the front end of the reactor.
In case Z: the popping occurs somewhere between entrance and exit.

Space time

$$\tau_X = \tau_Y = \tau_Z = \frac{V}{v_0} = \frac{1 \text{ liter}}{1 \text{ liter/min}} = 1 \text{ min}$$

Holding time

$$\bar{t}_X = \frac{1 \text{ liter}}{1 \text{ liter/min}} = 1 \text{ min}$$

$$\bar{t}_Y = \frac{1 \text{ liter}}{28 \text{ liter/min}} \cong 2 \text{ sec}$$

$\bar{t}_Z$ is somewhere between 2 and 60 s, depending on the kinetics

Note that the value of $\bar{t}$ depends on what happens in the reactor, while the value of τ is independent of what happens in the reactor.