Still we can have indifference curves of the following nature:

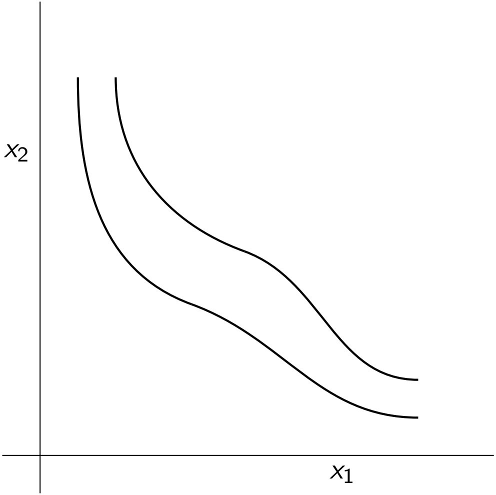

Figure: Violation of convexity

Convexity:
Consider two bundles x, y. If a consumer is indifferent between x and y bundle. Then any linear combination of x and y that is $\lambda x + (1 - \lambda)y$, $1 > \lambda > 0$ must be preferred to x and y. In other words average is preferred to extremes. Convexity along with all other conditions mentioned above ensures well behaved indifference curves.
The well behaved indifference curves are shown below.

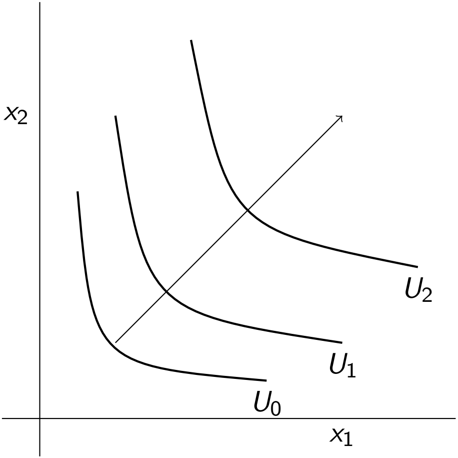

Figure: Indifference curves, $U_0 < U_1 < U_2$

We assume that the utility function is differentiable in x_1, x_2. It gives us following.
$\frac{\partial U(x_1, x_2)}{\partial x_1} = Mu_1$. It is the marginal utility from the consumption of good 1. Keeping the level of x_2 fixed, if we increase the consumption of good 1 by one unit , the additional utility we receive is $\frac{\partial U(x_1, x_2)}{\partial x_1} = Mu_1$.
It is assumed that marginal utility from a good always decreases although it is non-negative. This is called law of diminishing marginal utility.
Again $\frac{\partial U(x_1, x_2)}{\partial x_2} = Mu_2$, marginal utility from good 2. Keeping good 1 fixed at some level, if we increase the consumption of good 2 by one unit, the additional utility we receive is $\frac{\partial U(x_1, x_2)}{\partial x_2} = Mu_2$.

We look at the movement along the well-behaved indifference curve. We know well behaved indifference are convex in nature and downward sloping. It means from any point (x_1, x_2), if we increase x_1 we need to decrease x_2 to remain at the same level of utility or same indifference curves. Suppose utility is fixed at $\bar{U}$, taking total differentiation of it we get
$d\bar{U} = \frac{\partial U(x_1,x_2)}{\partial x_1} dx_1 + \frac{\partial U(x_1,x_2)}{\partial x_2} dx_2 = 0$, along an indifference curve.

$$\Rightarrow \frac{dx_2}{dx_1} = -\frac{\frac{\partial U(x_1,x_2)}{\partial x_1}}{\frac{\partial U(x_1,x_2)}{\partial x_2}} = -\frac{Mu_1}{Mu_2} = \text{marginal rate of substitution.}$$

This marginal rate of substitution is the rate at which the consumer is willing to substitute one good for the other at a fixed level of utility. If we increase the consumption of good 1 by one unit, how much amount of good 2 the consumer is willing to decrease to remain at the same level of utility. Convexity of preferences ensures that the marginal rate of substitution decreases as we increase the consumption of good 1.

The slope of the indifference curve is $\dfrac{dx_2}{dx_1} = -\dfrac{\frac{\partial U(x_1,x_2)}{\partial x_1}}{\frac{\partial U(x_1,x_2)}{\partial x_2}} = -\dfrac{Mu_1}{Mu_2}$

Budget Constraint

The consumers are constrained by the income they have. We represent income by m. The consumers have to buy the goods from the market. While buying they have to pay the price of the good. We assume that the price of the good 1 is p_1 and price of good 2 is p_2. From these two prices and income, we get the feasibility set of a consumer.
It is the budget set, $p_1x_1 + p_2x_2 \leq m$. p_1x_1 is the expenditure on good 1, p_2x_2 is the expenditure on good 2. Total expenditure should always be less than equal to total income m.
Example: suppose of good 1 is 10 and price of good 2 is 20 and income 10000. The budget set is $10x_1 + 20x_2 \leq 10000$.
It is shown in figure below.

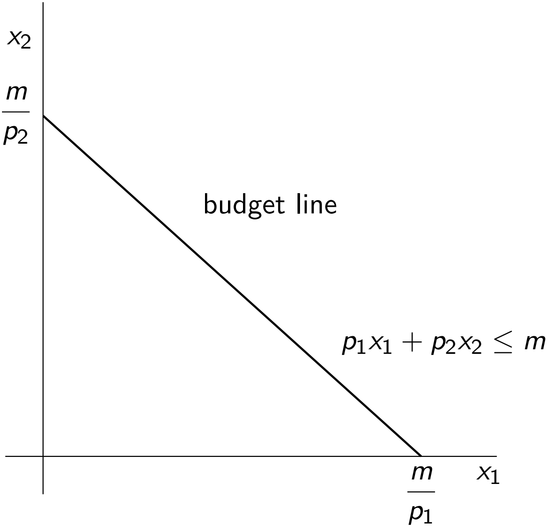

Figure: Budget set

- All the bundles of good 1 and 2 on or below the budget line are affordable to the consumer.
- If the price of good 1 that is p_1 increases the budget line will change as shown in figure 5. Now lesser bundles will be available.

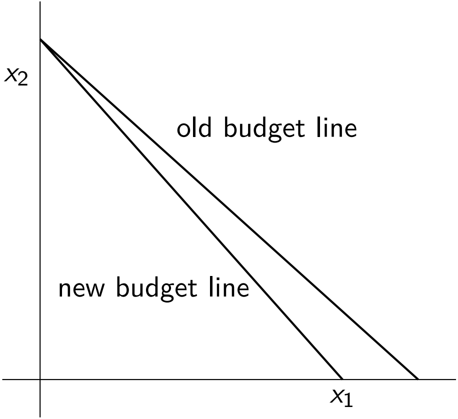

Figure: Price of good 1, p_1 increases.

If price of good 2 increases, the budget set will change. It is shown in figure below

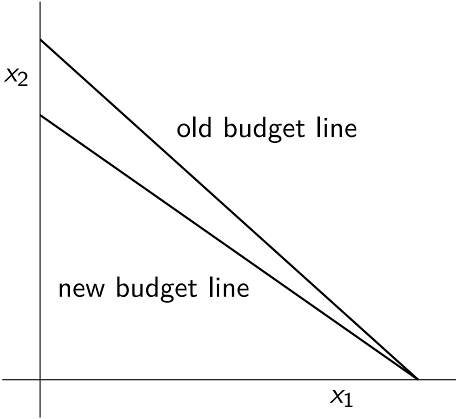

Figure: Price of good 2, p_2 increases.

When income m increases, the budget line will shift outward. If there is decrease in income, the budget line will shift inward. It is shown in figure below.

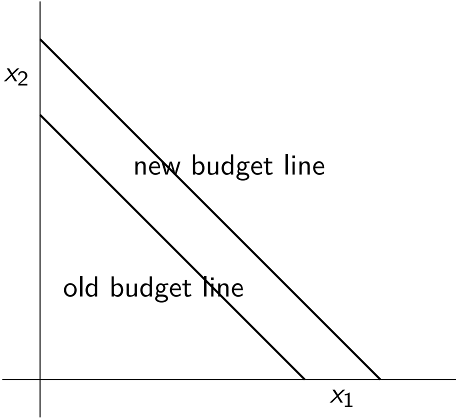

Figure: When income increases.

The slope of the budget line is $\frac{dx_2}{dx_1} = -\frac{p_1}{p_2}$. This is the exchange rate. The rate at which market is allowing to exchange one for other. If you want one more units of good 1 how many units of good 2 must be given up.

- Now we come to the question which bundle is actually chosen by a consumer.
- First we assume that the consumers have a utility function. The utility functions are such that it generates well behaved indifference curves. It means all the assumptions mentioned earlier on preferences and utility functions are true.
- The feasible set or the budget set is $p_1x_1 + p_2x_2 \leq m$. The consumer has to choose a bundle from this budget set.
- The consumer will choose that bundle which provides maximum utility. Thus, the choice of consumption bundle is a constraint maximization problem. It is presented in the following way;
  Maximizes $U(x_1, x_2)$ (objective function)
  Subject to $p_1.x_1 + p_2.x_2 \leq m$ (budget constraint).

We assume that Utility functions are well behaved in nature.

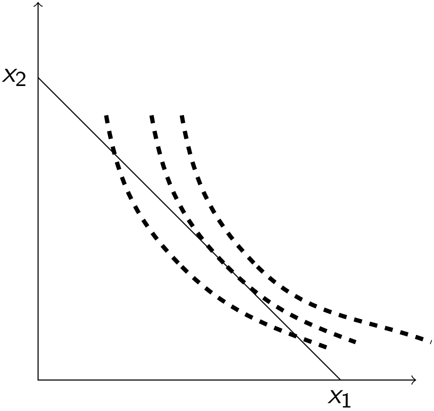

Figure: Utility maximization for a given budget.

Derivation of Demand function

- At the point where utility is maximized subject to a budget constraint, we the following condition is satisfied;
  The slope of indifference curves= slope of budget constraint
  $$-\frac{Mu_1}{Mu_2} = -\frac{\frac{\partial U(x_1,x_2)}{\partial x_1}}{\frac{\partial U(x_1,x_2)}{\partial x_2}} = -\frac{p_1}{p_2}$$
- Based on the maximization of utility subject to a budget constraint, we derive the demand curve for each good.

- The price of good 1 is decreasing so the budget line has moved in north east direction. We keep the price of good 2 and income level fixed. We plot the optimal quantity chosen at these different prices of good 1 in the lower panel. This gives us the demand curve of good 1.
- The demand curve of good 1 is downward sloping. As price of good 1 increases the quantity demanded decreases keeping other things constant. Keeping other things constant means income and price of good 2 are fixed and utility function is same.

The main reasons for having downward sloping demand curve are following:

- The slope of the budget line changes. The ratio $\dfrac{p_1}{p_2}$ increases when p_1 rises. So the consumer needs to give up more of good 2 to increase consumption of good 1 by one unit. Suppose the real income remains same in that case the old bundle is not utility maximising any more. At the new utility maximising bundle, the amount of good 1 will decrease as more amounts of good 2 need to be given up to increase one unit of good 1. This is substitution effect.
- The real income income falls ($\frac{m}{p_1}$ falls) that is, to buy the same amount of good 1 as earlier, the consumer requires higher amount of income. Due to fall in real income, the demand for good 1 will fall. This is income effect. It is true for normal goods.
- The income effect and substitution effect together leads to downward sloping demand curve of a normal good.