Questions on Production and Cost

1. Why does a production function experience diminishing marginal returns to labour in the short run?

2. Can an isoquant ever slop upward? Explain.

3. Do the following production functions exhibit increasing, constant, or decreasing returns to scale? What happens to marginal product of each individual factor as that factor is increased and the other factor held constant?

   (a) $Q = 3L + 2K$.

   (b) $Q = (2L + 2K)^{\frac{1}{2}}$.

   (c) $Q = 4L^{\frac{1}{2}} + 4K$.

4. If the short run average variable cost of a firm constant, what shape of the long run average curve is implied by this phenomenon? Explain.

5. A firm has a fixed production cost of Rs 5000 and a constant marginal cost of production of Rs 500 per unit produced.

   (a) What is the firm's total cost function? Average cost?

   (b) If the firm wanted to minimize the average total cost, would it choose to be very large or small? Explain.

6. Suppose that a firm's production function is $q = 10L^{\frac{1}{2}}K^{\frac{1}{2}}$. The cost of a unit of labor is Rs 20 and cost of a unit of capital is Rs 80.

   (a) The firm is currently producing 100 units of output. Determine the cost minimizing quantities of labour and capital.

   (b) The firm wants to increase output to 140 units. If capital is fixed is fixed at 5 units? Determine the cost minizing quantities of labour and capital in the short run. What is the cost function of the firm?

   (c) In the long run capital is not fixed. Determine the cost minimizing level of labour and capital in the long run.

Answer Hints.

1. In short run, suppose capital is fixed in supply and labour is variable. We observe diminishing marginal product of labour as the units of labour employed increases after a level. It is mainly because as the number of labour increases there is congestion in the production because there is fixed capital. So the addition to output from the addition of new labour results in less amount of additional output. Consider an example where a farmer cultivates a fixed plot of land. Here land is the capital and farmer is labour. If the number of farmers are increased given the same size of land, after a point the increase in numbers of farmers employed in the farm will starts giving less marginal output. Because the land will be crowded. Thus, we see law of diminishing marginal product starts operating.

2. No, we cannot have an isoquant upward sloping. If we have an upward sloping isoquant. It means the output is remaining same (constant) when we are increasing the units of both the inputs. This is not possible. One of our assumption is that when all the inputs are increased the output always increases.

3. (a) It is constant return to scale. Suppose all the inputs are increased by a scale of $\lambda, \lambda > 0$, then we have $\lambda q = 3.\lambda L + 2\lambda K = \lambda(3L + 2k)$. The marginal product of labour is $\frac{\partial q}{\partial L} = 3$, it is constant. The marginal product of capital is $\frac{\partial q}{\partial K} = 2$, it is constant.

   (b) The product function exhibits decreasing return to scale. If both labour and capital are increased by λ, $\lambda > 1$ times, the output increases by less than λ times. $(2\lambda L + 2\lambda K)^{\frac{1}{2}} = \lambda^{\frac{1}{2}}(2L + 2K)^{\frac{1}{2}} = \lambda^{\frac{1}{2}}q$. The marginal product of labour is $\frac{\partial q}{\partial L} = \frac{1}{2(2L+2K)^{\frac{1}{2}}}2$, it is decreasing. The marginal product of capital is $\frac{\partial q}{\partial K} = \frac{1}{2(2L+2K)^{\frac{1}{2}}}2$, it is decreasing.

   (c) The production function exhibits decreasing returns to scale. If both labour and capital are increased by $\lambda, \lambda > 1$ times, the output increases by less than λ times. Suppose one unit of labour and capital is used so out is 8. Now doubled the units of labour and capital that is two units of each input, the out we get is $4(\sqrt{2} + 2)$. It is less than two times the output. The marginal product of labour is $\frac{\partial q}{\partial L} = \frac{2}{\sqrt{L}}$, it is decreasing. The marginal product of capital is $\frac{\partial q}{\partial K} = 4$, it is constant.

4. Suppose there is no fixed cost then the long run average cost will be same as the constant average variable cost. Suppose there is a fixed cost in the short run. The average cost in the short run is shown in figure 1. The fixed cost is variable in the long run. The firm can employ variable amount of the factor which is fixed in short run. The firm minimizes is cost of production. The firm chooses that input combination which cost least for a given level of output. So the average cost in the long run will always lie below the short run average cost or will be same as the short run average cost curve. The firm can always use the short run input combination even in the long run, if it is the least cost combination. In this the shape will be same as the short run average cost.

5. (a) The total cost function the firm is $TC = 5000 + 500q$, where $q = \ output$. The average cost is $AC = \frac{5000}{q} + 500$.

   (b) The average cost of the firm in given in figure 2. If the firm wants to minimize its average cost, it has to produce a very high amount of output because average cost curve is asymptotic to the line $AC = 500$.

6. (a) The production function of the firm is $q = 10L^{\frac{1}{2}}K^{\frac{1}{2}}$. The unit cost labour is Rs 20 and unit cost of capital is rs 80. The firm is producing 100 units of output. The firm solves the following problem:
   Minimize $20L + 80k$ subject $100 = 10L^{\frac{1}{2}}K^{\frac{1}{2}}$.
   The Lagrange of the above problem is:
   $\mathcal{L} = 20L + 80k + \lambda(100 - 10L^{\frac{1}{2}}K^{\frac{1}{2}})$, where λ is Lagrange multiplier.
   At the optimum point we have,
   $\frac{\partial \mathcal{L}}{\partial L} = 20 - \lambda 5\sqrt{\frac{K}{L}} = 0$.
   $\frac{\partial \mathcal{L}}{\partial L} = 20 - \lambda 5\sqrt{\frac{K}{L}} = 0$.
   $\frac{\partial \mathcal{L}}{\partial \lambda} = 100 - 10L^{\frac{1}{2}}K^{\frac{1}{2}} = 0$.
   From the first two equation we have, $L = 4K$. Substituting it in the third equation we have. $K = 5$ and $L = 20$.

   (b) If capital is fixed, $K = 5$. The firm has to produce 140 units of output. One can use Kuhn Tucker method to solve the problem. But we can use the following method also. Lets us first check whether the cost minimizing input combination for producing 140 units of output is constrained by the fixed amount of capital. First solve the cost minimization problem when output is 140 and capital is not fixed. We can use the same equation s as in answer (a) and get $L = 4K$. Substitute it in the equation $140 = 10L^{\frac{1}{2}}K^{\frac{1}{2}}$. We get $K = 7$ and $L = 28$. But capital is fixed at the level 5. So in this case the cost minimizing amount of capital is 5 units. We can find the amount of labour as $140 = 10 \times (\sqrt{5}\sqrt{L})$, $L = \frac{196}{5}$.

   (c) The cost minimizing input combination is $L = 28$, $K = 7$, when 140 units of output is produced.

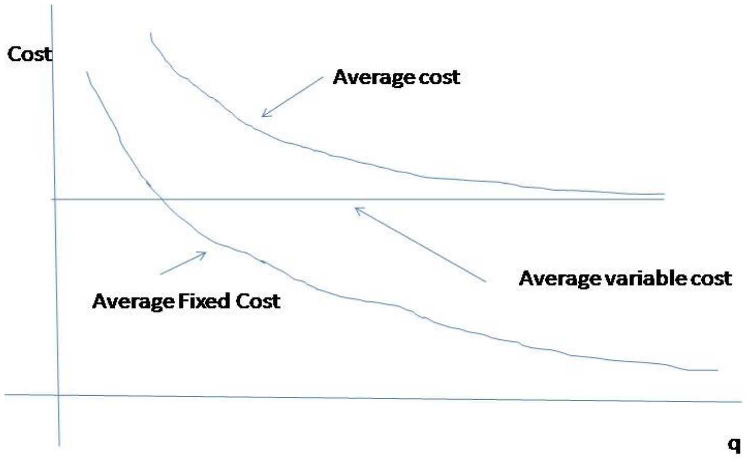

Figure 1, Cost in short run

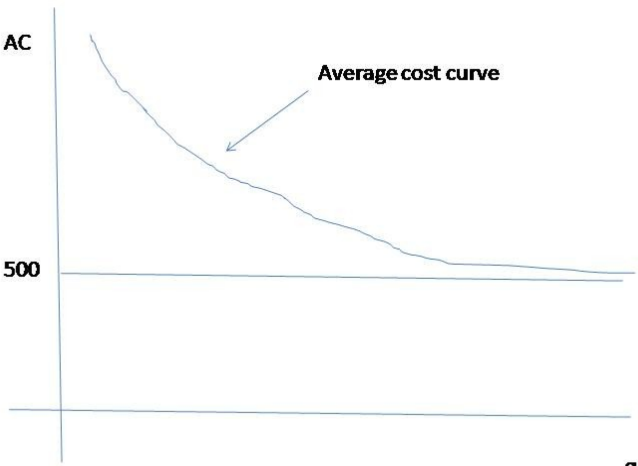

Figure 2