Production and Technology

- Inputs or factors of productions are used to produce output. Inputs; labour, land, machines etc are used to produce output. In a plot of land, farmers using their labour along with some machines produce food grain.
- The process of production is defined through a function. It is called production function.
- The amount of maximum output we get by employing some amount of inputs (labour and machines) is determined by technology.
- Suppose a good can be produced using a single input labour. It is something like this, $f(l) = al$, $a > 0$, here $l =$ labour and al units of output.

- if 10 units of labour are employed, the maximum output is $a10$. It is possible to produce $a9$ units of good using 10 units of labour. But we cannot get $a12$ units of output using 10 units of labour. This is the role of the technology.
- The production set and production function are shown in figure below.
- Another example of single input production function, $y = f(l) = al^{\alpha}$, $a > 0$ and $0 < \alpha < 1$. Here l is the single input labour and y is the output.
- Technological constraint means only certain combination of inputs are feasible ways to produce a given amount of output.

- Suppose two inputs are required to produce. In this course we rae going to take only two inputs - labour and capital (machines).
- Examples; 1 unit of labour and 2 units of capital produces 4 units of output and 2 units of labour and 1 units of capital also produces 4 units of output.
- These two inputs are written as a vector with two coordinates or elements, (l, k). Each combination (l, k) is called a technique of production. We represent in the following way.

Some properties;

- Technology must be monotonic: if any input is increased, the output must increase. Atleast same amount of output can be produced.
- Free disposal: Inputs can be disposed costlessly.
- Technology must be convex: it means if $(l_1, k_1$ units of labour- capital produce y_1 units of output and (l_2, k_2) units of labour and capital produces y_1 units of output then $(\theta l_1 + (1-\theta)l_2, \theta k_1 + (1-\theta)k_2)$ units of labour and capital must produce atleast y_1 units of output. This gives us that following types of combinations of (l, k) gives same amount of output.
- This curves are called isoquants. It is similar to indifference curves. Isoquant gives combinations of labour and capital (l, k) that give same amount of output.

Suppose $(100, 200)$ units of labour and capital gives 100 units of output. $(200, 100)$ also gives 100 units of output. If you have $(150, 150)$ units of labour and capital then we must be able to produce 100 units. We can produce 50 units of output using the $(50, 100)$ combination and another 50 units using $(100, 50)$ combination.

- Isoquants are drawn for a fixed amount of output. The output level increases in the north east direction.
- Some examples of production function with two inputs labour (l) and capital (k).
- $y = f(l, k) = l^{\alpha} k^{\beta}$, $0 < \alpha < 1$ and $0 < \beta < 1$, This is Cobb-Douglas production function.
- $y = \min\{l, k\}$ fixed proportion production function.
- $y = l + k$ perfect substitutes

- We assume that production function $y = f(l, k)$ is differentiable in (l, k).
- Derivation of $f(l, k)$ with respect to l is $\dfrac{\partial f(l, k)}{\partial l} = MP_l$ marginal product of labour.
- It means, the additional output we get if labour is increased by one more unit keeping the amount of capital fixed.
- Derivative of $f(l, k)$ with respect to k is $\dfrac{\partial f(l, k)}{\partial k} = MP_k$ marginal product of capital. Additional output we get if capital is increased by one more unit keeping the amount of labour fixed.

- The marginal product of labour MP_l decreases as more and more labour is employed keeping the amount of capital fixed.
- The marginal product of capital MP_k decreases as more and more capital is employed keeping the amount of labour fixed.
- This is law of diminishing marginal product of inputs.
- Suppose a plot of land is used of cultivation. The cultivation is done using labour, raw material and some machines. Suppose only labour is increased and everything else have been kept fixed. The additional output we are going to get goes on decreasing as we employ more and more labour.
- It is mainly because all other factors are fixed, so additional labour cannot be utilized upto its potential. Our land plot is also fixed. It can be ploughed only given number of times. So labour per machines will be more.

- In short run some of the inputs or factors of production are fixed. Cannot be increased or decreased immediately. Like land, and in many cases machines (capital) etc. Some inputs are variable in short like labour.
- In short run, since some inputs are fixed and to increase output we can only increase labour. The law of diminishing marginal product of labour operates in such situations.
- In the long run, all the inputs or factors of production are variable.

By taking total differential of the production function we get,

$dy = \dfrac{\partial f(l,k)}{\partial l}.dl + \dfrac{\partial f(l,k)}{\partial k}.dk$ In the movement along an isoquant curve, the amount of output is same, so $dy = 0$. This implies

$\dfrac{dk}{kl} = -\dfrac{\frac{\partial f(l,k)}{\partial l}}{\frac{\partial f(l,k)}{\partial k}} = -\dfrac{MP_l}{MP_k}=$ marginal rate of technical substitution.

This is the slope of the isoquant curve. As we go on increasing l, we need to give up less and less amount of k because of operation of law of diminishing marginal product.

Returns to scale:

- Increasing returns to scale; if all the inputs are increased by factor θ and $\theta > 1$, the output must increase by a factor more than θ.
- Suppose the production function is $y = f(l, k)$, we increase (l, k) by a factor θ, then $f(\theta l, \theta k) > \theta y$ where $\theta > 1$. If we double all the inputs the output must be more than double.
- Example; $y = l^{.7}k^{.8}$, if we double both the inputs we get $(2l)^{.7}(2k)^{.8}$ outputs. It is $2^{1.5}l^{.7}k^{.8}$. Increase in output is more than two times.

- Constant returns to scale: If all inputs are increased by a factor θ and $\theta > 0$, the output must increase θ times.
- Suppose the production function is $y = f(l, k)$, we increase (l, k) by a factor θ, then $f(\theta l, \theta k) = \theta y$ where $\theta > 0$. If we double all the inputs the output must be double.
- Example; $y = l^{.6}k^{.4}$, if we double both the inputs we get $(2l)^{.6}(2k)^{.4}$ outputs. It is $2l^{.6}k^{.4}$. Increase in output is two times.

- Decreasing returns to scale: If all inputs are increased by a factor θ and $\theta > 1$, the increase in output is less θ times.
- Suppose the production function is $y = f(l, k)$, we increase (l, k) by a factor θ, then $f(\theta l, \theta k) < \theta y$ where $\theta > 1$. If we double all the inputs the output must be less than double.
- Example; $y = l^{.3}k^{.4}$, if we double both the inputs we get $(2l)^{.3}(2k)^{.4}$ outputs. It is $2^{.7}l^{.3}k^{.4}$. Increase in output is less than two times.