

## INSTRUCTIONS

1. Write your name and roll number on the answerscript.
2. Your writing should be legible and neat.
3. This Mid-Sem has 7 questions, for a total of 30 marks.

## QUESTIONS

[5marks] 1. Fill in the blanks:

- (A) Consider a project where you invest an amount of 1 lakh today, receiving amounts of 2 lakhs after one year and 1 lakh after two years. Then, the present value of the project (in lakhs), with the annual risk-free rate (compounded annually) being  $r = 5\%$ , equals \_\_\_\_\_.
- (B) Consider a two year bond which trades at 1,000, with a face value of 1,000. The bond pays coupons of 100 at the end of the first and the second year. Then the (positive) annual yield rate (in percentage) of the bond equals \_\_\_\_\_.
- (C) Consider an European call and an European put option, both with the same expiration of one year and same strike price of  $X = 95$ . If the current stock price is  $S(0) = 90$ , and the price of the call and the put option are  $C^E = 2.25$  and  $P^E = 1.90$ , respectively, then the (continuously compounded) annual risk-free rate (in percentage) equals \_\_\_\_\_.
- (D) Consider an American call option on an underlying asset with current price  $S(0) = 100$  and the strike price being  $X = 105$ . The price  $C^A$  of this American option is 2.95. If  $P^A$  denotes the price of an American put option on the same underlying, with the same maturity, and the same strike price, as the American call option, then the largest possible value of  $P^A$  equals \_\_\_\_\_.
- (E) Consider a one-year forward contract on an asset with  $S(0) = 100$  and the annual risk-free rate (continuously compounded) is  $r = 6\%$ , with the forward price  $F(0,1)$ . Now, if it is assumed that the asset pays a dividend of 10 at time  $t = \frac{1}{2}$  (six months), then the forward price becomes  $F_{div}(0,1)$ . Then the value of  $\frac{F(0,1)}{F_{div}(0,1)}$  equals \_\_\_\_\_.

[4marks] 2. Consider two investment opportunities:

- (A) Investing 10,000 today, to receive 5,000 after three months, 4,500 after six months and 4,500 after nine months.
- (B) Investing 11,480 today, to receive 5,500 after three months, 5,000 after six months and 5,000 after nine months.

Assuming that the annual rate of interest is 5% with the compounding being done quarterly (every three months), determine the present value of Investment (A) and Investment (B). Hence state which of the Investments (A) and (B) is a better investment.

[5marks] 3. If  $X' < X''$ , then prove that:

(A)  $C_E(X') \geq C_E(X'')$ .

(B)  $C_E(X') - C_E(X'') \leq e^{-rT}(X'' - X')$ .

Handwritten calculations for part 3:

$$\frac{1}{(1+y)^2} = \frac{1}{(1.0125)^2} = \frac{1}{1.02515625} \approx 0.97537$$

$$1.02515625 = 1 + 0.02515625 = 1 + 0.0125^2$$



- [6marks] 4. Determine the value of the American put option at each of the nodes, for a binomial model with the parameters:

$$S(0) = 80, X = 75, N = 2, U = 0.2, D = -0.05 \text{ and } R = 0.1.$$

- [4marks] 5. Prove that the forward price with a non-dividend paying stock as the underlying is given by:  
 $F(0, T) = S(0)e^{rT}$ .

- [2marks] 6. Consider a long forward contract, with  $F(0, T) = 100$ ,  $T = 25 \ln\left(\frac{10}{9}\right)$  and  $r = 4\%$ . If the stock price at time  $t = 1$  is  $S(1) = 93$ , then determine the value of the forward contract at time  $t = 1$ .

- [4marks] 7. Consider the following margin account Table for the long position in a futures contract (with margin requirement at 10% and margin call at 5%). Determine the values of  $F_1$ ,  $F_2$ ,  $F_3$  and  $F_4$ .

$n$	$f(n, T)$	Cash	Margin 1	Payment	Margin 2
0	100	Opening	0	-10	10
1	95	-5	$F_1$	0	5
2	90	$F_2$	0	-9	$F_3$
3	95	5	14	$F_4$	9.5

$$F_1 = 5$$

$$F_2 = -5$$

$$F_3 = 9$$

$$F_4 = 5.5 \underline{4.5}$$

$$\underline{9.5}$$

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