BT209

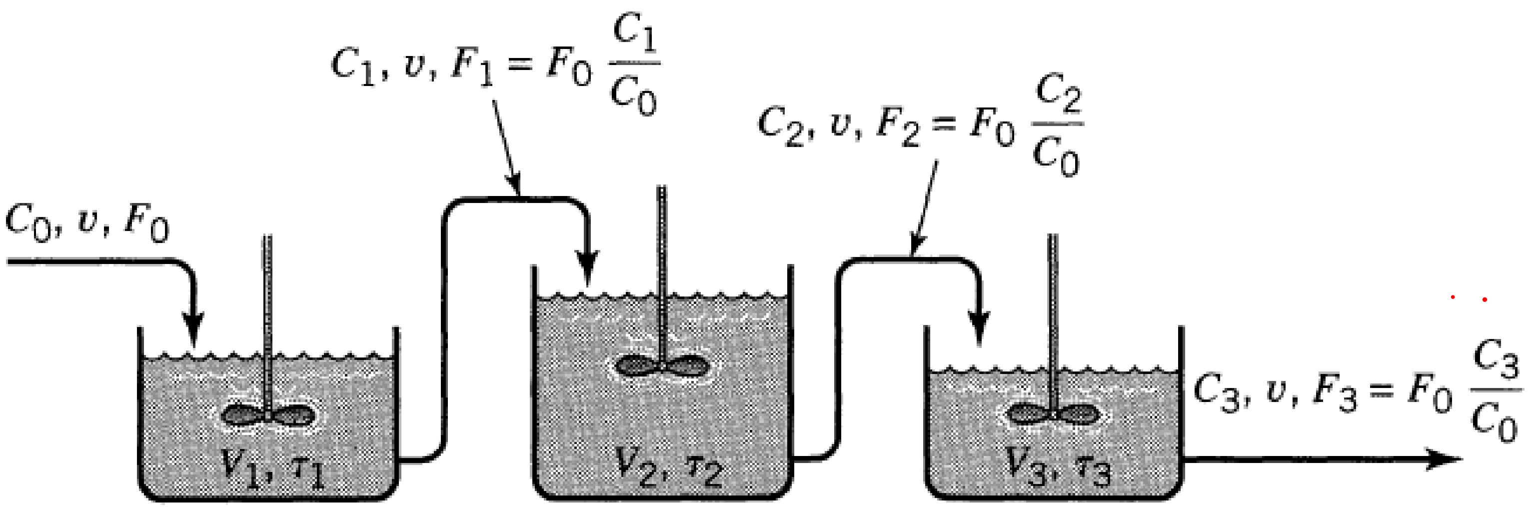
Bioreaction Engineering

15/03/2023

Mixed Flow Reactors of Different Sizes in Series

- For arbitrary kinetics in mixed flow reactors of different size, two types of questions may be asked:
  - how to find the outlet conversion from a given reactor system, and
  - the inverse question, how to find the best setup to achieve a given conversion.

✓ Different procedures are used for these two problems.

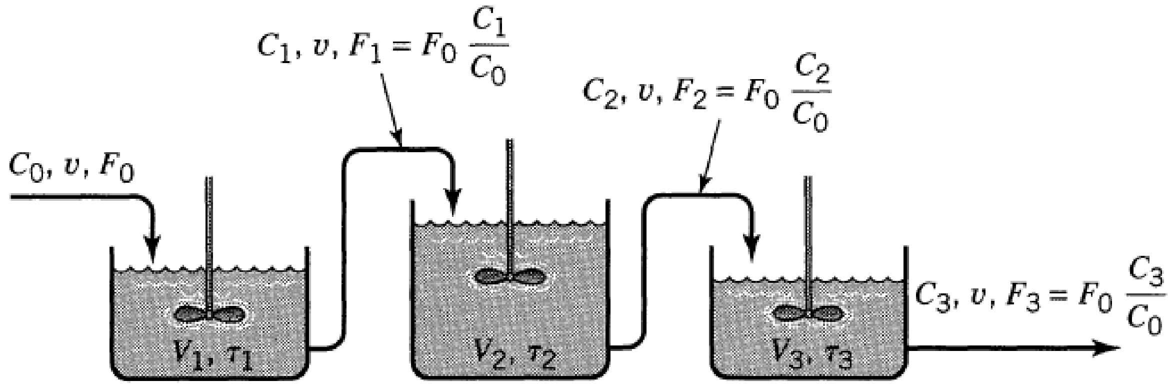


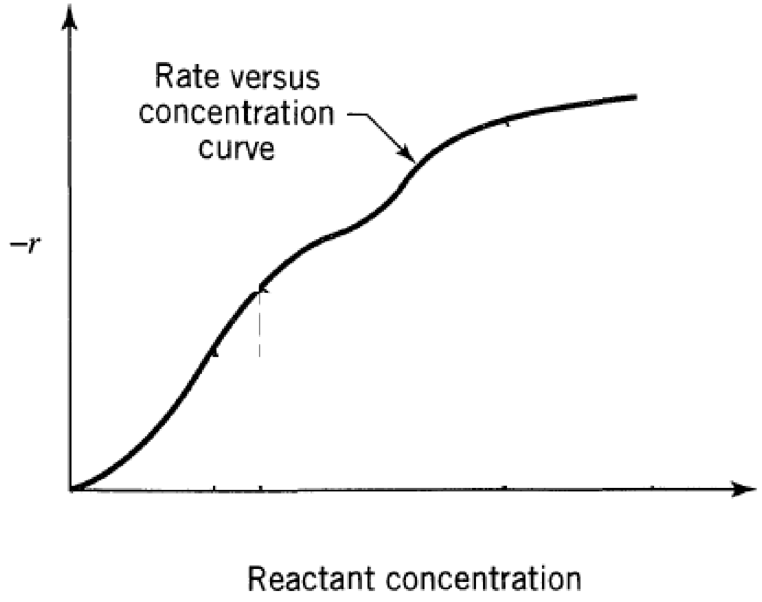
Finding the conversion in a given system

- Finding the outlet composition from a series of mixed flow reactors of various sizes for reactions with negligible density change

A graphical procedure

Need: ***r*** versus C curve for component A to represent the reaction rate at various concentrations





$$\tau_1 = \bar{t}_1 = \frac{V_1}{v} = \frac{C_0 - C_1}{(-r)_1}$$

$$-\frac{1}{\tau_2} = \frac{(-r)_2}{C_2 - C_1}$$

$$-\frac{1}{\tau_1} = \frac{(-r)_1}{C_1 - C_0}$$

For i^{th} reactor $$-\frac{1}{\tau_i} = \frac{(-r)_i}{C_i - C_{i-1}}$$

Cont.

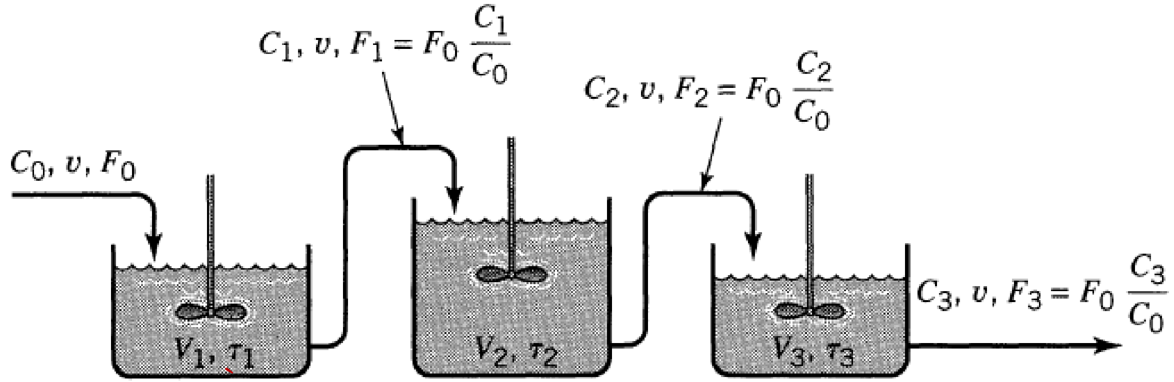
- Plot the C versus r curve for component A

- To find the conditions in the first reactor note that the inlet concentration Co is known (point L), that C_1 and $(-r)_1$ correspond to a point on the curve to be found (point M), and that the slope of the line LM = MN/NL = $(-r)_1/(C_1 - C_0)$ = -(l/T_1) from Equation.

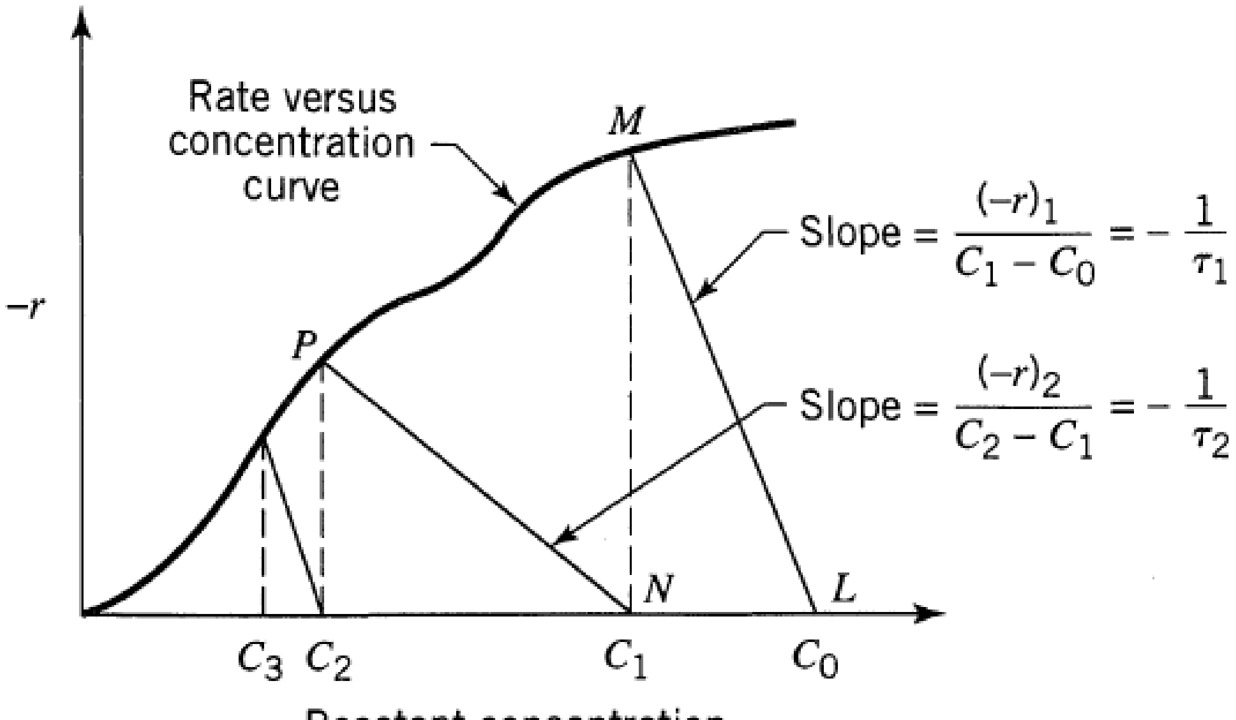
- Hence, from Co draw a line of slope -(l/τ_1) until it cuts the rate curve; this gives C_1.

- Similarly, we find from Equation of 2nd reactor that a line of slope -(1/ τ_2) from point N cuts the curve at P, giving the concentration C_2 of material leaving the second reactor.

- This procedure is then repeated as many times as needed.

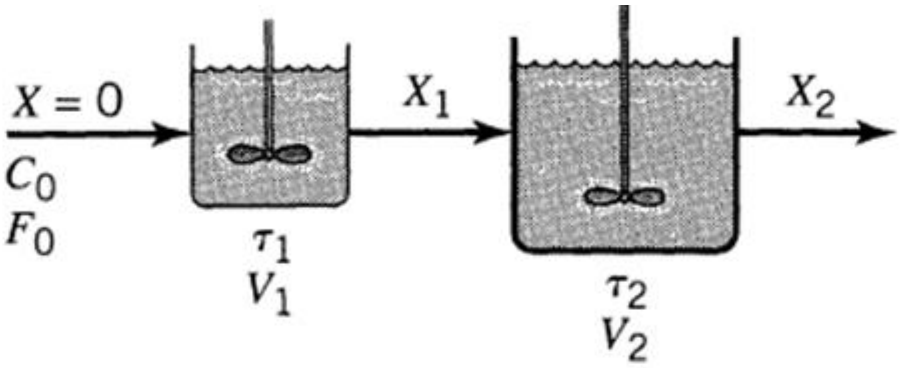


$$-\frac{1}{\tau_1} = \frac{(-r)_1}{C_1 - C_0} \qquad -\frac{1}{\tau_2} = \frac{(-r)_2}{C_2 - C_1}$$



Determining the Best System for a Given Conversion

Suppose we want to **find the minimum size of two mixed flow reactors in series** to achieve a specified conversion of feed which reacts with arbitrary but known kinetics.

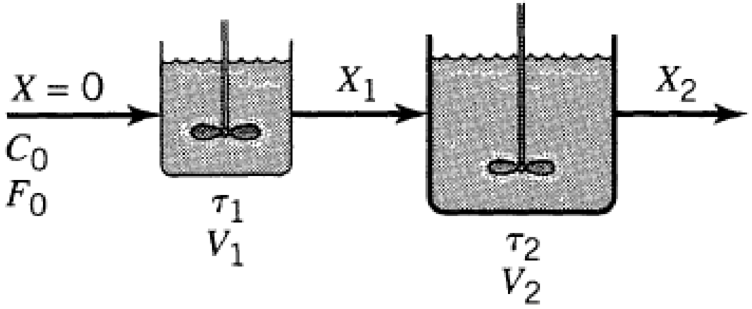


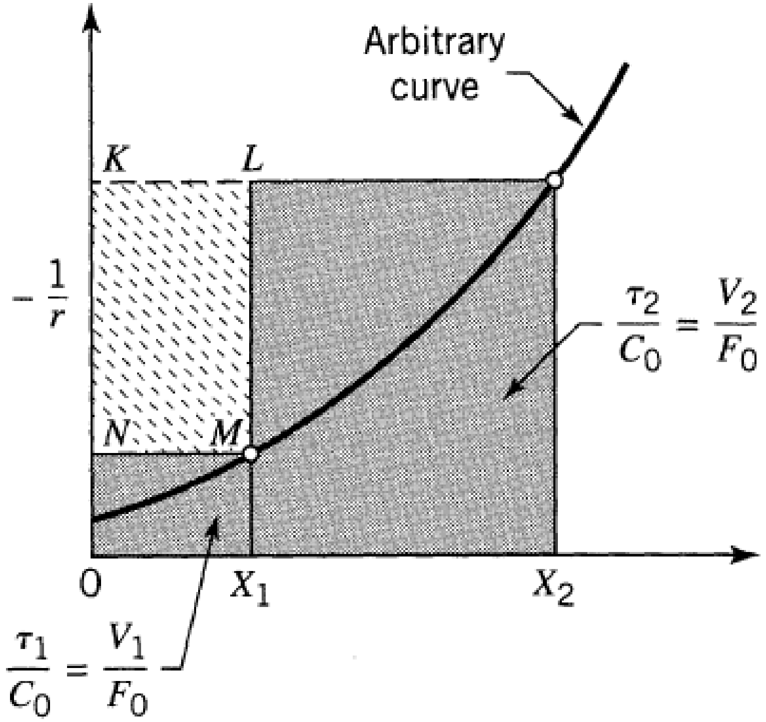
$$\frac{\tau_1}{C_0} = \frac{X_1}{(-r)_1} \qquad \frac{\tau_2}{C_0} = \frac{X_2 - X_1}{(-r)_2}$$

Objective: $V_1 + V_2$ to be minimum for a given conversion X_2

Cont..

$$\frac{\tau_1}{C_0} = \frac{V_1}{F_0} = \frac{X_1}{(-r)_1} \qquad \frac{\tau_2}{C_0} = \frac{V_2}{F_0} = \frac{X_2 - X_1}{(-r)_2}$$



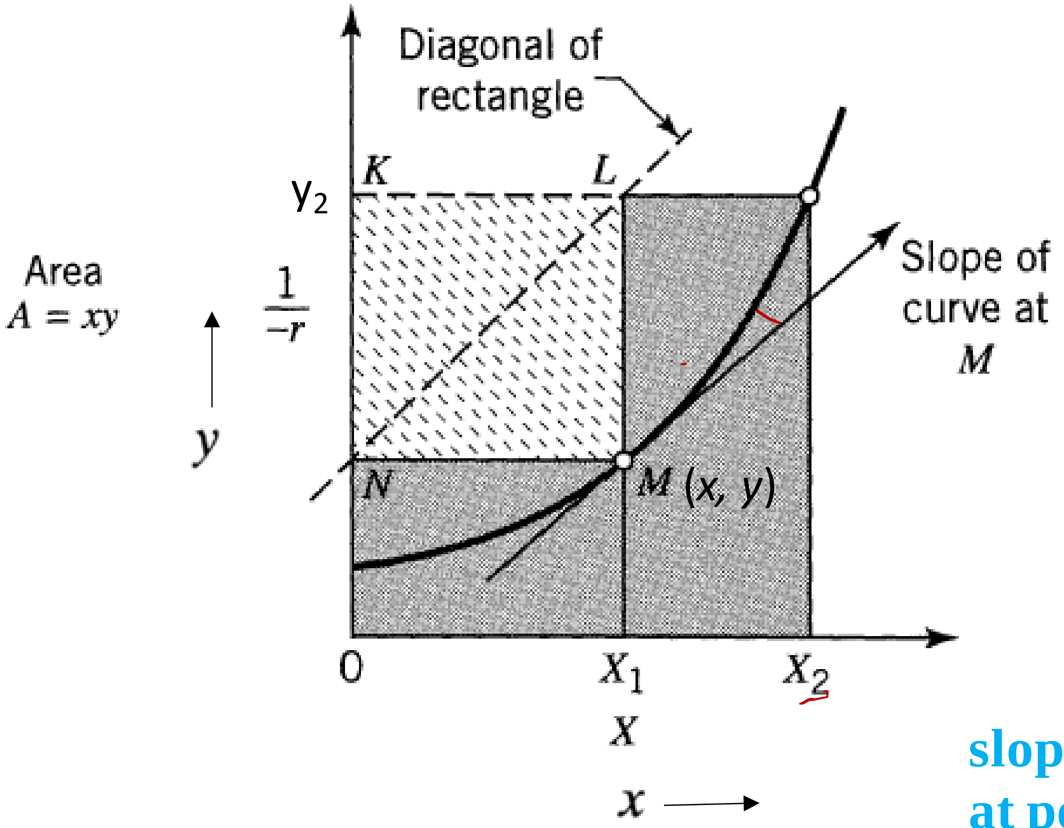


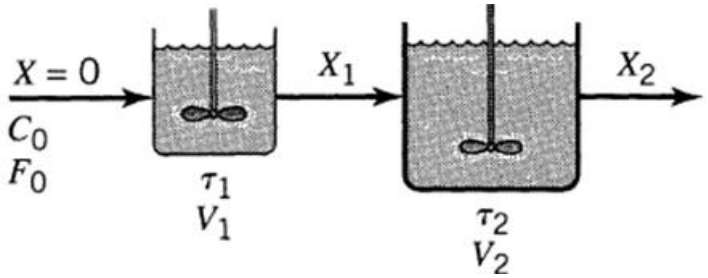
Objective: **minimized** V_1+V_2 for a given conversion X_2

Or **minimized** total shaded area $(V_1/F_0+V_2/F_0)$

Or **maximized** area of rectangle KLMN

Maximized area of KLMN





Area of KLMN, $A=(y_2-y)x$

For maximum A

$dA= (y_2\,dx - ydx - xdy) = 0$

$$\frac{dy}{dx} = \frac{y_2 - y}{x} = \frac{KN}{MN}$$

slope of the curve at point M

slope of the diagonal NL

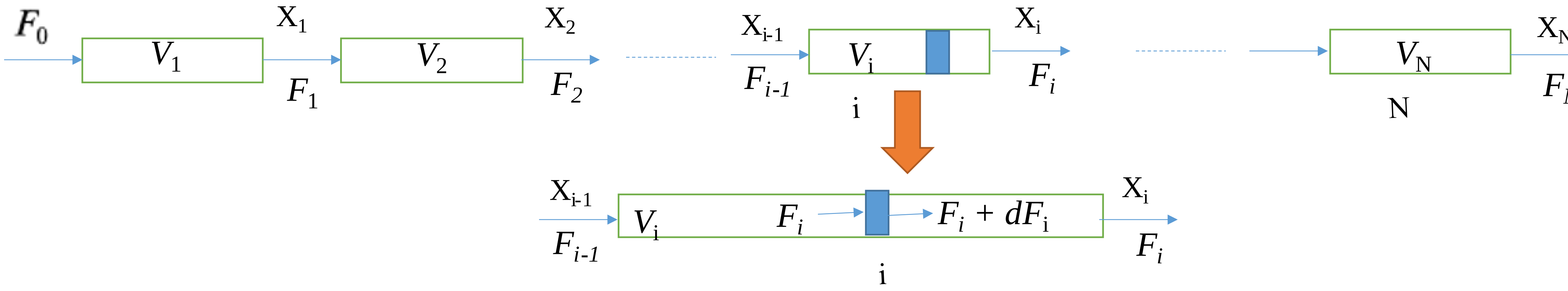
- Condition: Area of KLMN is maximized when M is at that point where the **slope of the curve at point M equals the slope of the diagonal NL of the rectangle**.

- Depending on the shape of the curve, there may be more than one or there may be no "best" point. However, for n^{th}-order kinetics, $n>0$, there always is just one "best" point.

Plug flow reactor in series

- Consider N plug flow reactors connected in series, and let $X_1, X_2,, \ldots ,X_N$, be the fractional conversion of component A leaving reactor 1, 2, . . . , N.

- Basing the material balance on the feed rate of A to the first reactor, we find for the ith reactor



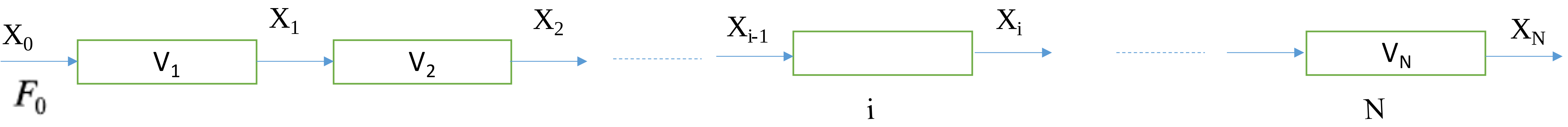
$$F_i = (F_i + dF_i\ ) + (-r)\ dV$$
$$= F_i + d[F_0\ (1-X_i)] + (-r)dV$$

$$\boxed{F_0\ dX_i = (-r)dV}$$

$$\boxed{\frac{V_i}{F_0} = \int_{X_{i-1}}^{X_i} \frac{dX}{-r}}$$

NOT F_{i-1}

Cont..



or for the N reactors in series

$$\frac{V}{F_0} = \sum_{i=1}^{N} \frac{V_i}{F_0} = \frac{V_1 + V_2 + \cdots + V_N}{F_0}$$

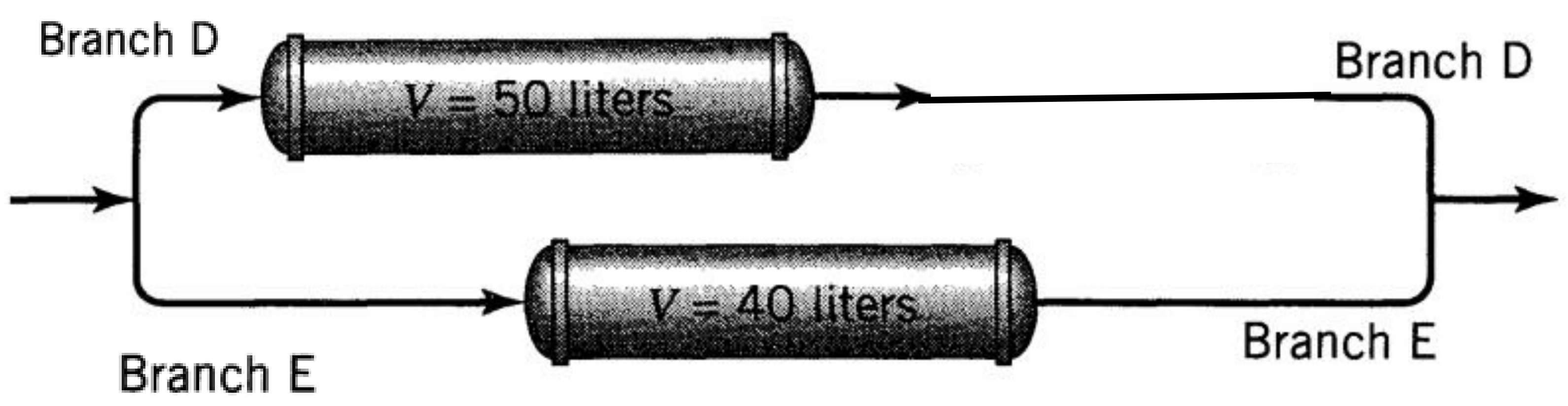
$$= \int_{X_0=0}^{X_1} \frac{dX}{-r} + \int_{X_1}^{X_2} \frac{dX}{-r} + \cdots + \int_{X_{N-1}}^{X_N} \frac{dX}{-r} = \int_0^{X_N} \frac{dX}{-r}$$

$$\boxed{\frac{V_i}{F_0} = \int_{X_{i-1}}^{X_i} \frac{dX}{-r}}$$

Hence, N plug flow reactors in series with a total volume V gives the same conversion as a single plug flow reactor of volume V.

- For the optimum hook up of plug flow reactors connected in series combination, we can treat the whole system as a single plug flow reactor of volume equal to the total volume of the individual units

PFR in parallel



•For the optimum hook up of plug flow reactors connected in parallel , we can treat the whole system as a single plug flow reactor of volume equal to the total volume of the individual units if the feed is distributed in such a manner that fluid streams that meet have the same composition.

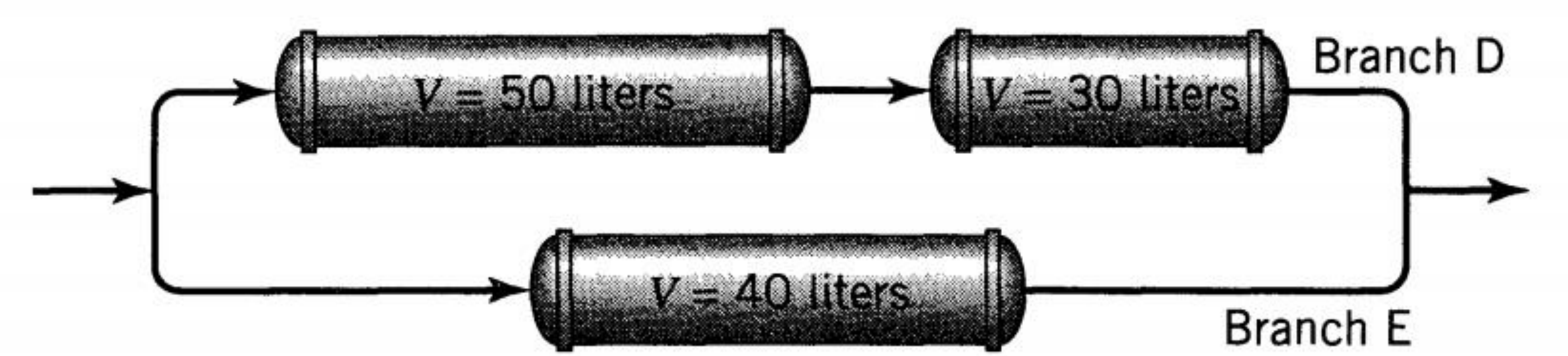
•Thus, for reactors in parallel V/F or τ must be the same for each parallel line.

•Any other way of feeding is less efficient

PFR in series-parallel

The reactor setup shown in Fig. consists of three plug flow reactors in two parallel branches. Branch D has a reactor of volume 50 liters followed by a reactor of volume 30 liters. Branch E has a reactor of volume 40 liters.

What fraction of the feed should go to branch D?



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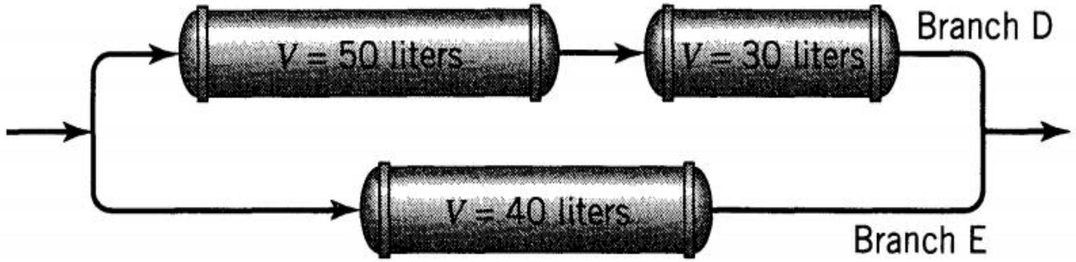
Branch D consists of two reactors in series; hence, it may be considered to be a single reactor of volume



$$V_D = 50 + 30 = 80 \text{ liters}$$

Now for reactors in parallel V/F must be identical if the conversion is to be the same in each branch. Therefore,

$$\left(\frac{V}{F}\right)_D = \left(\frac{V}{F}\right)_E$$

or

$$\frac{F_D}{F_E} = \frac{V_D}{V_E} = \frac{80}{40} = \underline{\underline{2}}$$

Therefore, two-thirds of the feed must be fed to branch D.

Determining the Best System for a Given Conversion

For the most effective use of a given set of ideal reactors we have the following general rules:

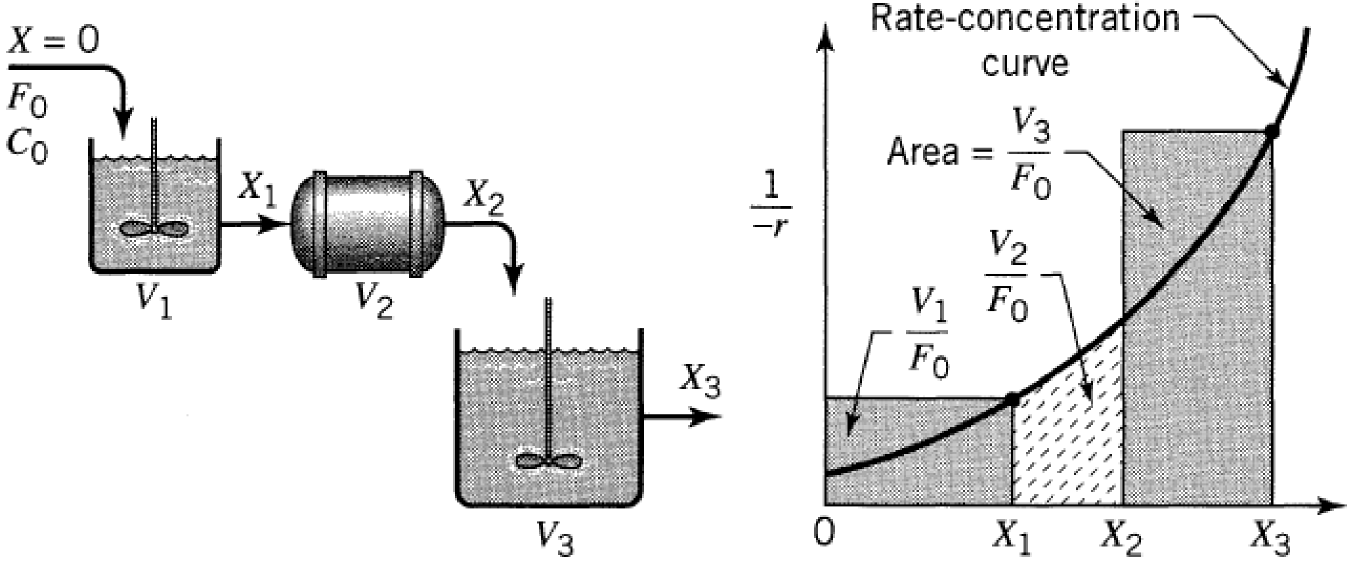
1. For a reaction whose rate-concentration curve rises monotonically (any nth-order reaction, $n>0$) the reactors should be connected in series.

   - They should be ordered so as to keep the concentration of reactant as high as possible if the rate-concentration curve is concave ($n>1$), and as low as possible if the curve is convex ($n<1$).
   - The ordering of units should be PFR, small CSTR, large CSTR, for $n>1$
   - the reverse order should be used when $n<1$

2. For reactions where the rate-concentration curve passes through a maximum or minimum the arrangement of units depends on the actual shape of curve, the conversion level desired, and the units available. No simple rules can be suggested.

Reactors of different types are put in series

If reactors of different types are put in series, such as a mixed flow reactor followed by a plug flow reactor which in turn is followed by another mixed flow reactor, we may write for the three reactors



$$\frac{V_1}{F_0} = \frac{X_1 - X_0}{(-r)_1}, \quad \frac{V_2}{F_0} = \int_{X_1}^{X_2} \frac{dX}{-r}, \quad \frac{V_3}{F_0} = \frac{X_3 - X_2}{(-r)_3}$$

- This allows us to predict the overall conversions for such systems, or conversions at intermediate points between the individual reactors.