CONDUCTION…..

Fourier's Law of Heat Conduction

In general,

$$\textbf{Rate of a transfer process} = \frac{\textbf{Driving force}}{\textbf{Resistance}}$$

- The transfer of heat by conduction also follows this basic equation and is written as *Fourier's law* for heat conduction in fluids or solids.

$$q_x = -kA\frac{dT}{dx}$$

- where q_x is the heat-transfer rate in the x direction in watts (W),
- A is the cross sectional area normal to the direction of flow of heat in m^2,
- T is temperature in K, and
- k is the thermal conductivity in W/m K
- q_x/A is called as heat flux in W/m^2.
- The quantity dT/dx is the temperature gradient in the x direction.
- The minus sign is required because if the heat flow is positive in the given direction, the temperature decreases in this direction.

Steady state Conduction thro a flat slab or wall

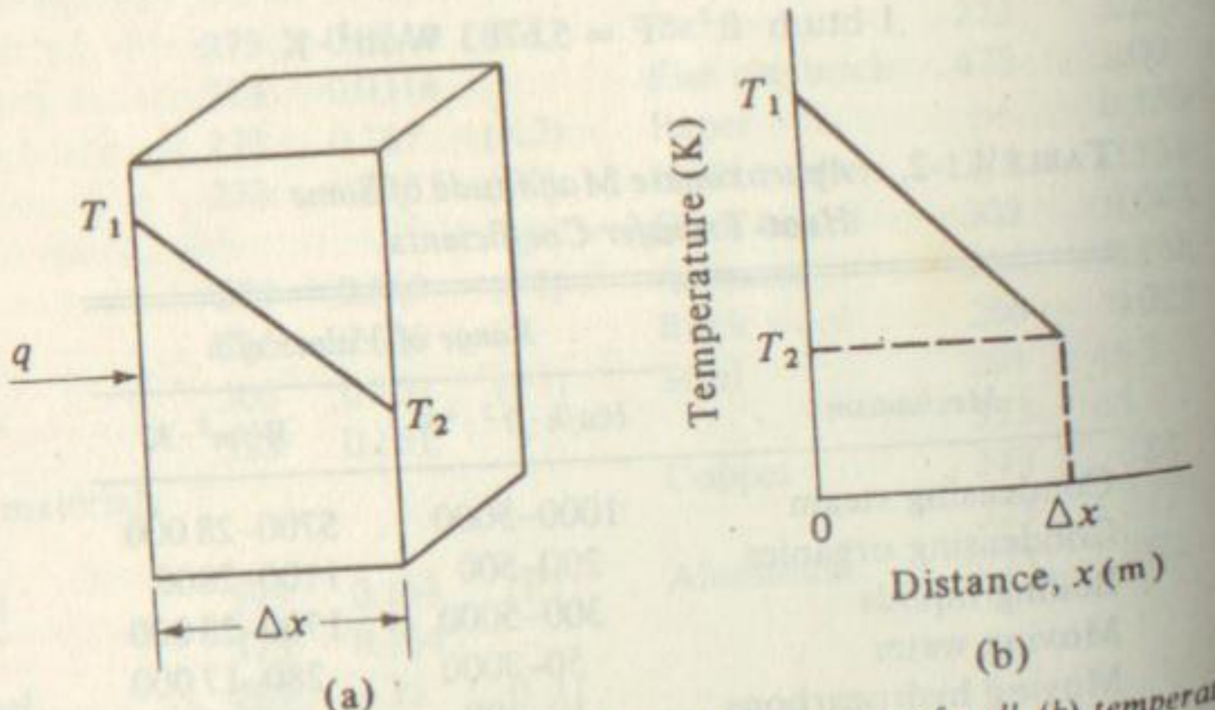


FIGURE 4.2-1. *Heat conduction in a flat wall: (a) geometry of wall, (b) temperature plot.*

Steady state Conduction thro a flat slab or wall

- For a flat slab or wall where the cross-sectional area A and k are constant, We can integrate the Fourier's eqn from x_1 to x_2 and T_1 to T_2

$$q_x = -kA\frac{dT}{dx}$$

$$\frac{q_x}{A}\int_{x_1}^{x_2} dx = -k\int_{T_1}^{T_2} dT$$

For a flat slab….

$$\boxed{\Rightarrow \frac{q}{A} = \frac{k}{(x_2 - x_1)}(T_1 - T_2)}$$

- If thermal conductivity is not a constant but varies linearly varies with temperature as given by k = a+bT (a,b are constants)

$$\Rightarrow \frac{q}{A} = \frac{a + b\frac{(T_1 + T_2)}{2}}{(x_2 - x_1)}(T_1 - T_2)$$

$$take\, k_m = a + b\frac{(T_1 + T_2)}{2}$$

$$\boxed{\therefore \frac{q}{A} = \frac{k_m}{(x_2 - x_1)}(T_1 - T_2)}$$

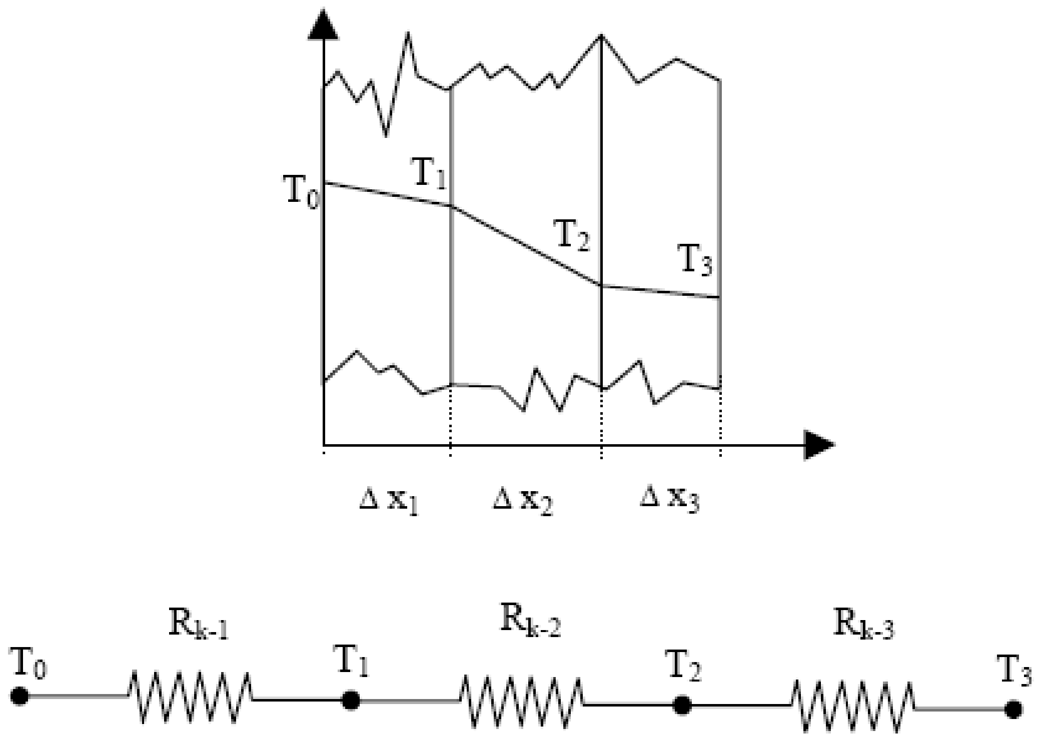
Prob 1

1. A rectangular slab, 2 cm thick, is measured to be 100 ^{o}C on one side and 96.2 ^{o}C on the other side. The slab is 20 cm by 20 cm. Calculate the rate of heat transfer through the slab if the conductivity of the slab is 170 W/m K.

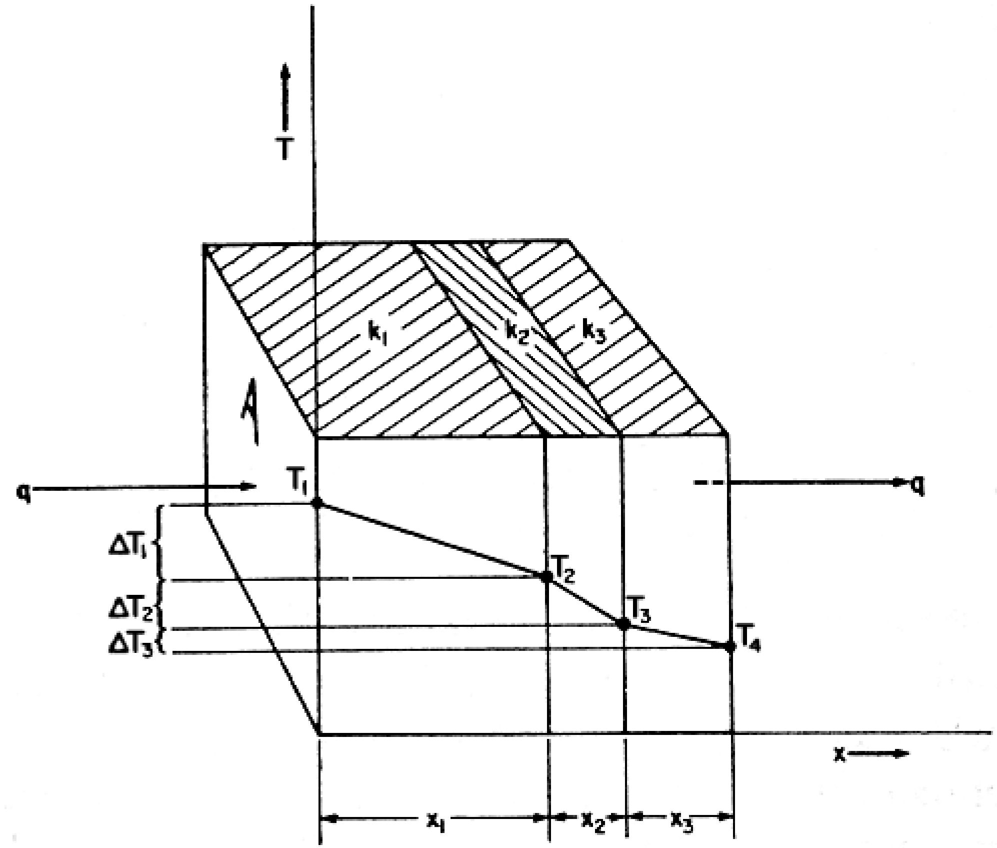
Ans: q = 1292 W

COMPOSITE WALL (material in series)

- In the case where there is a multilayer wall of more than one material present as shown in Fig.
- The temperature profiles in the three materials 1, 2, and 3 are shown.



Composite wall with material in series.



$$q = \frac{A\,\Delta T_1 k_1}{x_1} = \frac{A\,\Delta T_2 k_2}{x_2} = \frac{A\,\Delta T_3 k_3}{x_3}$$

- By Fourier's law of conduction,

Heat transfer rate through material 1

$$q_1 = \frac{k_1 \cdot A}{\Delta x_1} \cdot (T_0 - T_1) = \frac{(T_0 - T_1)}{R_{k-1}}$$

Heat transfer rate through material 2

$$q_2 = \frac{k_2 \cdot A}{\Delta x_2} \cdot (T_1 - T_2) = \frac{(T_1 - T_2)}{R_{k-2}}$$

Heat transfer rate through material 3

$$q_3 = \frac{k_3 \cdot A}{\Delta x_3} \cdot (T_2 - T_3) = \frac{(T_2 - T_3)}{R_{k-3}}$$

As the system is steady-state and no internal heat generated, the heat flows enter and exit each layer are equal. Therefore:

$$q_1 = q_2 = q_3 = q_x$$

- Rearranging and adding the above eqns…
- We get,

$$\boxed{q_x = \frac{T_0 - T_3}{R_{k-1} + R_{k-2} + R_{k-3}} = \frac{T_0 - T_3}{\Sigma R_n}}$$

Conduction Through a Hollow Cylinder

- In many instances in the process industries, heat is being transferred through the walls of a thick-walled cylinder as in a pipe that may or may not be insulated
- Consider the hollow cylinder in Fig. with an inside radius of r_1, where the temperature is T_1, an outside radius of r_2 having a temperature of T_2, and a length of L m.

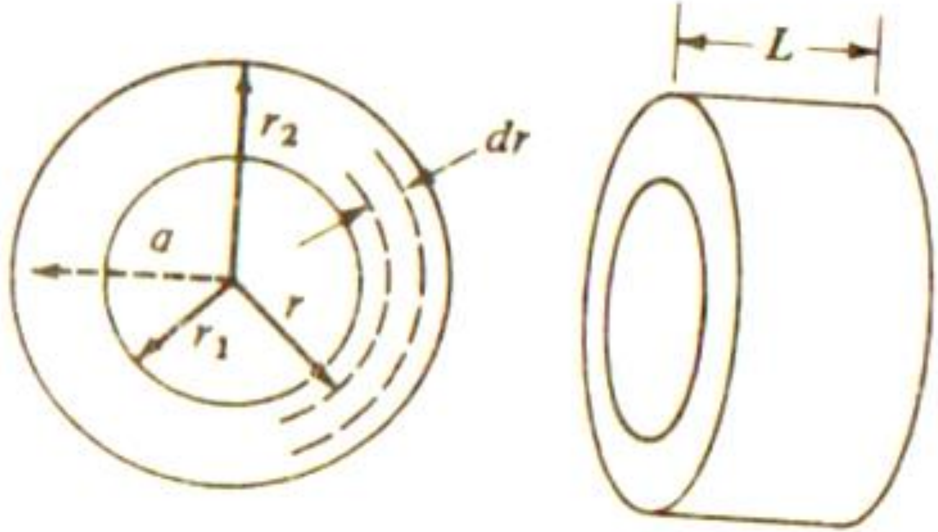


FIGURE 4.2-2. *Heat conduction in a cylinder.*

- Heat is flowing radially from the inside surface to the outside.
- Rewriting Fourier's law, with distance dr instead of dx,

$$\frac{q}{A} = -k\,\frac{dT}{dr}$$

- The CSA normal to the heat flow is $A = 2\pi rL$……sub. and integrating…..

$$\boxed{q = k\frac{2\pi L}{\ln(r_2/r_1)}(T_1 - T_2)}$$

$$multiplying\ num., and\ den., by\ (r_2 - r_1)$$

$$q = kA_{lm}\frac{(T_1 - T_2)}{r_2 - r_1}$$

$$\boxed{\Rightarrow q = \frac{(T_1 - T_2)}{(r_2 - r_1)/(kA_{lm})} = \frac{(T_1 - T_2)}{R}}$$

$$where,\ A_{lm} = \frac{(2\pi Lr_2) - (2\pi Lr_1)}{\ln(2\pi Lr_2/2\pi Lr_1)} = \frac{A_2 - A_1}{\ln(A_2/A_1)}$$

$$and\ R = \frac{(r_2 - r_1)}{kA_{lm}}$$

Conduction Through a Hollow Sphere

- The CSA normal to the heat flow is A $=4\pi r^2$……sub. and integrating…..

$$\boxed{q = \frac{4\pi k}{(1/r_1 - 1/r_2)}(T_1 - T_2)}$$

Convective-Heat-Transfer Coefficient

- It is well known that a hot piece of material will cool faster when air is blown or forced by the object.

- When the fluid outside the solid surface is in forced or natural convective motion, we express the rate of heat transfer from the solid to the fluid, or vice versa, by the following equation

$$\boxed{q = hA(T_w - T_f)}$$

- where q is the heat-transfer rate in W, A is the area in m^2, T_w is the temperature of the solid surface in K.

- T_f is the average or bulk temperature of the fluid flowing by in K, and *‘h’ is the convective heat-transfer coefficient in $W/m^2\,K$.*

- The coefficient ‘h’ is a function of the system geometry, fluid properties, flow velocity, and temperature difference.

- In many cases, empirical correlations are available to predict this coefficient, since it often cannot be predicted theoretically.

- Since we know that when a fluid flows by a surface there is a thin, almost stationary layer or film of fluid adjacent to the wall which presents most of the resistance to heat transfer, *we often call the coefficient ‘h’ a film coefficient.*

- 1 btu/h.ft^2.°F = 5.6783 W/m^{2}K