Tutorial-03

Q1. A hiker caught in a thunderstorm loses heat when her clothing becomes wet. She is packing emergency rations that, if completely metabolized, will release 30 kJ of heat per gram of rations consumed. How much rations must the hiker consume to avoid a reduction in body temperature of 4.0 K as a result of heat loss? Assume the heat capacity of the body equals that of water. Assume the hiker weighs 55 kg. (heat capacity of water at constant pressure C_P is 75.3 $JK^{-1}mol^{-1}$)

We start by calculating the heat that corresponds to a temperature decrease of 4 K. Using $q = C_p \Delta T$, we obtain:

$$q_{4K} = C_p \Delta T = (75.3\ \mathrm{J\,K^{-1}\,mol^{-1}}) \times (4.0\ \mathrm{K}) \times \frac{1}{(18.02 \times 10^{-3}\ \mathrm{kg\,mol^{-1}})} = 17000\ \mathrm{J\,kg^{-1}}$$

We then determine the heat lost for a 55 kg person as:

$$q_{person} = q_{4K}\, m = (17000\ \mathrm{J\,kg^{-1}}) \times (55\ \mathrm{kg}) = 9.2 \times 10^5\ \mathrm{J}$$

And finally the mass of rations that needs to be consumed is given by:

$$m_{rations} = \frac{q_{person}}{q_{rations}} = \frac{(9.2 \times 10^5\ \mathrm{J})}{(3.0 \times 10^4\ \mathrm{J\,g^{-1}})} = \underline{31.0\ \mathrm{g}}$$

Q3. Glucose and fructose are simple sugars with the molecular formula $C_6H_{12}O_6$. Sucrose, or table sugar, is a complex sugar with molecular formula $C_{12}H_{22}O_{11}$ that consists of a glucose unit covalently bound to a fructose unit (a water molecule is given off as a result of the reaction between glucose and fructose to form sucrose). (a) Calculate the energy released as heat when a typical table sugar cube of mass 1.5 g is burned in air. (b) To what height could you climb on the energy a table sugar cube provides assuming 25 per cent of the energy is available for work? (c) The mass of a typical glucose tablet is 2.5 g. Calculate the energy released as heat when a glucose tablet is burned in air. (d) To what height could YOU climb on the energy a cube provides assuming 25 per cent of the energy is available for work?
(Mol wt of sugar = 342.3 g/mol, Mol wt of glucose = 180 g/mol)
Enthalpy of combustion of sugar = -5645 kJ/mol, glucose= −2808 kJ/mol, Body mass = 65 Kg.)

(a) The magnitude of the energy released as heat is

$$q = -n\Delta_c H^\circ = -\frac{1.5\,\mathrm{g}}{342.3\,\mathrm{g\,mol^{-1}}} \times (-5645\ \mathrm{kJ\,mol^{-1}}) = \boxed{+25\ \mathrm{kJ}}$$

(b) Effective work available is $= 25\ \mathrm{kJ} \times 0.25 = 6.2\ \mathrm{kJ}$

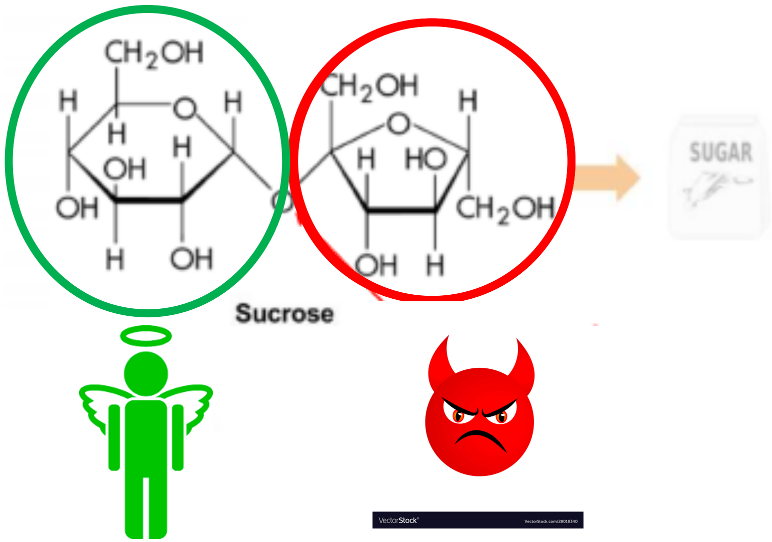
Because $w = mgh$, and $m \approx 65$ kg

$$h = \frac{6.2 \times 10^3\ \mathrm{J}}{65\ \mathrm{kg} \times 9.81\ \mathrm{m\,s^{-2}}} = \boxed{9.7\ \mathrm{m}}$$

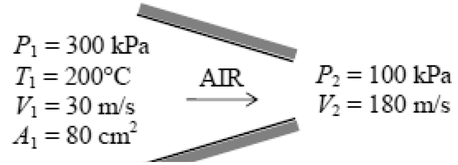
(c) The energy released as heat is

$$q = -\Delta_r H = -n\Delta_c H^\circ = -\left(\frac{2.5\ \mathrm{g}}{180\ \mathrm{g\,mol^{-1}}}\right) \times (-2808\ \mathrm{kJ\,mol^{-1}}) = \boxed{39\ \mathrm{kJ}}$$

(d) If one-quarter of this energy were available as work a 65 kg person could climb to a height h given by



Q4. Air enters an adiabatic nozzle steadily at 300 kPa, 200°C, and 30 m/s and leaves at 100 kPa and 180 m/s. The inlet area of the nozzle is 80 cm_2. Determine (*a*) the mass flow rate through the nozzle, (*b*) the exit temperature of the air, and (*c*) the exit area of the nozzle. (Gas constant for Air= 0.287 $\text{Kpa.m}^3/\text{Kg.K}$, C_p(air) = 1.02 $\text{KJ/Kg.}^0\text{C}$)



Assumptions **1** This is a steady-flow process since there is no change with time. **2** Air is an ideal gas with constant specific heats. **3** Potential energy changes are negligible. **4** The device is adiabatic and thus heat transfer is negligible. **5** There are no work interactions.

(*a*)

$$v_1 = \frac{RT_1}{P_1} = \frac{(0.287\ \text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(473\ \text{K})}{300\ \text{kPa}} = 0.4525\ \text{m}^3/\text{kg}$$

$$\dot{m} = \frac{1}{v_1}A_1V_1 = \frac{1}{0.4525\ \text{m}^3/\text{kg}}(0.008\ \text{m}^2)(30\ \text{m/s}) = \mathbf{0.5304\ kg/s}$$

(*b*)

$$\Delta\dot{U}_{CV} = \dot{q} + \dot{w}_{cv} + \dot{m}\,\Delta\left(\bar{H} + \tfrac{1}{2}v^2 + zg\right) \quad (1^{st}\text{ law open system})$$

$$0 = 0 + 0 + \dot{m}\,\Delta\left(\bar{H} + \tfrac{1}{2}v^2 + 0\right)$$

$$=)\ \mathbf{0} = \Delta\bar{H} + \Delta\left(\tfrac{1}{2}v^2\right)$$

$$=)\ 0 = C_p(\text{air})\,(T_1 - T_2) + \tfrac{1}{2}\left(v_1^2 - v_2^2\right)$$

$$=)\ 0 = C_p(\text{air})\ (T_1 - T_2) + \tfrac{1}{2}\ (v_1^2 - v_2^2)$$

$$=)\ 0 = (1.02\ \text{KJ/Kg.}^0\text{C})\ (200 - T2)^0\text{C} + (1/2)\ [\ (30\ \text{m/s})^2 - (180\ \text{m/s})^2]^*\ \left(\frac{1\ \text{KJ/Kg}}{1000\ \text{m}^2/\text{s}^2}\right)$$

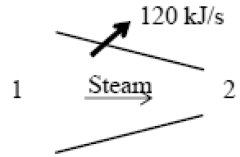
$$=)\ T2 = 184.6^0\text{C}$$

(*c*) The specific volume of air at the nozzle exit is

$$\upsilon_2 = \frac{RT_2}{P_2} = \frac{(0.287\ \text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(184.6+273\ \text{K})}{100\ \text{kPa}} = 1.313\ \text{m}^3/\text{kg}$$

$$\dot{m} = \frac{1}{\upsilon_2}A_2V_2 \longrightarrow 0.5304\ \text{kg/s} = \frac{1}{1.313\ \text{m}^3/\text{kg}}A_2(180\ \text{m/s}) \rightarrow A_2 = 0.00387\ \text{m}^2 = \mathbf{38.7\ cm^2}$$

Q5. Steam at 5 MPa and 400°C enters a nozzle steadily with a velocity of 80 m/s, and it leaves at 2 MPa and 300°C. The inlet area of the nozzle is 50 cm^2, and heat is being lost at a rate of 120 kJ/s. Determine (*a*) the mass flow rate of the steam, (*b*) the exit velocity of the steam, and (*c*) the exit area of the nozzle.



Given:
$v1 = 0.057838\ m^3/kg$
$H1 = 3196.7$ KJ/kg
$v2 = 0.12551\ m^3/kg$
$H2 = 3024.2$ KJ/kg

Assumptions 1 This is a steady-flow process since there is no change with time. 2 Potential energy changes are negligible. 3 There are no work interactions.

(*a*) There is only one inlet and one exit, and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$. The mass flow rate of steam is

$$\dot{m} = \frac{1}{v_1} V_1 A_1 = \frac{1}{0.057838\ \mathrm{m^3/kg}}(80\ \mathrm{m/s})(50 \times 10^{-4}\ \mathrm{m^2}) = \mathbf{6.92\ kg/s}$$

(b) $\Delta \dot{U}_{CV} = \dot{q} + \dot{w}_{cv} + \dot{m}\,\Delta\,(\bar{H} + \frac{1}{2} v^2 + zg)$ (1st law open system)

$$=)\ 0 = \dot{q} + 0 + \dot{m}\,\Delta\,(\bar{H} + \tfrac{1}{2} v^2 + 0)$$
$$=)\ \mathbf{0} = \dot{q} + \dot{m}\,[\,\Delta\bar{H} + \Delta\tfrac{1}{2} v^2\,]$$
$$=)\ 0 = \dot{q} + \dot{m}\,[\,(H1 - H2) + \tfrac{1}{2}(v_1^2 - v_2^2)\,]$$

$$=)\ 0 = \dot{q} + \dot{m}\ [\ (H1 - H2) + ½\ (\ v_1^{2}\ - v_2^{2})\ ]$$

$$=)\ 0 = -120\ KJ/s + (6.92\ kg/s)\ [(3196.7\ KJ/kg - 3024.2\ KJ/kg\ )$$
$$+ ½\ (\ (80\ m/s)2\ - v_2^{2})\ \left(\frac{1\ KJ/Kg}{1000\ m^2/s^2}\right)]$$

$$=)120\ KJ/s =(6.92\ kg/s)[(3196.7\ KJ/kg - 3024.2\ KJ/kg\ )+ ½\ (\ (80\ m/s)2\ - v_2^{2})\ \left(\frac{1\ KJ/Kg}{1000\ m^2/s^2}\right)$$

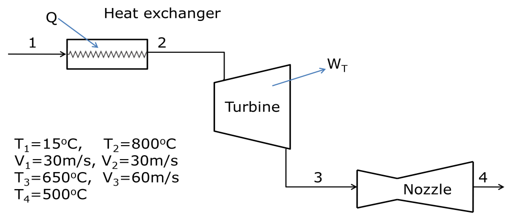
$$-120\ \text{kJ/s} = (6.916\ \text{kg/s})\left(3024.2 - 3196.7 + \frac{V_2^2 - (80\ \text{m/s})^2}{2}\left(\frac{1\ \text{kJ/kg}}{1000\ \text{m}^2/\text{s}^2}\right)\right)$$

It yields $\quad V_2 = \mathbf{562.7\ m/s}$

(*c*) The exit area of the nozzle is determined from

$$\dot{m} = \frac{1}{\upsilon_2} V_2 A_2 \longrightarrow A_2 = \frac{\dot{m}\upsilon_2}{V_2} = \frac{(6.916\ \text{kg/s})(0.12551\ \text{m}^3/\text{kg})}{562.7\ \text{m/s}} = \mathbf{15.42 \times 10^{-4}\ m^2}$$

Q6. Air at a temperature of 15^0C passes through a heat exchanger at a velocity of 30 m/s where its temperature is raised to 800^0C. It then passes through a turbine with the same velocity of 30 m/s and expands until the temperature falls to 650^oC. On leaving the turbine, the air is taken at a velocity of 60 m/s to a nozzle where it expands until its temperature has fallen to 500^0C. If the air flow rate is 2 kg/s, find (a) rate of heat transfer from the heat exchanger (b) the power output from the turbine (c) velocity at nozzle exit assuming no heat loss • Assume cp= 1.005 kJ/kg K



$$\Delta \dot{U}_{CV} = \dot{Q} + \dot{W} + \dot{m}\,(\overline{h_1} - \overline{h_2}\,) + \tfrac{1}{2}\,\dot{m}(V_1^2 - V_2^2) + \dot{m}\,g(z_1 - z_2)$$

- Applying the energy equation across 1-2 (heat exchanger)

$$\dot{Q} + \dot{W} = \dot{m}\left[h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)\right]$$

For a heat exchanger, this reduces to,

$$\dot{Q}_{1-2} = \dot{m}(h_2 - h_1) = \dot{m}\,c_p(T_2 - T_1)$$

$$= 2 \times 1.005 \times (1073.16 - 288.16) = 1580\ \text{kJ/s}$$

- The rate of heat exchanger to the air in the heat exchanger is 1580 kJ/s

- The energy equation the turbine 2-3

$$\dot{W} = \dot{m}\left[h_2 - h_3 + \frac{V_2^2 - V_3^2}{2}\right]$$

$$\dot{W} = 2 \times \left[1.005 \times (1073.16 - 923.16) + \frac{(30^2 - 60^2)}{2}\right]$$

$$= 298.8\ \text{kW}$$

- The power output from the turbine is 298.8 kW

- For the nozzle (3-4)

$$\frac{V_3^2}{2} + h_3 = \frac{V_4^2}{2} + h_4$$

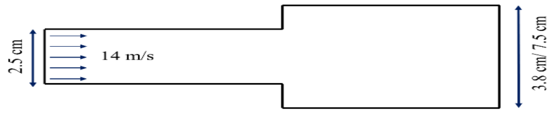
$$\frac{60^2}{2} + 1.005 \times (923.16) = \frac{V_4^2}{2} + 1.005 \times (773.16)$$

$$\therefore V_4 = 554\ m/s$$

- The velocity at the exit from the nozzle is 554 m/s.

Q7

Water at 301.15 K (28°C) flows in a straight horizontal pipe in which there is no exchange of either heat or work with the surroundings. Its velocity is 14 m s^{-1} in a pipe with aninternal diameter of 2.5 cm until it flows into a section where the pipe diameter abruptly increases. What is the temperature change of the water if the downstream diameter is 3.8 cm? If it is 7.5 cm? What is the maximum temperature change for an enlargement in the pipe?



Solution:

Energy Balance equation for Steady-State Flow Process is

$$\Delta H + \frac{\Delta u^2}{2} + g\Delta z = Q + W,$$

As there is no exchange of either heat or work with the surroundings, therefore above equation reduces to

$\Delta H + \frac{\Delta u^2}{2} = 0 - -(1)$, as $\Delta z = 0$ as there is no change in height of the pipe during flow.

We know that $C_p = \frac{\Delta H}{\Delta T}$, therefore $\Delta H = C_p \Delta T$

We know that for an incompressible fluid, therefore the mass flow rate is constant

$$\dot{m} = u_1 A_1 \rho = u_2 A_2 \rho \text{, therefore } u_1 A_1 = u_2 A_2$$

$$u_2 = \frac{u_1 A_1}{A_2} \text{, therefore } u_2 = u_1 (\frac{D_1^2}{D_2^2}) \text{ where } A_1 = \frac{\pi D_1^2}{4} \text{ and } A_2 = \frac{\pi D_2^2}{4}$$

$$\Delta u^2 = u_1^2 (\frac{D_1^4}{D_2^4} - 1)$$

Now equation 1 can be rewritten as

$$2C_p \Delta T + u_1^2 (\frac{D_1^4}{D_2^4} - 1) = 0$$

$$\Delta T = \frac{u_1^2}{2C_p}(1 - \frac{D_1^4}{D_2^4})$$

Given that C_p= 4.18 kJ kg^{-1} $°C^{-1}$, u_1= 14m/s , D_1= 2.15 cm

a) When D_2= 3.8 cm

$$\Delta T = \frac{u_1^2}{2C_p}(1 - \frac{D_1^4}{D_2^4}) = \frac{14^2}{2 \times 4.18}(1 - \frac{2.5^2}{3.8^2}) = 0.0019°C$$

b) When D_2 changes to 7.5 cm, then

$$\Delta T = \frac{u_1^2}{2C_p}(1 - \frac{D_1^4}{D_2^4}) = \frac{14^2}{2 \times 4.18}(1 - \frac{2.5^2}{7.5^2}) = 0.023°C$$

c) When D_2 changes to infinite cm, then change in temperature

$$\Delta T = \frac{u_1^2}{2C_p}(1 - \frac{D_1^4}{D_2^4}) = \frac{14^2}{2 \times 4.18}(1 - \frac{2.5^2}{\infty^2}) = 0.023°C$$

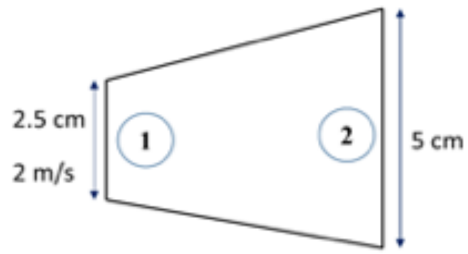
Show this shows changing of exit pipe diameter more than 7.5 cm does not have any further effect on the exit temperature of the water.

Q8

Problem 3 - An incompressible *(ρ= constant)* liquid flows steadily through a conduit of circular cross-section and increasing diameter. At location 1, the diameter is 2.5 cm and the velocity is 2 m s^{-1}, at location 2, the diameter is 5 cm.

(a) What is the velocity at location 2?

(b) What is the kinetic-energy change (J kg) of the fluid between locations 1 and 2?



We know that for an incompressible fluid, ρ= constant, therefore the mass flow rate is the same at location 1 and 2; then,

$$\dot{m} = u_1 A_1 \rho = u_2 A_2 \rho$$

Where $A_1 = \dfrac{\pi D_1^2}{4}$ and $A_2 = \dfrac{\pi D_2^2}{4}$

Then $u_1 D_1^2 = u_2 D_2^2$, therefore $u_2 = u_1 (\dfrac{D_1^2}{D_2^2})$

Given D_1= 2.5cm, u_1= 2m/s and D_2= 5cm

Then, $u_2 = u_1 (\dfrac{D_1^2}{D_2^2}) = 2 \times (\dfrac{2.5^2}{5^2}) = 0.5\,\text{m/s}$

c) The kinetic energy change $\Delta E_{\text{K.E}} = \frac{1}{2}(u_2^2 - u_1^2)$, taking unit mass of fluid.

$$\Delta E_{\text{K.E}} = \frac{1}{2}(0.5^2 - 2^2) = -1.875\,\text{J/kg}$$

Next: 2nd Law

Quiz (BT 202)

Date: 9th Sept 2022 (Friday)
Time: 5 PM – 6 PM
Venue: Lecture Hall 1
Syllabus: Up to Lecture 8 (L7.pdf)
10 Marks

Q2. A sample of the sugar D-ribose ($C_5H_{10}O_5$) of mass 0.727 g was placed in a constant- volume calorimeter and then ignited in the presence of excess oxygen. The temperature rose by 0.910 K. In a separate experiment in the same calorimeter, the combustion of 0.825 g of benzoic acid, for which the internal energy of combustion is -3251 kJ/mol , gave a temperature rise of 1.940 K. Calculate the internal energy of combustion of D- ribose and its enthalpy of formation.

Mol wt of Benzoic acid = 122.12 g/mol

Mol wt of D-ribose = 150.13 g/mol

ΔH ($C+O_2 \rightarrow CO_2$) = -393.51 KJ/mol

ΔH ($H_2+1/2\ O_2 \rightarrow H_2O$) = -285.83 KJ/mol

The calorimeter is a constant-volume instrument as described in the text (Section 2.4), therefore

$$\Delta U = q_V$$

The calorimeter constant is determined from the data for the combustion of benzoic acid

$$\Delta U = \left(\frac{0.825\ \text{g}}{122.12\ \text{g mol}^{-1}}\right) \times (-3251\ \text{kJ mol}^{-1}) = -21.9\bar{6}\ \text{kJ}$$

Since $\Delta T = 1.940$ K, $C = \frac{|q|}{\Delta T} = \frac{21.9\bar{6}\ \text{kJ}}{1.940\ \text{K}} = 11.3\bar{2}\ \text{kJ K}^{-1}$

For D-ribose, $\Delta U = -C\Delta T = -(11.3\bar{2}\ \text{kJ K}^{-1}) \times (0.910\ \text{K})$

Therefore, $\Delta_r U = \frac{\Delta U}{n} = -(11.3\bar{2}\ \text{kJ K}^{-1}) \times (0.910\ \text{K}) \times \left(\frac{150.13\ \text{g mol}^{-1}}{0.727\ \text{g}}\right) = -212\bar{7}\ \text{kJ mol}^{-1}$

The combustion reaction for D-ribose is

$$C_5H_{10}O_5(s) + 5\ O_2(g) \rightarrow 5\ CO_2(g) + 5\ H_2O(l)$$

Since there is no change in the number of moles of gas, $\Delta_r H = \Delta U$ [2.21]

The enthalpy of formation is obtained from the sum

	$\Delta H/(\text{kJ mol}^{-1})$
$5\ CO_2(g) + 5\ H_2O(l) \rightarrow C_5H_{10}O_5(s) + 5\ O_2(g)$	2127
$5\ C(s) + 5\ O_2(g) \rightarrow 5\ CO_2(g)$	$5 \times (-393.51)$
$5\ H_2(g) + \frac{2}{5}\ O_2(g) \rightarrow 5\ H_2O(l)$	$5 \times (-285.83)$
$5\ C(s) + 5\ H_2(g) + \frac{2}{5}\ O_2(g) \rightarrow C_5H_{10}O_5(s)$	-1270

Hence, $\Delta_f H = \boxed{-1270\ \text{kJ mol}^{-1}}$