ENERGY LOSSES IN FLOW

- Energy losses can occur through friction in pipes, bends and fittings, and in equipment
  - Friction in Pipes
  - Energy Losses in Bends and Fittings
  - Pressure Drop through Equipment
  - Calculation of Pressure Drops in Flow Systems

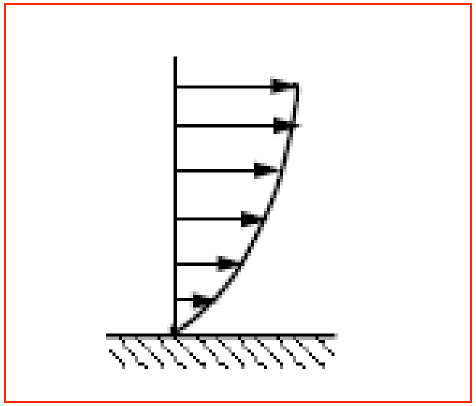
Friction losses in Pipes

Pressure loss due to friction in a pipeline

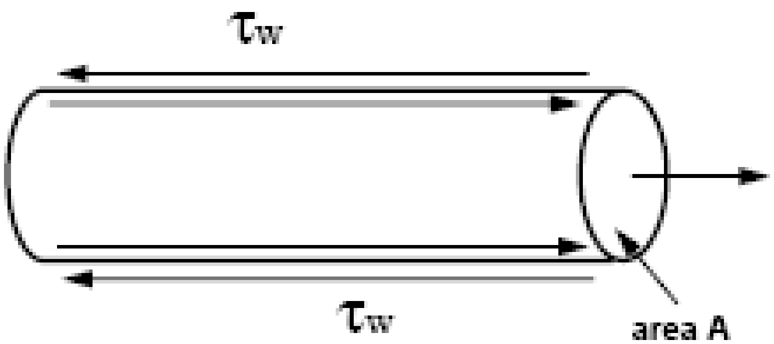
Because fluids are viscous,
energy is lost by flowing fluids due to friction.

In a real flowing fluid shear stress slows the flow.

velocity profile:



Consider a cylindrical element of incompressible fluid flowing in the pipe,



The <u>driving force</u> due to pressure

driving force = Pressure force at 1 - pressure force at 2

$$\boxed{\Delta p\, A = \Delta p \frac{\pi d^2}{4}}$$

The <u>retarding force</u> is due to the shear stress

$$= \text{shear stress} \times \text{area over which it acts}$$

$$= \tau_w \times \text{area of pipe wall}$$

$$= \tau_w \pi d L$$

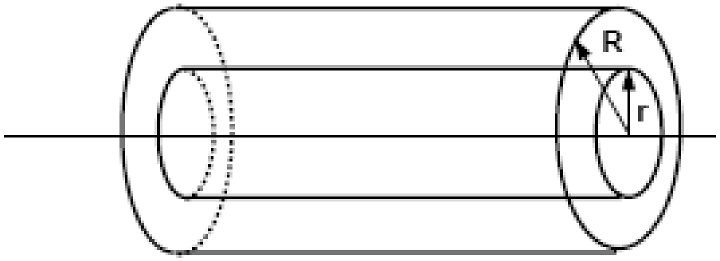
As the flow is in equilibrium,

driving force = retarding force

$$\Delta p \frac{\pi d^2}{4} = \tau_w \pi d L$$

$$\boxed{\Delta p = \frac{\tau_w 4 L}{d}}$$

What is the variation of shear stress in the flow?



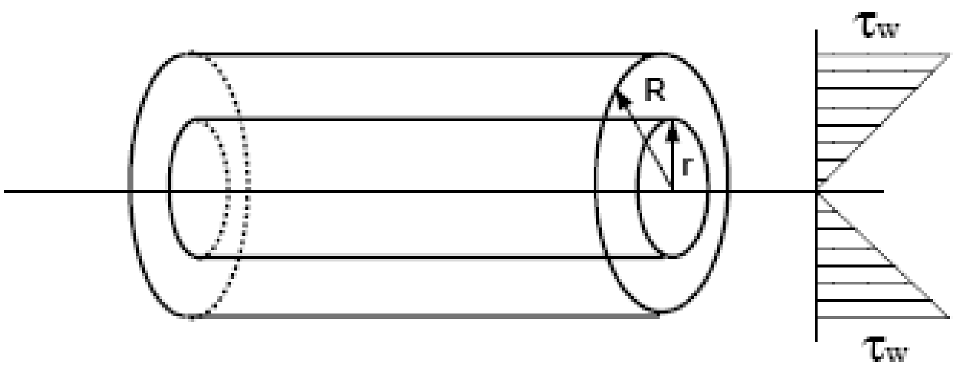
At the wall

$$\tau_w = \frac{R}{2}\frac{\Delta p}{L}$$

At a radius r

$$\tau = \frac{r}{2}\frac{\Delta p}{L}$$

$$\boxed{\tau = \tau_w \frac{r}{R}}$$



Pressure loss during laminar flow in a pipe

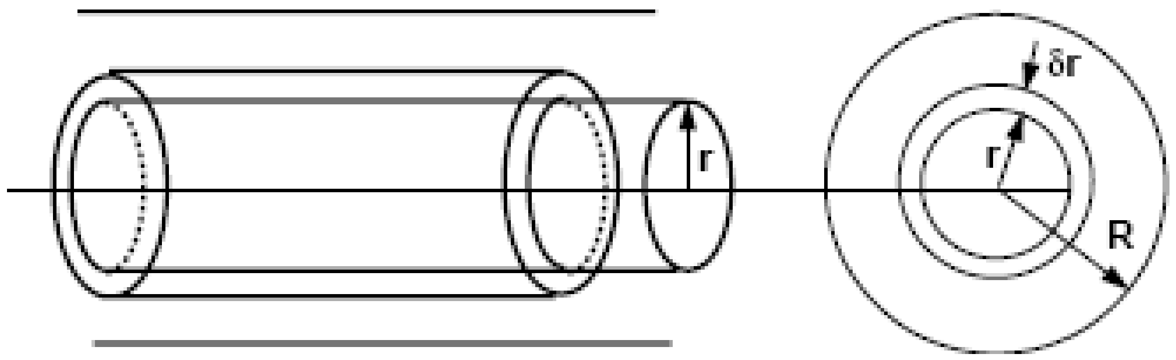
HAGEN-POISEUILLE eqn.

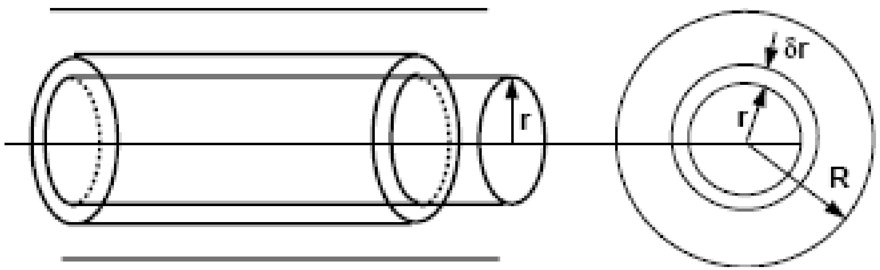
In laminar flow the paths of individual particles of fluid do not cross.

Flow is like a series of concentric cylinders sliding over each other.

And the stress on the fluid in laminar flow is entirely due to viscose forces.

As before, consider a cylinder of fluid, length L, radius r, flowing steadily in the centre of a pipe.





The fluid is in equilibrium,
shearing forces equal the pressure forces.

$$\tau \, 2\pi r \; L = \Delta p \; A = \Delta p \pi r^2$$

$$\tau = \frac{\Delta p}{L} \frac{r}{2}$$

Newtons law of viscosity says $\tau = \mu \dfrac{du}{dy}$,

We are measuring from the pipe centre, so

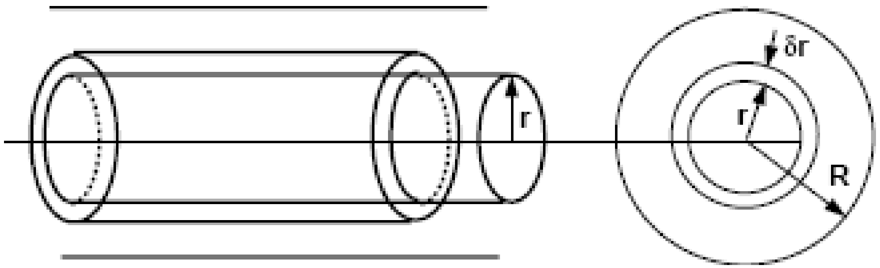
$$\tau = -\mu \frac{du}{dr}$$

$$\frac{\Delta p}{L} \frac{r}{2} = -\mu \frac{du}{dr}$$

$$\frac{du}{dr} = -\frac{\Delta p}{L} \frac{r}{2\mu}$$

In an integral form this gives an expression for velocity,

$$u = -\frac{\Delta p}{L} \frac{1}{2\mu} \int r \, dr$$



The value of velocity at a point distance r from the centre

$$u_r = -\frac{\Delta p}{L}\frac{r^2}{4\mu} + C$$

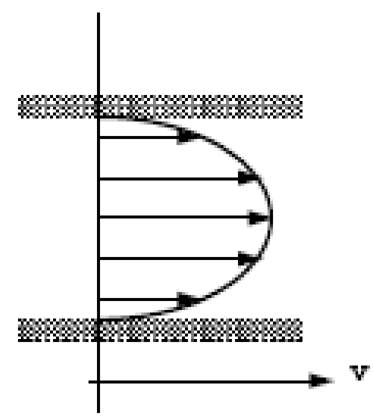
At $r = R$ (the pipe wall) $u = 0$;

$$C = \frac{\Delta p}{L}\frac{R^2}{4\mu}$$

At a point r from the pipe centre when the flow is laminar:

$$u_r = \frac{\Delta p}{L}\frac{1}{4\mu}\left(R^2 - r^2\right)$$

This is a parabolic profile
(of the form $y = ax^2 + b$)
so the velocity profile in the pipe looks similar to



What is the discharge in the pipe?

The flow in an annulus of thickness δr

$$\delta Q = u_r A_{annulus}$$

$$A_{annulus} = \pi(r + \delta r)^2 - \pi r^2 \approx 2\pi r\,\delta r$$

$$\delta Q = \frac{\Delta p}{L}\frac{1}{4\mu}\left(R^2 - r^2\right)2\pi r\,\delta r$$

$$Q = \frac{\Delta p}{L}\frac{\pi}{2\mu}\int_0^R \left(R^2 r - r^3\right) dr$$

$$= \frac{\Delta p}{L}\frac{\pi R^4}{8\mu} = \frac{\Delta p\,\pi d^4}{L128\mu}$$

$$Q = \frac{\Delta p}{L}\frac{\pi d^4}{128\mu}$$

This is the Hagen-Poiseuille Equation for laminar flow in a pipe

$$Average\ velocity = \overline{u} = \frac{Q}{A}$$

$$= \frac{Q}{\pi D^2 / 4}$$

$$\boxed{\overline{u} = \frac{\Delta P D^2}{32 L \mu}}$$

Another form of HAGEN-POISEUILLE Eqn.

Maximum velocity…..

- We know, for laminar flow….. the average velocity is given by, Hagen-Poiseuille's eqn….

$$\boxed{\bar{u} = \frac{\Delta P D^2}{32 L \mu}}$$

- We know the velocity profile, $u_r = \frac{\Delta P}{L}\frac{1}{4\mu}(R^2 - r^2)$
- And @ r = 0…..$u_r = u_{max}$

$$\boxed{\therefore u_{max} = \frac{\Delta P R^2}{4 \mu L}}$$

$$\boxed{\therefore \frac{\bar{u}}{u_{max}} = 0.5}$$

Prob 1

- A small capillary with an ID of $2.22x10^{-3}$ m and a length 0.317m is being used to continuously measure the flow rate of a liquid having a density of 875 kg/m^3 and viscosity $1.3x10^{-3}$ Pa-s. The pressure drop reading across the capillary during flow is 0.0655m of water (density $996kg/m^3$). What is the flow rate?
- u = 0.275m/s
- $Q = 1.066x10^{-6}m^3/s$
- $N_{Re} = 473$