BT209

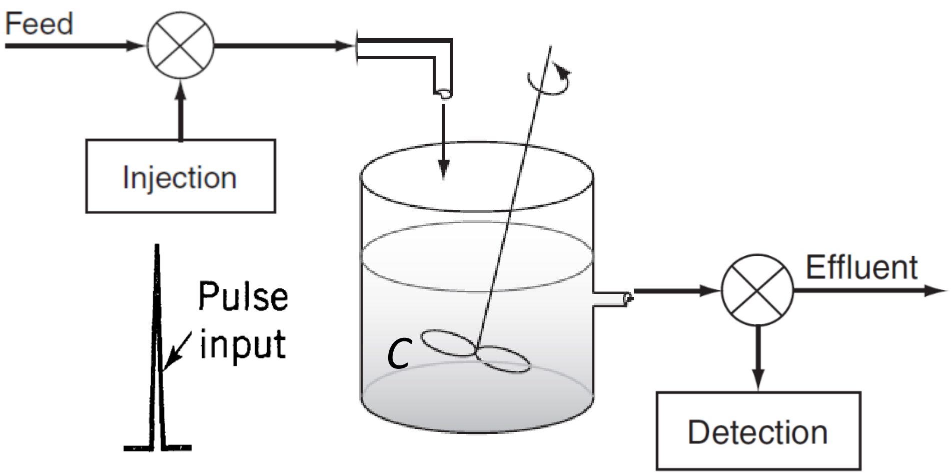
Bioreaction Engineering

26/04/2023

Non ideal flow reactor
(RTD)

RTD of ideal CSTR

- In an ideal CSTR (perfectly mixed) the concentration of any substance in the effluent stream is identical to the concentration throughout the reactor.



A material balance on an **inert tracer** that has been injected **as a pulse** at time ***t*=0 into a CSTR yields for *t* > 0**

$$\text{In} - \text{Out} = \text{Accumulation}$$

$$\overbrace{0} - \overbrace{vC} = \overbrace{V\frac{dC}{dt}}$$

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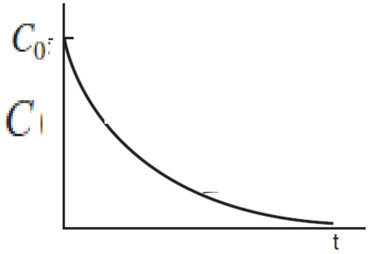
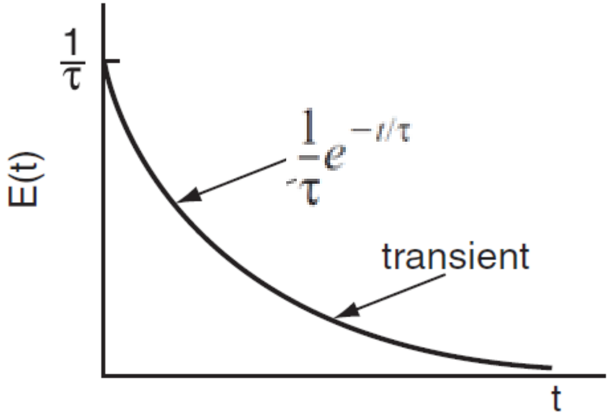


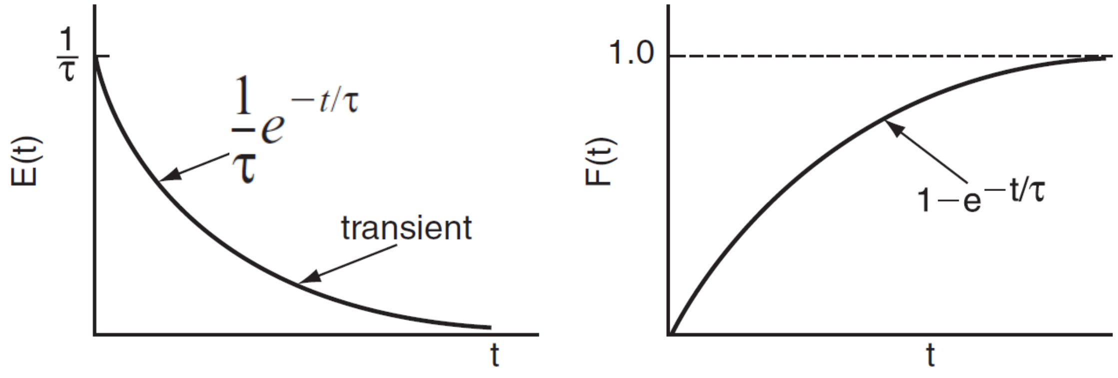
$$C(t) = C_0 e^{-t/\tau}$$

$$\boxed{E(t) = \frac{C(t)}{\int_0^\infty C(t)\,dt} = \frac{C_0 e^{-t/\tau}}{\int_0^\infty C_0 e^{-t/\tau}\,dt} = \frac{e^{-t/\tau}}{\tau}}$$



Cont.

F-curve of ideal CSTR: $$F(t) = \int_0^t E(t)dt = 1 - e^{-t/\tau}$$



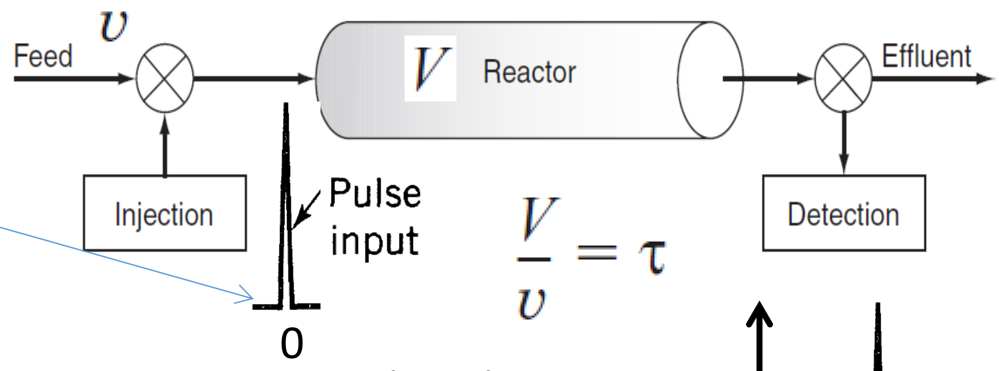
Mean residence time of ideal CSTR: $$t_m = \int_0^\infty tE(t)\,dt = \int_0^\infty \frac{t}{\tau}\,e^{-t/\tau}\,dt = \tau$$

RTD of ideal PFR

- In ideal PFR, all the atoms leaving the reactor have spent precisely the same amount of time within the reactors.

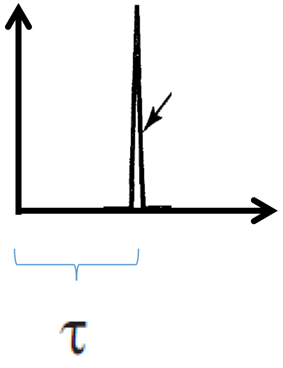
- For pulse input of tracer, the tracer outlet concentration distribution will be same as input pulse (spike of infinite height and zero width) but the spice occur at $t=\tau$

- The RTD function in such a case is a spike of infinite height and zero width, whose area is equal to 1



$$\frac{V}{v} = \tau$$

- ➢ Ideal pulse input (shot)
- ✓ spike of infinite height
- ✓ and zero width,



❑ Mathematically, this spike is represented by the Dirac delta function:

$$E(t) = \delta(t - \tau)$$

spice occur at $t=\tau$

PROPERTY

Area under the curve: $\int_0^\infty \delta(t - t_0)\,dt = 1$

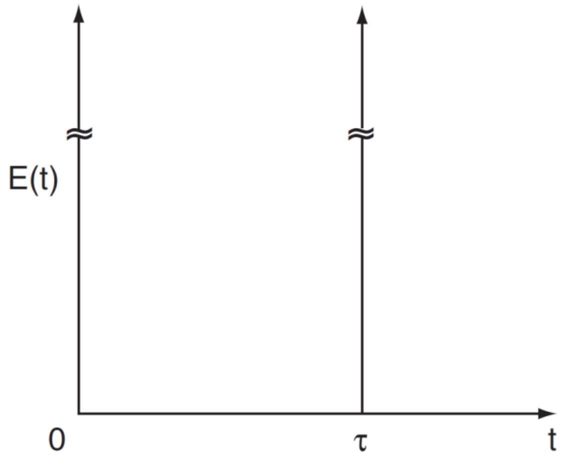
Any integration with a δ function: $\int_0^\infty \delta(t - t_0) f(t)\,dt = f(t_0)$

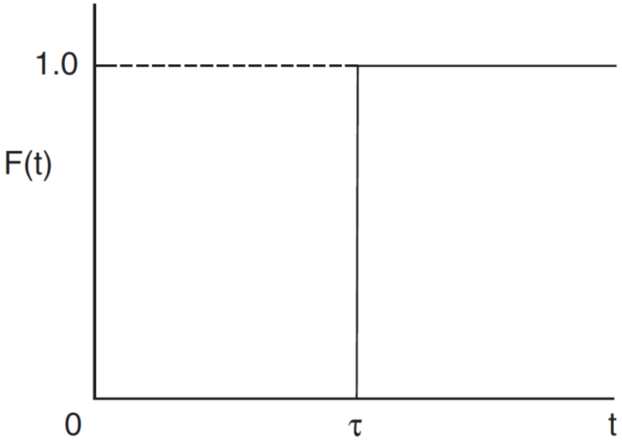
Cont..

Mean residence time of ideal PFR: $$t_m = \int_0^{\infty} tE(t)\,dt = \int_0^{\infty} t\,\delta(t-\tau)\,dt = \tau$$

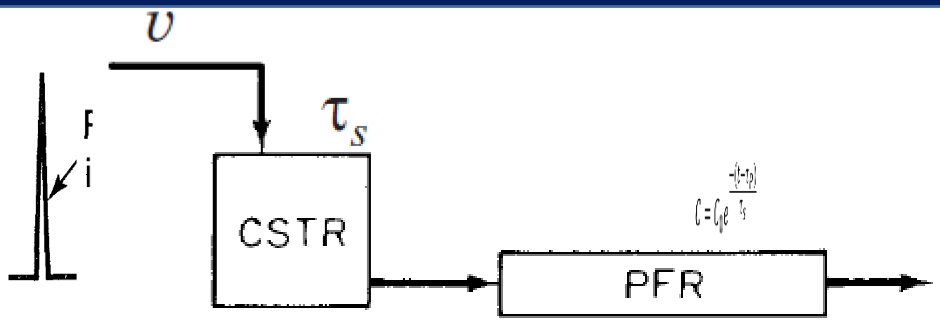
Variance of ideal PFR: $$\sigma^2 = \int_0^{\infty} (t-\tau)^2\,\delta(t-\tau)\,dt = 0$$

F-curve of ideal PFR: $$F(t) = \int_0^{t} E(t)dt = \int_0^{t} \delta(t-\tau)dt$$





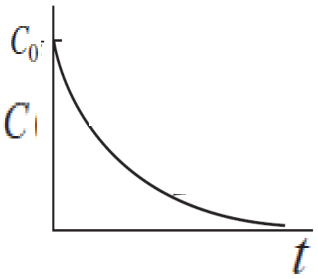
RTD of ideal CSTR followed by ideal PFR



Tracer inpu
at t=0

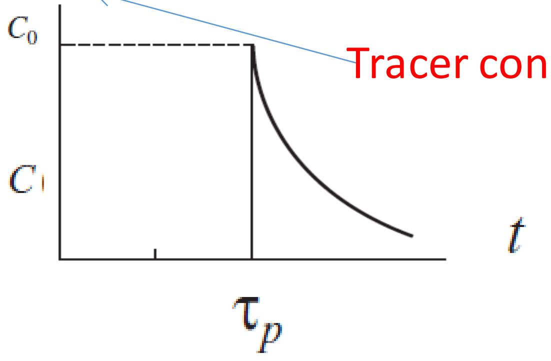
$$C = C_0 e^{-t/\tau_s}$$

Tracer concentration



$$C = C_0 e^{\frac{-(t-\tau_P)}{\tau_s}}$$

Tracer concentration

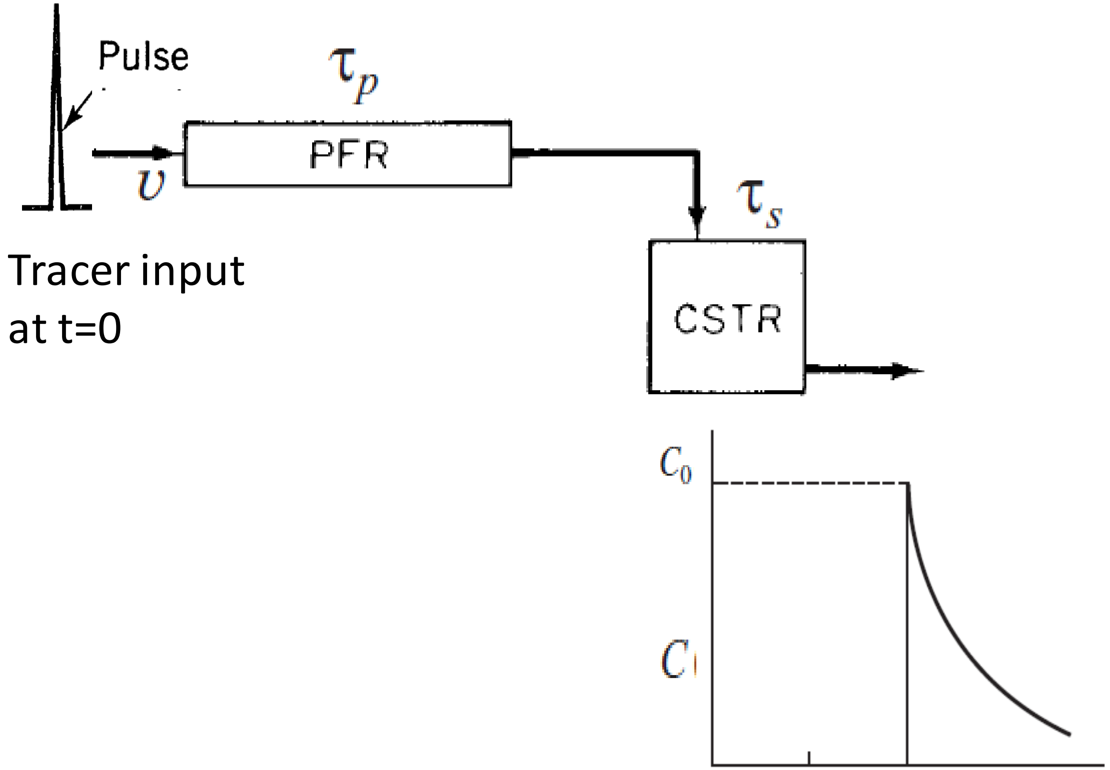


$$E(t) = \frac{C(t)}{\int_0^\infty C(t)dt} = \frac{C(t)}{\int_0^{\tau_P} C(t) + \int_{\tau_P}^\infty C(t)}$$

$$= \frac{C(t)}{0 + \int_{\tau_P}^\infty C(t)dt}$$

$$\boxed{E(t) = \begin{cases} 0 & t < \tau_p \\ \dfrac{e^{-(t-\tau_p)/\tau_s}}{\tau_s} & t \geq \tau_p \end{cases}}$$

RTD of ideal PFR followed by ideal CSTR

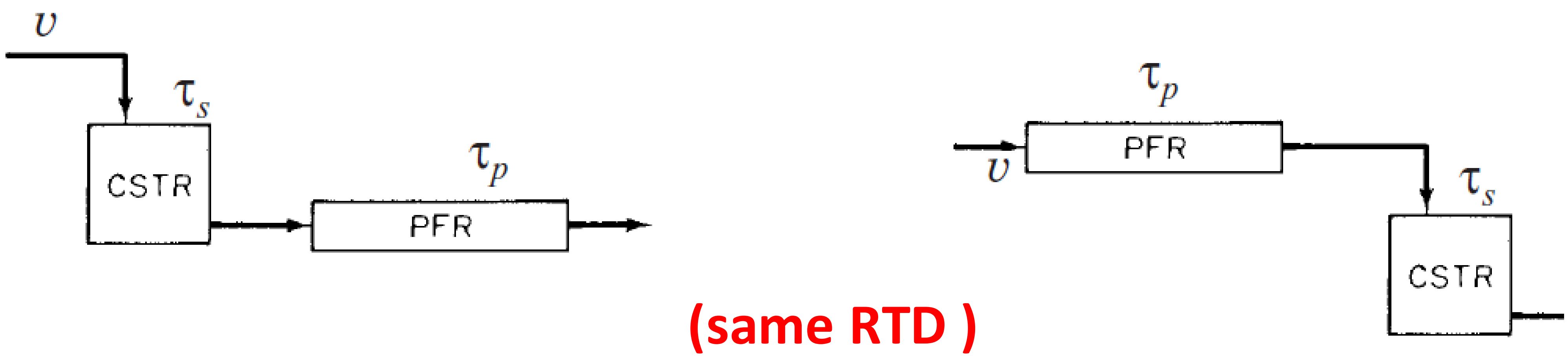


$$E(t) = \begin{cases} 0 & t < \tau_p \\ \dfrac{e^{-(t-\tau_p)/\tau_s}}{\tau_s} & t \geq \tau_p \end{cases}$$

(same RTD as CSTR→PFR)

- ❑ RTD is specific to a reactor system but same RTD does not mean same reactor system

Why mixing model?



(same RTD)

✓ Same RTD, but does not give same **conversion** for a reactant A (CHECK IT, Except 1st order)

- **The RTD is not a complete description of structure for a particular reactor or system of reactors**
- **Mixing is another important parameter (late mixing, early mixing)**

- **Need mixing model to predict conversion or size of reactor apart from RTD and kinetics**