Artificial Bee Colony Algorithm

Prakash Kotecha
Debasis Maharana & Remya Kommadath
Department of Chemical Engineering
Indian Institute of Technology Guwahati

Swarm Intelligence

- "Any attempt to design algorithms or distributed problem-solving devices inspired by the collective behaviour of social insect colonies and other animal societies"*1

- Examples of Swarms
  - bees swarming around their hive
  - ant colony as a swarm with ants as individual agents
  - flock of birds is a swarm of birds
  - immune system is a swarm of cells
  - crowd is a swarm of people

- Properties of swarm intelligent behaviour
  - self-organization
    - interactions are executed on the basis of purely local information without any relation to the global pattern
    - positive feedback, negative feedback, fluctuations and multiple interactions
  - division of labour
    - tasks performed simultaneously by specialized individuals

- Particle Swarm Optimization models the social behaviour of bird flocking or fish schooling

Swarm Intelligence: From Natural to Artificial Systems, New York, NY: Oxford University Press, 1999
An idea based on honey bee swarm for numerical optimization, Technical report-TR06, Erciyes University, Engineering Faculty, Computer Engineering Department 2005

Artificial Bee Colony Algorithm

AN IDEA BASED ON HONEY BEE SWARM FOR NUMERICAL OPTIMIZATION

(TECHNICAL REPORT-TR06, OCTOBER, 2005)

Dervis KARABOGA

karaboga@erciyes.edu.tr

Erciyes University, Engineering Faculty

Computer Engineering Department

Kayseri/Türkiye

https://abc.erciyes.edu.tr/publ.htm

Proposed in 2005

Journal of Global Optimization

November 2007, Volume 39, Issue 3, pp 459–471 | Cite as

A powerful and efficient algorithm for numerical function optimization: artificial bee colony (ABC) algorithm

Authors   Authors and affiliations

Dervis Karaboga, Bahriye Basturk

Original Paper

First Online: 13 April 2007

2007

Contents lists available at ScienceDirect

Information Sciences

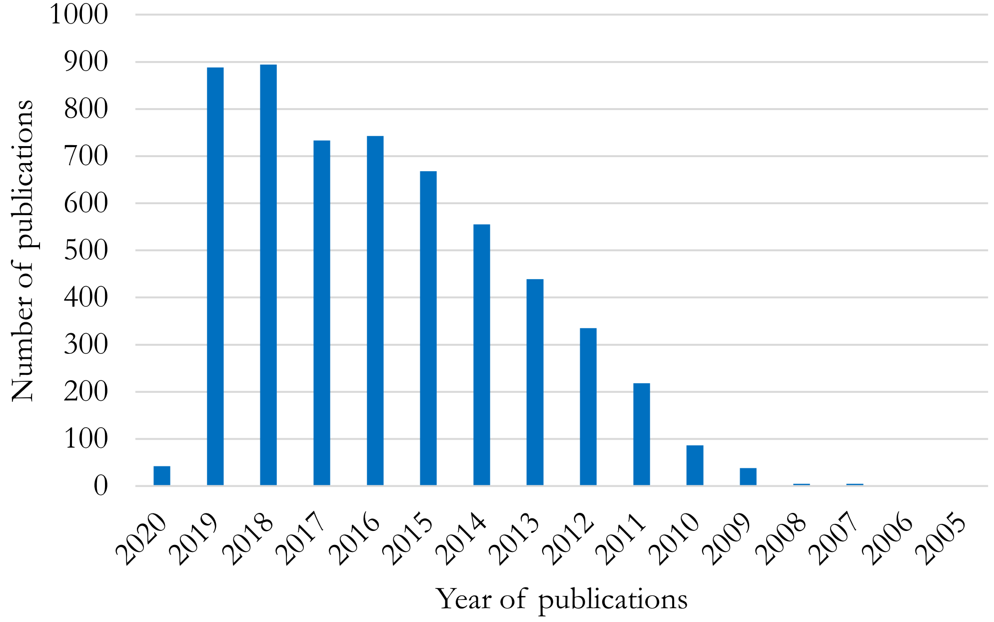
journal homepage: www.elsevier.com/locate/ins

ELSEVIER

On clarifying misconceptions when comparing variants of the Artificial Bee Colony Algorithm by offering a new implementation

Marjan Mernik [a,*], Shih-Hsi Liu [b], Dervis Karaboga [c], Matej Črepinšek [a]

2015



Around 6000 publications (Scopus, Dec 2019)

Artificial Bee Colony Algorithm

AN IDEA BASED ON HONEY BEE SWARM FOR NUMERICAL OPTIMIZATION

(TECHNICAL REPORT-TR06, OCTOBER, 2005)

Dervis KARABOGA

karaboga@erciyes.edu.tr

Erciyes University, Engineering Faculty

Computer Engineering Department

Kayseri/Türkiye

https://abc.erciyes.edu.tr/publ.htm

Proposed in 2005

Journal of Global Optimization

November 2007, Volume 39, Issue 3, pp 459–471 | Cite as

A powerful and efficient algorithm for numerical function optimization: artificial bee colony (ABC) algorithm

Authors Authors and affiliations

Dervis Karaboga, Bahriye Basturk

2007

Contents lists available at ScienceDirect

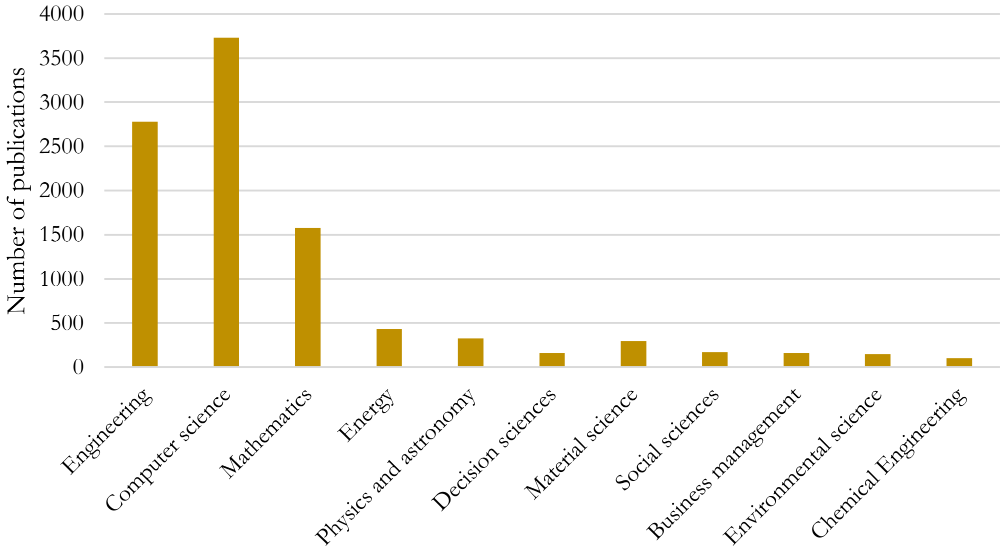
Information Sciences

journal homepage: www.elsevier.com/locate/ins

On clarifying misconceptions when comparing variants of the Artificial Bee Colony Algorithm by offering a new implementation

Marjan Mernik [a,*], Shih-Hsi Liu [b], Dervis Karaboga [c], Matej Črepinšek [a]

2015



Around 6000 publications (Scopus, Dec 2019)

Artificial Bee Colony Algorithm

AN IDEA BASED ON HONEY BEE SWARM FOR NUMERICAL OPTIMIZATION

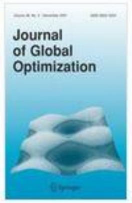
(TECHNICAL REPORT-TR06, OCTOBER, 2005)

Dervis KARABOGA
karaboga@erciyes.edu.tr

Erciyes University, Engineering Faculty
Computer Engineering Department
Kayseri/Türkiye

https://abc.erciyes.edu.tr/publ.htm

Proposed in 2005



Journal of Global Optimization
November 2007, Volume 39, Issue 3, pp 459–471 | Cite as

A powerful and efficient algorithm for numerical function optimization: artificial bee colony (ABC) algorithm

Authors Authors and affiliations

Dervis Karaboga, Bahriye Basturk

Original Paper
First Online: 13 April 2007

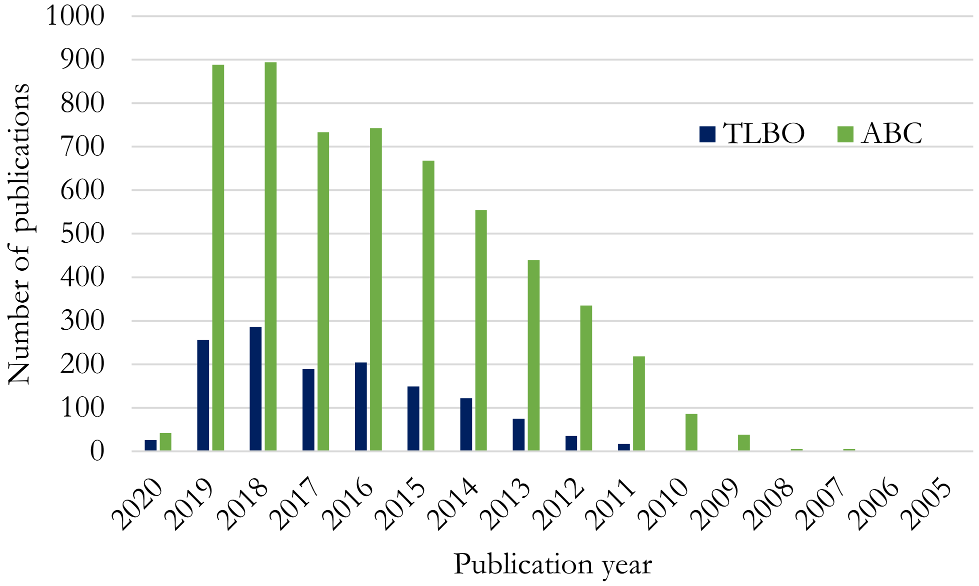
2007

Contents lists available at ScienceDirect
Information Sciences
journal homepage: www.elsevier.com/locate/ins
ELSEVIER

On clarifying misconceptions when comparing variants of the Artificial Bee Colony Algorithm by offering a new implementation

Marjan Mernik [a,*], Shih-Hsi Liu [b], Dervis Karaboga [c], Matej Črepinšek [a]

2015



Around 6000 publications (Scopus, Dec 2019)

Components of Honey Bee Swarms

- **Food Sources:**
  - value depends on its proximity, richness, and the ease of extraction
  - can be represented with a single quantity "profitability"
- **Employed foragers:**
  - currently exploiting a food source
  - contains information on distance, profitability and direction from the nest
  - shares information with a certain probability
  - takes nectar to the hive and unloads
    - abandons food source, becomes an uncommitted follower
    - dances, recruits, returns
    - continues to forage at the food source
- **Unemployed foragers:**
  - **Onlookers:** watch the waggle dances to become a recruit and starts searching for a food source
  - **Scout:** starts searching around the nest spontaneously

An idea based on honey bee swarm for numerical optimization, Technical report-TR06, Erciyes University, Engineering Faculty, Computer Engineering Department 2005

Artificial Bee Colony Algorithm (ABC)

➢Employed bee phase

- Employed bees try to identify better food source than the one associated with it
- Generate a new solution using a partner solution
- *Greedy selection:* Accept new solution if it is better than the current solution

➢Onlooker bee phase

- Select a food source with a probability related to nectar amount
- Generate a new solution using a partner solution
- *Greedy selection:* Accept new solution if it is better than the current solution

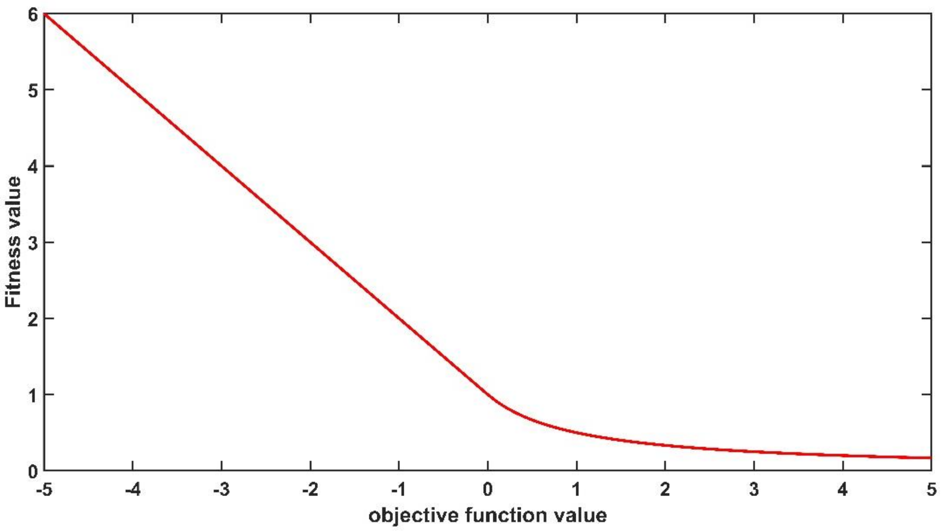
➢Scout bee phase

- Exhausted food source is abandoned
- Discard and generate new solution

Fitness evaluation and greedy selection

➢Fitness of a solution is evaluated as

$$fit = \begin{cases} \dfrac{1}{1+f} & if \;\; f \geq 0 \\ 1+|f| & if \;\; f < 0 \end{cases}$$



➢Greedy selection to update the solution

$$\left.\begin{array}{l} X = X_{new} \\ f = f_{new} \end{array}\right\} if \;\; fit_{new} > fit$$

X and f remains the same if $fit_{new} < fit$

X	Current solution
X_{new}	Newly generated solution
f	Objective function value of a solution
f_{new}	Objective function value of new solution
fit	Fitness of a solution
fit_{new}	Fitness of new solution

Employed bee phase: Generation of new solution

- Number of employed bees is equal to number of food sources
- All solutions get an opportunity to generate a new solution in the employed bee phase
- A partner is randomly selected to generate a new solution
- Partner and the current solution should not be the same
- New solution is generated by modifying a randomly selected variable

$$X_{new}^{j} = X^{j} + \phi\left(X^{j} - X_{p}^{j}\right)$$

- Bound the newly generated solution

$$X_{new}^{j} = lb \quad if \;\; X_{new}^{j} < lb$$
$$X_{new}^{j} = ub \quad if \;\; X_{new}^{j} > ub$$

Example: Let X = [2 1 6 9], X_p = [0 4 7 2] and j=2

Assume $\phi = -0.1$

$$X_{new}^{2} = 1 + (-0.1)(1-4) = 1.3$$
$$X_{new} = [2 \quad 1.3 \quad 6 \quad 9]$$

Employed bee phase: Selection of new solution

- Evaluate the objective function and fitness of newly generated solution
- Perform greedy selection to update current solution
- *trial* counter is used to track the number of failures encountered by each solution
- Increase the *trial* of current solution by one, if the new solution is inferior
- Reset the *trial* to zero if a better solution is generated

Pseudocode of Employed Bee Phase

Input: Objective function, lb, ub, N_p, food source (P), objective function value (f), fitness (fit), trial

for i = 1 to N_p

Randomly select a partner (p) such that $i \neq p$

Randomly select a variable j and **modify j^th^** variable

Bound X_{new}^{j}

Generation $X_{new}^{j} = X^{j} + \phi\left(X^{j} - X_{p}^{j}\right)$

Evaluate the objective function (f_{new}) and fitness (fit_{new})

Accept X_{new}, if $fit_{new} > fit_i$ and set $trial_i = 0$. Else increase $trial_i$ by 1

Selection

end

Determination of probability value

- Probability value of each solution to undergo onlooker phase is determined as

$$prob_i = 0.9\left(\frac{fit_i}{\max(fit)}\right) + 0.1$$

$prob_i$	Probability of i^th^ solution
fit_i	Fitness of i^th^ solution
N_P	Number of food sources

- Probability values of all solutions are determined before onlooker phase.

- A solution with higher fitness value will have higher probability.

- Fitter solution may undergo onlooker bee phase for more than once.

https://abc.erciyes.edu.tr/software.htm

Onlooker bee phase

➢ Generates N_P (=5) new solutions

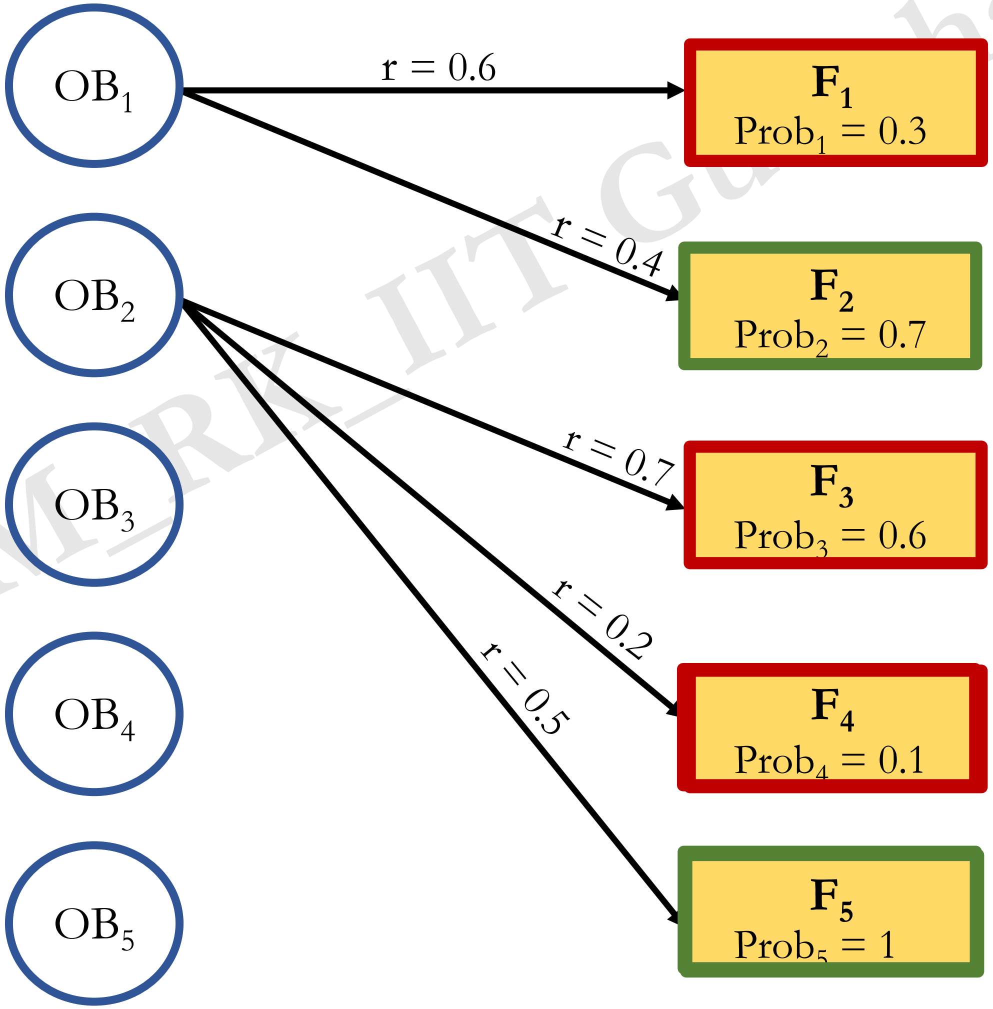
If **r < prob**, generate new solution

$$X^j_{new} = X^j + \phi\left(X^j - X^j_p\right)$$

$$Prob_i = 0.9\left(\frac{Fit_i}{\max(Fit)}\right) + 0.1$$

$$fit_{new} = \begin{cases} \dfrac{1}{1+f_{new}} & if \;\; f_{new} \geq 0 \\ 1+\left|f_{new}\right| & if \;\; f_{new} < 0 \end{cases}$$

Onlooker Bee	Food source
OB_1	F_2
OB_2	F_5



- Generate new X and bound it
- Perform greedy selection
- Update if new X has better fitness and reset trial to 0, else increase $trial_2$ by 1

- Generate new X and bound it
- Perform greedy selection
- Update if new X has better fitness and reset trial to 0, else increase $trial_5$ by 1

Onlooker bee phase

- Generates N_P (=5) new solutions

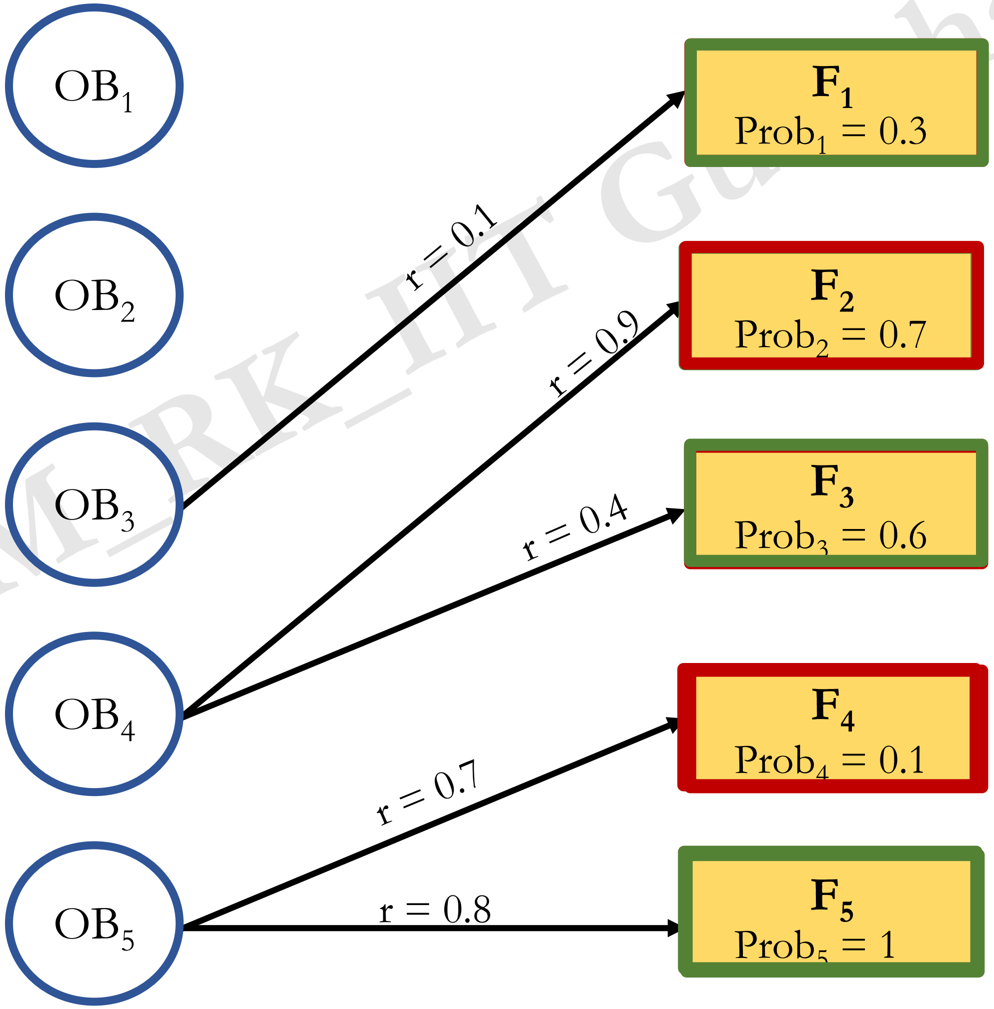
If **r < prob**, generate new solution

$$X^j_{new} = X^j + \phi\left(X^j - X^j_p\right)$$

$$Prob_i = 0.9\left(\frac{Fit_i}{\max\left(Fit\right)}\right) + 0.1$$

$$fit_{new} = \begin{cases} \dfrac{1}{1+f_{new}} & if\ \ f_{new} \ge 0 \\ 1+\left|f_{new}\right| & if\ \ f_{new} < 0 \end{cases}$$

Onlooker Bee	Food source
OB_1	F_2
OB_2	F_5
OB_3	F_1
OB_4	F_3
OB_5	F_5



- Generate new X and bound it
- Perform greedy selection
- Update if new X has better fitness and reset trial to 0 else increase $trial_1$ by 1

- Generate new X and bound it
- Perform greedy selection
- Update if new X has better fitness and reset trial to 0, else increase $trial_3$ by 1

- Generate new X and bound it
- Perform greedy selection
- Update if new X has better fitness and reset trial to 0, else increase $trial_5$ by 1

Onlooker bee phase

➢ Generates N_P (=5) new solutions

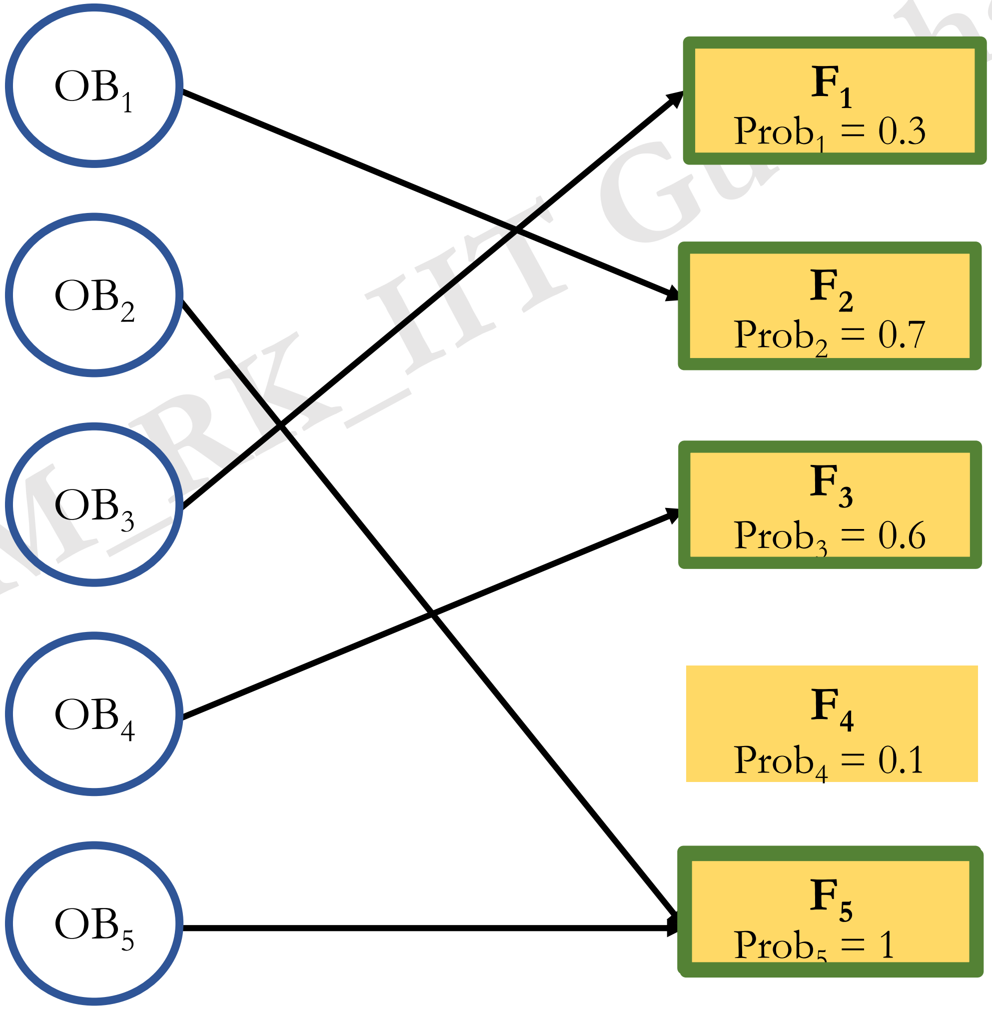
If **r < prob**, generate new solution

$$X^{j}_{new} = X^{j} + \phi\left(X^{j} - X^{j}_{p}\right)$$

$$Prob_i = 0.9\left(\frac{Fit_i}{\max(Fit)}\right) + 0.1$$

$$fit_{new} = \begin{cases} \dfrac{1}{1+f_{new}} & if \;\; f_{new} \geq 0 \\ 1+\left|f_{new}\right| & if \;\; f_{new} < 0 \end{cases}$$

Onlooker Bee	Food source
OB_1	F_2
OB_2	F_5
OB_3	F_1
OB_4	F_3
OB_5	F_5



Pseudocode of Onlooker Bee Phase

Input: Objective function, lb, ub, N_p, P, f, fit, prob, trial

Set m=0 and n = 1

While m < N_p

  Generate a random number r

  if r < $prob_n$

    Select a random partner (p) such that n $\neq$ p

    Randomly select a variable j and modify j^{th} variable

    Bound X_{new}^{j}

    Evaluate the objective function (f_{new}) and fitness (fit_{new})

    Accept X_{new}, if fit_{new} > fit_n and set $trial_n$ = 0. Else increase $trial_n$ by 1

    m = m + 1

  end

  n = n + 1

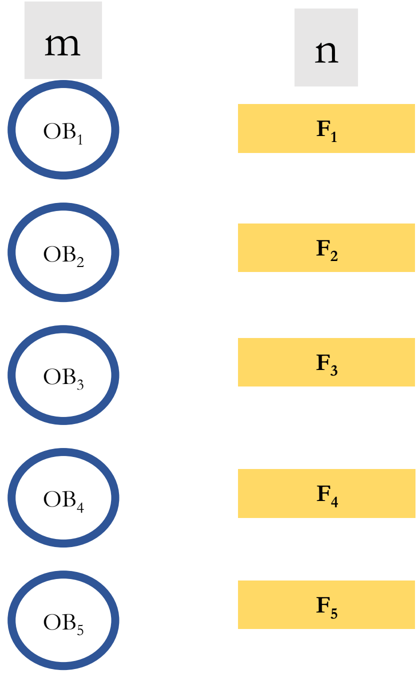
  Reset n = 1 if the value of n is greater than N_p

end

$$X_{new}^{j} = X^{j} + \phi\left(X^{j} - X_{p}^{j}\right)$$

Generation

Selection



limit: user-specified parameter

- *limit* is a user specified integer value
- Every solution is associated with an individual *trial* counter
- If the value of *trial* is greater than *limit*, the solution can potentially enter the scout phase
- The *trial* counter of abandoned solution is reset to zero
- The value of *limit* can be set as *limit* = N_p x D where *D* is the dimension of the problem

On the Importance of the Artificial Bee Colony Control Parameter 'Limit', Information technology and control, Volume 46, Issue 4, PP. 566-604, 2017

Scout phase

- Solutions with *trial* greater than *limit* are the candidates to be discarded
- One solution with its *trial* greater than *limit* is replaced with new random solution
- *trial* counter of newly included solution is reset to zero
- In one iteration, scout phase
  - Occurs only when the *trial* counter of at least one solution is greater than *limit*
  - Performed only on one solution with *trial* counter greater than *limit*
  - Can eliminate the best solution from the population due to the *limit*
  - Memorize the best solution before performing scout phase

Pseudocode of Scout Bee Phase

Input: Objective function, lb, ub, trial, limit, P

1. Identify the food source (k) whose *trial* greater than *limit*

2. Replace X_k from P as

$$X_k = lb + (ub - lb) r$$

3. Evaluate objective function (f_k) and assign fitness (fit_k)

Pseudocode

Selection of solution to perform scout phase

limit = 5

Case 1		Case 2		Case 3	
F_1 trial = 4	$trial \leq 5$	F_1 trial = 6	$trial > 5$	F_1 trial = 2	Randomly select one
F_2 trial = 0	$trial \leq 5$	F_2 trial = 0		F_2 trial = 8	scout phase
F_3 trial = 2	$trial \leq 5$	F_3 trial = 7	scout phase	F_3 trial = 8	Max(trial)
F_4 trial = 6	scout phase	F_4 trial = 2		F_4 trial = 8	Max(trial)
F_5 trial = 5	$trial \leq 5$	F_5 trial = 6	$trial > 5$	F_5 trial = 0	

Pseudocode of ABC

1. **Input**: Objective function, lb, ub, N_p, T and limit
2. Initialize a random population (P)
3. Evaluate objective function (f) and fitness (fit)
4. Set the trial counter of all food sources equal to zero

for t = 1 to T

Perform **Employed Bee Phase** of all food sources

Determine the probability of each food source

Perform **Onlooker Bee Phase** to generate N_p food sources

Memorize the best food source

if trial of any food source is greater than limit

Perform **Scout Bee Phase** of exhausted food source

end

end

Working of ABC: Sphere function

Consider $\quad \min \quad f(x)=\sum_{i=1}^{4} x_i^2; \quad 0 \le x_i \le 10, \quad i=1,2,3,4$ $\quad f(x)=x_1^2+x_2^2+x_3^2+x_4^2$

Decision variables: x_1, x_2, x_3 and x_4

- Step 1: Fix the swarm size S (= 10), number of cycles T (= 10) and *limit* = 1
- Step 2: Determine the no. of employed bees, onlooker bees and food sources, $N_P = S/2$
- Step 3: Generate random solutions within the domain of the decision variables

$$P=\begin{bmatrix} 4 & 0 & 1 & 8 \\ 3 & 1 & 9 & 7 \\ 0 & 3 & 1 & 5 \\ 2 & 1 & 4 & 9 \\ 1 & 2 & 8 & 3 \end{bmatrix} \qquad f=\begin{bmatrix} 81 \\ 140 \\ 35 \\ 102 \\ 78 \end{bmatrix}$$

Working of ABC: Sphere function

- Step 4: Calculate fitness of the population

$$f^1 = 81 \Rightarrow fit^1 = \frac{1}{1+81} = 0.0122$$

$$f^2 = 140 \Rightarrow fit^2 = \frac{1}{1+140} = 0.0071$$

$$f = \begin{bmatrix} 81 \\ 140 \\ 35 \\ 102 \\ 113 \end{bmatrix} \Rightarrow fit = \begin{bmatrix} 0.0122 \\ 0.0071 \\ 0.0278 \\ 0.0097 \\ 0.0127 \end{bmatrix}$$

$$P = \begin{bmatrix} 4 & 0 & 1 & 8 \\ 3 & 1 & 9 & 7 \\ 0 & 3 & 1 & 5 \\ 2 & 1 & 4 & 9 \\ 1 & 2 & 8 & 3 \end{bmatrix} \quad f = \begin{bmatrix} 81 \\ 140 \\ 35 \\ 102 \\ 78 \end{bmatrix}$$

$$fit = \begin{cases} \dfrac{1}{1+f} & if \;\; f \geq 0 \\ 1+|f| & if \;\; f < 0 \end{cases}$$

- Step 5: Generate initial *trial* vector for the population

$$t = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad P = \begin{bmatrix} 4 & 0 & 1 & 8 \\ 3 & 1 & 9 & 7 \\ 0 & 3 & 1 & 5 \\ 2 & 1 & 4 & 9 \\ 1 & 2 & 8 & 3 \end{bmatrix} \quad f = \begin{bmatrix} 81 \\ 140 \\ 35 \\ 102 \\ 78 \end{bmatrix} \quad fit = \begin{bmatrix} 0.0122 \\ 0.0071 \\ 0.0278 \\ 0.0097 \\ 0.0127 \end{bmatrix}$$

Employed bee phase: first solution

- Step 7: Select a random variable to change

  Let the variable be 3, $X^1 = [4 \quad 0 \quad 1 \quad 8]$

- Step 8: Select a random partner

  Let the partner be 4, $X^4 = [2 \quad 1 \quad 4 \quad 9]$

- Step 9: Create a new food location

  Let $\phi = 0.81$

  $$X_{new}^{1,3} = 1 + (0.81)(1-4) \Rightarrow X_{new}^{1,3} = -1.43$$

  $$X_{new}^{1} = [4 \quad 0 \quad -1.43 \quad 8]$$

$$P = \begin{bmatrix} \boxed{4 & 0 & 1 & 8} \\ 3 & 1 & 9 & 7 \\ 0 & 3 & 1 & 5 \\ 2 & 1 & 4 & 9 \\ 1 & 2 & 8 & 3 \end{bmatrix} \quad f = \begin{bmatrix} \boxed{81} \\ 140 \\ 35 \\ 102 \\ 78 \end{bmatrix} \quad fit = \begin{bmatrix} \boxed{0.0122} \\ 0.0071 \\ 0.0278 \\ 0.0097 \\ 0.0127 \end{bmatrix}$$

$$X_{new}^{j} = X^{j} + \phi\left(X^{j} - X_{p}^{j}\right)$$

Employed bee phase: first solution

$$X_{new}^{1} = \begin{bmatrix} 4 & 0 & -1.43 & 8 \end{bmatrix}$$

$$\boxed{0 \le x_i \le 10}$$

- Step 10: x_3 violates lower bound

$$X_{new}^{1} = \max\left(X_{new}^{1}, lb^{1}\right)$$

$$X_{new}^{1} = \max\left(-1.43, 0\right) = 0$$

$$X_{new}^{1} = \begin{bmatrix} 4 & 0 & 0 & 8 \end{bmatrix}$$

$$P = \begin{bmatrix} 4 & 0 & 1 & 8 \\ 3 & 1 & 9 & 7 \\ 0 & 3 & 1 & 5 \\ 2 & 1 & 4 & 9 \\ 1 & 2 & 8 & 3 \end{bmatrix} \quad f = \begin{bmatrix} 81 \\ 140 \\ 35 \\ 102 \\ 78 \end{bmatrix} \quad fit = \begin{bmatrix} 0.0122 \\ 0.0071 \\ 0.0278 \\ 0.0097 \\ 0.0127 \end{bmatrix}$$

Employed bee phase: first solution

- Step 11: Evaluate the fitness $X_{new}^{1}=\begin{bmatrix}4 & 0 & 0 & 8\end{bmatrix}$

$$P=\begin{bmatrix}4 & 0 & 1 & 8\\ 3 & 1 & 9 & 7\\ 0 & 3 & 1 & 5\\ 2 & 1 & 4 & 9\\ 1 & 2 & 8 & 3\end{bmatrix} \quad f=\begin{bmatrix}81\\ 140\\ 35\\ 102\\ 78\end{bmatrix} \quad fit=\begin{bmatrix}0.0122\\ 0.0071\\ 0.0278\\ 0.0097\\ 0.0127\end{bmatrix} \quad t=\begin{bmatrix}0\\ 0\\ 0\\ 0\\ 0\end{bmatrix}$$

$$f\left(X_{new}^{1}\right)=4^2+0+0+8^2=80$$

$$\boxed{f\left(x\right)=\sum_{i=1}^{4} x_i^2}$$

$$fit(X_{new}^{1})=\frac{1}{1+80}=0.0123$$

- Step 12: Perform greedy selection

$$X^{1}=\begin{bmatrix}4 & 0 & 1 & 8\end{bmatrix}, \quad fit^{1}=0.0122$$

$$X_{new}^{1}=\begin{bmatrix}4 & 0 & 0 & 8\end{bmatrix}, \quad fit_{new}^{1}=0.0123$$

$$f_{new}^{1}>f^{1}$$

$$X^{1}=X_{new}^{1}=\begin{bmatrix}4 & 0 & 0 & 8\end{bmatrix}$$

$$f^{1}=f_{new}^{1}=80$$

$$fit^{1}=fit_{new}^{1}=0.0123$$

$$\boxed{\text{Reset } trial\left(1\right) \text{ to } 0}$$

$$P=\begin{bmatrix}4 & 0 & 0 & 8\\ 3 & 1 & 9 & 7\\ 0 & 3 & 1 & 5\\ 2 & 1 & 4 & 9\\ 1 & 2 & 8 & 3\end{bmatrix} \quad f=\begin{bmatrix}80\\ 140\\ 35\\ 102\\ 78\end{bmatrix} \quad fit=\begin{bmatrix}0.0123\\ 0.0071\\ 0.0278\\ 0.0097\\ 0.0127\end{bmatrix} \quad t=\begin{bmatrix}0\\ 0\\ 0\\ 0\\ 0\end{bmatrix}$$

Employed bee phase: second solution

$$P = \begin{bmatrix} 4 & 0 & 0 & 8 \\ 3 & 1 & 9 & 7 \\ 0 & 3 & 1 & 5 \\ 2 & 1 & 4 & 9 \\ 1 & 2 & 8 & 3 \end{bmatrix} \quad f = \begin{bmatrix} 80 \\ \boxed{140} \\ 35 \\ 102 \\ 78 \end{bmatrix} \quad fit = \begin{bmatrix} 0.0123 \\ \boxed{0.0071} \\ 0.0278 \\ 0.0097 \\ 0.0127 \end{bmatrix}$$

- Step 7: Select a random variable to change

  Let the variable be 1, $X^2 = \begin{bmatrix} 3 & 1 & 9 & 7 \end{bmatrix}$

- Step 8: Select a random partner

  Let the partner be 3, $X^3 = \begin{bmatrix} 0 & 3 & 1 & 5 \end{bmatrix}$

- Step 9: Create a new food location

$$X_{new}^{j} = X^{j} + \phi\left(X^{j} - X_{p}^{j}\right)$$

Let $\phi = 0.19$

$$X_{new}^{2,1} = 3 + (0.19)(3-0) \Rightarrow X_{new}^{1,3} = 3.57$$

$$X_{new}^{2} = \begin{bmatrix} 3.57 & 1 & 9 & 7 \end{bmatrix}$$

Employed bee phase: second solution

- Step 11: Evaluate the fitness

$$f(x)=\sum_{i=1}^{4} x_i^2$$

$$X_{new}^2=\begin{bmatrix}3.57 & 1 & 9 & 7\end{bmatrix}$$

$$f\left(X_{new}^2\right)=3.57^2+1^2+9^2+7^2=143.74$$

$$fit(X_{new}^2)=\frac{1}{1+143.74}=0.0069$$

$$P=\begin{bmatrix}4&0&0&8\\3&1&9&7\\0&3&1&5\\2&1&4&9\\1&2&8&3\end{bmatrix}\quad f=\begin{bmatrix}81\\140\\35\\102\\78\end{bmatrix}\quad fit=\begin{bmatrix}0.0123\\0.0071\\0.0278\\0.0097\\0.0127\end{bmatrix}\quad t=\begin{bmatrix}0\\0\\0\\0\\0\end{bmatrix}$$

- Step 12: Perform greedy selection

$$X^2=\begin{bmatrix}3 & 1 & 9 & 7\end{bmatrix},\quad fit^2=0.0071 \qquad fit_{new}^2 < fit^2$$

$$X_{new}^2=\begin{bmatrix}3.57 & 1 & 9 & 7\end{bmatrix},\quad fit_{new}^2=0.0069$$

No change in X^2, f^2, fit^2

Increase $trial(2)$ by 1

$$P=\begin{bmatrix}4&0&0&8\\3&1&9&7\\0&3&1&5\\2&1&4&9\\1&2&8&3\end{bmatrix}\quad f=\begin{bmatrix}80\\140\\35\\102\\78\end{bmatrix}\quad fit=\begin{bmatrix}0.0123\\0.0071\\0.0278\\0.0097\\0.0127\end{bmatrix}\quad t=\begin{bmatrix}0\\1\\0\\0\\0\end{bmatrix}$$

Employed bee phase: third solution

$$P = \begin{bmatrix} 4 & 0 & 0 & 8 \\ 3 & 1 & 9 & 7 \\ 0 & 3 & 1 & 5 \\ 2 & 1 & 4 & 9 \\ 1 & 2 & 8 & 3 \end{bmatrix} \quad f = \begin{bmatrix} 80 \\ 140 \\ 35 \\ 102 \\ 78 \end{bmatrix} \quad fit = \begin{bmatrix} 0.0123 \\ 0.0071 \\ 0.0278 \\ 0.0097 \\ 0.0127 \end{bmatrix} \quad t = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Employed bee phase

Variable to change = 1

Partner solution = 1

ϕ = -0.56

$$P = \begin{bmatrix} 4 & 0 & 0 & 8 \\ 3 & 1 & 9 & 7 \\ 0 & 3 & 1 & 5 \\ 2 & 1 & 4 & 9 \\ 1 & 2 & 8 & 3 \end{bmatrix} \quad f = \begin{bmatrix} 80 \\ 140 \\ 35 \\ 102 \\ 78 \end{bmatrix} \quad fit = \begin{bmatrix} 0.0123 \\ 0.0071 \\ 0.0278 \\ 0.0097 \\ 0.0127 \end{bmatrix} \quad t = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

Determine the population and fitness

Employed bee phase: fourth solution

$$P = \begin{bmatrix} 4 & 0 & 0 & 8 \\ 3 & 1 & 9 & 7 \\ 0 & 3 & 1 & 5 \\ 2 & 1 & 4 & 9 \\ 1 & 2 & 8 & 3 \end{bmatrix} \quad f = \begin{bmatrix} 80 \\ 140 \\ 35 \\ 102 \\ 78 \end{bmatrix} \quad fit = \begin{bmatrix} 0.0123 \\ 0.0071 \\ 0.0278 \\ 0.0097 \\ 0.0127 \end{bmatrix} \quad t = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

Employed bee phase

Variable to change = 2

Partner solution = 3

ϕ = -0.6

$$P = \begin{bmatrix} 4 & 0 & 0 & 8 \\ 3 & 1 & 9 & 7 \\ 0 & 3 & 1 & 5 \\ 2 & 1 & 4 & 9 \\ 1 & 2 & 8 & 3 \end{bmatrix} \quad f = \begin{bmatrix} 80 \\ 140 \\ 35 \\ 102 \\ 78 \end{bmatrix} \quad fit = \begin{bmatrix} 0.0123 \\ 0.0071 \\ 0.0278 \\ 0.0097 \\ 0.0127 \end{bmatrix} \quad t = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

Determine the population and fitness

Employed bee phase: fifth solution

$$P = \begin{bmatrix} 4 & 0 & 0 & 8 \\ 3 & 1 & 9 & 7 \\ 0 & 3 & 1 & 5 \\ 2 & 1 & 4 & 9 \\ 1 & 2 & 8 & 3 \end{bmatrix} \quad f = \begin{bmatrix} 80 \\ 140 \\ 35 \\ 102 \\ 78 \end{bmatrix} \quad fit = \begin{bmatrix} 0.0123 \\ 0.0071 \\ 0.0278 \\ 0.0097 \\ 0.0127 \end{bmatrix} \quad t = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

Employed bee phase

Variable to change = 4

Partner solution = 3

$\phi = 0.81$

$$P = \begin{bmatrix} 4 & 0 & 0 & 8 \\ 3 & 1 & 9 & 7 \\ 0 & 3 & 1 & 5 \\ 2 & 1 & 4 & 9 \\ 1 & 2 & 8 & 1.38 \end{bmatrix} \quad f = \begin{bmatrix} 80 \\ 140 \\ 35 \\ 102 \\ 70.9 \end{bmatrix} \quad fit = \begin{bmatrix} 0.0123 \\ 0.0071 \\ 0.0278 \\ 0.0097 \\ 0.0139 \end{bmatrix} \quad t = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

Determine the population and fitness

ABC: Food source information

- Step 11: Calculate the probability values

$$\text{Prob} = 0.9\left(\frac{\text{Fitness}}{\max(\text{Fitness})}\right) + 0.1$$

$$P = \begin{bmatrix} 4 & 0 & 0 & 8 \\ 3 & 1 & 9 & 7 \\ 0 & 3 & 1 & 5 \\ 2 & 1 & 4 & 9 \\ 1 & 2 & 8 & 1.38 \end{bmatrix} \quad f = \begin{bmatrix} 80 \\ 140 \\ 35 \\ 102 \\ 70.9 \end{bmatrix} \quad fit = \begin{bmatrix} 0.0123 \\ 0.0071 \\ 0.0278 \\ 0.0097 \\ 0.0139 \end{bmatrix} \quad t = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

$$\text{Prob} = \begin{bmatrix} 0.9\left(\frac{0.0123}{0.0278}\right) + 0.1 \\ 0.9\left(\frac{0.0071}{0.0278}\right) + 0.1 \\ 0.9\left(\frac{0.0278}{0.0278}\right) + 0.1 \\ 0.9\left(\frac{0.0097}{0.0278}\right) + 0.1 \\ 0.9\left(\frac{0.0139}{0.0278}\right) + 0.1 \end{bmatrix} = \begin{bmatrix} 0.5 \\ 0.33 \\ 1 \\ 0.41 \\ 0.55 \end{bmatrix}$$

Onlooker bee phase: first bee

$$\text{prob} = \begin{bmatrix} 0.5 & 0.33 & 1 & 0.41 & 0.55 \end{bmatrix}$$

$$P = \begin{bmatrix} 4 & 0 & 0 & 8 \\ 3 & 1 & 9 & 7 \\ 0 & 3 & 1 & 5 \\ 2 & 1 & 4 & 9 \\ 1 & 2 & 8 & 1.38 \end{bmatrix} \quad f = \begin{bmatrix} 80 \\ 140 \\ 35 \\ 102 \\ 70.9 \end{bmatrix} \quad fit = \begin{bmatrix} 0.0123 \\ 0.0071 \\ 0.0278 \\ 0.0097 \\ 0.0139 \end{bmatrix} \quad t = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

- Step 1: Food source to be selected is 1
- Step 2: Select a random number, r = 0.39
- Step 3: Check if $r < \text{prob}$ $\quad 0.39 < 0.5$
- Step 4: Select a random variable to change

  Let the variable be 4, $X^1 = \begin{bmatrix} 4 & 0 & 0 & 8 \end{bmatrix}$

$$X^j_{new} = X^j + \phi\left(X^j - X^j_p\right)$$

- Step 5: Select a random partner

  Let the partner be 3, $X^3 = \begin{bmatrix} 0 & 3 & 1 & 5 \end{bmatrix}$

Onlooker bee phase: first bee

$$\text{prob} = \begin{bmatrix} 0.5 & 0.33 & 1 & 0.41 & 0.55 \end{bmatrix}$$

$$P = \begin{bmatrix} 4 & 0 & 0 & 8 \\ 3 & 1 & 9 & 7 \\ 0 & 3 & 1 & 5 \\ 2 & 1 & 4 & 9 \\ 1 & 2 & 8 & 1.38 \end{bmatrix} \quad f = \begin{bmatrix} 80 \\ 140 \\ 35 \\ 102 \\ 70.9 \end{bmatrix} \quad fit = \begin{bmatrix} 0.0123 \\ 0.0071 \\ 0.0278 \\ 0.0097 \\ 0.0139 \end{bmatrix} \quad t = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

- Step 6: Create a new food location

Let $\phi = -0.68$

$$X_{new}^{1,4} = 8 + (-0.68)(8-5) \Rightarrow X_{new}^{1,4} = 5.96$$

$$X_{new}^{1} = \begin{bmatrix} 4 & 0 & 0 & 5.96 \end{bmatrix}$$

- Step 7: Evaluate the fitness

$$f\left(X_{new}^{1}\right) = 4^2 + 0 + 0 + 5.96^2 = 51.52$$

$$fit(X_{new}^{1}) = \frac{1}{1+51.52} = 0.019$$

$$f(x) = \sum_{i=1}^{4} x_i^2$$

$$X_{new}^{j} = X^{j} + \phi\left(X^{j} - X_{p}^{j}\right)$$

Onlooker bee phase: first bee

- Step 8: Perform greedy selection

$$\text{prob} = \begin{bmatrix} 0.5 & 0.33 & 1 & 0.41 & 0.55 \end{bmatrix}$$

$$X^1 = \begin{bmatrix} 4 & 0 & 0 & 8 \end{bmatrix}, \quad fit^1 = 0.0123$$

$$X^1_{new} = \begin{bmatrix} 4 & 0 & 0 & 5.96 \end{bmatrix}, \quad fit^1_{new} = 0.019$$

$$f^1_{new} > f^1$$

$$P = \begin{bmatrix} 4 & 0 & 0 & 8 \\ 3 & 1 & 9 & 7 \\ 0 & 3 & 1 & 5 \\ 2 & 1 & 4 & 9 \\ 1 & 2 & 8 & 1.38 \end{bmatrix} \quad f = \begin{bmatrix} 80 \\ 140 \\ 35 \\ 102 \\ 70.9 \end{bmatrix} \quad fit = \begin{bmatrix} 0.0123 \\ 0.0071 \\ 0.0278 \\ 0.0097 \\ 0.0139 \end{bmatrix} \quad t = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

$$X^1 = X^1_{new} = \begin{bmatrix} 4 & 0 & 0 & 5.96 \end{bmatrix}$$

$$f^1 = f^1_{new} = 51.52$$

$$fit^1 = fit^1_{new} = 0.019$$

$$P = \begin{bmatrix} 4 & 0 & 0 & 5.96 \\ 3 & 1 & 9 & 7 \\ 0 & 3 & 1 & 5 \\ 2 & 1 & 4 & 9 \\ 1 & 2 & 8 & 1.38 \end{bmatrix} \quad f = \begin{bmatrix} 51.52 \\ 140 \\ 35 \\ 102 \\ 70.9 \end{bmatrix} \quad fit = \begin{bmatrix} 0.019 \\ 0.0071 \\ 0.0278 \\ 0.0097 \\ 0.0139 \end{bmatrix} \quad t = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

Reset $trial(1)$ to 0

Onlooker bee phase: second bee

$$\text{prob} = \begin{bmatrix} 0.5 & 0.33 & 1 & 0.41 & 0.55 \end{bmatrix}$$

$$P = \begin{bmatrix} 4 & 0 & 0 & 5.96 \\ 3 & 1 & 9 & 7 \\ 0 & 3 & 1 & 5 \\ 2 & 1 & 4 & 9 \\ 1 & 2 & 8 & 1.38 \end{bmatrix} \quad f = \begin{bmatrix} 51.52 \\ 140 \\ 35 \\ 102 \\ 70.9 \end{bmatrix} \quad fit = \begin{bmatrix} 0.019 \\ 0.0071 \\ 0.0278 \\ 0.0097 \\ 0.0139 \end{bmatrix} \quad t = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

- Step 1: Food source to be selected is 2
- Step 2: Select a random number, r = 0.2
- Step 3: Check if $r < \text{prob}$ $\boxed{0.2 < 0.33}$
- Step 4: Let the variable to change be 3
- Step 5: Let the random partner be 5

Let $\phi = -0.32$

Generate, bound,

Evaluate objective function, fitness

Greedy selection, update, reset t to 0

$$P = \begin{bmatrix} 4 & 0 & 0 & 5.96 \\ 3 & 1 & 8.68 & 7 \\ 0 & 3 & 1 & 5 \\ 2 & 1 & 4 & 9 \\ 1 & 2 & 8 & 1.38 \end{bmatrix} \quad f = \begin{bmatrix} 51.52 \\ 134.34 \\ 35 \\ 102 \\ 70.9 \end{bmatrix} \quad fit = \begin{bmatrix} 0.019 \\ 0.0074 \\ 0.0278 \\ 0.0097 \\ 0.0139 \end{bmatrix} \quad t = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

Onlooker bee phase: third bee

$$\text{prob} = \begin{bmatrix} 0.5 & 0.33 & 1 & 0.41 & 0.55 \end{bmatrix}$$

- Step 1: Food source to be selected is 3
- Step 2: Select a random number, $r = 0.57$
- Step 3: Check if $r < \text{prob}$ $\quad 0.57 < 1$
- Step 4: Let the variable to change be 2
- Step 5: Let the random partner be 4

$$P = \begin{bmatrix} 4 & 0 & 0 & 5.96 \\ 3 & 1 & 8.68 & 7 \\ 0 & 3 & 1 & 5 \\ 2 & 1 & 4 & 9 \\ 1 & 2 & 8 & 1.38 \end{bmatrix} \quad f = \begin{bmatrix} 51.52 \\ 134.34 \\ 35 \\ 102 \\ 70.9 \end{bmatrix} \quad fit = \begin{bmatrix} 0.019 \\ 0.0074 \\ 0.0278 \\ 0.0097 \\ 0.0139 \end{bmatrix} \quad t = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

Let $\phi = 0.07$

Generate, bound,

Evaluate objective function, fitness

Greedy selection, increase t by 1

$$P = \begin{bmatrix} 4 & 0 & 0 & 5.96 \\ 3 & 1 & 8.68 & 7 \\ 0 & 3 & 1 & 5 \\ 2 & 1 & 4 & 9 \\ 1 & 2 & 8 & 1.38 \end{bmatrix} \quad f = \begin{bmatrix} 51.52 \\ 134.34 \\ 35 \\ 102 \\ 70.9 \end{bmatrix} \quad fit = \begin{bmatrix} 0.019 \\ 0.0074 \\ 0.0278 \\ 0.0097 \\ 0.0139 \end{bmatrix} \quad t = \begin{bmatrix} 0 \\ 0 \\ 2 \\ 1 \\ 0 \end{bmatrix}$$

Onlooker bee phase: fourth bee

$\text{prob} = \begin{bmatrix} 0.5 & 0.33 & 1 & 0.41 & 0.55 \end{bmatrix}$

- Step 1: Food source to be selected is 4
- Step 2: Select a random number, r = 0.95
- Step 3: Check if $r < \text{prob}$ $\quad 0.95 < 0.41$

$$P = \begin{bmatrix} 4 & 0 & 0 & 5.96 \\ 3 & 1 & 8.68 & 7 \\ 0 & 3 & 1 & 5 \\ 2 & 1 & 4 & 9 \\ 1 & 2 & 8 & 1.38 \end{bmatrix} \quad f = \begin{bmatrix} 51.52 \\ 134.34 \\ 35 \\ 102 \\ 70.9 \end{bmatrix} \quad fit = \begin{bmatrix} 0.019 \\ 0.0074 \\ 0.0278 \\ 0.0097 \\ 0.0139 \end{bmatrix} \quad t = \begin{bmatrix} 0 \\ 0 \\ 2 \\ 1 \\ 0 \end{bmatrix}$$

No new solution to be generated using this food source

$$P = \begin{bmatrix} 4 & 0 & 0 & 5.96 \\ 3 & 1 & 8.68 & 7 \\ 0 & 3 & 1 & 5 \\ 2 & 1 & 4 & 9 \\ 1 & 2 & 8 & 1.38 \end{bmatrix} \quad f = \begin{bmatrix} 51.52 \\ 134.34 \\ 35 \\ 102 \\ 70.9 \end{bmatrix} \quad fit = \begin{bmatrix} 0.019 \\ 0.0074 \\ 0.0278 \\ 0.0097 \\ 0.0139 \end{bmatrix} \quad t = \begin{bmatrix} 0 \\ 0 \\ 2 \\ 1 \\ 0 \end{bmatrix}$$

Onlooker bee phase: fourth bee

$\text{prob} = [0.5 \quad 0.33 \quad 1 \quad 0.41 \quad 0.55]$

- Step 1: Food source to be selected is 5
- Step 2: Select a random number, r = 0.3
- Step 3: Check if $r < \text{prob}$ $\quad 0.3 < 0.55$
- Step 4: Let the variable to change be 1
- Step 5: Let the random partner be 2

$$P = \begin{bmatrix} 4 & 0 & 0 & 5.96 \\ 3 & 1 & 8.68 & 7 \\ 0 & 3 & 1 & 5 \\ 2 & 1 & 4 & 9 \\ 1 & 2 & 8 & 1.38 \end{bmatrix} \quad f = \begin{bmatrix} 51.52 \\ 134.34 \\ 35 \\ 102 \\ 70.9 \end{bmatrix} \quad fit = \begin{bmatrix} 0.019 \\ 0.0074 \\ 0.0278 \\ 0.0097 \\ 0.0139 \end{bmatrix} \quad t = \begin{bmatrix} 0 \\ 0 \\ 2 \\ 1 \\ 0 \end{bmatrix}$$

Let $\phi = 0.7$

Generate, bound,

Evaluate objective function, fitness

Greedy selection, update, reset t to 0

$$P = \begin{bmatrix} 4 & 0 & 0 & 5.96 \\ 3 & 1 & 8.68 & 7 \\ 0 & 3 & 1 & 5 \\ 2 & 1 & 4 & 9 \\ 0 & 2 & 8 & 1.38 \end{bmatrix} \quad f = \begin{bmatrix} 51.52 \\ 134.34 \\ 35 \\ 102 \\ 69.9 \end{bmatrix} \quad fit = \begin{bmatrix} 0.019 \\ 0.0074 \\ 0.0278 \\ 0.0097 \\ 0.0141 \end{bmatrix} \quad t = \begin{bmatrix} 0 \\ 0 \\ 2 \\ 1 \\ 0 \end{bmatrix}$$

Next food source to be selected $= 1$

Onlooker bee phase: fifth bee

$\text{prob} = \begin{bmatrix} 0.5 & 0.33 & 1 & 0.41 & 0.55 \end{bmatrix}$

- Step 1: Select food source, i = 1
- Step 2: Select a random number, r = 0.41
- Step 3: Check if $r < \text{prob}$ $\boxed{0.41 < 0.5}$
- Step 4: Let the variable to change be 1
- Step 5: Let the random partner be 2

Let $\phi = -0.87$

$$P = \begin{bmatrix} 4 & 0 & 0 & 5.96 \\ 3 & 1 & 8.68 & 7 \\ 0 & 3 & 1 & 5 \\ 2 & 1 & 4 & 9 \\ 1 & 2 & 8 & 1.38 \end{bmatrix} \quad f = \begin{bmatrix} 51.52 \\ 134.34 \\ 35 \\ 102 \\ 70.9 \end{bmatrix} \quad fit = \begin{bmatrix} 0.019 \\ 0.0074 \\ 0.0278 \\ 0.0097 \\ 0.0141 \end{bmatrix} \quad t = \begin{bmatrix} 0 \\ 0 \\ 2 \\ 1 \\ 0 \end{bmatrix}$$

Generate, bound, objective function, fitness greedy selection, reset t to 0

$$P = \begin{bmatrix} 3.13 & 0 & 0 & 5.96 \\ 3 & 1 & 8.68 & 7 \\ 0 & 3 & 1 & 5 \\ 2 & 1 & 4 & 9 \\ 0 & 2 & 8 & 1.38 \end{bmatrix} \quad f = \begin{bmatrix} 45.32 \\ 134.34 \\ 35 \\ 102 \\ 69.9 \end{bmatrix} \quad fit = \begin{bmatrix} 0.0216 \\ 0.0074 \\ 0.0278 \\ 0.0097 \\ 0.0141 \end{bmatrix} \quad t = \begin{bmatrix} 0 \\ 0 \\ 2 \\ 1 \\ 0 \end{bmatrix}$$

Scout bee phase:

Memorize the best solution: Store best population member and its objective function

$$B = \begin{bmatrix} 0 & 3 & 1 & 5 \end{bmatrix} \quad fit = \begin{bmatrix} 0.0278 \end{bmatrix} \quad f = \begin{bmatrix} 35 \end{bmatrix}$$

$$P = \begin{bmatrix} 3.13 & 0 & 0 & 5.96 \\ 3 & 1 & 8.68 & 7 \\ 0 & 3 & 1 & 5 \\ 2 & 1 & 4 & 9 \\ 0 & 2 & 8 & 1.38 \end{bmatrix} \quad f = \begin{bmatrix} 45.32 \\ 134.34 \\ 35 \\ 102 \\ 69.9 \end{bmatrix} \quad fit = \begin{bmatrix} 0.0216 \\ 0.0074 \\ 0.0278 \\ 0.0097 \\ 0.0141 \end{bmatrix} \quad t = \begin{bmatrix} 0 \\ 0 \\ 2 \\ 1 \\ 0 \end{bmatrix}$$

- Step 1: Select one solution for which *trials* is greater than *limit*

$$X^3 = \begin{bmatrix} 0 & 3 & 1 & 5 \end{bmatrix}, \quad fit^3 = 0.0278, \quad trial(3) = 2$$

- Step 2: Replace with a new random solution

$$X^3 = \begin{bmatrix} 9.94 & 9.71 & 8 & 6.02 \end{bmatrix},$$

$$f^3 = 293.33, \quad fit^3 = 0.0034,$$

$$trial(3) = 0$$

$$P = \begin{bmatrix} 3.13 & 0 & 0 & 5.96 \\ 3 & 1 & 8.68 & 7 \\ 9.94 & 9.71 & 8 & 6.02 \\ 2 & 1 & 4 & 9 \\ 0 & 2 & 8 & 1.38 \end{bmatrix} \quad f = \begin{bmatrix} 45.32 \\ 134.34 \\ 293.33 \\ 102 \\ 69.9 \end{bmatrix} \quad fit = \begin{bmatrix} 0.0216 \\ 0.0074 \\ 0.0034 \\ 0.0097 \\ 0.0141 \end{bmatrix} \quad t = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

Satisfaction of termination condition

$$\min \quad f(x)=\sum_{i=1}^{4} x_i^2; \quad 0 \le x_i \le 10, \quad i=1,2,3,4$$

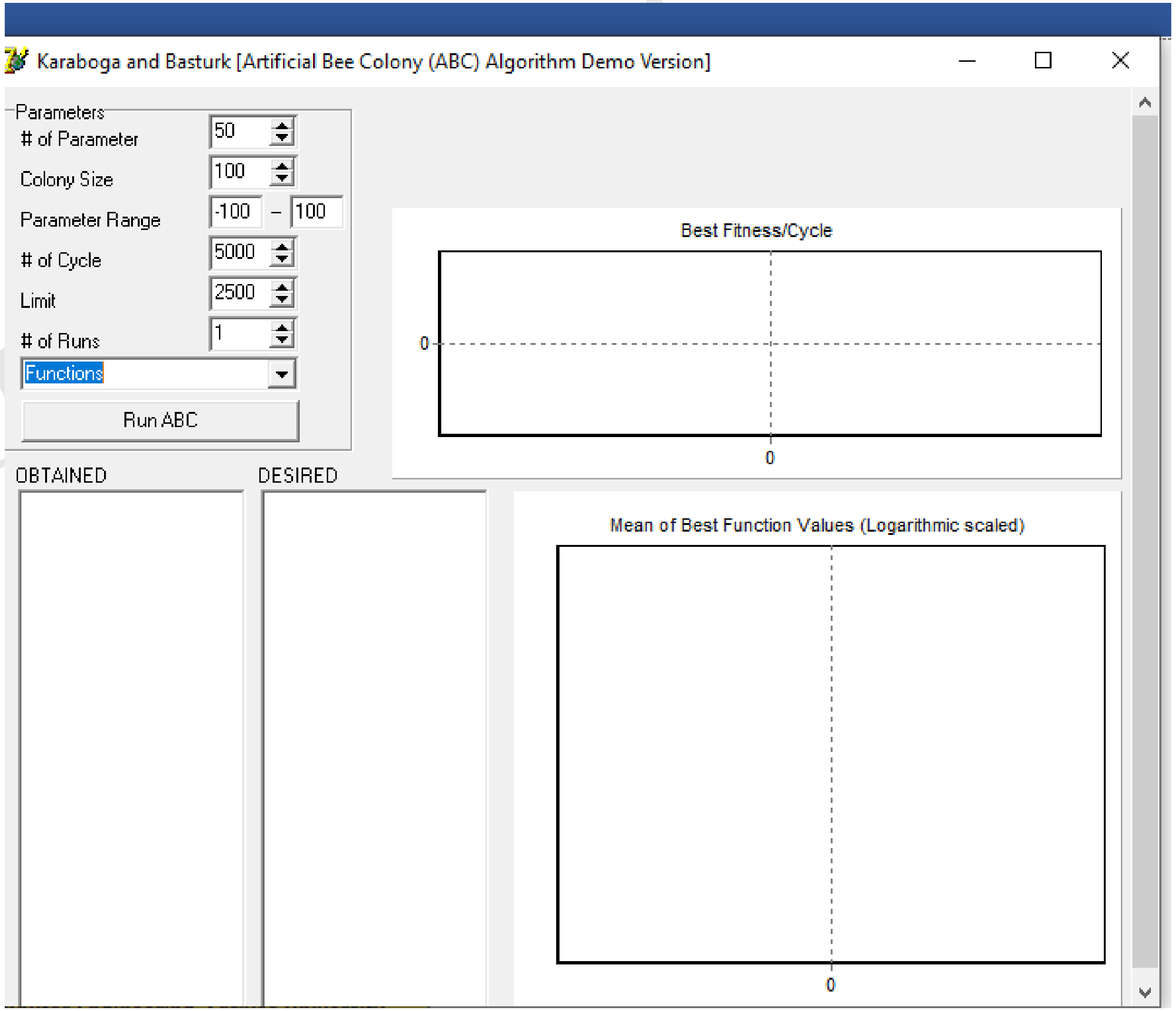
After completion of 10 iterations

$$P=\begin{bmatrix} 4.51 & 0 & 0 & 0 \\ 1.44 & 0 & 3.04 & 5.75 \\ 7.38 & 5.63 & 7.41 & 2.24 \\ 9.34 & 5.76 & 1.85 & 9.82 \\ 1.04 & 5.14 & 7.98 & 5.19 \end{bmatrix} \quad f=\begin{bmatrix} 20.34 \\ 44.38 \\ 146.08 \\ 220.27 \\ 118.12 \end{bmatrix} \quad fit=\begin{bmatrix} 0.0469 \\ 0.0220 \\ 0.0068 \\ 0.0045 \\ 0.0084 \end{bmatrix} \quad t=\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

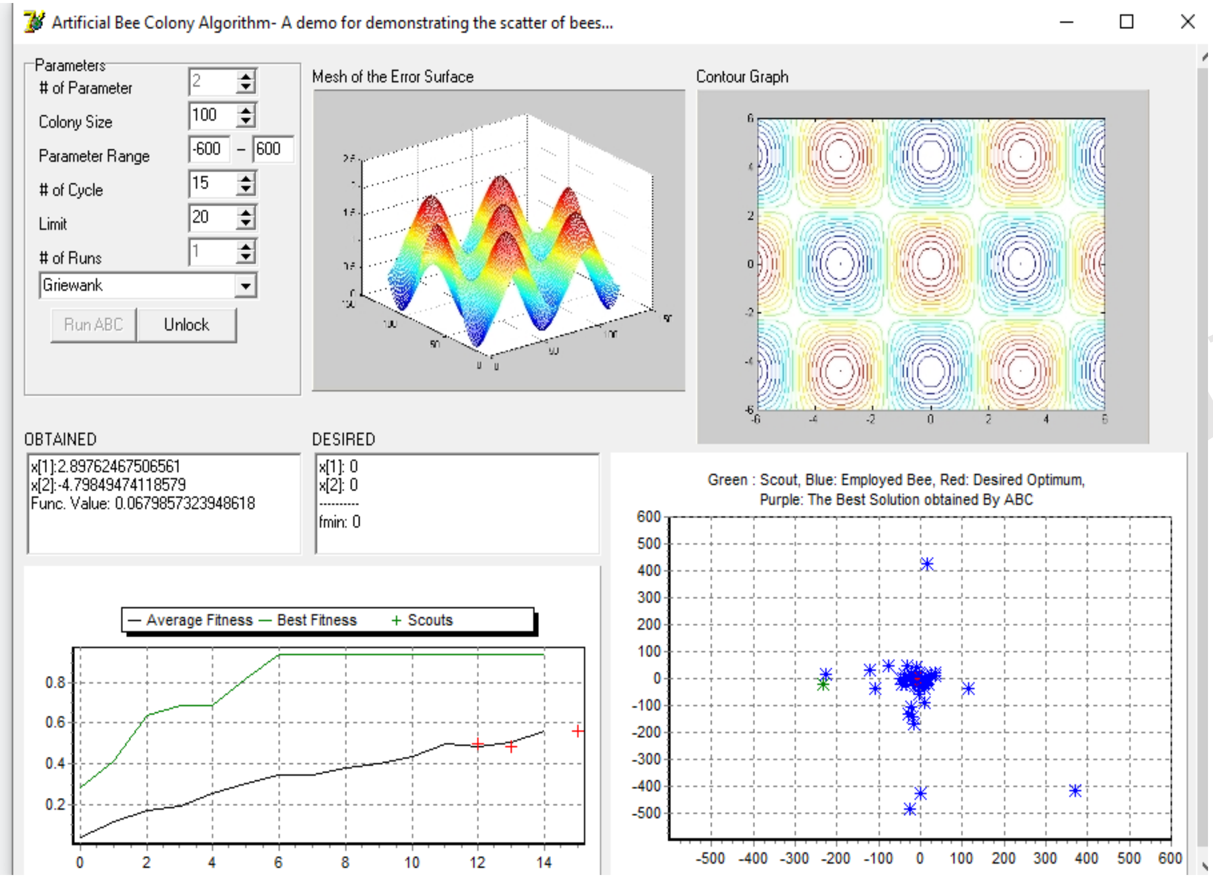
$$B=\begin{bmatrix} 4.51 & 0 & 0 & 0 \end{bmatrix} \qquad fit=\begin{bmatrix} 0.0469 \end{bmatrix} \qquad f=\begin{bmatrix} 20.34 \end{bmatrix}$$

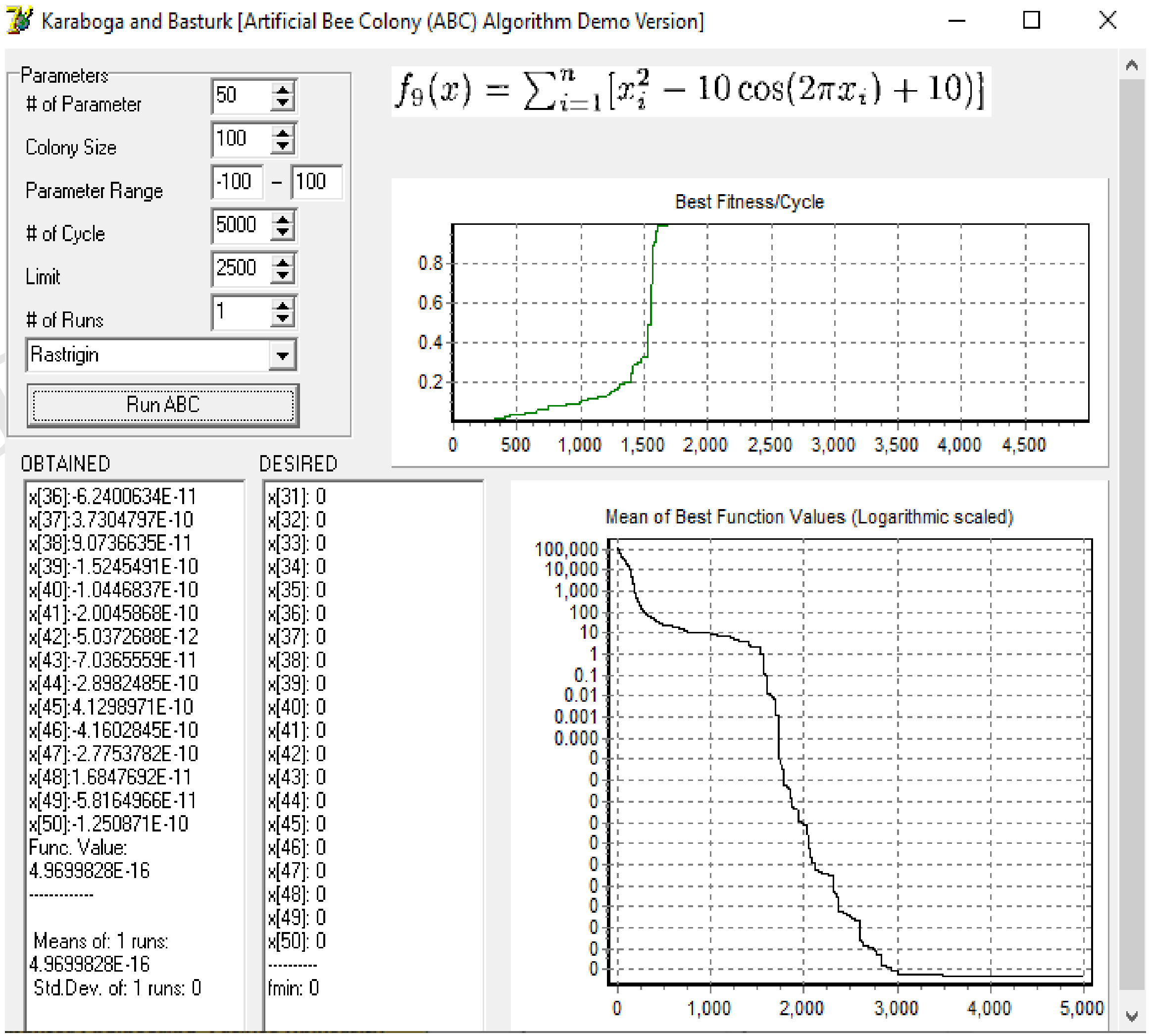
The minimum value of the function is **0**

Homepage of ABC

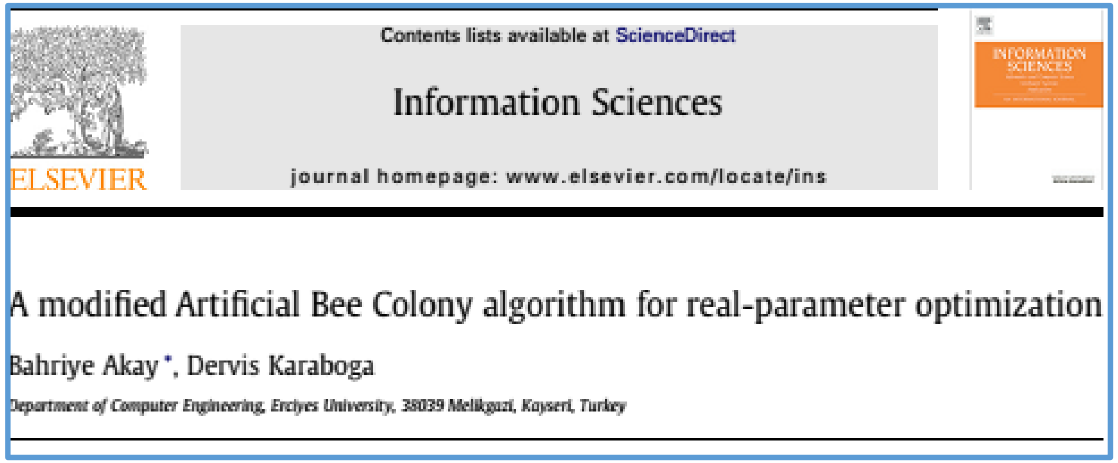


https://abc.erciyes.edu.tr/software.htm


Artificial Bee Colony Algorithm- A demo for demonstrating the scatter of bees...
Parameters
# of Parameter 2
Colony Size 100
Parameter Range -600 – 600
# of Cycle 15
Limit 20
# of Runs 1
Griewank
Run ABC
Unlock
Mesh of the Error Surface
Contour Graph
OBTAINED
x[1]:2.89762467506561
x[2]:-4.79849474118579
Func. Value: 0.0679857323948618
DESIRED
x[1]: 0
x[2]: 0
fmin: 0
Average Fitness
Best Fitness
Scouts
Green : Scout, Blue: Employed Bee, Red: Desired Optimum,
Purple: The Best Solution obtained By ABC


Karaboga and Basturk [Artificial Bee Colony (ABC) Algorithm Demo Version]
Parameters
# of Parameter 50
Colony Size 100
Parameter Range -100 – 100
# of Cycle 5000
Limit 2500
# of Runs 1
Rastrigin
Run ABC
$f_9(x) = \sum_{i=1}^{n} [x_i^2 - 10\cos(2\pi x_i) + 10)]$
Best Fitness/Cycle
OBTAINED
x[36]:-6.2400634E-11
x[37]:3.7304797E-10
x[38]:9.0736635E-11
x[39]:-1.5245491E-10
x[40]:-1.0446837E-10
x[41]:-2.0045868E-10
x[42]:-5.0372688E-12
x[43]:-7.0365559E-11
x[44]:-2.8982485E-10
x[45]:4.1298971E-10
x[46]:-4.1602845E-10
x[47]:-2.7753782E-10
x[48]:1.6847692E-11
x[49]:-5.8164966E-11
x[50]:-1.250871E-10
Func. Value:
4.9699828E-16
Means of: 1 runs:
4.9699828E-16
Std.Dev. of: 1 runs: 0
DESIRED
x[31]: 0
x[32]: 0
x[33]: 0
x[34]: 0
x[35]: 0
x[36]: 0
x[37]: 0
x[38]: 0
x[39]: 0
x[40]: 0
x[41]: 0
x[42]: 0
x[43]: 0
x[44]: 0
x[45]: 0
x[46]: 0
x[47]: 0
x[48]: 0
x[49]: 0
x[50]: 0
fmin: 0
Mean of Best Function Values (Logarithmic scaled)

Modified ABC


Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

ELSEVIER

INFORMATION SCIENCES

A modified Artificial Bee Colony algorithm for real-parameter optimization

Bahriye Akay *, Dervis Karaboga

Department of Computer Engineering, Erciyes University, 38039 Melikgazi, Kayseri, Turkey

Parameters: swarm size, number of cycles, limit

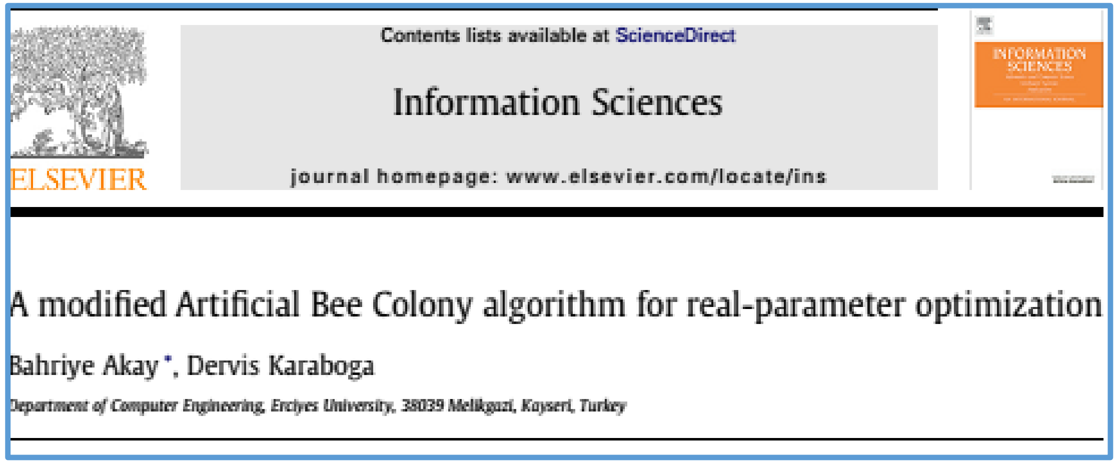
$$v_{ij} = x_{ij} + \phi_i \left(x_{ij} - x_{kj} \right) \qquad \phi_i \in [-1,1]$$

Frequency of the perturbation

$$v_{ij} = \begin{cases} x_{ij} + \phi_{ij} \left(x_{ij} - x_{kj} \right), & if \quad R_{ij} < MR \\ x_{ij}, & otherwise \end{cases} \qquad \phi_{ij} \in [-1,1] \quad R_{ij} \in [0,1]$$

Parameters:
swarm size, number of cycles, limit,

Modified ABC



Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

ELSEVIER

A modified Artificial Bee Colony algorithm for real-parameter optimization

Bahriye Akay *, Dervis Karaboga

Department of Computer Engineering, Erciyes University, 38039 Melikgazi, Kayseri, Turkey

Parameters: swarm size, number of cycles, limit

$$v_{ij} = x_{ij} + \phi_i \left(x_{ij} - x_{kj} \right) \qquad \phi_i \in [-1,1]$$

Frequency of the perturbation

$$v_{ij} = \begin{cases} x_{ij} + \phi_{ij}\left(x_{ij} - x_{kj}\right), & if \;\; R_{ij} < MR \\ x_{ij}, & otherwise \end{cases} \qquad \phi_{ij} \in [-1,1] \qquad R_{ij} \in [0,1]$$

Parameters: swarm size, number of cycles, limit

$$\phi_i \in [-SF, SF]$$

Parameters: Swarm size, number of cycles, limit, modification rate, scaling factor

Rechneberg's 1/5 mutation rule: Decrease SF if ratio of successful mutations to all mutations in m cycles, **ϕ**(m) < 1/5

Magnitude of the perturbation

$$SF(t+1) = \begin{cases} 0.85 SF(t), & if \;\; \phi(m) < 1/5 \\ SF(t)/0.85, & if \;\; \phi(m) > 1/5 \\ SF(t), & if \;\; \phi(m) = 1/5 \end{cases}$$

Parameters: swarm size, number of cycles, limit, modification rate, scaling factor, number of cycles (m) for changing SF

Pseudocode of ABC

1. **Input**: Fitness function, lb, ub, N_p, T and limit
2. Initialize a random population (P)
3. Evaluate objective function (f) of P and assign fitness (fit) ← FE = N_p
4. Set the trial counter of all food sources equal to zero

for t = 1 to T

Perform **Employed Bee Phase** of all food sources ← FE = N_p

Determine the probability of each food source

Perform **Onlooker Bee Phase** to generate N_p food sources ← FE = N_p

Memorize the best food source determined

if trial of any food source is greater than limit

Perform **Scout Bee Phase** of exhausted food source ← FE = 1

end

end

$$\left(N_p + 2N_pT\right) \le FE \le N_p + \left(2N_p + 1\right)T$$

For T iterations

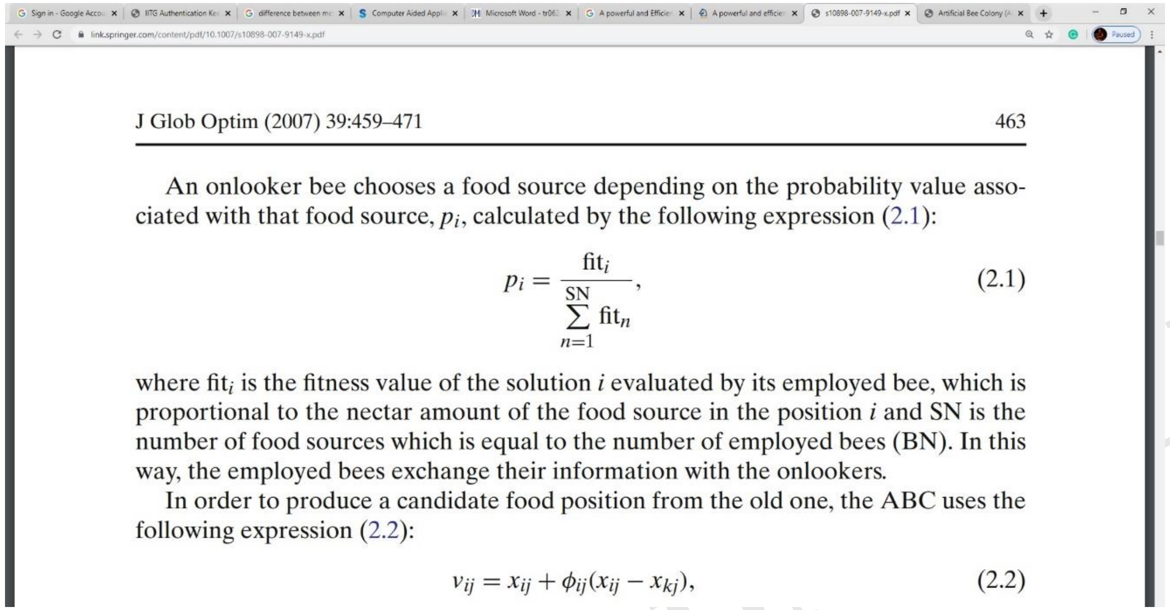
Scout phase is encountered in all iterations

$$Min \quad FE = N_p + 2N_pT$$

Scout phase is encountered in all iterations

$$Max \quad FE = N_p + \left(2N_p + 1\right)T$$

Probability calculation



J Glob Optim (2007) 39:459–471 463

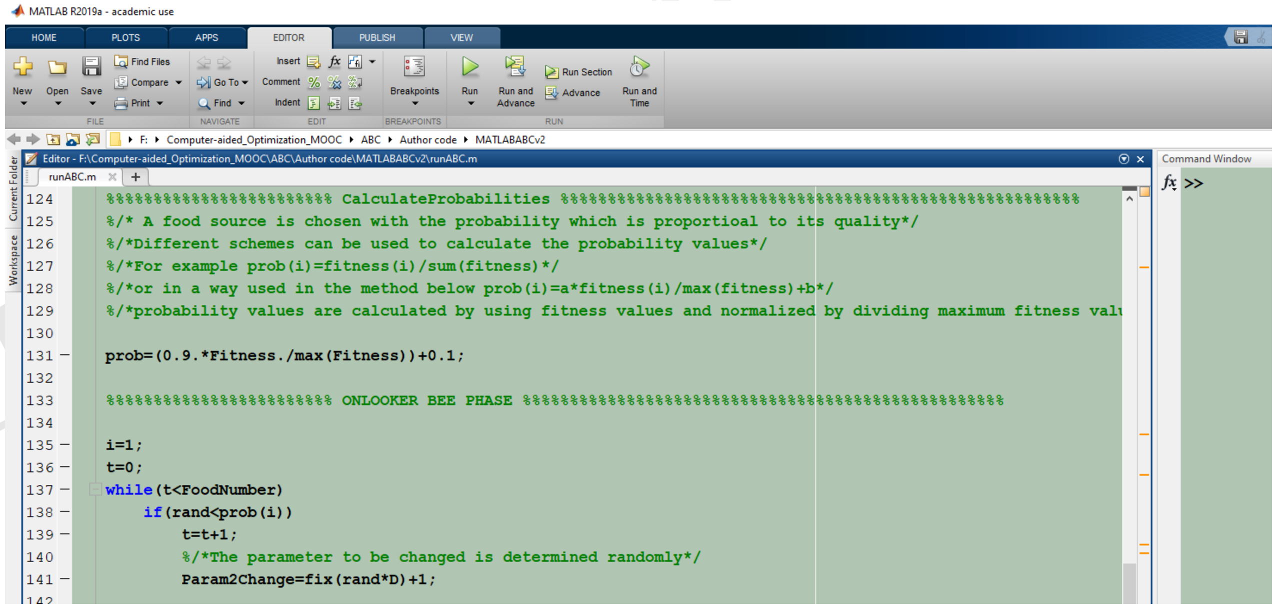
An onlooker bee chooses a food source depending on the probability value associated with that food source, p_i, calculated by the following expression (2.1):

$$p_i = \frac{\text{fit}_i}{\sum_{n=1}^{SN} \text{fit}_n}, \tag{2.1}$$

where fit_i is the fitness value of the solution i evaluated by its employed bee, which is proportional to the nectar amount of the food source in the position i and SN is the number of food sources which is equal to the number of employed bees (BN). In this way, the employed bees exchange their information with the onlookers.

In order to produce a candidate food position from the old one, the ABC uses the following expression (2.2):

$$v_{ij} = x_{ij} + \phi_{ij}(x_{ij} - x_{kj}), \tag{2.2}$$



$$prob_i = \frac{fit_i}{\sum_{i=1}^{N_p} fit_i}$$

$$prob_i = 0.9\left(\frac{fit_i}{\max(fit)}\right) + 0.1$$

A powerful and Efficient Algorithm for Numerical Function Optimization: Artificial Bee Colony (ABC) Algorithm, Journal of Global Optimization, 39(3), 459-171, 2007

Matlab Code: https://abc.erciyes.edu.tr/

Further reading

- An idea based on honey bee swarm for numerical optimization, Technical report-TR06, Erciyes University, Engineering Faculty, Computer Engineering Department 2005
- A powerful and Efficient Algorithm for Numerical Function Optimization: Artificial Bee Colony (ABC) Algorithm, Journal of Global Optimization, 39 (3), 459-171, 2007
- On clarifying misconceptions when comparing variants of the Artificial Bee Colony Algorithm by offering a new implementation, Information Sciences, 291, 115-127, 2015
- A Review on Artificial Bee Colony Algorithms and Their Applications to Data Clustering. Cybernetics and Information Technologies, 17 (3), 3-28, 2017
- A multi-objective artificial bee colony algorithm, Swarm and Evolutionary Computation, 2, 39-52, 2012

Comparison of techniques

	TLBO	PSO	DE	ABC
Phases	Teacher, Learner	No phases (Position and velocity update)	No phases (Mutation and crossover)	Employee, Onlooker and Scout
Convergence	Monotonic	Monotonic (with g_{best} & p_{best})	Monotonic	Monotonic (with globalized memory)
Parameters	Population size, termination criteria	Population size, termination criteria, w, c_1 and c_2	Population size, termination criteria, F	Population size, termination criteria, limit
Generation of solution	Using other solutions, mean and best solution	Using velocity vector, p_{best} and g_{best}	Using other solutions	Using other solutions
Best solution	Part of population	Need not be part of population	Part of population	Need not be part of population
Fitness function	Objective function	Objective function	Objective function	Inversely related to objective function
Population update	Twice	Once	Once	Twice or thrice (Scout phase)
Selection	Greedy	Always accept new solution into the population (μ, λ)	Greedy	Greedy and (μ, λ) (in scout phase)
#FE	$N_p + 2N_pT$	$N_p + N_pT$	$N_p + N_pT$	Max #FE = $N_p + 2N_pT + T$

Thank You !!!