MA 201, MATHEMATICS III, JULY-NOVEMBER, 2022 SESSION

QUIZ II — NOVEMBER 4, 2022 — TIME: 8:00 TO 8:45 AM

MAXIMUM MARKS: 10

INSTRUCTIONS

1. No one will be allowed to enter the exam hall after 8:10 AM.
2. No one will be allowed to sit in any room not assigned to him/her.
3. Invigilators will not respond to any query regarding question(s).
4. No one will be allowed to go out of the exam hall before submission of answer script. In other words, there will not be any bio-break.
5. No one will be allowed to leave the exam hall between 8:35-8:45 AM (till all the answer scripts are collected).

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1. By eliminating the arbitrary functions f and g from $u(x,y) = f(xy) + x\,g(y/x)$, $x \neq 0$, construct an appropriate partial differential equation (PDE). [2 Marks]

2. "A set of semi-linear first-order PDEs is a subset of a set of quasi-linear first-order PDEs." Explain this through the most general form of such PDEs having x, y as independent variables and u as the dependent variable. [1 Mark]

3. What is the "transversality condition" for first-order semi-linear PDEs? Describe the role of this condition in the method of characteristics. [2 Marks]

4. Determine the type of the PDE $y^2 u_{xx} - 2xyu_{xy} + x^2 u_{yy} = \dfrac{y^2}{x} u_x + \dfrac{x^2}{y} u_y$, $x, y \neq 0$ in terms of hyperbolic, parabolic, elliptic. [1 Mark]

5. Show that the curves $x - t =$ constant and $x + t =$ constant are the two characteristics of a hyperbolic PDE whose solution is $u(x,t) = \dfrac{1}{2}[f(x-t) + f(x+t)]$. [2 Marks]

6. Find the Fourier series expansion of the function $f(x) = x$, $-1 < x < 1$. Hence, find the value of $1 - \dfrac{1}{3} + \dfrac{1}{5} - \dfrac{1}{7} + \cdots$. [2+1 Marks]
