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General Algebraic Modelling System (GAMS)

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Generalized Algebraic Modelling System: https://www.youtube.com/watch?v=1KzuNmMuI0w
Solution of Production Planning Problem using GAMS & NEOS, MIRO: https://www.youtube.com/watch?v=Cu_e_Qgs1C8
Additional resources: tinyurl.com/sksopti, tinyurl.com/sksoptivid

General Algebraic Modelling System (GAMS)

- ➢ High-level modelling system for mathematical programming
- ➢ Specifically designed for modelling optimization problems
- ➢ Models written on one platform can run on others too
- ➢ All the solvers are supported on 64-bit windows and Linux OS
- ➢ Used in more than 120 countries

MODELLING

GAMS

SOLVERS	
LP	CPLEX, GUROBI, MOSEK, XPRESS
MILP	CPLEX, GUROBI, MOSEK, XPRESS
NLP	CONOPT, IPOPTH, KNITRO, MINOS, SNOPT
MINLP	ALPHAECP, ANTIGONE, BARON, DICOPT, OQNLP

	LP	MIP	NLP	MCP	MPEC	CNS	DNLP	MINLP	QCP	MIQCP	*Stoch.*	*Global*
ALPHAECP								✓		✓		
ANTIGONE 1.1			✓			✓	✓	✓	✓	✓		✓*
BARON	✓	✓	✓			✓	✓	✓	✓	✓		✓*
BDMLP	✓	✓										
BONMIN 1.8								✓		✓		
CBC 2.10	✓	✓										
CONOPT 3	✓		✓			✓	✓		✓			
CONOPT 4	✓		✓			✓	✓		✓			
COUENNE 0.5			✓			✓	✓	✓	✓	✓		✓*
CPLEX 12.10	✓	✓							✓	✓		
DECIS	✓										✓	
DICOPT								✓		✓		
GLOMIQO 2.3									✓	✓		✓*
GUROBI 9.0	✓	✓							✓	✓		
GUSS	✓	✓	✓	✓		✓	✓	✓	✓	✓		
IPOPT 3.12	✓		✓			✓	✓		✓			
KESTREL	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓		
KNITRO 11.1	✓		✓		✓	✓	✓	✓	✓	✓		
LGO	✓		✓				✓		✓			✓
LINDO 12.0	✓	✓	✓				✓	✓	✓	✓	✓	✓*
LINDOGLOBAL 12.0	✓	✓	✓				✓	✓	✓	✓		✓*
LOCALSOLVER 9.0		✓	✓			✓	✓	✓	✓	✓		
MILES				✓								
MINOS	✓		✓			✓	✓		✓			
MOSEK 9	✓	✓	✓				✓	✓	✓	✓		
MSNLP			✓				✓		✓			✓
NLPEC				✓	✓							
ODHCPLEX 4		✓								✓		
PATH				✓		✓						
SBB								✓		✓		
SCIP 6.0		✓	✓			✓	✓	✓	✓	✓		✓*
SNOPT	✓		✓			✓	✓		✓			
SOLVEENGINE	✓	✓										
SOPLEX 4.0	✓											
XA	✓	✓										
XPRESS 33.01	✓	✓							✓	✓		

* deterministic global solver

https://www.gams.com/products/introduction/
https://www.gams.com/latest/docs/S_MAIN.html

Demo version

- GAMS latest release can be downloaded from: https://www.gams.com/download/

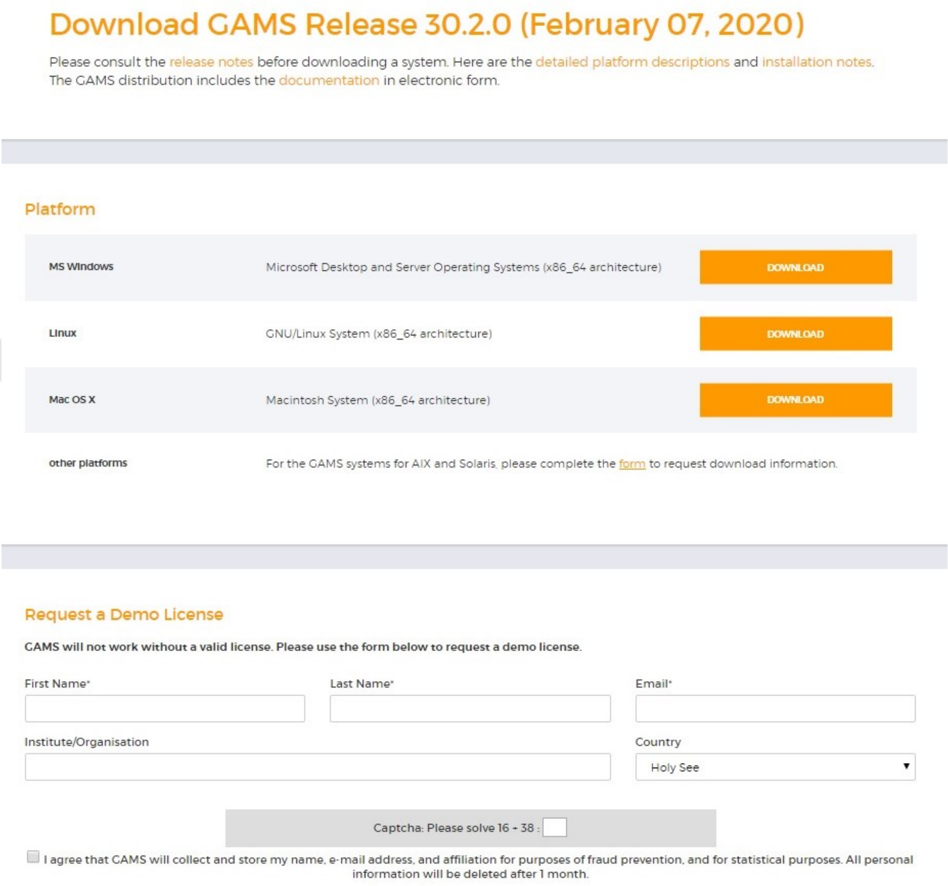
- Based on the operating system of your system, download the appropriate file

- Without license, compiler gives an error

- Model size limits with a demo license

  - For **linear models** (LP, RMIP, and MIP) GAMS will generate and solve models with up to **2000 constraints and 2000 variables**

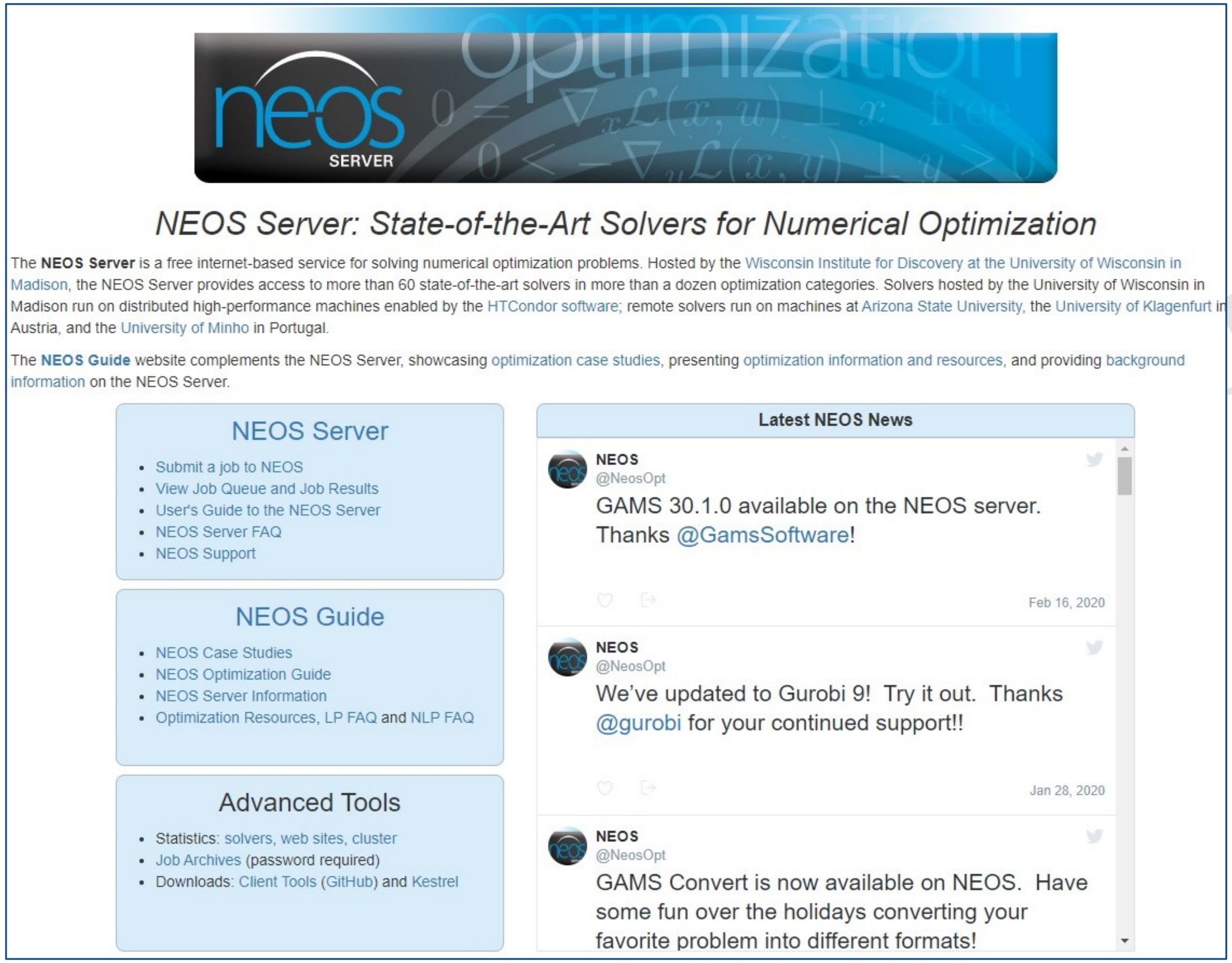
  - For all other model type GAMS will generate and solve models with up to **1000 constraints and 1000 variables**




```
G e n e r a l   A l g e b r a i c   M o d e l i n g   S y s t e m

**** No license specified and no gamslice.txt in system directory
**** Terminated due to a licensing error
**** Request a demo license at www.gams.com/download
```

Network-Enabled Optimization System (NEOS)



neos SERVER

NEOS Server: State-of-the-Art Solvers for Numerical Optimization

The **NEOS Server** is a free internet-based service for solving numerical optimization problems. Hosted by the Wisconsin Institute for Discovery at the University of Wisconsin in Madison, the NEOS Server provides access to more than 60 state-of-the-art solvers in more than a dozen optimization categories. Solvers hosted by the University of Wisconsin in Madison run on distributed high-performance machines enabled by the HTCondor software; remote solvers run on machines at Arizona State University, the University of Klagenfurt in Austria, and the University of Minho in Portugal.

The **NEOS Guide** website complements the NEOS Server, showcasing optimization case studies, presenting optimization information and resources, and providing background information on the NEOS Server.

NEOS Server

- Submit a job to NEOS
- View Job Queue and Job Results
- User's Guide to the NEOS Server
- NEOS Server FAQ
- NEOS Support

NEOS Guide

- NEOS Case Studies
- NEOS Optimization Guide
- NEOS Server Information
- Optimization Resources, LP FAQ and NLP FAQ

Advanced Tools

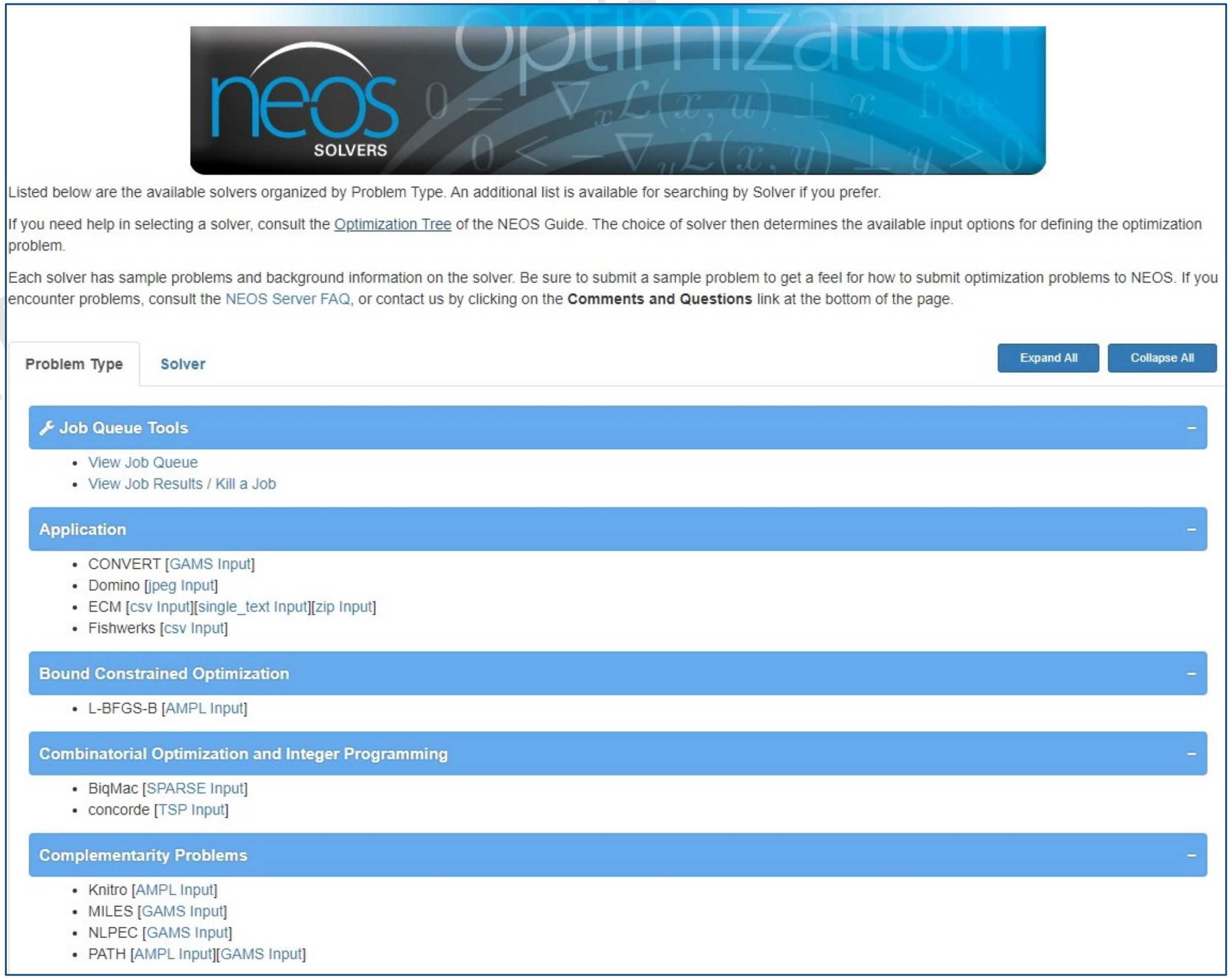
- Statistics: solvers, web sites, cluster
- Job Archives (password required)
- Downloads: Client Tools (GitHub) and Kestrel

Latest NEOS News

NEOS @NeosOpt
GAMS 30.1.0 available on the NEOS server. Thanks @GamsSoftware!
Feb 16, 2020

NEOS @NeosOpt
We've updated to Gurobi 9! Try it out. Thanks @gurobi for your continued support!!
Jan 28, 2020

NEOS @NeosOpt
GAMS Convert is now available on NEOS. Have some fun over the holidays converting your favorite problem into different formats!



neos SOLVERS

Listed below are the available solvers organized by Problem Type. An additional list is available for searching by Solver if you prefer.

If you need help in selecting a solver, consult the Optimization Tree of the NEOS Guide. The choice of solver then determines the available input options for defining the optimization problem.

Each solver has sample problems and background information on the solver. Be sure to submit a sample problem to get a feel for how to submit optimization problems to NEOS. If you encounter problems, consult the NEOS Server FAQ, or contact us by clicking on the **Comments and Questions** link at the bottom of the page.

Problem Type | Solver | Expand All | Collapse All

Job Queue Tools

- View Job Queue
- View Job Results / Kill a Job

Application

- CONVERT [GAMS Input]
- Domino [jpeg Input]
- ECM [csv Input][single_text Input][zip Input]
- Fishwerks [csv Input]

Bound Constrained Optimization

- L-BFGS-B [AMPL Input]

Combinatorial Optimization and Integer Programming

- BiqMac [SPARSE Input]
- concorde [TSP Input]

Complementarity Problems

- Knitro [AMPL Input]
- MILES [GAMS Input]
- NLPEC [GAMS Input]
- PATH [AMPL Input][GAMS Input]

https://neos-server.org/neos/

https://neos-server.org/neos/solvers/index.html

Network-Enabled Optimization System (NEOS)

- Allows to submit GAMS models to an online optimization service for executing on local machines and obtain a solution file
- Provides access to more than 60 state-of-the-art solvers in different optimization categories
- Contributing universities: University of Wisconsin in Madison, Arizona State University, the University of Klagenfurt in Austria, and the University of Minho in Portugal.
- Submit your model at https://neos-server.org/neos/solvers/index.html
- Global optimization: **scip** [AMPL Input][CPLEX Input][GAMS Input][MPS Input][OSIL Input][Python Input][ZIMPL Input]
- MILP: **FICO-Xpress** [AMPL Input][GAMS Input][MOSEL Input][MPS Input][NL Input]

Linear programming

- Minimize the cost of shipping goods from 2 canning plants to 3 markets, subject to supply and demand constraints.

- Details of distance between the plant and market (thousand miles), capacity of each plant and the demand of commodity in each market are given

Plants	Markets			Supply (cases)
	New York	Chicago	Topeka	
Seattle	2.5	1.7	1.8	**350**
San Diego	2.5	1.8	1.4	**600**
Demand (cases)	**325**	**300**	**275**	

- Freight in dollars per case per thousand miles is 90

Problem formulation

Parameters:

d_{ij}: distance between each plant and market

F : freight in dollars per case per thousand miles

a_i: supply of commodity in plant i (in cases)

b_j: demand for commodity at market j (in cases)

C_{ij}: cost per unit shipment between plant i and market j

$$C_{ij} = \frac{Fd_{ij}}{1000}$$

Plants	Markets			Supply (cases)
	New York	Chicago	Topeka	
Seattle	2.5	1.7	1.8	**350**
San Diego	2.5	1.8	1.4	**600**
Demand (cases)	**325**	**300**	**275**	

Decision variables: x_{ij} be the quantities of commodity transported from i^{th} plant to j^{th} market

Objective function: Minimize the total transportation cost (Z) in thousands of dollars

$$\boxed{Z = \sum_{i=1}^{2}\sum_{j=1}^{3} C_{ij}x_{ij}}$$

Problem formulation

Subject to

$$\boxed{\sum_{j=1}^{3} x_{ij} \le a_i \quad \forall i \in \{1,2\}} \Rightarrow \boxed{\text{Supply constraint of each plant}}$$

$$\boxed{\sum_{i=1}^{2} x_{ij} \ge b_j \quad \forall j \in \{1,2,3\}} \Rightarrow \boxed{\text{Demand constraint for each market}}$$

$$\boxed{x_{ij} \ge 0 \quad \forall i \in \{1,2\}; \forall j \in \{1,2,3\}} \Rightarrow \boxed{\text{Bounds}}$$

Plants	Markets			Supply (cases)
	New York	Chicago	Topeka	
Seattle	2.5	1.7	1.8	**350**
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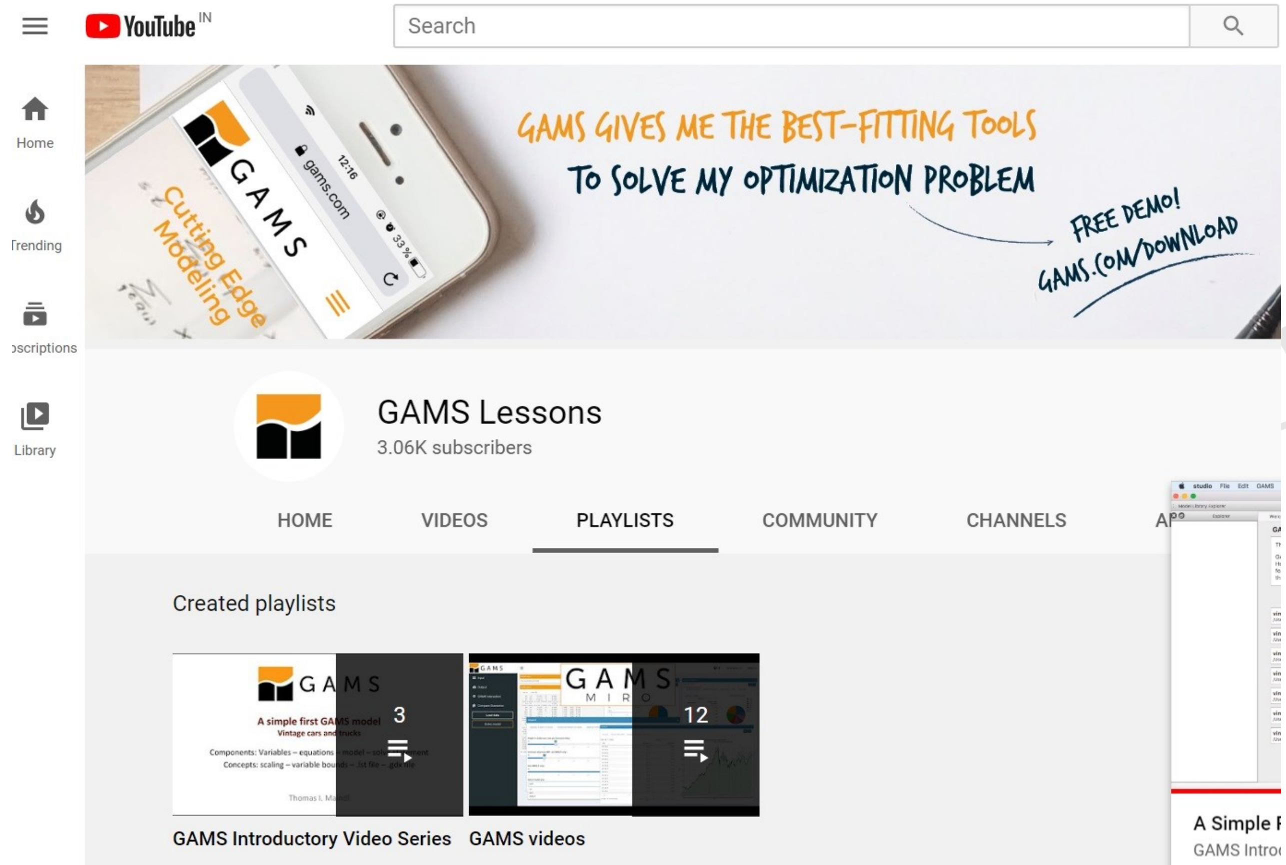
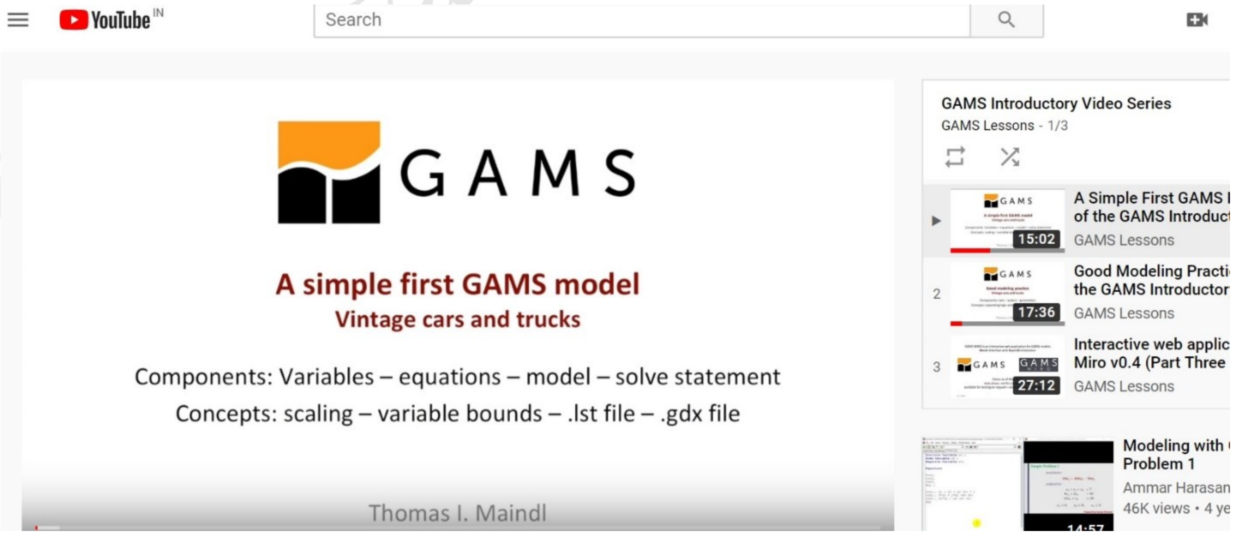
GAMS lessons





https://www.youtube.com/user/GAMSLessons/playlists
https://www.youtube.com/watch?v=ZDkW7QT81Ck&list=PLmUY6XPHlK1kTaepnyKTn7dcrCwsaox10

Structure of a GAMS model and output

	Component
Input	***Sets***
	Declaration
	Assignment of members
	Data (Parameters, Tables, Scalars)
	Declaration
	Assignment of values
	Variables
	Declaration
	Assignment of type
	Assignment of Variable Bounds and/or Initial Values (optional)
	Equations
	Declaration
	Definition
	Model and Solve Statements
	Display Statements (optional)

List file

	Component
Output	*Echo Prints*
	Reference Maps
	Equation Listings
	Status Reports
	Solution Reports

Log file

https://www.gams.com/latest/docs/UG_Tutorial.html

GAMS representation

```
Sets
       i   canning plants      / Seattle, San-Diego /
       j   markets             / New-York, Chicago, Topeka / ;
Parameters
       a(i)  capacity of plant i in cases
         /    Seattle     350
              San-Diego   600  /
       b(j)  demand at market j in cases
         /    New-York    325
              Chicago     300
              Topeka      275  / ;
Table  d(i,j)  distance in thousands of miles
                  New-York       Chicago      Topeka
    Seattle          2.5           1.7          1.8
    San-Diego        2.5           1.8          1.4  ;
Scalar f   freight in dollars per case per thousand miles  /90/ ;
Parameter
       c(i,j)  transport cost in thousands of dollars per case ;
c(i,j) = f * d(i,j) / 1000 ;
Variables
     x(i,j)    shipment quantities in cases
     z         total transportation costs in thousands of dollars ;
Positive variables x ;
Equations
     cost          define objective function
     supply(i)     observe supply limit at plant i
     demand(j)     satisfy demand at market j ;
cost ..          z   =e=  sum((i,j), c(i,j)*x(i,j)) ;
supply(i) ..     sum(j, x(i,j))   =l=   a(i) ;
demand(j) ..     sum(i, x(i,j))   =g=   b(j) ;
Model transport /all/ ;
Solve transport using LP minimizing z ;
```

Assignment of members

Assignment of values es, Scalars)

Assignment of variable type

Definition of equations

Model and Solve statement

$$Z = \sum_{i=1}^{2}\sum_{j=1}^{3} C_{ij}x_{ij}$$

$$\sum_{j=1}^{3} x_{ij} \le a_i \quad \forall i \in \{1,2\}$$

$$\sum_{i=1}^{2} x_{ij} \ge b_j \quad \forall j \in \{1,2,3\}$$

https://www.youtube.com/watch?v=98wJKIuRLPI

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cost ..        z  =e=  sum((i,j), c(i,j)*x(i,j)) ;
supply(i) ..   sum(j, x(i,j))  =l=  a(i) ;
demand(j) ..   sum(i, x(i,j))  =g=  b(j) ;
Model transport /all/ ;
Solve transport using LP minimizing z ;
```

Sets

$$Z = \sum_{i=1}^{2}\sum_{j=1}^{3} C_{ij}x_{ij}$$

$$\sum_{j=1}^{3} x_{ij} \le a_i \quad \forall i \in \{1,2\}$$

$$\sum_{i=1}^{2} x_{ij} \ge b_j \quad \forall j \in \{1,2,3\}$$

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GAMS representation

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demand(j) ..   sum(i, x(i,j))  =g=  b(j) ;
Model transport /all/ ;
Solve transport using LP minimizing z ;
```

Declaration of sets

$$Z = \sum_{i=1}^{2} \sum_{j=1}^{3} C_{ij} x_{ij}$$

$$\sum_{j=1}^{3} x_{ij} \le a_i \quad \forall i \in \{1,2\}$$

$$\sum_{i=1}^{2} x_{ij} \ge b_j \quad \forall j \in \{1,2,3\}$$

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GAMS representation

Assignment of members

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Model transport /all/ ;
Solve transport using LP minimizing z ;
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$$Z=\sum_{i=1}^{2}\sum_{j=1}^{3}C_{ij}x_{ij}$$

$$\sum_{j=1}^{3}x_{ij}\le a_i \qquad \forall i\in\{1,2\}$$

$$\sum_{i=1}^{2}x_{ij}\ge b_j \qquad \forall j\in\{1,2,3\}$$

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Model transport /all/ ;
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```

Data (Parameters, Tables, Scalars)

$$Z = \sum_{i=1}^{2}\sum_{j=1}^{3} C_{ij} x_{ij}$$

$$\sum_{j=1}^{3} x_{ij} \le a_i \qquad \forall i \in \{1,2\}$$

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Model transport /all/ ;
Solve transport using LP minimizing z ;
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Declaration of data

$$Z = \sum_{i=1}^{2}\sum_{j=1}^{3} C_{ij} x_{ij}$$

$$\sum_{j=1}^{3} x_{ij} \le a_i \quad \forall i \in \{1,2\}$$

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Model transport /all/ ;
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Assignment of values

$$Z = \sum_{i=1}^{2}\sum_{j=1}^{3} C_{ij} x_{ij}$$

$$\sum_{j=1}^{3} x_{ij} \le a_i \quad \forall i \in \{1,2\}$$

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Equations
     cost        define objective function
     supply(i)   observe supply limit at plant i
     demand(j)   satisfy demand at market j ;
cost ..        z  =e=  sum((i,j), c(i,j)*x(i,j)) ;
supply(i) ..   sum(j, x(i,j))  =l=  a(i) ;
demand(j) ..   sum(i, x(i,j))  =g=  b(j) ;
Model transport /all/ ;
Solve transport using LP minimizing z ;
```

Variables

$$Z = \sum_{i=1}^{2}\sum_{j=1}^{3} C_{ij} x_{ij}$$

$$\sum_{j=1}^{3} x_{ij} \le a_i \quad \forall i \in \{1,2\}$$

$$\sum_{i=1}^{2} x_{ij} \ge b_j \quad \forall j \in \{1,2,3\}$$

https://www.youtube.com/watch?v=98wJKIuRLPI

GAMS representation

```
Sets
       i   canning plants   / Seattle, San-Diego /
       j   markets          / New-York, Chicago, Topeka / ;
Parameters
       a(i)  capacity of plant i in cases
         /    Seattle     350
              San-Diego   600  /
       b(j)  demand at market j in cases
         /    New-York    325
              Chicago     300
              Topeka      275  / ;
Table  d(i,j)  distance in thousands of miles
                  New-York       Chicago      Topeka
    Seattle          2.5            1.7          1.8
    San-Diego        2.5            1.8          1.4  ;
Scalar f  freight in dollars per case per thousand miles  /90/ ;
Parameter
       c(i,j)  transport cost in thousands of dollars per case ;
c(i,j) = f * d(i,j) / 1000 ;
Variables
     x(i,j)  shipment quantities in cases
     z       total transportation costs in thousands of dollars ;
Positive variables x ;
Equations
     cost          define objective function
     supply(i)     observe supply limit at plant i
     demand(j)     satisfy demand at market j ;
cost ..          z  =e=  sum((i,j), c(i,j)*x(i,j)) ;
supply(i) ..   sum(j, x(i,j))  =l=  a(i) ;
demand(j) ..   sum(i, x(i,j))  =g=  b(j) ;
Model transport /all/ ;
Solve transport using LP minimizing z ;
```

Declaration of equations

Definition of equations

$$Z = \sum_{i=1}^{2}\sum_{j=1}^{3} C_{ij}x_{ij}$$

$$\sum_{j=1}^{3} x_{ij} \le a_i \quad \forall i \in \{1,2\}$$

$$\sum_{i=1}^{2} x_{ij} \ge b_j \quad \forall j \in \{1,2,3\}$$

https://www.youtube.com/watch?v=98wJKIuRLPI

GAMS representation

```
Sets
       i   canning plants   / Seattle, San-Diego /
       j   markets          / New-York, Chicago, Topeka / ;
Parameters
       a(i)  capacity of plant i in cases
         /    Seattle     350
              San-Diego   600  /
       b(j)  demand at market j in cases
         /    New-York    325
              Chicago     300
              Topeka      275  / ;
Table  d(i,j)  distance in thousands of miles
                  New-York       Chicago      Topeka
    Seattle          2.5           1.7          1.8
    San-Diego        2.5           1.8          1.4  ;
Scalar f  freight in dollars per case per thousand miles  /90/ ;
Parameter
       c(i,j)  transport cost in thousands of dollars per case ;
c(i,j) = f * d(i,j) / 1000 ;
Variables
     x(i,j)  shipment quantities in cases
     z       total transportation costs in thousands of dollars ;
Positive variables x ;
Equations
     cost         define objective function
     supply(i)    observe supply limit at plant i
     demand(j)    satisfy demand at market j ;
cost ..        z  =e=  sum((i,j), c(i,j)*x(i,j)) ;
supply(i) ..   sum(j, x(i,j))  =l=  a(i) ;
demand(j) ..   sum(i, x(i,j))  =g=  b(j) ;
Model transport /all/ ;
Solve transport using LP minimizing z ;
```

Model and Solve statement

$$Z = \sum_{i=1}^{2}\sum_{j=1}^{3} C_{ij} x_{ij}$$

$$\sum_{j=1}^{3} x_{ij} \le a_i \quad \forall i \in \{1,2\}$$

$$\sum_{i=1}^{2} x_{ij} \ge b_j \quad \forall j \in \{1,2,3\}$$

https://www.youtube.com/watch?v=98wJKIuRLPI

GAMS representation

```
Sets
       i   canning plants   / Seattle, San-Diego /
       j   markets          / New-York, Chicago, Topeka / ;
Scalar f  freight in dollars per case per thousand miles  /90/ ;

Table  d(i,j)  distance in thousands of miles
                  New-York       Chicago      Topeka
    Seattle          2.5           1.7          1.8
    San-Diego        2.5           1.8          1.4  ;

Parameter
a(i)  capacity of plant i in cases
        /    Seattle     350
             San-Diego   600  /
b(j)  demand at market j in cases
        /    New-York    325
             Chicago     300
             Topeka      275  /
c(i,j)  transport cost in thousands of dollars per case;
       c(i,j) = f * d(i,j) / 1000
Variables
     x(i,j)  shipment quantities in cases
     z       total transportation costs in thousands of dollars ;
Positive variables x ;
Equations
     cost        define objective function
     supply(i)   observe supply limit at plant i
     demand(j)   satisfy demand at market j ;
cost ..        z  =e=  sum((i,j), c(i,j)*x(i,j)) ;
supply(i) ..   sum(j, x(i,j))  =l=  a(i) ;
demand(j) ..   sum(i, x(i,j))  =g=  b(j) ;

Model transport /cost, supply, demand/ ;
Solve transport using LP minimizing z ;
```

Specify the equations to be included

$$Z = \sum_{i=1}^{2}\sum_{j=1}^{3} C_{ij} x_{ij}$$

$$\sum_{j=1}^{3} x_{ij} \le a_i \quad \forall i \in \{1,2\}$$

$$\sum_{i=1}^{2} x_{ij} \ge b_j \quad \forall j \in \{1,2,3\}$$

Output

```
Compilation
Equation Listing   SOLVE transport Using LP From line 34
Equation
Column Listing     SOLVE transport Using LP From line 34
Column
Model Statistics   SOLVE transport Using LP From line 34
Solution Report    SOLVE transport Using LP From line 34
SolEQU
SolVAR
```

```
GAMS 30.2.0  r482c588 Released Feb  7, 2020 WEX-WEI x86 64bit/MS
G e n e r a l   A l g e b r a i c   M o d e l i n g   S y s t e m
C o m p i l a t i o n


   1  Sets
   2         i   canning plants    / Seattle, San-Diego /
   3         j   markets           / New-York, Chicago, Topeka /
   4  Scalar f  freight in dollars per case per thousand miles
   5
   6  Table  d(i,j)  distance in thousands of miles
   7                    New-York       Chicago      Topeka
   8      Seattle          2.5           1.7          1.8
   9      San-Diego        2.5           1.8          1.4  ;
  10
  11  Parameter
  12  a(i)   capacity of plant i in cases
  13            /     Seattle     350
  14                  San-Diego   600  /
  15  b(j)   demand at market j in cases
  16            /     New-York    325
  17                  Chicago     300
  18                  Topeka      275  /
  19  c(i,j)   transport cost in thousands of dollars per case;
  20           c(i,j) = f * d(i,j) / 1000
  21  Variables
  22       x(i,j)  shipment quantities in cases
  23       z       total transportation costs in thousands of do
  24  Positive variables x ;
  25  Equations
  26       cost        define objective function
  27       supply(i)   observe supply limit at plant i
  28       demand(j)   satisfy demand at market j ;
  29  cost ..          z  =e=  sum((i,j), c(i,j)*x(i,j)) ;
  30  supply(i) ..   sum(j, x(i,j))   =l=  a(i) ;
  31  demand(j) ..   sum(i, x(i,j))   =g=  b(j) ;
  32
  33  Model transport /cost, supply, demand/ ;
```

```
        Demo license for demonstration and instructional purposes only
--- Starting compilation
--- Transport.gms(34) 3 Mb
--- Starting execution: elapsed 0:00:00.002
--- Transport.gms(20) 4 Mb
--- Generating LP model transport
--- Transport.gms(34) 4 Mb
---    6 rows  7 columns  19 non-zeroes
--- Executing CPLEX: elapsed 0:00:00.051

IBM ILOG CPLEX   30.2.0 r482c588 Released Feb 07, 2020 WEI x86 64bit/MS Window
*** This solver runs with a demo license. No commercial use.
Cplex 12.10.0.0

Reading data...
Starting Cplex...
Space for names approximately 0.00 Mb
Use option 'names no' to turn use of names off
Version identifier: 12.10.0.0 | 2019-11-26 | 843d4de2ae
CPXPARAM_Advance                                        0
CPXPARAM_Simplex_Display                                2
CPXPARAM_Threads                                        1
CPXPARAM_Parallel                                       1
CPXPARAM_Simplex_Limits_Iterations                      2000000000
CPXPARAM_TimeLimit                                      1000
CPXPARAM_Tune_TimeLimit                                 200
Tried aggregator 1 time.
LP Presolve eliminated 1 rows and 1 columns.
Reduced LP has 5 rows, 6 columns, and 12 nonzeros.
Presolve time = 0.00 sec. (0.00 ticks)

Iteration      Dual Objective            In Variable           Out Variable
     1              73.125000    x(Seattle.New-York)  demand(New-York) slack
     2             119.025000     x(Seattle.Chicago)   demand(Chicago) slack
     3             153.675000     x(San-Diego.Topeka)   demand(Topeka) slack
     4             153.675000  x(San-Diego.New-York)  supply(Seattle) slack
LP status(1): optimal
Cplex Time: 0.05sec (det. 0.01 ticks)

Optimal solution found.
Objective :          153.675000

--- Restarting execution
```

Compilation details

List file

Process log file

Output: List file

Echo prints

```
Sets
        i   canning plants   / Seattle, San-Diego /
        j   markets          / New-York, Chicago, Topeka / ;
Parameters
        a(i)  capacity of plant i in cases
          /    Seattle     350
               San-Diego   600  /
        b(j)  demand at market j in cases
          /    New-York    325
               Chicago     300
               Topeka      275  / ;
Table  d(i,j)  distance in thousands of miles
                   New-York       Chicago      Topeka
    Seattle          2.5           1.7          1.8
    San-Diego        2.5           1.8          1.4  ;
Scalar f  freight in dollars per case per thousand miles  /90/ ;
Parameter
        c(i,j)  transport cost in thousands of dollars per case ;
c(i,j) = f * d(i,j) / 1000 ;
Variables
      x(i,j)  shipment quantities in cases
      z       total transportation costs in thousands of dollars ;
Positive variables x ;
Equations
      cost         define objective function
      supply(i)    observe supply limit at plant i
      demand(j)    satisfy demand at market j ;
cost ..        z  =e=  sum((i,j), c(i,j)*x(i,j)) ;
supply(i) ..   sum(j, x(i,j))  =l=  a(i) ;
demand(j) ..   sum(i, x(i,j))  =g=  b(j) ;
Model transport /all/ ;
Solve transport using LP minimizing z ;
```

GAMS model

```
Sets
        i   canning plants   / Seattle, San-Diego /
        j   markets          / New-York, Chicago, Topeka / ;
Parameters
        a(i)  capacity of plant i in cases
          /    Seattle     350
               San-Diego   600  /
        b(j)  demand at market j in cases
          /    New-York    325
               Chicago     300
               Topeka      275  / ;
Table  d(i,j)  distance in thousands of miles
                   New-York       Chicago      Topeka
    Seattle          2.5           1.7          1.8
    San-Diego        2.5           1.8          1.4  ;
Scalar f  freight in dollars per case per thousand miles  /90/ ;
Parameter
        c(i,j)  transport cost in thousands of dollars per case ;
c(i,j) = f * d(i,j) / 1000 ;
Variables
      x(i,j)  shipment quantities in cases
      z       total transportation costs in thousands of dollars ;
Positive variables x ;
Equations
      cost         define objective function
      supply(i)    observe supply limit at plant i
      demand(j)    satisfy demand at market j ;
cost ..        z  =e=  sum((i,j), c(i,j)*x(i,j)) ;
supply(i) ..   sum(j, x(i,j))  =l=  a(i) ;
demand(j) ..   sum(i, x(i,j))  =g=  b(j) ;
Model transport /all/ ;
Solve transport using LP minimizing z ;
```

Output: List file

Equation Listing

```
---- cost  =E=  define objective function

cost..  - 0.225*x(Seattle,New-York) - 0.153*x(Seattle,Chicago)

      - 0.162*x(Seattle,Topeka) - 0.225*x(San-Diego,New-York)

      - 0.162*x(San-Diego,Chicago) - 0.126*x(San-Diego,Topeka) + z =E= 0 ;

      (LHS = 0)


---- supply  =L=  observe supply limit at plant i

supply(Seattle)..  x(Seattle,New-York) + x(Seattle,Chicago) + x(Seattle,Topeka)
      =L= 350 ;  (LHS = 0)

supply(San-Diego)..  x(San-Diego,New-York) + x(San-Diego,Chicago)

      + x(San-Diego,Topeka) =L= 600 ;  (LHS = 0)


---- demand  =G=  satisfy demand at market j

demand(New-York)..  x(Seattle,New-York) + x(San-Diego,New-York) =G= 325 ;

      (LHS = 0, INFES = 325 ****)

demand(Chicago)..  x(Seattle,Chicago) + x(San-Diego,Chicago) =G= 300 ;

      (LHS = 0, INFES = 300 ****)

demand(Topeka)..  x(Seattle,Topeka) + x(San-Diego,Topeka) =G= 275 ;

      (LHS = 0, INFES = 275 ****)
```

GAMS model

```
Sets
       i   canning plants    / Seattle, San-Diego /
       j   markets           / New-York, Chicago, Topeka / ;
Parameters
       a(i)  capacity of plant i in cases
         /    Seattle     350
              San-Diego   600  /
       b(j)  demand at market j in cases
         /    New-York    325
              Chicago     300
              Topeka      275  / ;
Table  d(i,j)  distance in thousands of miles
                    New-York       Chicago      Topeka
    Seattle            2.5            1.7          1.8
    San-Diego          2.5            1.8          1.4  ;
Scalar f  freight in dollars per case per thousand miles  /90/ ;
Parameter
       c(i,j)  transport cost in thousands of dollars per case ;
c(i,j) = f * d(i,j) / 1000 ;
Variables
     x(i,j)  shipment quantities in cases
     z       total transportation costs in thousands of dollars ;
Positive variables x ;
Equations
     cost        define objective function
     supply(i)   observe supply limit at plant i
     demand(j)   satisfy demand at market j ;
cost ..        z  =e=  sum((i,j), c(i,j)*x(i,j)) ;
supply(i) ..   sum(j, x(i,j))  =l=  a(i) ;
demand(j) ..   sum(i, x(i,j))  =g=  b(j) ;
Model transport /all/ ;
Solve transport using LP minimizing z ;
```

Output: List file

Model Statistics

```
MODEL STATISTICS

BLOCKS OF EQUATIONS           3     SINGLE EQUATIONS            6
BLOCKS OF VARIABLES           2     SINGLE VARIABLES            7
NON ZERO ELEMENTS            19
```

Status Reports

```
               S O L V E      S U M M A R Y

     MODEL    transport              OBJECTIVE  z
     TYPE     LP                     DIRECTION  MINIMIZE
     SOLVER   CPLEX                  FROM LINE  32

**** SOLVER STATUS       1 Normal Completion
**** MODEL STATUS        1 Optimal
**** OBJECTIVE VALUE                 153.6750

 RESOURCE USAGE, LIMIT          0.047      1000.000
 ITERATION COUNT, LIMIT         4     2000000000
```

GAMS model

```
Sets
       i   canning plants     / Seattle, San-Diego /
       j   markets            / New-York, Chicago, Topeka / ;
Parameters
       a(i)  capacity of plant i in cases
         /    Seattle     350
              San-Diego   600  /
       b(j)  demand at market j in cases
         /    New-York    325
              Chicago     300
              Topeka      275  / ;
Table  d(i,j)  distance in thousands of miles
                  New-York       Chicago      Topeka
    Seattle          2.5           1.7          1.8
    San-Diego        2.5           1.8          1.4  ;
Scalar f  freight in dollars per case per thousand miles  /90/ ;
Parameter
       c(i,j)  transport cost in thousands of dollars per case ;
c(i,j) = f * d(i,j) / 1000 ;
Variables
     x(i,j)  shipment quantities in cases
     z       total transportation costs in thousands of dollars ;
Positive variables x ;
Equations
     cost        define objective function
     supply(i)   observe supply limit at plant i
     demand(j)   satisfy demand at market j ;
cost ..         z  =e=  sum((i,j), c(i,j)*x(i,j)) ;
supply(i) ..    sum(j, x(i,j))  =l=  a(i) ;
demand(j) ..    sum(i, x(i,j))  =g=  b(j) ;
Model transport /all/ ;
Solve transport using LP minimizing z ;
```

Output: List file

Solution Reports

```
                         LOWER     LEVEL     UPPER    MARGINAL

---- EQU cost              .         .         .        1.000

  cost  define objective function

---- EQU supply  observe supply limit at plant i

               LOWER     LEVEL     UPPER    MARGINAL

Seattle        -INF     350.000   350.000     EPS
San-Diego      -INF     550.000   600.000      .

---- EQU demand  satisfy demand at market j

              LOWER     LEVEL     UPPER    MARGINAL

New-York    325.000   325.000     +INF      0.225
Chicago     300.000   300.000     +INF      0.153
Topeka      275.000   275.000     +INF      0.126

---- VAR x  shipment quantities in cases

                          LOWER     LEVEL     UPPER    MARGINAL

Seattle  .New-York          .       50.000     +INF       .
Seattle  .Chicago           .      300.000     +INF       .
Seattle  .Topeka            .         .        +INF      0.036
San-Diego.New-York          .      275.000     +INF       .
San-Diego.Chicago           .         .        +INF      0.009
San-Diego.Topeka            .      275.000     +INF       .
```

Report Summary

```
**** REPORT SUMMARY :        0     NONOPT
                             0 INFEASIBLE
                             0  UNBOUNDED
```

GAMS model

```
Sets
       i   canning plants     / Seattle, San-Diego /
       j   markets            / New-York, Chicago, Topeka / ;
Parameters
       a(i)  capacity of plant i in cases
         /    Seattle     350
              San-Diego   600   /
       b(j)  demand at market j in cases
         /    New-York    325
              Chicago     300
              Topeka      275   / ;
Table  d(i,j)  distance in thousands of miles
                  New-York       Chicago      Topeka
    Seattle          2.5           1.7          1.8
    San-Diego        2.5           1.8          1.4  ;
Scalar f  freight in dollars per case per thousand miles  /90/ ;
Parameter
       c(i,j)  transport cost in thousands of dollars per case ;
c(i,j) = f * d(i,j) / 1000 ;
Variables
     x(i,j)  shipment quantities in cases
     z       total transportation costs in thousands of dollars ;
Positive variables x ;
Equations
     cost         define objective function
     supply(i)    observe supply limit at plant i
     demand(j)    satisfy demand at market j ;
cost ..        z  =e=  sum((i,j), c(i,j)*x(i,j)) ;
supply(i) ..   sum(j, x(i,j))  =l=  a(i) ;
demand(j) ..   sum(i, x(i,j))  =g=  b(j) ;
Model transport /all/ ;
Solve transport using LP minimizing z ;
```

Output: Log file

```
--- Job Transport.gms Start 03/08/20 18:33:13 30.2.0 r482c588 WEX-WEI x86 64bit/MS Windows
--- GAMS Parameters defined
    Input C:\Users\PKotecha\Documents\gamsdir\projdir\Transport.gms
    PageSize 0
    ScrDir C:\Users\PKotecha\Documents\gamsdir\projdir\225a\
    SysDir C:\GAMS\win64\30.2\
    LogOption 3
    ErrMsg 1
    ErrorLog 99
    IDE 1
    LstTitleLeftAligned 1
GAMS 30.2.0   Copyright (C) 1987-2020 GAMS Development. All rights reserved
Licensee: GAMS Demo license for Remya Kommadath          G200303|0002CO-GEN
          Indian Institute of Technology Guwahati, India           DL003328
          remyakommadath@gmail.com, Remya Kommadath
          Demo license for demonstration and instructional purposes only
--- Starting compilation
--- Transport.gms(33) 3 Mb
--- Starting execution: elapsed 0:00:00.004
--- Transport.gms(20) 4 Mb
--- Generating LP model transport
--- Transport.gms(33) 4 Mb
---   6 rows  7 columns  19 non-zeroes
--- Executing CPLEX: elapsed 0:00:00.021

IBM ILOG CPLEX   30.2.0 r482c588 Released Feb 07, 2020 WEI x86 64bit/MS Window
*** This solver runs with a demo license. No commercial use.
Cplex 12.10.0.0

Reading data...
Starting Cplex...
Space for names approximately 0.00 Mb
Use option 'names no' to turn use of names off
Version identifier: 12.10.0.0 | 2019-11-26 | 843d4de2ae
CPXPARAM_Advance                                 0
CPXPARAM_Simplex_Display                         2
CPXPARAM_Threads                                 1
CPXPARAM_Parallel                                1
CPXPARAM_Simplex_Limits_Iterations               2000000000
CPXPARAM_TimeLimit                               1000
CPXPARAM_Tune_TimeLimit                          200
Tried aggregator 1 time.
LP Presolve eliminated 1 rows and 1 columns.
Reduced LP has 5 rows, 6 columns, and 12 nonzeros.
Presolve time = 0.00 sec. (0.00 ticks)

Iteration      Dual Objective            In Variable           Out Variable
     1              73.125000    x(Seattle.New-York) demand(New-York) slack
     2             119.025000     x(Seattle.Chicago)  demand(Chicago) slack
     3             153.675000    x(San-Diego.Topeka)   demand(Topeka) slack
     4             153.675000  x(San-Diego.New-York)  supply(Seattle) slack
LP status(1): optimal
Cplex Time: 0.00sec (det. 0.01 ticks)

Optimal solution found.
Objective :         153.675000

--- Restarting execution
--- Transport.gms(33) 2 Mb
--- Reading solution for model transport
*** Status: Normal completion
--- Job Transport.gms Stop 03/08/20 18:33:13 elapsed 0:00:00.194
```


Details:

- ➢Location of the .gms file
- ➢GAMS version and license details
- ➢Settings for execution
- ➢Objective obtained in each iteration
- ➢Time taken for the completion
- ➢Reason for exit
- ➢Objective function value

Mixed Integer Linear Programming

- Reddy Mikks company produces interior and exterior paints from raw materials, M1 and M2.
- Daily demand for interior paint cannot exceed that for exterior paint by more than 1 unit.
- Maximum daily demand for the interior paint is 2 units.
- Determine optimum quantity of interior and exterior paints that maximizes total daily profit.

	Exterior paint	Interior paint	Availability
M1	6	4	24
M2	1	2	6
Profit	5	4	

Operations research an introduction, H.A. Taha

Mixed Integer Linear Programming

Let x_1 = Units of exterior paint produced daily

x_2 = Units of interior paints produced daily

Decision variables

Maximize Profit, $Z = 5x_1 + 4x_2$

Objective function

Subject to

$$6x_1 + 4x_2 \leq 24$$
$$x_1 + 2x_2 \leq 6$$

Raw material constraints

$$x_2 \leq x_1 + 1$$

Demand constraint

Constraints

$$x_1, x_2 \geq 0$$
$$x_2 \leq 2$$

Bound constraints

	Ext. paint	Int. paint	Availability
M1	6	4	24
M2	1	2	6
Profit	5	4	

Daily demand for interior paint cannot exceed that for exterior paint by more than 1 Unit.

GAMS representation

```
Variables
        x1      Units of exterior paint produced daily
        x2      Units of interior paint produced daily
        Z       Total Profit;

integer variables
        x1      Units of exterior paint produced daily
        x2      Units of interior paint produced daily

Equations
        Profit  Objective function
        Raw1    Constraint on raw material 1
        Raw2    Constraint on raw material 2
        Demand  Demand constriant;

Profit ..          Z =e= 5*x1 + 4*x2;
Raw1 ..         6*x1 + 4*x2 =l=  24;
Raw2 ..          x1 + 2*x2 =l=    6;
Demand ..       -x1 +  x2 =l=    1;

x2.up = 2;

Model PaintProblem /all/ ;
Solve PaintProblem using mip maximizing z ;
```

Declaration of variables

Assignment of variable type

Declaration and definition of equations

Upper bound of the variables

Model and Solve statement

Maximize $Z = 5x_1 + 4x_2$

$6x_1 + 4x_2 \leq 24$

$x_1 + 2x_2 \leq 6$

$x_2 \leq x_1 + 1$

$x_2 \leq 2$

x_1 and x_2 are integers

Output: List file

```
MODEL STATISTICS

BLOCKS OF EQUATIONS           4     SINGLE EQUATIONS              4
BLOCKS OF VARIABLES           3     SINGLE VARIABLES              3
NON ZERO ELEMENTS             9     DISCRETE VARIABLES            2
```

Status Reports

```
               S O L V E      S U M M A R Y

     MODEL   MILPproblem          OBJECTIVE  Z
     TYPE    MIP                  DIRECTION  MAXIMIZE
     SOLVER  CPLEX                FROM LINE  23

**** SOLVER STATUS     1 Normal Completion
**** MODEL STATUS      1 Optimal
**** OBJECTIVE VALUE             20.0000

 RESOURCE USAGE, LIMIT          0.250      1000.000
 ITERATION COUNT, LIMIT         0      2000000000
```

Solution Reports

```
                      LOWER     LEVEL     UPPER    MARGINAL

---- EQU Profit         .         .         .        1.000
---- EQU Raw1         -INF     24.000    24.000       .
---- EQU Raw2         -INF      4.000     6.000       .
---- EQU Demand       -INF     -4.000     1.000       .

  Profit  Objective function
  Raw1  Constraint on raw material 1
  Raw2  Constraint on raw material 2
  Demand  Demand constriant

                      LOWER     LEVEL     UPPER    MARGINAL

---- VAR x1             .       4.000   100.000      5.000
---- VAR x2             .         .       2.000      4.000
---- VAR Z            -INF     20.000     +INF        .
```

GAMS model

```
Variables
              x1       Units of exterior paint produced daily
              x2       Units of interior paint produced daily
              Z        Total Profit;

integer variables
              x1       Units of exterior paint produced daily
              x2       Units of interior paint produced daily;

Equations
              Profit   Objective function
              Raw1     Constraint on raw material 1
              Raw2     Constraint on raw material 2
              Demand   Demand constriant;

Profit ..             Z =e= 5*x1 + 4*x2;
Raw1 ..            6*x1 + 4*x2 =l=  24;
Raw2 ..              x1 + 2*x2 =l=   6;
Demand ..           -x1 +   x2 =l=   1;

x2.up = 2;

Model PaintProblem /all/ ;
Solve PaintProblem using mip maximizing z ;
```

Non Linear Programming

Minimize $f(x) = x_1$

Objective function

Subject to

$$x_1 x_2 = 1$$

$$\left(\frac{x_3 x_4}{x_1}\right) = 4.8$$

$$\left(\frac{x_5 x_6}{x_2}\right) = 0.98$$

$$x_6 x_4 = 1$$

$$x_1 + 10^{-7} x_3 = x_2 + 10^{-5} x_5$$

$$2x_1 + 10^{-7} x_3 + 10^{-2} x_6 = 2x_2 + 10^{-5} x_5 + 10^{-2} x_4$$

Constraints

Variables	Initial value
x_1	1
x_2	1
x_3	1
x_4	1
x_5	1
x_6	1

https://www.gams.com/latest/gamslib_ml/libhtml/gamslib_wall.html

GAMS representation

```
Variables x1, x2, x3, x4, x5, x6 ;

Equations r1, r2, r3, r4, b1, b2 ;

r1..  x1 * x2 =e= 1 ;
r2..  x3 * x4 / x1  =e= 4.8 ;
r3..  x5 * x6 / x2  =e= .98 ;
r4..  x6 * x4 =e= 1 ;
b1..  x1 + 1e-7*x3 =e= x2 + 1e-5*x5 ;
b2..  2 * x1 + 1e-7*x3 + 1e-2*x6 =e= 2 * x2 + 1e-5*x5 + 1e-2*x4 ;

Model wall / all / ;

x1.l=1; x2.l=1; x3.l=1; x4.l=1; x5.l=1; x6.l=1;

solve wall using nlp minimizing x1;
```

Starting point of the variables

$$minimize \quad f(x) = x_1$$

$$x_1 x_2 = 1$$

$$\left(\frac{x_3 x_4}{x_1}\right) = 4.8$$

$$\left(\frac{x_5 x_6}{x_2}\right) = 0.98$$

$$x_6 x_4 = 1$$

$$x_1 + 10^{-7} x_3 = x_2 + 10^{-5} x_5$$

$$2x_1 + 10^{-7} x_3 + 10^{-2} x_6 = 2x_2 + 10^{-5} x_5 + 10^{-2} x_4$$

Variables	Initial value
x_1	1
x_2	1
x_3	1
x_4	1
x_5	1
x_6	1

https://www.gams.com/latest/gamslib_ml/libhtml/gamslib_wall.html

Output: List file

Status Reports

```
MODEL STATISTICS

BLOCKS OF EQUATIONS           6     SINGLE EQUATIONS          6
BLOCKS OF VARIABLES           6     SINGLE VARIABLES          6
NON ZERO ELEMENTS            20     NON LINEAR N-Z           10
DERIVATIVE POOL              20     CONSTANT POOL            16
CODE LENGTH                  22
```

```
               S O L V E      S U M M A R Y

     MODEL   wall                 OBJECTIVE  x1
     TYPE    NLP                  DIRECTION  MINIMIZE
     SOLVER  BARON                FROM LINE  16

**** SOLVER STATUS     1 Normal Completion
**** MODEL STATUS      2 Locally Optimal
**** OBJECTIVE VALUE               -1.0000

 RESOURCE USAGE, LIMIT          0.100      1000.000
 ITERATION COUNT, LIMIT         0     2000000000
 EVALUATION ERRORS              0              0
```

Solution Reports

```
                     LOWER     LEVEL     UPPER    MARGINAL

---- EQU r1          1.000     1.000     1.000     -0.500
---- EQU r2          4.800     4.800     4.800       .
---- EQU r3          0.980     0.980     0.980 -5.005E-6
---- EQU r4          1.000     1.000     1.000 -2.331E-6
---- EQU b1            .         .         .        0.501
---- EQU b2            .         .         .    -2.572E-4

                     LOWER     LEVEL     UPPER    MARGINAL

---- VAR x1          -INF     -1.000     +INF        .
---- VAR x2          -INF     -1.000     +INF        .
---- VAR x3          -INF     -4.798     +INF        .
---- VAR x4          -INF      1.000     +INF        .
---- VAR x5          -INF     -0.980     +INF        .
---- VAR x6          -INF      1.000     +INF        .
```

GAMS model

```
Variables x1, x2, x3, x4, x5, x6 ;

Equations r1, r2, r3, r4, b1, b2 ;

r1..  x1 * x2 =e= 1 ;
r2..  x3 * x4 / x1  =e= 4.8 ;
r3..  x5 * x6 / x2  =e= .98 ;
r4..  x6 * x4 =e= 1 ;
b1..  x1 + 1e-7*x3 =e= x2 + 1e-5*x5 ;
b2..  2 * x1 + 1e-7*x3 + 1e-2*x6 =e= 2 * x2 + 1e-5*x5 + 1e-2*x4 ;

Model wall / all / ;

x1.l=1; x2.l=1; x3.l=1; x4.l=1; x5.l=1; x6.l=1;

solve wall using nlp minimizing x1;
```

Mixed Integer Nonlinear Programming

Minimize $Z = \left(-3 + x_1\right)^2 + \left(-2 + x_2\right)^2 + \left(4 + x_3\right)^2$ **Objective function**

Subject to

$$\left.\begin{aligned} &\sqrt{x_3} + x_1 + 2x_2 \geq 10 \\ &0.24x_1^2 - x_2 + 0.26x_3 \geq -3 \\ &x_2^2 - \frac{1}{x_3^3\sqrt{x_3}} - 4x_1 \geq -12 \end{aligned}\right\} \text{Constraints}$$

$$\left.\begin{aligned} &x_1, x_2 \leq 200 \ \left(\textit{Integer variables}\right) \\ &0.001 \leq x_3 \leq 200 \ \left(\textit{Continuous variable}\right) \end{aligned}\right\} \text{Bound constraints}$$

Variables	Initial value
x_1	1
x_2	1
x_3	1

http://www.minlplib.org/nvs08.html

GAMS representation

```
Variables  i1,i2,x3,objvar;

Integer Variables  i1,i2;

Equations  e1,e2,e3,e4;


e1.. sqrt(x3) + i1 + 2*i2 =G= 10;

e2.. 0.24*sqr(i1) - i2 + 0.26*x3 =G= -3;

e3.. sqr(i2) - 1/(POWER(x3,3)*sqrt(x3)) - 4*i1 =G= -12;

e4.. -(sqr((-3) + i1) + sqr((-2) + i2) + sqr(4 + x3)) + objvar =E= 0;

* set non-default bounds
i1.up = 200;
i2.up = 200;
x3.lo = 0.001; x3.up = 200;

* set non-default levels
i1.l = 1;
i2.l = 1;
x3.l = 1;

Model m / all /;
Solve m using MINLP minimizing objvar;
```

$$\textit{Minimize } Z = \left(-3+x_1\right)^2 + \left(-2+x_2\right)^2 + \left(4+x_3\right)^2$$

subject to

$$\sqrt{x_3} + x_1 + 2x_2 \geq 10$$

$$0.24x_1^2 - x_2 + 0.26x_3 \geq -3$$

$$x_2^2 - \frac{1}{x_3^3\sqrt{x_3}} - 4x_1 \geq -12$$

Bounds of the variables

$$x_1, x_2 \leq 200 \ \left(\textit{Integer variables}\right)$$

$$0.001 \leq x_3 \leq 200 \ \left(\textit{Continuous variable}\right)$$

Starting point of the variables

Variables	Initial value
x_1	1
x_2	1
x_3	1

Output: List file

Status Reports

```
MODEL STATISTICS

BLOCKS OF EQUATIONS            4     SINGLE EQUATIONS           4
BLOCKS OF VARIABLES            4     SINGLE VARIABLES           4
NON ZERO ELEMENTS             13     NON LINEAR N-Z             7
DERIVATIVE POOL               20     CONSTANT POOL             19
CODE LENGTH                   37     DISCRETE VARIABLES         2
```

```
               S O L V E      S U M M A R Y

     MODEL   m                    OBJECTIVE  objvar
     TYPE    MINLP                DIRECTION  MINIMIZE
     SOLVER  DICOPT               FROM LINE  27

**** SOLVER STATUS     1 Normal Completion
**** MODEL STATUS      8 Integer Solution
**** OBJECTIVE VALUE               23.4497

 RESOURCE USAGE, LIMIT          0.529      1000.000
 ITERATION COUNT, LIMIT       210     2000000000
 EVALUATION ERRORS              0             0
```

Solution Reports

```
                     LOWER     LEVEL     UPPER    MARGINAL

---- EQU e1          10.000    10.795     +INF        .
---- EQU e2          -3.000     1.004     +INF        .
---- EQU e3         -12.000   -12.000     +INF       0.334
---- EQU e4             .         .         .        1.000

                     LOWER     LEVEL     UPPER    MARGINAL

---- VAR i1             .        4.000   200.000     3.337
---- VAR i2             .        3.000   200.000    -0.005
---- VAR x3            0.001     0.631   200.000      .
---- VAR objvar        -INF     23.450     +INF       .
```

GAMS model

```
Variables   i1,i2,x3,objvar;

Integer Variables  i1,i2;

Equations   e1,e2,e3,e4;


e1.. sqrt(x3) + i1 + 2*i2 =G= 10;

e2.. 0.24*sqr(i1) - i2 + 0.26*x3 =G= -3;

e3.. sqr(i2) - 1/(POWER(x3,3)*sqrt(x3)) - 4*i1 =G= -12;

e4.. -(sqr((-3) + i1) + sqr((-2) + i2) + sqr(4 + x3)) + objvar =E= 0;

* set non-default bounds
i1.up = 200;
i2.up = 200;
x3.lo = 0.001; x3.up = 200;

* set non-default levels
i1.l = 1;
i2.l = 1;
x3.l = 1;

Model m / all /;
Solve m using MINLP minimizing objvar;
```

GAMS modelling: General remarks

- Rule governing the ordering of statements is that an entity of the model cannot be referenced before it is declared to exist.
- GAMS statements may be laid out typographically in almost any style that is appealing to the user
  - Multiple lines per statement as well as multiple statements in one line are allowed
- Should terminate every statement with a semicolon
- An asterisk (*) in the first column of a line means that the line will not be processed, but treated as a comment
- GAMS is not case sensitive

https://www.gams.com/latest/docs/UG_Tutorial.html

Common errors

- When compiler encounters error in the input file,
  - A coded error message is inserted in the echo print, immediately after the line containing error
  - Error message starts with **** and contains $ sign followed by a numerical error code

- Reserved words could not be used as identifiers

```
set q quarterly time periods / spring, sos1, fall, wtr / ;
```

```
    In echo print
    1 set q quarterly time periods / spring, sos1, fall, wtr / ;
****                                               $160
```

GAMS compiler indicates that something is wrong with the set element sos1

```
Error Message
160 Unique element expected....
```

Problem is that sos1 is a reserved word, which can not be used as identifiers

https://www.gams.com/latest/docs/UG_Tutorial.html

Common errors

➤Preceding to direct assignment or equation definition a semicolon must be inserted

```
Parameter c(i,j) transport cost in 1000s of dollars per case
          c(i,j) = f * d(i,j) / 1000
```

Error message in echo print

```
16 Parameter c(i,j) transport cost in 1000s of dollars per case
17 c(i,j) = f * d(i,j)/1000
****        $97          $195,96,409
Error Message
96 Blank needed between identifier and text
      (-or- illegal character in identifier)
      (-or- check for missing ';' on previous line)
97 Explanatory text can not start with '$', '=', or '..'
      (-or- check for missing ';' on previous line)
195 Symbol redefined with a different type
409 Unrecognizable item - skip to find a new statement
    looking for a ';' or a key word to get started again
```

No of error messages = 4
Fix the first error ($97)

Detail description of first error

Error message appropriately advises us to check the preceding line for a missing semicolon

```
Parameter c(i,j) transport cost in 1000s of dollars per case;
          c(i,j) = f * d(i,j) / 1000
```

Inserted a semicolon

https://www.gams.com/latest/docs/UG_Tutorial.html

Common errors

➢Spelling mistakes in the identifiers should not be there

```
sets
     i canning plants /seattle, san-diego /
     j markets /new-york, chicago, topeka / ;

table d(i,j) distance in thousand of miles
                 new-york chicago topeka
   seatle             2.5     1.7     1.8
****   $170
san-diego 2.5 1.8 1.4 ;

Error Message
170 Domain violation for element
```

Spelling of ‘Seattle’ is different in the set declaration and in the table

https://www.gams.com/latest/docs/UG_Tutorial.html

GAMS model of production planning problem

```
 SETS
j process /1*54/, lev Level /l,m,h/, k RawType /r1,r2/;

TABLE rm(k,j) Raw material process j at type k
        1        2        3        4        5        6        7        8
r1    0.948   0.9432   0.949   0.9546   0.955    1.045    1.05     0.5103
r2      0        0        0        0       0        0        0        0

TABLE c(lev,j) Capacity process j at Level lev
        1      2      3       4      5      6       7       8       9
l      70     75    77.5     70    47.5    40      40      45      40
m     135    150    155     145    95      80      80      90      80
h     270    300    310     290    190    160     160     180      160


Table PC(lev,j) production cost of process j at Level lev
    1        2        3        4        5        6        7        8        9
l  50.7     56.8     56.9     51.7     38.2     38.5     31.8     37.8     38.
m  90.1     103.8    103.7    97.6     69.8     65.2     57.1     57.7     65.
h  170.7    196.2    195.7    184.8    130.4    120.7    105.5    94.9     119

Table IC(lev,j) investment cost for process j at Level lev
      1          2         3         4         5         6         7         8
l    55         58        60.2      55.1      43.3      66.2      40        106.6
m    81.1       85.1      86.8      83.1      66.8      92.8      61.4      151.7
h    131.6      132.4     134.1     132       104.3     153.2     95.1      231.5

parameters
SP(j) selling price of process j
/1    0.975, 2    0.975, 3    0.975, 4   0.975, 5    0.975, 6   0.78, 7    0.78, 8

parameter
R(k) available feedstock
/ r1   500, r2    500/;

Scalar B budget /1000/;
```


Sale Price	Product	Process	Capacity			Production cost			Investment cost			Raw material required		
			l_j	m_j	h_j	cl_j	cm_j	ch_j	il_j	im_j	ih_j	*rm*1	*rm*2	*rm*3
0.975	T1	P1	70	135	270	50.7	90.1	170.7	55	81.1	131.6	0.948	0	0
		P2	75	150	300	56.8	103.8	196.2	58	85.1	132.4	0.9432	0	0
		P3	77.5	155	310	56.9	103.7	195.7	60.2	86.8	134.1	0.949	0	0
0.975	T2	P4	70	145	290	51.7	97.6	184.8	55.1	83.1	132	0.9546	0	0
		P5	47.5	95	190	38.2	69.8	130.4	43.3	66.8	104.3	0.955	0	0
0.780	T3	P6	40	80	160	38.5	65.2	120.7	66.2	92.8	153.2	1.045	0	0
		P7	40	80	160	31.8	57.1	105.5	40	61.4	95.1	1.05	0	0
0.735	T4	P8	45	90	180	37.8	57.7	94.9	106.6	151.7	231.5	0.5103	0	0
1.450	T5	P9	40	80	160	38.5	65.6	119.1	82.8	125.4	207	0.6289	0	0
1.130	T6	P10	90	180	360	92.2	159.2	290.9	233.5	390.7	698.7	0.8648	0	0
		P11	90	180	360	86.7	154.1	287.7	185.8	304.5	537.1	0.9546	0	0
		P12	90	180	360	95.8	175	330.9	119	179.4	289.2	0.8265	0	0
		P13	90	180	360	87.5	157.2	294.9	212.3	362.7	657.7	0.7875	0	0
		P14	90	180	360	105.9	196.6	375.2	109.8	164.3	263.1	0.8101	0	0
		P15	90	180	360	93.1	131.1	239.4	221.7	376.1	672.7	0.8782	0	0
0.830	T7	P16	50	100	200	41.4	68.7	117.2	115.5	180.4	287.4	0.815	0	0
		P17	50	100	200	34.9	62	111.6	63.7	100.2	156.3	0.6994	0	0
0.450	T8	P18	60	120	240	36.6	62.1	120.8	23.1	33.2	50.7	0.3784	0	0

GAMS model of production planning problem

```
SETS
j process /1*54/, lev Level /l,m,h/, k RawType /r1,r2/;
```

Declaration of sets

```
TABLE rm(k,j) Raw material process j at type k
        1       2        3       4        5       6       7      8
r1    0.948   0.9432   0.949   0.9546   0.955   1.045   1.05   0.5103
r2    0       0        0       0        0       0       0      0
```

Amount of raw material required in each process

```
TABLE c(lev,j) Capacity process j at Level lev
      1     2     3      4     5      6     7     8     9
l     70    75    77.5   70    47.5   40    40    45    40
m     135   150   155    145   95     80    80    90    80
h     270   300   310    290   190    160   160   180   160
```

Production capacity of processes in each production level

```
Table PC(lev,j) production cost of process j at Level lev
   1       2       3       4       5       6       7       8      9
l  50.7    56.8    56.9    51.7    38.2    38.5    31.8    37.8   38.
m  90.1    103.8   103.7   97.6    69.8    65.2    57.1    57.7   65.
h  170.7   196.2   195.7   184.8   130.4   120.7   105.5   94.9   119
```

Production cost of each process at different level

```
Table IC(lev,j) investment cost for process j at Level lev
      1       2       3       4       5       6       7      8
l     55      58      60.2    55.1    43.3    66.2    40     106.6
m     81.1    85.1    86.8    83.1    66.8    92.8    61.4   151.7
h     131.6   132.4   134.1   132     104.3   153.2   95.1   231.5
```

Investment cost of each process at different level

```
parameters
SP(j) selling price of process j
/1  0.975, 2  0.975, 3  0.975, 4  0.975, 5  0.975, 6  0.78, 7  0.78, 8
```

Selling price of products produced by each process

```
parameter
R(k) available feedstock
/ r1  500, r2  500/;
```

Raw materials available

```
Scalar B budget /1000/;
```

Budget available

GAMS model of production planning problem

```
VARIABLES X(j),Y(j),Z(j),L(j),M(j),H(j),OBJ;

POSITIVE VARIABLES L(j),M(j),H(j);

BINARY VARIABLES Y(j),Z(j);

EQUATIONS
PROFIT,Eqn4(j),Eqn1(j),Eqn2(j),Eqn3(j),Eqn5(j),Eqn7(k),Eqn6;


PROFIT.. OBJ =e= sum(j,SP(j)*X(j))- sum(j,PC('l',j)*L(j)+PC('m',j)*M(j)+PC('h',j)*H(j));

Eqn4(j)..X(j)=e=(c('l',j)*L(j)+c('m',j)*M(j)+c('h',j)*H(j));

Eqn1(j) ..L(j)=l=Y(j);

Eqn2(j)..H(j) =l= 1-Y(j);

Eqn3(j)..L(j)+M(j)+H(j) =e= Z(j);

Eqn5(j)..X(j) =l= 100000*Z(j);

Eqn7(k)..sum(j,rm(k,j)*X(j)) =l= R(k);

Eqn6..sum(j,IC('l',j)*L(j)+IC('m',j)*M(j)+IC('h',j)*H(j)) =l= B;

model petrochemical /all/;

solve petrochemical using mip maximizing OBJ;

display X.l, L.l, M.l, H.l;
```

To display the variables X, L, M and H

$$Max\ profit = \sum_{j=1}^{J} SP_j X_j - \sum_{j=1}^{J}\left(cl_j L_j + cm_j M_j + ch_j H_j\right)$$

$$L_j \le Y_j \qquad (1)$$

$$H_j \le 1 - Y_j \qquad (2)$$

$$L_j + M_j + H_j = Z_j \qquad (3)$$

$$X_j = l_j L_j + m_j M_j + h_j H_j \qquad (4)$$

$$X_j \le 10000 Z_j \qquad (5)$$

$$\sum_{j=1}^{J}\left(il_j L_j + im_j M_j + ih_j H_j\right) \le B \qquad (6)$$

$$\sum_{j=1}^{J} rm_{jk} X_j \le R_k \qquad k = 1,\ldots,K \qquad (7)$$

$$\left.\begin{array}{l} Y_j, Z_j = 0 \text{ or } 1 \\ X_j, L_j, M_j, H_j \ge 0 \end{array}\right\} \quad \forall j = 1,2,\ldots,J$$

Result analysis

---- VAR X

	LOWER	LEVEL	UPPER	MARGINAL
1	-INF	.	+INF	.
2	-INF	.	+INF	.
3	-INF	310.0000 ✓	+INF	.
4	-INF	215.5982 ✓	+INF	.
5	-INF	.	+INF	.
6	-INF	.	+INF	.
7	-INF	.	+INF	.
8	-INF	.	+INF	.
9	-INF	.	+INF	.
10	-INF	.	+INF	.
11	-INF	.	+INF	.
12	-INF	.	+INF	.
13	-INF	.	+INF	.
14	-INF	.	+INF	.
15	-INF	.	+INF	.
16	-INF	.	+INF	.
17	-INF	.	+INF	.
18	-INF	.	+INF	.
19	-INF	.	+INF	.
20	-INF	.	+INF	.
21	-INF	.	+INF	.
22	-INF	.	+INF	.
23	-INF	.	+INF	.
24	-INF	.	+INF	.
25	-INF	.	+INF	.
26	-INF	.	+INF	.
27	-INF	.	+INF	.
28	-INF	.	+INF	.
29	-INF	.	+INF	.
30	-INF	.	+INF	.
31	-INF	.	+INF	.
32	-INF	.	+INF	.
33	-INF	.	+INF	.
34	-INF	.	+INF	.
35	-INF	.	+INF	.
36	-INF	.	+INF	.
37	-INF	.	+INF	.
38	-INF	.	+INF	.
39	-INF	.	+INF	.
40	-INF	.	+INF	.
41	-INF	.	+INF	.
42	-INF	.	+INF	.
43	-INF	.	+INF	.
44	-INF	.	+INF	.
45	-INF	.	+INF	.
46	-INF	612.8191 ✓	+INF	.
47	-INF	450.0000 ✓	+INF	.
48	-INF	680.0000 ✓	+INF	.
49	-INF	.	+INF	.
50	-INF	.	+INF	.
51	-INF	.	+INF	.
52	-INF	.	+INF	.
53	-INF	.	+INF	.
54	-INF	.	+INF	.

- X indicates the amount of product produced by the corresponding process
- Processes used: P3, P4, P46, P47, and P48
- Rest of the processes remain unused

Process	P3	P4	P46	P47	P48
X	310	215.598	612.819	450	680
L	0	0	0	0	0
M	0	0.513	0.292	1	0
H	1	0.487	0.708	0	1
Z	1	1	1	1	1
Y	0	0	0	0	0

```
Solution satisfies tolerances.

MIP Solution:          712.504042    (4 iterations, 0 nodes)
Final Solve:           712.504042    (2 iterations)

Best possible:         764.379568
Absolute gap:           51.875527
Relative gap:            0.067866
```

Profit = 712.5040

GAMS model of production planning problem


```
VARIABLES X(j),Y(j),Z(j),L(j),M(j),H(j),OBJ;

POSITIVE VARIABLES L(j),M(j),H(j);

BINARY VARIABLES Y(j),Z(j);

EQUATIONS
PROFIT,Eqn4(j),Eqn1(j),Eqn2(j),Eqn3(j),Eqn5(j),Eqn7(k),Eqn6;


PROFIT.. OBJ =e= sum(j,SP(j)*X(j))- sum(j,PC('l',j)*L(j)+PC('m',j)*M(j)+PC('h',j)*H(j));

Eqn4(j)..X(j)=e=(c('l',j)*L(j)+c('m',j)*M(j)+c('h',j)*H(j));

Eqn1(j) ..L(j)=l=Y(j);

Eqn2(j)..H(j) =l= 1-Y(j);

Eqn3(j)..L(j)+M(j)+H(j) =e= Z(j);

Eqn5(j)..X(j) =l= 100000*Z(j);

Eqn7(k)..sum(j,rm(k,j)*X(j)) =l= R(k);

Eqn6..sum(j,IC('l',j)*L(j)+IC('m',j)*M(j)+IC('h',j)*H(j)) =l= B;

option optcr = 0.00001;

model petrochemical /all/;

solve petrochemical using mip maximizing OBJ;

display X.l, L.l, M.l, H.l, Y.l, Z.l;
```

Default value of optcr is change
Default value = 0.1

https://support.gams.com/solver:what_is_optca_optcr

Result analysis

---- VAR X

	LOWER	LEVEL	UPPER	MARGINAL
1	-INF	217.0992	+INF	.
2	-INF	.	+INF	.
3	-INF	310.0000	+INF	.
4	-INF	.	+INF	.
5	-INF	.	+INF	.
6	-INF	.	+INF	.
7	-INF	.	+INF	.
8	-INF	.	+INF	.
9	-INF	.	+INF	.
10	-INF	.	+INF	.
11	-INF	.	+INF	.
12	-INF	.	+INF	.
13	-INF	.	+INF	.
14	-INF	.	+INF	.
15	-INF	.	+INF	.
16	-INF	.	+INF	.
17	-INF	.	+INF	.
18	-INF	.	+INF	.
19	-INF	.	+INF	.
20	-INF	.	+INF	.
21	-INF	.	+INF	.
22	-INF	.	+INF	.
23	-INF	.	+INF	.
24	-INF	.	+INF	.
25	-INF	.	+INF	.
26	-INF	.	+INF	.
27	-INF	.	+INF	.
28	-INF	.	+INF	.
29	-INF	.	+INF	.
30	-INF	.	+INF	.
31	-INF	.	+INF	.
32	-INF	.	+INF	.
33	-INF	.	+INF	.
34	-INF	.	+INF	.
35	-INF	.	+INF	.
36	-INF	.	+INF	.
37	-INF	.	+INF	.
38	-INF	.	+INF	.
39	-INF	.	+INF	.
40	-INF	.	+INF	.
41	-INF	50.0000	+INF	.
42	-INF	.	+INF	.
43	-INF	.	+INF	.
44	-INF	.	+INF	.
45	-INF	.	+INF	.
46	-INF	613.4417	+INF	.
47	-INF	.	+INF	.
48	-INF	680.0000	+INF	.
49	-INF	450.0000	+INF	.
50	-INF	.	+INF	.
51	-INF	.	+INF	.
52	-INF	.	+INF	.
53	-INF	.	+INF	.
54	-INF	.	+INF	.

- X indicates the amount of product produced by the corresponding process
- Processes used: P1, P3, P41, P46, P48 and P49
- Rest of the processes remain unused

Process	P1	P3	P41	P46	P48	P49
X	217.099	310	50	613.442	680	450
L	0	0	0	0	0	0
M	0.392	0	0	0.289	0	1
H	0.608	1	1	0.711	1	0
Z	1	1	1	1	1	1
Y	0	0	0	0	0	0

Log file

```
Proven optimal solution.

MIP Solution:          726.006789    (1376 iterations, 410 nodes)
Final Solve:           726.006789    (4 iterations)

Best possible:         726.006789
Absolute gap:            0.000000
Relative gap:            0.000000
```

Profit = 726.0068

Result comparison (with and without option)

Process	P3	P4	P46	P47	P48
X	310	215.598	612.819	450	680
L	0	0	0	0	0
M	0	0.513	0.292	1	0
H	1	0.487	0.708	0	1
Z	1	1	1	1	1
Y	0	0	0	0	0

```
Solution satisfies tolerances.

MIP Solution:           712.504042    (4 iterations, 0 nodes)
Final Solve:            712.504042    (2 iterations)

Best possible:          764.379568
Absolute gap:            51.875527
Relative gap:             0.067866
```

Profit = 712.5040

option optcr = 0.00001;

Process	P1	P3	P41	P46	P48	P49
X	217.099	310	50	613.442	680	450
L	0	0	0	0	0	0
M	0.392	0	0	0.289	0	1
H	0.608	1	1	0.711	1	0
Z	1	1	1	1	1	1
Y	0	0	0	0	0	0

```
Proven optimal solution.

MIP Solution:           726.006789    (1376 iterations, 410 nodes)
Final Solve:            726.006789    (4 iterations)

Best possible:          726.006789
Absolute gap:             0.000000
Relative gap:             0.000000
```

Profit = 726.0068

Options

	Description	Equation	Value
Best integer	best solution that satisfies all integer requirements found so far	–	10
Best estimate	provides a bound for the optimal integer solution	–	15
Absolute gap	distance between best integer and optimal solution	$\lvert best\ estimate - best\ integer \rvert$	15-10 = 5
OPTCA **Default value = 0**	If OPTCA $\geq$ Absolute gap, then algorithm terminates		
Relative gap	Measure of relative quality of a solution with respect to best estimate	$\frac{Absolute\ gap}{max(\lvert best\ estimate\rvert, \lvert best\ integer\rvert)}$	$\frac{5}{\max(\lvert 15\rvert, \lvert 10\rvert)} = 0.33$
OPTCR **Default value = 0.1**	If OPTCR $\geq$ Relative gap, then algorithm terminates		
Relative gap in cplex	Measure of relative quality of a solution with respect to best estimate	$\frac{Absolute\ gap}{10^{-10} + \lvert best\ integer\rvert}$	$\frac{5}{10^{-10} + \lvert 10\rvert} = 0.5$

https://support.gams.com/solver:what_is_optca_optcr
https://www.gams.com/latest/docs/S_CPLEX.html

Result comparison of production planning

Resources [B,R1,R2]	GAMS		Metaheuristic Technique					
			Without correction			With correction		
	default optcr	optcr = 0.00001	TLBO	DE	PSO	TLBO	DE	PSO
[1000, 500, 500]	712.50	726.01	400.59	1.17E+20	546.28	699.38	690.82	710.04
[1000, 1000, 1000]	834.30	834.30	622.51	2.00E+20	639.80	790.78	816.72	750.45
[2000, 500, 500]	1133.15	1173.11	757.77	419.19	647.70	1066.3	1092.3	857.91
[2000, 1000, 1000]	1452.82	1452.82	1077.50	463.96	922.38	1360.27	1375.50	1297.05

Thank You !!!