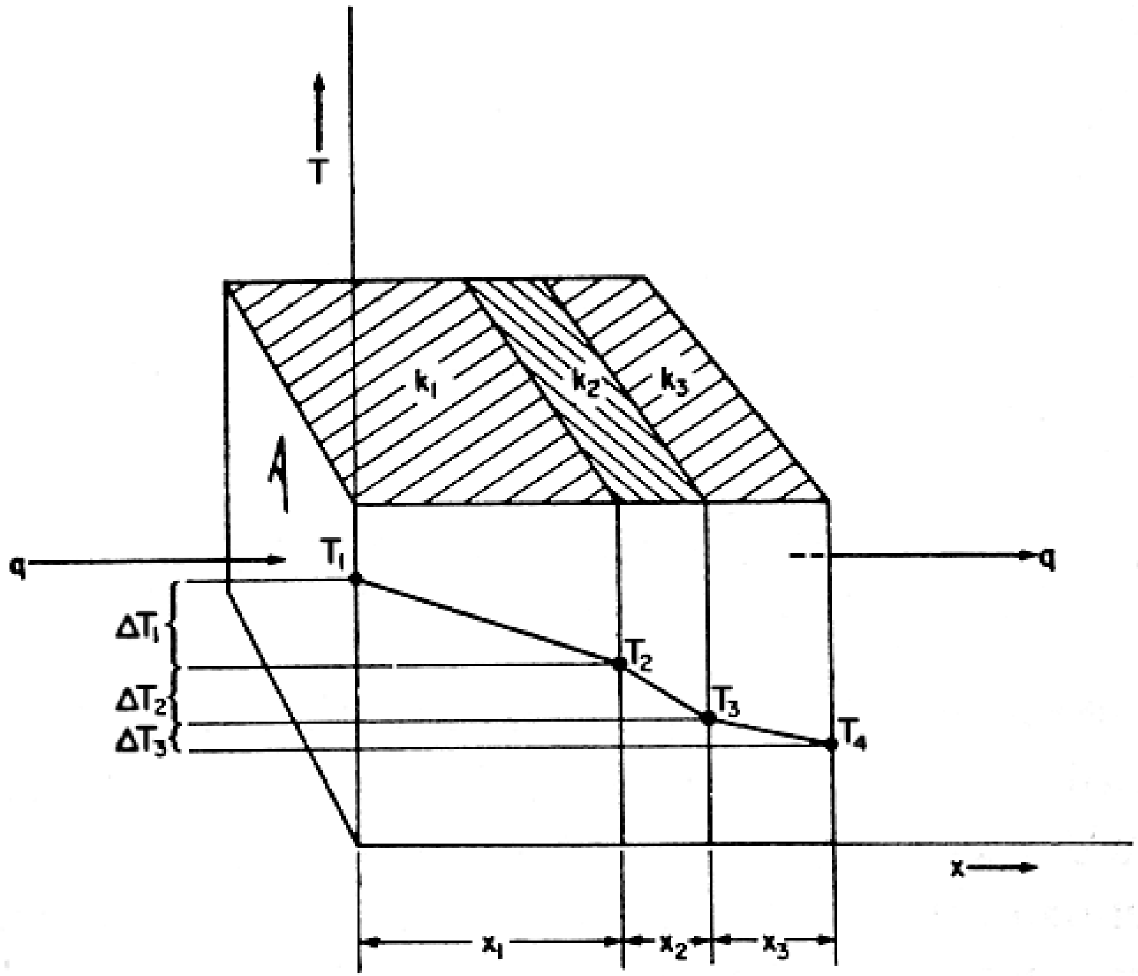
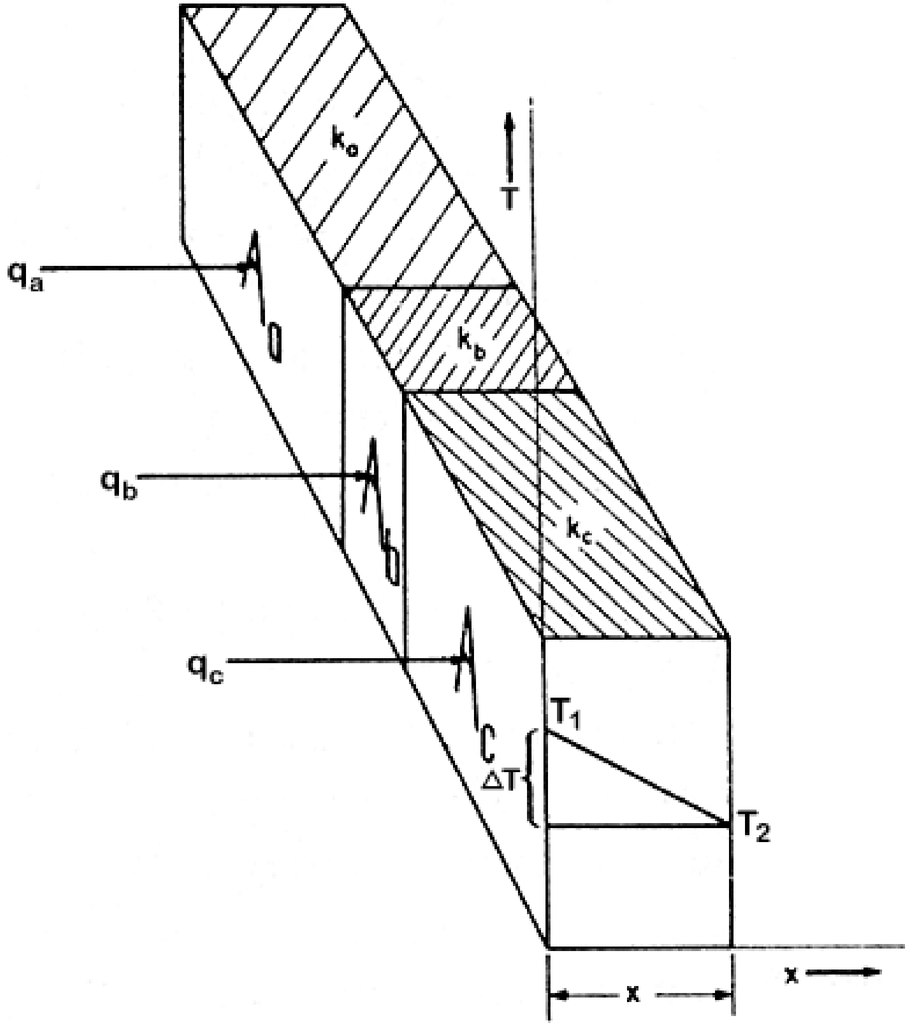
Heat conductances in parallel!!!!

Heat conductances in series



$$q = \frac{A \Delta T_1 k_1}{x_1} = \frac{A \Delta T_2 k_2}{x_2} = \frac{A \Delta T_3 k_3}{x_3}$$

Heat conductances in parallel



$$q_a = \frac{A_a \Delta T k_a}{x} \quad q_b = \frac{A_b \Delta T k_b}{x} \quad q_c = \frac{A_c \Delta T k_c}{x}$$

- Suppose that two plane solids A and B are placed side by side in parallel and the direction of heat flow is perpendicular to the plane of the exposed surface of each solid.

- Then the total heat flow is the sum of the heat flow through solid A plus that thro B

- Writing Fourier’s law for each solid and summing….

$$\boxed{q_T = q_A + q_B = \frac{k_A A_A}{\Delta x_A}(T_1 - T_2) + \frac{k_B A_B}{\Delta x_B}(T_3 - T_4)}$$

- Where, q_T is the total heat flow,
- T_1 and T_2 are the front and rear surface temp for solid A and
- T_3 and T_4 are those for solid B
- If we assume that T_1=T_3 (front temps the same for A and B) and
- T_2=T_4 (equal rear temps)….

$$q_T = \frac{(T_1 - T_2)}{\Delta x_A / k_A A_A} + \frac{(T_1 - T_2)}{\Delta x_B / k_B A_B} = \left(\frac{1}{R_A} + \frac{1}{R_B}\right)(T_1 - T_2)$$

$$\boxed{q_T = \Sigma\left\{\frac{1}{R}\right\}(T_1 - T_2) = \frac{\Delta T}{1/\Sigma\left\{\frac{1}{R}\right\}}}$$

For PARALLEL!!!!

- An example would be an insulated wall (A) of a brick oven where steel reinforcing members (B) are in parallel and penetrate the wall.

- Even though the are A_B of the steel would be small compared to the insulated brick area A_A, the higher conductivity of the metal (which could be several hundred times larger than that of the brick) could allow a large portion of the heat lost to be conducted by the steel

- Hope u remember….for conductances in series…..

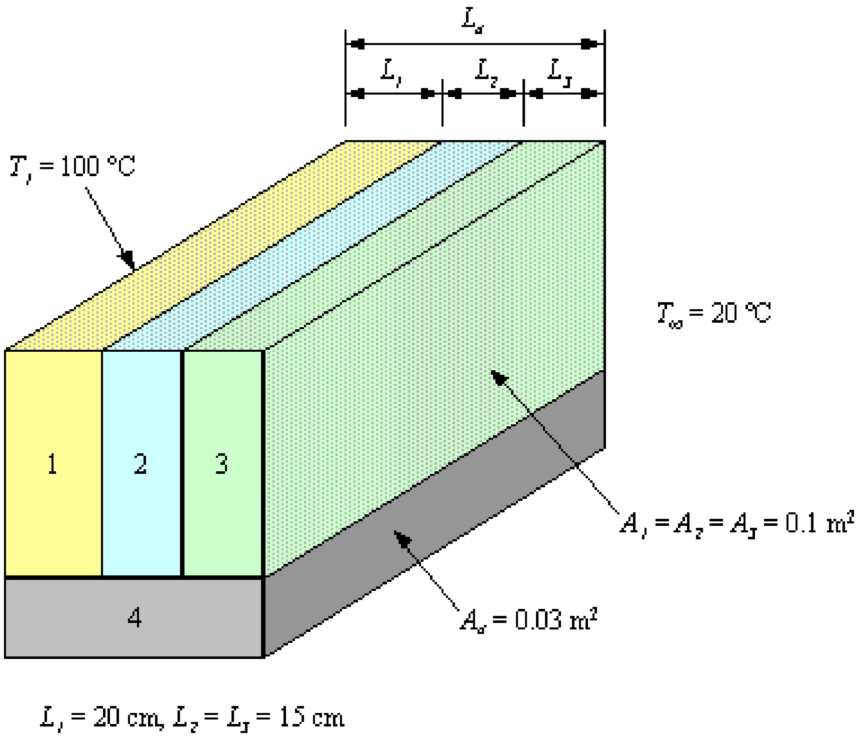
$$\boxed{q_T = \frac{\Delta T}{\Sigma\{R\}}}$$

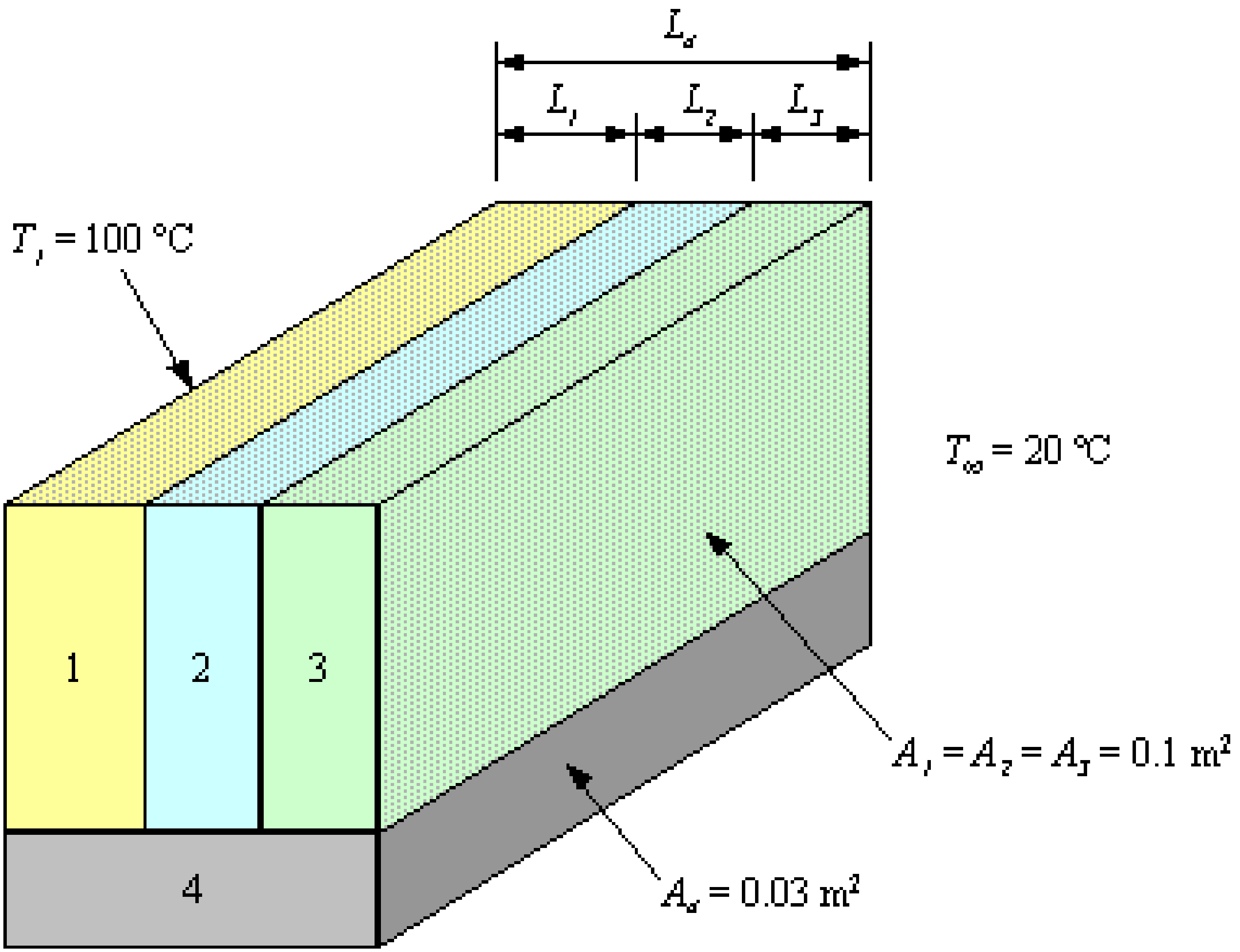
- But for conductances in parallel….

$$\boxed{q_T = \frac{\Delta T}{1/\Sigma\left\{\dfrac{1}{R}\right\}}}$$

Prob 1

- Consider a composite structure shown on below: Conductivities of the layer are: $k1 = k3 = 10$ W/mK, $k2 = 16$ W/mK, and $k4 = 46$ W/mK. The convection coefficient on the right side of the composite is 30 W/m^{2}K. Calculate the total resistance and the heat flow through the composite.





L_1 = 20 cm, L_2 = L_3 = 15 cm

$$q = \frac{T_{initial} - T_{final}}{R_{total}}$$

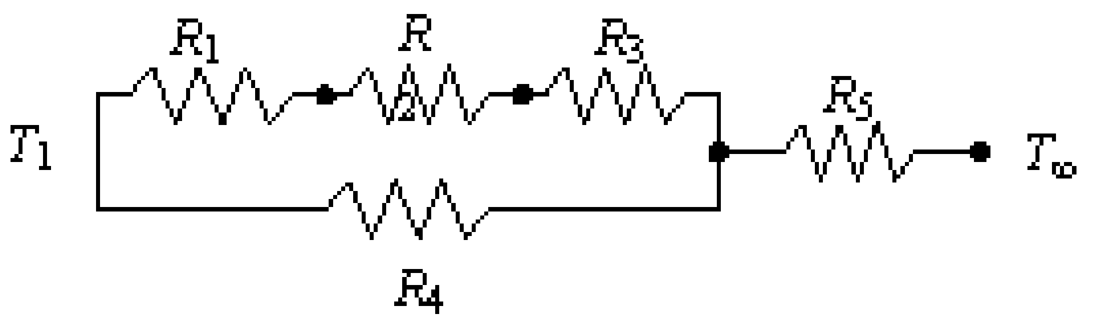
$$R_{conduction} = \frac{L}{kA}$$

$$R_{series} = \sum R$$

$$\frac{1}{R_{parallel}} = \sum \frac{1}{R}$$

$$R_{convection} = \frac{1}{hA}$$

- First, draw the thermal circuit for the composite.
- The circuit must span between the two *known* temperatures; that is, $T1$ and $T\infty$.



$$R_1 = \frac{L}{kA} = \frac{0.2}{(10)(0.1)} = 0.2$$

Similarly, $R_2 = 0.09$, $R_3 = 0.15$, and $R_4 = 0.36$

$$R_5 = \frac{1}{hA} = \frac{1}{(30)(0.13)} = 0.26$$

- To find the total resistance, an equivalent resistance for layers 1, 2, and 3 is found first.
- These three layers are combined in series:

$$
\begin{aligned}
R_{1,2,3} &= \sum R \\
&= R_1 + R_2 + R_3 \\
&= 0.2 + 0.09 + 0.15 \\
&= 0.44
\end{aligned}
$$

The equivalent resistor $R_{1,2,3}$ is in parallel with R_4:

$$\frac{1}{R_{1,2,3,4}} = \frac{1}{R_{1,2,3}} + \frac{1}{R_4}$$

$$= \frac{1}{0.44} + \frac{1}{0.36}$$

$$= 5.05$$

$$R_{1,2,3,4} = 5.05^{-1}$$

$$= 0.20$$

- Finally, R1,2,3,4 is in series with R5.
- The total resistance of the circuit is:

$$R_{total} = R_{1,2,3,4} + R_5 = 0.46$$

$$q = \frac{T_1 - T_\infty}{R_{total}}$$

$$= \frac{100 - 20}{0.46}$$

$$= \mathbf{173.9\ W.}$$