

Questions on Complete Information Static Games

1. Consider a two player and three strategy game and pay-offs are given in figure 1. Player 1's strategy set is $\{L_1, L_2, L_3\}$ and strategy set of player 2 is $\{R_1, R_2, R_3\}$. Find the solution through iterated elimination of strictly/weakly dominated strategies. Find the pure strategy Nash equilibrium.

	R_1	R_2	R_3
L_1	0, 0	1, 0	1, 1
L_2	1, 1	1, 1	3, 0
L_3	1, 1	2, 1	2, 2

Figure 1: Pay-off matrix of the game between player 1 and player 2.

2. Consider a two player and three strategy game and pay-offs are given in figure 2. Player 1's strategy set is $\{A, B, C\}$ and strategy set of player 2 is $\{E, F, G\}$. Find the pure strategy Nash equilibrium.

	E	F	G
A	1, 1	2, 1	4, 1
B	2, 1	5, 5	3, 6
C	1, 4	6, 3	0, 0

Figure 2: Pay-off matrix of the game between player 1 and player 2.

3. Two people have to divide Rs 10 between themselves. They use the following procedure. Each person names a number of Rs (a non negative real number), at most Rs 10. If the sum of the amounts that the people name is at most 10, then each person receives the amount of money she names (and the remainder is destroyed). If the sum of the amounts that the people name exceeds 10 and the amounts named are different, then the person who named the smaller amount receives that amount and the other person receives the remaining money. If the sum of the amounts that the people name exceeds 10 and the amounts named are same, then each person receives Rs 5. Both person prefers to have higher portion of Rs 10. Determine the best response of each player to each other player's actions, plot them in a diagram and find the Nash equilibria of the game.
4. Give examples of two player pure strategy games for the following situations.
- (a) The game has a unique Nash equilibrium which is not a weakly dominant strategy equilibrium.
 - (b) The game has a unique Nash equilibrium which is a weakly dominant strategy equilibrium but not a strongly dominant strategy equilibrium.
 - (c) The game has one strongly dominant or one weakly dominant strategy equilibrium and a second one which is only a Nash equilibrium

Answer Hints

1. The game cannot be reduced beyond the following form

	R_1	R_2	R_3
L_2	1, 1	1, 1	3, 0
L_3	1, 1	2, 1	2, 2

The Nash equilibrium of the game is $\{L_2, R_1\}$.

2. Nash equilibrium is $\{A, G\}$.

3. Suppose person 1 name X_1 rupees , $0 \leq x_1 < 10$.
Suppose person 2 name X_2 rupees , $0 \leq x_2 < 10$.

Person 1 receives in the following way:

x_1 , if $x_1 + x_2 \leq 10$.

x_1 , if $x_1 < x_2$ and $x_1 + x_2 > 10$.

$10 - x_2$, if $x_1 > x_2$ and $x_1 + x_2 > 10$.

0.5, if $x_1 = x_2$ and $x_1 + x_2 > 10$.

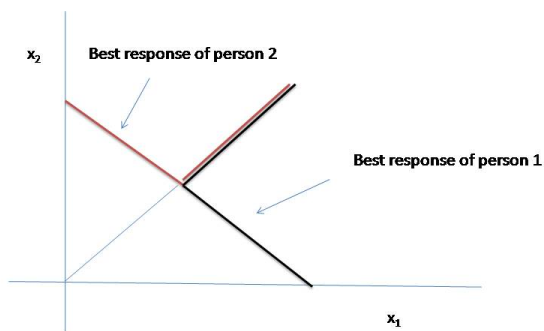
Person 2 receives in the following way:

x_2 , if $x_1 + x_2 \leq 10$.

x_2 , if $x_2 < x_1$ and $x_1 + x_2 > 10$.

$10 - x_1$, if $x_2 > x_1$ and $x_1 + x_2 > 10$.
 0.5 , if $x_1 = x_2$ and $x_1 + x_2 > 10$.

The best response function person 1 is: $r_1(x_2) = 10 - x_2$ for $x_2 \leq 5$. If person 2 name anything less than 5 than the best response of player 1 is name the remaining that is, $10 - x_2$. If person 1 name anything greater than $10 - x_2$ than also it will be $10 - x_2$ when $x_2 \leq 5$. $r_1(x_2) = x_2$ for $x_2 > 5$. If person 1 name anything greater than x_2 , it will receive $10 - x_2$. When $x_2 > 5$, the portion of person 1 will be less than 5, if it names anything greater than x_2 . Similarly we get the best response of person 2, $r_2(x_1) = 10 - x_1$ for $x_1 \leq 5$. $r_2(x_1) = x_1$ when $x_1 > 5$.



4. (a) The following game has a unique Nash equilibrium $\{A, E\}$ which is not a weakly dominant strategy equilibrium.

	E	F
A	5, 5	3, 2
B	3, 6	4, 4
C	1, 2	2, 3,

- (b) The following has a a unique Nash equilibrium $\{A, E\}$ which is a weakly dominant strategy equilibrium but not a strongly dominant strategy equilibrium.

	E	F
A	5, 5	3, 2
B	3, 4	4, 4
C	2, 5	5, 3

- (c) The following game has one strongly dominant or one weakly dominant strategy equilibrium $\{A, E\}$ and a second one which is only a Nash equilibrium $\{C, D\}$

	E	F
A	5, 5	3, 2
B	3, 6	4, 4
C	1, 5	5, 5