Teaching Learning Based Optimization

Prakash Kotecha
Debasis Maharana & Remya Kommadath
Department of Chemical Engineering
Indian Institute of Technology Guwahati

Outline

- Generic framework of metaheuristic algorithms
- Sanitized Teaching Learning Based Optimization (s-TLBO)
- Detailed working of s-TLBO with an example
- Various types of convergence curves
- Statistical analysis of multiple runs
- Preliminary comparison of algorithms
- Issues in TLBO
- Variants of TLBO

Terminologies

Optimization	Metaheuristic techniques
Decision variables	marks, subjects, position, gene
Solution	population member, learner, chromosome, child, parent, particle, bee, moth, flame, stream
Set of Solutions	population, class, moths, flames, water body, swarm
Objective function value	nectar amount, energy, fitness*
Iteration	generation, cycles

Metaheuristic techniques and optimization problem

Metaheuristic techniques
1) Generate **randomly** a single solution/a set of solutions 2) Based on the fitness of the current solution/set of solutions, **suggests other solutions with the help of "intelligent" operators**

$X = [5 \quad 2 \quad 3]$

Decision Variables →

← **Fitness**

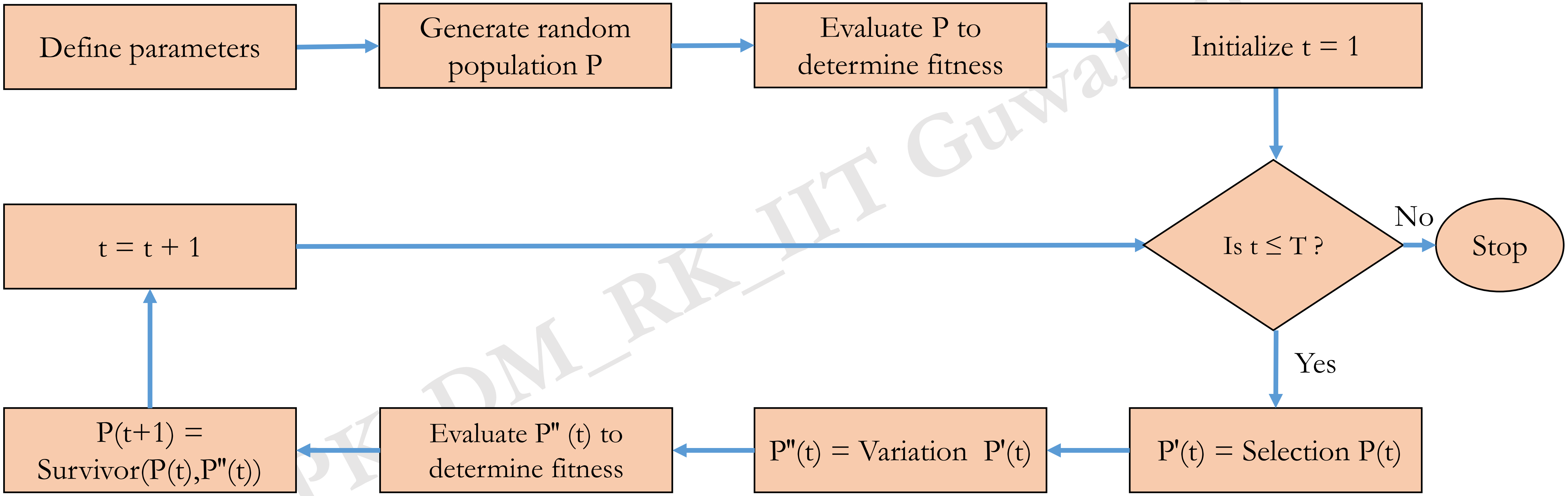
$f = -52$

Optimization problem
Evaluate fitness function $f = x_1^2 - 8x_1x_2 + x_3$

Generalized Scheme for metaheuristic techniques

Problem: Fitness function, Bounds of decision variables

Technique: Population size, Maximum iteration (T)

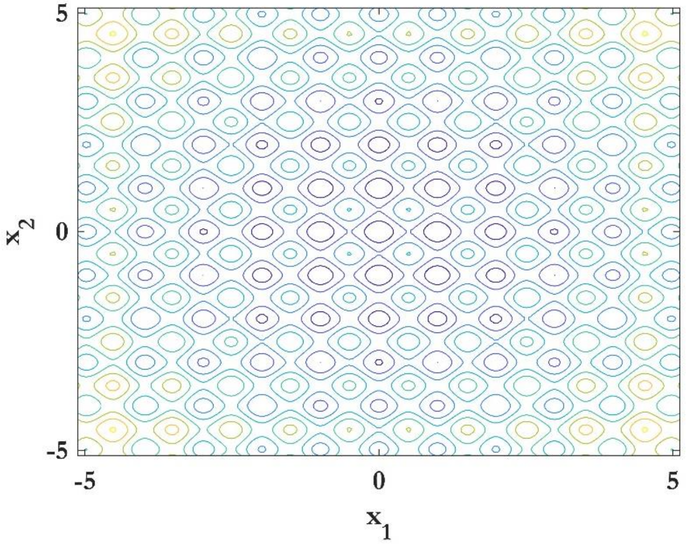


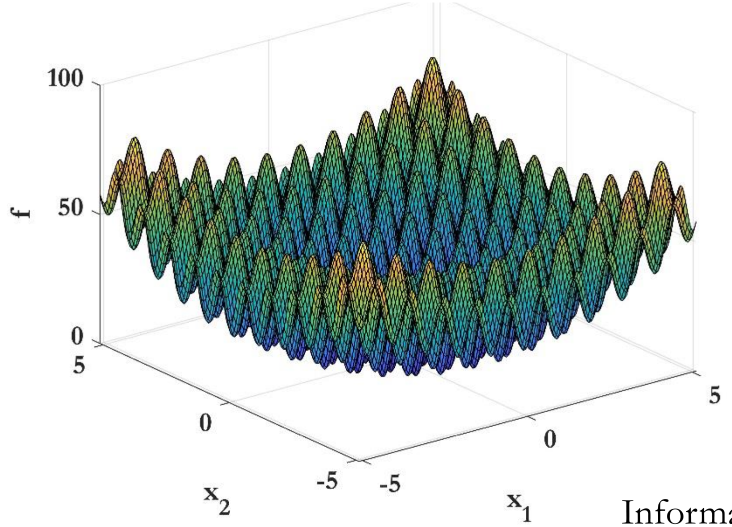
Performance of metaheuristic techniques

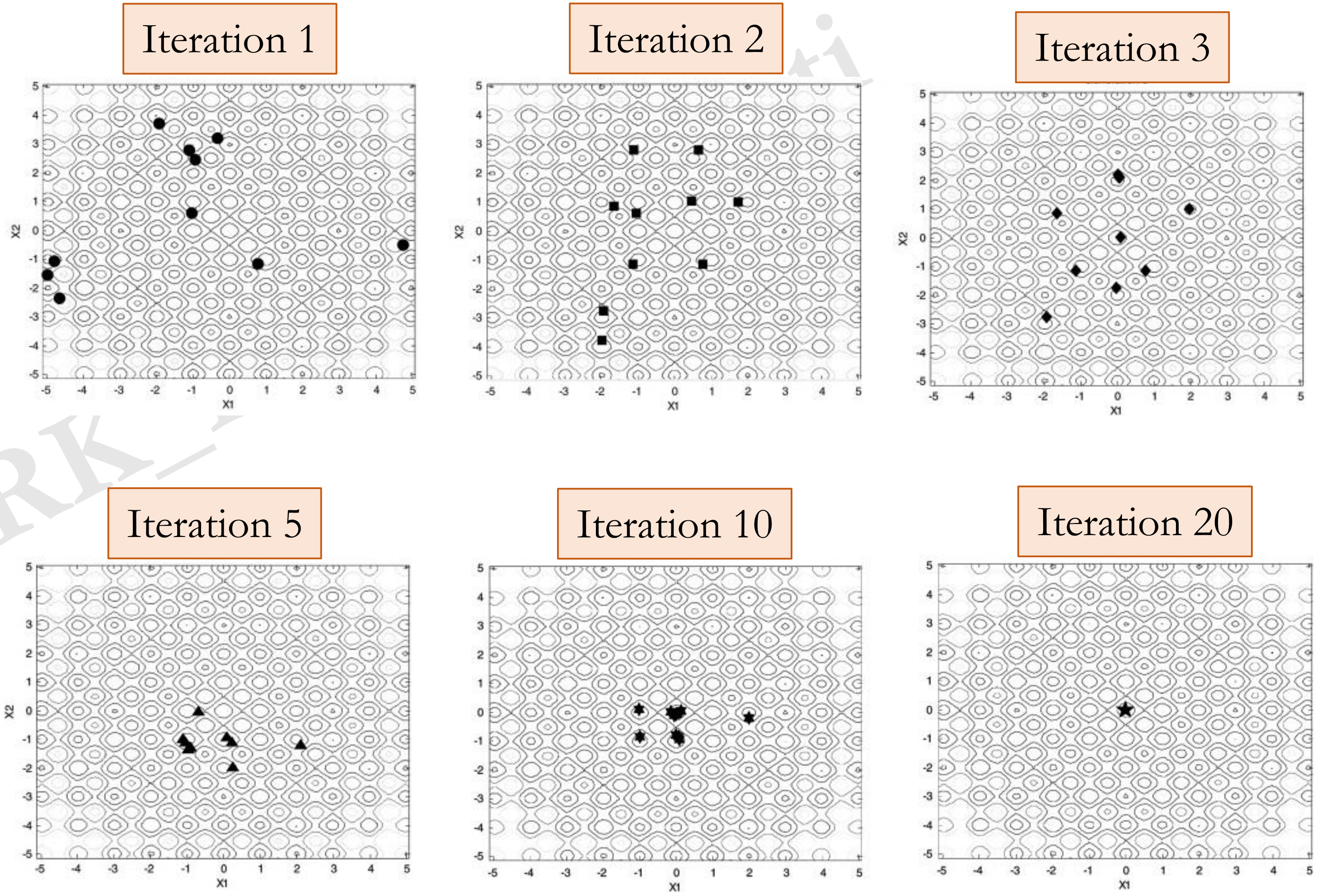
Rastrigin functions

$$\min f(x) = \sum_{i=1}^{D}\left[x_i^2 - 10\cos(2\pi x_i) + 10\right] \qquad -5.12 \le x_i \le 5.12$$

Number of decision variables, D = 2







Information Sciences, Volume 183, Issue 1, 2012, Pages 1-15, Elsevier: https://doi.org/10.1016/j.ins.2011.08.006

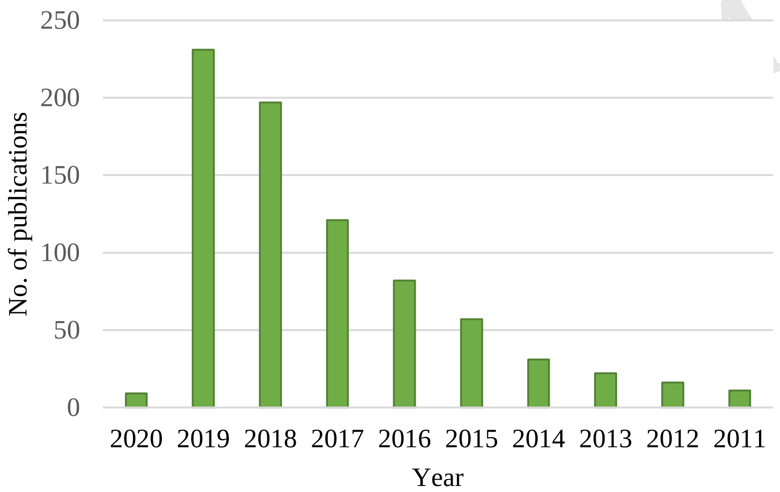
Teaching Learning Based Optimization (TLBO)

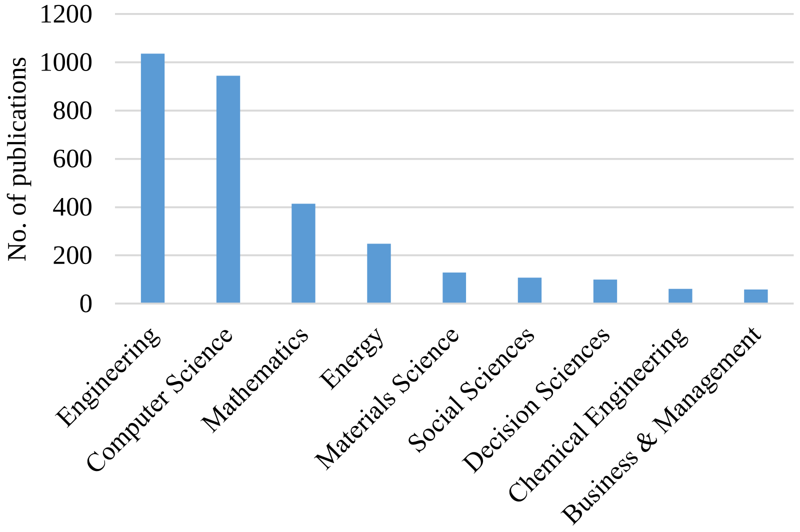
- Teaching-learning-based optimization: A novel method for constrained mechanical design optimization problems, *Computer-Aided Design*, Volume 43, Issue 3, 2011

- Teaching-learning-based optimization algorithm for unconstrained and constrained real-parameter optimization problems, *Engineering Optimization*, Volume 44, 2012

- Teaching-learning-based optimization: An optimization method for continuous non-linear large scale problems, *Information Sciences*, Volume 183, Issue 1, 2012

- Codes of TLBO: https://drive.google.com/file/d/0B96X2BLz4rx-VUQ3OERMZGFhUjg/view?usp=sharing





Around 1700 publications (Scopus, Dec 2019)

Teaching Learning Based Optimization (TLBO)

- Stochastic population based technique proposed by Rao et. al. in 2011
- Inspiration: Knowledge transfer in a classroom environment
- Required parameters: Population size and number of iterations
- Algorithm constitutes of two phases
  - **Teacher Phase**
    - New solution is generated using the best solution and mean of the population
    - Greedy selection: Accept new solution if better than the current solution
  - **Learner Phase**
    - New solution is generated using a partner solution
    - Greedy selection
- Each solution undergoes teacher phase followed by learner phase

Iteration 1			Iteration 2			Iteration 3			....	Iteration T		
T_1L_1	T_2L_2	T_3L_3	T_1L_1	T_2L_2	T_3L_3	T_1L_1	T_2L_2	T_3L_3		T_1L_1	T_2L_2	T_3L_3

Working of sanitized TLBO: Sphere function

- Consider

$$\min \quad f(x)=\sum_{i=1}^{4} x_i^2; \quad 0 \le x_i \le 10, \quad i=1,2,3,4$$

- Decision variables: x_1, x_2, x_3 and x_4

$$f(x)=x_1^2+x_2^2+x_3^2+x_4^2$$

- Step 1: Fix the population size, $N_P = 5$
- Step 2: Fix the maximum iterations, T = 10
- Step 3: Generate random solutions within the domain of the decision variables

$$P=\begin{bmatrix} 4 & 0 & 0 & 8 \\ 3 & 1 & 9 & 7 \\ 0 & 3 & 1 & 5 \\ 2 & 1 & 4 & 9 \\ 6 & 2 & 8 & 3 \end{bmatrix} \qquad f=\begin{bmatrix} 80 \\ 140 \\ 35 \\ 102 \\ 113 \end{bmatrix}$$

Teacher Phase: Generation of new solution

- New solution is generated with the help of teacher and mean of the population

- Teacher: Solution corresponding to the best fitness value

- Each variable in a solution (X) is modified as

$$X_{new} = X + r\left(X_{best} - T_f X_{mean}\right)$$

  - T_f is the same for all variables of a solution
  - r to be selected for each variable

X	Current solution
X_{new}	New solution
X_{best}	Teacher
X_{mean}	Mean of the population
T_f	Teaching factor, either 1 or 2
r	Random number between 0 and 1

Teacher Phase

- Step 4: Select Teacher, $X_{best} = [0 \;\; 3 \;\; 1 \;\; 5]$

$$P = \begin{bmatrix} 4 & 0 & 0 & 8 \\ 3 & 1 & 9 & 7 \\ \boxed{0 \;\; 3 \;\; 1 \;\; 5} \\ 2 & 1 & 4 & 9 \\ 6 & 2 & 8 & 3 \end{bmatrix} \qquad f = \begin{bmatrix} 80 \\ 140 \\ \boxed{35} \\ 102 \\ 113 \end{bmatrix}$$

- Step 5: Determine mean of the class,

$$X_{mean} = [\, 3.0 \;\; 1.4 \;\; 4.4 \;\; 6.4 \,]$$

$$\text{Mean} = \frac{[\,15 \;\; 7 \;\; 22 \;\; 32]}{5} = [3.0 \;\; 1.4 \;\; 4.4 \;\; 6.4]$$

- Step 6: Teacher phase of first student, ([4 0 0 8])

$$\boxed{X_{new} = X + r\left(X_{best} - T_f X_{mean}\right)}$$

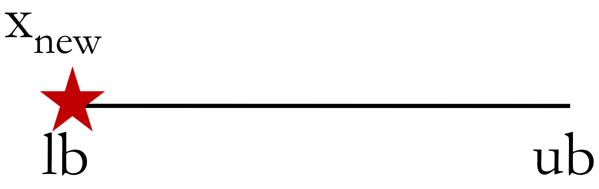
Let r = [0.8 0.2 0.7 0.4], and $T_f = 2$

Current solution Random number Best solution Mean

$$X^1_{new} = [4 \;\; 0 \;\; 0 \;\; 8] + [0.8 \;\; 0.2 \;\; 0.7 \;\; 0.4] \times ([0 \;\; 3 \;\; 1 \;\; 5] - 2 \times [3.0 \;\; 1.4 \;\; 4.4 \;\; 6.4])$$

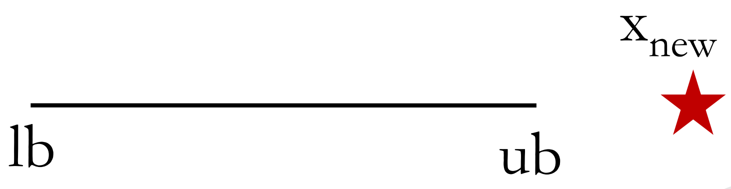
$$X^1_{new} = \left[-0.80 \quad 0.04 \quad -5.46 \quad 4.88\right]$$

Teacher Phase: Bounding of solution



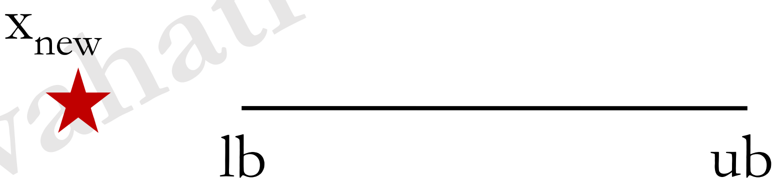
x_{new} is within bounds

No bounding required

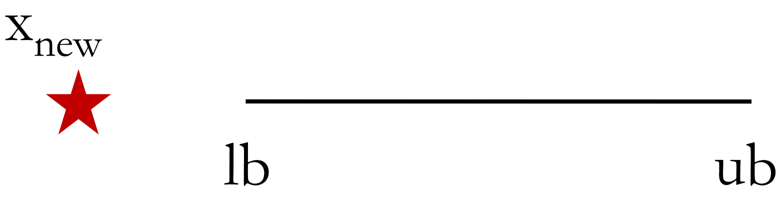


x_{new} violates the upper bound

Shift x_{new} to upper bound



x_{new} violates the lower bound



Shift x_{new} to lower bound

Teacher Phase

$$X^1_{new} = [-0.80 \quad 0.04 \quad -5.46 \quad 4.88]$$

$$\boxed{0 \le x_i \le 10}$$

$$P = \begin{bmatrix} 4 & 0 & 0 & 8 \\ 3 & 1 & 9 & 7 \\ 0 & 3 & 1 & 5 \\ 2 & 1 & 4 & 9 \\ 6 & 2 & 8 & 3 \end{bmatrix} \qquad f = \begin{bmatrix} 80 \\ 140 \\ 35 \\ 102 \\ 113 \end{bmatrix}$$

- Step 7: x_1 and x_3 violates lower bound

$$X^1_{new} = \max\left(X^1_{new}, lb\right)$$

$$X^1_{new} = \max\left([-0.80 \quad 0.04 \quad -5.46 \quad 4.88],[0 \quad 0 \quad 0 \quad 0]\right)$$

$$\max\left(-0.80, 0\right) = 0 \qquad \max\left(-5.46, 0\right) = 0$$

$$X^1_{new} = [0 \quad 0.04 \quad 0 \quad 4.88]$$

Teacher Phase: Selection of solution

- Evaluate fitness (f_{new}) of the new solution (X_{new}) generated in teacher phase
- Perform greedy selection to update the population

$$\left.\begin{aligned} X &= X_{new} \\ f &= f_{new} \end{aligned}\right\} \text{if} \;\; f_{new} < f$$

X and f remains the same if $f_{new} > f$

Teacher Phase

- Step 8: Evaluate the fitness of bounded solution

$$X^1_{new} = \begin{bmatrix} 0 & 0.04 & 0 & 4.88 \end{bmatrix}$$

$$f\left(X^1_{new}\right) = 0 + 0.04^2 + 0 + 4.88^2 = 23.82$$

$$\boxed{f(x) = \sum_{i=1}^{4} x_i^2}$$

$$P = \begin{bmatrix} 4 & 0 & 0 & 8 \\ 3 & 1 & 9 & 7 \\ 0 & 3 & 1 & 5 \\ 2 & 1 & 4 & 9 \\ 6 & 2 & 8 & 3 \end{bmatrix} \qquad f = \begin{bmatrix} 80 \\ 140 \\ 35 \\ 102 \\ 113 \end{bmatrix}$$

- Step 9: Perform greedy selection to update the population

$$X^1 = \begin{bmatrix} 4 & 0 & 0 & 8 \end{bmatrix}, \quad f^1 = 80$$

$$X^1_{new} = \begin{bmatrix} 0 & 0.04 & 0 & 4.88 \end{bmatrix}, \quad f^1_{new} = 23.82$$

$$f^1_{new} < f^1$$

$$X^1 = X^1_{new} = \begin{bmatrix} 0 & 0.04 & 0 & 4.88 \end{bmatrix}$$

$$f^1 = f^1_{new} = 23.82$$

$$P = \begin{bmatrix} 0 & 0.04 & 0 & 4.88 \\ 3 & 1 & 9 & 7 \\ 0 & 3 & 1 & 5 \\ 2 & 1 & 4 & 9 \\ 6 & 2 & 8 & 3 \end{bmatrix} \qquad f = \begin{bmatrix} 23.82 \\ 140 \\ 35 \\ 102 \\ 113 \end{bmatrix}$$

Learner Phase: Generation of new solution

- New solution is generated with the help of a partner solution
- Partner solution: Randomly selected solution from the population
- Each variable of solution is modified as

$$X_{new} = X + r\left(X - X_p\right) \quad if \;\; f < f_p$$
$$X_{new} = X - r\left(X - X_p\right) \quad if \;\; f \geq f_p$$

X	Current solution
X_{new}	New solution
X_p	Partner solution
f	Fitness of current solution
f_{new}	Fitness of partner solution
r	Random number between 0 and 1

Learner Phase

- Step 10: Select the partner solution for X^1

  Let the partner be X^4

  r = [0.9 0.1 0.2 0.5]

  $$X^1 = [0 \quad 0.04 \quad 0 \quad 4.88] \quad \text{and} \quad X^4 = [2 \quad 1 \quad 4 \quad 9]$$

$$P = \begin{bmatrix} 0 & 0.04 & 0 & 4.88 \\ 3 & 1 & 9 & 7 \\ 0 & 3 & 1 & 5 \\ 2 & 1 & 4 & 9 \\ 6 & 2 & 8 & 3 \end{bmatrix} \qquad f = \begin{bmatrix} 23.82 \\ 140 \\ 35 \\ 102 \\ 113 \end{bmatrix}$$

$$X_{new} = X + r\left(X - X_p\right) \quad if \;\; f < f_p \qquad -(1)$$

$$X_{new} = X - r\left(X - X_p\right) \quad if \;\; f \geq f_p \qquad -(2)$$

- Step 11: Learner phase of X^1

  $$f\left(X^1\right) = 23.82 < f\left(X^4\right) = 102$$

  Equation 1 is selected

$$X^1_{new} = \underbrace{[0 \;\; 0.04 \;\; 0 \;\; 4.88]}_{\text{Current solution}} + \underbrace{[0.9 \;\; 0.1 \;\; 0.2 \;\; 0.5]}_{\text{Random number}} \times \left(\underbrace{[0 \;\; 0.04 \;\; 0 \;\; 4.88]}_{\text{Current solution}} - \underbrace{[2 \;\; 1 \;\; 4 \;\; 9]}_{\text{Partner solution}} \right)$$

$$X^1_{new} = [-1.80 \;\; -0.06 \;\; -0.80 \;\; 2.82]$$

Learner Phase: Bounding and selection of solution

- Bound the newly generated variables, if required

$$x = lb \quad if \;\; x < lb$$
$$x = ub \quad if \;\; x > ub$$

- Evaluate fitness of new solution (f_{new}) generated using learner phase equation

- Perform greedy selection to update the population member

$$\left.\begin{aligned} X &= X_{new} \\ f &= f_{new} \end{aligned}\right\} if \;\; f_{new} < f$$

X and f remains the same if $f_{new} > f$

Learner Phase

$$X^1_{new} = \begin{bmatrix} -1.8 & -0.056 & -0.8 & 2.82 \end{bmatrix}$$

$$0 \le x_i \le 10$$

$$P = \begin{bmatrix} 0 & 0.04 & 0 & 4.88 \\ 3 & 1 & 9 & 7 \\ 0 & 3 & 1 & 5 \\ 2 & 1 & 4 & 9 \\ 6 & 2 & 8 & 3 \end{bmatrix} \qquad f = \begin{bmatrix} 23.82 \\ 140 \\ 35 \\ 102 \\ 113 \end{bmatrix}$$

- Step 12: x_1 , x_2 and x_3 violates lower bound

$$X^1_{new} = \max\left(X^1_{new}, lb\right)$$

$$X^1_{new} = \max\left(\begin{bmatrix} -1.8 & -0.056 & -0.8 & 2.82 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}\right)$$

$$X^1_{new} = \begin{bmatrix} 0 & 0 & 0 & 2.82 \end{bmatrix}$$

Learner Phase

- Step 13: Evaluate the fitness of bounded solution

$$X_{new}^{1} = \begin{bmatrix} 0 & 0 & 0 & 2.82 \end{bmatrix}$$

$$f\left(X_{new}^{1}\right) = 0 + 0 + 0 + 2.82^2 = 7.95$$

$$f(x) = \sum_{i=1}^{4} x_i^2$$

$$P = \begin{bmatrix} 0 & 0.04 & 0 & 4.88 \\ 3 & 1 & 9 & 7 \\ 0 & 3 & 1 & 5 \\ 2 & 1 & 4 & 9 \\ 6 & 2 & 8 & 3 \end{bmatrix} \qquad f = \begin{bmatrix} 23.82 \\ 140 \\ 35 \\ 102 \\ 113 \end{bmatrix}$$

- Step 14: Perform greedy selection to update the population

$$X^{1} = \begin{bmatrix} 0 & 0.04 & 0 & 4.88 \end{bmatrix}, \quad f^{1} = 23.82$$

$$X_{new}^{1} = \begin{bmatrix} 0 & 0 & 0 & 2.82 \end{bmatrix}, \quad f_{new}^{1} = 7.95$$

$$f_{new}^{1} < f^{1}$$

$$X^{1} = X_{new}^{1} = \begin{bmatrix} 0 & 0 & 0 & 2.82 \end{bmatrix}$$

$$f^{1} = f_{new}^{1} = 7.95$$

$$P = \begin{bmatrix} 0 & 0 & 0 & 2.82 \\ 3 & 1 & 9 & 7 \\ 0 & 3 & 1 & 5 \\ 2 & 1 & 4 & 9 \\ 6 & 2 & 8 & 3 \end{bmatrix} \qquad f = \begin{bmatrix} 7.95 \\ 140 \\ 35 \\ 102 \\ 113 \end{bmatrix}$$

Teacher Phase: Second solution

$$P = \begin{bmatrix} \boxed{0 \quad 0 \quad 0 \quad 2.82} \\ 3 \quad 1 \quad 9 \quad 7 \\ 0 \quad 3 \quad 1 \quad 5 \\ 2 \quad 1 \quad 4 \quad 9 \\ 6 \quad 2 \quad 8 \quad 3 \end{bmatrix} \qquad f = \begin{bmatrix} \boxed{7.95} \\ 140 \\ 35 \\ 102 \\ 113 \end{bmatrix}$$

- Step 1: Select Teacher , $X_{best} = [0 \;\; 0 \;\; 0 \;\; 2.82]$
- Step 2: Determine mean of the population

$$X_{mean} = [\,2.2 \;\; 1.4 \;\; 4.4 \;\; 5.36\,]$$

$$X_{new} = X + r\left(X_{best} - T_f X_{mean}\right)$$

- Step 3: Teacher phase of second student, ([3 1 9 7])

Let r = [0.9 0.3 0.8 0.4] and $T_f = 1$

$$X^2_{new} = [3\;1\;9\;7] + [0.9 \;\; 0.3 \;\; 0.8 \;\; 0.4] \times ([0 \;\; 0 \;\; 0 \;\; 2.82] - 1 \times [2.2 \;\; 1.4 \;\; 4.4 \;\; 5.36])$$

$$= [1.02 \;\; 0.58 \;\; 5.48 \;\; 5.98]$$

Teacher Phase: Second solution

- Step 4: Evaluate the fitness of bounded solution

$$X^2_{new} = \begin{bmatrix}1.02 & 0.58 & 5.48 & 5.98\end{bmatrix}$$

$$\boxed{f(x) = \sum_{i=1}^{4} x_i^2}$$

$$f\left(X^2_{new}\right) = 1.02^2 + 0.58^2 + 5.48^2 + 5.98^2 = 67.17$$

$$P = \begin{bmatrix} 0 & 0 & 0 & 2.82 \\ 3 & 1 & 9 & 7 \\ 0 & 3 & 1 & 5 \\ 2 & 1 & 4 & 9 \\ 6 & 2 & 8 & 3 \end{bmatrix} \qquad f = \begin{bmatrix} 7.95 \\ 140 \\ 35 \\ 102 \\ 113 \end{bmatrix}$$

- Step 5: Perform greedy selection to update the population

$$X^2 = \begin{bmatrix}3 & 1 & 9 & 7\end{bmatrix}, \quad f^2 = 140$$

$$X^2_{new} = \begin{bmatrix}1.02 & 0.58 & 5.48 & 5.98\end{bmatrix}, \quad f^2_{new} = 67.17$$

$$f^2_{new} < f^2$$

$$X^2 = X^2_{new} = \begin{bmatrix}1.02 & 0.58 & 5.48 & 5.98\end{bmatrix}$$

$$f^2 = f^2_{new} = 67.17$$

$$P = \begin{bmatrix} 0 & 0 & 0 & 2.82 \\ 1.02 & 0.58 & 5.48 & 5.98 \\ 0 & 3 & 1 & 5 \\ 2 & 1 & 4 & 9 \\ 6 & 2 & 8 & 3 \end{bmatrix} \qquad f = \begin{bmatrix} 7.95 \\ 67.17 \\ 35 \\ 102 \\ 113 \end{bmatrix}$$

Learner Phase: Second solution

- Step 6: Select the partner solution for X^2

Let the partner be X^5

r = [0.09 0.7 0.1 0.6]

$$P = \begin{bmatrix} 0 & 0 & 0 & 2.82 \\ 1.02 & 0.58 & 5.48 & 5.98 \\ 0 & 3 & 1 & 5 \\ 2 & 1 & 4 & 9 \\ 6 & 2 & 8 & 3 \end{bmatrix} \qquad f = \begin{bmatrix} 7.95 \\ 67.17 \\ 35 \\ 102 \\ 113 \end{bmatrix}$$

$X^2 = [1.02 \quad 0.58 \quad 5.48 \quad 5.98]$ and $X^5 = [6 \quad 2 \quad 8 \quad 3]$

- Step 7: Learner phase of X^2

$$X_{new} = X + r\left(X - X_p\right) \quad if \ f < f_p \quad -(1)$$

$$X_{new} = X - r\left(X - X_p\right) \quad if \ f \geq f_p \quad -(2)$$

$$f\left(X^2\right) = 67.17 < f\left(X^5\right) = 113$$

X^2_{new} = [1.02 0.58 5.48 5.98] + [0.09 0.7 0.1 0.6] x ([1.02 0.58 5.48 5.98] – [6 2 8 3])

X^2_{new} = [0.57 -0.41 5.23 7.77] $\xrightarrow{\text{Bounding}}$ X^2_{new} = [0.57 0 5.23 7.77]

Learner Phase: Second solution

- Step 8: Evaluate the fitness of bounded solution

$$X^2_{new} = \begin{bmatrix} 0.57 & 0 & 5.23 & 7.77 \end{bmatrix}$$

$$f\left(X^2_{new}\right) = 0.57^2 + 0 + 5.23^2 + 7.77^2 = 88.05$$

$$P = \begin{bmatrix} 0 & 0 & 0 & 2.82 \\ 1.02 & 0.58 & 5.48 & 5.98 \\ 0 & 3 & 1 & 5 \\ 2 & 1 & 4 & 9 \\ 6 & 2 & 8 & 3 \end{bmatrix} \qquad f = \begin{bmatrix} 7.95 \\ 67.17 \\ 35 \\ 102 \\ 113 \end{bmatrix}$$

- Step 9: Perform greedy selection to update the population

$$X^2 = \begin{bmatrix} 1.02 & 0.58 & 5.48 & 5.98 \end{bmatrix}, \quad f^2 = 67.17$$

$$X^2_{new} = \begin{bmatrix} 1.02 & 0 & 4.26 & 7.77 \end{bmatrix}, \quad f^2_{new} = 88.05$$

$$f^1_{new} > f^1$$

$$P = \begin{bmatrix} 0 & 0 & 0 & 2.82 \\ 1.02 & 0.58 & 5.48 & 5.98 \\ 0 & 3 & 1 & 5 \\ 2 & 1 & 4 & 9 \\ 6 & 2 & 8 & 3 \end{bmatrix} \qquad f = \begin{bmatrix} 7.95 \\ 67.17 \\ 35 \\ 102 \\ 113 \end{bmatrix}$$

Learner phase does not yield a better solution

Teacher Phase: Third solution

- Step 1: Select Teacher, $X_{best} = [0 \;\; 0 \;\; 0 \;\; 2.82]$
- Step 2: Determine mean of the class

$$X_{mean} = [\,1.80 \;\; 1.32 \;\; 3.7 \;\; 5.16]$$

$$P = \begin{bmatrix} 0 & 0 & 0 & 2.82 \\ 1.02 & 0.58 & 5.48 & 5.98 \\ 0 & 3 & 1 & 5 \\ 2 & 1 & 4 & 9 \\ 6 & 2 & 8 & 3 \end{bmatrix} \qquad f = \begin{bmatrix} 7.95 \\ 67.17 \\ 35 \\ 102 \\ 113 \end{bmatrix}$$

- Step 3: Teacher phase of third solution, **([0 3 1 5])**

Let r = [0.8 0.41 0.02 0.1] and $T_f = 2$

$X^3_{new} = [-2.88 \;\; 1.92 \;\; 0.85 \;\; 4.25] \longrightarrow$ Bounding $\longrightarrow X^3_{new} = [0 \;\; 1.92 \;\; 0.85 \;\; 4.25]$

$$f\left(X^3_{new}\right) = 22.47$$

$$f^3_{new} < f^3 \qquad X^3 = X^3_{new} = [0 \;\; 1.92 \;\; 0.85 \;\; 4.25]$$

$$f^3 = f^3_{new} = 22.47$$

$$P = \begin{bmatrix} 0 & 0 & 0 & 2.82 \\ 1.02 & 0.58 & 5.48 & 5.98 \\ 0 & 1.92 & 0.85 & 4.25 \\ 2 & 1 & 4 & 9 \\ 6 & 2 & 8 & 3 \end{bmatrix} \qquad f = \begin{bmatrix} 7.95 \\ 67.17 \\ 22.47 \\ 102 \\ 113 \end{bmatrix}$$

Learner Phase: Third solution

- Step 4: Select the partner solution for X^3

  Let the partner be X^1

  Let r = [0.8 0.4 0.3 0.3]

  $X^3 = [0 \quad 1.92 \quad 0.85 \quad 4.25]$ and $X^1 = [0 \quad 0 \quad 0 \quad 2.82]$

$$P = \begin{bmatrix} 0 & 0 & 0 & 2.82 \\ 1.02 & 0.58 & 5.48 & 5.98 \\ 0 & 1.92 & 0.85 & 4.25 \\ 2 & 1 & 4 & 9 \\ 6 & 2 & 8 & 3 \end{bmatrix} \quad f = \begin{bmatrix} 7.95 \\ 67.17 \\ 22.47 \\ 102 \\ 113 \end{bmatrix}$$

$$X_{new} = X + r\left(X - X_p\right) \quad if \;\; f < f_p \quad -(1)$$

$$X_{new} = X - r\left(X - X_p\right) \quad if \;\; f \geq f_p \quad -(2)$$

- Step 5: Learner phase of X^3

  $f\left(X^3\right) = 22.47 > f\left(X^1\right) = 7.95$

  X^3_{new} = [0 1.92 0.85 4.25] - [0.8 0.4 0.3 0.3] x ([0 1.92 0.85 4.25] – [0 0 0 2.82])

  X^3_{new} = [0 1.15 0.6 3.82]

  $f\left(X^3_{new}\right) = 16.27$

$$P = \begin{bmatrix} 0 & 0 & 0 & 2.82 \\ 1.02 & 0.58 & 5.48 & 5.98 \\ 0 & 1.15 & 0.6 & 3.82 \\ 2 & 1 & 4 & 9 \\ 6 & 2 & 8 & 3 \end{bmatrix} \quad f = \begin{bmatrix} 7.95 \\ 67.17 \\ 16.27 \\ 102 \\ 113 \end{bmatrix}$$

Fourth solution

$$P = \begin{bmatrix} 0 & 0 & 0 & 2.82 \\ 1.02 & 0.58 & 5.48 & 5.98 \\ 0 & 1.15 & 0.6 & 3.82 \\ 2 & 1 & 4 & 9 \\ 6 & 2 & 8 & 3 \end{bmatrix} \quad f = \begin{bmatrix} 7.95 \\ 67.17 \\ 16.27 \\ 102 \\ 113 \end{bmatrix}$$

Teacher phase:

r = [0.9 0.95 0.5 0.8]

$T_f = 2$

Determine the population and fitness

(round to two decimal places)

$$P = \begin{bmatrix} 0 & 0 & 0 & 2.82 \\ 1.02 & 0.58 & 5.48 & 5.98 \\ 0 & 1.15 & 0.6 & 3.82 \\ 0 & 0 & 0 & 1.82 \\ 6 & 2 & 8 & 3 \end{bmatrix} \quad f = \begin{bmatrix} 7.95 \\ 67.17 \\ 16.27 \\ 3.31 \\ 113 \end{bmatrix}$$

Learner phase:

r = [0.9 0.7 0.1 0.6]

Partner = 2

Fifth Solution

$$P = \begin{bmatrix} 0 & 0 & 0 & 2.82 \\ 1.02 & 0.58 & 5.48 & 5.98 \\ 0 & 1.15 & 0.6 & 3.82 \\ 0 & 0 & 0 & 1.82 \\ 6 & 2 & 8 & 3 \end{bmatrix} \quad f = \begin{bmatrix} 7.95 \\ 67.17 \\ 16.27 \\ 3.31 \\ 113 \end{bmatrix}$$

Teacher phase:

r = [0.6 0.85 0.8 0.89]

$T_f = 1$

Determine the population and fitness

(round to two decimal places)

$$P = \begin{bmatrix} 0 & 0 & 0 & 2.82 \\ 1.02 & 0.58 & 5.48 & 5.98 \\ 0 & 1.15 & 0.6 & 3.82 \\ 0 & 0 & 0 & 1.82 \\ 1.55 & 1.32 & 5.23 & 2.21 \end{bmatrix} \quad f = \begin{bmatrix} 7.95 \\ 67.17 \\ 16.27 \\ 3.31 \\ 36.39 \end{bmatrix}$$

Learner phase:

r = [0.7 0.2 0.1 0.3]

Partner = 3

Satisfaction of termination condition

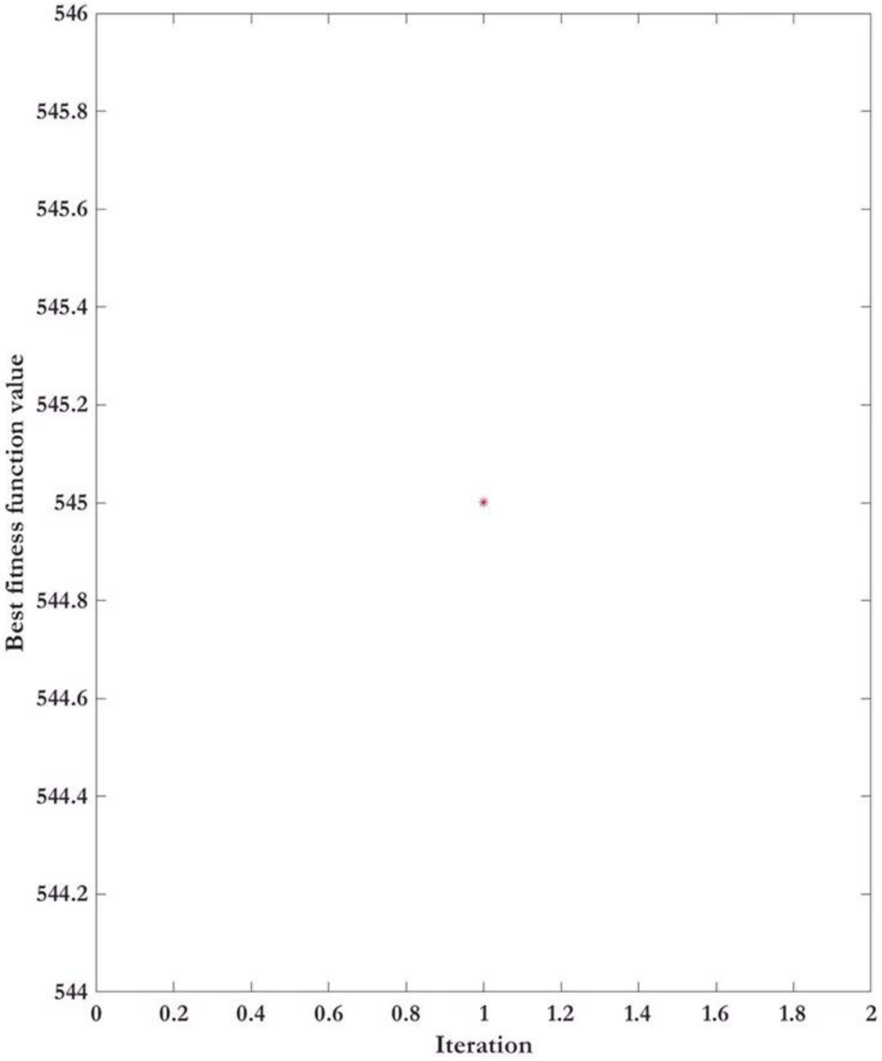
$$\min \quad f(x) = \sum_{i=1}^{4} x_i^2; \quad 0 \le x_i \le 10, \quad i = 1, 2, 3, 4$$

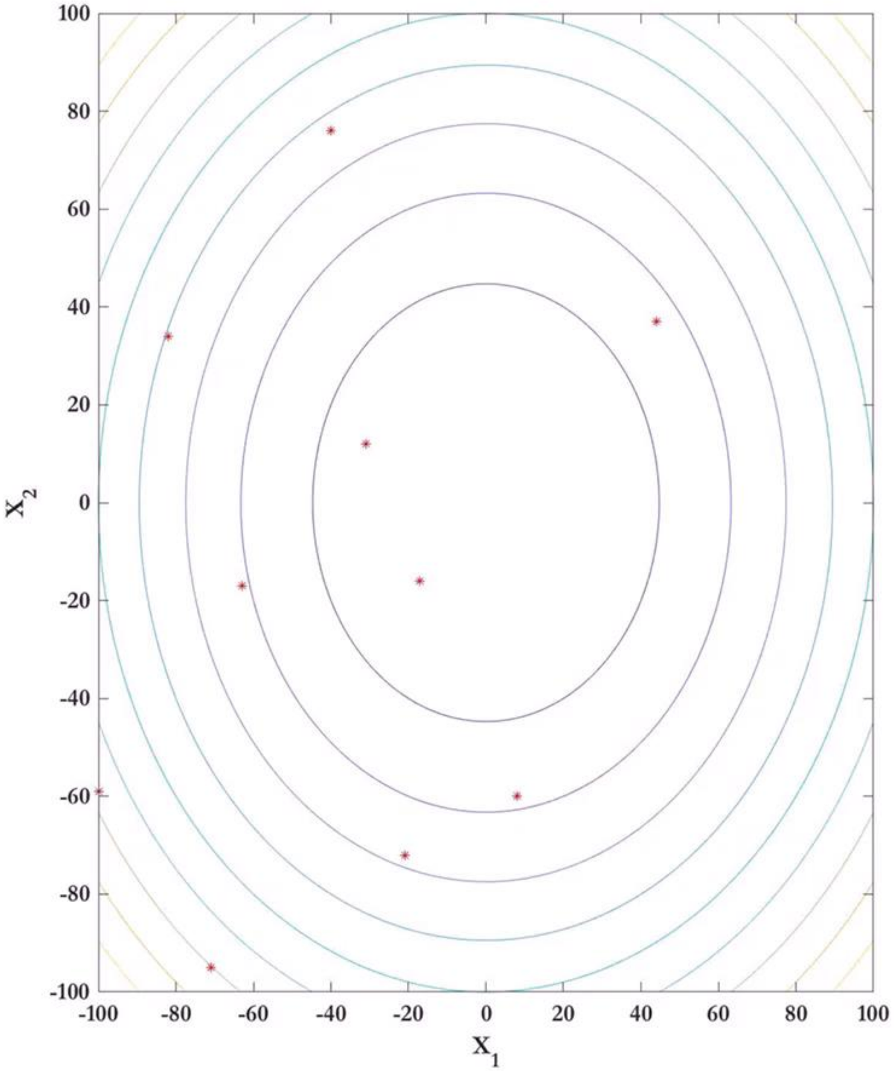
After completion of 10 iterations

$$P = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.01 & 0 & 0 & 0.05 \\ 0 & 0.01 & 0 & 0.06 \\ 0 & 0 & 0 & 0 \\ 0.01 & 0.02 & 0.11 & 0.04 \end{bmatrix} \qquad f = \begin{bmatrix} 0 \\ 0.0026 \\ 0.0037 \\ 0 \\ 0.0142 \end{bmatrix}$$

The minimum value of the function is **0**

Performance of s-TLBO





Pseudocode

1. Input: Fitness function, lb, ub, N_p, T
2. Initialize a random population (P)
3. Evaluate fitness of P
4. for t = 1 to T

for i = 1 to N_P

Choose X_{best}

Determine X_{mean}

$X_{new} = X_i + r\,(X_{best} - T_f\,X_{mean})$

Bound X_{new} and evaluate its fitness f_{new}

Accept X_{new} if it is better than X_i

Choose any solution randomly , X_p

Determine X_{new} as

if $f_i < f_p$

$X^i_{new} = X_i + r\,(X_i - X_p)$

else

$X^i_{new} = X_i - r\,(X_i - X_p)$

end

Bound X_{new} and evaluate its fitness f_{new}

Accept X_{new} if it is better than X_i

end

end

FE = N_P

FE = 1

FE = 1

One iteration FE = $2N_P$

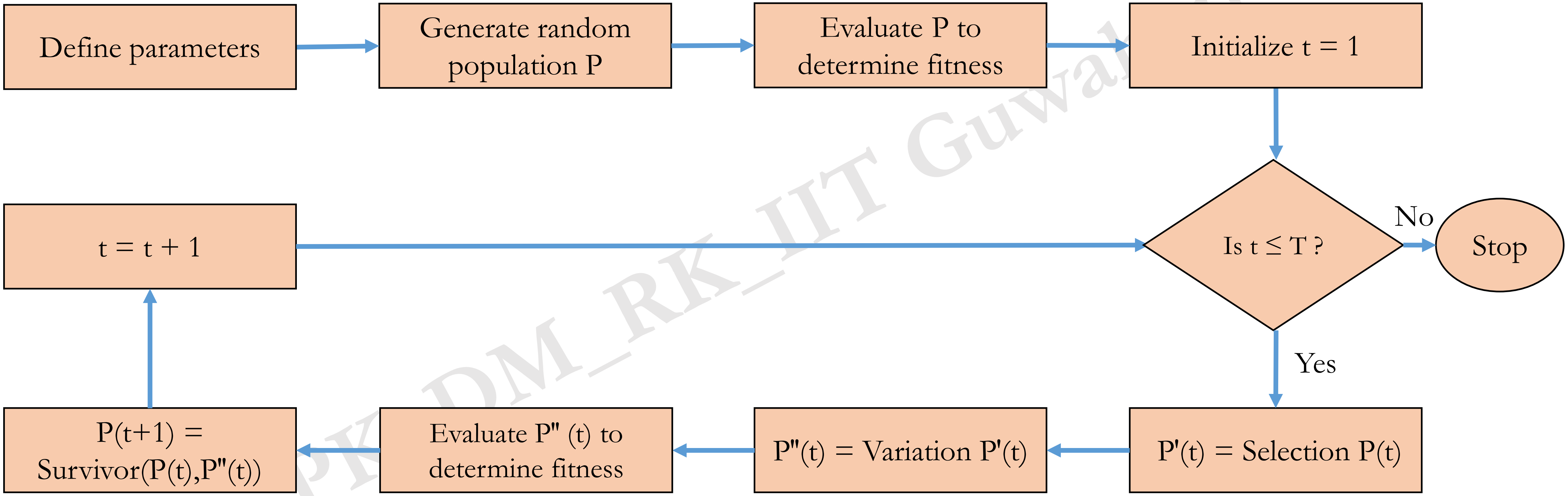
T iterations FE = $2N_P$ T

FE = $N_P + 2N_P$ T

Generalized Scheme for metaheuristic techniques

Problem: Fitness function, Bounds of decision variables

Technique: Population size, Maximum iteration (T)



Convergence curve: Iteration vs. Best fitness

Iteration 0

$$P_0 = \begin{bmatrix} 14 & -9 \\ 19 & 6 \\ 8 & 12 \end{bmatrix} \qquad f = \begin{bmatrix} 277 \\ 397 \\ \boxed{208} \end{bmatrix}$$

Iteration 1

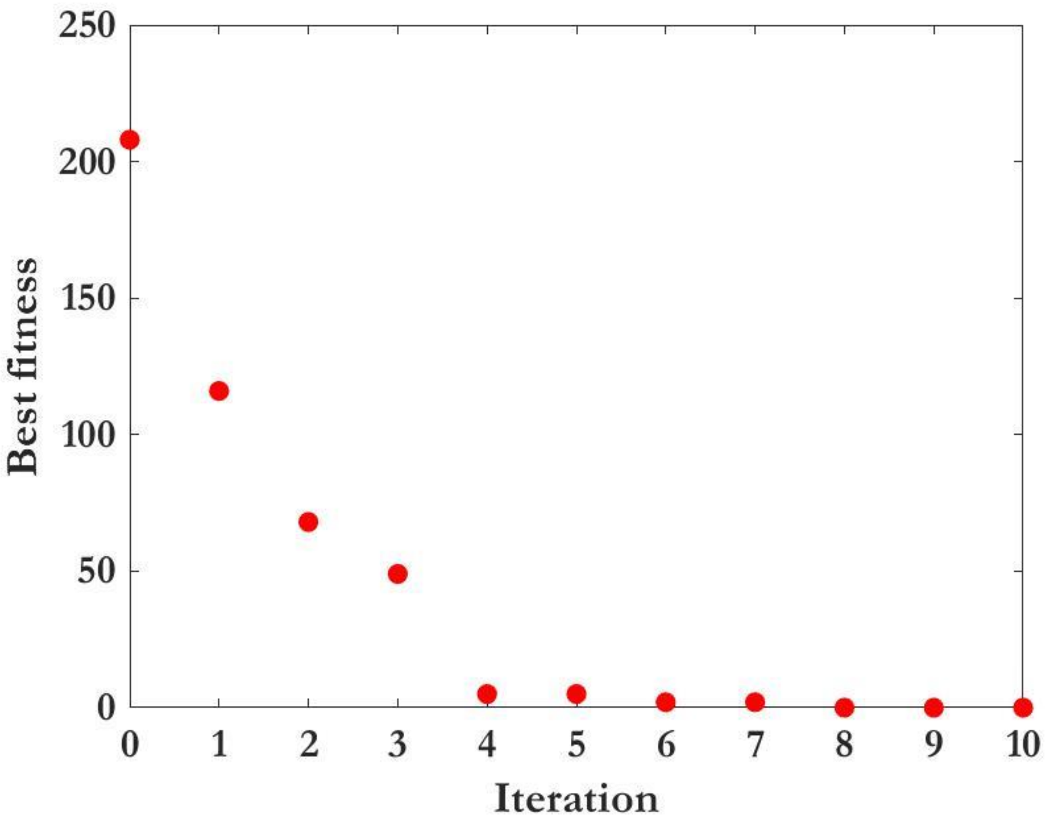
$$P_1 = \begin{bmatrix} 10 & -4 \\ 13 & 10 \\ 6 & 11 \end{bmatrix} \qquad f = \begin{bmatrix} \boxed{116} \\ 269 \\ 157 \end{bmatrix}$$

Iteration 2

$$P_2 = \begin{bmatrix} 10 & -4 \\ 8 & 2 \\ 5 & 8 \end{bmatrix} \qquad f = \begin{bmatrix} 116 \\ \boxed{68} \\ 89 \end{bmatrix}$$

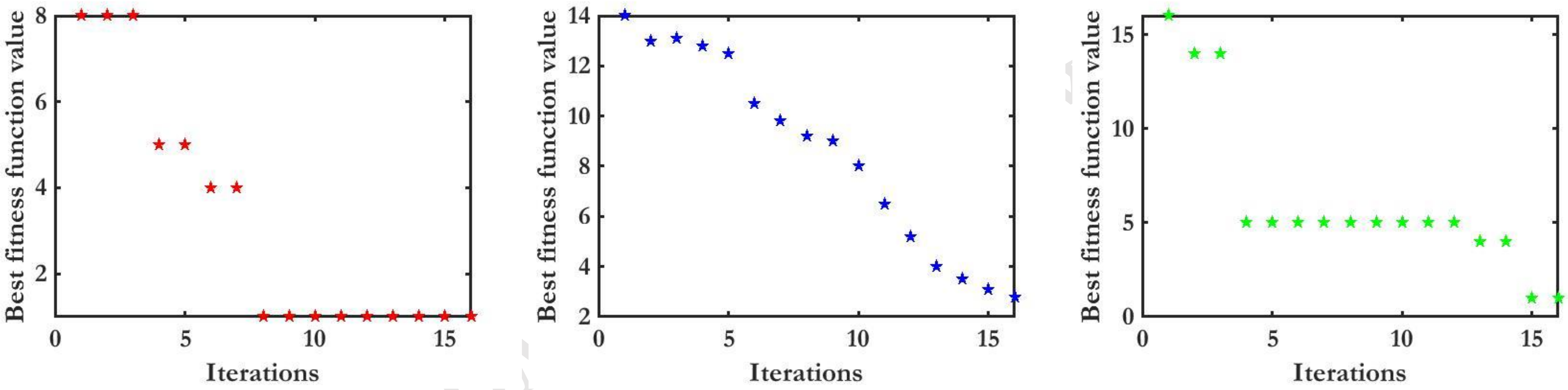
Iteration 10

$$P_{10} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & -1 \end{bmatrix} \qquad f = \begin{bmatrix} \boxed{0} \\ 0 \\ 1 \end{bmatrix}$$



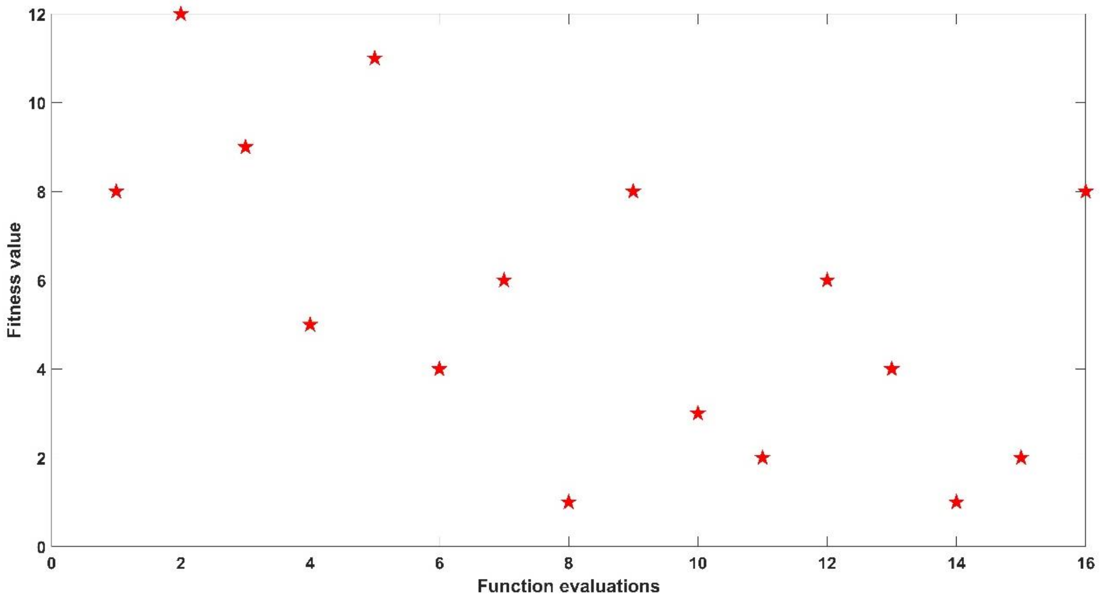
Cases of convergence

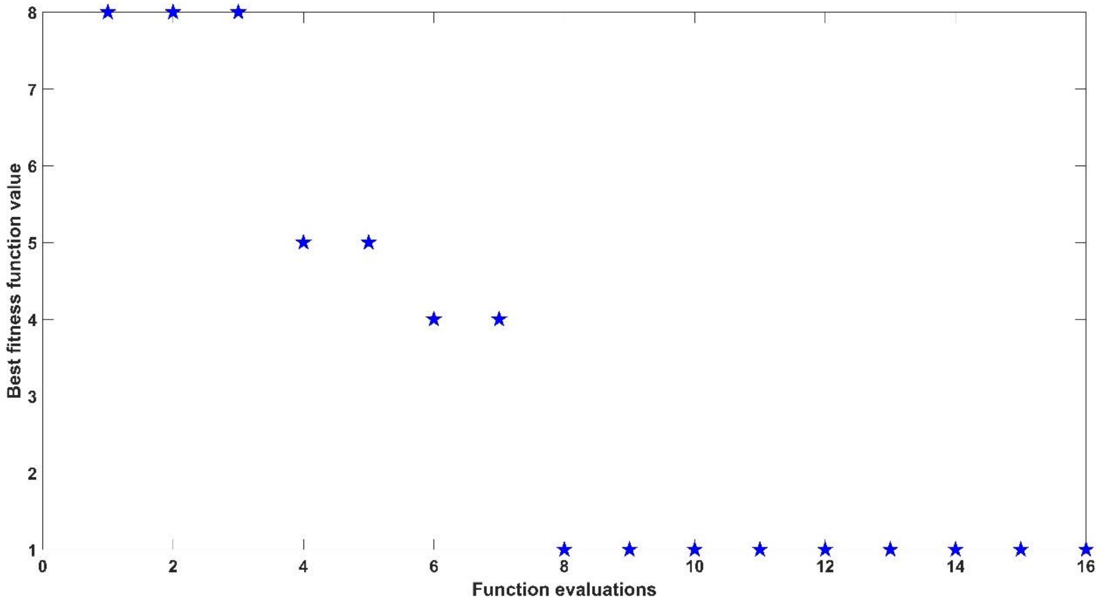
T = 16



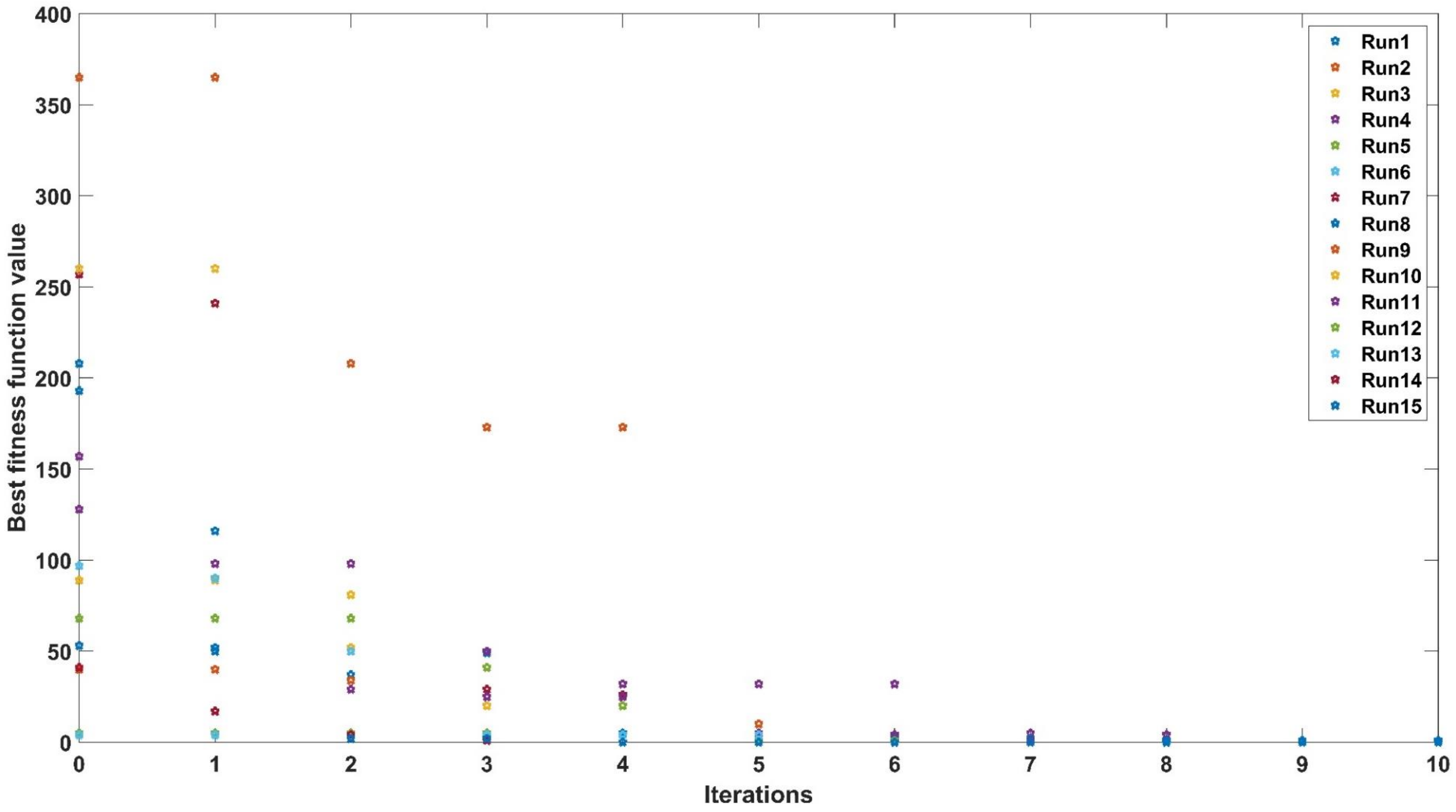
Convergence curve: # FE vs. Best fitness value

# FE	Fitness value	Best fitness value
1	8	8
2	12	8
3	9	8
4	5	5
5	11	5
6	4	4
7	6	4
8	1	1
9	8	1
10	3	1
11	2	1
12	6	1
13	4	1
14	1	1
15	2	1
16	8	1





Multiple runs and statistical table

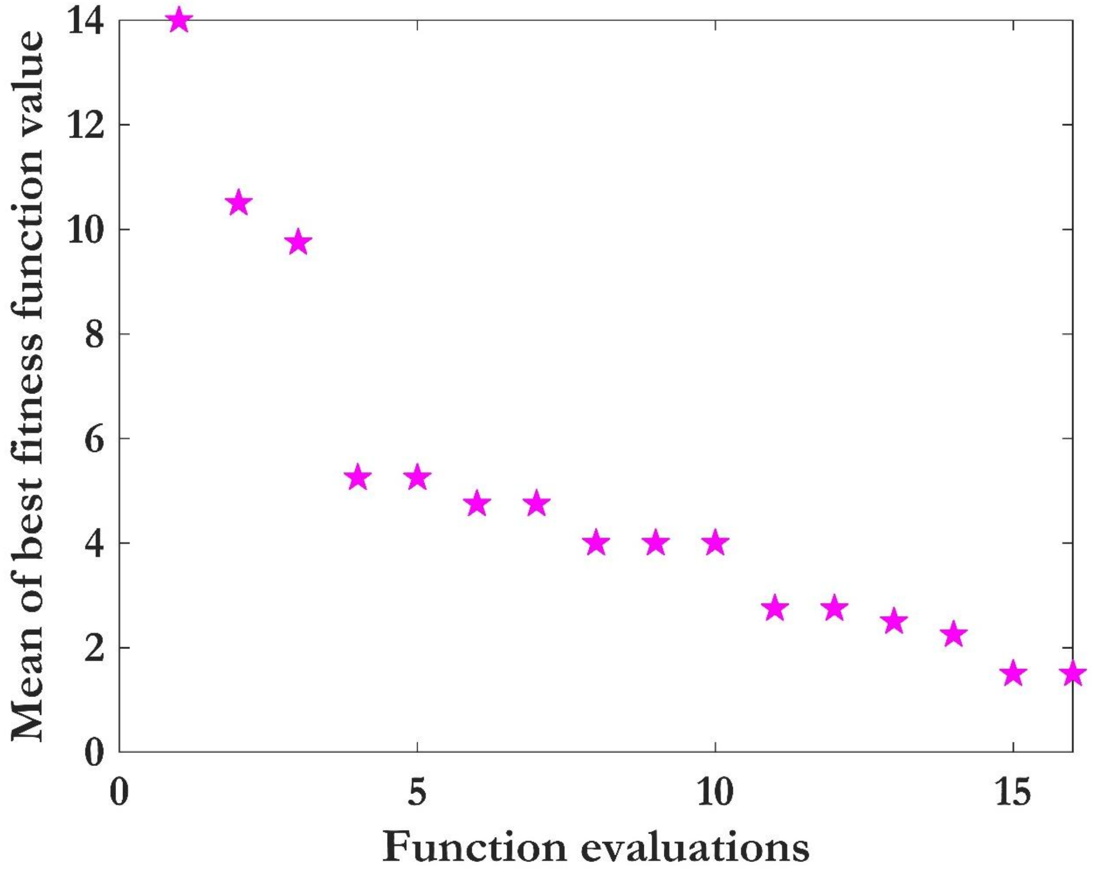


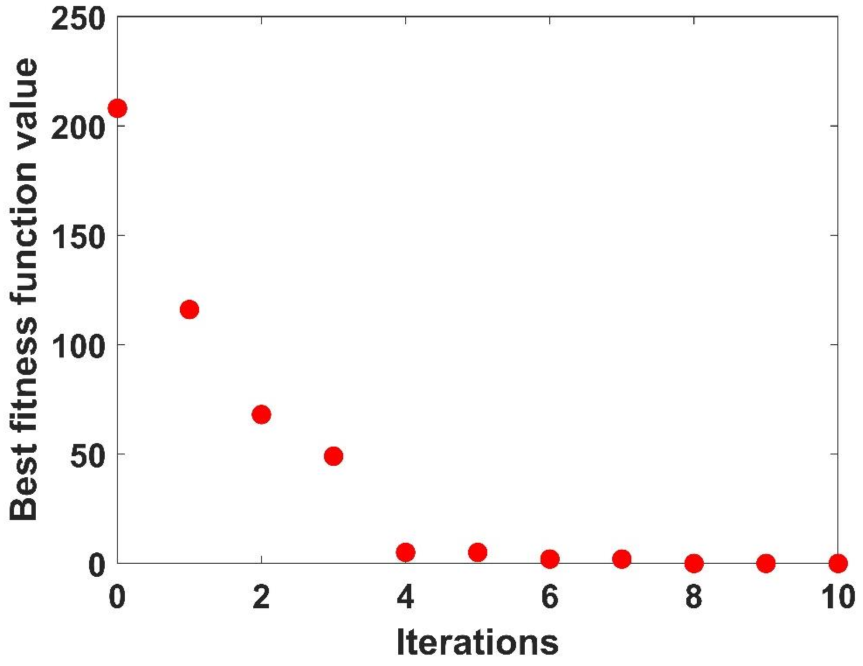
Best	Worst	Mean	Median	Standard Deviation
0	4	0.533	0	1.06

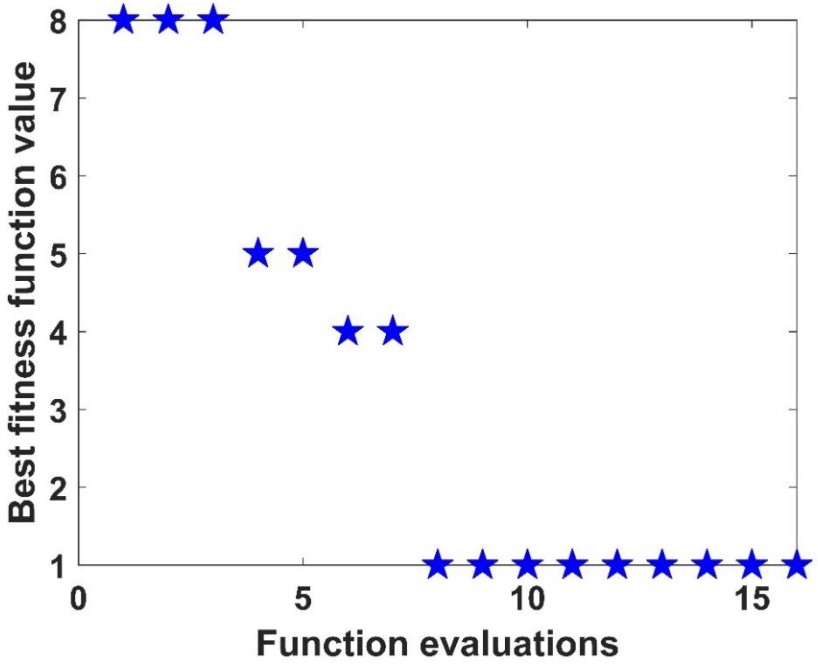
Run	Best fitness after 10 iterations
1	0
2	0
3	0
4	1
5	0
6	0
7	1
8	0
9	1
10	0
11	4
12	0
13	1
14	0
15	0

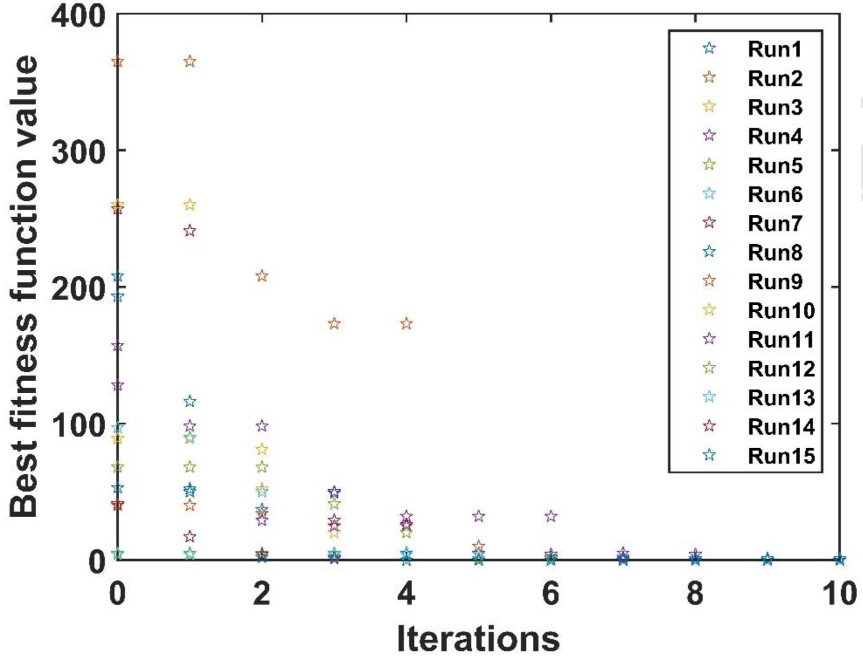
Mean convergence curve

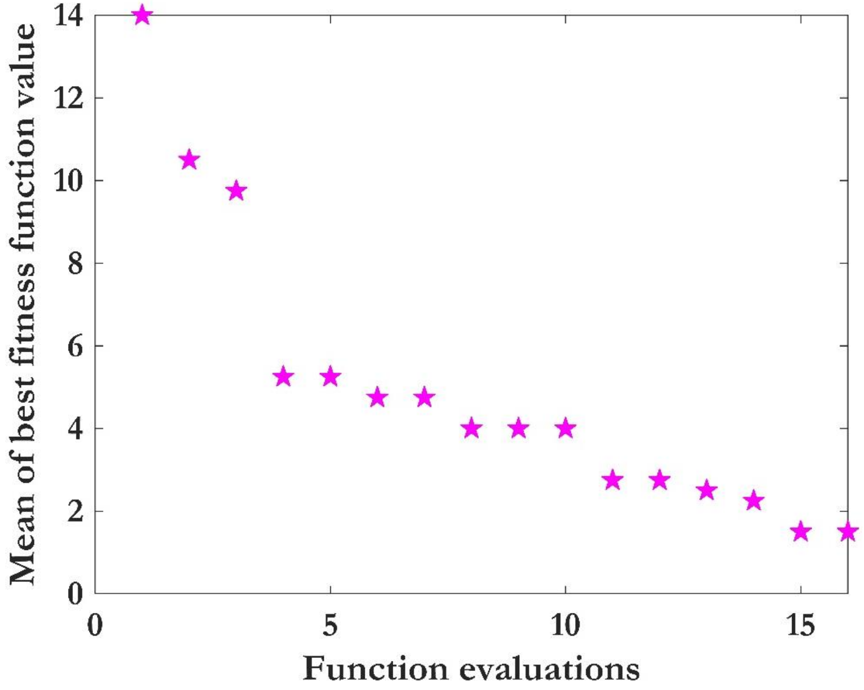
# FE	Run 1 F value	Run 1 Best value	Run 2 F value	Run 2 Best value	Run 3 F value	Run 3 Best value	Run 4 F value	Run 4 Best value	Mean
1	8	8	12	12	20	20	16	16	14
2	12	8	8	8	12	12	14	14	10.5
3	9	8	9	8	9	9	14	14	9.75
4	5	5	6	6	5	5	5	5	5.25
5	11	5	11	6	11	5	11	5	5.25
6	4	4	5	5	5	5	5	5	4.75
7	6	4	6	5	6	5	6	5	4.75
8	1	1	7	5	7	5	7	5	4
9	8	1	8	5	8	5	8	5	4
10	3	1	5	5	6	5	7	5	4
11	2	1	3	3	5	2	5	5	2.75
12	6	1	6	3	6	2	6	5	2.75
13	4	1	4	3	4	2	4	4	2.5
14	1	1	6	3	1	1	4	4	2.25
15	2	1	4	3	2	1	1	1	1.5
16	8	1	3	3	8	1	8	1	1.5




Best fitness function value
Iterations
0
50
100
150
200
250
0
2
4
6
8
10


Best fitness function value
Function evaluations
1
2
3
4
5
6
7
8
0
5
10
15


Best fitness function value
Iterations
0
100
200
300
400
0
2
4
6
8
10
Run1
Run2
Run3
Run4
Run5
Run6
Run7
Run8
Run9
Run10
Run11
Run12
Run13
Run14
Run15


Mean of best fitness function value
Function evaluations
0
2
4
6
8
10
12
14
0
5
10
15

Comparison of algorithms

Algorithm 1

Function	Best	Worst	Mean	Median	Standard Deviation
Function 1	26	30	27.2	28	1.46
Function 2	18	21	18.12	18	0.6
Function 3	60	137	120.68	131	27.08
Function 4	46	51	47.24	47	1.36
Function 5	235	250	239.24	238	4.21

Algorithm 2

Function	Best	Worst	Mean	Median	Standard Deviation
Function 1	26	58	43.4	57	15.78
Function 2	18	21	20.6	21	0.99
Function 3	136	141	139	139	1.43
Function 4	46	50	48.4	48	1.00
Function 5	254	263	259	259	2.54

	Algorithm 1	Algorithm 2	Identical
Best	2	0	3
Worst	3	1	1
Mean	5	0	0
Median	5	0	0
Std. Dev.	2	3	0

Function	Best Solution
Function 1	26
Function 2	18
Function 3	60
Function 4	46
Function 5	235

Issues in the implementation of TLBO

Journals & Books Register

Download PDF Share Export

Information Sciences
Volume 183, Issue 1, 15 January 2012, Pages 1-15

Teaching–Learning-Based Optimization: An optimization method for continuous non-linear large scale problems

R.V. Rao, V.J. Savsani, D.P. Vakharia

Journals & Books Register

Download PDF Share Export

Information Sciences
Volume 212, 1 December 2012, Pages 79-93

A note on teaching–learning-based optimization algorithm

Matej Črepinšek[a], Shih-Hsi Liu[b], Luka Mernik[c]

Show more

https://doi.org/10.1016/j.ins.2012.05.009

Get rights and content

Journals & Books Register

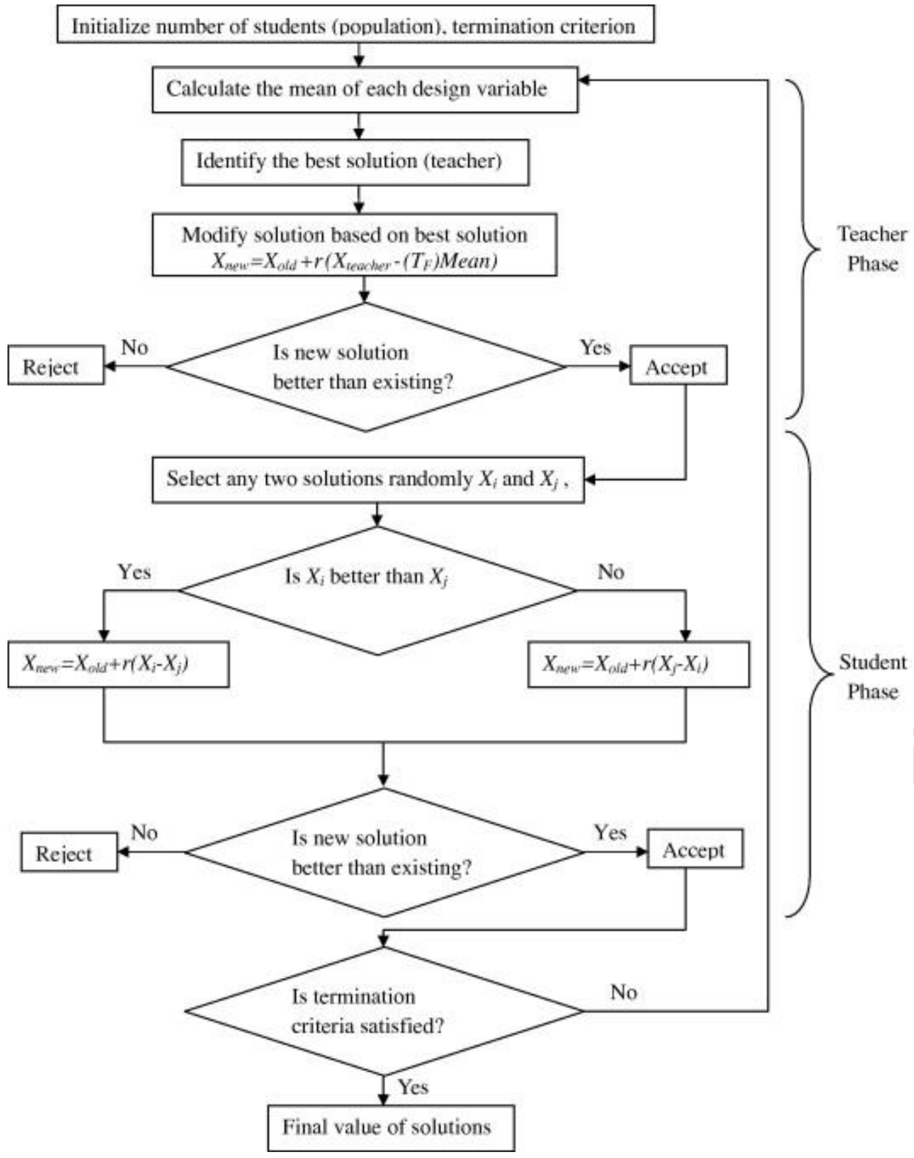
Download PDF Share Export

Information Sciences
Volume 229, 20 April 2013, Pages 159-169

Comments on "A note on teaching–learning-based optimization algorithm"

Gajanan Waghmare

Issues in the implementation of TLBO



for i = 1:N_P
    Perform the Teacher Phase of i^{th} solution
    Update the population
    Perform the Learner Phase of i^{th} solution
    Update the population
end

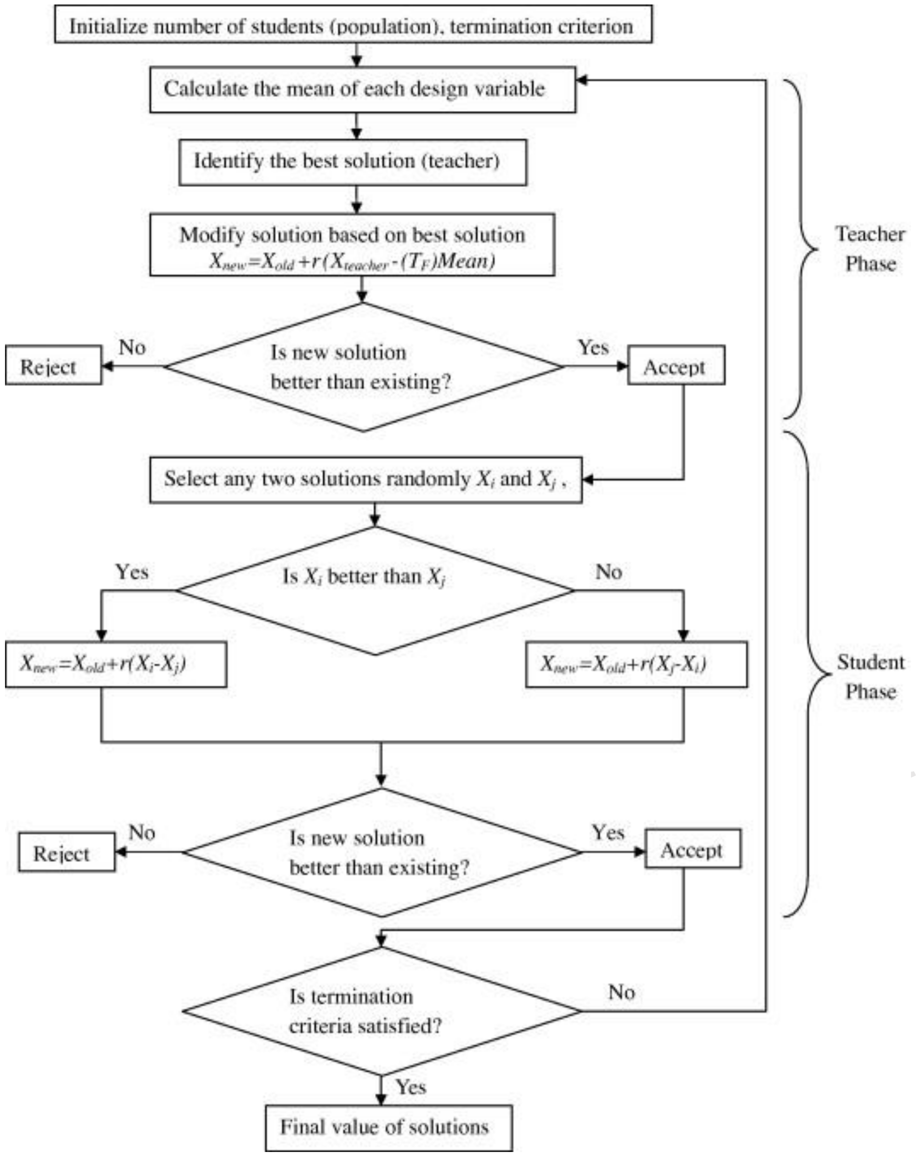
Iteration 1			Iteration 2			Iteration 3			....	Iteration T		
T_1L_1	T_2L_2	T_3L_3	T_1L_1	T_2L_2	T_3L_3	T_1L_1	T_2L_2	T_3L_3		T_1L_1	T_2L_2	T_3L_3

for i = 1:N_P
    Perform the Teacher Phase of i^{th} solution
    Update the population
end
for i = 1:N_P
    Perform the Learner Phase of i^{th} solution
    Update the population
end

Iteration 1		Iteration 2		Iteration 3		....	Iteration T	
$T_1T_2T_3$	$L_1L_2L_3$	$T_1T_2T_3$	$L_1L_2L_3$	$T_1T_2T_3$	$L_1L_2L_3$		$T_1T_2T_3$	$L_1L_2L_3$

Computer-Aided Design, Volume 43, Issue 3, 2011, Pages 303-315, Elsevier: https://doi.org/10.1016/j.cad.2010.12.015

Issues in Implementation of TLBO



1. Input: Fitness function, lb, ub, N_p, T
2. Initialize a random population (P)
3. Evaluate fitness of P
4. for t = 1 to T

    for i = 1 to N_P

      Choose X_{best}

      Determine X_{mean}

      $X_{new} = X_i + r\,(X_{best} - T_f X_{mean})$

      Bound X_{new} and evaluate its fitness f_{new}

      Accept X_{new} if it is better than X_i

      Choose any solution randomly , X_p

      Determine X_{new} as

      if $f_i < f_p$

        $X_{new} = X_i + r\,(X_i - X_p)$

      else

        $X_{new} = X_i + r\,(X_i - X_p)$

      end

      Bound X_{new} and evaluate its fitness f_{new}

      Accept X_{new} if it is better than X_i

    end

end

Computer-Aided Design, Volume 43, Issue 3, 2011, Pages 303-315, Elsevier: https://doi.org/10.1016/j.cad.2010.12.015

Duplicates

- Two solutions with identical set of decision variables

S1 and S2 are identical solutions if the values of decision variables are identical. Comparison of S1 and S2 should **NOT** be done after sort the variables

Tag	Solution	f	Tag	Solution	f
S1	[2 5 4]	-7	Sorted S1	[2 4 5]	-7
S2	[4 2 5]	-3	Sorted S2	[2 4 5]	-7

$$f = x_1 - x_2 - x_3$$

- S1 and S3 are not identical solutions if the decision variables are not identical but their objective function values are identical. S1 and S3 are realizations

Tag	Solution	f
S1	[2 5 4]	-7
S3	[0 5 2]	-7

$$f = x_1 - x_2 - x_3$$

- Occurrence of duplicates can be very rare, especially in higher dimension problems

Difference between TLBO and s-TLBO

- Duplicate removal
  - Included in TLBO. Duplicates identified by sorting the solutions.
  - No duplicate removal in sanitized TLBO
- Number of times the fitness function is evaluated
  - Is stochastic in TLBO as it depends on the duplicates
  - Deterministic in Sanitized TLBO ($2N_pT + N_p$)
- Partners
  - Multiple solutions can have the same partner in TLBO
  - Every member has an unique partner

Elitist TLBO (ETLBO): Variant of TLBO

- Elitism: replacement of worst solutions with the elite solutions.
- Incorporated in every iteration at the end of learner phase.
- Procedure to generate new solutions is same as in TLBO.
- Algorithm parameters: population size, number of iterations and **elite size.**
- Elite size specifies the number of worst solutions which have to be replaced.
- Duplicate removal is performed after replacing worst solutions with elite solutions.

An elitist teaching-learning-based optimization algorithm for solving complex constrained optimization problems, International Journal of Industrial Engineering Computations, 3(4), 535-560, 2012

Improved TLBO: Variant of TLBO

- Divides the population into groups.
- Incorporates tutorial learning in teacher phase.
- Incorporates self-learning in learner phase.
- Number of teachers in the population is equal to number of groups.
- Solution corresponding to best fitness value is chief teacher.
- Other teachers are selected based on the fitness value of chief teacher and their fitness.
- An adaptive teaching factor is introduced.
- Elitism and duplicate removal are incorporated.

An improved teaching-learning-based optimization algorithm for solving unconstrained optimization problems, Scientia Iranica, 20 (3), 710-720, 2013.

TLBO Codes

- ➢MATLAB Code: by inventors https://sites.google.com/site/tlborao/tlbo-code

  (includes duplicate removal, duplicates identified after sorting)

- ➢MATLAB Code of Sanitized TLBO:
  https://in.mathworks.com/matlabcentral/fileexchange/65628-teaching-learning-based-optimization

- ➢MATLAB Code: https://yarpiz.com/83/ypea111-teaching-learning-based-optimization

  (entire class undergoes teacher phase first)

- ➢JAVA Code: https://github.com/maciejj04/TLBO

Further reading

- Teaching-learning-based optimization: An optimization method for continuous non-linear large scale problems, *Information Sciences*, Volume 183, Issue 1, 2012
- A note on teaching-learning-based optimization algorithm, *Information Sciences*, Volume 212, Pages 79-93, 2012
- Comments on "A note on teaching-learning-based optimization algorithm", *Information Sciences*, Volume 229, Pages 159-169, 2013
- Teaching-Learning-Based Optimization (TLBO) Algorithm and its engineering applications. *Springer International Publishing, Switzerland*, 2016
- A survey of teaching-learning-based optimization, *Neurocomputing*, Volume 335, Pages 366-383, 2019
- Multi-objective optimization using teaching-learning-based optimization algorithm, *Engineering Applications of Artificial Intelligence*, Volume 26, Issue 4, Pages 1291-1300, 2013

Closure

- Generic framework of metaheuristic algorithms
- Sanitized Teaching Learning Based Optimization (s-TLBO)
- Detailed working of s-TLBO with an example
- Various types of convergence curves
- Statistical analysis of multiple runs
- Preliminary comparison of algorithms
- Issues in TLBO
- Variants of TLBO
- Implementation of s-TLBO in MATLAB

Thank You !!!