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Case Study: Production Planning

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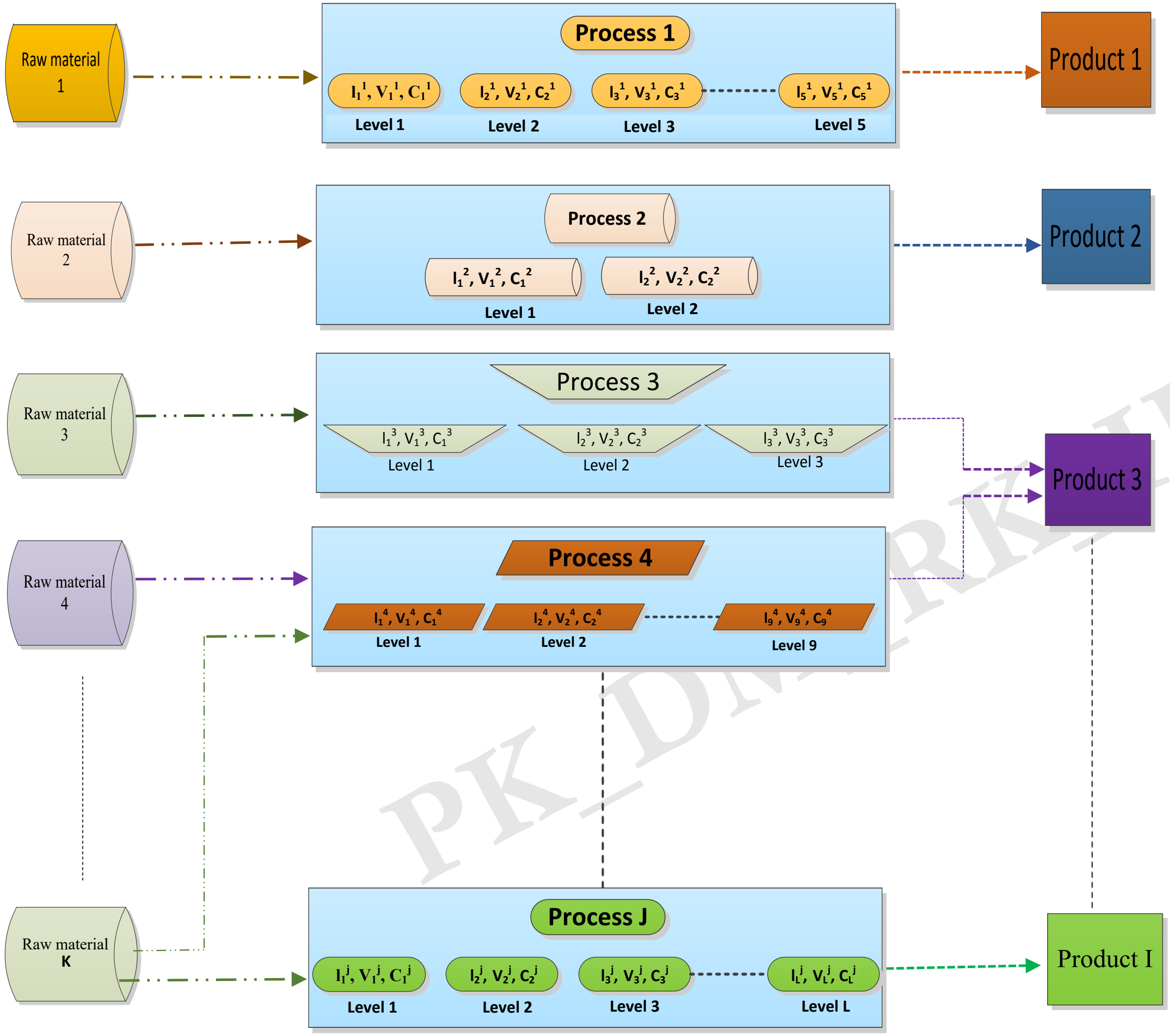
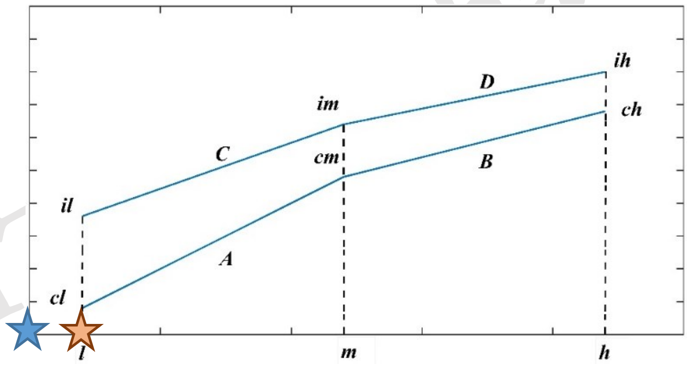
Case study: https://www.youtube.com/watch?v=PRjExZxWsNc
Implementation on MATLAB: https://www.youtube.com/watch?v=VjTDGJUQAcg
Constraint handling using correction approach: https://www.youtube.com/watch?v=HwWvfQ9QbYo
Single-Level Production Planning in Petrochemical Industries Using Novel Computational Intelligence Algorithms
MILP formulation of Production Planning Problem: https://www.youtube.com/watch?v=9Y99qRwObtk
Additional resources: tinyurl.com/sksopti, tinyurl.com/sksoptivid

Production Planning: Problem Definition



K type of raw materials, J processes, T different products.

A product can be produced by more than one process.



Production cost and investment costs are known at different production capacity levels.

Cost between successive known levels are a linear function of the quantity that is produced.

- If produced, production below the minimum level or greater than the maximum level is NOT possible.
- Limited amount of budget is available.
- Limited amount of raw materials are available.
- Not all products need to be produced.
- **Maximize** the profit (diff. b/w total selling price and production costs)

Engineering Optimization, 2002, 34(6), 671–687

Case Study: Production Planning Industry

Sale Price (monetary unit/ unit of product)	Product	Process	Capacity (units of product/yr) l_j	m_j	h_j	Production cost (monetary unit/yr) cl_j	cm_j	ch_j	Investment cost (monetary unit) il_j	im_j	ih_j	Raw material required (per unit of product) $rm1$	$rm2$	$rm3$
0.975	T1	P1	70	135	270	50.7	90.1	170.7	55	81.1	131.6	0.948	0	0
		P2	75	150	300	56.8	103.8	196.2	58	85.1	132.4	0.9432	0	0
		P3	77.5	155	310	56.9	103.7	195.7	60.2	86.8	134.1	0.949	0	0
0.975	T2	P4	70	145	290	51.7	97.6	184.8	55.1	83.1	132	0.9546	0	0
		P5	47.5	95	190	38.2	69.8	130.4	43.3	66.8	104.3	0.955	0	0
0.780	T3	P6	40	80	160	38.5	65.2	120.7	66.2	92.8	153.2	1.045	0	0
		P7	40	80	160	31.8	57.1	105.5	40	61.4	95.1	1.05	0	0
0.735	T4	P8	45	90	180	37.8	57.7	94.9	106.6	151.7	231.5	0.5103	0	0
1.450	T5	P9	40	80	160	38.5	65.6	119.1	82.8	125.4	207	0.6289	0	0
1.130	T6	P10	90	180	360	92.2	159.2	290.9	233.5	390.7	698.7	0.8648	0	0
		P11	90	180	360	86.7	154.1	287.7	185.8	304.5	537.1	0.9546	0	0
		P12	90	180	360	95.8	175	330.9	119	179.4	289.2	0.8265	0	0
		P13	90	180	360	87.5	157.2	294.9	212.3	362.7	657.7	0.7875	0	0
		P14	90	180	360	105.9	196.6	375.2	109.8	164.3	263.1	0.8101	0	0
		P15	90	180	360	93.1	131.1	239.4	221.7	376.1	672.7	0.8782	0	0
0.830	T7	P16	50	100	200	41.4	68.7	117.2	115.5	180.4	287.4	0.815	0	0
		P17	50	100	200	34.9	62	111.6	63.7	100.2	156.3	0.6994	0	0
0.450	T8	P18	60	120	240	36.6	62.1	120.8	23.1	33.2	50.7	0.3784	0	0

Case Study: Production Planning Industry

Sale Price (monetary unit/ unit of product)	Product	Process	Capacity (units of product/yr)			Production cost (monetary unit/yr)			Investment cost (monetary unit)			Raw material required (per unit of product)		
			l_j	m_j	h_j	cl_j	cm_j	ch_j	il_j	im_j	ih_j	*rm1*	*rm2*	*rm3*
0.74	T9	P19	100	200	400	67.6	125.2	237.2	117.6	186	307.5	0	0	0
		P20	50	100	300	33	63.1	163.8	62.5	114	209.6	0	0	0
1.25	T10	P21	25	50	100	28.7	48.3	86	73.1	101.1	148	0	0	0
		P22	25	50	100	24	43.1	79.5	46.5	70.7	110.1	0	0	0
		P23	125	250	500	63.8	123.5	241	49.2	74.4	112.8	0	0	0
0.43	T11	P24	125	250	500	68.5	134.5	264	79.1	144.2	258.1	0	0	0
		P25	250	500	1000	101.5	195	377	134	229.9	392.2	0	0.4678	0
0.6	T12	P26	90	180	360	50.3	90	165.6	142.6	234.8	397.5	0	0.7267	0
0.69	T13	P27	67.5	135	200	53.9	101.2	146.4	82.7	133.6	181.3	0	0.393	0
		P28	70	135	270	42.1	75.1	141.8	56.9	84.5	131.5	0	1.02	0
0.86	T14	P29	70	135	270	44.6	77.5	147.7	63.4	84.5	136.9	0	1.02	0
		P30	70	135	270	44.6	78.8	148	66.5	96.2	147.7	0	1.02	0
		P31	100	200	400	55.7	106.8	208.4	51.4	83	144.5	0	0.9461	0
0.9	T15	P32	75	150	300	48.3	90.2	172.8	46.9	66	98.6	0	0.9387	0
		P33	122.5	245	490	92	174	336.2	82.4	116.6	175.6	0	0.943	0
0.87	T16	P34	50	100	200	34.9	63.9	120.4	72	117.7	199.7	0	1.06	0
		P35	182.5	365	540	63.2	111.4	156.6	125.6	195.9	259.6	0	0	0
0.48	T17	P36	182.5	365	540	60.3	103	142.6	116.4	168.2	213.5	0	0	0
		P37	180	360	550	64.7	110.2	154.6	133.2	196.3	248.9	0	0	0

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Case Study: Production Planning Industry

Sale Price (monetary unit/ unit of product)	Product	Process	Capacity (units of product/yr)			Production cost (monetary unit/yr)			Investment cost (monetary unit)			Raw material required (per unit of product)		
			l_j	m_j	h_j	cl_j	cm_j	ch_j	il_j	im_j	ih_j	*rm1*	*rm2*	*rm3*
0.16	T18	P38	300	430	590	48.3	65	85	210.9	278.2	356	0	0	6.35
		P39	300	430	590	52.8	71.4	92.7	243.5	322.4	412.6	0	0	5.928
		P40	105	170	340	19.4	27.4	47.3	87	119.5	196.7	0	0	6.678
0.5	T19	P41	15	25	50	6.6	9.7	17.7	15.3	20.2	32.7	0	0	0
		P42	15	25	50	6.9	10.6	19.4	17.9	26.2	44.9	0	0	0
0.15	T20	P43	415	830	1660	55.2	96.3	184.3	224.6	365.5	682.1	0	0	7.867
		P44	415	830	1660	56.5	100.5	194.3	228.5	384.6	727.6	0	0	7.778
		P45	415	830	1660	51.9	98	187.6	199.1	371.5	702.9	0	0	7.661
0.76	T21	P46	225	450	680	105.8	204.8	306	116.9	190	265.1	0	0.2891	0
		P47	225	450	680	108	209.7	313.5	115.6	191.8	266.8	0	0.2878	0
		P48	225	450	680	105.6	202.5	302.6	125.2	192.7	269	0	0.2843	0
		P49	225	450	680	106.7	206.1	308.1	125.2	202	285.5	0	0.2874	0
0.7	T22	P50	12.5	25	50	9.4	16.4	28.4	26	40.8	63.9	0	0	0
		P51	12.5	25	50	9	15.4	27.3	27.7	39.6	56.9	0	0	0
0.735	T23	P52	45	90	180	36.8	64	118.7	108.8	157.2	251.6	0	0	0
0.68	T24	P53	125	250	500	81.4	145.8	275.5	208.1	308.6	515.5	0	0	0
		P54	125	250	500	78.4	145	277	170.5	267.3	452.7	0	0	0

Decisions

- Products that need to be produced
- Processes to be used for producing selected products
- Amount of production from the processes that have been selected for producing a particular product

Selection of Decision Variables: Approach 1

$$\overbrace{\qquad\qquad}^{\mathbf{X}}$$

Product			Process				Production quantity			
T1	T2	T3	P1	P2	…	P6	$x1$	$x2$	…	$x6$
Binary variables			Binary variables				Continuous variables: $0 \le x(j) \le h(j)$			

$\cancel{l(j) \le x_j \le h\,(j)}$

Products			Process						Production quantity					
1	1	0	1	0	1	1	1	0	6	0	10	5	20	0
T1	T2	T3	P1	P2	P3	P4	P5	P6	$x1$	$x2$	$x3$	$x4$	$x5$	$x6$

Product	Process	Production level		
		l	m	h
T1	P1	5	10	20
	P2	8	13	22
T2	P3	4	9	20
	P4	2	7	20
	P5	10	15	25
T3	P6	3	8	20

Consistent

Selection of Decision Variables: Approach 1

X

Product			Process				Production quantity			
T1	T2	T3	P1	P2	…	P6	$x1$	$x2$	…	$x6$
Binary variables			Binary variables				Continuous variables: $0 \le x(j) \le h(j)$			

Products			Process						Production quantity					
1	1	0	1	0	1	1	1	0	6	0	10	5	20	0
T1	T2	T3	P1	P2	P3	P4	P5	P6	$x1$	$x2$	$x3$	$x4$	$x5$	$x6$

T1 and T2 produced

P1, P3, P4 and P5 are used

Quantity produced

T1	T2		
P1	P3	P4	P5
6	10	5	20

Consistent

Product	Process	Production level		
		l	m	h
T1	P1	5	10	20
	P2	8	13	22
T2	P3	4	9	20
	P4	2	7	20
	P5	10	15	25
T3	P6	3	8	20

Selection of Decision Variables: Approach 1

$\mathbf{X}$

Product			Process				Production quantity			
T1	T2	T3	P1	P2	…	P6	$x1$	$x2$	…	$x6$
Binary variables			Binary variables				Continuous variables: $0 \leq x(j) \leq h(j)$			

Products			Process						Production quantity					
1	0	1	1	0	0	1	0	0	6	8	10	5	20	10
T1	T2	T3	P1	P2	P3	P4	P5	P6	$x1$	$x2$	$x3$	$x4$	$x5$	$x6$

Process P2 not used

Process P2 produces 8 units

Inconsistent

Product	Process	Production level		
		l	m	h
T1	P1	5	10	20
	P2	8	13	22
T2	P3	4	9	20
	P4	2	7	20
	P5	10	15	25
T3	P6	3	8	20

Selection of Decision Variables: Approach 1

$\mathbf{X}$

Product			Process				Production quantity			
T1	T2	T3	P1	P2	…	P6	$x1$	$x2$	…	$x6$
Binary variables			Binary variables				Continuous variables: $0 \leq x(j) \leq h(j)$			

Products			Process						Production quantity					
1	0	1	1	0	0	1	0	0	6	8	10	5	20	10
T1	T2	T3	P1	P2	P3	P4	P5	P6	$x1$	$x2$	$x3$	$x4$	$x5$	$x6$

T2 is not produced

P4 for T2 is active

35 units of T2 is produced using P3, P4 and P5

Inconsistent

Product	Process	Production level		
		l	m	h
T1	P1	5	10	20
	P2	8	13	22
T2	P3	4	9	20
	P4	2	7	20
	P5	10	15	25
T3	P6	3	8	20

Selection of Decision Variables: Approach 2

$$\overbrace{\hspace{20em}}^{\mathbf{X}}$$

Product			Process				Production quantity			
T1	T2	T3	P1	P2	…	P6	$x1$	$x2$	…	$x6$
Binary variables			Binary variables				Continuous variables: $0 \leq x(j) \leq h(j)$			

Products			Process						Production quantity					
1	0	1	1	0	0	1	0	0	6	8	10	5	20	10
T1	T2	T3	P1	P2	P3	P4	P5	P6	$x1$	$x2$	$x3$	$x4$	$x5$	$x6$

T3 is produced

P6 for T3 is inactive

10 units of T3 is produced

Inconsistent

Product	Process	Production level		
		l	m	h
T1	P1	5	10	20
	P2	8	13	22
T2	P3	4	9	20
	P4	2	7	20
	P5	10	15	25
T3	P6	3	8	20

Selection of Decision Variables: Approach 2

X

x_1	x_2	x_3	x_4	x_5	………	x_J

x_1: Amount of product produced by process 1

x_5: Amount of product produced by process 5

x_J: Amount of product produced by process J

Domain of decision variables: $0 \le x_j \le h_j \;\; \forall j = 1, 2, \ldots, J$

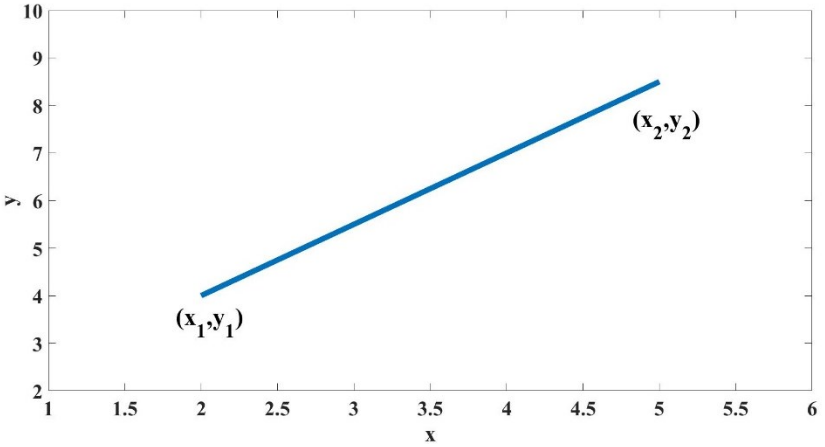
J is the total number of processes

X	12	9	5	18	14	7

Product	Process used	Total amount
T1	P1, P2	$x_1 + x_2 = 12 + 9 = 21$
T2	P3, P4, P5	$x_3 + x_4 + x_5 = 5 + 18 + 14 = 37$
T3	P6	$x_6 = 7$

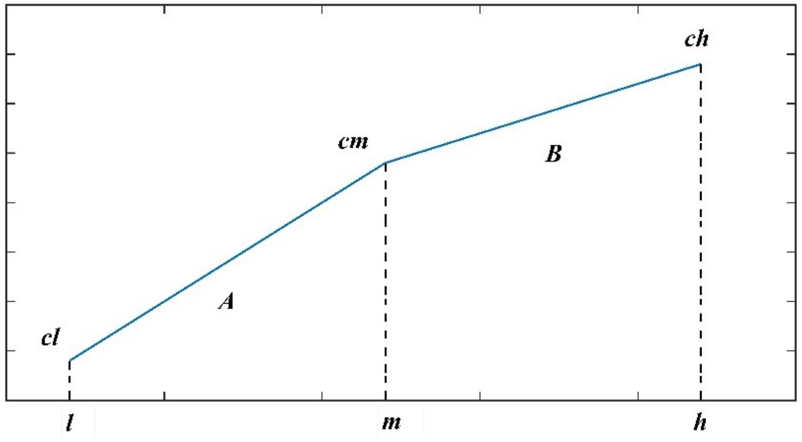
Product	Process	Production level		
		l	m	h
T1	P1	5	10	20
	P2	8	13	22
T2	P3	4	9	20
	P4	2	7	20
	P5	10	15	25
T3	P6	3	8	20

Cost determination



$$\frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1}$$

$$y=y_1+\frac{y_2-y_1}{x_2-x_1}\left(x-x_1\right)$$



Production cost between lower and medium level

A

$$x=X$$
$$x_1=l, x_2=m$$
$$y_1=cl, y_2=cm$$

$$c=cl+\frac{cm-cl}{m-l}\left(X-l\right)$$

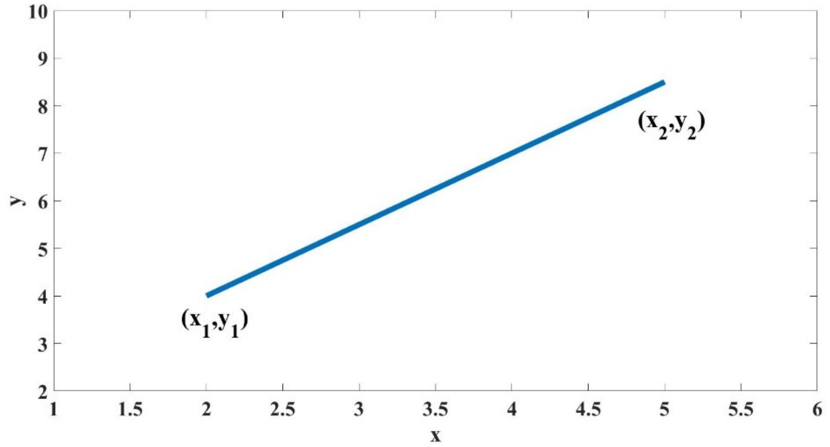
Production cost between medium and high level

B

$$x=X$$
$$x_1=m, x_2=h$$
$$y_1=cm, y_2=ch$$

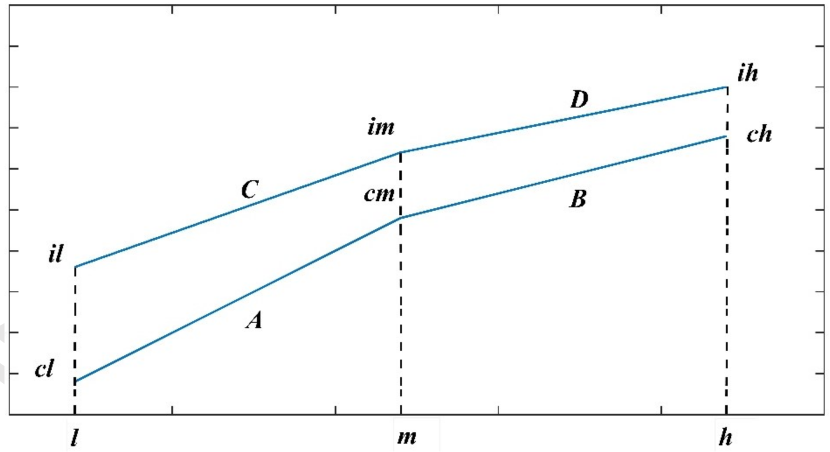
$$c=cm+\frac{ch-cm}{h-m}\left(X-m\right)$$

Cost determination



$$\frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1}$$

$$y=y_1+\frac{y_2-y_1}{x_2-x_1}(x-x_1)$$



Production cost between lower and medium level

A

$x=X$

$x_1=l, x_2=m$

$y_1=cl, y_2=cm$

$$PC=cl+\frac{cm-cl}{m-l}(X-l)$$

Production cost between medium and high level

B

$x=X$

$x_1=m, x_2=h$

$y_1=cm, y_2=ch$

$$PC=cm+\frac{ch-cm}{h-m}(X-m)$$

Investment cost between lower and medium level

C

$x=X$

$x_1=l, x_2=m$

$y_1=il, y_2=im$

$$IC=il+\frac{im-il}{m-l}(X-l)$$

Investment cost between medium and high level

D

$x=X$

$x_1=m, x_2=h$

$y_1=im, y_2=ih$

$$IC=im+\frac{ih-im}{h-m}(X-m)$$

Cost determination

One process

Production cost between lower and medium level

$$x = X$$
$$x_1 = l, x_2 = m$$
$$y_1 = cl, y_2 = cm$$

$$PC = cl + \frac{cm - cl}{m - l}(X - l)$$

Production cost between medium and high level

$$x = X$$
$$x_1 = m, x_2 = h$$
$$y_1 = cm, y_2 = ch$$

$$PC = cm + \frac{ch - cm}{h - m}(X - m)$$

Investment cost between lower and medium level

$$x = X$$
$$x_1 = l, x_2 = m$$
$$y_1 = il, y_2 = im$$

$$IC = il + \frac{im - il}{m - l}(X - l)$$

Investment cost between medium and high level

$$x = X$$
$$x_1 = m, x_2 = h$$
$$y_1 = im, y_2 = ih$$

$$IC = im + \frac{ih - im}{h - m}(X - m)$$

Process *j*

Production cost between lower and medium level

$$x(j) = X(j)$$
$$x_1 = l(j), x_2 = m(j)$$
$$y_1 = cl(j), y_2 = cm(j)$$

$$PC(j) = cl(j) + \frac{cm(j) - cl(j)}{m(j) - l(j)}\left(X(j) - l(j)\right)$$

Production cost between medium and high level

$$x(j) = X(j)$$
$$x_1 = m(j), x_2 = h(j)$$
$$y_1 = cm(j), y_2 = ch(j)$$

$$PC(j) = cm(j) + \frac{ch(j) - cm(j)}{h(j) - m(j)}\left(X(j) - m(j)\right)$$

Investment cost between lower and medium level

$$x(j) = X(j).$$
$$x_1 = l(j), x_2 = m(j)$$
$$y_1 = il(j), y_2 = im(j)$$

$$IC(j) = il(j) + \frac{im(j) - il(j)}{m(j) - l(j)}\left(X(j) - l(j)\right)$$

Investment cost between medium and high level

$$x(j) = X(j)$$
$$x_1 = m(j), x_2 = h(j)$$
$$y_1 = im(j), y_2 = ih(j)$$

$$IC(j) = im(j) + \frac{ih(j) - im(j)}{h(j) - m(j)}\left(X(j) - m(j)\right)$$

Profit calculation

X

6	0	10	5	20	0

Products	Processes	Production level			Production cost			Investment cost			Raw material required		Selling price
		l	m	h	cl	cm	ch	il	im	ih	rm1	rm2	
T1	P1	5	10	20	10	20	30	50	60	70	0.6	0.8	10
	P2	8	13	22	12	22	31	52	62	71	0.5	1.2	10
T2	P3	4	9	20	8	18	29	55	65	76	0.4	0.6	30
	P4	2	7	20	10	20	33	58	68	81	0.7	0.9	30
	P5	10	15	25	12	22	32	60	70	80	0.9	1.1	30
T3	P6	3	8	20	15	25	37	54	64	76	0.8	1.3	50

Solution	6	0	10	5	20	0	Total
Production cost	12	0	19	16	27	0	74
Investment cost	52	0	66	64	75	0	257
Raw material 1	3.6	0	4	3.5	18	0	29.1
Raw material 2	4.8	0	6	4.5	22	0	37.3
Revenue	60	0	300	150	600	0	1110
Profit	48	0	281	134	573	0	1036

$$PC(j) = cl(j) + \frac{cm(j) - cl(j)}{m(j) - l(j)}\left(X - l(j)\right).$$

$$PC(1) = 10 + \frac{20-10}{10-5}\left(6-5\right) = 12$$

$$PC(j) = cm(j) + \frac{ch(j) - cm(j)}{h(j) - m(j)}\left(X - m(j)\right).$$

$$PC(3) = 18 + \frac{29-18}{20-9}\left(10-9\right) = 19$$

Profit calculation

Products	Processes	Production level			Production cost			Investment cost			Raw material required		Selling price
		l	m	h	cl	cm	ch	il	im	ih	rm1	rm2	
T1	P1	5	10	20	10	20	30	50	60	70	0.6	0.8	10
	P2	8	13	22	12	22	31	52	62	71	0.5	1.2	10
T2	P3	4	9	20	8	18	29	55	65	76	0.4	0.6	30
	P4	2	7	20	10	20	33	58	68	81	0.7	0.9	30
	P5	10	15	25	12	22	32	60	70	80	0.9	1.1	30
T3	P6	3	8	20	15	25	37	54	64	76	0.8	1.3	50

X

6	0	10	5	20	0

Solution	6	0	10	5	20	0	Total
Production cost	12	0	19	16	27	0	74
Investment cost	52	0	66	64	75	0	257
Raw material 1	3.6	0	4	3.5	18	0	29.1
Raw material 2	4.8	0	6	4.5	22	0	37.3
Revenue	60	0	300	150	600	0	1110
Profit	48	0	281	134	573	0	1036

$$IC(j) = il(j) + \frac{im(j) - il(j)}{m(j) - l(j)}\left(X - l(j)\right).$$

$$IC(1) = 50 + \frac{60 - 50}{10 - 5}(6 - 5) = 52$$

$$IC(j) = im(j) + \frac{ih(j) - im(j)}{h(j) - m(j)}\left(X - m(j)\right).$$

$$IC(3) = 65 + \frac{76 - 65}{20 - 9}(10 - 9) = 66$$

Profit calculation

X

6	0	10	5	20	0

Products	Processes	Production level			Production cost			Investment cost			Raw material required		Selling price
		l	m	h	cl	cm	ch	il	im	ih	rm1	rm2	
T1	P1	5	10	20	10	20	30	50	60	70	0.6	0.8	10
	P2	8	13	22	12	22	31	52	62	71	0.5	1.2	10
T2	P3	4	9	20	8	18	29	55	65	76	0.4	0.6	30
	P4	2	7	20	10	20	33	58	68	81	0.7	0.9	30
	P5	10	15	25	12	22	32	60	70	80	0.9	1.1	30
T3	P6	3	8	20	15	25	37	54	64	76	0.8	1.3	50

Solution	6	0	10	5	20	0	Total
Production cost	12	0	19	16	27	0	74
Investment cost	52	0	66	64	75	0	257
Raw material 1	3.6	0	4	3.5	18	0	29.1
Raw material 2	4.8	0	6	4.5	22	0	37.3
Revenue	60	0	300	150	600	0	1110
Profit	48	0	281	134	573	0	1036

X	Raw material 1 used
6	6 x 0.6 = 3.6
0	0
10	10 x 0.4 = 4
5	5 x 0.7 = 3.5
20	20 x 0.9 = 18
0	0

Profit calculation

X

6	0	10	5	20	0

Products	Processes	Production level			Production cost			Investment cost			Raw material required		Selling price
		l	m	h	cl	cm	ch	il	im	ih	rm1	rm2	
T1	P1	5	10	20	10	20	30	50	60	70	0.6	0.8	10
	P2	8	13	22	12	22	31	52	62	71	0.5	1.2	10
T2	P3	4	9	20	8	18	29	55	65	76	0.4	0.6	30
	P4	2	7	20	10	20	33	58	68	81	0.7	0.9	30
	P5	10	15	25	12	22	32	60	70	80	0.9	1.1	30
T3	P6	3	8	20	15	25	37	54	64	76	0.8	1.3	50

Solution	6	0	10	5	20	0	Total
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Raw material 1	3.6	0	4	3.5	18	0	29.1
Raw material 2	4.8	0	6	4.5	22	0	37.3
Revenue	60	0	300	150	600	0	1110
Profit	48	0	281	134	573	0	1036

X	Revenue
6	6 x 10 = 60
0	0
10	10 x 30 = 300
5	5 x 30 = 150
20	20 x 30 = 600
0	0

Profit calculation

X

6	0	10	5	20	0

Products	Processes	Production level			Production cost			Investment cost			Raw material required		Selling price
		l	m	h	cl	cm	ch	il	im	ih	rm1	rm2	
T1	P1	5	10	20	10	20	30	50	60	70	0.6	0.8	10
	P2	8	13	22	12	22	31	52	62	71	0.5	1.2	10
	P3	4	9	20	8	18	29	55	65	76	0.4	0.6	30
T2	P4	2	7	20	10	20	33	58	68	81	0.7	0.9	30
	P5	10	15	25	12	22	32	60	70	80	0.9	1.1	30
T3	P6	3	8	20	15	25	37	54	64	76	0.8	1.3	50

Solution	6	0	10	5	20	0	Total
Production cost	12	0	19	16	27	0	74
Investment cost	52	0	66	64	75	0	257
Raw material 1	3.6	0	4	3.5	18	0	29.1
Raw material 2	4.8	0	6	4.5	22	0	37.3
Revenue	60	0	300	150	600	0	1110
Profit	48	0	281	134	573	0	1036

X	Profit
6	60 – 12 = 48
0	0
10	300 – 19 = 281
5	150 – 16 = 134
20	600 – 27 = 573
0	0

Domain constraint

- Quantity produced by a process can be **zero** or should be **greater than or equal to its low level production capacity**.
- Penalty incurred for a violated variable is $P^{domain}(j) = \begin{cases} 10^5 & if\ 0 < X(j) < l(j) \\ 0 & if\ l(j) \le X(j) \le h(j) \end{cases} \quad \forall j = 1,2,\dots,J$

X	12	20	5	4	18	0

- **P1 produces T1 between M and H - valid**
- **P2 produces T1 between M and H - valid**

- **P3 produces T2 between L and M - valid**
- **P4 produces T2 between L and M - valid**
- **P5 produces T2 between M and H - valid**

- **P6 has not produced T3**

Product	Process	*l*	*m*	*h*
T1	P1	5	10	20
	P2	8	13	22
T2	P3	4	9	20
	P4	2	7	20
	P5	10	15	25
T3	P6	3	8	20

Values of all variable are within their domains - Feasible solution with respect to the domain

$$P^{domain} = \sum_{j=1}^{J} P^{domain}(j) = 0$$

Domain constraint

- Quantity produced by a process can be **zero** or should be **greater than or equal to its low level production capacity**.
- Penalty incurred for a violated variable is $P^{domain}(j) = \begin{cases} 10^5 & if\ 0 < X(j) < l(j) \\ 0 & if\ l(j) \le X(j) \le h(j) \end{cases} \quad \forall j = 1,2,\ldots,J$

X	4	5	2	1	5	2

- **P1 produces T1 between 0 and L - not valid**
- **P2 produces T1 between 0 and L - not valid**

- **P3 produces T2 between 0 and L - not valid**
- **P4 produces T2 between 0 and L - not valid**
- **P5 produces T2 between 0 and L - not valid**

- **P6 produces T3 between 0 and L - not valid**

Product	Process	*l*	*m*	*h*
T1	P1	5	10	20
	P2	8	13	22
T2	P3	4	9	20
	P4	2	7	20
	P5	10	15	25
T3	P6	3	8	20

Values of all variables are in the invalid region – Infeasible solution with respect to the domain

$$P^{domain} = \sum_{j=1}^{J} P^{domain}(j) = 6 \times 10^5$$

Domain constraint

- Quantity produced by a process can be **zero** or should be **greater than or equal to its low level production capacity**.
- Penalty incurred for a violated variable is $P^{domain}(j)=\begin{cases}10^5 & if\ 0<X(j)<l(j)\\ 0 & if\ l(j)\le X(j)\le h(j)\end{cases}\quad \forall j=1,2,\ldots,J$

X	9	7	8	0	6	18

- **P1 produces T1 between L and M - valid**
- **P2 produces T1 between 0 and L - not valid**

- **P3 produces T2 between L and M - valid**
- **P4 is not produced T2**
- **P5 produces T2 between 0 and L - not valid**

- **P6 produces T3 between L and M - valid**

Product	Process	*l*	*m*	*h*
T1	P1	5	10	20
	P2	8	13	22
T2	P3	4	9	20
	P4	2	7	20
	P5	10	15	25
T3	P6	3	8	20

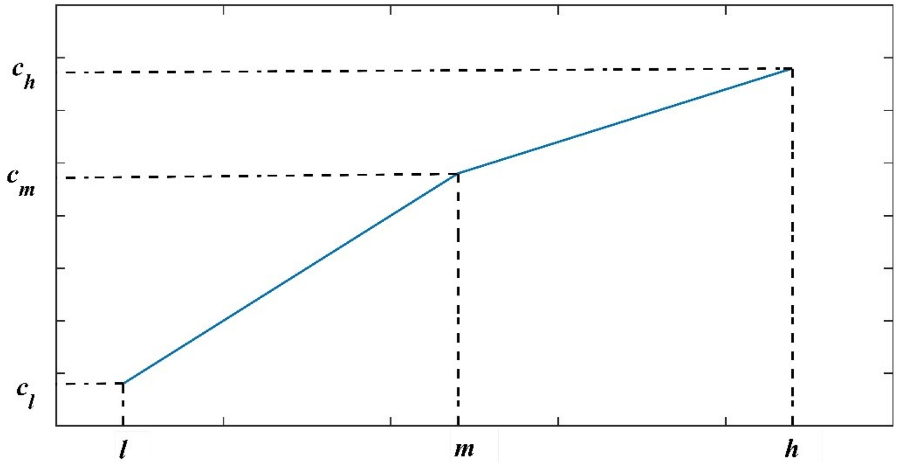
Values of some variables are in the invalid region – Infeasible solution with respect to the domain

$$P^{domain}=\sum_{j=1}^{J}P^{domain}(j)=2\times10^5$$

Production cost

- Production cost of j^{th} process can be determined as

$$PC(j)=\begin{cases} cl(j)+\dfrac{cm(j)-cl(j)}{m(j)-l(j)}\left(X(j)-l(j)\right) & \text{if } l(j)\le X(j)\le m(j) \\ cm(j)+\dfrac{ch(j)-cm(j)}{h(j)-m(j)}\left(X(j)-m(j)\right) & \text{if } m(j)\le X(j)\le h(j) \end{cases} \quad \forall j=1,2,\ldots,J$$



- Permissible production for each process is known.

- Production cost cannot be determined if production not in the permissible range.

J	Total number of processes
X(j)	Quantity produced by jth process
cl(j)	Production cost of j^{th} process at level *l*
cm(j)	Production cost of j^{th} process at level *m*
ch(j)	Production cost of j^{th} process at level *h*
l(j)	Low level production capacity of j^{th} process
m(j)	Medium level production capacity of j^{th} process
h(j)	High level production capacity of j^{th} process

Production cost

Let the solution be

X	9	7	8	15	6	18

Values of some variables are in the invalid region – Infeasible solution with respect to the domain

- Production cost of variables violating their domains cannot be calculated.

18	-	16	28	-	35

- Total production cost = 18 + 16 + 28 + 35 = **97**

Product	Process	*l*	*m*	*h*	*cl*	*cm*	*ch*
T1	P1	5	10	20	50	60	70
	P2	8	13	22	52	62	71
T2	P3	4	9	20	55	65	76
	P4	2	7	20	58	68	81
	P5	10	15	25	60	70	80
T3	P6	3	8	20	54	64	76

$$PC(j)=\begin{cases} cl(j)+\dfrac{cm(j)-cl(j)}{m(j)-l(j)}\left(X(j)-l(j)\right) & \text{if } l(j)\le X(j)\le m(j) \\ cm(j)+\dfrac{ch(j)-cm(j)}{h(j)-m(j)}\left(X(j)-m(j)\right) & \text{if } m(j)\le X(j)\le h(j) \end{cases}$$
$$\forall j=1,2,\ldots,J$$

Production cost

Let the solution be

X	4	5	2	1	5	2

Values of all variables are in the invalid region - Infeasible solution with respect to the domain

- Production cost cannot be calculated.

Product	Process	*l*	*m*	*h*	*cl*	*cm*	*ch*
T1	P1	5	10	20	50	60	70
	P2	8	13	22	52	62	71
T2	P3	4	9	20	55	65	76
	P4	2	7	20	58	68	81
	P5	10	15	25	60	70	80
T3	P6	3	8	20	54	64	76

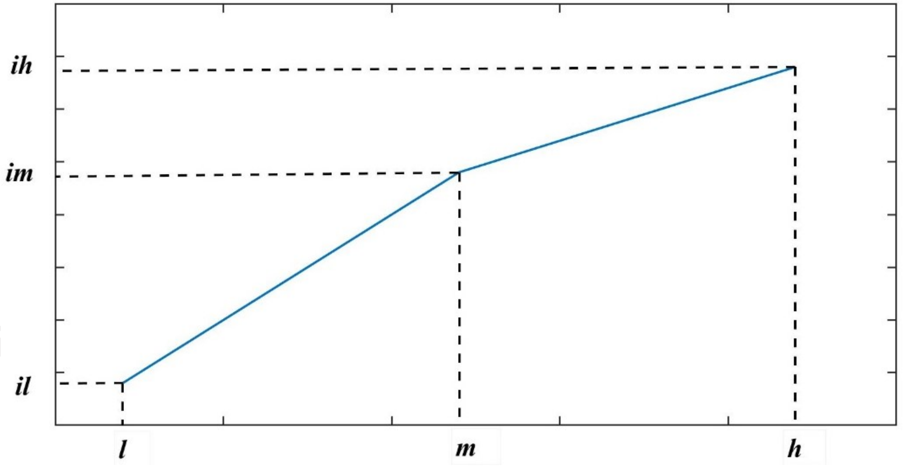
$$PC(j)=\begin{cases} cl(j)+\dfrac{cm(j)-cl(j)}{m(j)-l(j)}\left(X(j)-l(j)\right) & \text{if } l(j)\le X(j)\le m(j) \\ cm(j)+\dfrac{ch(j)-cm(j)}{h(j)-m(j)}\left(X(j)-m(j)\right) & \text{if } m(j)\le X(j)\le h(j) \end{cases}$$

$$\forall j=1,2,\ldots,J$$

Investment cost and budget

- Investment cost of j[th] process can be determined as

$$IC(j)=\begin{cases} il(j)+\left(\dfrac{im(j)-il(j)}{m(j)-l(j)}\right)(X(j)-l(j)) & \text{if } l(j)\le X(j)\le m(j) \\ im(j)+\left(\dfrac{ih(j)-im(j)}{h(j)-m(j)}\right)(X(j)-m(j)) & \text{if } m(j)\le X(j)\le h(j) \end{cases} \quad \forall j=1,2,\dots,J$$



- Investment cost of the entire production plan should not exceed the available budget.
- Violation incurs penalty (P^I)

$$P^I=\begin{cases} \left(B-\sum_{j=1}^{J} IC(j)\right)^2 & \text{if } \sum_{j=1}^{J} IC(j)>B \\ 0 & \text{otherwise} \end{cases}$$

J	Total number of processes
X(j)	Quantity produced by jth process
il(j)	Investment cost of j[th] process at level *l*
im(j)	Investment cost of j[th] process at level *m*
ih(j)	Investment cost of j[th] process at level *h*
l(j)	Low level production capacity of j[th] process
m(j)	Medium level production capacity of j[th] process
h(j)	High level production capacity of j[th] process
B	Budget available

Investment cost and budget

Let the available budget be 400 monetary units

X	**12**	**20**	**5**	**4**	**18**	**0**

Values of all variables are within their domains -
Feasible solution with respect to the domain

- Investment cost corresponding to each process is as

62	**69**	**57**	**62**	**73**	**0**

- Total investment cost = **323**

Total investment cost < Available budget
Feasible solution with respect to the budget constraint

$$\boxed{P^I = 0}$$

Product	Process	*l*	*m*	*h*	*il*	*im*	*ih*
T1	P1	5	10	20	50	60	70
	P2	8	13	22	52	62	71
T2	P3	4	9	20	55	65	76
	P4	2	7	20	58	68	81
	P5	10	15	25	60	70	80
T3	P6	3	8	20	54	64	76

$$IC(j)=\begin{cases} il(j)+\left(\dfrac{im(j)-il(j)}{m(j)-l(j)}\right)\left(X(j)-l(j)\right) & \text{if } l(j)\le X(j)\le m(j) \\ im(j)+\left(\dfrac{ih(j)-im(j)}{h(j)-m(j)}\right)\left(X(j)-m(j)\right) & \text{if } m(j)\le X(j)\le h(j) \end{cases}$$
$$\forall j=1,2,\ldots,J$$

$$P^I=\begin{cases}\left(B-\sum_{j=1}^{J} IC(j)\right)^2 & \text{if } \sum_{j=1}^{J} IC(j)>B \\ 0 & \text{otherwise}\end{cases}$$

Investment cost and budget

Let the available budget be 400 monetary units

X	20	21	20	19	23	20

Values of all variables are within their domains - Feasible solution with respect to the domain

- Investment cost to each process is as

70	70	76	80	78	76

- Total investment cost = **450**

Total investment cost > Available budget
Infeasible solution with respect to the budget constraint

$$P^I = (400-450)^2 = 2500$$

Product	Process	*l*	*m*	*h*	*il*	*im*	*ih*
T1	P1	5	10	20	50	60	70
	P2	8	13	22	52	62	71
T2	P3	4	9	20	55	65	76
	P4	2	7	20	58	68	81
	P5	10	15	25	60	70	80
T3	P6	3	8	20	54	64	76

$$IC(j) = \begin{cases} il(j) + \left(\dfrac{im(j)-il(j)}{m(j)-l(j)}\right)(X(j)-l(j)) & \text{if } l(j) \le X(j) \le m(j) \\ im(j) + \left(\dfrac{ih(j)-im(j)}{h(j)-m(j)}\right)(X(j)-m(j)) & \text{if } m(j) \le X(j) \le h(j) \end{cases}$$
$$\forall j = 1,2,\ldots,J$$

$$P^I = \begin{cases} \left(B - \sum_{j=1}^{J} IC(j)\right)^2 & \text{if } \sum_{j=1}^{J} IC(j) > B \\ 0 & \text{otherwise} \end{cases}$$

Investment cost and budget

Let the available budget be 400 monetary units

X	4	5	2	1	5	2

Values of all variables are in invalid region - Infeasible solution with respect to the domain

- Investment cost cannot be calculated.

Product	Process	*l*	*m*	*h*	*il*	*im*	*ih*
T1	P1	5	10	20	50	60	70
	P2	8	13	22	52	62	71
T2	P3	4	9	20	55	65	76
	P4	2	7	20	58	68	81
	P5	10	15	25	60	70	80
T3	P6	3	8	20	54	64	76

$$IC(j)=\begin{cases} il(j)+\left(\dfrac{im(j)-il(j)}{m(j)-l(j)}\right)(X(j)-l(j)) & \text{if } l(j)\le X(j)\le m(j) \\ im(j)+\left(\dfrac{ih(j)-im(j)}{h(j)-m(j)}\right)(X(j)-m(j)) & \text{if } m(j)\le X(j)\le h(j) \end{cases}$$
$$\forall j=1,2,\dots,J$$

$$P^I=\begin{cases}\left(B-\sum_{j=1}^{J} IC(j)\right)^2 & \text{if } \sum_{j=1}^{J} IC(j)>B \\ 0 & \text{otherwise}\end{cases}$$

Investment cost and budget

Let the available budget be 400 monetary units

X	**9**	**7**	**8**	**15**	**6**	**18**

Values of some variables are in the invalid region - Infeasible solution with respect to the domain

- Investment cost for domain violating variables is not calculated.

58	**-**	**63**	**76**	**-**	**74**

- Total investment cost = 58 + 63 + 76 + 74 = **271**

Total investment cost < Available budget
Feasible solution with respect to the budget constraint

$$\boxed{P^I = 0}$$

Product	Process	*l*	*m*	*h*	*il*	*im*	*ih*
T1	P1	5	10	20	50	60	70
	P2	8	13	22	52	62	71
T2	P3	4	9	20	55	65	76
	P4	2	7	20	58	68	81
	P5	10	15	25	60	70	80
T3	P6	3	8	20	54	64	76

$$IC(j) = \begin{cases} il(j) + \left(\dfrac{im(j) - il(j)}{m(j) - l(j)}\right)\left(X(j) - l(j)\right) & \text{if } l(j) \le X(j) \le m(j) \\ im(j) + \left(\dfrac{ih(j) - im(j)}{h(j) - m(j)}\right)\left(X(j) - m(j)\right) & \text{if } m(j) \le X(j) \le h(j) \end{cases}$$
$$\forall j = 1, 2, \ldots, J$$

$$P^I = \begin{cases} \left(B - \sum_{j=1}^{J} IC(j)\right)^2 & \text{if } \sum_{j=1}^{J} IC(j) > B \\ 0 & \text{otherwise} \end{cases}$$

Raw material

Let the available raw material be 120 units

X	12	20	5	4	18	0

Values of all variables are within their domains -
Feasible solution with respect to the domain

- Amount of raw material required for each process is

24	26	4	6	45	0

- Total raw material required = **105**

Total raw material required < Available raw material
Feasible solution with respect to the raw material constraint

$$\boxed{P^R = 0}$$

Product	Process	*l*	*m*	*h*	Raw material required (rm)
T1	P1	5	10	20	2
	P2	8	13	22	1.3
T2	P3	4	9	20	0.8
	P4	2	7	20	1.5
	P5	10	15	25	2.5
T3	P6	3	8	20	1

$$P^R(k) = \begin{cases} \left(R(k) - \sum_{j=1}^{J} rm(j) X(j) \right)^2 & \text{if } R(k) < \sum_{j=1}^{J} rm(j) X(j) \\ 0 & \text{if } R(k) \geq \sum_{j=1}^{J} rm(j) X(j) \end{cases} \quad \forall k = 1, 2, \ldots, K$$

Raw material

Let the available raw material be 120 units

X	**20**	**10**	**20**	**16**	**24**	**20**

Values of all variables are within their domains - Feasible solution with respect to the domain

- Amount of raw material required for each process is

40	**13**	**16**	**24**	**60**	**20**

- Total raw material required = **173**

Total raw material required > Available raw material Infeasible solution with respect to the raw material constraint

$$P^{I} = (120-173)^2 = 2809$$

Product	Process	*l*	*m*	*h*	Raw material required (rm)
T1	P1	5	10	20	2
	P2	8	13	22	1.3
T2	P3	4	9	20	0.8
	P4	2	7	20	1.5
	P5	10	15	25	2.5
T3	P6	3	8	20	1

$$P^{R}(k) = \begin{cases} \left(R(k) - \sum_{j=1}^{J} rm(j)X(j)\right)^2 & if \;\; R(k) < \sum_{j=1}^{J} rm(j)X(j) \\ 0 & if \;\; R(k) \geq \sum_{j=1}^{J} rm(j)X(j) \end{cases}$$

$$\forall k = 1, 2, \dots, K$$

Raw material

Let the available raw material be 120 units

X	4	5	2	1	5	2

Values of all variables are in the invalid region
Infeasible solution with respect to the domain

➢ Amount of raw material required cannot be calculated.

Product	Process	*l*	*m*	*h*	Raw material required (rm)
T1	P1	5	10	20	2
	P2	8	13	22	1.3
T2	P3	4	9	20	0.8
	P4	2	7	20	1.5
	P5	10	15	25	2.5
T3	P6	3	8	20	1

$$P^R(k)=\begin{cases}\left(R(k)-\sum_{j=1}^{J} rm(j)X(j)\right)^2 & if\ \ R(k)<\sum_{j=1}^{J} rm(j)X(j)\\ 0 & if\ \ R(k)\geq\sum_{j=1}^{J} rm(j)X(j)\end{cases}$$
$$\forall k=1,2,\dots,K$$

Raw material

Let the available raw material be 120 units

X	**9**	**7**	**10**	**18**	**6**	**18**

Values of some variables are in the invalid region -
Infeasible solution with respect to the domain

➢Raw material required for the domain violating variables is not calculated.

18	**-**	**8**	**27**	**-**	**18**

➢Total raw material required = 18 + 8 + 27 + 18 = **71**

Total raw material required < Available raw material
Feasible solution with respect to the raw material constraint

$$\boxed{P^R = 0}$$

Product	Process	*l*	*m*	*h*	Raw material required (rm)
T1	P1	5	10	20	2
	P2	8	13	22	1.3
T2	P3	4	9	20	0.8
	P4	2	7	20	1.5
	P5	10	15	25	2.5
T3	P6	3	8	20	1

$$P^R(k) = \begin{cases} \left(R(k) - \sum_{j=1}^{J} rm(j)X(j)\right)^2 & \text{if } R(k) < \sum_{j=1}^{J} rm(j)X(j) \\ 0 & \text{if } R(k) \geq \sum_{j=1}^{J} rm(j)X(j) \end{cases}$$

$$\forall k = 1, 2, \ldots, K$$

Determination of Profit

- Profit calculation

$$Profit = \sum_{j=1}^{J} \left(SP(j)X(j) - PC(j) \right)$$

$SP(j)$: Selling price for product produced using j^{th} process

$PC(j)$: Production cost for product produced using j^{th} process

$X(j)$: Quantity of product produced from j^{th} process

X

6	4	10	5	20	0

X	Production cost	Revenue	Profit
6	12	60	48
4	-	-	-
10	19	300	281
5	16	150	134
20	27	600	573
0	0	0	0

Total Profit = 1036

Products	Processes	Production level			Production cost			Investment cost			Raw material required		Selling price
		l	*m*	*h*	*cl*	*cm*	*ch*	*il*	*im*	*ih*	*rm1*	*rm2*	
T1	P1	5	10	20	10	20	30	50	60	70	0.6	0.8	10
	P2	8	13	22	12	22	31	52	62	71	0.5	1.2	10
T2	P3	4	9	20	8	18	29	55	65	76	0.4	0.6	30
	P4	2	7	20	10	20	33	58	68	81	0.7	0.9	30
	P5	10	15	25	12	22	32	60	70	80	0.9	1.1	30
T3	P6	3	8	20	15	25	37	54	64	76	0.8	1.3	50

Determination of fitness function value

$$P = \lambda\left(\left(\sum_{j=1}^{J} P^{domain}(j)\right) + \left(\sum_{k=1}^{K} P^{R}(k)\right) + \left(P^{I}\right)\right)$$

$f = Profit - \lambda(P)$ — **Maximization**

$f = -Profit + \lambda(P)$ — **Minimization**

$SP(j)$: Selling price for product produced using j^{th} process

$PC(j)$: Production cost for product produced using j^{th} process

$X(j)$: Quantity of product produced from j^{th} process

Products	Processes	Production level			Production cost			Investment cost			Raw material required		Selling price
		l	m	h	cl	cm	ch	il	im	ih	rm1	rm2	
T1	P1	5	10	20	10	20	30	50	60	70	0.6	0.8	10
	P2	8	13	22	12	22	31	52	62	71	0.5	1.2	10
T2	P3	4	9	20	8	18	29	55	65	76	0.4	0.6	30
	P4	2	7	20	10	20	33	58	68	81	0.7	0.9	30
	P5	10	15	25	12	22	32	60	70	80	0.9	1.1	30
T3	P6	3	8	20	15	25	37	54	64	76	0.8	1.3	50

X	Domain constraint	Penalty Budget violation	Penalty Raw Material violation		Profit
20	0	5476	506.25	0	170
21	0				180
2	10^5				0
19	0				538
23	0				660
20	0				963

P = 10^{15} (10^5 + 5476 + 506.25 + 0)

f = −2511 + 10^{15} (10^5 + 5476 + 506.25 + 0)

$\lambda = 10^{15}$

$f = 1.06 \times 10^{20}$

X	20	21	2	19	23	20	Total	Available
IC	70	70	0	80	78	76	374	**300**
rm1	12	10.5	0	13.3	20.7	16	72.5	**50**
rm2	16	25.2	0	17.1	25.3	26	109.6	**120**

Products	Processes	Production level			Production cost			Investment cost			Raw material required		Selling price
		l	m	h	cl	cm	ch	il	im	ih	rm1	rm2	
T1	P1	5	10	20	10	20	30	50	60	70	0.6	0.8	10
	P2	8	13	22	12	22	31	52	62	71	0.5	1.2	10
T2	P3	4	9	20	8	18	29	55	65	76	0.4	0.6	30
	P4	2	7	20	10	20	33	58	68	81	0.7	0.9	30
	P5	10	15	25	12	22	32	60	70	80	0.9	1.1	30
T3	P6	3	8	20	15	25	37	54	64	76	0.8	1.3	50

Available budget = 300

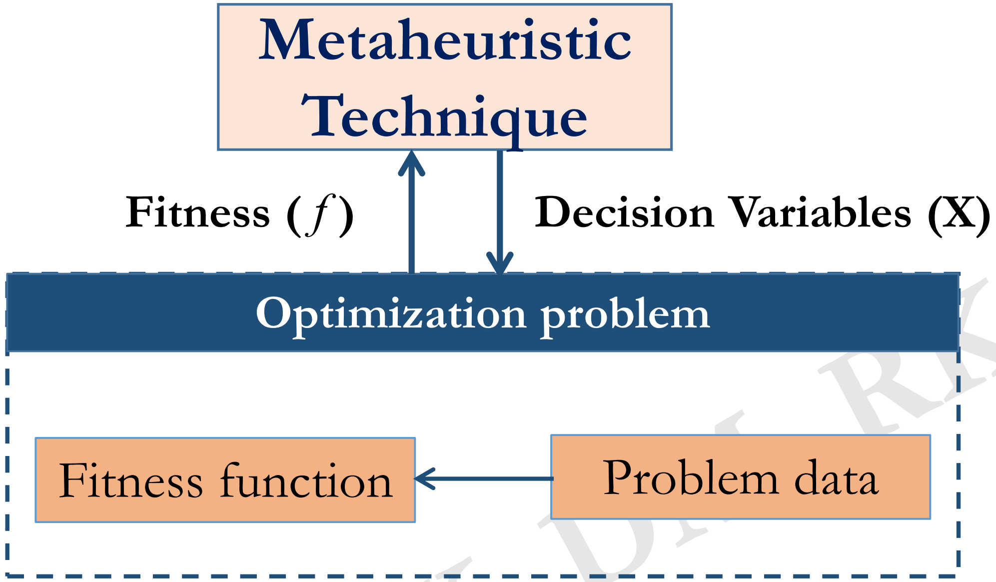
Available raw material 1 = 50

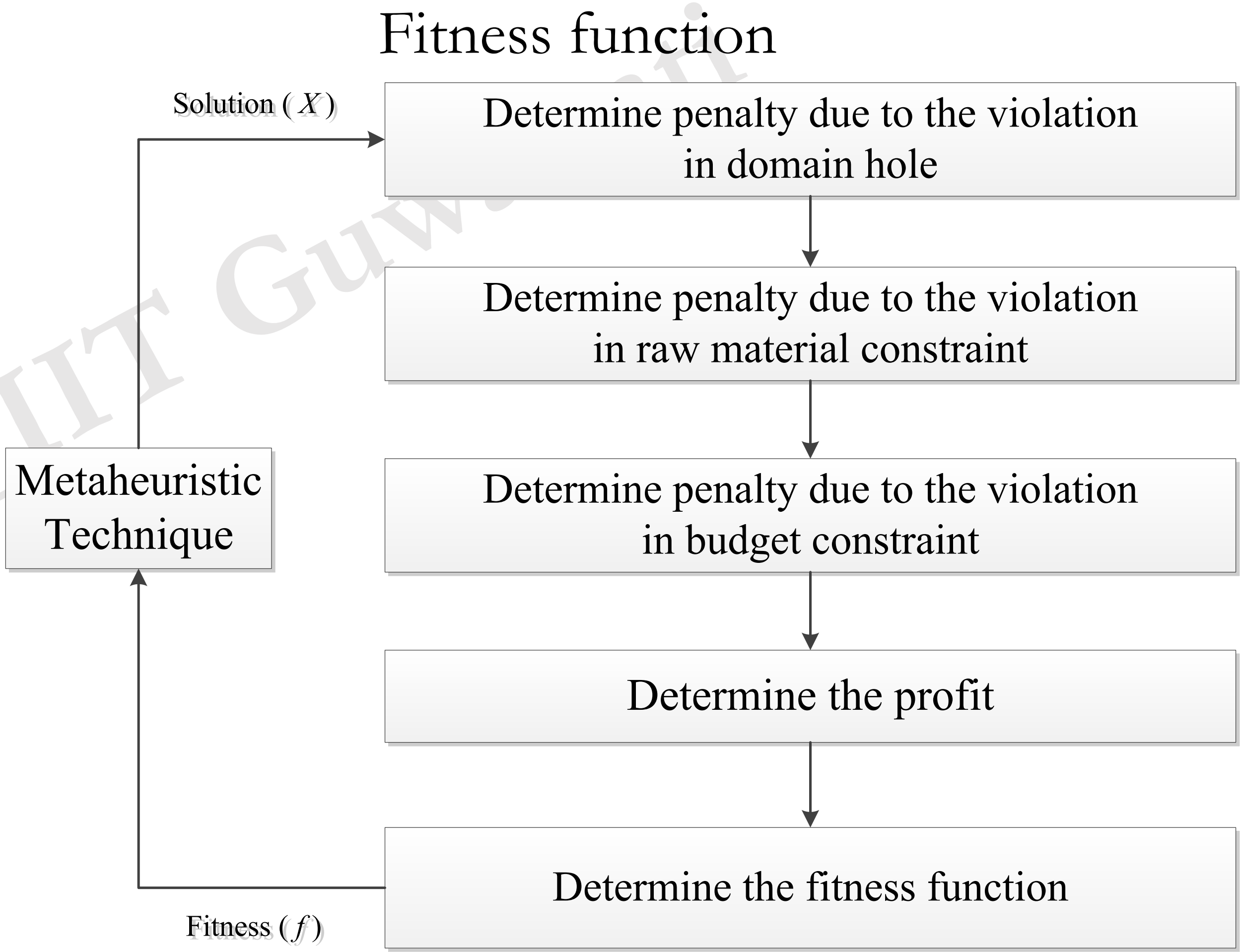
Available raw material 2 = 50

$\lambda = 10^{15}$

	X = [6 10 5 20 0 0]	X = [18 15 8 10 5 20]
Penalty Domain Hole	0	1×10^5
Penalty Investment Cost	0	1764
Penalty Raw Material 1	0	0
Penalty Raw Material 2	0	492.84
Total Penalty	0	1.02×10^6
Total Production Cost	71	128
Total Revenue	910	1870
Max. Profit (Objective function)	839	1742
Min. Fitness	–839	1.02×10^{20}

Metaheuristic techniques and optimization problem





Metaheuristic techniques and optimization problem

Solution (X)

Metaheuristic Technique

Fitness (f)

Fitness function

Determine penalty due to the violation in domain hole

Determine penalty due to the violation in raw material constraint

Determine penalty due to the violation in budget constraint

Determine the profit

$$Profit = \sum_{j=1}^{J}\left(SP(j)X(j) - PC(j)\right)$$

Determine the fitness function

$$f = -Profit + \lambda(P)$$

$$P = \lambda\left(\left(\sum_{j=1}^{J} P^{domain}(j)\right) + \left(\sum_{k=1}^{K} P^{R}(k)\right) + \left(P^{I}\right)\right)$$

$$P^{domain}(j) = \begin{cases} 10^5 & \text{if } 0 < X^j < l^j \\ 0 & \text{if } l^j \le X^j \le h^j \end{cases} \quad \forall j = 1,2,\ldots,J$$

$$P^{R}(k) = \begin{cases} \left(R(k) - \sum_{j=1}^{J} rm(j)X(j)\right)^2 & \text{if } R(k) < \sum_{j=1}^{J} rm(j)X(j) \\ 0 & \text{if } R(k) \ge \sum_{j=1}^{J} rm(j)X(j) \end{cases}$$

$$\forall k = 1,2,\ldots,K$$

$$IC(j) = \begin{cases} il(j) + \left(\dfrac{im(j) - il(j)}{m(j) - l(j)}\right)\left(X(j) - l(j)\right) & \text{if } l(j) \le X(j) \le m(j) \\ im(j) + \left(\dfrac{ih(j) - im(j)}{h(j) - m(j)}\right)\left(X(j) - m(j)\right) & \text{if } m(j) \le X(j) \le h(j) \end{cases}$$

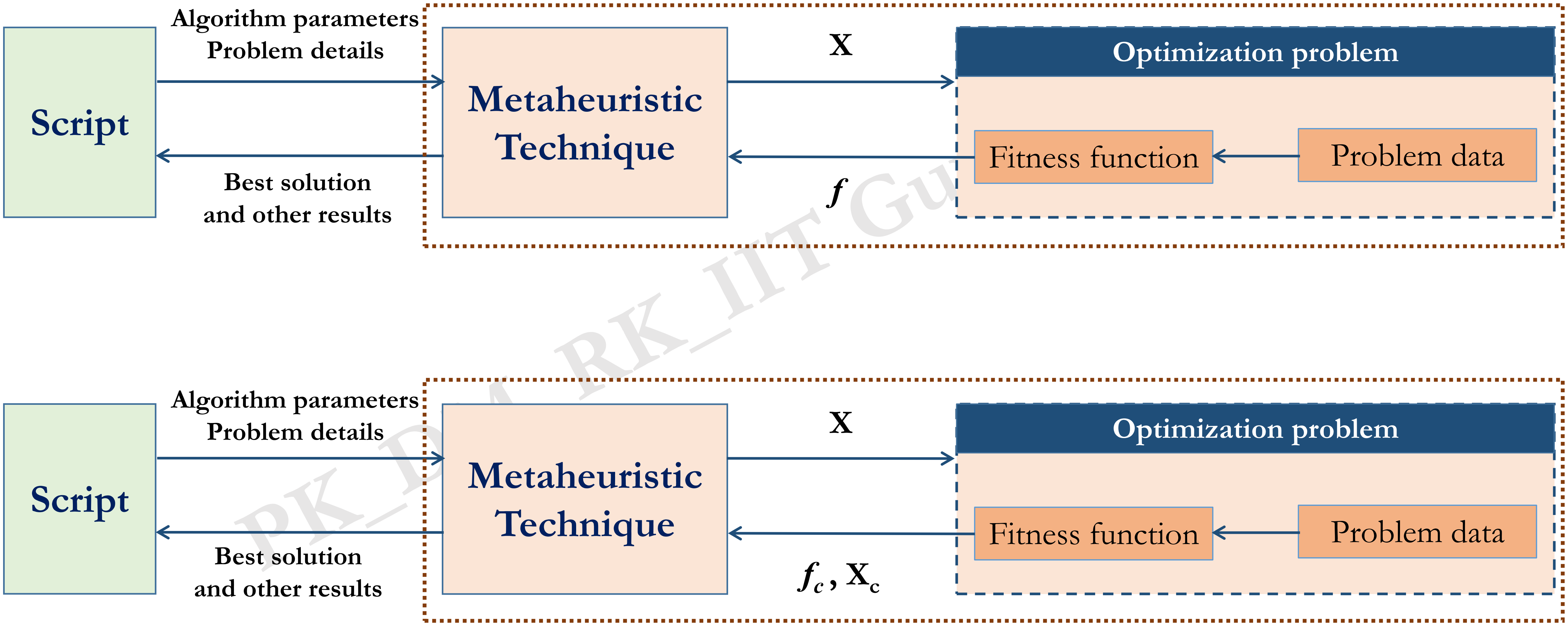
$$\forall j = 1,2,\ldots,J$$

$$P^{I} = \begin{cases} \left(B - \sum_{j=1}^{J} IC(j)\right)^2 & \text{if } \sum_{j=1}^{J} IC(j) > B \\ 0 & \text{otherwise} \end{cases}$$

$$PC(j) = \begin{cases} cl(j) + \dfrac{cm(j) - cl(j)}{m(j) - l(j)}\left(X(j) - l(j)\right) & \text{if } l(j) \le X(j) \le m(j) \\ cm(j) + \dfrac{ch(j) - cm(j)}{h(j) - m(j)}\left(X(j) - m(j)\right) & \text{if } m(j) \le X(j) \le h(j) \end{cases}$$

$$\forall j = 1,2,\ldots,J$$

Metaheuristic techniques and optimization problem



Different correction approaches

Processes		P1	P2	P3	P4	P5
Low level capacity (l)		5	9	1	3	4
Decision variables	X	12	6	2	19	2
Approach 1 (Fix it to zero)	X_c	12	**0**	2	19	**0**
Approach 2 (Fix it to low level)	X_c	12	**9**	2	19	**4**
Approach 3 (Fix randomly)	X_c	12	**0**	2	19	**4**

r = 0.3 (P2) r = 0.8 (P5)

Approach 1

$$x_i = \begin{cases} 0 & if \quad x_i < l_i \quad and \quad x_i \neq 0 \\ x_i & else \end{cases}$$

$$\forall i = \{1, 2, ..., D\}$$

where D is the problem dimension

Approach 2

$$x_i = \begin{cases} l_i & if \quad x_i < l_i \quad and \quad x_i \neq 0 \\ x_i & else \end{cases}$$

$$\forall i = \{1, 2, ..., D\}$$

where D is the problem dimension

Approach 3

$$x_i = \begin{cases} 0 & if \quad x_i < l_i \quad and \quad x_i \neq 0 \quad and \quad r \leq 0.5 \\ l_i & if \quad x_i < l_i \quad and \quad x_i \neq 0 \quad and \quad r > 0.5 \\ x_i & else \end{cases}$$

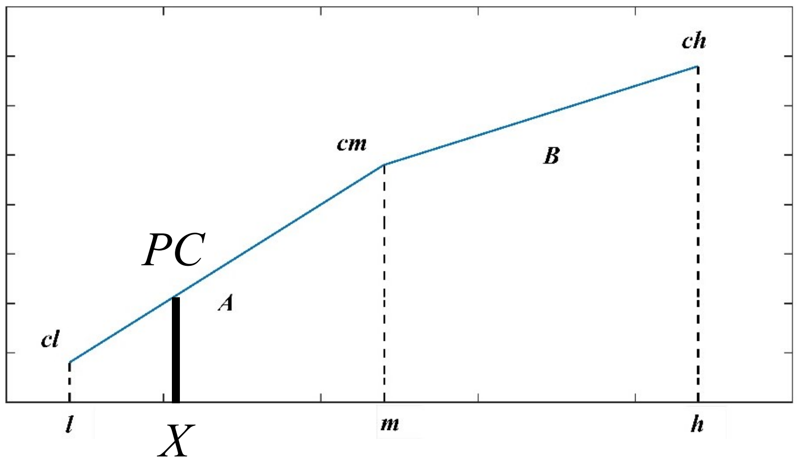
$$\forall i = \{1, 2, ..., D\}$$

where D is the problem dimension

R. Kommadath, Technical report, IIT Guwahati, 2019

MILP Formulation of Production Planning Problem

Mathematical formulation



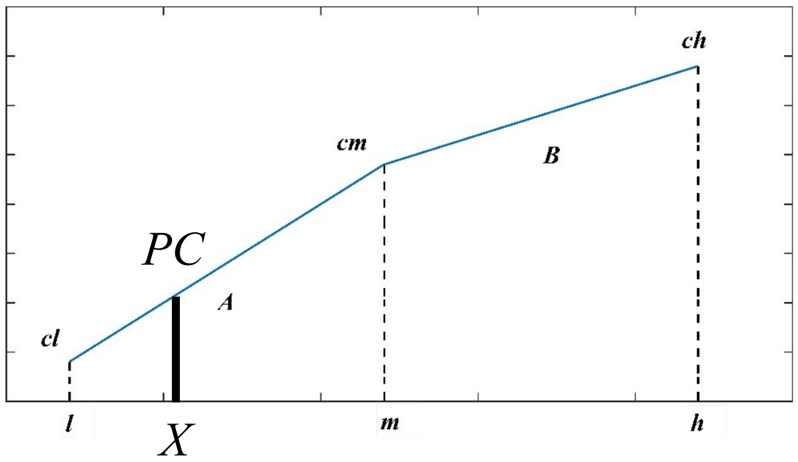
$$if\ \ X \geq l\ and\ \ X \leq m \Rightarrow PC = cl + \frac{cm - cl}{m - l}(X - l)$$

$$if\ \ X \geq m\ and\ \ X \leq h \Rightarrow PC = cm + \frac{ch - cm}{h - m}(X - m)$$

$$(X = 0) \vee (l \leq X \leq h)$$

$$X = 0 \Rightarrow PC = 0, IC = 0$$

Mathematical formulation



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$$Y = \begin{cases} 1, & X \le m \\ 0, & X \ge m \end{cases} \qquad Z = \begin{cases} 1, & X > 0 \\ 0, & X = 0 \end{cases}$$

$$L \le Y \qquad (1)$$

$$H \le 1 - Y \qquad (2)$$

$$L + M + H = Z \qquad (3)$$

$$X = l \cdot L + m \cdot M + h \cdot H \qquad (4)$$

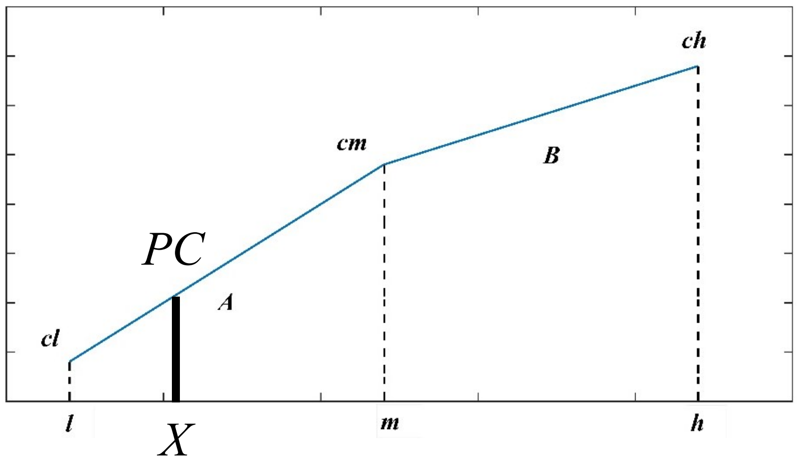
$$PC = cl \cdot L + cm \cdot M + ch \cdot H \qquad (5)$$

$$X_j \le U \cdot Z_j \qquad (6)$$

$L, M, H \in \mathbb{R}$

U is a large number

Mathematical formulation



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$$\boxed{if\ \ X \ge l\ and\ X \le m \Rightarrow Y = 1, Z = 1}$$

$$L + M = 1 \ \Rightarrow \ M = 1 - L$$

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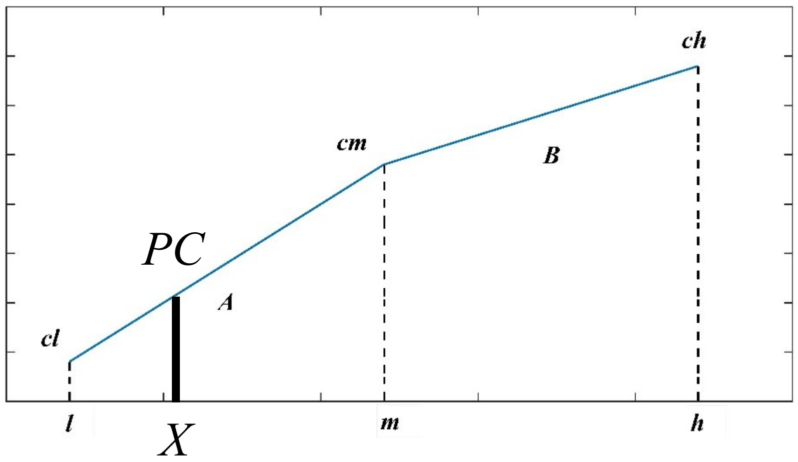

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$$PC = \frac{cl \cdot X - cm \cdot X + cm \cdot l - cl \cdot l + cl \cdot l - cl \cdot m}{l - m}$$

$$PC = \frac{X \cdot (cl - cm) + l \cdot (cm - cl)}{l - m} + \frac{cl \cdot (l - m)}{l - m}$$

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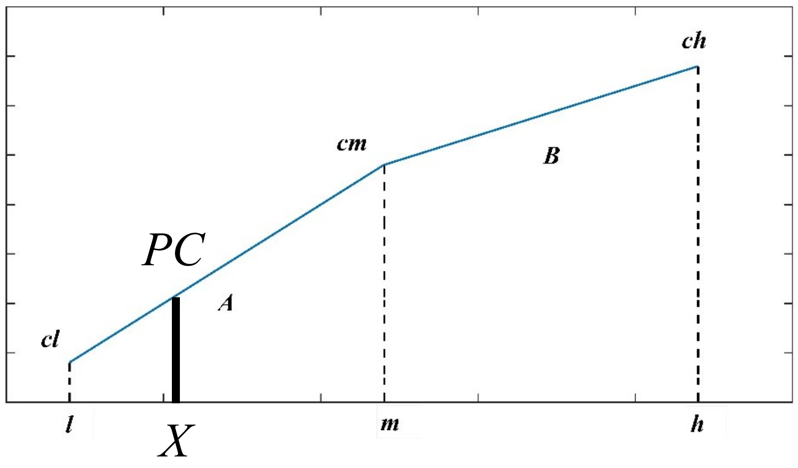
Mathematical formulation



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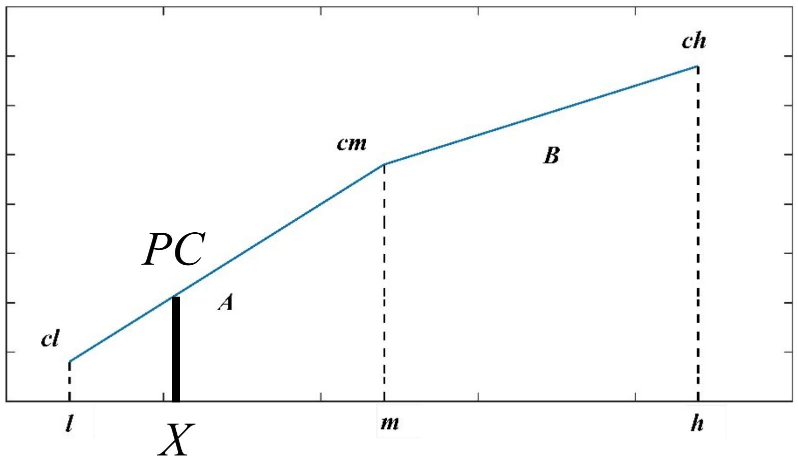
Mathematical formulation



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$$(X = 0) \vee (l \le X \le h)$$

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$$PC = cm \cdot M + ch \cdot H$$

$$PC = \left(\frac{cm - ch}{m - h}\right) \cdot (X - m) + cm$$

Mathematical formulation

Y, Z : Binary variables
$0 \le L, M, H \le 1$

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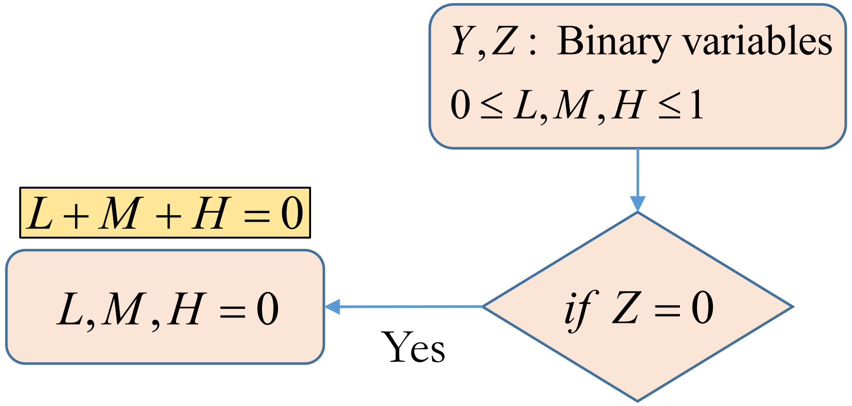
Mathematical formulation



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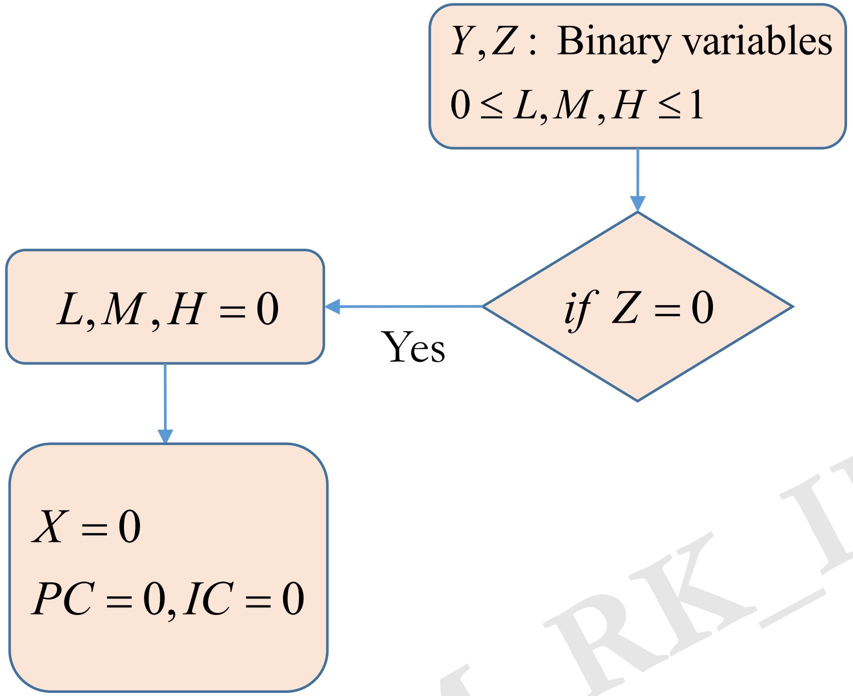

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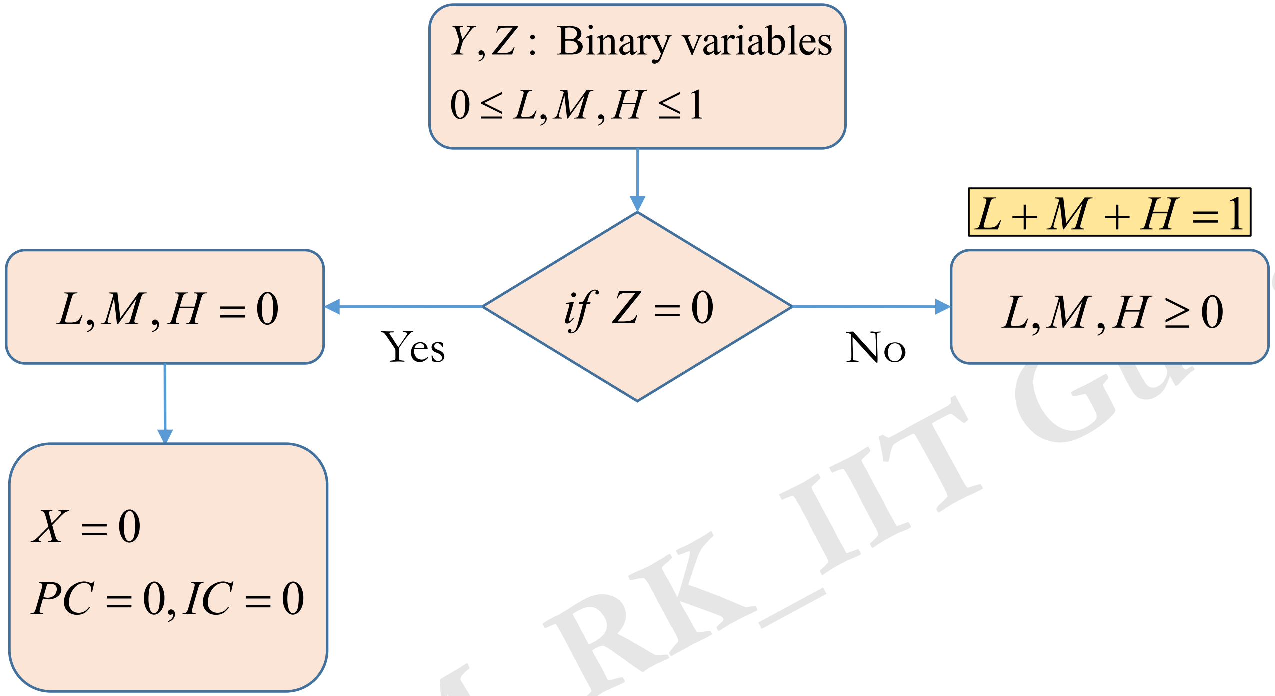

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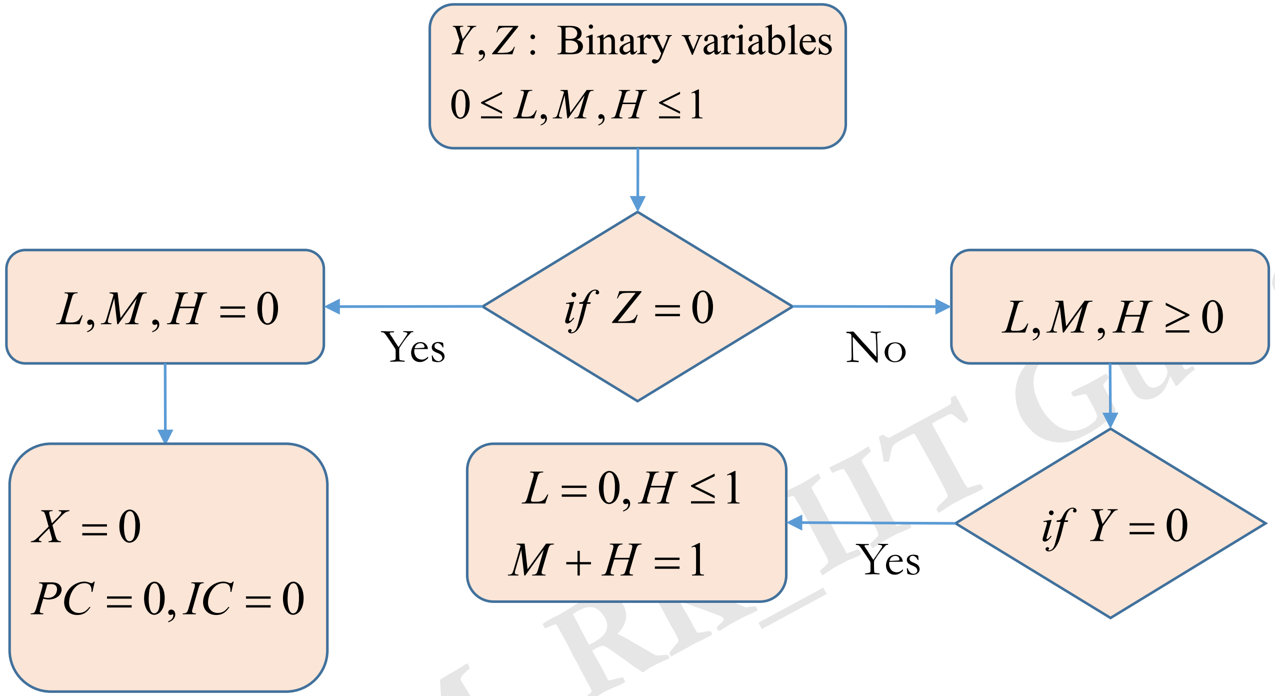
Mathematical formulation



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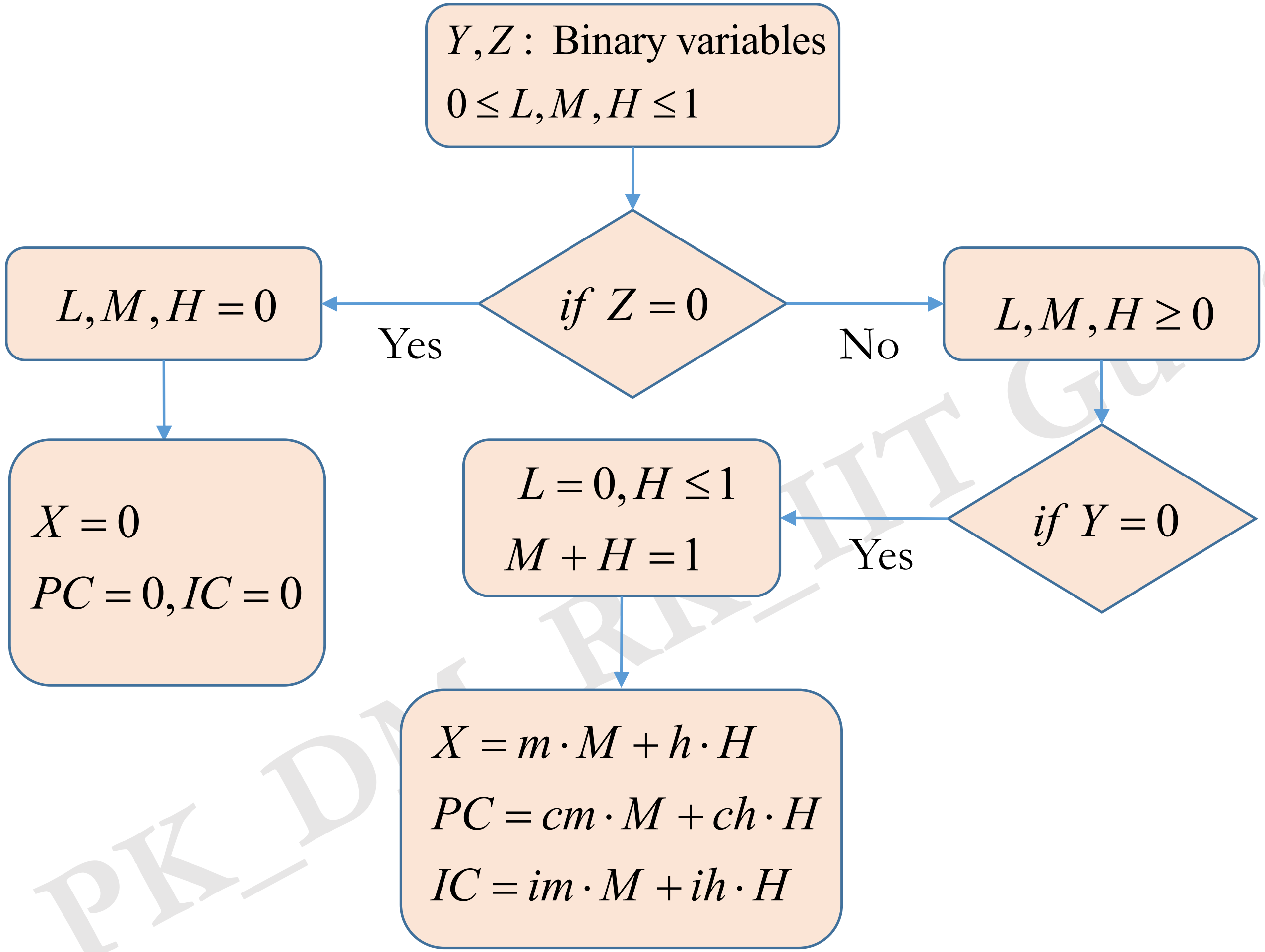

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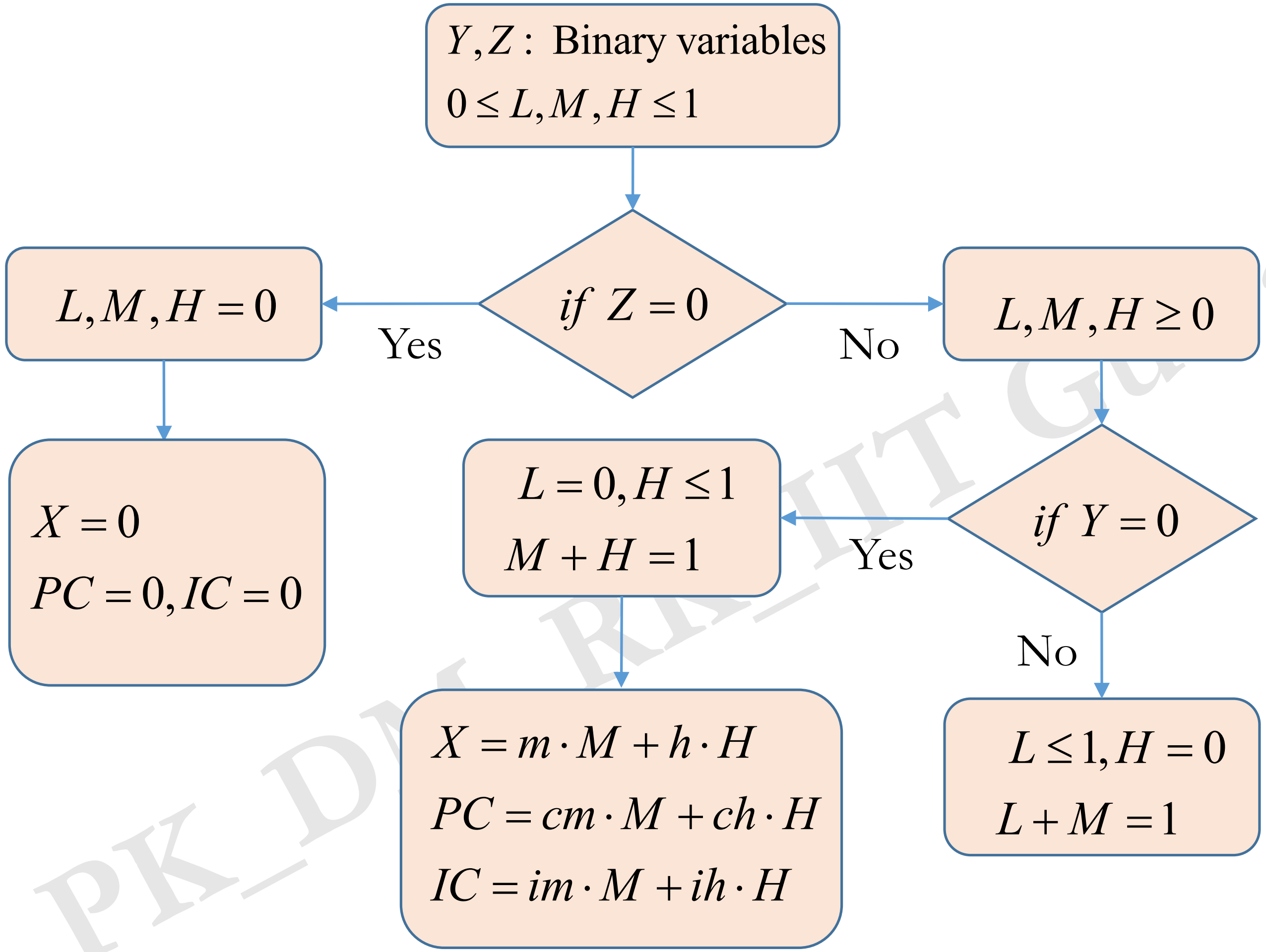

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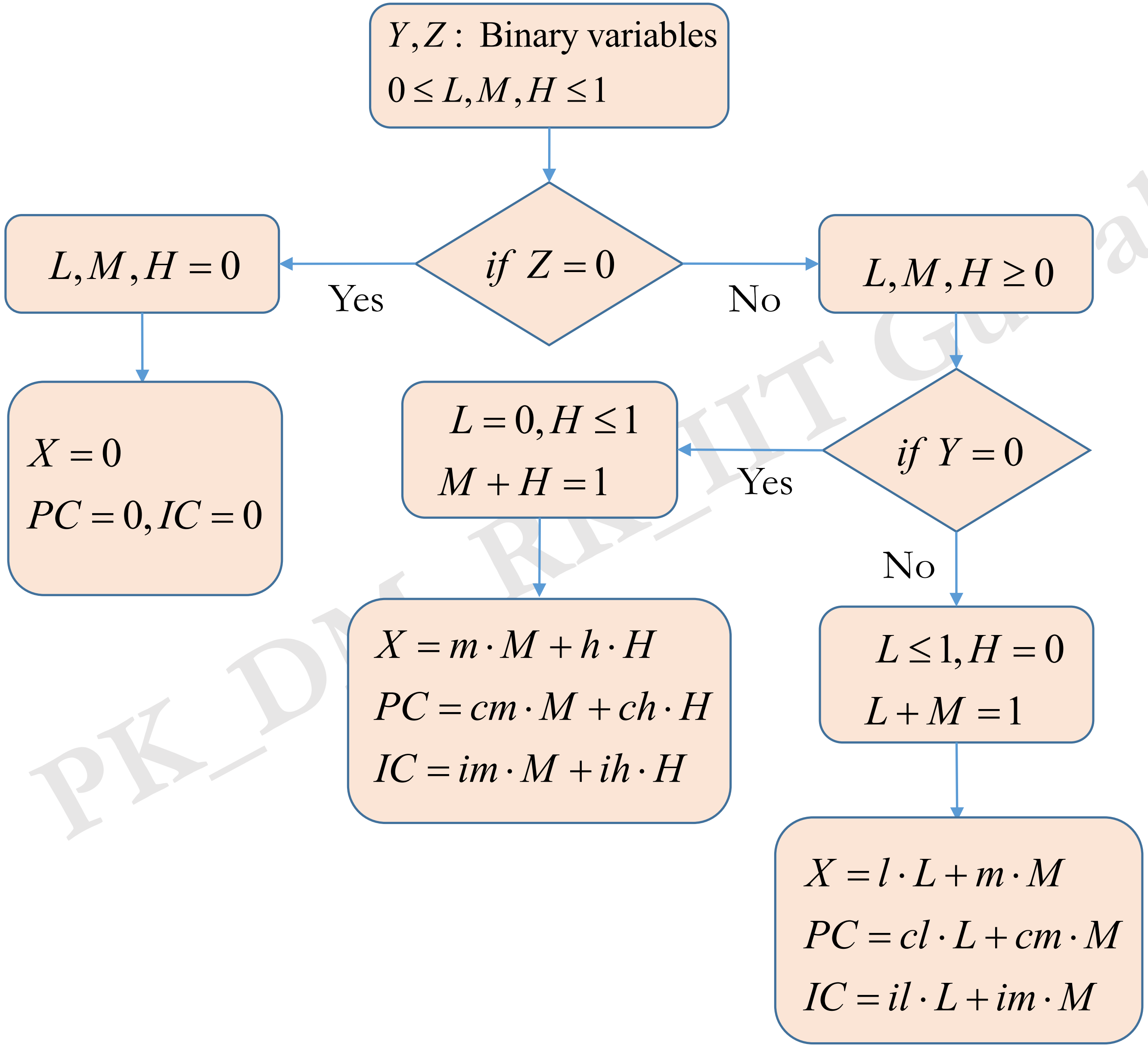

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Mathematical formulation

Y, Z : Binary variables
$0 \le L, M, H \le 1$

if $Z = 0$

Yes: $L, M, H = 0$ → $X = 0$, $PC = 0, IC = 0$

No: $L, M, H \ge 0$ → if $Y = 0$

Yes: $L = 0, H \le 1$, $M + H = 1$ → $X = m \cdot M + h \cdot H$, $PC = cm \cdot M + ch \cdot H$, $IC = im \cdot M + ih \cdot H$

No: $L \le 1, H = 0$, $L + M = 1$ → $X = l \cdot L + m \cdot M$, $PC = cl \cdot L + cm \cdot M$, $IC = il \cdot L + im \cdot M$

$$L \le Y \quad (1)$$
$$H \le 1 - Y \quad (2)$$
$$L + M + H = Z \quad (3)$$
$$X = l \cdot L + m \cdot M + h \cdot H \quad (4)$$
$$X \le U \cdot Z \quad (5)$$
$$PC = cl \cdot L + cm \cdot M + ch \cdot H \quad (6)$$
$$IC = il \cdot L + im \cdot M + ih \cdot H \quad (7)$$

$\boldsymbol{l \le X \le m}$

$L + M + 0 = 1 \Rightarrow M = 1 - L$

$X = l \cdot L + m \cdot M + 0 \Rightarrow X = l \cdot L + m \cdot (1 - L)$

$L = \left(\frac{X - m}{l - m}\right)$

$\boxed{PC = cl \cdot L + cm \cdot M}$ $\boxed{IC = il \cdot L + im \cdot M}$

$PC = cl \cdot \left(\frac{X - m}{l - m}\right) + cm \cdot \left(1 - \left(\frac{X - m}{l - m}\right)\right)$

$PC = cl \cdot \left(\frac{X - m}{l - m}\right) + cm \cdot \left(\frac{l - X}{l - m}\right)$

$PC = \frac{cl \cdot X - cm \cdot X + cm \cdot l - cl \cdot m}{l - m}$

Add and substract $cl \cdot l$ in numerator

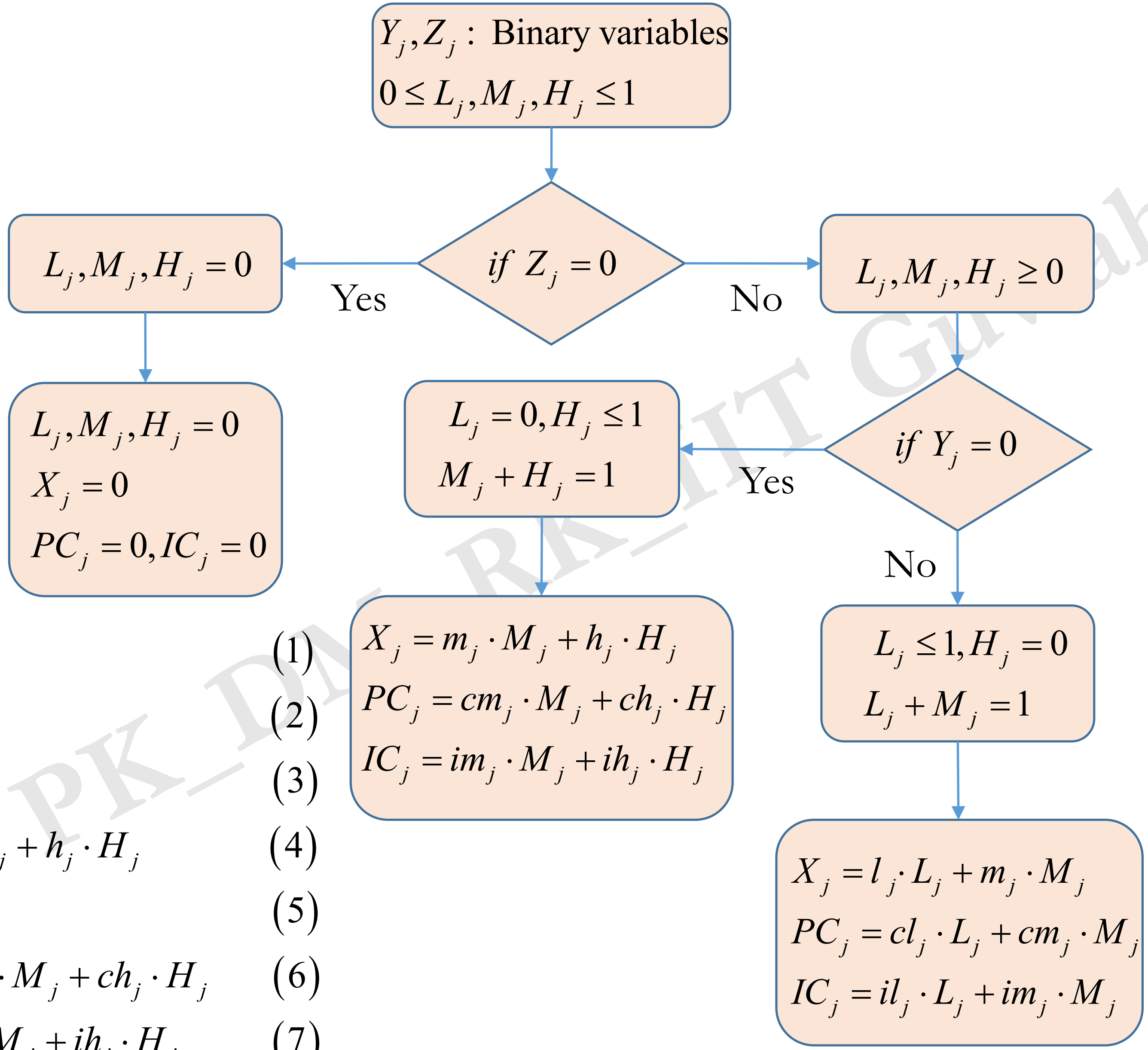
$PC = \frac{cl \cdot X - cm \cdot X + cm \cdot l - cl \cdot l + cl \cdot l - cl \cdot m}{l - m}$

$PC = \frac{X \cdot (cl - cm) + l \cdot (cm - cl)}{l - m} + \frac{cl \cdot (l - m)}{l - m}$

$PC = \left(\frac{cm - cl}{m - l}\right) \cdot (X - l) + cl$

$\boldsymbol{m \le X \le h}$ $\boxed{PC = cm \cdot M + ch \cdot H}$ $\boxed{IC = im \cdot M + ih \cdot H}$

Mathematical formulation



$$L_j \le Y_j \quad (1)$$
$$H_j \le 1 - Y_j \quad (2)$$
$$L_j + M_j + H_j = Z_j \quad (3)$$
$$X_j = l_j \cdot L_j + m_j \cdot M_j + h_j \cdot H_j \quad (4)$$
$$X_j \le U \cdot Z_j \quad (5)$$
$$PC_j = cl_j \cdot L_j + cm_j \cdot M_j + ch_j \cdot H_j \quad (6)$$
$$IC_j = il_j \cdot L_j + im_j \cdot M_j + ih_j \cdot H_j \quad (7)$$

$\boldsymbol{l_j \le X_j \le m_j}$

$$L_j + M_j + 0 = 1 \Rightarrow M_j = 1 - L_j$$
$$X_j = l_j \cdot L_j + m_j \cdot M_j + 0 \Rightarrow X_j = l_j \cdot L_j + m_j \cdot (1 - L_j)$$
$$L_j = \left(\frac{X_j - m_j}{l_j - m_j}\right)$$
$$\boxed{PC_j = cl_j \cdot L_j + cm_j \cdot M_j} \qquad \boxed{IC_j = il_j \cdot L_j + im_j \cdot M_j}$$
$$PC_j = cl_j \cdot \left(\frac{X_j - m_j}{l_j - m_j}\right) + cm_j \cdot \left(1 - \left(\frac{X_j - m_j}{l_j - m_j}\right)\right)$$
$$PC_j = cl_j \cdot \left(\frac{X_j - m_j}{l_j - m_j}\right) + cm_j \cdot \left(\frac{l_j - X_j}{l_j - m_j}\right)$$
$$PC_j = \frac{cl_j \cdot X_j - cm_j \cdot X_j + cm_j \cdot l_j - cl_j \cdot m_j}{l_j - m_j}$$

Add and substract $cl_j \cdot l_j$ in numerator

$$PC_j = \frac{cl_j \cdot X_j - cm_j \cdot X_j + cm_j \cdot l_j - cl_j \cdot l_j + cl_j \cdot l_j - cl_j \cdot m_j}{l_j - m_j}$$
$$PC_j = \frac{X_j \cdot (cl_j - cm_j) + l_j \cdot (cm_j - cl_j)}{l_j - m_j} + \frac{cl_j \cdot (l_j - m_j)}{l_j - m_j}$$
$$PC_j = \left(\frac{cm_j - cl_j}{m_j - l_j}\right) \cdot (X_j - l_j) + cl_j$$

$\boldsymbol{m_j \le X_j \le h_j}$ $\quad \boxed{PC_j = cm_j \cdot M_j + ch_j \cdot H} \quad \boxed{IC_j = im_j \cdot M_j + ih_j \cdot H_j}$

$$PC_1 = cl_1 \cdot L_1 + cm_1 \cdot M_1 + ch_1 \cdot H_1, \quad j = 1$$
$$PC_j = cl_j \cdot L_j + cm_j \cdot M_j + ch_j \cdot H_j, \quad \forall j = 1,2..J$$

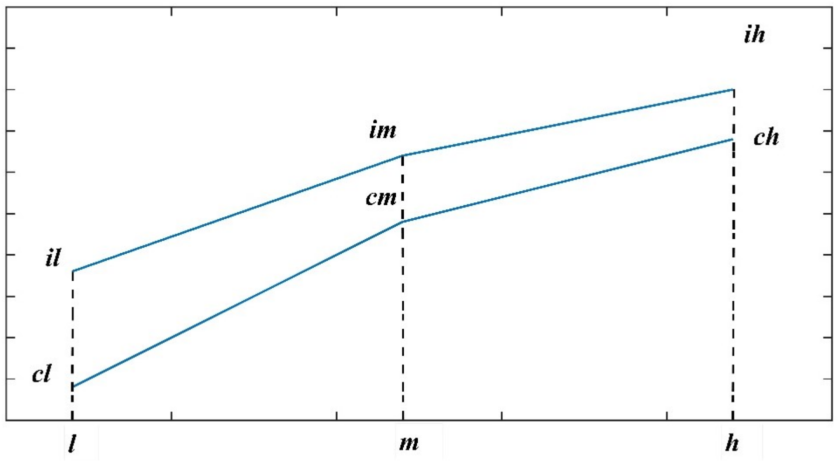
$$IC_1 = il_1 \cdot L_1 + im_1 \cdot M_1 + ih_1 \cdot H_1, \quad j = 1$$
$$IC_j = il_j \cdot L_j + im_j \cdot M_j + ih_j \cdot H_j, \quad \forall j = 1,2,...$$

Mathematical formulation

$$\text{Max profit } = \sum_{j=1}^{J}\left(SP_j \cdot X_j - PC_j\right)$$

$$PC_j = cl_j \cdot L_j + cm_j \cdot M_j + ch_j \cdot H_j \quad \forall j = 1,2,\ldots,J$$

Determining production cost



cl_j, cm_j and ch_j	Production cost for using process j at l_j, m_j and h_j capacity level.
SP_j	Selling price of product produced from process j
L_j, M_j and H_j	Portions of product produced using production levels l_j, m_j and h_j

Mathematical formulation

$$\text{Max profit } = \sum_{j=1}^{J}\left(SP_j \cdot X_j - PC_j\right)$$

$$PC_j = cl_j \cdot L_j + cm_j \cdot M_j + ch_j \cdot H_j \quad \forall j = 1,2,\ldots,J$$

Determining production cost

$$\sum_{j=1}^{J} IC_j \le B$$

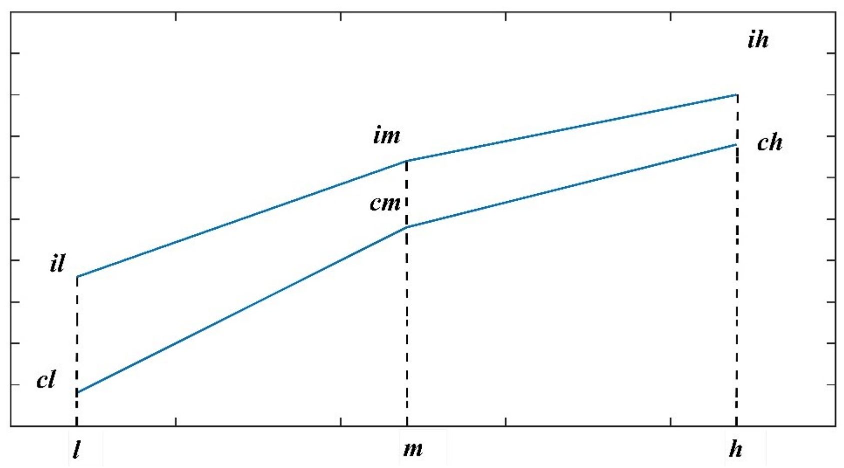
Constraints on investment cost

$$IC_j = il_j \cdot L_j + im_j \cdot M_j + ih_j \cdot H_j \quad \forall j = 1,2,\ldots,J$$

Determining investment cost

$$\sum_{j=1}^{J} rm_{jk} \cdot X_j \le R_k \qquad k = 1,\ldots,K$$

Constraints on raw material



cl_j, cm_j and ch_j	Production cost for using process j at l_j, m_j and h_j capacity level.
SP_j	Selling price of product produced from process j
il_j, im_j and ih_j	Investment cost for process j at l_j, m_j and h_j capacity level.
B	Total available budget
rm_{jk}	Amount of k type raw material consumed for the production of X_j
R_k	The total amount of k type raw material available in feedstock.
L_j, M_j and H_j	Portions of product produced using production levels l_j, m_j and h_j

Mathematical formulation

$$\text{Max profit } = \sum_{j=1}^{J}\left(SP_j \cdot X_j - PC_j\right)$$

$$PC_j = cl_j \cdot L_j + cm_j \cdot M_j + ch_j \cdot H_j \quad \forall j = 1,2,\ldots,J$$

Determining production cost

$$\sum_{j=1}^{J} IC_j \le B$$

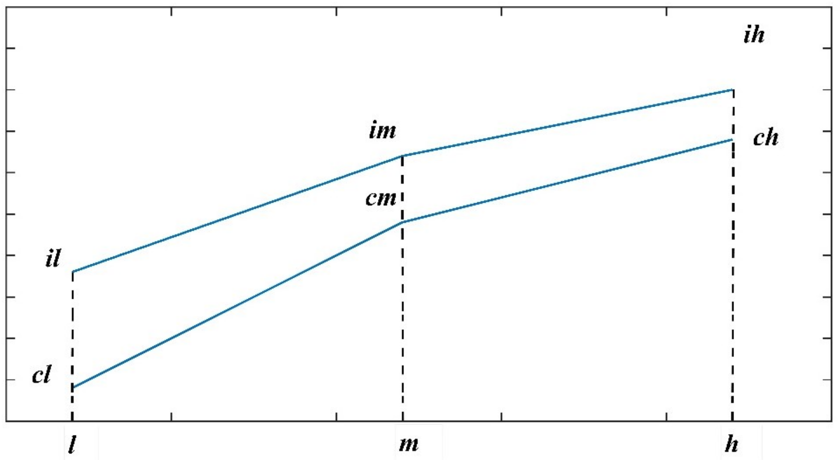
Constraints on investment cost

$$IC_j = il_j \cdot L_j + im_j \cdot M_j + ih_j \cdot H_j \quad \forall j = 1,2,\ldots,J$$

Determining investment cost

$$\sum_{j=1}^{J} rm_{jk} \cdot X_j \le R_k \qquad k = 1,\ldots,K$$

Constraints on raw material



$$L_j \le Y_j \qquad \forall j = 1,2,\ldots,J$$
$$H_j \le 1 - Y_j \qquad \forall j = 1,2,\ldots,J$$
$$L_j + M_j + H_j = Z_j \qquad \forall j = 1,2,\ldots,J$$
$$X_j = l_j \cdot L_j + m_j \cdot M_j + h_j \cdot H_j \qquad \forall j = 1,2,\ldots,J$$
$$X_j \le U \cdot Z_j \qquad \forall j = 1,2,\ldots,J$$
$$Y_j, Z_j = 0 \text{ or } 1 \qquad \forall j = 1,2,\ldots,J$$
$$X_j, L_j, M_j, H_j \ge 0 \qquad \forall j = 1,2,\ldots,J$$

cl_j, cm_j and ch_j	Production cost for using process j at l_j, m_j and h_j capacity level.
SP_j	Selling price of product produced from process j
il_j, im_j and ih_j	Investment cost for process j at l_j, m_j and h_j capacity level.
B	Total available budget
rm_{jk}	Amount of k type raw material consumed for the production of X_j
R_k	The total amount of k type raw material available in feedstock.
L_j, M_j and H_j	Portions of product produced using production levels l_j, m_j and h_j

Constraints on variables

Mathematical programming

$$\text{Max profit } = \sum_{j=1}^{J}\left(SP_j \cdot X_j - PC_j\right)$$

$$PC_j = cl_j \cdot L_j + cm_j \cdot M_j + ch_j \cdot H_j$$

$$\sum_{j=1}^{J} IC_j \le B$$

$$IC_j = il_j \cdot L_j + im_j \cdot M_j + ih_j \cdot H_j$$

$$\sum_{j=1}^{J} rm_{jk} \cdot X_j \le R_k \qquad \forall k = 1,\dots,K$$

$$L_j \le Y_j \qquad \forall j = 1,2,\dots,J$$

$$H_j \le 1 - Y_j \qquad \forall j = 1,2,\dots,J$$

$$L_j + M_j + H_j = Z_j \qquad \forall j = 1,2,\dots,J$$

$$X_j = l_j \cdot L_j + m_j \cdot M_j + h_j \cdot H_j \qquad \forall j = 1,2,\dots,J$$

$$X_j \le U \cdot Z_j \qquad \forall j = 1,2,\dots,J$$

$$Y_j, Z_j = 0 \text{ or } 1 \qquad \forall j = 1,2,\dots,J$$

$$X_j, L_j, M_j, H_j \ge 0 \qquad \forall j = 1,2,\dots,J$$

Metaheuristic techniques

$$\text{Min fitness } f = -\sum_{j=1}^{J}\left(SP_j \cdot X_j - PC_j\right) + \lambda(P), \quad 0 \le X_j \le h_j$$

$$PC_j = \begin{cases} cl_j + \dfrac{cm_j - cl_j}{m_j - l_j}\left(X_j - l_j\right) & l_j \le X_j \le m_j, \quad \forall j = 1,2,\dots,J \\ cm_j + \dfrac{ch_j - cm_j}{h_j - m_j}\left(X_j - m_j\right) & m_j \le X_j \le h_j, \quad \forall j = 1,2,\dots,J \\ 0 & X_j = 0, \quad \forall j = 1,2,\dots,J \end{cases}$$

$$P = \left(\left(\sum_{j=1}^{J} P^{domain}(j)\right) + \left(\sum_{k=1}^{K} P^{R}(k)\right) + \left(P^{I}\right)\right)$$

$$P^{domain}(j) = \begin{cases} 10^5 & \text{if } 0 < X_j < l_j \\ 0 & \text{otherwise} \end{cases} \quad \forall j = 1,2,\dots,J$$

$$P^{R}(k) = \begin{cases} \left(R(k) - \sum_{j=1}^{J} rm(j) \cdot X(j)\right)^2 & \text{if } R(k) < \sum_{j=1}^{J} rm(j) \cdot X(j) \\ 0 & \text{if } R(k) \ge \sum_{j=1}^{J} rm(j) \cdot X(j) \end{cases}$$

$$P^{I} = \begin{cases} \left(B - \sum_{j=1}^{J} IC(j)\right)^2 & \text{if } \sum_{j=1}^{J} IC(j) > B \\ 0 & \text{otherwise} \end{cases}$$

$$IC_j = \begin{cases} il_j + \dfrac{im_j - il_j}{m_j - l_j}\left(X_j - l_j\right) & l_j \le X_j \le m_j, \quad \forall j = 1,2,\dots,J \\ im_j + \dfrac{ih_j - im_j}{h_j - m_j}\left(X_j - m_j\right) & m_j \le X_j \le h_j, \quad \forall j = 1,2,\dots,J \\ 0 & X_j = 0, \quad \forall j = 1,2,\dots,J \end{cases}$$

Technique	No. of variables		No. of constraints
	Binary	Continuous	
Mathematical programming	54 x 2 = 108	54 x 4 = 216	54 x 5 + 2 + 1 = 273
Metaheuristic techniques	0	54	54 + 2 + 1=57

Thank You !!!