EE 626: Quiz 2

Duration: 45 minutes

Marks:10

Date: Apr 16' 2024

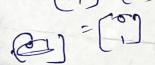
Note: No clarifications or discussions on the questions will be entertained during the examination session.

Recall, that the GMM for a d dimensional input x with M Gaussian distributions is defined as

$$\sum_{j=1}^{M} \pi_{j} N(\boldsymbol{\mu}_{j}, \boldsymbol{\Sigma}_{j})$$

where the index j referring to the j^{th} Gaussian is given by

$$N(\boldsymbol{\mu}_j, \boldsymbol{\Sigma}_j) = \frac{1}{(2\pi)^{d/2} |\boldsymbol{\Sigma}_j|^{0.5}} e^{-\frac{1}{2} (\mathbf{x} - \boldsymbol{\mu}_j)^T \boldsymbol{\Sigma}_j^{-1} (\mathbf{x} - \boldsymbol{\mu}_j)}$$



A GMM is trained using two Gaussian distributions on a two dimensional data. The mean of the first and second Gaussian is $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ -2 \end{bmatrix}$ respectively. Likewise, the covariance matrix of the first and second Gaussian is $\begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ respectively. Moreover, it is known that for a feature vector $\mathbf{x} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, its posterior probability of assignment with regards to the first Gaussian is 0.2.

- (i) Find the posterior probability of x with regards to the second Gaussian.
- (ii) Find the values of π_1 and π_2

$$[1+5.5=6.5 \text{ marks}]$$

2. Consider a one-dimensional two-category classification problem with prior, $P(\omega_1) = 0.6$. Three i.i.d training observations were collected: $D_1 = \{1, 2, 5\}$ and $D_2 = \{3, 4, 7\}$ for ω_1 and ω_2 , respectively. It is desired to classify a test pattern using the Parzen Window technique, discussed in class. Using the window function $\phi(x) = \frac{1}{\sqrt{2\pi}} \exp^{-\frac{x^2}{2}}$ with parameter of bandwidth h=1, classify the pattern x=4.5 by specifying the posterior probability under the zero-one loss paradigm.

