BT209

Bioreaction Engineering

26/04/2023_Tutorial

Non ideal flow reactor
(RTD)

Problem 1

A sample of tracer was injected as a pulse to a tubular reactor and the effluent concentration measured as a function of time, resulting the following data

t (min)	0	1	2	3	4	5	6	7	8	9	10	12	14
C (g/m^3)	0	1	5	8	10	8	6	4	3.0	2.2	1.5	0.6	0

(a) Construct figures showing C(t) and E(t) as functions of time.
(b) Determine both the fraction of material leaving the reactor that has spent between 3 and 6 min in the reactor, and
(c) Determine the fraction of material leaving the reactor that has spent 3 min or less in the reactor.
(d) Calculate mean residence time and variance of E curve

$$\int_0^{\infty} C(t)\,dt = \int_0^{10} C(t)\,dt + \int_{10}^{14} C(t)\,dt$$

$$\begin{aligned}\int_0^{10} C(t)\,dt &= \tfrac{1}{3}[1(0) + 4(1) + 2(5) + 4(8)\\ &\qquad + 2(10) + 4(8) + 2(6)\\ &\qquad + 4(4) + 2(3.0) + 4(2.2) + 1(1.5)]\\ &= 47.4\ \text{g}\cdot\text{min/m}^3\end{aligned}$$

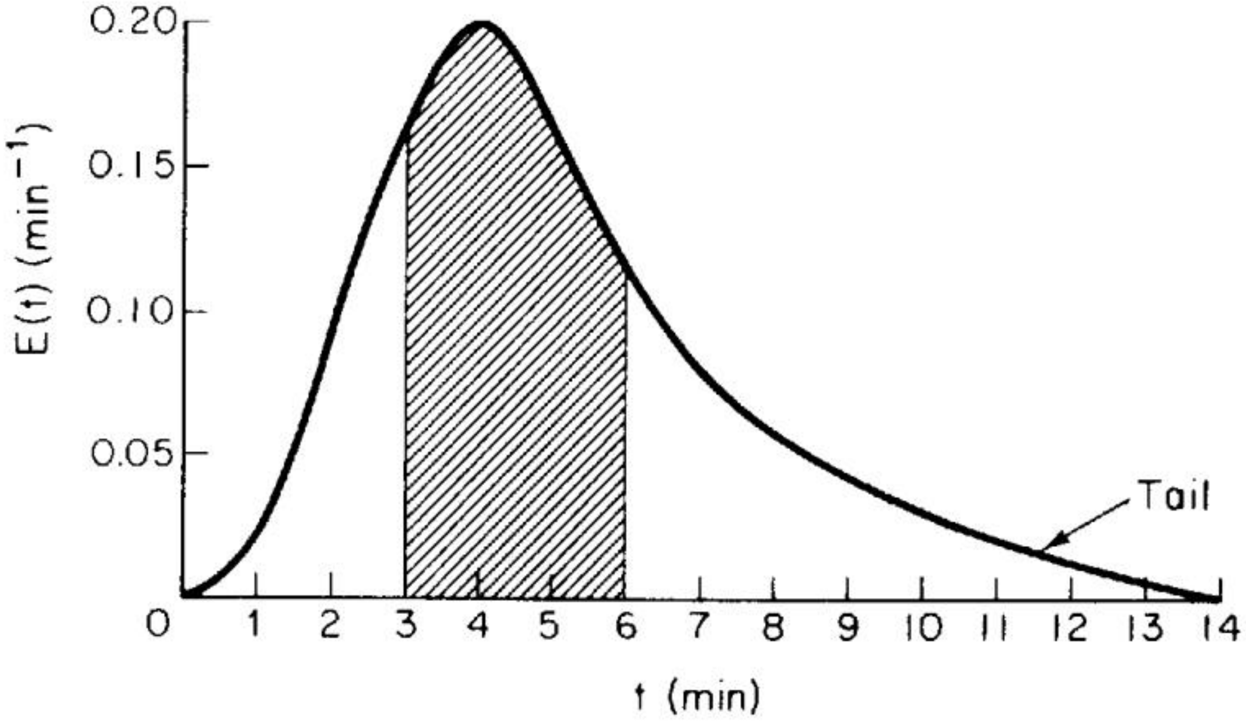
$$\int_{10}^{14} C(t)\,dt = \tfrac{2}{3}[1.5 + 4(0.6) + 0] = 2.6\ \text{g}\cdot\text{min/m}^3$$

$$\int_0^{\infty} C(t)\,dt = 50.0\ \text{g}\cdot\text{min/m}^3$$

$$E(t) = \frac{C(t)}{\int_0^\infty C(t)\,dt} = \frac{C(t)}{50\ \mathrm{g \cdot min/m^3}}$$

t (min)	1	2	3	4	5	6	7	8	9	10	12	14
$C(t)$ (g/m^3)	1	5	8	10	8	6	4	3	2.2	1.5	0.6	0
$E(t)$ (min^{-1})	0.02	0.1	0.16	0.2	0.16	0.12	0.08	0.06	0.044	0.03	0.012	0

b)

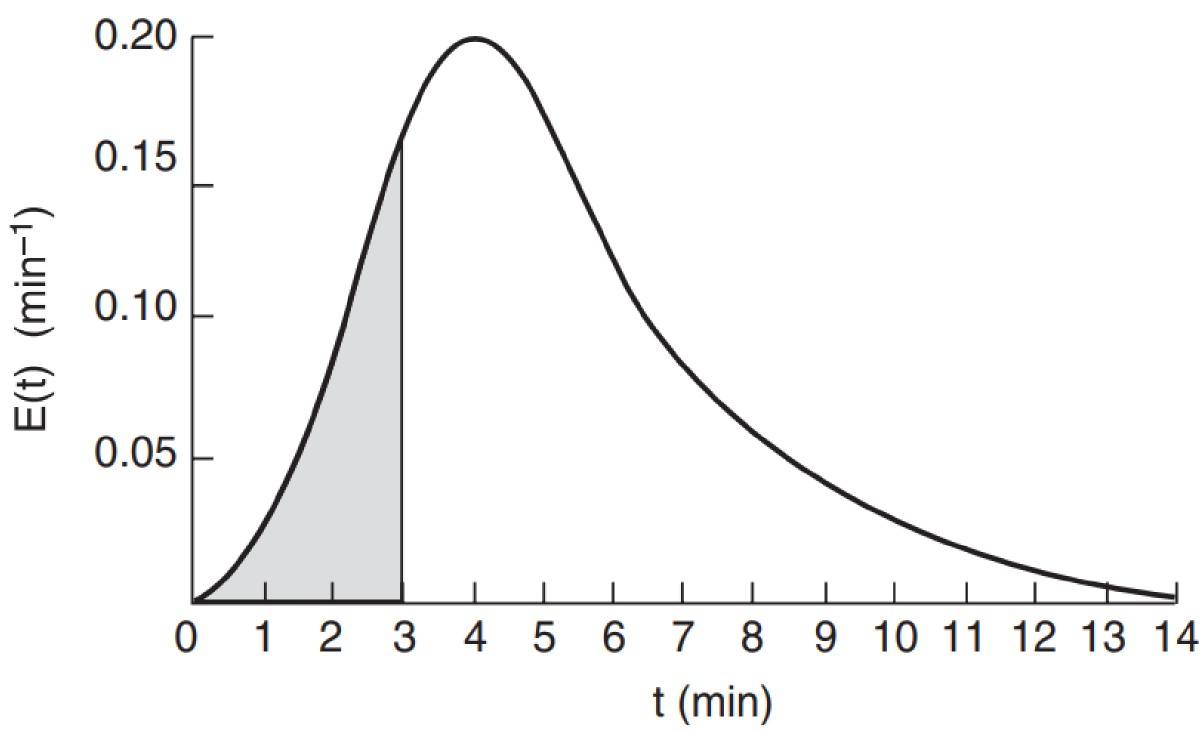


$$\int_3^6 E(t)\,dt = \text{shaded area}$$

$$= \tfrac{3}{8}\Delta t(f_1 + 3f_2 + 3f_3 + f_4)$$

$$= \tfrac{3}{8}(1)[0.16 + 3(0.2) + 3(0.16) + 0.12] = 0.51$$

Evaluating this area, we find that 51% of the material leaving the reactor spends between 3 and 6 min in the reactor.



Calculating the area under the curve, we see that 20% of the material has spent 3 min or less in the reactor

t	0	1	2	3	4	5	6	7	8	9	10	12	14
c	0	1	5	8	10	8	6	4	3	2.2	1.5	0.6	0
E	0	0.02	0.1	0.16	0.2	0.16	0.12	0.08	0.06	0.044	0.03	0.012	0
tE	0	0.02	0.2	0.48	0.8	0.8	0.72	0.56	0.48	0.4	0.3	0.14	0
t^2E	0	0.02	0.4	1.44	3.2	4	4.32	3.92	3.84	3.60	3.0	1.68	0

$$\therefore \int_0^\infty c(t)\,dt = 50\ \text{g.min}, \quad \bar{t} = \int_0^\infty tE\,dt = 5.15\ \text{min}$$

$$\int_0^\infty t^2E\,dt = \frac{1}{3}[1(0)+4(0.02)+2(0.4)+4(1.44)+2(2.2)+4(4.0)$$
$$+2(4.32)+4(3.92)+2(3.84)+4(3.6+1(3.0)]$$
$$+\frac{2}{3}[3.0+4(1.68)+0]$$
$$= 32.63\ \text{min}^2$$

$$\therefore \sigma^2 = \int_0^\infty (t-\bar{t})^2E\,dt = \int_0^\infty t^2E\,dt - \bar{t}^2$$
$$= 32.63-(5.15)^2 = 6.10\ \text{min}^2$$