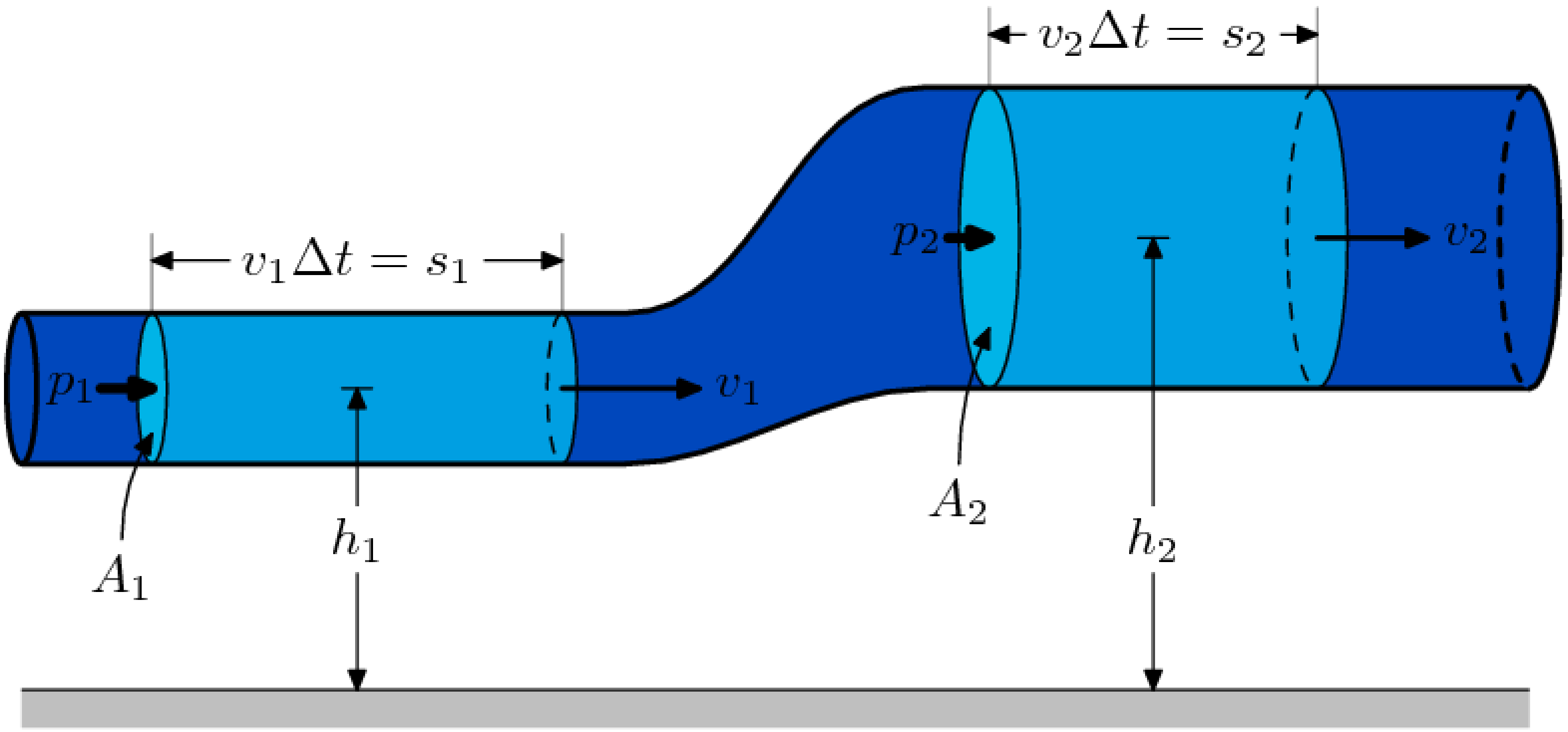
<u>**BERNOULLI'S EQUATION**</u>

- Bernoulli's equation states that the sum of all forms of energy in a fluid flowing along an enclosed path is the same at any two points in that path.
- <u>**Assumptions**</u>:
  - Flow is steady
  - Density is constant (incompressible)
  - Friction losses are negligible

BERNOULLI'S EQUATION



- By conservation of energy,

$$(\text{Energy})_1 = (\text{Energy})_2$$

$$\Delta(\text{Energy}) = 0$$

$$\{\Delta\,(\text{Press. forces}) + \Delta\,(\text{Kinetic Energy}) + \Delta\,(\text{Potential Energy})\} = 0$$

- Pressure forces = F S
- Kinetic Energy = ½ m v^2
- Potential energy = m g h
- Pressure forces @ 1 are given by,

$$F_1 S_1 = (\mathbf{p_1 A_1})(\mathbf{v_1 \Delta t})$$

$$= \frac{\boldsymbol{\rho}}{\boldsymbol{\rho}} A_1 v_1 \bullet \; p_1 \Delta t$$

$$= \frac{\dot{m}}{\rho} p_1 \Delta t$$

$$= \frac{m}{\rho \, \Delta t} p_1 \Delta t$$

$$\boxed{\therefore F_1 S_1 = \frac{m \; p_1}{\rho}}$$

- Sub. all the values in the energy balance equation:

$$F_1S_1 + \frac{1}{2}mv_1^2 + mgh_1 = F_2S_2 + \frac{1}{2}mv_2^2 + mgh_2$$

$$\Rightarrow m\frac{p_1}{\rho} + \frac{1}{2}mv_1^2 + mgh_1 = m\frac{p_2}{\rho} + \frac{1}{2}mv_2^2 + mgh_2$$

$$\boxed{\Rightarrow\Rightarrow \frac{p_1}{\rho} + \frac{1}{2}v_1^2 + gh_1 = \frac{p_2}{\rho} + \frac{1}{2}v_2^2 + gh_2}$$

Bernoulli's eqn:

$$\boxed{OR \quad \frac{p}{\rho} + \frac{v^2}{2} + gh = \text{constant}}$$

All are having units of J/kg

Applications of B.Eqn:

- B.Eqn often combined with continuity equation to find velocities & pressures in the flow streams

- Orifice meter; venturi meter
- Flow in pumps etc.,

Fluid friction……

- Fluid friction is defined as any conversion of mechanical energy into heat in a flowing stream

- Denoted by the letter h_f (J/kg)

- h_f represents all the friction generated per unit mass of fluid bet (1) & (2)

- B.Eqn becomes……..

$$\boxed{\frac{p_1}{\rho} + \frac{1}{2}v_1^2 + gh_1 = \frac{p_2}{\rho} + \frac{1}{2}v_2^2 + gh_2 + h_f}$$

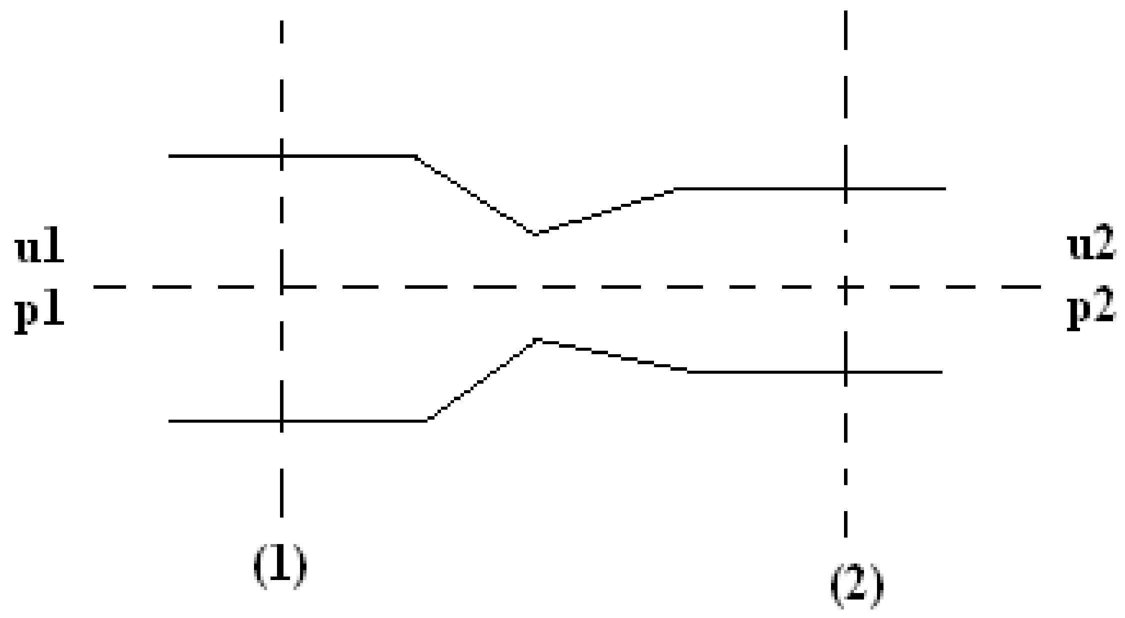
Pump work…….

- If a pump is used during flow, then the term "work done by the pump" should be added to B.Eqn

$$\boxed{\frac{p_1}{\rho} + \frac{1}{2}v_1^2 + gh_1 + \eta W_p = \frac{p_2}{\rho} + \frac{1}{2}v_2^2 + gh_2}$$

Prob 1

- A fluid of density 960 kg/m^3 is flowing steadily thro a tube as shown in the fig: The sections diameters are d_1=100mm & d_2=80mm. The press p_1 =200kN/m^2; u_1=5m/s. The tube is horizontal. What is the pressure at section(2)?



- By continuity equation: $\boxed{\dot{m} = \rho_1 v_1 A_1 = \rho_2 v_2 A_2}$

- v_2 = m/s

- From B.Eqn…. $\boxed{\dfrac{p_1}{\rho} + \dfrac{1}{2}v_1^2 + \cancel{gh_1} = \dfrac{p_2}{\rho} + \dfrac{1}{2}v_2^2 + \cancel{gh_2}}$

- p_2 = N/m^2

- By continuity equation: $\boxed{\dot{m} = \rho_1 v_1 A_1 = \rho_2 v_2 A_2}$

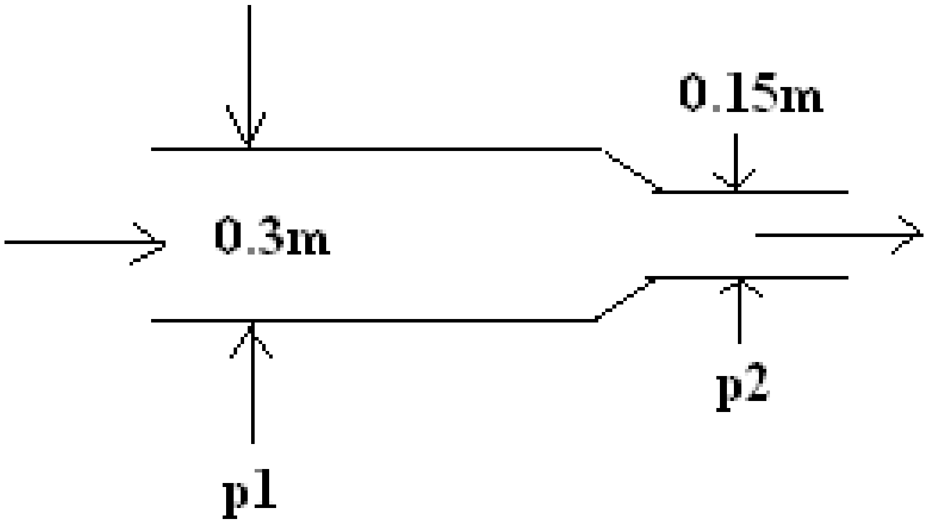
- v_2 = 7.8125 m/s

- From B.Eqn…. $\boxed{\frac{p_1}{\rho} + \frac{1}{2} v_1^2 + \cancel{g h_1} = \frac{p_2}{\rho} + \frac{1}{2} v_2^2 + \cancel{g h_2}}$

- p_2 = 182.703 x 10^3 N/m^2

Prob 2

- Gasoline(680 kg/m^3) flows from a 0.3m dia pipe in which the pressure is 300kPa into a 0.15m dia pipe in which the press is 120kPa. If the pipes are horizontal & viscous effects are negligible, determine the flow rate:



- By continuity equation:

$$\boxed{\dot{m} = \rho_1 v_1 A_1 = \rho_2 v_2 A_2}$$

- $v_2 = 4\ v_1$
- From B.Eqn….
- $v_1 = 5.94$ m/s
- Flow rate, $Q = A_1 v_1 = 0.4199\ m^3/s$