BT612 Systems Biology

Total Marks: 40 | **Mid-semester Examination** | **Time: 2 hr.**

July-November 2023

Instructions

1. Preferably write the answers following the order of questions
2. You MUST SHOW ALL the calculation STEPS. MARK the FINAL answer clearly.
3. Use standard mathematical notations/symbols only.
4. All graphs should be suitably labeled and axes clearly marked.

Q1. A) What is bimodal gene expression? What is the connection between bifurcation and bimodal gene expression?

B) What are the key assumptions behind the ODE-based modeling of a biochemical system? Explain those very briefly. Homon geneous Large [2 × 2.5]

Q2. The dynamics of two interacting cell populations are modeled using the following system of ODEs:

$$\frac{dx}{dt} = 4(1-x)x - xy$$

$$\frac{dy}{dt} = 2(1-y)y - xy$$

Find the steady state values of x and y when the initial condition is: $x = 0.2, y = 0.2$ at $t = 0$. Show all relevant calculations to justify your answer. [5]

Q3. A protein positively autoregulates its own expression. The following ODE is used to model its dynamics:

$$\frac{dP}{dt} = \frac{P^2}{1+P^2} - cP$$

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Find the condition in terms of c that makes this system bistable. A bistable system has two stable steady states. $P \geq 0$ and $c \geq 0$. [5]

Q4. "For the ODE, $\frac{dx}{dt} = rx + x^3$, r is the bifurcation parameter" – prove this statement using the concept of potential landscape. You must show the diagram of the potential landscape and all the relevant calculations. [5]

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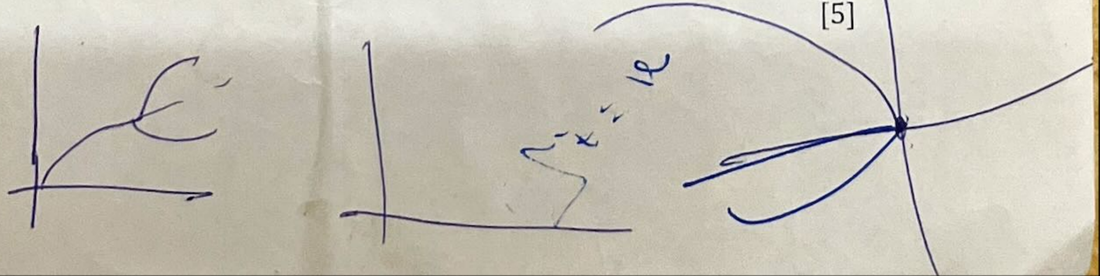

Q5. S is an input signal that induces X, and Y. Y inhibits X. This is a feedforward motif. Consider the steady state value of X as the output of this motif. The following ODEs are used to capture the dynamics of this motif:

$$\frac{dX}{dt} = k_1 S(1-X) - k_2 XY$$

$$\frac{dY}{dt} = k_3 S\frac{(1-Y)}{K_m + (1-Y)} - k_4 Y$$

Show that for $K_m \ll 1$ the output of this motif is independent of the input signal. $S > 0$. [5]

Q6. A reversible autocatalytic reaction is shown here,

$$A + X \underset{k_2}{\overset{k_1}{\rightleftharpoons}} 2X$$

$\frac{d[A]}{dt} = 0$

$\frac{dx}{dt} = k_1 x^2 - k_1 x$

Create an ODE-based model for this system. Consider that A is present in excess, and therefore, its concentration does not change with time. Use the concept of the Law of Mass Action to create the model. According to the Law of Mass Action, the rate of an elementary reaction is proportional to the product of the concentration of its reactants.

Find all steady states of this system and comment on their stabilities. [5]

Q7. A dynamical system is represented by the following system of ODEs:

$$\frac{dx}{dt} = -3x + 2y \quad \text{and} \quad \frac{dy}{dt} = x - 2y$$

What will be the behavior of a trajectory in the phase space when $t \to -\infty$? [5]

Q8. The following system of ODEs represents a mutual repressor circuit involving three molecules. Consider, a = 2, b = c = 0.1 and x, y, z ≥ 0.

$$\frac{dx}{dt} = a - bx - cxz; \quad \frac{dy}{dt} = a - by - cyx; \quad \frac{dz}{dt} = a - bz - czy;$$

Can this system show bifurcation when the rate of production of these molecules, a, is considered as the bifurcation parameter? Consider a is always greater than zero. [5]