Basic concepts of probability

Probability

The probability of an event

Imagine that you have 1000 songs on your phone, each of them recorded exactly once. When you push the “shuffle” button, the phone plays a song at random from the list of 1000. The probability that the first song played is your single favorite song is 1/1000, or 0.001. The probability that the first song played is not your favorite song is 999/1000, or 0.999. What exactly do these numbers mean?

The concept of probability rests on the notion of a **random trial**. A random trial is a process or experiment that has two or more possible outcomes whose occurrence cannot be predicted with certainty. Only one outcome is observed from each repetition of a random trial. In the phone songs example, a random trial consists of pushing the shuffle button once. The specified outcome is “your favorite song is played,” which is one of 1000 possible outcomes.

What is probability? To answer this, we need to define the **event** of interest and the list of all possible **outcomes** of a random trial.

Probability

1. Sample Space

The set of every possible outcome in a probability experiment (trial) is known as the sample space. For example, the sample space for tossing a coin is head and tail.

2. Experiment

A process or trial that provides a range of potential results is called an experiment. Any scenario where the results are uncertain may be the subject of the experiment. For example, the tossing of a coin, selecting a card from a deck of cards, throwing a die, etc.

3. Sample Point

It is one of the possible results. Consider the example of rolling a fair six-sided dice. The sample space for this experiment consists of six possible outcomes: 1, 2, 3, 4, 5 or 6. Each of these possible outcomes is a sample point.

4. Event

A subset of the sample space. For example: getting 1 as an outcome while rolling a die is an event.

5. Favorable outcome

An occurrence that has produced the desired consequence or an expected event.

Probability

Equally Likely Events
Equally likely events are those whose chances or probabilities of happening are equal. Both events are not related to one another. For example, there are equal possibilities of receiving either a head or a tail when we flip a coin.

Exhaustive Events
We call an event exhaustive when the set of all experiment results is the same as the sample space.

Mutually Exclusive Events
Events that are mutually exclusive cannot occur at the same time. For instance, when a coin is tossed then the result will be either head or tail, but we cannot get both the results.

Complimentary events
The non-occurring events. The complement of an event A is the event, not A (or $\bar{A}\ or\ A'$).

Probability

The probability of an event

The **probability** of an event is the *proportion* of all random trials in which the specified event occurs when the same random process is repeated over and over again independently and under the same conditions.

In an infinite number of random trials carried out in exactly the same way, the probability of an event is the fraction of the trials in which the event occurs.

The ***probability*** of an event is the proportion of times the event would occur if we repeated a random trial over and over again under the same conditions.

Probability is the measure of the likelihood that an event will occur. It is quantified as a number between 0 and 1.

$$P(E) = \text{Number of favorable outcomes} / \text{Total number of outcomes}$$

Probability

Example of an event:

Let us throw a die, then each throw of a die has six outcomes: {1, 2, 3, 4, 5, 6}.

A throw with a specific outcome is an **Event** e.g. e=1, 2, 3, 4, 5, 6

Probability is a numerical descriptions of **how likely** a specific face (e) will appear if we throw the dice.

Outcomes of two coin tosses: {HH, TT, HT, TH}
Examples of **events**: Two coin tosses with only one H: {HT, TH} and Two coin tosses with no H: {TT}

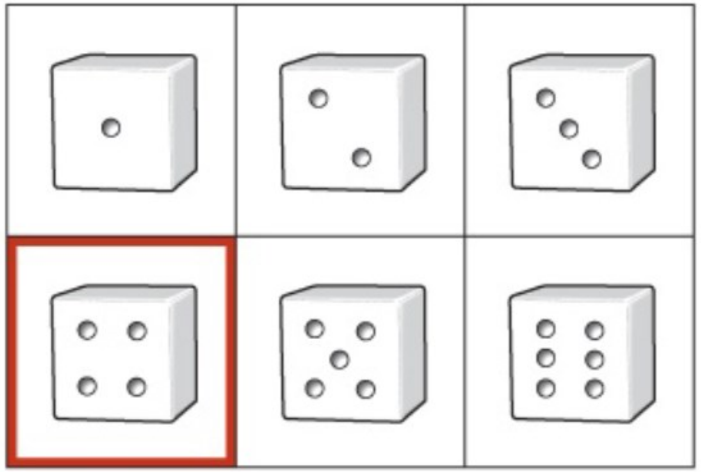
In general,
Probability is a numerical descriptions of **how likely** an **Event** is to occur.

The meaning of numerical descriptions: **Probability** of an event is the **proportion** of times the event would occur if the random trial is repeated a large number of times.

Probability

Venn diagrams

One useful way to think about the probabilities of events is with a graphical tool called a **Venn diagram**.



A Venn diagram for the possible outcomes of a roll of a six-sided die. The area corresponding to the event “the result is a four” is highlighted in red.

Probability

Example 1. There are 8 balls in a container, 4 are red, 1 is yellow and 3 are blue. What is the probability of picking a yellow ball?

The probability is equal to the number of yellow balls in the container divided by the total number of balls in the container, i.e. 1/8.

Example 2: A dice is rolled. What is the probability that an even number has been obtained?

When fair six-sided dice are rolled, there are six possible outcomes: 1, 2, 3, 4, 5, or 6.

Out of these, half are even (2, 4, 6) and half are odd (1, 3, 5).

Therefore, the probability of getting an even number is:
P(even) = number of even outcomes / total number of outcomes
P(even) = 3 / 6
P(even) = 1/2

Rules of Probability

Probability Theorems

Theorem 1: The sum of the probability of happening an event and not happening an event is equal to 1.

$$P(A) + P(\bar{A}) = 1$$

Theorem 2: The probability of an impossible event or the probability of an event not happening is always equal to 0.

$$P(\phi) = 0$$

Theorem 3: The probability of a sure event is always equal to 1.

$$P(A) = 1$$

Theorem 4: The probability of happening of any event always lies between 0 and 1.

$$0 \leq P(A) \leq 1$$

Theorem 5: If there are two events A and B, the probability of happening of event A or event B is

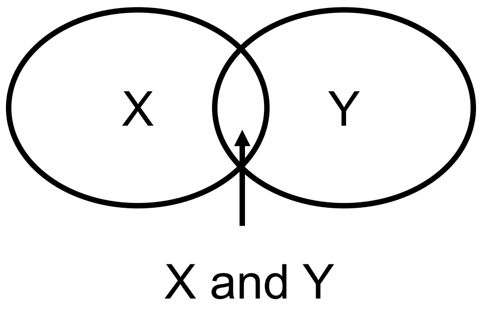
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Rules of Probability

Let us consider that X and Y are **NOT** mutually exclusive, then

$$P\,(X \text{ or } Y) = P(X) + P(Y) - P(X\ \textbf{and}\ Y)$$

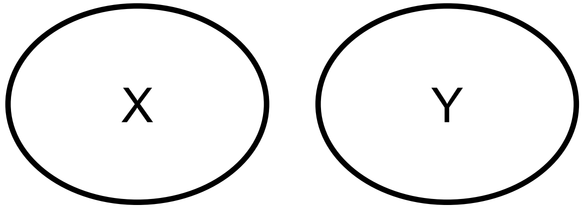
This is called **addition rule**.



Rules of Probability

Two events are m**utually exclusive** if both of those events can not occur at the same time.

Let us take X and Y are mutually exclusive, then probability of both happening simultaneously



P (X **and** Y) = 0

P (X **or** Y) = P(X) + P(Y)

Also for two mutually exclusive events A and B, we have P(A U B) = P(A) + P(B)

Rules of Probability

1. Whenever an event is the union of two events, say A and B, then

$$P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

2. Whenever an event is the complement of another event if A is an event, then

$$P(\text{not } A) = 1 - P(A) \text{ or } P(\bar{A}) = 1 - P(A)$$

$$P(A) + P(\bar{A}) = 1$$

3. Whenever an event is the intersection of two events, i.e., events A and B need to occur simultaneously. Then

$$P(A \text{ and } B) = P(A) \times P(B)$$

Rules of Probability

Independence and the multiplication rule

Two events are ***independent*** if the occurrence of one does not inform us about the probability that the second will occur.

The ***multiplication rule***: If two events *A* and *B* are independent, then P(A and B)= $P(A \cap B)$ = P(A)×P(B).

Similarly, if X, Y and Z are independent, P(X and Y and Z) = $P(x \cap y \cap z)$ = P(X) x P(Y) x P(Z). This is called **Product Rule**.

"And" versus "or"

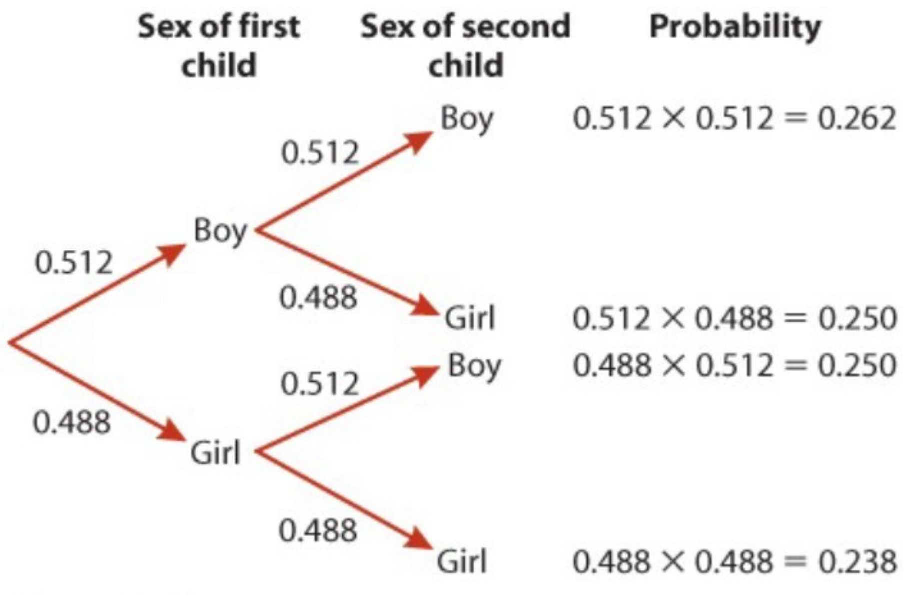
- The probability of A or B involves addition. That is, P(A or B) = P(A) + P(B), if the two events A and B are mutually exclusive.

- The probability of A and B involves multiplication. That is, P(A and B) = P(A) × P(B), if A and B are independent.

Rules of Probability

Probability trees

A **probability tree** is a diagram that can be used to calculate the probabilities of combinations of events resulting from multiple random trials.



A probability tree for all possible values of a two-child family.

Probability

Example 3. Fill in the blanks in the following table

	P(A)	P(B)	$P(A \cap B)$	P(A U B)
i	1/3	1/5	1/15	—
ii	0.35	—	0.25	0.6
iii	0.5	0.35	—	0.7

(i) P(A) = 1/3
P(B) = 1/5
P(A U B) = 1/15
P(A U B) = P(A) + P(B) – $P(A \cap B)$
= 7/15

(ii) P(A) = 0.35
$P(A \cap B)$ = 0.25
P(A U B) = 0.6
P(A U B) = P(A) + P(B) – $P(A \cap B)$
0.60 = 0.35 + P(B) – 0.25
0.60 – 0.35 + 0.25 = P(B)
P(B) = 0.50

(iii) P(A) = 0.5 and P(B) = 0.35 and P(A U B) = 0.7
$P(A \cap B)$ = P(A) + P(B) – P(A U B)
= 0.5 + 0.35 – 0.7
= 0.15

Probability

Example 4. A card is drawn from a well-shuffled pack of 52 cards. Find the probability of being spade or a king?

A card is drawn from well shuffled pack of 52 cards,
Hence, S(sample space) = ${}^{52}C_1$
= 52
Let A = event of choosing a card of spade
A = ${}^{13}C_1$
= 13
Let B = event of choosing a king
B = ${}^{4}C_1$
= 4

We can also have an event where the king drawn is of spade, hence, P(A n B) = 1

$P(A \cup B) = P(A) + P(B) – P(A \cap B)$
= 13/52 + 4/52 – 1/52
= 18/52
= 4/13

Probability

Example 5. In a single throw of two dice, find the probability that neither doublet nor the total is 9?

Two dices are thrown hence the sample space will be: $S = 6^2 = 36$

Let A be the event of choosing doublet: A = {(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)} => A = 6

Let B be the event of choosing sum equal to 9: B = {(3,6), (4,5), (5,4), (6,3)} => B = 4

Here event A and B can't occur together i.e. P(sum is 9 and dice are doublet) = $P(A \cap B) = 0$

Now,
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$= 6/36 + 4/36$
$= 10/36$
$= 5/18$

Now, the probability that neither double nor the total is 9,
$= 1 - P(A \cup B)$
$= 1 - 5/18$
$= 13/18$

Probability

Example 6. A natural number is chosen at random from 500. What is the probability that a number is divisible by 5 or 3?

Since a number is chosen from the first 500 natural numbers, Hence, n(S) = 500
Let A be event of choosing a number divisible by 3,
A = {3,6,9 ……………….., 498}
n(A) = 166 ($a_n = a + (n-1)*d$ where $a = 3$, $d = 3$, $t_n = 498$ => $498 = 3 + (n-1)3$ =>$n = 166$)

P(A) = 166/500
Let B be the event of choosing a number divisible by 5,
B = {5,10,15,20. . . . . .495,500} => n(B) = 100
P(B) = 100/500

We can also have a event where the number is divisible by 5 and 3
P(A n B) = {15,30,45. . . . 495}
n(A n B) = 33
P(A n B) = 33/500

Now, P(A U B) = P(A) + P(B) – P(A ∩ B)
= 166/500 + 100/500 – 33/500
= 233/500

Probability

Example 7. A die is thrown twice. What is the probability that one of the number comes up with 3?

A dice is thrown twice, Hence the sample space will be
n(S) = 36

Let A be the event of getting 3 at first throw –
n(A) = 6
P(A) = 6/36
= 1/6

Let B be the event of getting 3 at second throw-
n(B) = 6
P(B) = 6/36
= 1/6

Also, $P(A \cap B) = 1/36$

$P(A \cup B) = P(A) + P(B) – P(A \cap B)$
=1/6 + 1/6 – 1/36
= 11/36

Probability

Example 8. The probability that a student will pass in BT307 and BT308 is 0.5 and the probability that will pass in neither is 0.1. If the probability of passing in BT307 is 0.75, what is the probability of passing the BT308 examination?

Let A be the event of passing in the BT307 exam: P(A) = 0.75

Let B be the event of passing in the BT308 exam: P(B) = ?

The probability that a student will pass in BT307 and BT308 is 0.5: Hence, P(A n B) = 0.5

The probability that will pass in neither is 0.1: $P(\bar{A}n\bar{B})$ = 0.1
1 – P(A U B) = $P(\bar{A}n\bar{B})$
P(A U B) = 1 – 0.1
= 0.9

Now,
P(A U B) = P(A) + P(B) – P(A ∩ B)
0.9 = 0.75 + P(B) – 0.5
P(B) = 0.65

Probability

Example 9. A bag contains 8 blue balls and some pink balls. If the probability of drawing a pink ball is half of the probability of drawing a blue ball then find the number of pink balls in the bag.

Let us considered the number of pink balls be n.
The number of blue balls = 8.
Therefore, the total number of balls present in the bag = n + 8.

Now, the probability of drawing a pink ball, i.e. P(A) = n/n + 8
the probability of drawing blue ball, i.e. P(B) = 8/n + 8

According to the question, the probability of drawing pink ball is half of the probability of drawing the blue ball
So, P(A) = P(B)/2
n = 4.

So, number of pink balls present in the bag is 4.

Probability

Example 10. Two coins are tossed simultaneously for 360 times. The number of times ‘2 Tails’ appeared was three times ‘No Tail’ appeared and number of times ‘1 tail’ appeared is double the number of times ‘No Tail’ appeared. Find the probability of getting ‘Two tails’.

Total number of outcomes = 360

Let us considered the number of times ‘No Tail’ appeared be z

Then, number of times ‘2 Tails’ appeared = $3z$

Number of times ‘1 Tail’ appeared = $2z$

Now, $z + 2z + 3z = 360$
$6z = 360$
$z = 60$

Hence, the probability of getting ‘two tails’ = $(3 \times 60)/360 = 1/2$

Probability

Exercise:

Blood Group	Population distribution (%)
O	37
A	23
B	32
AB	8

What is the probability that a randomly chosen person can donate blood to a person of the blood group B?

Thank You