Monopoly Market

- Single seller, one firm which produces the good. Example: Railways, power supplier, water supplier.
- Single firm, so can decide the market price.
- Objective is to maximize profit by deciding output or price.

Model

- The general market demand function is $D(p) = q$, $D'(p) < 0$ for all p.
- There exists p^{Max} such that at $p = p^{Max}$, $D(p^{Max}) = 0$.
- At $p = 0$, $D(0) = q^{Max}$.
- We have inverse demand function, since demand function is strictly decreasing in price. It is given as $f(q) = p$. In the inverse demand function, at $p^{Max}, f(p^{Max}) = 0$ and at $p = 0,\ f(0) = q^{Max}$.

- The cost function of the firm is $c(q) + F$.
- $c(q)$ is the variable cost and F is the total cost. We also assume $c(0) = 0$
- We further assume that $c'(q) > 0$ and $c''(q) \geq 0$.
- $c'(q)$ is the marginal cost.
- It means that cost function is constant returns to scale (CRS) or decreasing returns to scale (DRS).

- The profit function of the monopolist is
  $\pi = f(q)q - c(q) - F$.
- The above profit function is function of monopolist output q. In this case the monopolist maximizes with respect to quantity q.
- We can also write the profit function as function of price.
  $\pi = D(p)p - c(D(p)) - F$.
- When profit is taken as function of price, the monopolist maximizes profit with respect to price.

- Suppose the profit function is a function of quantity. So $\pi = f(q)q - c(q) + F$. Since $f(q) and c(q)$ are differentiable so we use calculus to find the optimal q.
$\frac{d\pi}{dq} = f(q) + f'(q)q - c'(q)$
First order condition say gives that
$\frac{d\pi}{dq} = f(q) + f'(q)q - c'(q) = 0$
$\Rightarrow f(q) + f'(q)q = c'(q)$.
We get the optimal output q^* by solving the equation
$f(q) + f'(q)q = c'(q)$.
At q^*, $f(q^*) + f'(q^*)q^* - c'(q^*) = 0$

- The first order condition implies marginal revenue equal to marginal cost at the optimal point.
  $f(q) + f'(q)q =$ marginal revenue. It is derivative of total revenue.
- Marginal revenue (MR) is the additional increase in revenue received by the firm when it sells one more additional unit.
- It lies below the demand curve since the demand curve is downward sloping. To increase sells, the firm has to reduce price. So additional revenue at the margin falls.

$\cup$ shaped marginal cost

See class notes

- Second order condition;
$$\frac{d^2\pi}{dq^2} = 2f'(q) + f''(q)q - c''(q) < 0 \text{ at } q = q^*, \text{ the optimal point.}$$
From the assumption, $c''(q) \geq 0$, so $-c''(q) \leq 0$. $f'(q) < 0$.
- Further we assume that if $f''(q) > 0$ it should not be big enough. It means that Demand curve is downward sloping and convex to the origin to origin. If $f'(q) < 0$ and $f''(q) > 0$. We assume that it should not be highly convex for the seconder condition to be satisfied.
- If $f''(q) \leq 0$, we do not have any problem. The second order condition is always satisfied.

Market Power

The first order condition gives
$f(q) + f'(q)q = c'(q)$.
$\Rightarrow p = \dfrac{c'(q)}{(1 - \frac{1}{|\xi_d|})} = \dfrac{MC}{(1 - \frac{1}{|\xi_d|})}$, where $|\xi_d|$ is the price elasticity of demand.
If $|\xi_d| = 1$ then $p(1 - \frac{1}{|\xi_d|}) = c'(q)$ becomes $0 = c'(q)$. Firm lost it is power to set price. So the monopolist always produces when the price elasticity of demand is elastic.
Monopolist sets higher price for the goods with higher price elasticity of demand.

Inefficiency in Monopoly

See class notes

Natural Monopoly

The fixed cost is very high and marginal cost is relative is low.
See class notes