EE 626 : Quiz 2

Duration: 45 minutes **Marks:10** **Date: Apr 16' 2024**

Note: No clarifications or discussions on the questions will be entertained during the examination session.

1. Recall, that the GMM for a d dimensional input $\mathbf{x}$ with M Gaussian distributions is defined as

$$\sum_{j=1}^{M} \pi_j N(\mu_j, \Sigma_j)$$

where the index j refering to the j^{th} Gaussian is given by

$$N(\mu_j, \Sigma_j) = \frac{1}{(2\pi)^{d/2}|\Sigma_j|^{0.5}} e^{-\frac{1}{2}(\mathbf{x}-\mu_j)^T \Sigma_j^{-1}(\mathbf{x}-\mu_j)}$$

A GMM is trained using two Gaussian distributions on a two dimensional data. The mean of the first and second Gaussian is $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ -2 \end{bmatrix}$ respectively. Likewise, the covariance matrix of the first and second Gaussian is $\begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ respectively. Moreover, it is known that for a feature vector $\mathbf{x} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, its posterior probability of assignment with regards to the first Gaussian is 0.2.

(i) Find the posterior probability of $\mathbf{x}$ with regards to the second Gaussian.

(ii) Find the values of π_1 and π_2

[1 + 5.5 = 6.5 marks]

2. Consider a one-dimensional two-category classification problem with prior, $P(\omega_1) = 0.6$. Three i.i.d training observations were collected: $D_1 = \{1, 2, 5\}$ and $D_2 = \{3, 4, 7\}$ for ω_1 and ω_2, respectively. It is desired to classify a test pattern using the Parzen Window technique, discussed in class. Using the window function $\phi(x) = \frac{1}{\sqrt{2\pi}} \exp^{-\frac{x^2}{2}}$ with parameter of bandwidth h=1, classify the pattern $x = 4.5$ by specifying the posterior probability under the zero-one loss paradigm.

[3.5 marks]