BT209

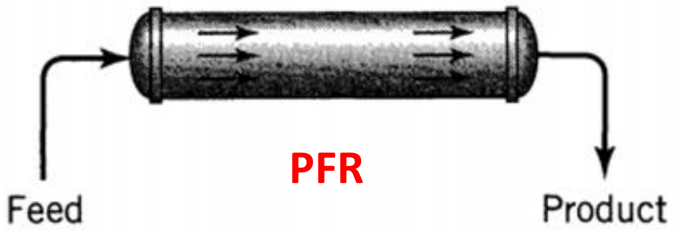
Bioreaction Engineering

13/02/2023

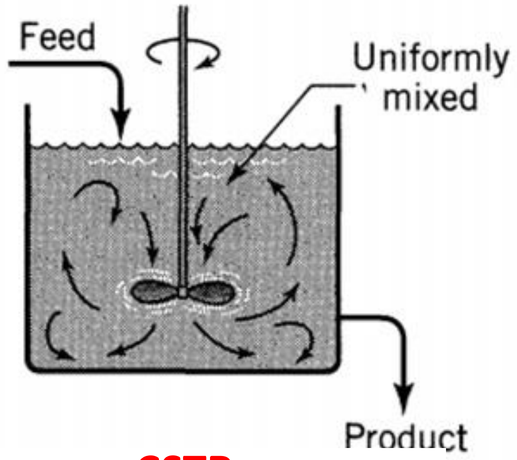
Space time and space velocity of FLOW REACTOR

❑ Reaction time t is the natural performance measure for a batch reactor,

❑ Space-time and space-velocity the proper performance measures of flow reactors.



PFR



CSTR

Space-time:

$$\tau = \frac{1}{s} = \begin{pmatrix} \text{time required to process one} \\ \text{reactor volume of feed measured} \\ \text{at specified conditions} \end{pmatrix} = [\text{time}]$$

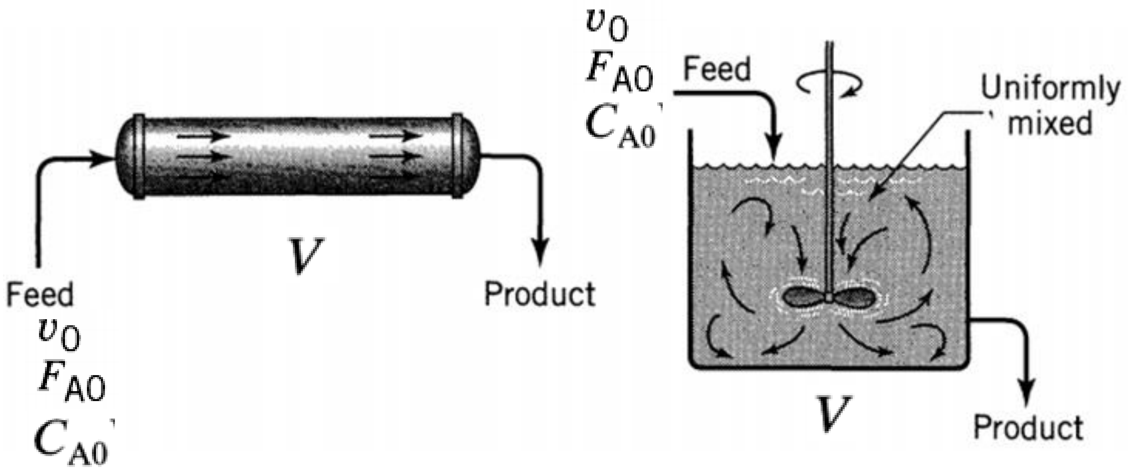
Space-velocity:

$$s = \frac{1}{\tau} = \begin{pmatrix} \text{number of reactor volumes of} \\ \text{feed at specified conditions which} \\ \text{can be treated in unit time} \end{pmatrix} = [\text{time}^{-1}]$$

Cont..

- Thus, a space-velocity of 5 hr^{-1} means that five reactor volumes of feed at specified conditions are being fed into the reactor per hour.

- A space-time of 2 min means that every 2 min one reactor volume of feed at specified conditions is being treated by the reactor.



V

$$\tau = \frac{1}{s} = \frac{C_{A0}V}{F_{A0}} = \frac{\left(\dfrac{\text{moles A entering}}{\text{volume of feed}}\right)(\text{volume of reactor})}{\left(\dfrac{\text{moles A entering}}{\text{time}}\right)}$$

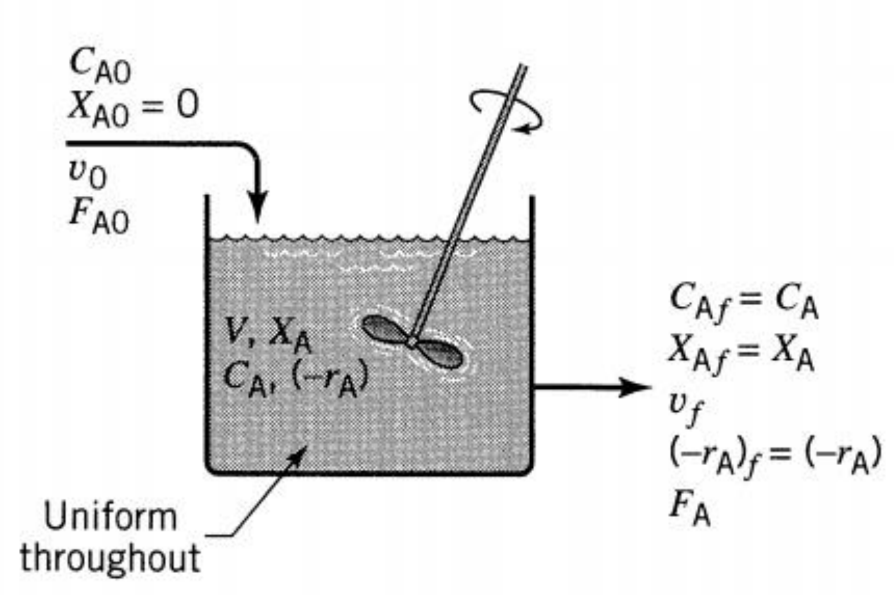
$$= \frac{V}{v_0} = \frac{(\text{reactor volume})}{(\text{volumetric feed rate})}$$

Steady sate continuous stirred tank reactor (CSTR)

- It is a reactor in which the contents are well stirred and uniform throughout .

- Thus, the exit stream from this reactor has the same composition as the fluid within the reactor.

- Also named as mixed flow reactor (MFR), the backmix reactor or the CFSTR (constant flow stirred tank reactor),



$$\begin{pmatrix}\text{rate of}\\ \text{reactant}\\ \text{flow into}\\ \text{element}\\ \text{of volume}\end{pmatrix} = \begin{pmatrix}\text{rate of}\\ \text{reactant}\\ \text{flow out}\\ \text{of element}\\ \text{of volume}\end{pmatrix} + \begin{pmatrix}\text{rate of reactant}\\ \text{loss due to}\\ \text{chemical reaction}\\ \text{within the element}\\ \text{of volume}\end{pmatrix} + \begin{pmatrix}\text{rate of}\\ \text{accumulation}\\ \text{of reactant}\\ \text{in element}\\ \text{of volume}\end{pmatrix}$$

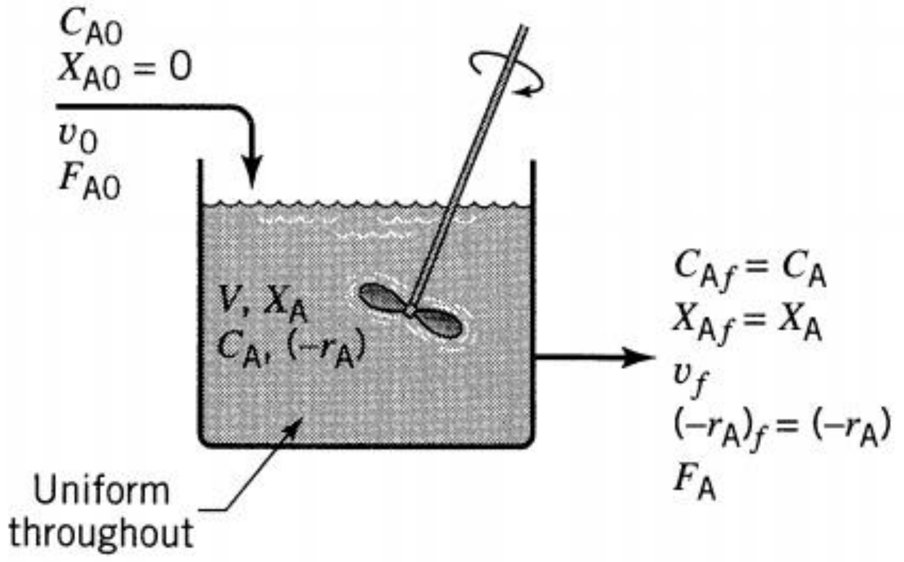
$$\text{input} = \text{output} + \text{disappearance by reaction} + \cancelto{=0}{\text{accumulation}}$$

Cont..

$$\text{input of A, moles/time} = F_{A0}(1 - X_{A0}) = F_{A0}$$

$$\text{output of A, moles/time} = F_A = F_{A0}(1 - X_A)$$

$$\begin{array}{c}\text{disappearance of A}\\ \text{by reaction,}\\ \text{moles/time}\end{array} = (-r_A)V = \left(\frac{\text{moles A reacting}}{\text{(time)(volume of fluid)}}\right)\left(\begin{array}{c}\text{volume of}\\ \text{reactor}\end{array}\right)$$



$$F_{A0}X_A = (-r_A)V$$

which on rearrangement becomes

$$\boxed{\begin{array}{c}\dfrac{V}{F_{A0}} = \dfrac{\tau}{C_{A0}} = \dfrac{X_A}{-r_A}\\[2ex] \tau = \dfrac{1}{s} = \dfrac{V}{v_0} = \dfrac{VC_{A0}}{F_{A0}} = \dfrac{C_{A0}X_A}{-r_A}\end{array}}$$

or $\qquad$ any ε_A

Cont..

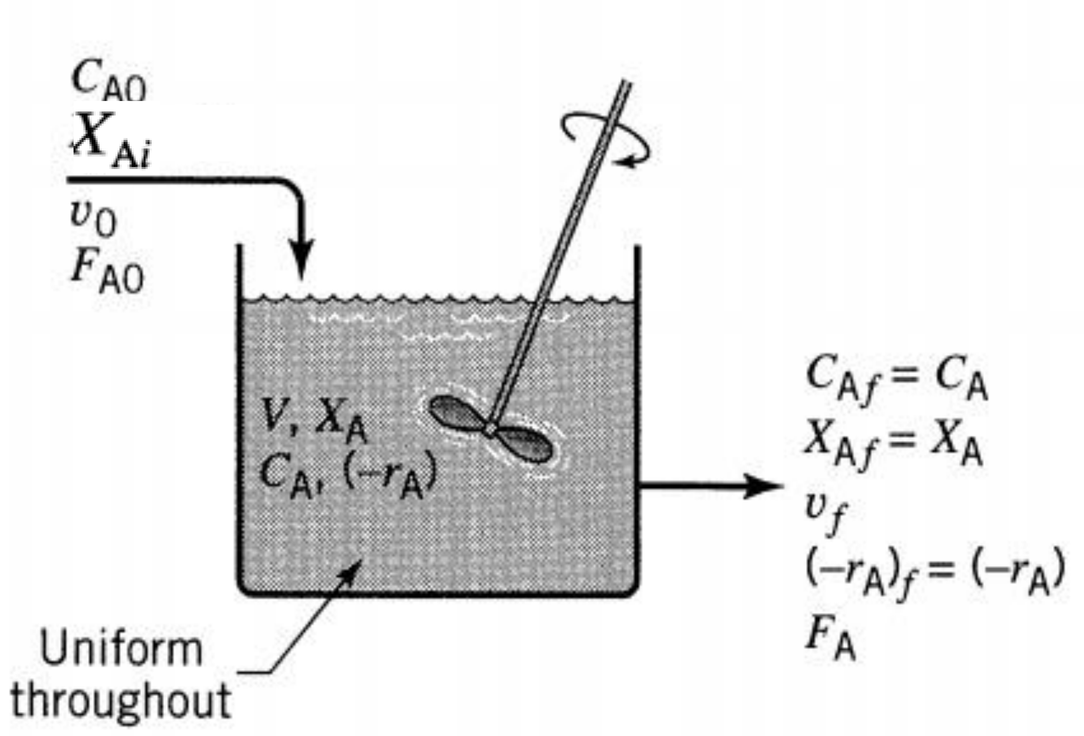
More generally, if the feed on which conversion is based, subscript 0, enters the reactor partially converted, subscript *i*, and leaves at conditions given by subscript *f*, we have

$$\frac{V}{F_{A0}} = \frac{\Delta X_A}{(-r_A)_f} = \frac{X_{Af} - X_{Ai}}{(-r_A)_f}$$

$$\tau = \frac{VC_{A0}}{F_{A0}} = \frac{C_{A0}(X_{Af} - X_{Ai})}{(-r_A)_f}$$

$$\boxed{\begin{aligned} \frac{V}{F_{A0}} &= \frac{\tau}{C_{A0}} = \frac{\Delta X_A}{-r_A} \\ \frac{V}{F_{A0}} &= \frac{\tau}{C_{A0}} = \frac{X_A}{-r_A} \end{aligned}}$$

Design equation of CSTR



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For the special case of constant-density systems $X_A = 1 - C_A/C_{A0}$, in which case the performance equation for mixed reactors can also be written in terms of concentrations or

$$\frac{V}{F_{A0}} = \frac{\tau}{C_{A0}} = \frac{X_A}{-r_A}$$

$$\boxed{\begin{aligned} \frac{V}{F_{A0}} &= \frac{X_A}{-r_A} = \frac{C_{A0} - C_A}{C_{A0}(-r_A)} \\ \tau = \frac{V}{v} &= \frac{C_{A0}X_A}{-r_A} = \frac{C_{A0} - C_A}{-r_A} \end{aligned}} \qquad \varepsilon_A = 0$$

- These expressions relate in a simple way the four terms X_A, $-r_A$, V, F_{AO};

- Thus, knowing any three allows the fourth to be found directly. In design, then, the size of reactor needed for a given duty or the extent of conversion in a reactor of given size is found directly.

Cont..

$$\frac{V}{F_{A0}} = \frac{\tau}{C_{A0}} = \frac{\Delta X_A}{-r_A} = \frac{X_A}{-r_A}$$

$$\tau = \frac{1}{s} = \frac{V}{v_0} = \frac{VC_{A0}}{F_{A0}} = \frac{C_{A0}X_A}{-r_A}$$

any ε_A

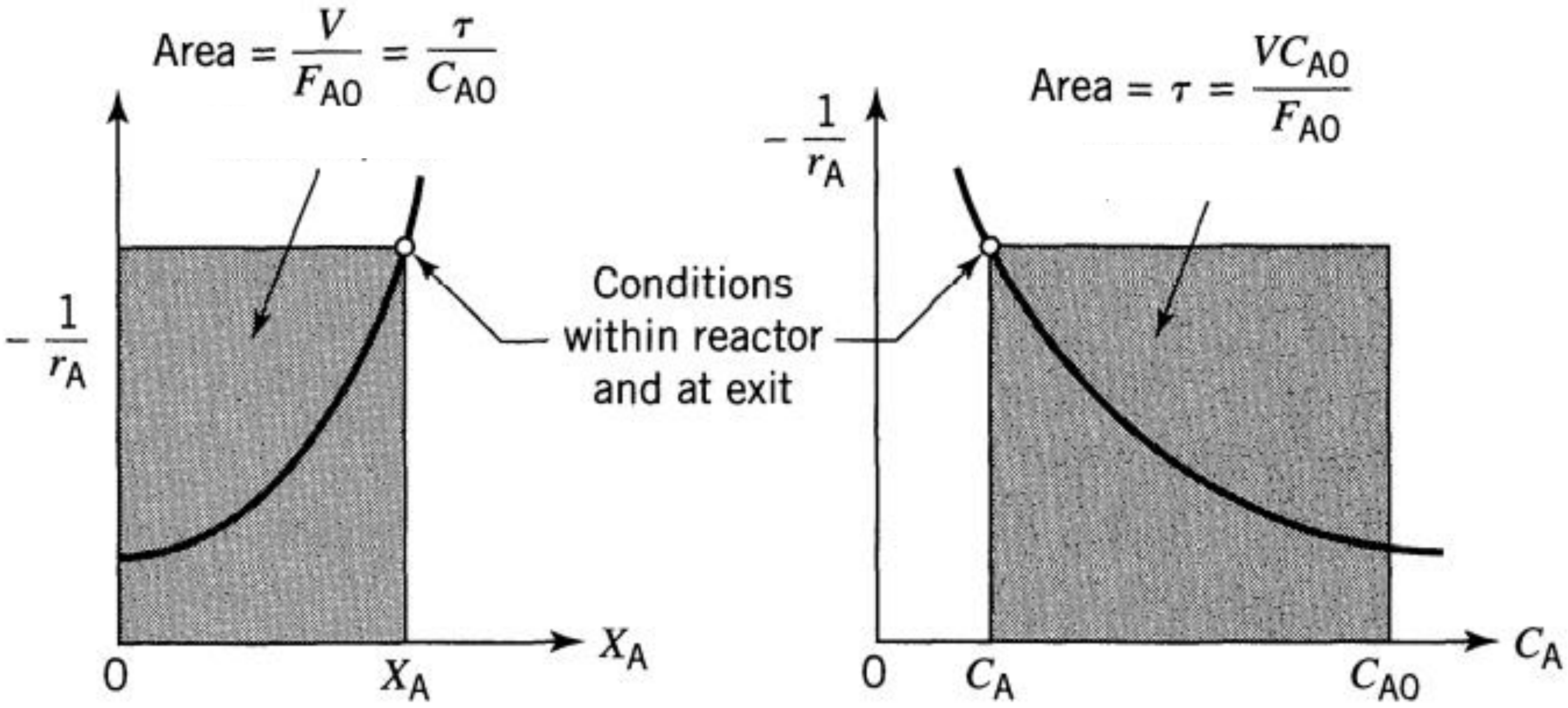
General case

$$\frac{V}{F_{A0}} = \frac{X_A}{-r_A} = \frac{C_{A0} - C_A}{C_{A0}(-r_A)}$$

$$\tau = \frac{V}{v} = \frac{C_{A0}X_A}{-r_A} = \frac{C_{A0} - C_A}{-r_A}$$

$\varepsilon_A = 0$

Constant-density systems only



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As an example, for constant density systems $C_A/C_{A0} = 1 - X_A$, thus the performance expression *for first-order reaction* becomes

$$k\tau = \frac{X_A}{1 - X_A} = \frac{C_{A0} - C_A}{C_A} \qquad \text{for } \varepsilon_A = 0$$

$\varepsilon_A = 0$

$$\frac{V}{F_{A0}} = \frac{X_A}{-r_A} = \frac{C_{A0} - C_A}{C_{A0}(-r_A)}$$

$$\tau = \frac{V}{v} = \frac{C_{A0}X_A}{-r_A} = \frac{C_{A0} - C_A}{-r_A}$$

On the other hand, for linear expansion

$$V = V_0(1 + \varepsilon_A X_A) \quad \text{and} \quad \frac{C_A}{C_{A0}} = \frac{1 - X_A}{1 + \varepsilon_A X_A}$$

thus *for first-order reaction* the performance expression

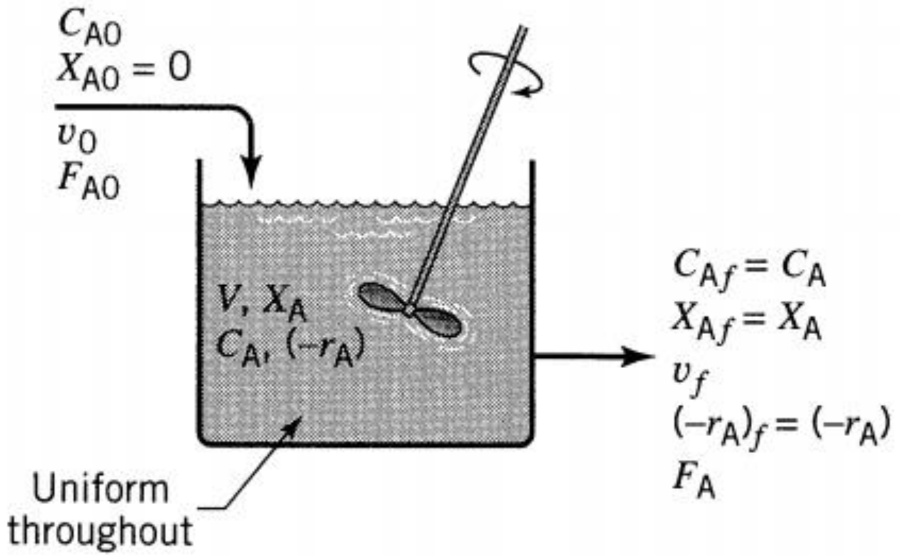
$$k\tau = \frac{X_A(1 + \varepsilon_A X_A)}{1 - X_A} \qquad \text{for any } \varepsilon_A$$

Cont..

For *second-order reaction*, A → products, $-r_A = kC_A^2$, $\varepsilon_A = 0$,

$$\boxed{\begin{aligned} \frac{V}{F_{A0}} &= \frac{X_A}{-r_A} = \frac{C_{A0} - C_A}{C_{A0}(-r_A)} \\ \tau = \frac{V}{v} &= \frac{C_{A0}X_A}{-r_A} = \frac{C_{A0} - C_A}{-r_A} \end{aligned}}$$

$\varepsilon_A = 0$



$$\boxed{k\tau = \frac{C_{A0} - C_A}{C_A^2} \quad \text{or} \quad C_A = \frac{-1 + \sqrt{1 + 4k\tau C_{A0}}}{2k\tau}}$$