

Molecular Dynamics

Analysis of MD trajectory (Part 2)

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Heat-Capacity

$$C_V = \left(\frac{\partial U}{\partial T} \right)_V$$

C_V = Heat Capacity at constant volume

U = Internal Energy = (Kinetic Energy + Potential Energy)

T = Temperature

Method I

Simulations

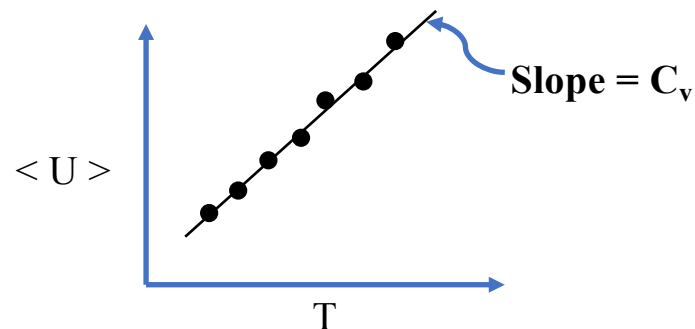
(N, V, T_1)

(N, V, T_2)

(N, V, T_3)

.....

(N, V, T_7)



Method II

Single simulation at (N, V, T)

$$C_V = \frac{\langle (U - \langle U \rangle)^2 \rangle}{K_B T^2}$$

More accurate

Less round of error than,

$$\langle U^2 \rangle - \langle U \rangle^2$$

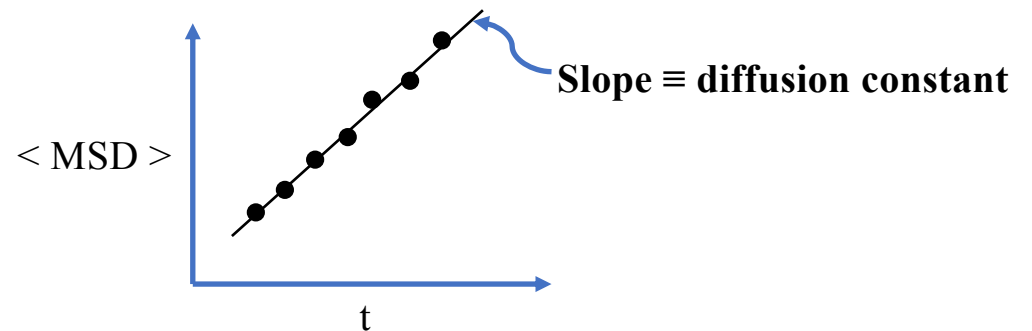
Diffusion coefficient (D)

Step A: Calculate the mean square displacement out of a single trajectory

$$\langle MSD \rangle = \langle \frac{1}{N} \sum_{i=1}^N (r_i(t) - r_i(t=0))^2 \rangle$$

Step B: Apply Einstein Equation: $3 D = \lim_{t \rightarrow \infty} \frac{\langle MSD \rangle}{2 t}$

*Dimension
1, 2, 3 etc*



Diffusion coefficient (D)

“D” describes how quickly one material can diffuse through another material.

- Increasing temperature increases the diffusion coefficient.
- Viscosity decreases diffusion coefficient
- **depends on size and shape of molecule**

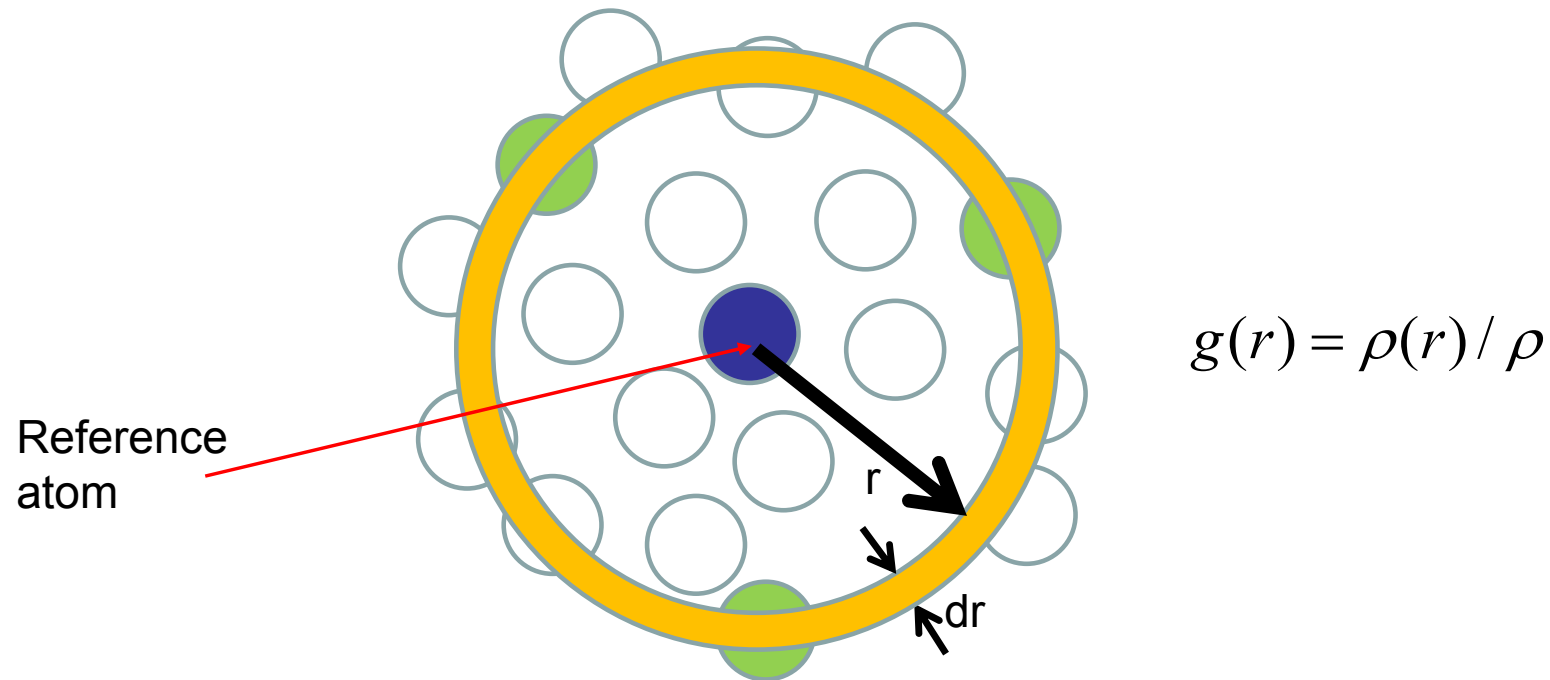
Radial Distribution Function

Many slides taken from:

https://ocw.mit.edu/courses/materials-science-and-engineering/3-021j-introduction-to-modeling-and-simulation-spring-2012/part-i-lectures-readings/MIT3_021JS12_P1_L4.pdf

Formal approach: Radial distribution function (RDF)

Ratio of density of atoms at distance r (in control area dr) by overall density = relative density of atoms as function of radius



Formal approach: Radial distribution function (RDF)

The radial distribution function is defined as

$$g(r) = \underbrace{\rho(r)}_{\text{Local density}} / \underbrace{\rho}_{\text{Overall density of atoms (volume)}}$$

Provides information about the density of atoms at a given radius r ; $\rho(r)$ is the local density of atoms

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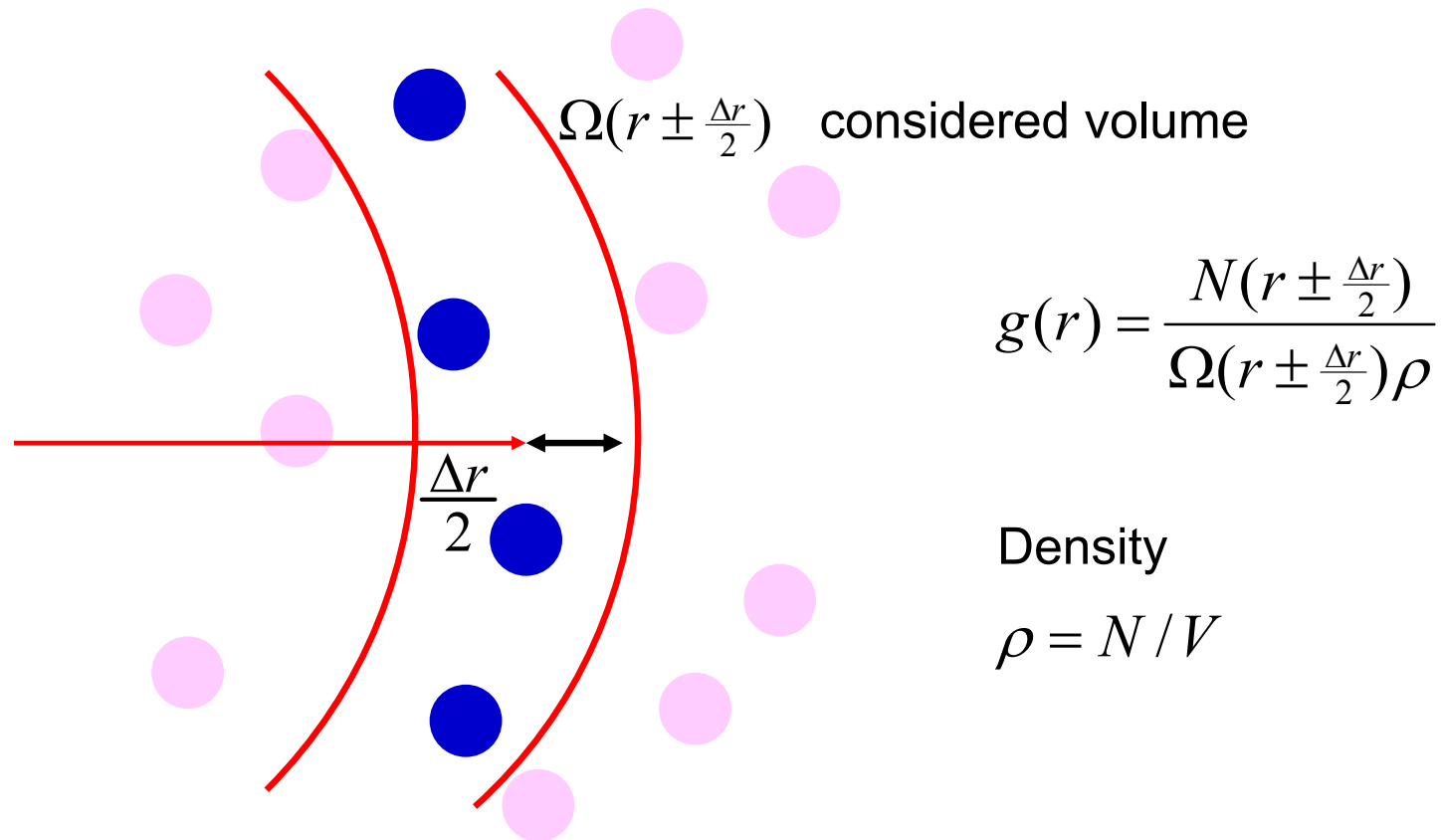
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Discrete:

$$g(r) = \frac{\overbrace{\langle N(r \pm \frac{\Delta r}{2}) \rangle}^{\text{Number of atoms in the interval } r \pm \frac{\Delta r}{2}}}{\underbrace{\Omega(r \pm \frac{\Delta r}{2})}_{\text{Volume of this shell } (dr)}} \frac{1}{\rho}$$

$g(r)2\pi r^2 dr$ = Number of particles that lie in a spherical shell of radius r and thickness dr

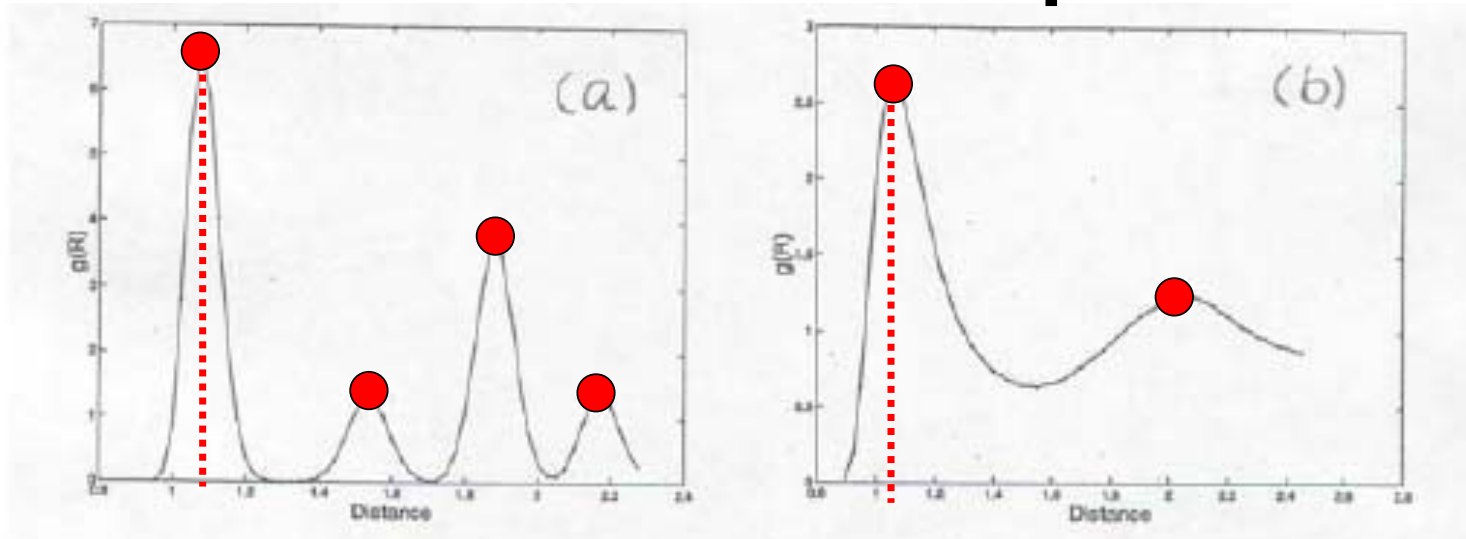
Radial distribution function



Note: RDF can be measured experimentally using x-ray or neutron-scattering techniques

Radial distribution function:

Which one is solid / liquid?



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Interpretation: A peak indicates a **particularly favored separation distance** for the neighbors to a given particle. Thus, RDF reveals details about the atomic structure of the system being simulated.

Java applet:

<http://physchem.ox.ac.uk/~rkt/lectures/liqsolns/liquids.html>

Radial distribution function: Solid versus liquid versus gas

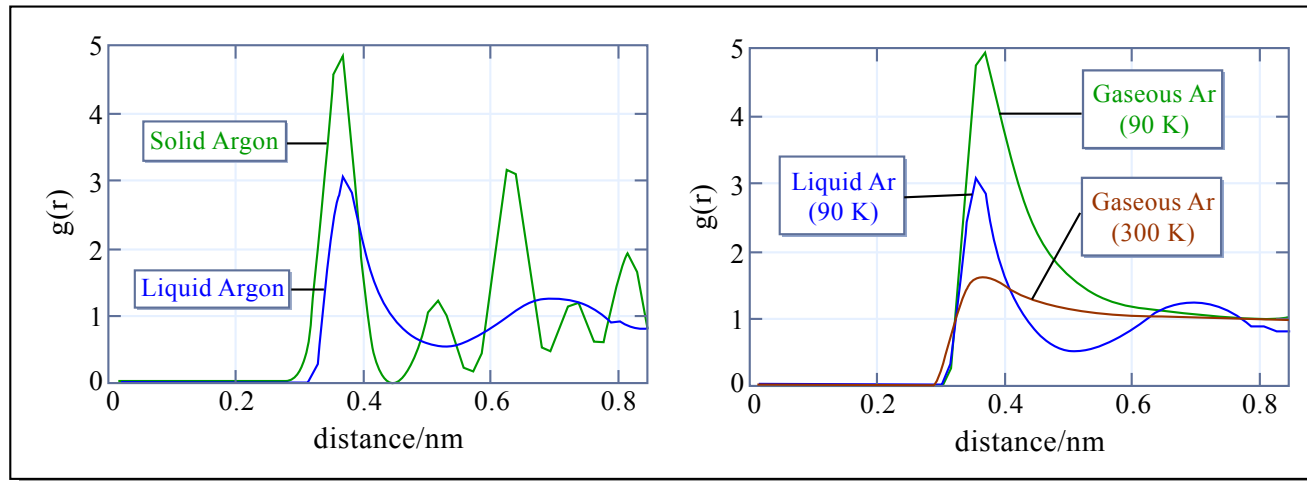


Image by MIT OpenCourseWare.

Note: The first peak corresponds to the nearest neighbor shell, the second peak to the second nearest neighbor shell, etc.

Notes: Radial distribution function (RDF)

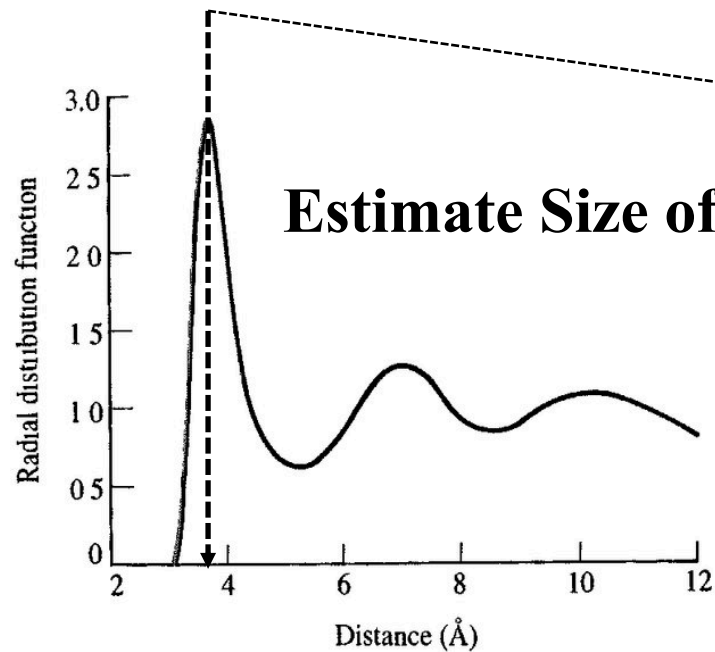
- Pair correlation function (consider only pairs of atoms)
- Provides structural information
- Can provide information about dynamical change of structure, but not about transport properties (how fast atoms move)

Additional comments:

- Describes how - on average - atoms in a system are radially packed around each other
- Particularly **effective way of describing the structure of disordered molecular systems (liquids)**
- In liquids there is **continual movement of the atoms and a single snapshot of the system shows only the instantaneous disorder it is extremely useful to be able to deal with the average structure**

Radial Distribution Function

Argon Liquid

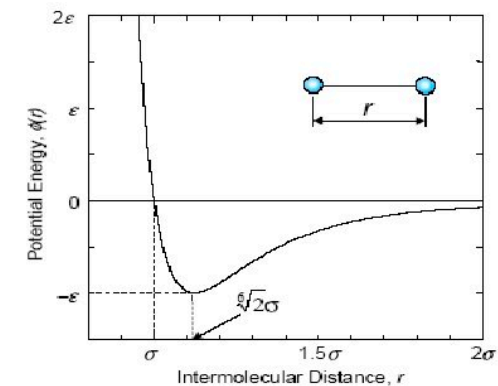


https://www.researchgate.net/publication/228387451_Molecular_dynamics_of_triglycerides_atomistic_and_coarse-grained_approaches/figures?lo=1

$$U_{LJ} = 4\epsilon \left[\left(\frac{\sigma}{r} \right)^{12} - \left(\frac{\sigma}{r} \right)^6 \right]$$

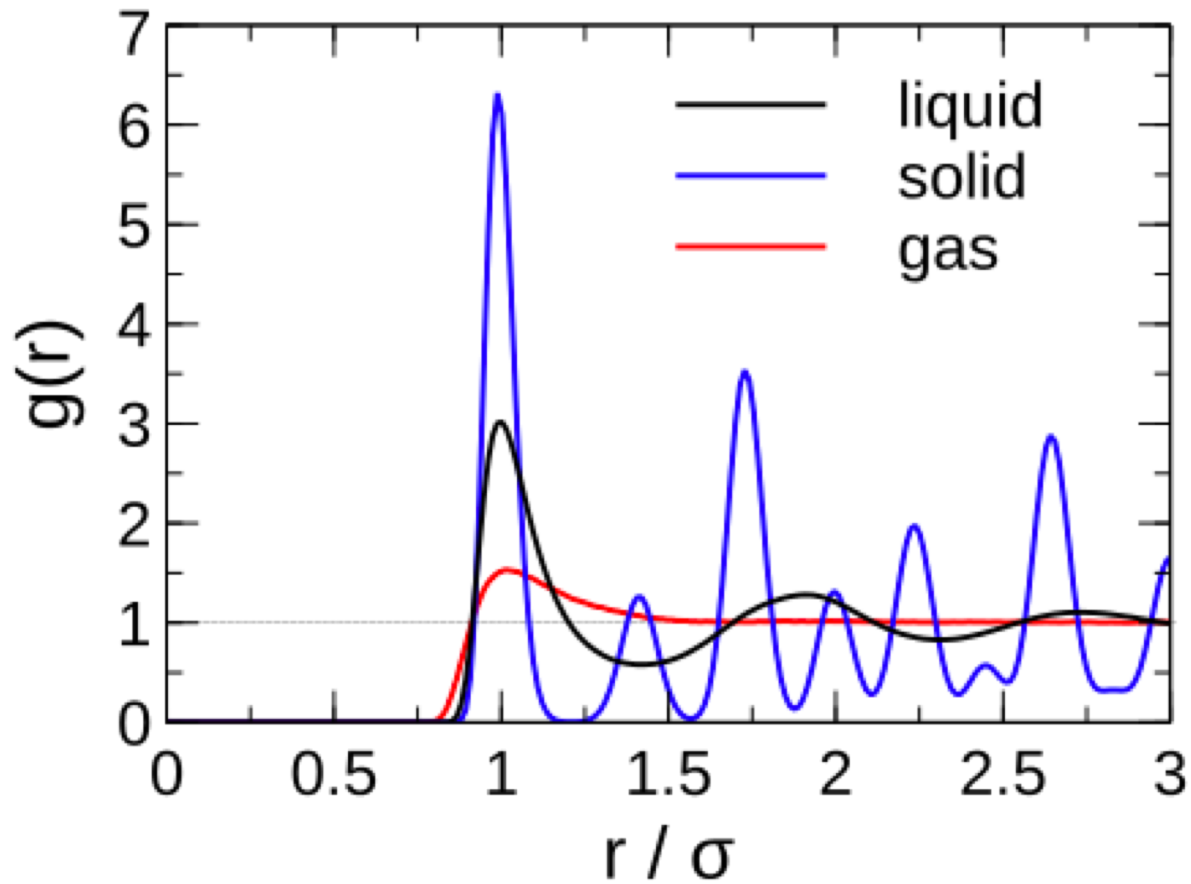
Lennard-Jones Potential: Noble Gas Atoms

	σ [nm]	ϵ [J]	ϵ/k_B [K]
Ne	0.274	0.50×10^{-21}	36.2
Ar	0.340	1.67×10^{-21}	121
Kr	0.359	2.25×10^{-21}	163
Xe	0.398	3.20×10^{-21}	232



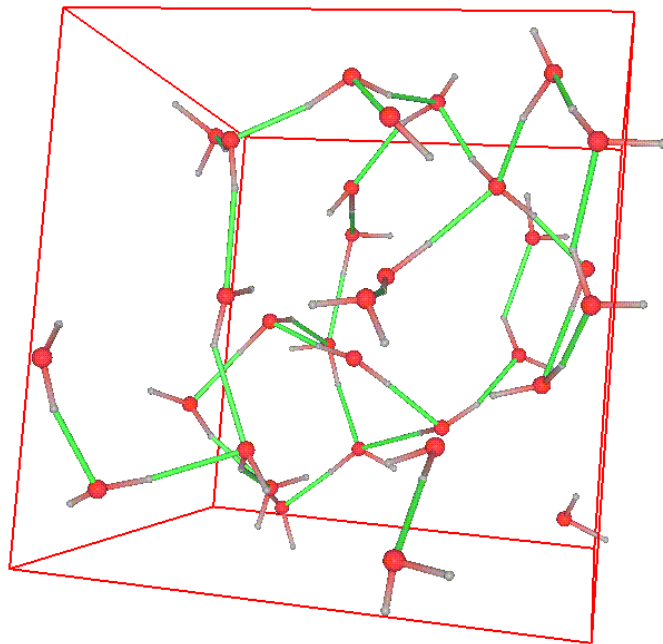
<https://slidetodoc.com/crystal-binding-bonding-part-vi-more-on-van/>

The radial distribution functions of argon
solid ($T = 50 \text{ K}$), liquid ($T = 80 \text{ K}$), and gas ($T = 300 \text{ K}$).



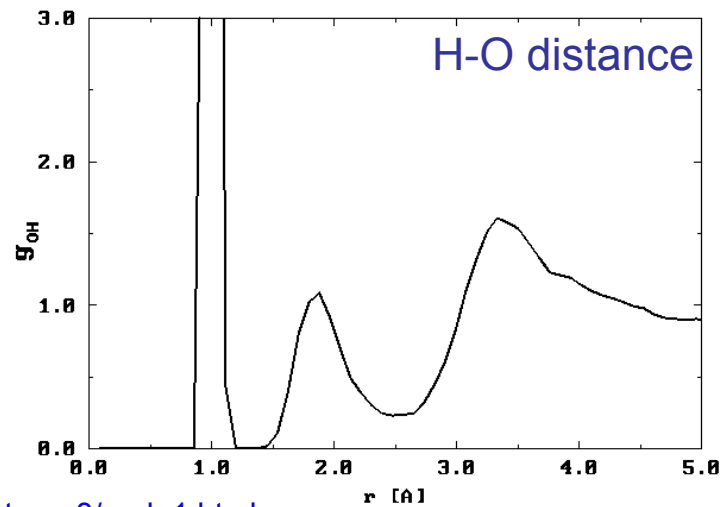
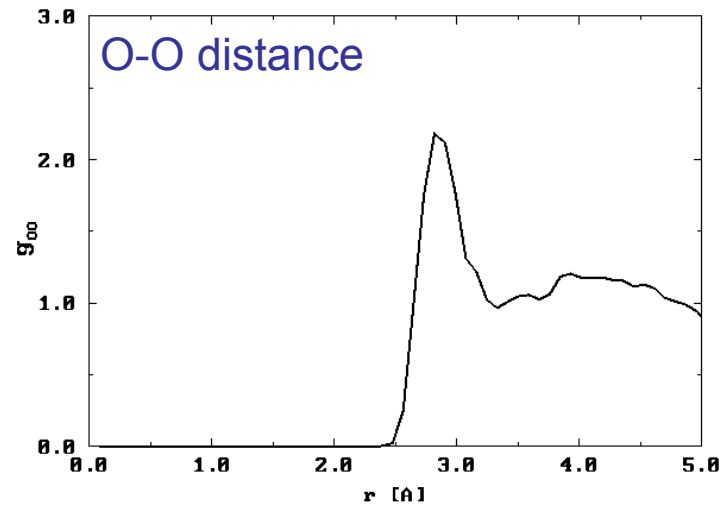
➤ Sharp peaks at regular interval indicate order

RDF of water (H_2O)

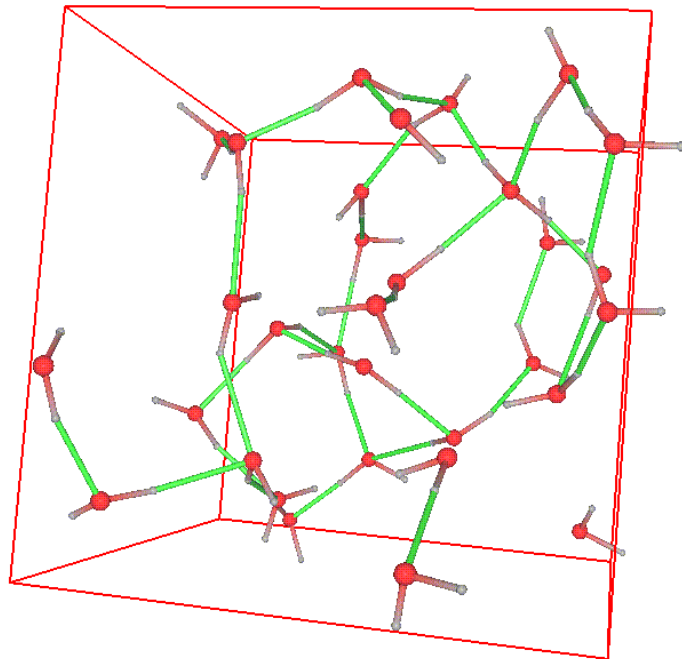
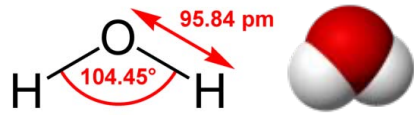


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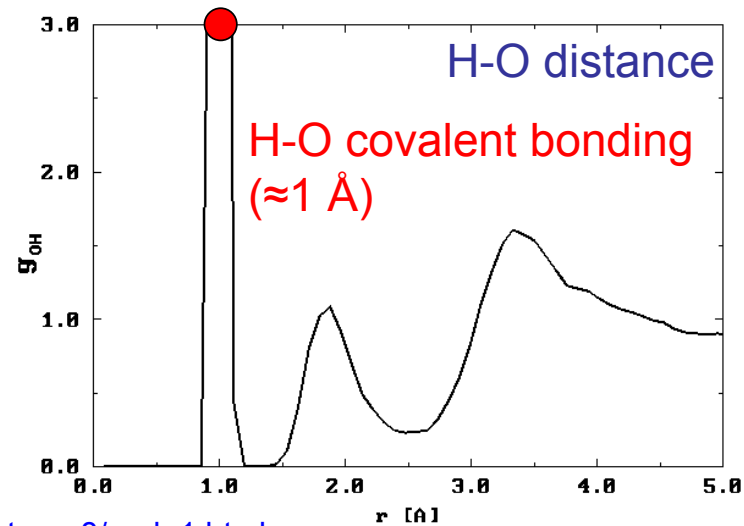
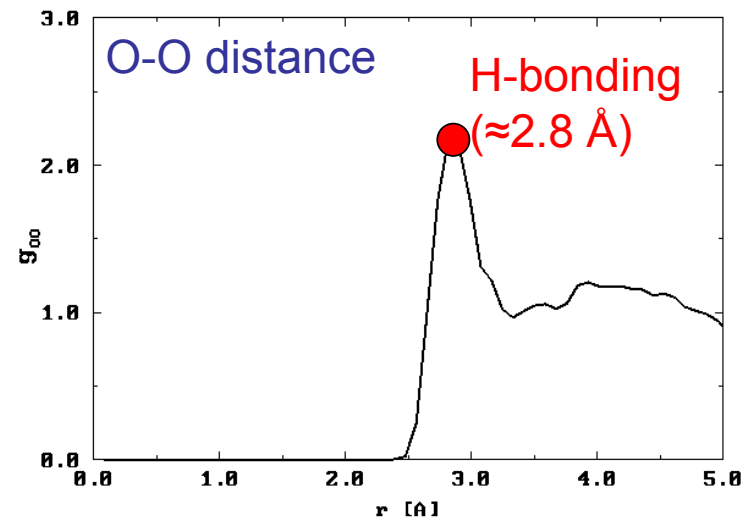
http://www.nyu.edu/classes/tuckerman/stat.mech/lectures/lecture_8/node1.html



RDF of water (H_2O)



Images courtesy of Mark Tuckerman. Used with permission.



**Radial
Distribution
Function, $g(r)$**

Fourier Transform

**Structure
factor, $S(q)$**

**Scattering
Experiment**

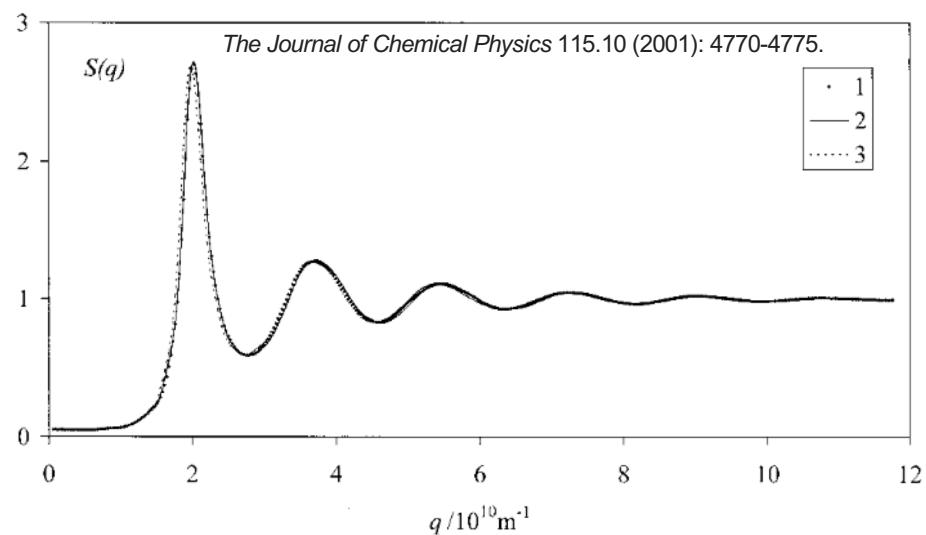


FIG. 4. Structure factor of liquid argon. (1) Experiment, (2) simulation results with our final LJ parameter set, (3) simulation results with the usual LJ parameter set.

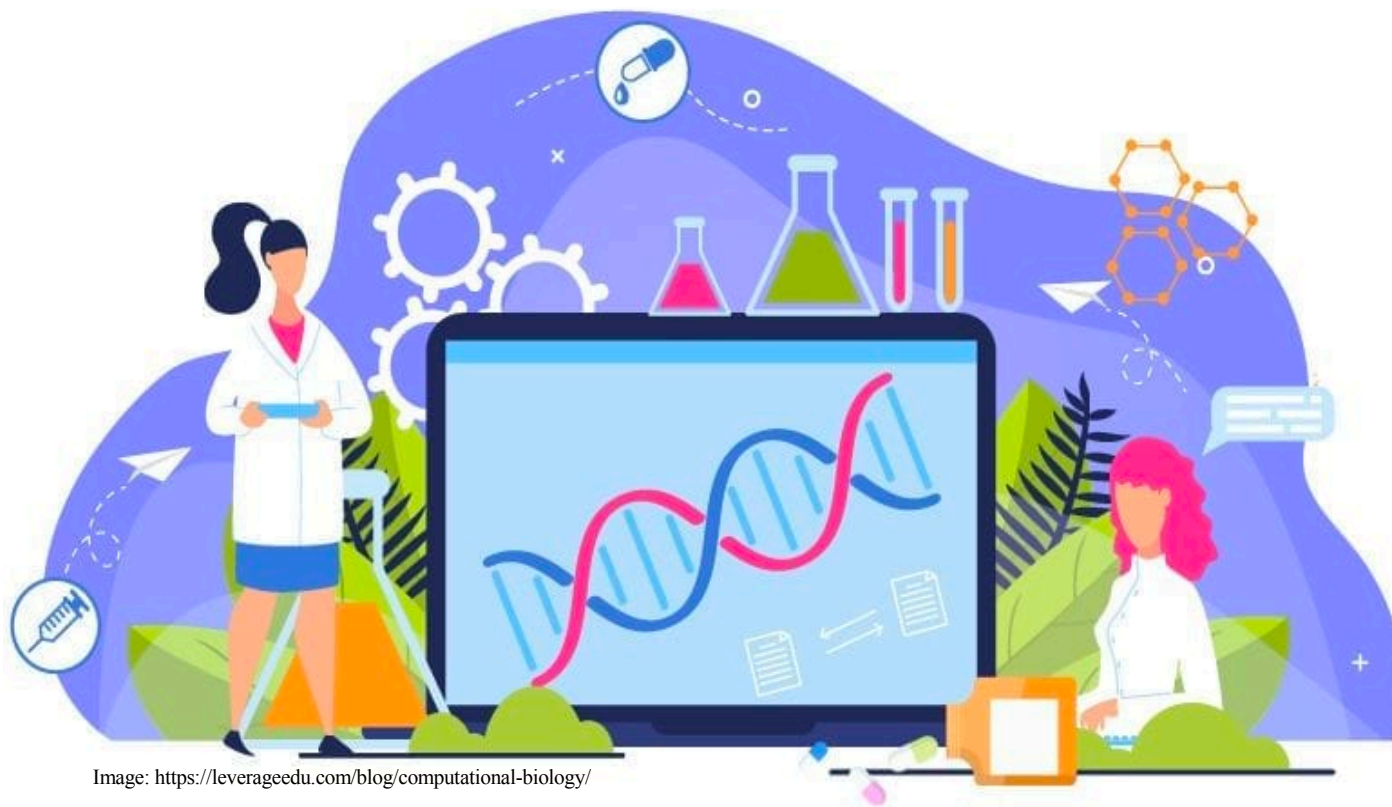


Image: <https://leverageedu.com/blog/computational-biology/>

Next : Connection between MD trajectory and Free Energy