HS239: Economics of Uncertainty and Information

PS 5: 2024

1. In a society, there are two types of people. If Q amount of good is consumed, type I people derive utility $U_1 = 4\ln(Q) - T$, while type II people derive utility $U_2 = 2\ln(Q) - T$. The cost of providing Q amount is $C = 0.5 * Q$. If they do not buy anything, they get a utility of zero. It is known that $\frac{1}{3}$ part of the society are type I people.

a) Suppose the monopolist has identified the types and wish to extract all consumer surplus from them. What should be the quantity and per unit price charged to each group?

b) If the monopolist cannot identify the types, and yet continue to offer the menu in (a), calculate monopolists' profit. Who benefits, and how?

2. A firm hires two kinds of workers: alphas and betas. One cannot distinguish between them a priori. Alpha will produce \$3000 worth of output per month and a beta produces \$2500 worth of output per month. The firm proposes an entrance test to separate alphas from betas. For each question that they get right, alphas spend half an hour to study and betas spend an hour. For either type, an hour of study is as bad as giving up 20\$. A worker will be paid \$3000 if the get at least 40 answers correct and \$2500 otherwise. What is the equilibrium of the scheme?

3. An employer has hired someone to undertake a project. If the project fails, it will lose 20000. If it succeeds, the project would gain \$100000.The employee can work (effort =1) or shirk (effort =0). If he works, the project will succeed only half the time. If he shirks, the project will fail for sure. The agent's utility is \$10000 less if he works. The reservation utility of the agent is \$5000. The employer is choosing between whether to pay the employee a fixed wage of \$20000 or a performance related pay in which the wage is 0\$ if the project fails and 40000\$ if the project succeeds. If both parties are risk neutral, what compensation scheme should the employer use?

4. It is known that some fraction d of all new cars are defective. Defective cars cannot be identified as such except by sellers. Each consumer is risk neutral and values a non defective car at \$16000. New cars sell for \$14000 and used cars sell for \$2000. If cars do not depreciate physically with use (consumers value an defective new car at per with an used car) , what is the proportion d of defective new cars?

5. Professor P hires a teaching assistant , Mr. A. Payoff function of P is $x - s$, where x is the number of hours taught by A and s is the total wage to A. If Mr.A teaches for x hours, his utility is$\left(s - \frac{x^2}{2}\right)$,and the reservation utility is zero.

a) If Professor P chooses x and s to maximize his utility subject to the constraint that Mr. A is willing to work for him, how many hours will Mr. A teach and what will be the payment?

b) Suppose the wage schedule is linear: $s = ax + b$. P chooses a, b, but A chooses x. What values of a and b should Professor choose, given that he cannot directly monitor x?

Answers

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1.*a) In essence, the monopolist should offer a type contingent quantity bundle* Q_i *and type contingent "price" of the bundle* T_i. *If the monopolist knows who is who, he will extract all utility (consumer surplus) from the types. So for type* θ_i, *he will maximize* $T_i - 0.5Q_i$ *such that* $\theta_i \ln(Q_i) - T_i = 0$, *that is* $T_i = \theta_i \ln Q_i$. *So his objective function is* $\theta_i \ln(Q_i) - 0.5Q_i$, *which is maximum at* $Q_i = 2\theta_i$. *Thus.* $T_i = \theta_i \ln(2\theta_i)$. *Putting the value of* θ_i *in both cases, we get*

$$\begin{aligned} Q_1 &= 8, T_1 = 4 * \ln(8.0) = 8.32 \rightarrow \frac{T_1}{Q_1} = \frac{8.32}{8.0} = 1.04 \\ Q_2 &= 4; T_2 = 2 * \ln(4.0) = 2.77 \rightarrow \frac{T_2}{Q_2} = \frac{2.77}{4} = 0.69 \end{aligned}$$

Thus, people with higher demand pays higher unit price.

b) Suppose the monopolist cannot distinguish between the types and continues to offer the menu $\{(8, 8.32), (4, 2.77)\}$*. If type 2 people take type 1 menu their utility is* $2 * \ln(8) - 8.32 = -4.16 < 0$*, so they are not going to take this menu. If type* 1 *people take type 2 menu their utility is* $4 * \ln(4) - 2.77 = 2.78 > 0$*. So they will all take type* 2 *menu and enjoy a surplus of* 2.78*. This group benefits, while the other group remains where they are.*

The monopolist obviously gets lower profit. If he could separate the types as in (a), his expected profit is $0.33(8.32 - 0.5 * 8) + 0.67 * (2.77 - 0.5 * 4) = 1.95$*, but now if types pool together he gets* $0.33(2.77 - 0.5 * 4) + .67(2.77 - 0.5 * 4) = 0.77$.

2. *Let* a *be the number of correctly answered questions. For type* α*, the cost of signaling is* $c_a = \frac{20a}{2} = 20a$*. Similarly, for the* β *type, the cost is* $c_\beta = 20a$*. The wage schedule is*

$$\begin{aligned} w(a) &= 2500, a < 40 \\ &= 3000, a \geq 40 \end{aligned}$$

For the α *type, the payoff function is*

$$\begin{aligned} V_\alpha &= 2500 - 10a, a < 40 \\ &= 3000 - 10a, a \geq 40 \end{aligned}$$

Thus the α *type can choose to answer either* 0 *or* 40 *questions. Notice that* $V_\alpha(40) = 2600 > V_\alpha(0) = 2500$*, hence* α *type will always choose* $a = 40$.

For β *type, the payoff is*

$$\begin{aligned} V_\beta &= 2500 - 20a, a < 40 \\ &= 3000 - 20a, a \geq 40 \end{aligned}$$

So, β type also chooses a to be either zero or 40 Note that $V_\beta(40) = 2200 < V_\beta(0) = 2200$.

So, in equilibrium, α type will choose $a = 40$ and β type will choose $a = 0$; the wages being $(3000, 2500)$ on choice of a. This is one separating equilibrium of the scheme.

3. *Let e be effort, and it has only two values, $e = 0$ and $e = 1$. The probability of success as a function of e is $p(0) = 0$ and $p(e) = 0.5$. The project profit is $\pi_H = 100000$ or $\pi_L = -20000$.Thus, the expected benefit of the project is $E\pi(e) = p(e) * 100000 + (1 - p(e))(-20000)$. Here, $E\pi(1) = 40000 > E\pi(0) = -20000$. So the employer will prefer high effort.*

Suppose the remuneration schedule is $w(\pi)$. Then the risk neutral agents' utility as a function of e is given by

$$v(\pi, e) = p(e)\, w(\pi_H) + (1 - p(e))\, w(\pi_L) - e * 10000$$

If the wages are fixed at \$20000, the agents' utility is $v(\pi, e) = 20000 - e * 10000$, and hence he must choose $e = 0$.

Under performance related pay, the agents' utility is $v(\pi, e) = p(e) * 40000 + (1 - p(e)) * 0 - 10000$. With $e = 1$, $v(\pi, 1) = 0.5 * 40000 - 10000 = 10000$ and with $e = 0$,$v(\pi, 0) = 0$. Note that both ICC $(10000 > 0)$ and PC $(10000 > 5000)$ are satisfied. Thus the worker will take the job offer and maximize work effort.

It remains to check that the employer will also benefit. Note that the employer will maximize the net payoff

$$\mathit{\Pi}(e) = p(e)(\pi_H - w_H) + (1 - p(e))(\pi_L - w_L)$$

Under fixed wage scheme, $e = 0$, $\pi(e) = 0$ and $\pi_L = -2000$. Hence $\Pi(0) = -40000$. Under performance related pay, $e = 1$ and $\Pi(1) = 40000 - 0.5 * 40000 = 20000$. Hence the employer also benefits from performance related payment..

4. *In the market equilibrium, the market price of new cars equals the expected value of new cars. Thus we have* $(1-d)*16000+d*$*(value of a defective new car)=market value of a new car. By the conditions, value of a defective new car=\$2000, hence*$(1-d)*16000+d*2000=14000$. *Solving this, we get* $d=\frac{1}{7}$.

5.a) In this case, Mr P chooses x, s to maximise $(x-s)$ such that $s-\frac{x^2}{2}\geq 0$. Thus, P should choose an x such that $s-\frac{x^2}{2}=0$. Thus, the principal chooses x in such a way that $x-\frac{x^2}{2}$ is maximised. Solving this, one gets $x^*=1$.

b) Given that the wage schedule is linear, the agent chooses his x given a, b. Thus, the agent maximises $s-\dfrac{x^2}{2}=(ax+b)-\dfrac{x^2}{2}$. Maximising this, one gets $x=a$. This, then, becomes the ICC.

Now principal maximises $x-s=a-(ax+b)=a-a^2-b$ such that $s-\frac{x^2}{2}=\frac{a^2}{2}+b=0$. Maximising w.r.t. a and b, one gets $a^*=1$. From the PC, $b^*=-0.5$. Thus the wage schedule is $s=x-0.5$.