EE 657 : Quiz 2

Duration: 50 minutes   Marks: 12   Date: April 11' 2016

Note: The question paper is quite explanatory. As far as possible, no clarifications or discussions on the questions will be entertained!

1. Consider a Support Vector Machine and the training data from two categories:

$$\omega_1 : \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \end{bmatrix} \qquad \omega_2 : \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

   (a) Compute the weight vector that maximizes the margin of separation of the samples of the categories.

   (b) What is the margin of separation.

   (c) What are the support vectors.

[2.5+1+1.5]

2. You are given a data set $\{\mathbf{x}_i, y_i\}_{i=1}^{N}$ of size N. Each input $\mathbf{x}_i$ is d-dimensional and its corresponding target y_i takes one of the 2 values (+1 or -1). The least squares version of the SVM classifier is obtained by reformulating the minimization problem of the traditional problem as:

$$\min_{\mathbf{w},b,\xi} \quad \frac{1}{2}\mathbf{w}^T\mathbf{w} + \frac{C}{2}\sum_{i=1}^{N}\xi_i^2$$

subject to the constraints:

$$y_i(\mathbf{w}^T\mathbf{x}_i + b] = 1 - \xi_i \qquad i = 1, 2, 3.....N$$

Here, C is a penalty factor set by the user. The variable b represents the bias and is a real-valued scalar.

   (a) Write down the Lagrangian Function $L(\mathbf{w}, b, \boldsymbol{\xi}\ )$ corresponding to this optimization problem. Here $\boldsymbol{\xi} = \{\xi_1, \xi_2, ...\xi_N\}$.

   (b) State the Lagrangian conditions, that are to be satisfied at optimality .

[1+2.5]

3. Consider a one-dimensional two-category classification problem with priors, $P(\omega_1) = 2/5$ and $P(\omega_2) = 3/5$. Three i.i.d training observations were collected: $D_1 = \{1, 5, 6\}$ and $D_2 = \{3, 4, 8\}$ for ω_1 and ω_2, respectively. It is desired to classify a test pattern using the Parzen Window technique, discussed in class. Using the window function $\phi(x) = \frac{1}{2}\exp^{-|x|}$, classify the pattern $x = 4.5$. It is known that: the misclassification of samples of ω_1 to ω_2 incurs a loss twice to that of ω_2 to ω_1. Correct classifications are assigned zero loss. Assume , in addition that the loss incurred in misclassifying the pattern to ω_1 is 0.5.

[2]

4. Find the direction $\mathbf{w}$ that maximizes the projection of the line linking the means $\boldsymbol{\mu}_1$ and $\boldsymbol{\mu}_2$ of two categories ω_1 and ω_2. (Use Cauchy Schwartz inequality!)

[1.5]