## Quiz 2

1. You have Bernoulli utility function $v(c) = -e^{-c}$ . Your initial wealth is Rs $c_0$. You can invest $c_1$ of this in an asset that will return to you either double the amount you invested or half the amount you invested, with equal probabilities. The rest, $c_0 - c_1$: does not earn any interest rate. Calculate the elasticity of your optimal choice of $c_1$ with respect to $c_0$. (10)

(**PS3, Q1 with a slight change**)

Ans Key: My optimization exercise is (given $p = .5$)

$$\max_{c_1} 0.5 * v(c_0 + c_1) + 0.5 * v\left(c_0 - \frac{c_1}{2}\right)$$

FOC is

$$\begin{aligned}
v'(c_0 + c_1) &= \frac{1}{2}v'\left(c_0 - \frac{c_1}{2}\right) \\
&\rightarrow \quad e^{-c_0} * e^{-c_1} = \frac{1}{2}e^{-c_0} * e^{\frac{c_1}{2}} \\
&\rightarrow \quad e^{-c_1} = \frac{1}{2}e^{\frac{c_1}{2}} \\
&\rightarrow \quad e^{\frac{3c_1}{2}} = 2 \\
&\rightarrow \quad \frac{3c_1}{2} = \ln 2 \\
&\rightarrow \quad c_1 = \frac{2}{3}\ln 2.0 = 0.46
\end{aligned}$$

Thus, required elasticity

$$\begin{aligned}
\frac{dc_1/c_1}{dc_0/c_0} &= \frac{dc_1}{dc_0} * \frac{c_0}{c_1} \\
&= 0 * \frac{c_0}{0.46} = 0
\end{aligned}$$

2. Discuss the shape of Markowitz's Bullet in case of two risky assets with $\blacksquare = -1$

**(PS 3)**

we have the following facts with $\rho = -1$ (Derivation needed)

- The slopes have different signs at $a = 0$ and $a = 1$.
- Minimum variance portfolio occurs at $(0, \mu_0)$.
- The portfolio line represents a straight line, since both $\mu$ and $\sigma$ varies monotonically with $a$.

Combining the facts, the portfolio curve looks like following (a rocket, rather than a bullet)

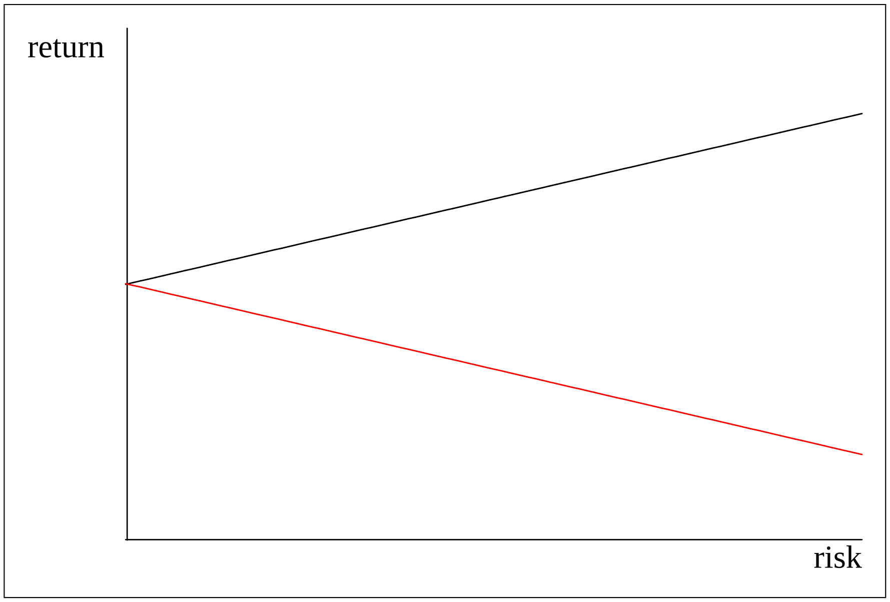


Fig 1: Markowitz's 'Rocket'

3. In a state space approach with two outcomes, explain diagrammatically how we can demonstrate (a) Certainty equivalence (b) risk premium. You may assume a Bernoulli utility function that exhibits positive but diminishing marginal utility.

**Ans Key:** Straight from lecture notes. I expect clarity of the diagram.

**Quiz 3**

The question deals with the model of tax evasion done in class. Assume that $p = 0.25$, $1 - p = 0.75$; $F = 2$, $t = 0.3, Y = 1000$; $U(Y) = \ln(Y)$. Here, $p$ is the probability of detection. What is the amount of tax evaded?

**Ans**

Here,

$$
\begin{aligned}
Y_{nc} &= 1000 - 0.3 * X \\
Y_c &= (1 - t)\, Y - Ft(Y - X) \\
&= .7 * 1000 - 2 * .3 * 1000 + 2 * .3 * X \\
&= 100 + .6X
\end{aligned}
$$

The agent's objective function is

$$\max_X \; .25 * \ln (100 + .6X) + .75 * \ln (1000 - .3X)$$

FOC is

$$\frac{.25 * .6}{100 + .6X} = \frac{.75 * .3}{1000 - .3X}$$

Solving which, we get $X = 708.33$

Thus, the amount of tax evaded $= t(Y - X) = .3 * (1000 - 708.33) = 87.50$

2. MCQ.

a) In a model of sharecropping, higher disturbance in production will create higher share for tenant (True/false)

*False, as the share does not depend on disturbance term* $(\sigma^2)$.

b) The utility function $u = \sqrt{x}$ would exhibit precautionary savings. (True/ False)

*True, as* $u''' = \dfrac{3}{8x^{\frac{5}{2}}} > 0$. *Thus,* $u'$ *is convex.*

c) The utility function , $u = \alpha x - \frac{\beta}{2}x^2$, $\alpha, \beta > 0$ , would exhibit precautionary savings.(True/false)

*False, as* $u''' = 0$

d) In a model of sharecropping, higher disturbance in production will create higher fixed wage or lower rent for tenant. (True/false) .

*True.* A*ssume to contract to tenant is* $sY - R$*,* $R > 0$ *implying rent.. In equilibrium,* $\bar{U} = se - R - \frac{\beta_T}{2} s^2 \sigma^2 - ce^2$ *If everything is optimally set up, higher* $\sigma^2$ *means* $R$ *to fall (becomes a lower positive value or higher negative value*)

e) Higher tax rates would induce more tax compliance if the utility function is CARA/DARA/IARA. (Tick)

*DARA*

f) In the model of tax evasion with a constant moral cost of hiding income, the proportion of people under-declaring their income will go up with higher income. (T/F)

False. *Suppose the moral cost is* $\phi$*. Then the agent does not evade if* $\phi > k_0 = t(1 - p - Fp) * u'((1-t)Y)$*. We have* $\frac{dk_0}{dY} = t(1 - p - Fp) * (1 - t) * u''((1-t)Y) < 0$*. Therefore, the percentage of people who declare their income goes up, underdeclaration goes down.*

g) In a standard model of tax evasion, suppose $F = 2.5$ and $p$ (audit probability) is $p = .25$. One must *evade/not-evade*

*For evasion ,* $p(1 + F) < 1$. Here, $p(1 + F) = .25 * (1 + 2.5) = 0.875$ *Therefore everybody will evade.*

h) In a model of sharecropping, if the tenant becomes more risk averse, then she demands higher/lower share of output (Tick).

*Lower share, as with increased share, the risk increases.*

i) A gentleman has CARA utility function. His income is $Y$ and under-declaration is $E = Y - X$. The ratio $E/Y$, with increasing income, will (a) stays constant, (b) increase, (c) decrease. (Tick)

*With CARA,* $E = Y - X$ does not change as $Y$ increases. *Thus,* $\frac{E}{Y} = 1 - \frac{X}{Y}$ *will decrease as* $Y$ *increases.*

j) For a tax administrator, which policy is more easy to adopt? (a) increasing audit probability, (b) increasing fine rate, (c) creating more jails. (Tick)

*Increasing fine rate.*