EE 626 : Quiz 1

Duration: 45 minutes **Date: Feb 11, 2025** **Marks:15**

1. Consider a one-dimensional two-category classification problem with priors, $P(\omega_1) = 3/5$ and $P(\omega_2) = 2/5$, where the continuous class conditional densities have the form

$$p(x|\omega_i) = \frac{x}{\theta_i^2} e^{\frac{-x^2}{2\theta_i^2}} \qquad x \geq 0$$

The parameters θ_i for $i = 1, 2$ are positive but unknown. Assume zero one loss function,

(i) The following three i.i.d training observations were collected: $\mathbb{D}_1 = \{1, 2\sqrt{2}, 3\}$ and $\mathbb{D}_2 = \{3, \sqrt{11}, 4\}$ for ω_1 and ω_2, respectively. Using this information, find the maximum-likelihood values for θ_1 and θ_2.

(ii) Based on your answer to part (i), determine the Bayes threshold (decision boundary) x^*.

(iii) Compute the average probability of a pattern to be classified to ω_1.

[4 + 3 + 4 = 11 marks]

2. In a two class problem, the feature vectors $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ in each class are normally distributed with identical covariance matrix : $\mathbf{\Sigma} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$. The respective mean vectors $\boldsymbol{\mu}_1$ and $\boldsymbol{\mu}_2$ (for class ω_1 and ω_2) are $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$. Assuming that the class ω_1 is probable with 0.65, classify the feature vector $\begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$, according to the Bayes minimum probability classifier, by specifying the posterior probability.

[4 marks]

The normal distribution $N(\boldsymbol{\mu}, \mathbf{\Sigma})$ in d-dimensions is given by

$$p(\mathbf{x}) = \frac{1}{(2\pi)^{d/2}|\mathbf{\Sigma}|^{0.5}} e^{-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})^T\mathbf{\Sigma}^{-1}(\mathbf{x}-\boldsymbol{\mu})}$$