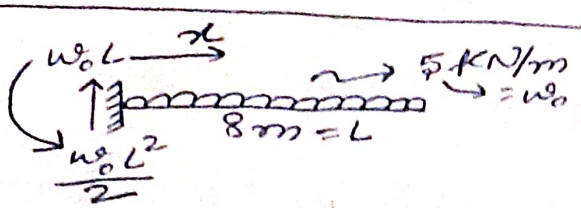
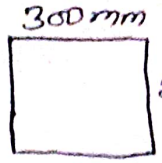
AMAN RAJ – 21010401-6 – CE616 (Assignment – 2)

1.





$E = 200\ GPa$.

$$I = \frac{300 \times (500)^3}{12} = 3.125 \times 10^9\ mm^4.$$

$$-EI\frac{d^2y}{dx^2} = -\frac{w_0 L^2}{2} + w_0 L x - \frac{w_0 x^2}{2}$$

$$\Rightarrow EI\frac{dy}{dx} = \frac{w_0 L^2}{2}x - \frac{w_0 L x^2}{2} + \frac{w_0 x^2}{6} + C_1$$

$$\Rightarrow EI\,y(x) = \frac{w_0 L^2 x^2}{4} - \frac{w_0 L x^3}{6} + \frac{w_0 x^4}{24} + C_1 x + C_2$$

$$y(0) = 0 \quad \& \quad \left.\frac{dy}{dx}\right)_{x=0} = 0.$$

$$\Rightarrow \boxed{C_1 = 0} \quad \& \quad \boxed{C_2 = 0}$$

$$\Rightarrow \boxed{y(x) = \frac{w_0}{EI}\left[\frac{x^4}{24} - \frac{Lx^3}{6} + \frac{L^2x^2}{4}\right]}$$

$$\Rightarrow \phi(x) = \frac{y(x)}{y(L)} = \frac{\frac{x^4}{24} - \frac{Lx^3}{6} + \frac{L^2x^2}{4}}{L^4\left[\frac{1}{24} - \frac{1}{6} + \frac{1}{4}\right]}$$

$$\Rightarrow \boxed{\phi(x) = \frac{1}{3L^4}\left[x^4 - 4Lx^3 + 6L^2x^2\right]}$$

$$\Rightarrow m^* = \int_0^L m(x)\,[\phi(x)]^2\,dx$$

$$= \frac{\rho A}{9L^8}\int_0^L \left(x^8 + 16L^2x^6 + 36L^4x^4 - 8Lx^7 - 48L^3x^5 + 12L^2x^6\right)dx$$

$$= \frac{\rho AL}{9}\left[\frac{1}{9} + \frac{16}{7} + \frac{36}{5} - \frac{8}{8} - \frac{48}{6} + \frac{12}{7}\right]$$

$$\Rightarrow \boxed{m^* = 0.257\,\rho AL}\ \text{Ans.}$$

$$\Rightarrow \boxed{K_M = \frac{m^*}{m} = 0.257}\ \text{Ans.}$$

$$k^* = \int_0^L E(x)\, I(x)\, [\phi''(x)]^2\, dx$$

$$= \frac{EI}{9L^8} \int_0^L (12x^2 - 24Lx + 12L^2)^2\, dx$$

$$= \frac{EI}{9L^8} \int_0^L (144x^4 + 576L^2x^2 + 144L^4 + 288x^2L^2 - 576Lx^2 - 576L^3x)\, dx$$

$$= \frac{EI}{9L^3} \left[\frac{144}{5} + \frac{576}{3} + 144 + \frac{288}{3} - \frac{576}{4} - \frac{576}{2} \right]$$

$$\Rightarrow \boxed{k^* = \frac{3.2\,EI}{L^3} \approx \frac{3EI}{L^3}}$$

$$F^*(x) = \int_0^L F(x)\, \phi(x)\, dx$$

$$= \int_0^L \frac{w_0}{3L^4} (x^4 - 4Lx^3 + 6L^2x^2)\, dx$$

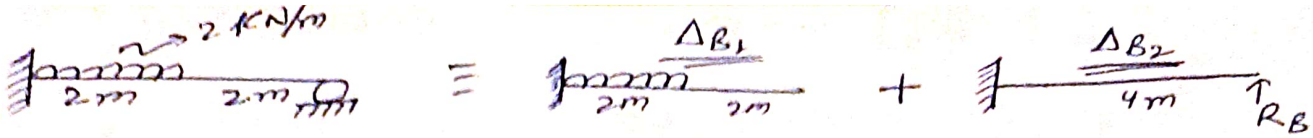
$$= \frac{w_0 L}{3} \left(\frac{1}{5} - \frac{4}{4} + \frac{6}{3} \right)$$

$$\Rightarrow \boxed{F^*(x) = 0.4\, w_0 L}$$

$$\Rightarrow \boxed{K_L = \frac{F^*}{F} = 0.4}$$

$$\Rightarrow \boxed{K_{LM} = \frac{K_M}{K_L} = \frac{0.257}{0.4} = 0.643}$$

2.



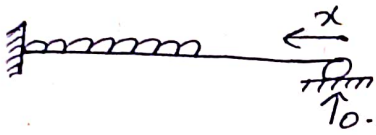
$$\Delta_{B_1} = \int_0^2 \frac{-(2x \times \frac{x}{2})(x+2)\,dx}{EI} = \frac{-1}{EI}\int_0^2 (x^3+2x^2)\,dx$$

$$= \frac{-1}{EI}\left[\frac{2^4}{4} + 2\times\frac{2^3}{3}\right] = -\frac{28}{3EI}.$$

$$\& \quad \Delta_{B_2} = \frac{R_B \times 4^3}{3EI} = \frac{64 R_B}{3EI}.$$

$$\Delta_{B_1} + \Delta_{B_2} = 0 \Rightarrow \boxed{R_B = 0.44\ KN}$$

⇒



$$\Rightarrow -EI\frac{d^2y}{dx^2} = 0.44x - \langle x-2\rangle \cdot \frac{\langle x-2\rangle}{1}$$

$$\Rightarrow EI\frac{d^2y}{dx^2} = \langle x-2\rangle^2 - 0.44x$$

$$\Rightarrow EI\frac{dy}{dx} = \frac{\langle x-2\rangle^3}{3} - 0.22x^2 + C_1$$

$$\Rightarrow EI\,y(x) = \frac{\langle x-2\rangle^4}{12} - \frac{0.22}{3}x^3 + C_1 x + C_2$$

$$y(4) = 0 \quad \& \quad y'(4) = 0$$

$$\Rightarrow \frac{8}{3} - (0.22\times 16) + C_1 = 0 \Rightarrow \boxed{C_1 = 0.85}$$

$$\& \quad \frac{16}{12} - \frac{0.22}{3}\times 64 + 4\times 0.85 + C_2 = 0$$

$$\Rightarrow \boxed{C_2 = -0.05}$$

$$\Rightarrow \boxed{y(x) = \frac{1}{EI}\left[\frac{\langle x-2\rangle^4}{12} - \frac{0.22}{3}x^3 + 0.85x - 0.05\right]}$$

$y_{max.} \Rightarrow y'(x) = 0.$

$x > 2 \Rightarrow \dfrac{(x^3 - 8 - 6x^2 + 12x)}{3} - 0.22x^2 + 0.85 = 0$

$\Rightarrow x = 4\,m$ (α), $0.69\,m$ (α), $1.97\,m$ (α)

$x < 2 \Rightarrow -0.22x^2 + 0.85 = 0 \Rightarrow x = 1.97\,m$ ✓

$$\Rightarrow \phi(x) = \frac{\langle x-2\rangle^4}{12} - \frac{0.22x^3}{3} + 0.85x - 0.05$$

$$y(1.97) = 1.064$$

$$\Rightarrow \boxed{\phi(x) = 0.078\langle x-2\rangle^4 - 0.069x^3 + 0.799x - 0.047}$$

$$\Rightarrow m^* = \rho A \int_0^4 [\phi(x)]^2 dx$$

$$= \rho A \left[\int_0^2 (-0.069x^3 + 0.799x - 0.047)^2 dx + \int_2^4 (0.078(x-2)^4 - 0.069x^3 + 0.799x - 0.047)^2 dx \right]$$

$$= (0.81 + 0.964)\rho A = \left(\frac{1.774}{4}\right)\rho A L$$

$$\Rightarrow \boxed{m^* = 0.443\,\rho A L}$$ Ans. $\Rightarrow \boxed{K_M = 0.443}$ Ans.

$$K^* = EI \int_0^4 [\phi''(x)]^2 dx$$

$$= EI\left[\int_0^2 (-0.069\times 3\times 2\times x)^2 dx + \int_2^4 (0.078\times 4\times 3\langle x-2\rangle^2 - 0.069\times 3\times 2x)^2 dx\right]$$

$$= EI[0.457 + 1.573] = \frac{EI}{L^3}(129.92)$$

$$\Rightarrow \boxed{K^* = \frac{130\,EI}{L^3}}$$ Ans.

$$F^*(x) = \int_0^4 F(x)\phi(x)\,dx = \int_0^2 (0)\cdot\phi(x)\,dx + \int_2^4 w_0\,\phi(x)\,dx$$

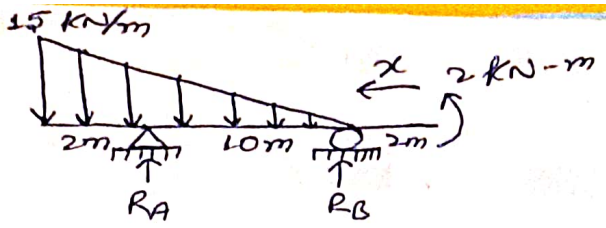
$$= \int_2^4 w_0 [0.078(x-2)^4 - 0.069x^3 + 0.799x - 0.047]\,dx$$

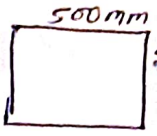
$$= 1.0592\,w_0 = \boxed{0.265\,w_0 L = F^*(x)}$$ Ans.

$$\Rightarrow \boxed{K_L = 0.265}$$ Ans.

$$\Rightarrow \boxed{K_{LM} = \frac{K_L}{K_M} = 0.5977}$$ Ans.

3.





$I = \frac{500 \times 500^3}{12}\ mm^4$

$= 5.21 \times 10^9\ mm^4$

$E = 5000\sqrt{f_{ck}} = 5000\sqrt{50}$

$= 35355\ MPa$

$\Sigma M_A = 0 \Rightarrow (90 \times 2) - 10R_B - 2 = 0 \Rightarrow \boxed{R_B = 17.8\ KN}$

$\Rightarrow \boxed{R_A = 72.2\ KN}$

$$\Rightarrow -EI\frac{d^2y}{dx^2} = 2 + 17.8\langle x-2\rangle - \frac{1}{2}\times\langle x-2\rangle^2 \times 1.25 \times \frac{\langle x-2\rangle}{3} + 72.2\langle x-12\rangle$$

$$\Rightarrow EI\frac{dy}{dx} = -2x - 8.9\langle x-2\rangle^2 + \frac{\langle x-2\rangle^4}{24}\times 1.25 - \frac{72.2}{2}\langle x-12\rangle^2 + C_1$$

$$\Rightarrow EI\,y(x) = -x^2 - \frac{8.9}{3}\langle x-2\rangle^3 + \frac{\langle x-2\rangle^5}{96} - \frac{72.2}{6}\langle x-12\rangle^3 + C_1x + C_2$$

$y(2) = 0$ & $y(12) = 0$.

$\Rightarrow -4 + 2C_1 + C_2 = 0 \Rightarrow \boxed{2C_1 + C_2 = 4}$

& $-144 - \frac{8.9}{3}\times 1000 + \frac{10^5}{96} + 12C_1 + C_2 = 0$

$\Rightarrow \boxed{12C_1 + C_2 = 2069}$

$\Rightarrow C_1 = 206.5$ & $C_2 = -409$

$$\Rightarrow \boxed{y(x) = \frac{1}{EI}\left[\frac{\langle x-2\rangle^5}{96} - \frac{8.9}{3}\langle x-2\rangle^3 - \frac{72.2}{6}\langle x-12\rangle^3 - x^2 + 206.5x - 409\right]}$$

$\hookrightarrow y_{max.} = y(7.04) = 649.27$

$$\Rightarrow \boxed{\phi(x) = 10^{-5}\left[1.6\langle x-2\rangle^5 - 456.9\langle x-2\rangle^3 - 1853.36\langle x-12\rangle^3 - 154.02x^2 + 31805x - 62994\right]}$$

$$\rightarrow m^* = \rho A \times 10^{-10}\left[\int_0^2 (-154.02x^2 + 31805x - 62994)^2\,dx + \int_2^{12}(1.6(x-2)^5 - 456.9(x-2)^3 - 154.02x^2 + 31805x - 62994)^2\,dx + \int_{12}^{14}(1.6(x-2)^5 - 456.9(x-2)^3 - 1853.36(x-12)^3 - 154.02x^2 + 31805x - 62994)^2\,dx\right]$$

$$\rightarrow m^* = \left(\frac{0.263 + 4.96 + 0.2}{14}\right) \rho A L$$

$$\rightarrow \boxed{m^* = 0.387\ \rho A L} \quad \rightarrow \boxed{K_M = 0.387}$$

$$\rightarrow K^* = EI \int_0^{14} [\phi''(x)]^2 dx$$

$$= EI \times 10^{-10} \left[\int_0^2 (-308.04)^2 dx + \int_2^{12} (-308.04 + 32(x-2)^3 - 2741.4(x-2))^2 dx + \int_{12}^{14} (-308.04 + 32(x-2)^3 - 2741.4(x-2) - 11120.16(x-12))^2 dx \right]$$

$$= \frac{EI}{L^3} \left(0.19 \times 10^{-4} + 7.2 \times 10^{-4} + 495.1 \times 10^{-4}\right) \times 14^3$$

$$\rightarrow \boxed{K^* = 137.88 \frac{EI}{L^3}}$$

$$\rightarrow F^*(x) = \int_0^{14} F(x)\, \phi(x)\, dx$$

$$= 10^{-5} \int_2^{12} [1.25(x-2)] [1.6(x-2)^5 - 456.9(x-2)^3 - 154.02x^2 + 31805x - 62994]\, dx$$

$$+ 10^{-5} \int_{12}^{14} [1.25(x-2)] [1.6(x-2)^5 - 456.9(x-2)^3 - 154.02x^2 + 31805x - 62994 - 1853.36(x-12)^3]\, dx$$

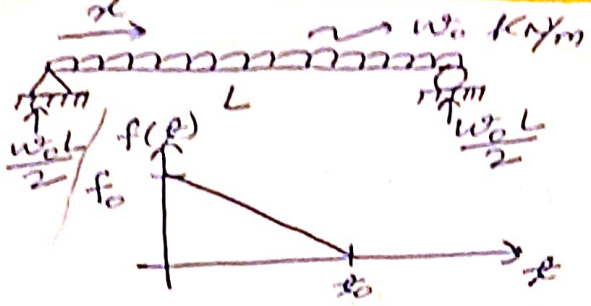
$$= (-783414 + 3948708.69) \times 10^{-5} = 3.17 \times 10^6 \times 10^{-5}$$

$$= \left(\frac{3.17 \times 10^0}{90}\right) w_0 L$$

$$\rightarrow \boxed{F^*(x) = 0.352\ w_0 L} \quad \rightarrow \boxed{K_L = 0.352}$$

$$\rightarrow \boxed{K_{LM} = \frac{K_L}{K_M} = \frac{0.352}{0.387} = 0.91}$$

4.



$E = 200$ GPa

$I = \frac{\pi d^4}{64}$

Let $w_0(x) = f_0$ (max.)

$$-EI\frac{d^2y}{dx^2} = \frac{w_0 L}{2}x - \frac{w_0 x^2}{2}$$

$$\Rightarrow EI\frac{dy}{dx} = \frac{w_0 x^3}{6} - \frac{w_0 L x^2}{4} + C_1$$

$$\Rightarrow EI\,y(x) = \frac{w_0 x^4}{24} - \frac{w_0 L x^3}{12} + C_1 x + C_2$$

$y(0) = 0$ & $y(L) = 0$

$\Rightarrow \boxed{C_2 = 0}$ & $\boxed{C_1 = -\frac{w_0 L^3}{24} + \frac{w_0 L^3}{12} = \frac{w_0 L^3}{24}}$

$$\Rightarrow \boxed{y(x) = \frac{w_0}{EI}\left[\frac{x^4}{24} - \frac{L x^3}{12} + \frac{L^3 x}{24}\right]}$$

$$\Rightarrow \phi(x) = \frac{\frac{x^4}{24} - \frac{L x^3}{12} + \frac{L^3 x}{24}}{L^4\left[\frac{1}{24\times16} - \frac{1}{12\times8} + \frac{1}{24\times2}\right]}$$

$$\Rightarrow \boxed{\phi(x) = \frac{16}{5L^4}\left[x^4 - 2Lx^3 + L^3 x\right]}$$

$$\Rightarrow m^* = \int_0^L m\,[\phi(x)]^2\,dx$$

$$= \frac{\rho A \times 256}{25 L^8}\int_0^L (x^8 + 4L^2x^6 + L^6x^2 - 4Lx^7 - 4L^4x^4 + 2L^3x^5)\,dx$$

$$= \rho AL \times \frac{256}{25}\left[\frac{1}{9} + \frac{4}{7} + \frac{1}{3} - \frac{4}{8} - \frac{4}{5} + \frac{2}{6}\right]$$

$$\Rightarrow \boxed{m^* = 0.504\,\rho AL}$$ Ans.

where, $\rho = 7850\ Kg/m^3$, $A = \frac{\pi d^2}{4}$

$$\Rightarrow \boxed{K_m = \frac{m^*}{m} = 0.504}$$ Ans.

$$K^* = EI \int_0^L [\phi''(x)]^2 dx = EI \int_0^L (12x^2 - 12Lx)^2 \times \frac{256}{25L^8} dx$$

$$= \frac{256\,EI}{25L^8} \int_0^L (144x^4 + 144L^2x^2 - 288Lx^3) dx$$

$$= \frac{256\,EI}{25L^3} \left[\frac{144}{5} + \frac{144}{3} - \frac{288}{4} \right]$$

$$\Rightarrow \boxed{K^* = \frac{49.15\,EI}{L^3} \approx \frac{48\,EI}{L^3}} \text{ Ans.}$$

$$F^*(x) = \int_0^L F(x)\,\phi(x)\,dx$$

$$= w_0 \times \frac{16}{5L^4} \int_0^L (x^4 - 2Lx^3 + L^3x)\,dx$$

$$= \frac{16}{5} w_0 L \left[\frac{1}{5} - \frac{2}{4} + \frac{1}{2} \right] = 0.64\,w_0 L$$

$$\Rightarrow \boxed{F^* = 0.64\,f_0 L} \text{ Ans.}$$

$$\Rightarrow \boxed{K_L = 0.64} \text{ Ans.}$$

$$\& \quad \boxed{K_{LM} = \frac{K_L}{K_M} = 0.127} \text{ Ans.}$$