structural design. The following idealized discussion is presented to develop a parallel between the deterministic approaches in terms of the capacity reduction factor and load factors with risk-based design. Referring to Figure 7.1, where the uncertainties in the load and resistance variables are expressed in the form of the probability density functions, we can express the measure of risk in terms of the probability of the *failure event* or $P(R < S)$ as:

$$\begin{aligned} p_f &= P(\text{failure}) = P(R < S) \\ &= \int_0^{\infty}\left[\int_0^{s} f_R(r)dr\right]f_S(s)ds \qquad (7.2) \\ &= \int_0^{\infty} F_R(s)f_S(s)ds \end{aligned}$$

where $F_R(s)$ is the CDF of R evaluated at s. Equation 7.2 states that when the load is $S = s$, the probability of failure is $F_R(s)$, and since the load is a random variable, the integration needs to be carried out for all the possible values of S, with their respective likelihoods represented by the PDF of S. Equation 7.2 can be considered to be the basic equation of the risk-based design concept. The CDF of R or the PDF of S may not always be available in explicit form, and thus the integration of Equation 7.2 may not be practical. However, Equation 7.2 can be evaluated easily, without performing the integration, for some special cases. They are considered first in the following sections.

7.4.1 Load and Resistance Normal Variables: Single Load Case

Consider a structure with resistance R subjected to a single load S. The structure is subjected to one load at a time (i.e, dead load alone, live load alone, wind load alone, seismic load alone, etc.). If both R and S are normal variables, that is, $N(\mu_R, \sigma_R)$ and $N(\mu_S, \sigma_S)$, then another random variable Z can be introduced as

$$Z = R - S. \qquad (7.3)$$

Since it is quite reasonable to assume that R and S are statistically independent, based on the discussion in Section 6.3.1.2, we can infer that Z is also a normal random variable, that is, $N(\mu_R - \mu_S, \sqrt{\sigma_R^2 + \sigma_S^2})$. Then, Equation 7.2 can be used to define the probability of failure as

$$p_f = P(Z < 0)$$

or

$$p_f = \Phi\left[\frac{0 - (\mu_R - \mu_S)}{\sqrt{\sigma_R^2 + \sigma_S^2}}\right]$$

or

$$p_f = 1 - \Phi\left[\frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}}\right] \qquad (7.4)$$

where Φ is the CDF of the standard normal variate. To develop the explicit expression for the risk-based design format, Equation 7.4 can be rewritten as

$$\mu_R \geq \mu_S + \Phi^{-1}(1 - p_f)\sqrt{\sigma_R^2 + \sigma_S^2} \qquad (7.5)$$

where $\Phi^{-1}(1 - p_f)$ is the value of the standard normal variate at the probability level $(1 - p_f)$. Introducing $\beta = \Phi^{-1}(1 - p_f)$, and considering the equality condition, we can rewrite Equation 7.5 as

$$\mu_R = \mu_S + \beta\sqrt{\sigma_R^2 + \sigma_S^2}. \tag{7.6}$$

From Equation 7.4,

$$\beta = \Phi^{-1}(1 - p_f) = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}}. \tag{7.7}$$

If β is large, p_f will be small, implying that the underlying risk is small.

To eliminate the square root sign to separate R and S in Equation 7.6, a parameter ε can be introduced as

$$\varepsilon = \frac{\sqrt{\sigma_R^2 + \sigma_S^2}}{\sigma_R + \sigma_S}. \tag{7.8}$$

ε can be considered to be approximately 0.75 in most cases. Substituting Equation 7.8 into Equation 7.7, one obtains

$$\beta = \frac{\mu_R - \mu_S}{\varepsilon(\sigma_R + \sigma_S)}. \tag{7.9a}$$

After the variables are separated, the equation becomes

$$\mu_R - \varepsilon\beta\sigma_R = \mu_S + \varepsilon\beta\sigma_S$$

or

$$(1 - \varepsilon\beta\delta_R)\mu_R = (1 + \varepsilon\beta\delta_S)\mu_S \tag{7.9b}$$

where

$$\delta_R = \frac{\sigma_R}{\mu_R} \quad \text{and} \quad \delta_S = \frac{\sigma_S}{\mu_S}.$$

Before considering the nominal safety factor, using Equation 7.9b, we can introduce the concept of the *central safety factor* by taking the ratio of the mean values of the load and resistance (refer to Figure 7.1):

$$\bar{\zeta} = \frac{\mu_R}{\mu_S} = \frac{1 + \varepsilon\beta\delta_S}{1 - \varepsilon\beta\delta_R}. \tag{7.10}$$

Referring to Equation 7.9b, we can express the capacity reduction factor corresponding to the central safety factor as

$$\bar{\phi} = 1 - \varepsilon\beta\delta_R. \tag{7.11a}$$

The corresponding load factor can be shown to be:

$$\bar{\gamma} = 1 + \varepsilon\beta\delta_S. \tag{7.11b}$$

To define the nominal safety factor, the nominal or characteristic values of the load and resistance need to be introduced as

$$R_N = \mu_R(1 - k_R\delta_R) \tag{7.12a}$$

$$S_N = \mu_S(1 + k_S\delta_S). \tag{7.12b}$$

Again, the nominal value of R is k_R standard deviations below the mean and the nominal value of S is k_S standard deviations above the mean, as shown in Figure 7.1. The conventional or nominal safety factor, as in Equation 7.1, becomes

$$\zeta = \frac{R_N}{S_N} = \frac{\mu_R}{\mu_S} \bullet \frac{(1 - k_R\delta_R)}{(1 + k_S\delta_S)} = \left(\frac{1 + \varepsilon\beta\delta_S}{1 + k_S\delta_S}\right)\left(\frac{1 - k_R\delta_R}{1 - \varepsilon\beta\delta_R}\right). \tag{7.13}$$

Thus, after the variables are separated, the nominal capacity reduction factor and the load factor, in terms of $\phi R_N \geq \gamma_{S_N}$, can be shown to be

$$\phi = \frac{1 - \varepsilon\beta\delta_R}{1 - k_R\delta_R} \tag{7.14a}$$

$$\gamma = \frac{1+\varepsilon\beta\delta_S}{1+k_S\delta_S}. \tag{7.14b}$$

It is clear from these two equations that the probability-based capacity reduction factor and load factor convey more information than the corresponding deterministic factors. Both of them depend on four factors: ε, β, δ_R or δ_S, and k_R or k_S. Equation 7.14a indicates that for normal engineering design when ($\varepsilon\beta$) is expected to be greater than k_R, the capacity reduction factor ϕ will be less than one, which is commonly assumed in a deterministic design. It also indicates that the capacity reduction factor is a function of β, representing the acceptable risk for the structure being considered. If β is large, implying that the acceptable risk is small, the capacity reduction factor is expected to be small given that the other parameters remain the same. For small acceptable risk, more conservatism is introduced in estimating the resistance by using a low ϕ factor. The capacity reduction factor also depends on the uncertainty in the resistance, δ_R. It can be shown that if the uncertainty in R is large, the ϕ factor will be smaller. It depends on how conservatively the nominal resistance value, represented by the parameter k_R, is selected. If k_R is large, indicating that the nominal resistance was selected very conservatively, then the ϕ factor will approach unity, as expected. The capacity reduction factor is also a function of ε, representing the uncertainty in both the resistance and load variables.

The load factor for normal engineering design ($\varepsilon\beta > k_S$), represented by Equation 7.14b, is expected to be greater than one. As stated earlier, its value depends on four factors. The load factor depends on the uncertainty in the load under consideration. If different loads have different amounts of uncertainty, the load factors are expected to be different while all the other parameters remain the same. The uncertainty in the dead load is expected to be smaller than that in the live load. Thus, the load factor for the dead load will be smaller than that of the live load. If β is large, implying that the acceptable risk for the project is small, the corresponding load factor will also be large. If the design load is selected very conservatively, implying that k_S is large, then the load factor will approach unity, as expected. If ε is large, the load factor will also be large.

In deterministic design, the capacity reduction factor and load factors are determined subjectively based on judgment, intuition, and experience; in probabilistic designs, they can be estimated explicitly project by project considering the specific conditions, giving more control to the design engineers.

7.4.2 Load and Resistance Normal Variables: Multiple Load Case

In engineering design, components need to be designed to meet the maximum demand considering all possible loads that may act on them during their lifetime. Considering one load at a time may not be sufficient; the likely combinations of loads need to be considered. It is extremely unlikely that all possible loads will act simultaneously on a structure, but some of them will act together. For example, in a typical structural design, some of the common load combinations are dead plus live loads, dead plus live plus wind loads, dead plus live plus seismic loads, and dead plus wind loads. Thus, it is essential that multiple load effects be considered when estimating load and resistance factors.

To consider the effect of multiple loadings, S can be represented as $S = S_1 + S_2 + ... + S_n$. An obvious choice in this case is to combine the multiple load effects into one, as was discussed extensively in Chapter 6. Since S is a linear function of other random variables, its mean and variance can be estimated using Equations 6.32 and 6.33 or 6.34. Then, assuming S is a normal random variable and using the discussion in the previous section, we can estimate the resistance and load factors. The load factor will give a composite value considering the effect of loads in that particular combination; it will not give the individual load factors.

To estimate the individual load factors, the following procedure can be followed. For the multiple loads case, assuming $S_1, S_2, \ldots, S_n$ are statistically independent as in Equation 6.34, Equation 7.6 can be rewritten as:

$$\mu_R = (\mu_{S_1} + \mu_{S_2} + \ldots + \mu_{S_n}) + \beta\sqrt{\sigma_R^2 + \left(\sigma_{S_1}^2 + \sigma_{S_2}^2 + \ldots + \sigma_{S_n}^2\right)}.$$

Introducing the parameter ε, similar to Equation 7.8, results in

$$\begin{aligned}\mu_R &= (\mu_{S_1} + \mu_{S_2} + \ldots + \mu_{S_n}) \\ &+ \varepsilon\beta\left(\sigma_R + \sqrt{\sigma_{S_1}^2 + \sigma_{S_2}^2 + \ldots + \sigma_{S_n}^2}\right).\end{aligned} \tag{7.15}$$

Again, to help separate the load variables, and using the information on the individual mean and standard deviation of the loads, we can eliminate the square root sign in Equation 7.15 can by introducing a parameter ε_{nn}, similar to ε in Equation 7.8, as

$$\varepsilon_{nn} = \frac{\sqrt{\sigma_{S_1}^2 + \sigma_{S_2}^2 + \ldots + \sigma_{S_n}^2}}{\sigma_{S_1} + \sigma_{S_2} + \ldots + \sigma_{S_n}}. \tag{7.16}$$

Equation 7.15 can be rewritten as

$$\mu_R = (\mu_{S_1} + \mu_{S_2} + \ldots + \mu_{S_n}) + \varepsilon\beta[\sigma_R + \varepsilon_{nn}(\sigma_{S_1} + \sigma_{S_2} + \ldots + \sigma_{S_n})]$$

or

$$\begin{aligned}(1 - \varepsilon\beta\delta_R)\mu_R &= (1 + \varepsilon\varepsilon_{nn}\beta\delta_{S_1})\mu_{S_1} \\ &+ (1 + \varepsilon\varepsilon_{nn}\beta\delta_{S_2})\mu_{S_2} + \ldots + (1 + \varepsilon\varepsilon_{nn}\beta\delta_{S_n})\mu_{S_n}.\end{aligned} \tag{7.17}$$

The form of Equation 7.17 is identical to that of Equation 7.9b. By following the same logic as for the single load case, we can show that for the case of multiple loads, the central and nominal capacity reduction factors can still be evaluated from Equations 7.11a and 7.14a, respectively. However, the central and nominal load factors for the ith load, similar to Equations 7.11b and 7.14b, respectively, become

$$\bar{\gamma}_i = 1 + \varepsilon\varepsilon_{nn}\beta\delta_{S_i} \tag{7.18a}$$

and

$$\gamma_i = \frac{1 + \varepsilon\varepsilon_{nn}\beta\delta_{S_i}}{1 + k_{S_i}\delta_{S_i}}. \tag{7.18b}$$

Suppose a structural member is subjected to dead load (D) and live load (L) only. For design purposes, the following relationship needs to be satisfied:

$$\phi R_N \ge \gamma_D D + \gamma_L L. \tag{7.19}$$

If the safety margin is defined by Equation 7.3, then ϕ, γ_D, and γ_L can be estimated by Equations 7.14a and 7.18b, respectively.

EXAMPLE 7.1

A simply supported steel beam with a 30-foot span has been designed to carry a dead load of 70 psf and a live load of 100 psf, as shown in Figure 7.2. The beams are spaced 10 feet apart and are continuously laterally supported by the concrete slab. Using A36 steel and the American Institute of Steel Construction (AISC)'s *Manual of Load and Resistance Factor Design*, an engineer suggests a steel section of W14 × 61. Based on the preceding discussion, what will be the corresponding resistance and load factors?

To calculate these factors, some additional information is required. Consider that the nominal dead load and live load are both selected to be two standard deviations above the corresponding mean values, and the nominal resistance of the steel section is selected to be two standard deviations below the mean value. Further assume that the uncertainties in the dead load and live load in terms of COV are 0.13 and 0.37, respectively. The uncertainty in the resistance of the steel section, considering the uncertainties in material properties, fabrication, and modeling, is 0.13. These COVs are typical values reported in the literature. Consider first that all the variables are normal random variables.

$$\text{Nominal dead load} = D_N$$

$$= \mu_D + 2\sigma_D = \mu_D\left(1 + 2\frac{\sigma_D}{\mu_D}\right) = \mu_D(1 + 2\delta_D)$$

or

$$70 = \mu_D(1 + 2 \times 0.13)$$

or

$$\mu_D = 55.56 \text{ psf} \quad \text{and} \quad \sigma_D = 0.13 \times 55.56 = 7.22 \text{ psf.}$$

Similarly, the mean and standard deviation of the live load can be shown to be

$$\mu_L = 57.47 \text{ psf} \quad \text{and} \quad \sigma_L = 0.37 \times 57.47 = 21.26 \text{ psf.}$$

It is quite logical to assume that the dead and live loads are statistically independent. Denoting $S = D + L$, and using Equations 6.21 and 6.22, we can show the mean and standard deviation of S to be

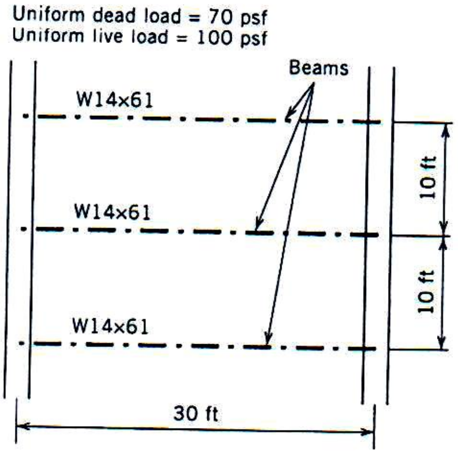

Figure 7.2 Resistance and Load Factor Evaluation for Beams

$$\mu_S = 55.56 + 57.47 = 113.03 \text{ psf}$$

and

$$\sigma_S = \sqrt{7.22^2 + 21.26^2} = 22.45 \text{ psf.}$$

Thus, $\delta_S = 22.45/113.03 = 0.199$.

For a simply supported beam of span l and subjected to a uniform load (per unit length) of S, the applied moment can be calculated using the formula $M = Sl^2/8$. Denoting the moment caused by the applied dead and live loads as M_A, we can estimate its mean and standard deviation as

$$\mu_{M_A} = (113.03 \times 10 \times 30^2 \times 12)/(8 \times 1{,}000) = 1{,}525.91 \text{ kip-in.}$$

and

$$\sigma_{M_A} = 0.199 \times 1{,}525.91 = 303.66 \text{ kip-in.}$$

$$\text{Nominal resistance} = R_N = \mu_R - 2\sigma_R.$$

The plastic moment capacity of a beam M_p can be considered to be the nominal moment capacity of the beam. Thus,

$$R_N = M_P = ZF_y = 102 \times 36 = 3{,}672 = \mu_R(1 - 2 \times 0.13)$$

or

$$\mu_R = 4{,}962.16 \text{ kip-in. and } \sigma_R = 0.13 \times 4{,}962.16 = 645.08 \text{ kip-in.}$$

Equation 7.4 can be used to estimate the probability of failure of the beam subjected to the dead and live loads considered here:

$$p_f = 1 - \Phi\left(\frac{4{,}962.16 - 1{,}525.91}{\sqrt{645.08^2 + 303.66^2}}\right)$$

$$= 1 - \Phi(4.82) = 0.72 \times 10^{-6}.$$

For this example β is 4.82. The resistance and load factors for this design can be estimated as discussed next. Using Equation 7.8, we can show that

$$\varepsilon = \frac{\sqrt{645.08^2 + 303.66^2}}{645.08 + 303.66} = 0.75.$$

Using Equation 7.16, we can calculate ε_{nn} as

$$\varepsilon_{nn} = \frac{\sqrt{7.22^2 + 21.26^2}}{7.22 + 21.26} = 0.79.$$

Using Equations 7.14a and 7.18b, we can estimate the resistance and load factors as

$$\phi = \frac{1 - 0.75 \times 4.82 \times 0.13}{1 - 2 \times 0.13} = 0.72$$

$$\gamma_D = \frac{1 + 0.75 \times 0.79 \times 4.82 \times 0.13}{1 + 2 \times 0.13} = 1.09$$

$$\gamma_L = \frac{1 + 0.75 \times 0.79 \times 4.82 \times 0.37}{1 + 2 \times 0.37} = 1.18.$$

Thus, the design equation is

$$0.72R = 1.09D + 1.18L$$

or

$$R = 1.51D + 1.64L.$$

7.4.3 Load and Resistance Lognormal Variables: Single Load Case

Considering the physical aspects of a design problem, R and S can be more appropriately considered to be statistically independent lognormal variables, that is, $LN(\lambda_R, \zeta_R)$ and $LN(\lambda_S, \zeta_S)$, since they cannot take negative values. In this case, another random variable Y can be introduced as

$$Y = R / S \tag{7.20a}$$

or

$$\ln Y = Z = \ln R - \ln S. \tag{7.20b}$$

The failure event can be defined as when $Y < 1.0$ or $Z < 0.0$. Since R and S are lognormal, $\ln R$ and $\ln S$ are normal (see Section 6.3.1.3); therefore, $\ln Y$ or Z is a normal random variable, that is, $Z \sim N\left(\lambda_R - \lambda_S, \sqrt{\zeta_R^2 + \zeta_S^2}\right)$. The probability of failure, similar to Equation 7.4, can be defined as

$$p_f = 1 - \Phi\left(\frac{\lambda_R - \lambda_S}{\sqrt{\zeta_R^2 + \zeta_S^2}}\right). \tag{7.21}$$

Using the relationships between the mean, standard deviation, and coefficient of variation and the parameters of the lognormal distribution (Equations 4.10 and 4.11), we can rewrite Equation 7.21 as

$$p_f = 1 - \Phi\left[\frac{\ln\left\{\left(\frac{\mu_R}{\mu_S}\right)\sqrt{\frac{1+\delta_S^2}{1+\delta_R^2}}\right\}}{\sqrt{\ln(1+\delta_R^2)(1+\delta_S^2)}}\right]. \tag{7.22}$$

If δ_R and δ_S are not large, say ≤ 0.30, Equation 7.22 can be simplified as

$$p_f \approx 1 - \Phi\left[\frac{\ln\left(\frac{\mu_R}{\mu_S}\right)}{\sqrt{\delta_R^2 + \delta_S^2}}\right]. \tag{7.23}$$

In this formulation, β as in Equation 7.7 can be shown to be

$$\beta = \Phi^{-1}(1 - p_f) = \frac{\ln\left\{\left(\frac{\mu_R}{\mu_S}\right)\sqrt{\frac{1+\delta_S^2}{1+\delta_R^2}}\right\}}{\sqrt{\ln(1+\delta_R^2)(1+\delta_S^2)}} \approx \frac{\ln\left(\frac{\mu_R}{\mu_S}\right)}{\sqrt{\delta_R^2 + \delta_S^2}}. \tag{7.24}$$

In many engineering problems of practical interest, the simplification suggested in Equation 7.23 may not be appropriate, since the uncertainty in many loads in terms of COV can be large (greater than 0.3). As in Equation 7.6, an alternative form of Equation 7.22 can be expressed as

$$\mu_R = \mu_S\left(\sqrt{\frac{1+\delta_R^2}{1+\delta_S^2}}\right)\exp\left[\beta\sqrt{\ln(1+\delta_R^2) + \ln(1+\delta_S^2)}\right] \tag{7.25}$$

where $\beta = \Phi^{-1}(1 - p_f)$, as given by Equation 7.24.

The parameter ε_l, similar to ε in Equation 7.8, can be introduced as:

$$\varepsilon_l = \frac{\sqrt{\ln(1+\delta_R^2)+\ln(1+\delta_S^2)}}{\sqrt{\ln(1+\delta_R^2)}+\sqrt{\ln(1+\delta_S^2)}}. \tag{7.26}$$

Proceeding as in the normal variables case in Section 7.4.1, the capacity reduction factor and the load factor corresponding to the central safety factor can be expressed as

$$\bar{\phi} = \frac{\exp\left[-\beta\varepsilon_l\sqrt{\ln(1+\delta_R^2)}\right]}{\sqrt{1+\delta_R^2}} \tag{7.27a}$$

and

$$\bar{\gamma} = \frac{\exp\left[\beta\varepsilon_l\sqrt{\ln(1+\delta_S^2)}\right]}{\sqrt{1+\delta_S^2}}. \tag{7.27b}$$

To obtain these factors with respect to the nominal safety factor, as in Equation 7.12, the nominal values of the resistance can be shown to be

$$\ln R_N = \ln \mu_R - k_R\delta_R$$

or

$$R_N = \mu_R \exp(-k_R\delta_R). \tag{7.28a}$$

Similarly, the nominal load can be expressed as

$$S_N = \mu_S \exp(k_S\delta_S). \tag{7.28b}$$

The nominal safety factor becomes

$$\zeta = \frac{R_N}{S_N} = \frac{\mu_R}{\mu_S}\frac{\exp(-k_R\delta_R)}{\exp(k_S\delta_S)}. \tag{7.29}$$

The corresponding nominal capacity reduction factor and load factor are

$$\phi = \bar{\phi}\exp(k_R\delta_R) \tag{7.30a}$$

and

$$\gamma = \bar{\gamma}\exp(-k_S\delta_S) \tag{7.30b}$$

$\bar{\phi}$ and $\bar{\gamma}$ can be estimated from Equations 7.27a and 7.27b, respectively.

7.4.4 Load and Resistance Lognormal Variables: Multiple Load Case

To consider the effect of statistically independent multiple load cases (i.e., when $S = S_1 + S_2 + \ldots + S_n$), the parameter ε_{nl}, similar to Equation 7.16, can be estimated by trial and error from the following equation as

$$\mu_S\left(\frac{\exp\left[\beta\varepsilon_l\sqrt{\ln(1+\delta_S^2)}\right]}{\sqrt{1+\delta_S^2}}\right) = \sum_{i=1}^{n}\mu_{S_i}\left(\frac{\exp\left[\beta\varepsilon_l\varepsilon_{nl}\sqrt{\ln(1+\delta_{S_i}^2)}\right]}{\sqrt{1+\delta_{S_i}^2}}\right). \tag{7.31}$$

The nominal capacity reduction factor can still be calculated by using Equation 7.30a; however, the load factor corresponding to the central safety factor for the ith load becomes

$$\bar{\gamma}_{S_i} = \frac{\exp\left[\beta \varepsilon_l \varepsilon_{nl} \sqrt{\ln(1+\delta_{S_i}^2)}\right]}{\sqrt{1+\delta_{S_i}^2}} \tag{7.32}$$

and the corresponding nominal load factor for the ith load is

$$\gamma_{S_i} = \bar{\gamma}_{S_i} \exp(-k_{S_i} \delta_{S_i}). \tag{7.33}$$

EXAMPLE 7.2

Example 7.1 is considered again, except that now both the load and resistance are assumed to be lognormal random variables. The means, standard deviations, and coefficients of variation of all the parameters were calculated earlier. In this case, the probability of failure of a beam can be calculated as

$$p_f = 1 - \Phi\left[\frac{\ln\left\{\left(\frac{4,962.16}{1,525.91}\right)\sqrt{\frac{(1+0.199^2)}{(1+0.13^2)}}\right\}}{\sqrt{\ln(1+0.13^2)(1+0.199^2)}}\right] = 1 - \Phi(5.05) = 0.2213 \times 10^{-6}.$$

In this case, β is 5.05. With Equation 7.26, ε_l can be calculated as

$$\varepsilon_l = \frac{\sqrt{\ln(1+0.13^2) + \ln(1+0.199^2)}}{\sqrt{\ln(1+0.13^2)} + \sqrt{\ln(1+0.199^2)}} = 0.72.$$

With Equation 7.31, ε_{nl} can be calculated by trial and error as:

$$113.03\left(\frac{\exp\left[5.05 \times 0.72 \times \sqrt{\ln(1+0.199^2)}\right]}{\sqrt{1+0.199^2}}\right)$$

$$= 55.56\left(\frac{\exp\left[5.05 \times 0.72 \times \varepsilon_{nl}\sqrt{\ln(1+0.13^2)}\right]}{\sqrt{1+0.13^2}}\right)$$

$$+ 57.47\left(\frac{\exp\left[5.05 \times 0.72 \times \varepsilon_{nl}\sqrt{\ln(1+0.37^2)}\right]}{\sqrt{1+0.37^2}}\right)$$

ε_{nl} is found to be 0.77. Equations 7.27a, 7.30a, 7.32, and 7.33 can be used to obtain the following information:

$$\bar{\phi} = \frac{\exp\left[-5.05 \times 0.72 \times \sqrt{\ln(1+0.13^2)}\right]}{\sqrt{1+0.13^2}} = 0.62$$

$$\bar{\gamma}_D = \frac{\exp\left[5.05 \times 0.72 \times 0.77 \times \sqrt{\ln(1 + 0.13^2)}\right]}{\sqrt{1 + 0.13^2}} = 1.42$$

$$\bar{\gamma}_L = \frac{\exp\left[5.05 \times 0.72 \times 0.77 \times \sqrt{\ln(1 + 0.37^2)}\right]}{\sqrt{1 + 0.37^2}} = 2.56$$

$$\phi = 0.62 \exp(2 \times 0.13) = 0.80$$

$$\gamma_D = 1.42 \exp(-2 \times 0.13) = 1.09$$

$$\gamma_L = 2.56 \exp(-2 \times 0.37) = 1.22$$

Thus, the design equation is

$$0.8R = 1.09D + 1.22L$$

or

$$R = 1.36D + 1.53L.$$

The two examples given here clearly indicate that it is not difficult to estimate the underlying resistance and load factors in a particular design for a set of assumptions. Whether these factors are acceptable is a different question. In actual building design codes, these factors are usually calibrated to satisfy current practice. Furthermore, assuming that the acceptable risk is going to remain the same for all the load combinations to be considered for the design of a structure, the engineer can calculate the corresponding resistance and load factors similarly for all load combinations. Maintaining uniform risk for different loads and load combinations is not practical in deterministic designs. The ability to design a structure for uniform risk with several loads and load combinations is one of the many desirable features of risk-based design.

The discussion and the examples clearly indicate that conventional safety factor–based deterministic designs, in terms of capacity reduction factor and load factors, and probability-based load and resistance factor designs are essentially parallel to each other. However, probabilistic design addresses the necessary design conservatism more explicitly, perhaps better and more comprehensively, through treatment of the uncertainty in the random variables, the conservatism used in selecting the design values, and the desired underlying reliability. Engineers are empowered to use judgment in selecting these factors on a case-by-case basis. This concept is the basis of all the reliability-based design codes being developed in different areas of engineering worldwide.

7.5 FUNDAMENTAL CONCEPT OF RELIABILITY ANALYSIS

The basic concept of the classical theory of structural reliability and risk-based design can now be presented more formally. We have seen that it is not difficult to calculate the underlying resistance and load factors for a given design, assuming an acceptable level of risk. However, it is more relevant to calculate the underlying risk of a given design, as is discussed in the following sections.

The first step in evaluating the reliability or probability of failure of a structure is to decide on specific performance criteria and the relevant load and resistance parameters,