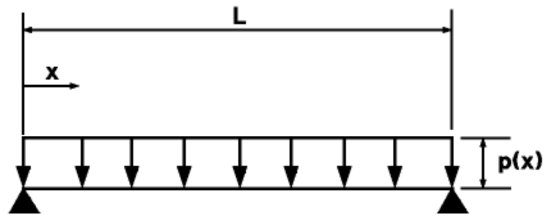
EXAMPLE 3A-3 (A)DYNAMIC DESIGN FACTORS FOR A ONE-WAY ELEMENT

Required: **The load, mass and load-mass factors for a structural steel beam in the elastic range, with a distributed load.**

Solution:

Step 1: Given structural steel beam shown in Figure 3A-8

Figure 3A-8



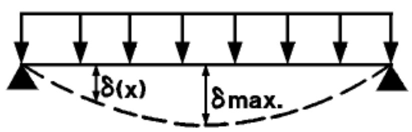
a. L = 120 in

b. Simply supported on both edges

c. $p(x)$ = 2,000 lb/in

$m(x)$ = 0.0055(lb-s^2/in^4)/in

Figure 3A-9



Step 2: Assumed deflected shape for elastic range is shown in Figure 3A-9

Step 3: The maximum deflection at the center is;

$$\delta(\max) = \frac{5p(\max)L^4}{384EI}$$

Step 4: Determine deflection function

$$\delta(x) = \frac{p(x)}{24EI}\left(L^3 - 2Lx^2 + x^3\right)$$

Step 5: Calculate the shape function using Equation 3-43

$$\phi = \frac{\delta(x)}{\delta_{\max}} = \frac{p(x)x}{24EI}\left(L^3 - 2Lx^2 + x^3\right)\left(\frac{384EI}{5p(x)L^4}\right)$$

$$= \frac{16}{5L^4}\left(L^3x - 2Lx^3 + x^4\right)$$

Step 6: a. Using Equation 3-42, determine equivalent force

$$F_E = \int_0^L p(x)\phi(x)dx = \int_0^{120} (2{,}000\,\text{lb}/\text{in})\frac{16}{5L^4}\left(L^3x - 2Lx^3 + x^4\right)dx$$

$$= \frac{6{,}400}{L^4}\left[\frac{L^3x^2}{2} - \frac{2Lx^4}{4} + \frac{x^5}{5}\right]_0^{120} = 1{,}280L$$

$$= 153{,}600\,\text{lb}$$

b. From Equation 3-41, find the load factor

$$K_L = \frac{F_E}{F} = \frac{153{,}600\,\text{lb}}{\left(2{,}000\,\text{lb}/\text{in} \times 120\,\text{in}\right)}$$

K_L = 0.64 in the elastic range

Step 7: a. Find the equivalent mass from Equation 3-48:

$$M_E = \int_0^L m(x)\phi^2(x)dx = 0.0055\left(\frac{256}{25L^8}\right)\left(\int_0^{120}\left(L^3x - 2Lx^3 + x^4\right)^2 dx\right)$$

$$= \frac{1.408}{25L^8}\left(\int_0^{120}\left(L^6x^2 - 4L^4x^4 + 2L^3x^5 + 4L^2x^6 - 4Lx^7 + x^8\right)dx\right)$$

$$= \frac{1.408}{25L^8}\left[\frac{L^6x^3}{3} - \frac{4L^4x^5}{5} + \frac{2L^3x^6}{6} + \frac{4L^2x^7}{7} - \frac{4Lx^8}{8} + \frac{x^9}{9}\right]_0^{120}$$

$$= 0.00277L$$

$$= 0.3325\ \text{lb}^2 - \text{s}^3/\text{in}$$

b. From Equation 3-47, calculate the mass factor:

$$K_M = \frac{M_E}{M} = \frac{0.3325\,\text{lb} - s^2/\text{in}^3}{\left(0.0055\ \text{lb} - s^2/\text{in}^4 \times 120\,\text{in}\right)}$$

K_M = 0.50 in the elastic range

Step 8: Calculate the load-mass factor as defined by Equation 3-51:

$K_{LM} = K_M/K_L$

$= 0.50/0.64$

K_{LM} = 0.78 in the elastic range