

HS239: Economics of Uncertainty and Information

PS 4: 2024

In Class, 23/10/2024

1. This question is designated to make you go through Marshallian inefficiency of sharecropping, step-by-step. Assume that an agricultural production process has production function $y = \sqrt{e}$ and cost as $c(e) = ke$, where e is the intensity of effort.

a) If the landlord tills his/her own land, what must be (i) the level of effort and (ii) landlord's income?

b) If the landlord hires a tenant, and opts for a fixed rent \bar{R} , what must be the i) tenant's income, (ii) landlord's income?

c) Now suppose the landlord demands $s\%$ of output, leaving $(1 - s)\%$ to the tenant. Re-do part (b).

d) Now assume that the landlord fixes \bar{R} to be what he was getting in part (c) and offers a fixed rent contract to tenant ($s = 0$). Re-do part b.

e) Argue why (d) is superior to (c).

2. In the model of sharecropping with risk-sharing, suppose we have the following specifications

$$\begin{aligned}U_L &= E(y_L) - \frac{1}{3}Var(y_L) \\U_T &= E(y_T) - \frac{4}{5}var(y_T) - c(e) \\c(e) &= \frac{k}{2}e^2\end{aligned}$$

Reservation utility of the agent = 10. The production function is $y = e + \varepsilon$, where ε is an rv with mean 0 and variance σ^2 . The landlord offers a sharecropping contract to the tenant: $y_T = sy - R$.

Characterise the optimal contract.

3. Assume an agent has the utility function $v = -e^{-Y}$. Show that evaded income $E = Y - X$ is independent of Y .

4. Assume an agent has the utility function $v = \ln(Y)$. Show that the proportion of income not declared $\left(\frac{X}{Y}\right)$ is constant.

5. Consider the model of consumption over time. Assume that the utility functions are $\ln c_i$. Without uncertainty, what are the optimal savings?

6. Continue with the above example. Now suppose income of period 0 is $(y_1 + 3)$ with probability .5 and $(y_1 - 3)$ with the complementary probability. How does your answer change in the above problem?

In 5 and 6, you may assume $y_0 = y_1 = 10, r = 5\%, \beta(1 + r) = 1$

7. Consider the model of consumption over time. Show that precautionary savings will be higher for agents with higher coefficient of absolute prudence $A(w) = -\frac{u'''}{u''}$