

EE 626: Pattern Recognition and Machine Learning

Quiz 1

Please outline all the steps systematically in the answer sheet.

1. It is known that the training samples $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ of a class ω_1 come from a two dimensional Gaussian probability density function of the form:

$$p(x_1, x_2) = K e^{\frac{-1}{8}(x_1^2 + x_2^2 - 2x_1 - 6x_2 + 10)}$$

- (a) Find the value of the normalizing constant K that ensures that this function does satisfy the condition of being a probability density function.
- (b) Compute the mean vector and covariance matrix corresponding to this distribution.
- (c) Suppose the training samples are transformed using the matrix $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$, compute the mean vector and covariance matrix corresponding to the transformed distribution. The normal distribution $N(\mu, \Sigma)$ in d -dimensions is given by :

$$p(\mathbf{x}) = \frac{1}{(2\pi)^{d/2} |\Sigma|^{0.5}} e^{-\frac{1}{2}(\mathbf{x} - \mu)^T \Sigma^{-1} (\mathbf{x} - \mu)}$$

[3 + 3 + 2.5 = 8.5 marks]

2. (a) Compute the maximum likelihood estimate of the parameter λ of the Poisson Distribution $P(X = k) = \frac{\lambda^k e^{-\lambda}}{k!}$ using the N training examples $\{x_1, x_2, x_3, \dots, x_N\}$

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- (b) The measured heights of two plant species (say A and B) have the normal distribution. The mean and variance for species A is 3 and 1 while for species B , mean and variance is 4 and 1. Moreover, A species is probable with 0.4. Find the probability that a measured height of 4.8 corresponds to plant species B . The normal distribution $N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ in d -dimensions is given by :

$$p(\mathbf{x}) = \frac{1}{(2\pi)^{d/2} |\boldsymbol{\Sigma}|^{0.5}} e^{-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1}(\mathbf{x}-\boldsymbol{\mu})}$$

[3 + 3.5 = 6.5 marks]