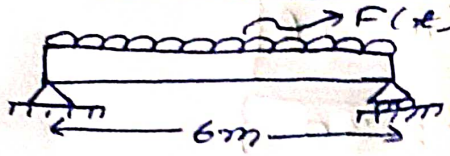
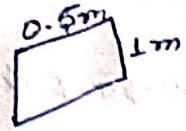
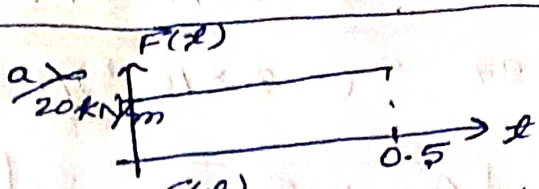
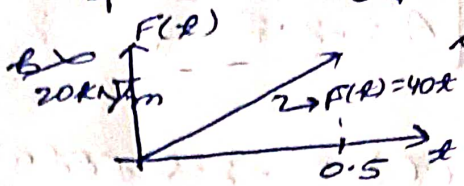
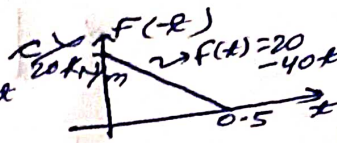
AMAN RAJ – 210104016 – CE-411 (ASS.3)











$$m^* = 0.504\,\rho A L = 0.504 \times \frac{25\times10^3}{9.81} \times 0.5\times1\times6 \text{ kg}$$

$$\Rightarrow \boxed{m^* = 3853.211 \text{ kg}}$$

$$F^*(t) = 0.64\, w_0 L$$

a) $\boxed{F^*(t) = 76.8 \text{ kN}}$

b) $\boxed{F^*(t) = 153.6\,t \text{ kN}}$

c) $\boxed{F^*(t) = 76.8(1-2t) \text{ kN}}$

$$k^* = \frac{48EI}{L^3} = \frac{48\times5000\sqrt{25}\times\frac{0.5\times1^3}{12}}{6^3\times(10^3)^3} = 231.48\times10^{-9}\ \frac{N}{mm}$$

$$\Rightarrow \boxed{k^* = 2.3148\times10^{-6}\ N/m}$$

$$\Rightarrow \omega = \sqrt{\frac{k^*}{m^*}} = 2.451\times10^{-4}\ /\text{sec}.$$

a) $u(t) = A\cos\omega t + B\sin\omega t + \frac{76.8\times10^3}{k^*}$

$u(0)=0 \Rightarrow A + \frac{76.8\times10^3}{k^*} = 0 \Rightarrow \boxed{A = -\frac{76.8\times10^3}{k^*}}$

$\dot{u}(t) = -A\omega\sin\omega t + B\omega\cos\omega t + 0$

$\dot{u}(0) = 0 \Rightarrow \boxed{B = 0}$

$$\Rightarrow \boxed{u(t) = \frac{76.8\times10^3}{k^*}\left[-\cos\omega t + 1\right]}$$

$\hookrightarrow 0 \le t \le 0.5$ sec.

for $t \ge 0.5$ sec.

$u(t) = A\cos\omega t + B\sin\omega t$

$\Rightarrow \dot{u}(t) = -A\omega\sin\omega t + B\omega\cos\omega t$

$\Rightarrow \frac{76.8\times10^3}{k^*}\left(1-\cos\frac{\omega}{2}\right) = A\cos\frac{\omega}{2} + B\sin\frac{\omega}{2}$

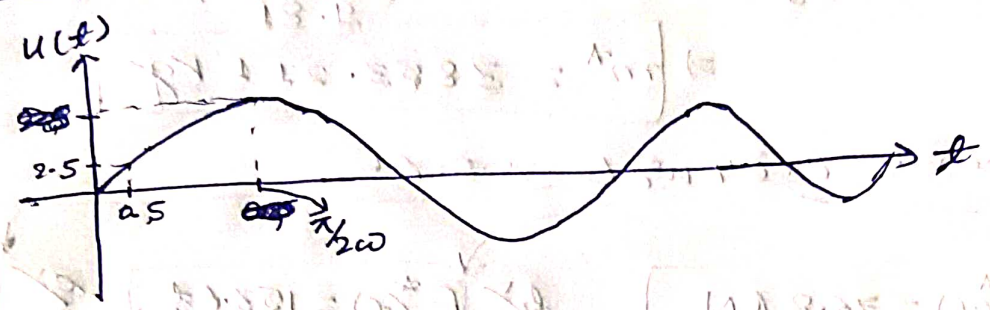
$\Rightarrow$ $2.4934 = 0.999A + 1.226\times10^{-4}B$

& $9.9697 = (-3\times10^{-8})A + (2.449\times10^{-4})B$

$\Rightarrow$ $A = -2.5$ & $B = 40709.30$

$\Rightarrow$ $u(t) = -2.5\cos\omega t + 40709.30\sin\omega t$

$\hookrightarrow t \geq 0.5$ sec.



b) $u(t) = A\sin\omega t + B\cos\omega t + \frac{153.6\times10^3}{K^*}t$

$u(0) = 0 \Rightarrow B = 0$

$\dot{u}(t) = A\omega\cos\omega t - B\omega\sin\omega t + \frac{153.6\times10^3}{K^*}$

$\dot{u}(0) = 0 \Rightarrow A = -\frac{153.6\times10^3}{\omega K^*}$

$\Rightarrow u(t) = \frac{153.6\times10^3}{K^*}\left[-\frac{\sin\omega t}{\omega} + t\right]$

$\hookrightarrow 0 < t \leq 0.5$ sec

For $t \geq 0.5$ sec,

$u = A\cos\omega t + B\sin\omega t$

$\Rightarrow \dot{u} = -A\omega\sin\omega t + B\omega\cos\omega t$

$\Rightarrow 0.999A + 1.226\times10^{-4}B = -2.70\times10^{12}$

& $(-3\times10^{-8})A + (2.449\times10^{-4})B = 6.63\times10^{8}$

$\Rightarrow A = -2.70\times10^{12}$ & $B = 2.70\times10^{12}$

$\Rightarrow u(t) = 2.79\times10^{12}[\sin\omega t - \cos\omega t]$

$\hookrightarrow t > 0.5$ sec

c) $u(t) = A\cos\omega t + B\sin\omega t + \frac{76.8(1-2t)}{k^*}$

$u(0) = 0 \Rightarrow \boxed{A = -\frac{76.8}{k^*}}$

$\dot{u}(t) = -A\omega\sin\omega t + B\omega\cos\omega t - \frac{153.6\,t}{k^*}$

$\dot{u}(0) = 0 \Rightarrow \boxed{B = 0}$

$\Rightarrow \boxed{u(t) = \frac{76.8}{k^*}\left[1 - 2t - \cos\omega t\right]}$

$\hookrightarrow 0 \le t \le 0.5$ sec.

For $t > 0.5$ sec,

$u(t) = A\cos\omega t + B\sin\omega t$

$\Rightarrow \dot{u}(t) = -A\omega\sin\omega t + B\omega\cos\omega t$

$\Rightarrow \boxed{0.999A + 1.226\times10^{-4}B = -40.676}$

& $\boxed{(-3\times10^{-8})A + (2.449\times10^{-4})B = -6.635\times10^{-5}}$

$\Rightarrow A = 3.32\times10^{5}$ & $B = -2.71\times10^{9}$

$\Rightarrow \boxed{u(t) = (3.32\times10^{5})\cos\omega t - (2.71\times10^{9})\sin\omega t}$

$\hookrightarrow t > 0.5$ sec