(A)

Time: 50 min | Quiz: BT612 Systems Biology | Marks: 10

Name: Roll No.

Instructions: Write the answers briefly, but you must show all the relevant steps in the calculations/derivations. Use conventional mathematical symbols/notations. No marks will be given for a partially correct answer. Marks will be deducted for irrelevant calculations/derivations.

Q1. In a cell, on average, two proteins reach the cell membrane from the cytoplasm in one minute. Consider this trafficking of protein from cytoplasm to membrane as a Poisson process.

a) Calculate the probability that R proteins will reach the membrane in two minutes. Here, R is the last digit of your roll number. For example, if your roll number is 240103020, then R = 0.

b) Calculate the variance in the number of proteins that reach the membrane in 10 minutes. [4 + 2]

Ans:

a) $\lambda = 2$ /min

$t = 2$ min

$\therefore$ Mean $\mu = \lambda t = 4$

Probability that R protein will reach,

$$P(R) = \frac{\mu^R}{\lfloor R} e^{-\mu} = \frac{(4)^R}{\lfloor R} e^{-4} \quad (*)$$

Plug the value of R as per your roll number and calculate the P(R).

b) $\lambda = 2$ /min, $t = 10$ min

$\therefore$ Mean $\mu = \lambda t = 20$

In Poisson distribution, mean = variance.

$\therefore$ variance $= \mu = 20 \quad (*)$

Q2. X is a continuous random variable such that $-\infty \le x \le \infty$. Its PDF is,

$$f_X(x) = \begin{cases} 3x^2, & \text{for } 0 \le x \le 1 \\ 0, & \text{otherwise} \end{cases}$$

Find the expectation of $(X + k)$. Here, $k > 0$. k is the last digit of your roll number. For example, if your roll number is 240103017, then $k = 7$. However, if the last digit of your roll number is zero, then $k = 1$. For example, if your roll number is 240103020, then $k = 1$. [4]

Ans:

$$E(X+k) = E(x) + k.$$

$$E(x) = \int_{-\alpha}^{+\alpha} x\, f_x(x)\, dx = \int_0^1 x\, f_x(x)\, dx$$

(as in other parts $f_x(x) = 0$)

$$\therefore E(x) = \int_0^1 x\, 3x^2\, dx$$

$$= 3\int_0^1 x^3\, dx = 3 \times \frac{x^4}{4}\Big|_0^1 = \frac{3}{4}$$

$$\therefore E(x+k) = E(x) + k = \frac{3}{4} + k \quad (A)$$

Plug the value of k as per your roll number and calculate $E(x+k)$

(B)

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Instructions: Write the answers briefly, but you must show all the relevant steps in the calculations/derivations. Use conventional mathematical symbols/notations. No marks will be given for a partially correct answer. Marks will be deducted for irrelevant calculations/derivations.

Q1. X is a continuous random variable such that $-\infty \le x \le \infty$. Its PDF is,

$$f_X(x) = \begin{cases} \dfrac{2-x}{2}, & \text{for } 0 \le x \le 2 \\ 0, & \text{otherwise} \end{cases}$$

Find the expectation of kX. Here, $k > 0$. k is the last digit of your roll number. For example, if your roll number is 240103017, then k = 7. However, if the last digit of your roll number is zero, then k = 1. For example, if your roll number is 240103020, then k = 1. [4]

Ans:

$$E(kX) = k\,E(X)$$

$$E(\alpha) = \int_{-\alpha}^{\alpha} x f_X(x)\,dx = \int_0^2 x f_X(x)\,dx$$

[as $f_X(x) = 0$ for other values]

$$\therefore E(X) = \int_0^2 x f_X(x)\,dx = \int_0^2 x\left(\frac{2-x}{2}\right)dx$$

$$= \int_0^2 \cancel{2x}\, x\,dx - \int_0^2 \frac{x^2}{2}\,dx$$

$$= \frac{1}{2}x^2\Big|_0^2 - \frac{1}{6}x^3\Big|_0^2$$

$$= \frac{4}{2} - \frac{8}{6} =$$

$$= 2 - \frac{4}{3} = \frac{2}{3}$$

$$\therefore E(kX) = k\,E(X) = \frac{2k}{3}$$ (*)

Plug the value of k as per your roll number.

Q2. In a cell, on average, two proteins reach the cell membrane from the cytoplasm in one minute. Consider this trafficking of protein from cytoplasm to membrane as a Poisson process.

a) Calculate the probability that R proteins will reach the membrane in two minutes. Here, R is the last digit of your roll number. For example, if your roll number is 240103020, then R = 0.

b) Calculate the variance in the number of proteins that reach the membrane in 15 minutes. [4 + 2]

Ans:

a) $\lambda = 2$ /min, $t = 2$ min

$\therefore$ Mean $\mu = \lambda t = 4$

$\therefore$ Probability that R proteins reach the membrane,

$$P(R) = \frac{\mu^R}{\lfloor R} e^{-\mu} = \frac{(4)^R}{\lfloor R} e^{-4}$$

Plug the value of R as per your roll number and calculate the final value of P(R).

b) $\lambda = 2$ /min, $t = 15$ min

$\therefore$ Mean $\mu = \lambda t = 30$

In Poisson distribution mean = variance.

$\therefore$ Variance in protein number,

$$\text{var} = \mu = 30.$$

C

Marks: 10 **Quiz: BT612 Systems Biology** Time: 50 min

Roll No. **Name:**

Instructions: Write the answers briefly, but you must show all the relevant steps in the calculations/derivations. Use conventional mathematical symbols/notations. No marks will be given for a partially correct answer. Marks will be deducted for irrelevant calculations/derivations.

Q1. We treated a suspension of *E. coli* with an antibiotic that kills the bacteria. Consider cell death as a Poisson process, with ~~four~~ 2 cells dying every minute, on average.

a) Calculate the probability that N cells will die in two minutes. Here, N is the last digit of your roll number. For example, if your roll number is 240103020, then N = 0.

b) Let M be the number of bacteria that died in ~~ten~~ 5 minutes. Calculate the variance M. [4 + 2]

Ans:

a) Given, $\lambda = 2$ /min

$t = 2$ min

$\therefore$ Mean $\mu = \lambda t = 4$

Probability of N death, in t.

$$p(N) = \frac{\mu^N}{\lfloor N} e^{-\mu} = \frac{4^N}{\lfloor N} e^{-4}$$ *

Plug the value of N, as per your roll number and calculate the value of p(N).

b) $\lambda = 2$ / min

$t = 5$ min

$\therefore$ Mean $= \mu = \lambda t = 10$

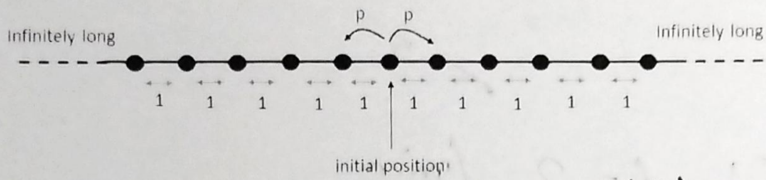
In poisson distribution Mean = variance

$\therefore$ Variance in number of death,

$var(M) = \mu = 10$

Q2. Assume that a protein is diffusing randomly in one dimension. We imagine it as a discrete hopping process. Consider time as discrete. At every time step, the protein jumps to a new site at a unit distance, either on the left or right. The probability of such a jump is equal in both directions. In the following diagram, these sites are shown as black-filled circles. This one-dimensional array of sites is infinitely long.

What is the probability that the protein will be at an N-unit distance away from the initial position after N steps? Here, N is the last two digits of your roll number. For example, if your roll number is 240103020, then N = 20. If your roll number is 240103017, then N = 17. [4]



Ans: The protein could be N unit distance away either on Left or on Right.

In either case, it must hops N times continuously in one direction.

∴ Probability of being N unit distance away, in N jumps,

$$p(N) = 2 \times p^N$$

As $P_L = P_R = p$, then $p = \frac{1}{2}$

$$\therefore p(N) = 2 \times \left(\frac{1}{2}\right)^N = \left(\frac{1}{2}\right)^{N-1}$$

Plug value of N as per your roll number. No need to calculate the final numerical value of p(N).

D

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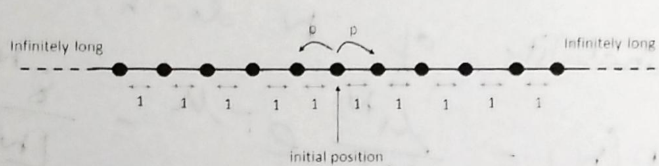
Q1. Assume that a protein is diffusing randomly in one dimension. We imagine it as a discrete hopping process. Consider time as discrete. At every time step, the protein jumps to a new site at a unit distance, either on the left or right. The probability of such a jump is equal in both directions. In the following diagram, these sites are shown as black-filled circles. This one-dimensional array of sites is infinitely long.

What is the probability that the protein will be at an N-unit distance away from the initial position after N steps? Here, N is the last two digits of your roll number. For example, if your roll number is 240103020, then N = 20. If your roll number is 240103017, then N = 17. [4]



Ans:

It can be N-unit distance away on left or on right. In either case that is achieved by repeated ~~hopping~~ N hops in left or right.

$\therefore$ The probability to be at N unit distance away from initial position,

$$P(N) = 2 \times p^N$$

As $p_L = p_R = p$, $p = \frac{1}{2}$

$$\therefore P(N) = 2 \times \left(\frac{1}{2}\right)^N = \left(\frac{1}{2}\right)^{N-1}$$

Plug the value of N as per your roll number. No need to calculate the final value.

Q2. We treated a suspension of *E. coli* with an antibiotic that kills the bacteria. Consider cell death as a Poisson process, with four cells dying every minute, on average.

a) Calculate the probability that N cells will die in two minutes. Here, N is the last digit of your roll number. For example, if your roll number is 240103020, then N = 0.

b) Let M be the number of bacteria that died in ten minutes. Calculate the variance M. [4 + 2]

Ans:

a) Given, $\lambda = 4$ /min

$t = 2$ min

$\therefore$ Mean death, $\mu = \lambda t = 8$

$\therefore$ Probability of N death,

$$P(N) = \frac{\mu^N}{\lfloor N} e^{-\mu} = \frac{8^N}{\lfloor N} e^{-8} *$$

Plug the value of N as per your roll number and calculate P (N)

b) In poisson distribution Mean = Variance

Here, $\lambda = 4$ /min

$t = 10$ min

$\therefore$ Mean, $\mu = \lambda t = 40$

$\therefore$ Variance of number of death

$= \mu = \cancel{40}$ 40