Biology : SYSTEM BIOLOGY.

Logistic Model

$$\frac{dx}{dt} = x\left(1-\frac{x}{c}\right)$$

$$\Rightarrow \frac{dx}{dt} = x(1-x)$$

$$x(1-x)=0$$

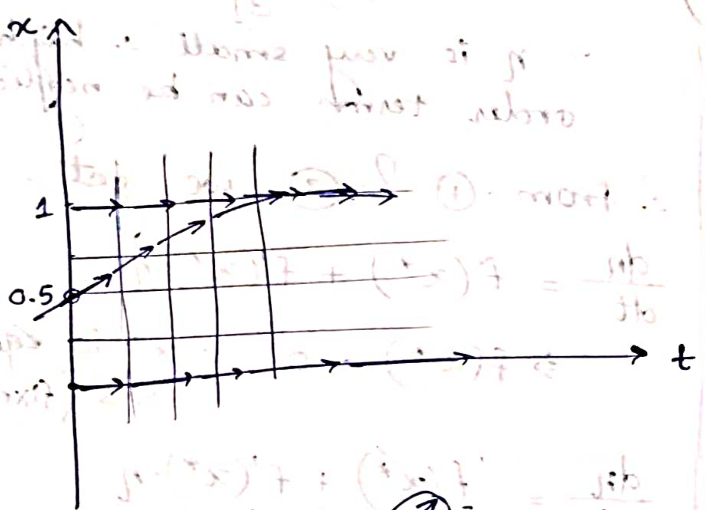
At steady state

x_s, x_{ss}, $x^* = 0, 1$.

Lets plot it:

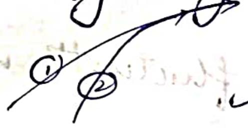
$\frac{dx}{dt} \rightarrow$ slope at pt. (can consider it as a vector.)

at $x=0 \Rightarrow dx/dt = 0$.



vector is +ve ↗.

Trajectories can't intersect
if intersect → system will become undeterministic.
they can merge.
but if they merge we can't say which path is followed ① or ②.



• Steady State

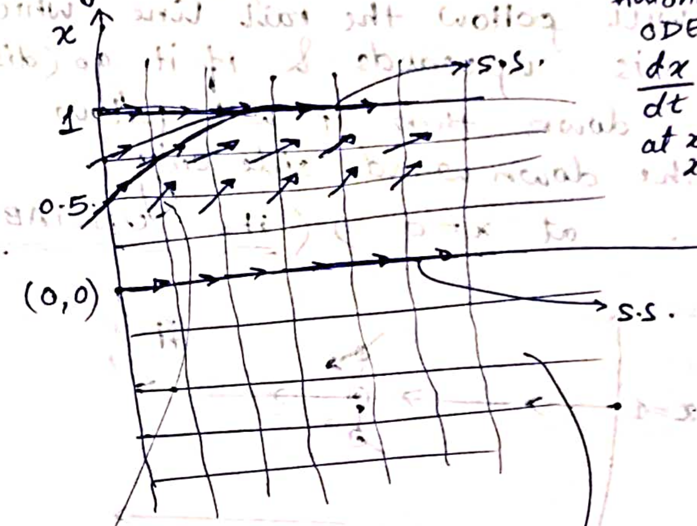
How x changes with time, looking for long-term.

$$\frac{dx}{dt} = f(x).$$

$\Rightarrow$ then at steady state/fixed pt is $\frac{dx}{dt} = 0$.

Logistic growth, $\frac{dx}{dt} = x(1-x)$ — Autonomous ODE

$\frac{dx}{dt} = 0$ at $x=0$, $x=1$.



when $x=0 \Rightarrow \frac{dx}{dt}$ = slope is zero.
∴ arrow will be horizontal
at $x=1$ also slope is zero ie arrow will be flat horizontal.

at $x=0.5 \Rightarrow$ arrow is ↗.

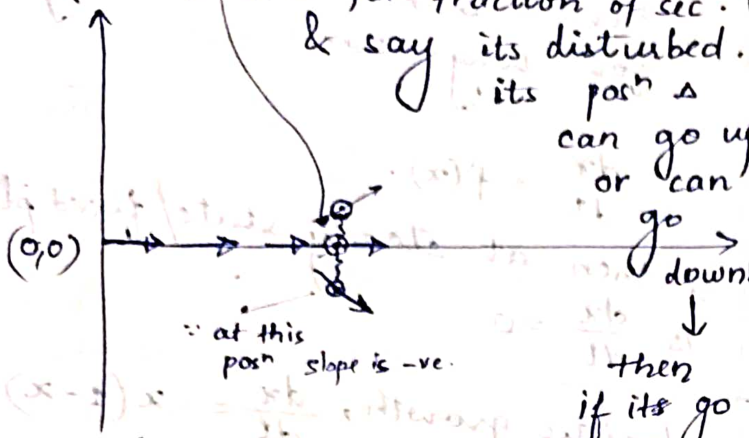
arrows are identical, becoz eqn is Autonomous.

This field called DIRECTION FIELD.

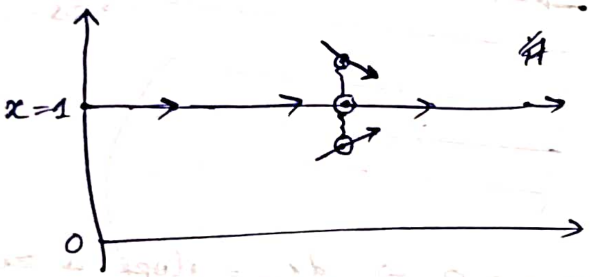
vector is called INTEGRAL CURVE.

Trajectories did not intersect.
we can say those lines collapse
Asymptotically collapse?!

→ fluctuations can occur / can Remains for fraction of sec. & say its disturbed. its posn Δ can go up or can go down then if its go up then it will follow the rail line which is upwards & if it go (disturbed) down then it will follow the downward side only.



∴ at $x=0$ ⇒ (its UNSTABLE)



at $x=1$ ⇒ if it goes up then it will have slope ~~at~~ -ve, arrow downwards.

if it goes down ie. if disturbed downward then it will go up ∴ arrow is up ie slope +ve

$x = 1.1$ ⇒ $\frac{dx}{dt} = x(1-x)$
$= 1.1\,(1-1.1)$
slope = -ve

$x = 0.8$ ⇒ $\frac{dx}{dt} = x(1-x)$
$= 0.8\,(1-0.8)$
= +ve.

∴ at $x=1$ ⇒ (its STABLE)

$\frac{dx}{dt} = f(x)$

⇒ x^* = fixed pt (s.s).

if its disturbed (if x^* is dis...

⇒ means fluctuatn is added which is (η) → linear Noise.

⇒ New disturbed posn x

$x = x^* + \eta$

$\frac{d\eta}{dt} = \frac{dx}{dt} = f(x) = f(x^* + \eta)$

Taylor series expansion of $f(x^* + \eta)$

$f(x^*+\eta) = f(x^*) + f'(x^*)\cdot\eta + f''(x^*)\ldots$
$+ f'''(x^*)\frac{\eta^3}{3!}$ — ②

∵ η is very small ∴ higher order terms can be neglected.

∴ from ① & ② we get:

$$\frac{d\eta}{dt} = f(x^*) + f'(x^*)\eta$$

⇒ $f(x^*) = 0$ ∵ x^* is s.s (fixed pt)

$$\frac{d\eta}{dt} = \cancel{f(x^*)} + f'(x^*)\cdot\eta$$

students make mistake. they differentiate wrt time but its $\frac{df(x^*)}{dx} = f'(x^*)$

$\frac{d\eta}{dt}$ – fluctuatn wrt time.

$$\frac{d\eta}{dt} = f'(x^*)\eta = \text{constant}$$

$$\int\frac{d\eta}{\eta} = f'(x^*)\int dt$$

$$\boxed{\eta(t) = \eta\, e^{f'(x^*)t}}$$

→ exponentially Δing.

$$\boxed{\eta(t) = \eta\, e^{f(x^*)t}}$$

$\to$ +ve: $\uparrow$ exponentially (increase)

$\to$ −ve: $\downarrow\downarrow$ exponentially (loose the noise)

$$\frac{dx}{dt} = x(1-x) = f(x)$$

$$f'(x) = \frac{d\,f(x)}{dx} = \_\_\_\_$$

substitute $x = x^*$

then $f'(x^*)$ will be const.

Example $\frac{dx}{dt} = x(1-x)$

$$\frac{dx}{dt} = f(x) = x - x^2$$

$$x^* = 1, 0$$

$\Rightarrow x^* = 0,\ f'(x^*) = 1 - 2x = 1$ its +ve > 0

$$\frac{d\eta}{dt} = \eta\, e^{1t} \to \text{unstable}$$

becoz $\frac{d\eta}{dt} > 0$.

$\Rightarrow x^* = 1,\ f'(x^*) = 1 - 2(1)$

$= -1$

$=$ its −ve < 0

$\to$ stable.

if $f'(x^*)$ is :

+ve $\Rightarrow$ unstable

−ve $\Rightarrow$ stable.

Example : $\frac{dx}{dt} = (1-x)^2$

step 1 : $\frac{dx}{dt} = 0 \Rightarrow x^* = 1$

x^* is stable / unstable.

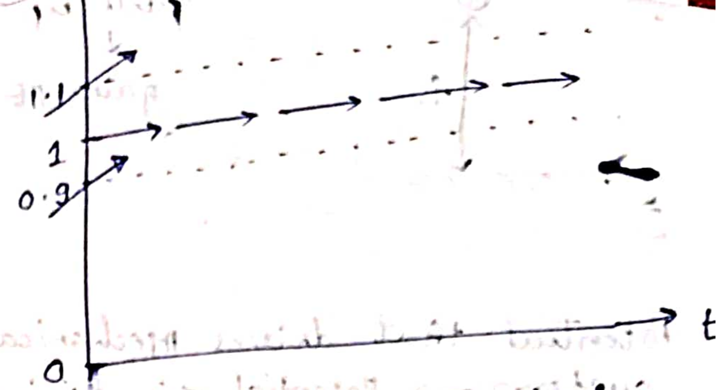
$f'(x^*) = -2(1-x)$

$f'(1) = -2(1-1) = 0 \to$ $f'(x^*)$ is zero means

Noise doesn't decay.

$\Downarrow$

Then check direct^n field diagram.



Take value up & one below :

at $x = 0.9 \Rightarrow \frac{dx}{dt} = (1-0.9)^2$

$= (0.1)^2$

$= +ve$.

at $x = 1.1 \Rightarrow \frac{dx}{dt} = (1-1.1)^2$

$= +ve$.

$\to$ at this side we collapse with asymptote

$\to$ at this side we run away from asym

$\Downarrow$

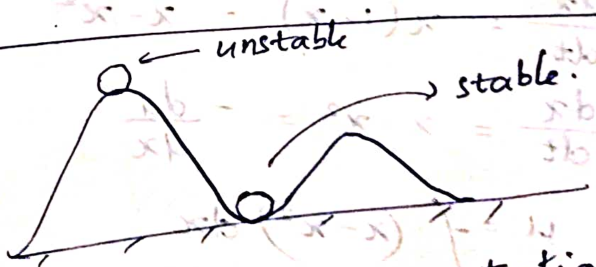
This is called : Semi Stable or Half - Stable.

Stability

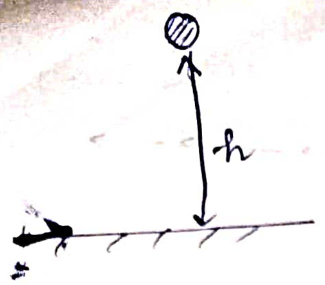
$\frac{dx}{dt} = f(x)$,

x^*: steady state $\to$ stable, $\to$ Unstable, $\to$ Semi-Stable

$f'(x)\big|_{x=x^*}$ $\to = 0$, $\to < 0$, $\to > 0$



stable $\Rightarrow$ Lower potential



if ball pull up

↓

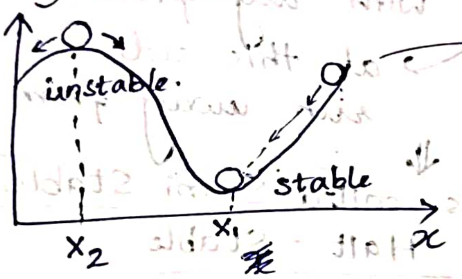
gains PE

Potential that drives Mechanical system → Potential ~~F~~ is driving F

$$dF = -\frac{du}{dx}$$

say; generalised dynamic system $\frac{dx}{dt} = f(x)$

& say it has potential



Equation of U?

$\frac{dx}{dt} = f(x)$: dynamical system

$$\frac{dx}{dt} = -\frac{du}{dx} = f(x)$$

$$u = -\int f(x)\,dx$$

$$\frac{dx}{dt} = x(1-x) = x - x^2$$

$$\frac{dx}{dt} = x - x^2 = -\frac{du}{dx}$$

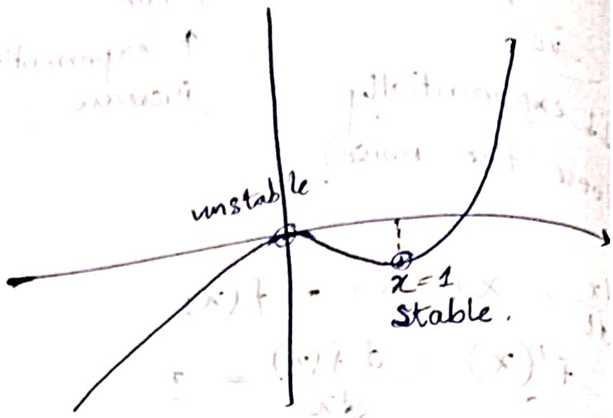
$$u = -\int (x-x^2)\,dx$$

$$u = -\frac{1}{2}x^2 + \frac{1}{3}x^3$$

↓

Now plot the graph.

$$u = -\frac{1}{2}x^2 + \frac{1}{3}x^3$$



Example: $\frac{dx}{dt} = (1-x^2)$

↓

derive Landscape for this sys.

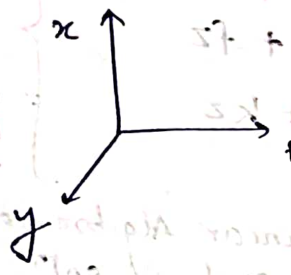
1D Autonomous system ODE
⇒ can't get oscillatⁿ?
⇒ will not give —''— Prove.

Example: Linear Homog. system.

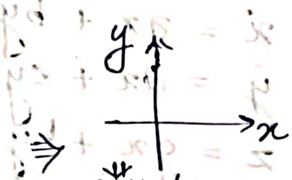
(:
$$\dot{x} = 1-y$$
$$\dot{y} = 1+x$$
} Need to Analyze the system

Need to find steady state.

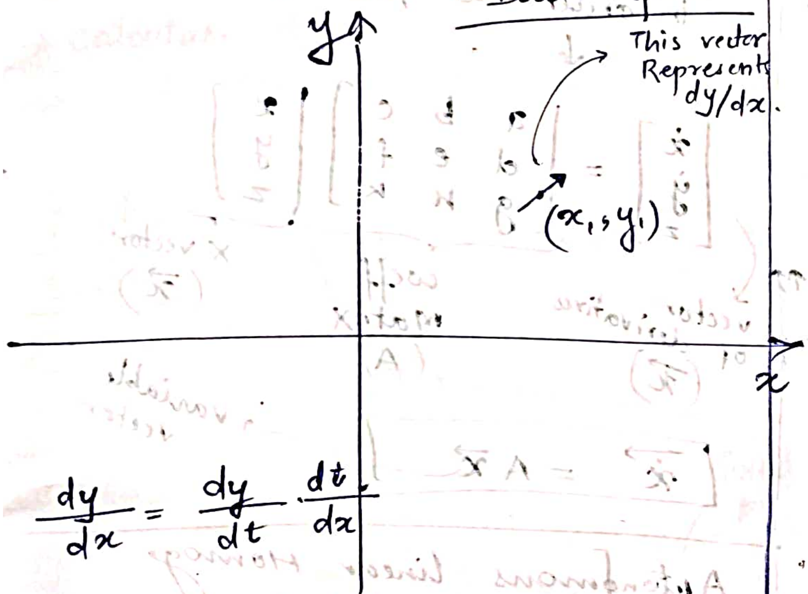
⇒ $\dot{x} = 1-y = 0 \Rightarrow y = 1$
$\dot{y} = 1+x = 0 \Rightarrow x = -1$.



⇒



called.
phase space
OR
state space.



$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$$

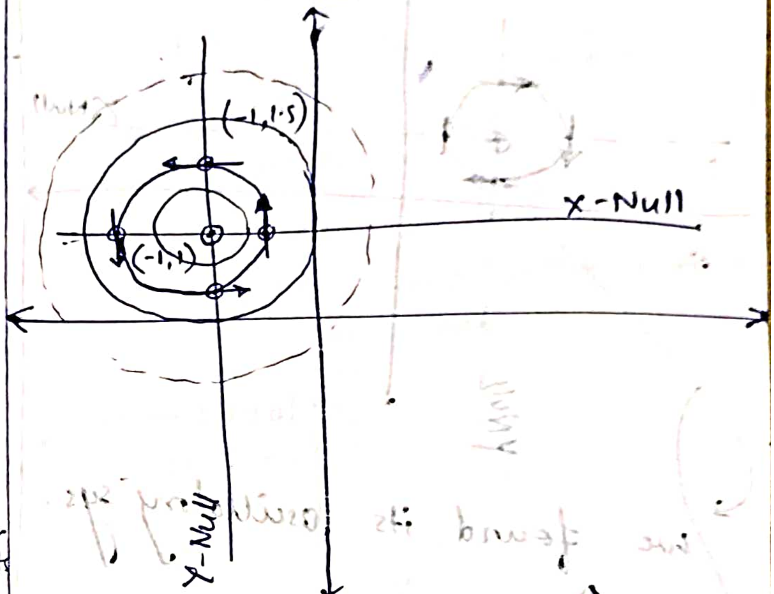
Arrow Head in directⁿ field was always in +ve t ∵ time ↑↑
But here how we will decide Arrow head?
→ Check sign of (dy/dt) & sign of (dx/dt)
Represents: How y & x Δ with time at that posⁿ.

Example

$$\dot{x} = 1-y$$
$$\dot{y} = 1+x$$

S.S: $\dot{x} = 0 \Rightarrow 1-y = 0$
This is called $\boxed{\text{X - null cline.}}$

$\dot{y} = 0 \Rightarrow 1+x = 0$
This is called $\boxed{\text{Y- Nullcline.}}$



(−1, 1) are my S.S. (X of 2 different Null clines.)
∴ (−1, 1) is fixed pt.

Now check Behaviour stable / unstable.

Taking pt on X-Nullcline.
Take pt. (−0.5, 1)

$$\left.\frac{dx}{dt}\right|_{(-0.5,1)} = 0$$

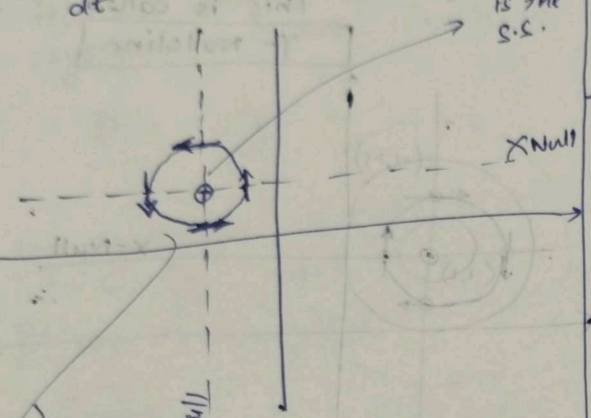
$$\left.\frac{dy}{dt}\right|_{(-0.5,1)} = 1+x = 1-0.5 = 0.5 = +ve$$
∴ slope

Take pt again
(−1, 1.5), (0, −1.5)
&
∞ amount of oscillations around ⇒ is this good?

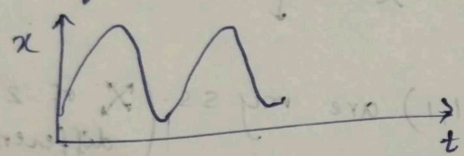
14/08/24 Phase plane
OR
State Space.

Null cline → zero slope.

$$\frac{dx}{dt} = 1-y$$
$$\frac{dy}{dt} = 1+x$$



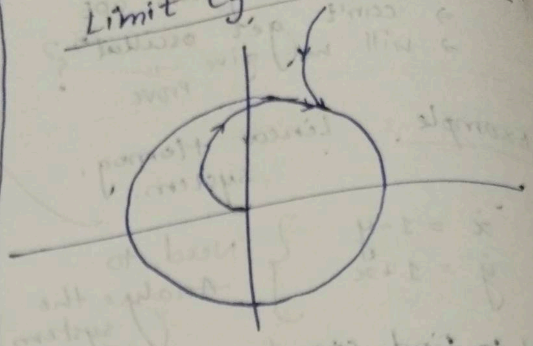
we found its oscillatory sys.



x ↑↑ & then ↓↓ : & then again ↑↑

Does this kind of system favourable in Biology?
→ No because?
→ Amplitude is Δing?
∴ Such kind of oscillations aren't favourable.

Limit Cycle



Generalised 3D system.

$$\dot{x} = ax + by + cz$$
$$\dot{y} = dx + ey + fz$$
$$\dot{z} = gx + hy + kz$$

Linear Homo Sys. No indep term

↓ convert to linear Algebra form in order to find solⁿ.

↓

$$\underbrace{\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix}}_{\substack{\text{vector of derivatives} \\ (\vec{\dot{x}})}} = \underbrace{\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & k \end{bmatrix}}_{\substack{\text{coeff Matrix} \\ (A)}} \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{\substack{\text{x vector} \\ (\vec{x})}}$$

$$\boxed{\vec{\dot{x}} = A\vec{x}}$$

variable vector

Autonomous linear Homog system of ODE.

$$\dot{x} = ax + by$$
$$\dot{y} = dx + ey$$
$$A = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

Steady state ⇒

$$\boxed{\vec{\dot{x}} = A\vec{x}} = 0 \text{ for S.S.}$$

$\dot{x} = ax + by$
$\dot{y} = dx + ey$

i.e. $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$A\vec{x} = 0$

$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} =$

$ax + by = 0$
$cx + dy = 0$

$cx - \frac{a}{b} dx$

$x [ad - b$

det A

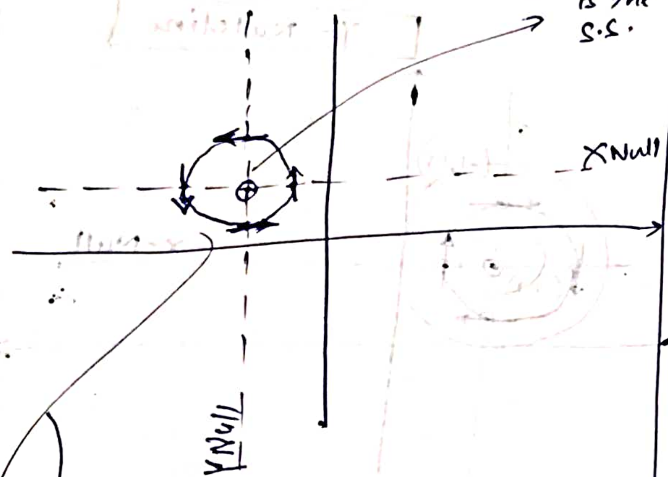
Trivial solⁿ

→ In order
→ calculate de
→ if det

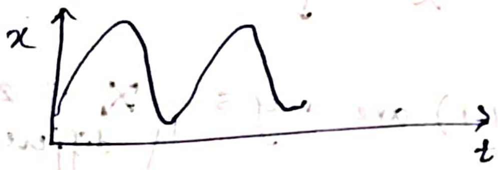
Fixed pt. is $\boxed{\text{stable}}$

14/08/24 Phase plane
OR
State Space. Null cline → zero slope.

$$\frac{dx}{dt} = 1 - y$$
$$\frac{dy}{dt} = 1 + x$$



we found its oscillatory sys.



x ↑↑ & then ↓↓ & then again ↑↑

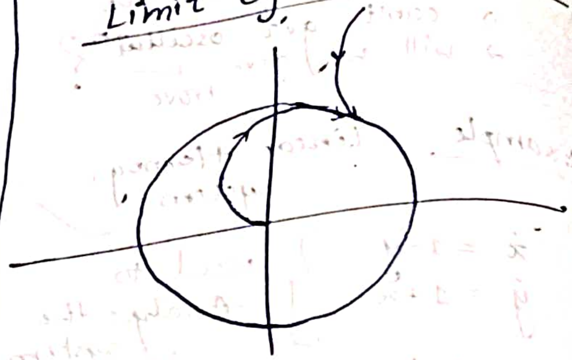
Does this kind of system favourable in Biology?

→ No because ?

Amplitude is Δing?

∴ Such kind of oscillations aren't favourable.

Limit Cycle ... Oscillation



Generalised 3D system:

$$\left.\begin{aligned}\dot{x} &= ax + by + cz\\ \dot{y} &= dx + ey + fz\\ \dot{z} &= gx + hy + kz\end{aligned}\right\}$$

Line Hom sys. No indep tem

↓

convert to linear Algebra form in order to find soln:

↓

$$\underbrace{\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{z}\end{bmatrix}}_{\text{vector of derivatives } (\vec{\dot{x}})} = \underbrace{\begin{bmatrix}a & b & c\\ d & e & f\\ g & h & k\end{bmatrix}}_{\text{coeff Matrix } (A)} \underbrace{\begin{bmatrix}x\\ y\\ z\end{bmatrix}}_{X \text{ vector } (\vec{x})}$$

$$\boxed{\vec{\dot{x}} = A\vec{x}}$$ variable vector.

Autonomous linear Homog system of ODE.

$$\dot{x} = ax + by$$
$$\dot{y} = dx + ey$$
$$A = \begin{vmatrix}a & b\\ c & d\end{vmatrix}$$

Steady state ⇒

$$\boxed{\vec{\dot{X}} = A\vec{x}} = 0$$ for S

$\dot{x} = ax + by$
$\dot{y} = dx + ey$

system of ODE

$\therefore A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$A\vec{x} = 0$

$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 0$

$ax + by = 0 \Rightarrow y = -\left(\frac{a}{b}\right)x$

$cx + dy = 0$

$cx - \frac{a}{b} dx = 0$

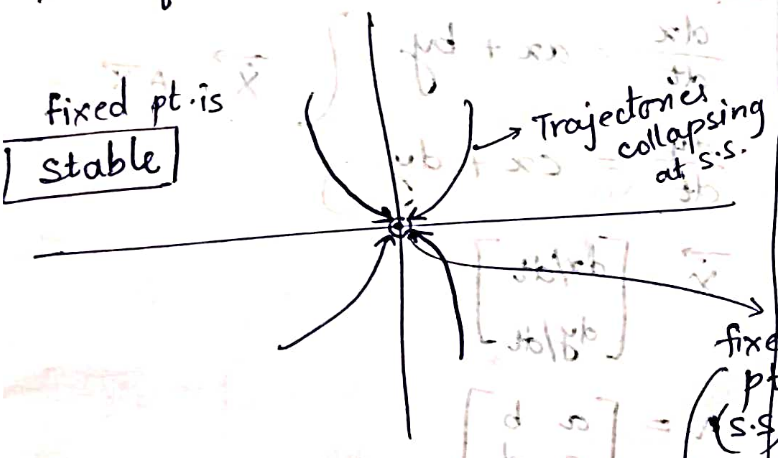
$x[ad - bc] = 0$

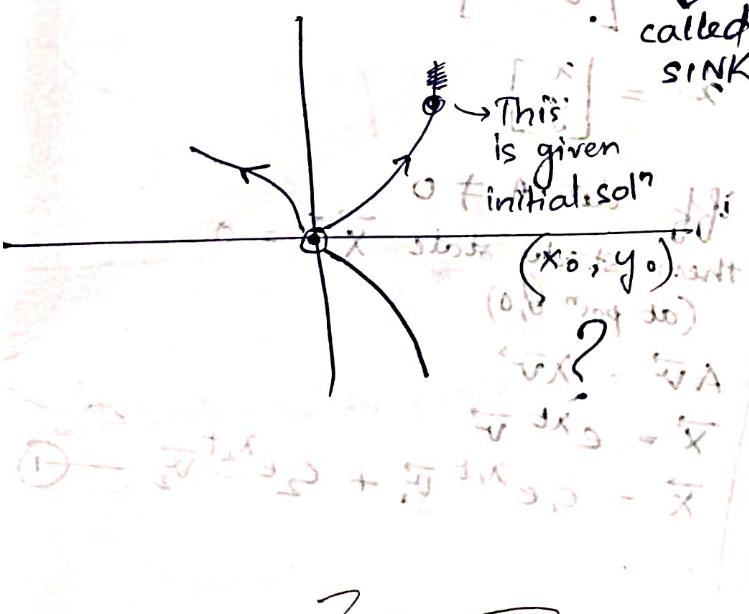
$\det A \neq 0$

Trivial sol^n we have

→ In order to find s.s.
→ calculate det
→ if $\det \neq 0 \Rightarrow$ sol^n is Trivial.

Fixed pt. is $\boxed{\text{stable}}$





$\vec{x} = \begin{bmatrix} x \\ y \end{bmatrix}$

$\boxed{\vec{x} = e^{\lambda t}\vec{v}}$ — ① → eigenvector of coeff. Matrix [A]

λ → eigenvalue.

Let's prove eq^n ①

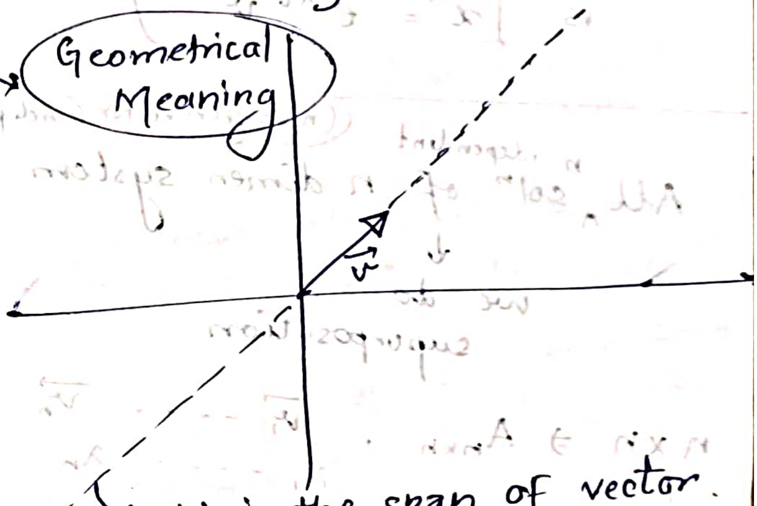
$\boxed{A\vec{v} = \lambda\vec{v}}$ — ②

$(\vec{v} \neq 0)$

λ → scalar

A is a operator does linear operations.

A does operation on $\vec{v}$ = eigenvector it gives same vector multiplied by scalar



This is the span of vector. When A does operation of eigenvector $\vec{v} \Rightarrow$ it can squeeze or enlarge or (flip) a direction but on that span of vector only.

$\left.\begin{aligned} \dot{x} &= ax + by \\ \dot{y} &= dx + ey \end{aligned}\right\}$ system

↓

its sol^n is

$\boxed{\vec{x} = e^{\lambda t}\vec{v}}$ when $\det \neq 0$

$\vec{v}$ → Not depend on time (const term)

$$\frac{d}{dt}(\vec{x}) = \dot{\vec{x}}$$

$$= \lambda e^{\lambda t}\vec{v}$$

$$= e^{\lambda t}\lambda v$$

$\lambda v \to A\vec{v}$

$$= e^{\lambda t}A\vec{v}$$

$$= A(e^{\lambda t}\vec{v})$$

$$= A\vec{x}$$

Thus proved $\boxed{\dot{\vec{x}} = A\vec{x}}$

---

general sol^n of a system where $\det \neq 0$ is $\boxed{\vec{x} = e^{\lambda t}\vec{v}}$

All n idependent sol^n of n dimen system (n eigen vector (indep)

↓

we do superposition.

$n \times n \ni A_{n\times n}$, $\vec{v_1} \cdots \vec{v_n}$

$\lambda_1 \cdots \lambda_n$

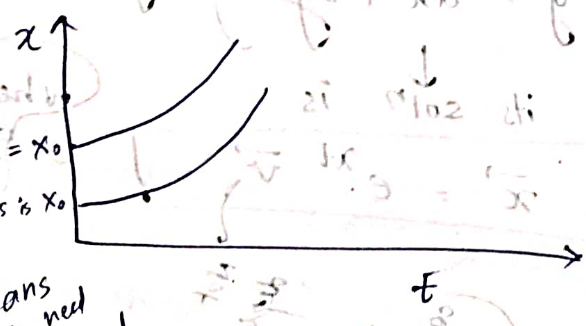
$$\boxed{\vec{X} = c_1 e^{\lambda_1 t}\vec{v_1} + c_2 e^{\lambda_2 t}\vec{v_2} + \cdots + c_n e^{\lambda_n t}\vec{v_n}}$$

This is generalised sol^n.

How to find c

say, $\frac{dx}{dt} = kx$

$x(t) = x_0 e^{kt}$



if $x = x_0$

if this is x_0

↓

means we need initial value

Let's consider 2 D. sol^n

$$\boxed{\vec{X} = c_1 e^{\lambda_1 t}\vec{v_1} + c_2 e^{\lambda_2 t}\vec{v_2}} \quad (1)$$

given, $t = 0$

$x(0) = x_0$

$y(0) = y_0$

at $t = 0$, $\vec{X} = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}$

$$\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = c_1\vec{v_1} + c_2\vec{v_2}$$

$$\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = c_1\begin{bmatrix} v_{11} \\ v_{12} \end{bmatrix} + c_2\begin{bmatrix} v_{21} \\ v_{22} \end{bmatrix}$$

$$\left.\begin{aligned} x_0 &= c_1 v_{11} + c_2 v_{21} \\ y_0 &= c_1 v_{12} + c_2 v_{22} \end{aligned}\right\}$$ Solve This we ge[t] c_1 & c_2

13/08/2[...]

L.H.S.D. Eq.

2nd order

$$\left.\begin{aligned} \frac{dx}{dt} &= ax + by \\ \frac{dy}{dt} &= cx + dy \end{aligned}\right\} \quad \dot{\vec{X}} = A\vec{X}$$

$$\dot{\vec{X}} = \begin{bmatrix} dx/dt \\ dy/dt \end{bmatrix}$$

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\vec{x} = \begin{bmatrix} x \\ y \end{bmatrix}$$

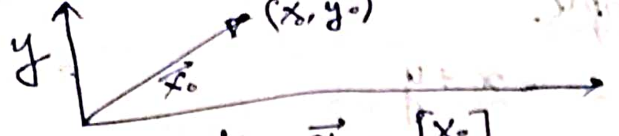
iff $\det A \neq 0$

then stedy state $\vec{X}^* = 0$ (at pos^n 0,0)

$A\vec{v} = \lambda\vec{v}$

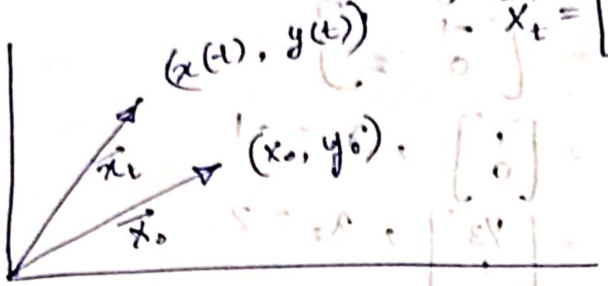
$\vec{X} = e^{\lambda t}\vec{v}$

$\vec{X} = c_1 e^{\lambda_1 t}\vec{v_1} + c_2 e^{\lambda_2 t}\vec{v_2}$



position vector $\vec{x}_0 = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}$

$\vec{x}_t = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$



$\vec{x}_0$ vector used to calculate c_1 & c_2

Example :

$$\frac{dx}{dt} = -x + y$$

$$\frac{dy}{dt} = -2y$$

Say initial posn $\vec{x} = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$

$$A = \begin{bmatrix} -1 & 1 \\ 0 & -2 \end{bmatrix}$$

$\det A = +2 - 1 = 1 \neq 0$

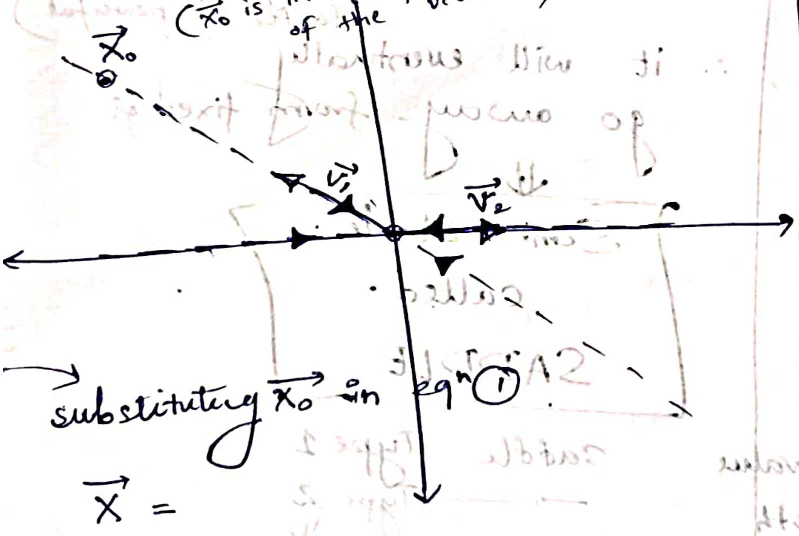
$\therefore$ steady state $= (0,0) = \vec{x}^*$

eigen vectors & corresponding eigen value.

$\vec{v}_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ $\lambda_1 = -2$

$\vec{v}_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\lambda_2 = -1$

($\vec{x}_0$ is in the span of the vector $\vec{v}_1$)



substituting $\vec{x}_0$ in eqn (1)

$\vec{x} =$

$\vec{x} = c_1 e^{\lambda_1 t} \vec{v}_1$

$\vec{x} = 2 e^{-2t} \vec{v}_1$

$\vec{x} = e^{-2t} (2\vec{v}_1)$

at $t=0 \Rightarrow e^0$ = becomes 1.

at $t=1 \Rightarrow e^{-2} \Rightarrow$ becomes < 1

as $t \uparrow\uparrow$ value $\downarrow\downarrow$

if I start at initial value then as t progress to $+\infty$ it will move towards zero along vector span.

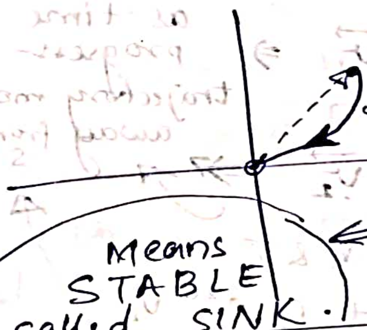
say initial value at $(3,0)$

$\vec{x} = c_2 e^{\lambda_2 t} \vec{v}_2$

$\vec{x} = e^{-1t} (3 v_2)$

eigen value -ve $\Rightarrow$ motion will shrink with time

say now start at anywhere other than vector span.



while shrinking it will take this path.

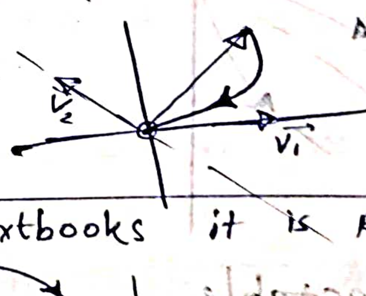
collapse at fixed pt.

Means STABLE called SINK.

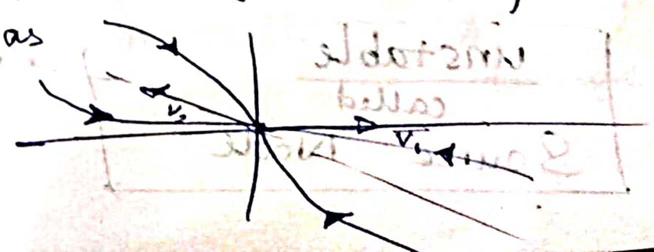
$\vec{x} = c_1 e^{\lambda_1 t} \vec{v}_1 + c_2 e^{\lambda_2 t} \vec{v}_2$

$c_1 e^{-2t} \vec{v}_1 + c_2 e^{-1t} \vec{v}_2$

This will shrink faster compared to this.



In textbooks it is Represented as



Example

$\frac{dx}{dt} = x + y$

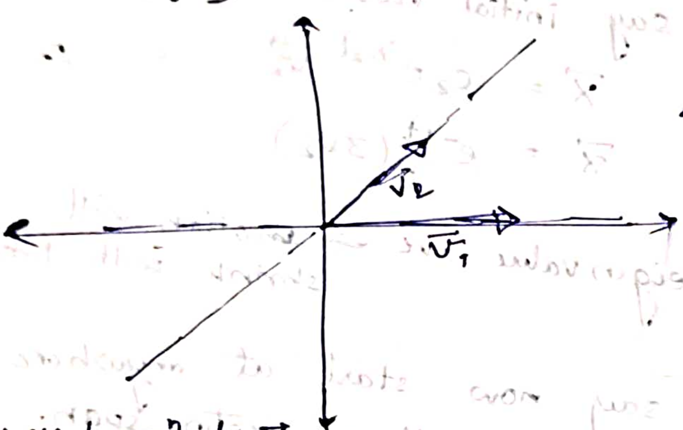
$\frac{dy}{dt} = 2y$

$A = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} \Rightarrow \det \neq 0$

$\therefore$ S.S $= (0,0)$

$\vec{v_1} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $\lambda_1 = 1$

$\vec{v_2} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $\lambda_2 = 2$.



initial posn at $\vec{v_1}$

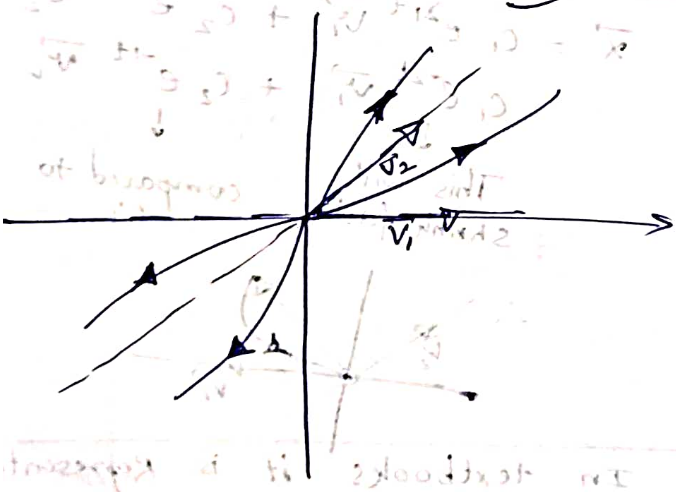
$\vec{X} = c_1 e^{1t} \vec{v_1} \Rightarrow$ as time progress trajectory move away from S.S.

init. posn at $\vec{v_2}$

$\vec{X} = c_2 e^{2t} \vec{v_2} \Rightarrow$

initial posn other at $\vec{v_1}$ & $\vec{v_2}$

$\vec{X} = c_1 e^{1t} \vec{v_1} + c_2 e^{2t} \vec{v_2}$



$\boxed{\text{unstable called Source Node}}$ — eigenvalue both +ve.

Example.

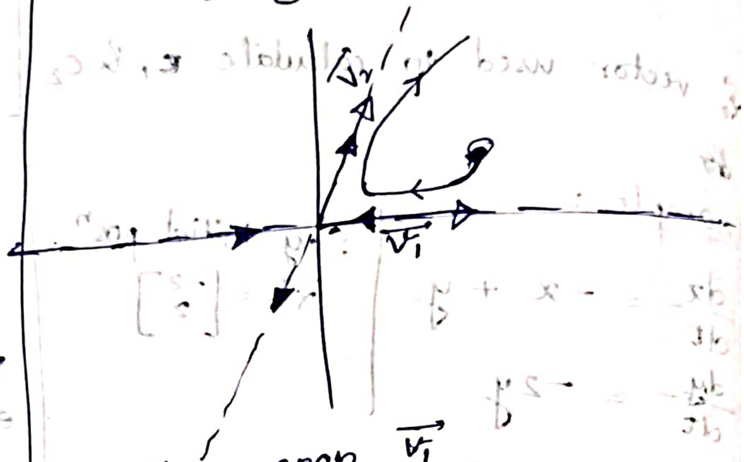
$\frac{dx}{dt} = -x + y$

$\frac{dy}{dt} = 2y$

$A = \begin{bmatrix} -1 & 1 \\ 0 & 2 \end{bmatrix}$

$\vec{v_1} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $\lambda_1 = -1$

$\vec{v_2} = \begin{bmatrix} 1/3 \\ 1 \end{bmatrix}$, $\lambda_2 = 2$.



start on span $\vec{v_1}$

$\vec{X} = c_1 e^{\lambda_1 t} \vec{v_1} = c_1 e^{-1t} \vec{v_1}$

as $t \uparrow\uparrow \Rightarrow \vec{X}$ will collapse at 0

start at $\vec{v_2}$

$\vec{X} = c_2 e^{\lambda_2 t} \vec{v_2}$

$\vec{X} = c_2 e^{2t} \vec{v_2}$

as $t \uparrow\uparrow \Rightarrow \vec{X}$ will move away

start in the middle

$\vec{X} = c_1 e^{-1t} \vec{v_1} + \underbrace{c_2 e^{2t} \vec{v_2}}_{\text{moving away faster (its powerful)}}$

$\therefore$ it will eventually go away from fixed pt.

$\Downarrow$

$\boxed{\text{Semi-stable called SADDLE}}$

saddle Type 1

Type 2 $\Rightarrow$ when if its 4D & 2 eigenvalue -ve 2 eigenvalue +ve when $\lambda_1, \lambda_2 < 0 \Rightarrow$ stable / sink

$\lambda_1, \lambda_2 > 0 \Rightarrow$ unstable / source

$\lambda_1 < 0$ & $\lambda_2 > 0 \Rightarrow$ saddle

In short we need sign of eigenvalues
So how to calculate sign of eigenvalues?

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$tr A = a + b$

$\det A = ad - bc$

eigenvalues: $\lambda = \dfrac{tr A \pm \sqrt{trA^2 - 4 \det A}}{2}$

L.H.S. ODE 20/8/24.

$\dot{\vec{x}} = A\vec{x}$

$A = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$

$\det A \neq 0 \rightarrow \vec{x}^* = (0,0)$

$A\vec{v} = \lambda \vec{v}$

$\vec{x} = c_1 e^{\lambda_1 t} \vec{v_1} + c_2 e^{\lambda_2 t} \vec{v_2}$

$\lambda_1, \lambda_2 < 0 \Rightarrow x^* \rightarrow$ stable

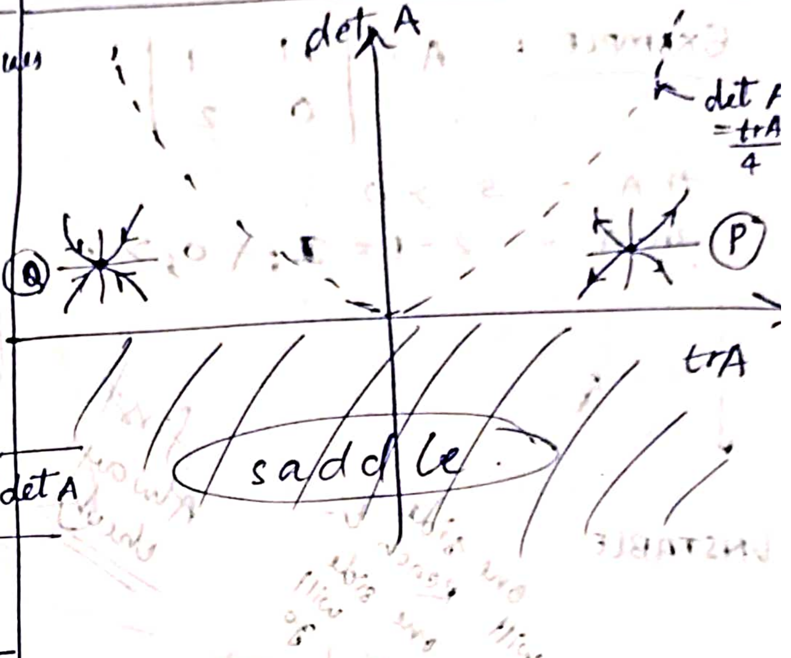
$\lambda_1, \lambda_2 > 0 \Rightarrow x^* \rightarrow$ unstable.

$\lambda_1 < 0, \lambda_2 > 0 \Rightarrow x^* \rightarrow$ saddle

$$\lambda = \frac{tr A \pm \sqrt{trA^2 - 4 \det A}}{2}$$

$\boxed{\det A = \frac{tr^2}{4}}$

Its a parabola.
$\rightarrow$ open to up since +ve trace.



we have ex:

$$A = \begin{vmatrix} -1 & 1 \\ 0 & -2 \end{vmatrix} \Bigg\} \begin{matrix} trA < 0 \\ \det A > 0 \end{matrix}$$

$tr A = -3$

$\det A = 2 - 0 = 2 \neq 0$

$\Rightarrow \boxed{trA^2 > 4 \det A}$

Cond^n: $trA < 0$, $\det A > 0$ & $trA^2 \geq 4 \det A$.

So the pt lies in 3rd quadr
So at pos^n Q: we can see that its SINK (stable)

$$\lambda = \frac{\overset{-ve}{trA} \pm \overset{+ve}{\sqrt{trA^2 - 4\det A}}}{2}$$

λ = all will be $-ve$.

λ_1 & $\lambda_2 \Rightarrow -ve$

$\therefore$ its stable.

because

det = 'multiplicⁿ' of eigenvalue

EXAMPLE : $A = \begin{vmatrix} +1 & 1 \\ 0 & 2 \end{vmatrix}$

$tr\,A = 3 > 0$

$det\,A = 2 - 0 = 2 ; \neq 0 ; > 0$

UNSTABLE

one side will reach & one side will go

eventually will go.

first Always check

⇒ if det A = −ve

⇓

means its saddle.

Example :

$A = \begin{vmatrix} -1 & 1 \\ 0 & 2 \end{vmatrix}$

$tr\,A > 0$

$det < 0$

$$\lambda = \frac{\overbrace{trA}^{+ve} \pm \overbrace{\sqrt{D}}^{\text{bigger than } trA}}{2}$$

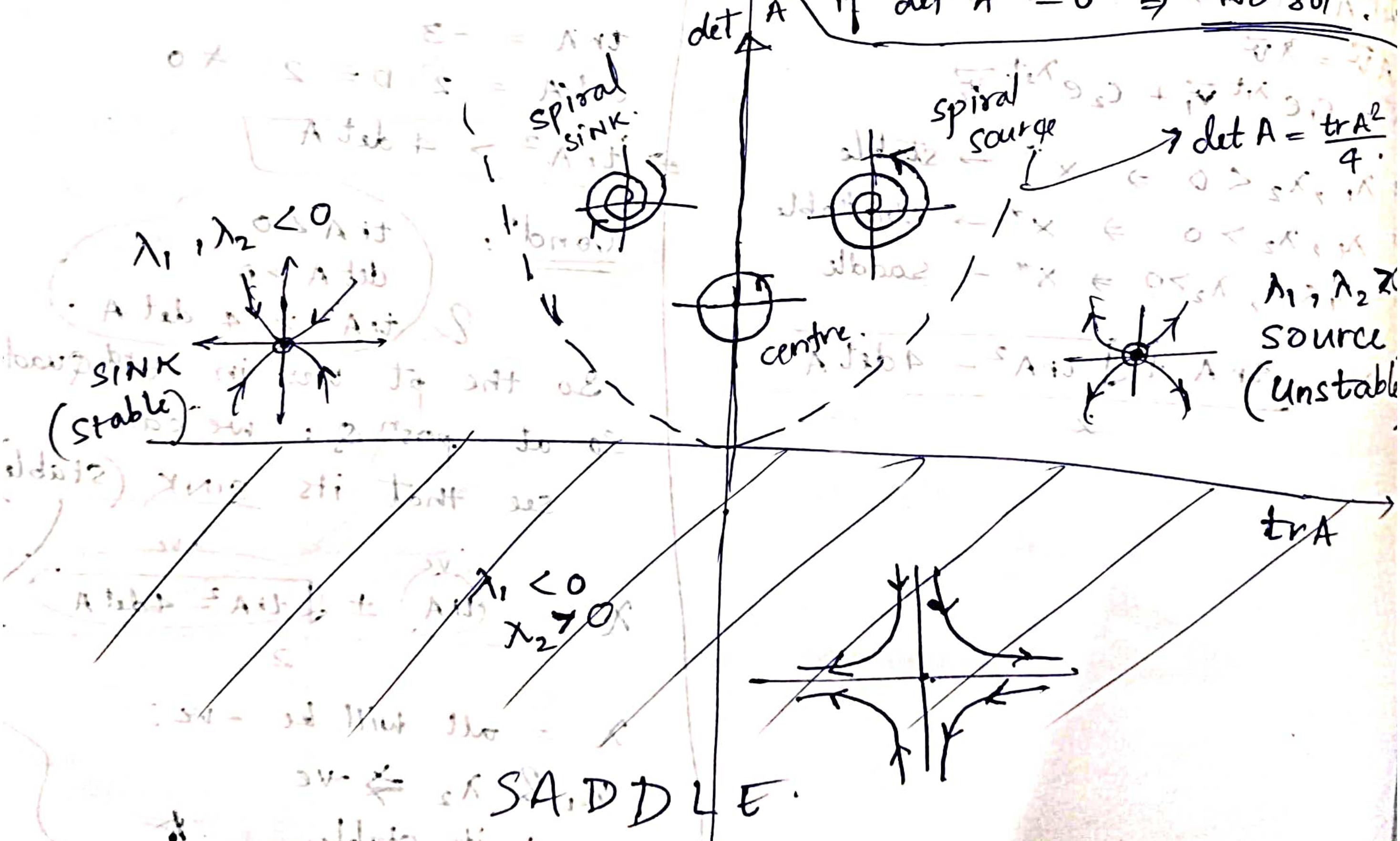
if det A = +ve

calculate trace A.

check $trA^2 > 4\,det\,A$?

$trA^2 < 4\,det\,A$

if det A = 0 ⇒ No solⁿ.



Now if $trA = 0$, $\det A > 0$ on this line

$\lambda_1 = zi$

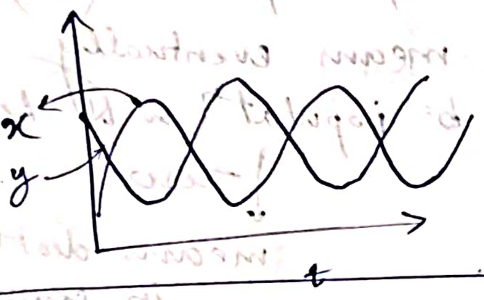
$\lambda_2 = -zi$ → means eigenvectors will also be vector.

eigenvalues complex, then what will be behaviour!

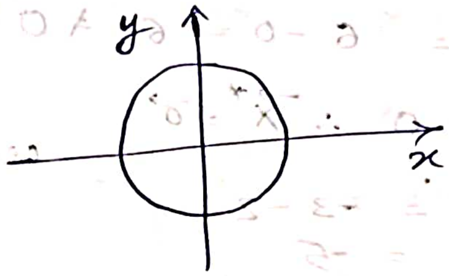
$\vec{x}$ is $\begin{bmatrix} x \\ y \end{bmatrix}$ → is a vector

sol^n of sys: $\boxed{\vec{x} = c_1 e^{zit}\,\vec{v_1} + c_2 e^{-zit}\,\vec{v_2}}$

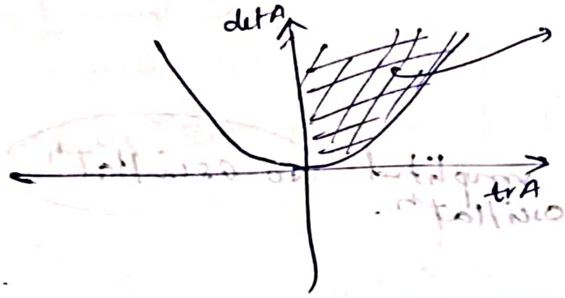
we have ⇒ $e^{iz} = \cos(z) + i\sin(z)$



⇒



x & y are having cos & sin terms

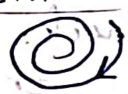


→ complex Zone

$e^{-zit}\,\vec{v}$

-ive sign exponential decrement

∴ spiral sink



$e^{zit}\,\vec{v}$

$e^{(r+zi)t}\,\vec{v}$

$e^{r}\,e^{zit}\,\vec{v}$ → giving circular Motion

→ giving exponential factor. away

∴ spiral source

what is phase potrait?

phase potrait will be SINK / spiral sink / saddle.

PROBLEM

(X) → (Y)
↓ ↓
ϕ ϕ

$$\frac{dx}{dt} = -3x$$

$$\frac{dy}{dt} = 2x - 2y$$

cell X → differ... into cell Y

both will eventually die.

Solⁿ:

$$A = \begin{bmatrix} -3 & 0 \\ 2 & -2 \end{bmatrix}$$

check det A = 6 − 0 = 6 ≠ 0 ⇒ ∴ fixed pt. at (0,0)

det A ≠ 0 ∴ $\vec{x}^* = \vec{0}$

means eventually cell populatⁿ will become zero.

means death in consi...

⇒ Now trace A = −3 − 2
= −5

means in 3rd quadrant.

stable

- Spiral sink → means damped oscillatⁿ
- SINK → bigger amplitude oscillatⁿ — No oscillatⁿ

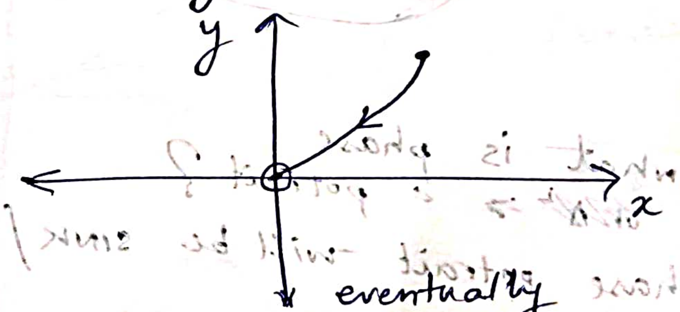
check:

⇒ $tr A^2 = (-5)^2 = 25$
$4 \det A = 4(6) = 24$

$tr A^2 > 4 \det A$

means ⇒ SINK (stable)

both eigen values Real −ve No ∴ system behavior.



eventually collapse here.

Stable SINK

21/08/24

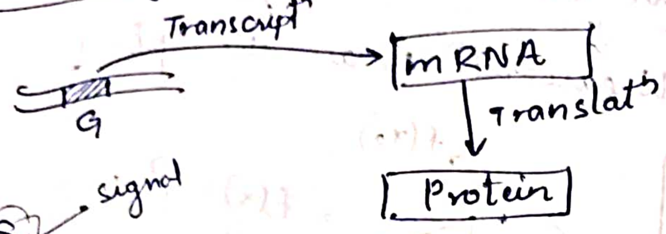
each cell has genome.

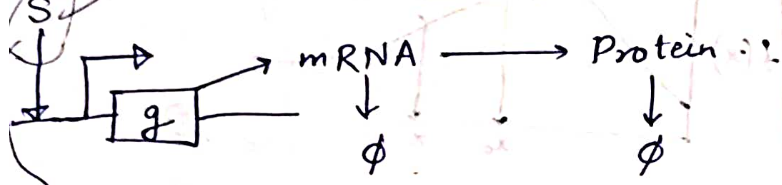
↓

The gene (by process of Transcript^n) we get mRNA.

↓

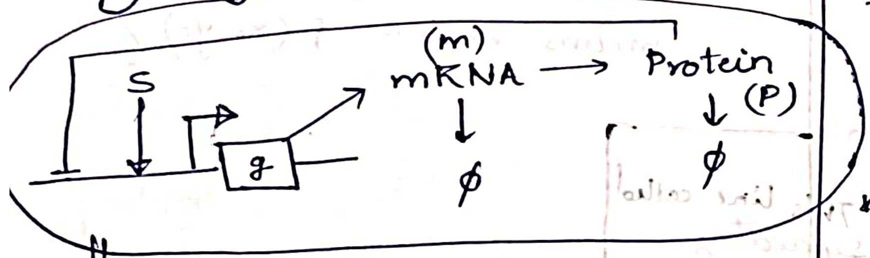
By Translat^n we get Protein.





signal which is Triggering the Rxn ⇒ ↓ (+vely)

-vely Regulated ⇒ ⊥ (Inhibit^n)



⇓

Negative Autoregulation.

Rxn Regulated by S & P.

$$\frac{dm}{dt} = \left(\frac{S}{1+S}\right) + \left(\frac{1}{1+P}\right)$$

also Rxn will decay at last

$$\therefore \boxed{\frac{dm}{dt} = \left(\frac{S}{1+S}\right) + \left(\frac{1}{1+P}\right) - m} \text{ ---- (1)}$$

(-m) term used.

$$\boxed{\frac{dP}{dt} = m - P} \text{ ---- (2)}$$

where $m, P \geq 0$

(1) & (2) are not L.H.S. ODE ⇒ can't get A

$$\Rightarrow \frac{dm}{dt} = \left(\frac{S}{1+S}\right) + \left(\frac{1}{1+P}\right) - m$$

$$\frac{dP}{dt} = m - P$$

At Steady State:

$$\frac{dm}{dt} = \frac{dP}{dt} = 0$$

$\because$ S is constant ⇒ $\frac{S}{1+S}$ ⇒ constant

$\therefore$ we can say $\frac{S}{1+S} = z$

$$\frac{dm}{dt} = z + \left(\frac{1}{1+p}\right) - m = 0 \text{ (for s.s)}$$

$$\frac{dP}{dt} = m - p = 0 \Rightarrow m = p.$$

$$z + \left(\frac{1}{1+p}\right) - p = 0 \text{ ---- (3)}$$

Say $S_1 = 1 \Rightarrow z = \frac{S}{1+S} = \frac{1}{1+1} = \frac{1}{2}$

substitute $z = \frac{1}{2}$ in (3) & solve quadratic.

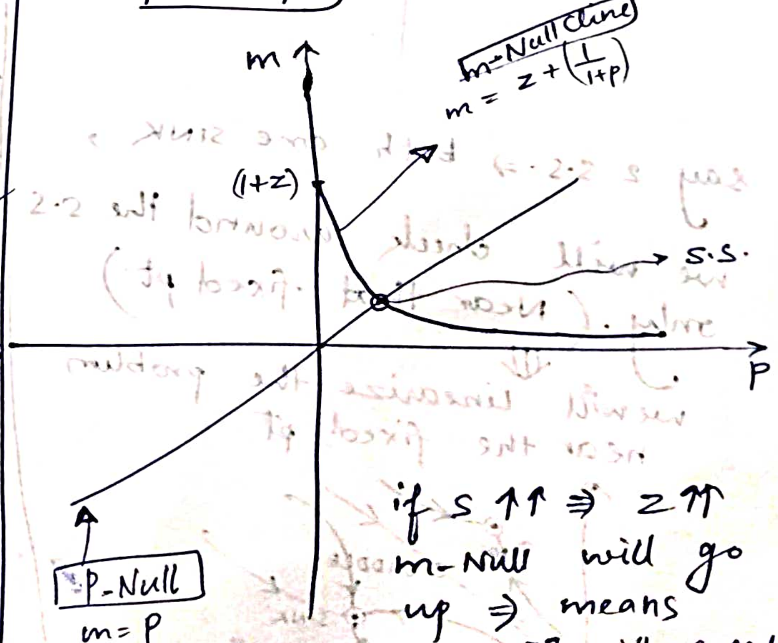
m - Null cline

$$\frac{dm}{dt} = \cancel{\left(\frac{S}{1+S}\right)}^{z} + \left(\frac{1}{1+P}\right) - m = 0$$

$$\boxed{z + \left(\frac{1}{1+p}\right) = m}$$

p - Null cline.

$$\boxed{m = p}$$

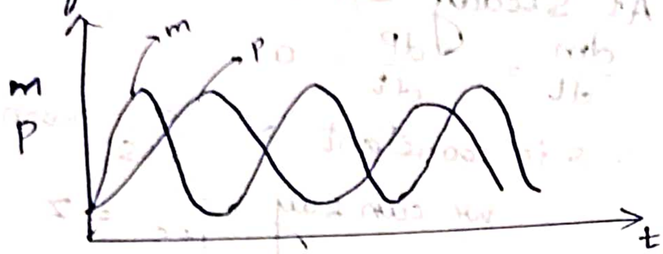


if S ↑↑ ⇒ z ↑↑

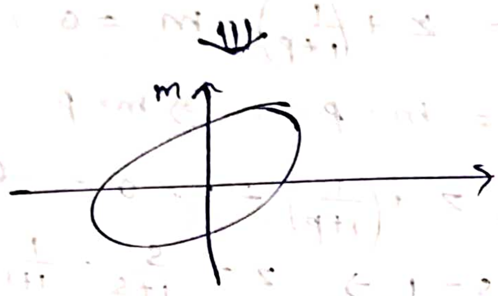
m-Null will go up ⇒ means one ✗ will p-Null

-ve feedback $\Rightarrow$ gives rise to oscillatⁿ

if oscillations are present



$\Downarrow$



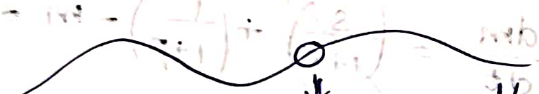
centre type.

---

if its oscillatⁿ $\Rightarrow$ means N. Linear

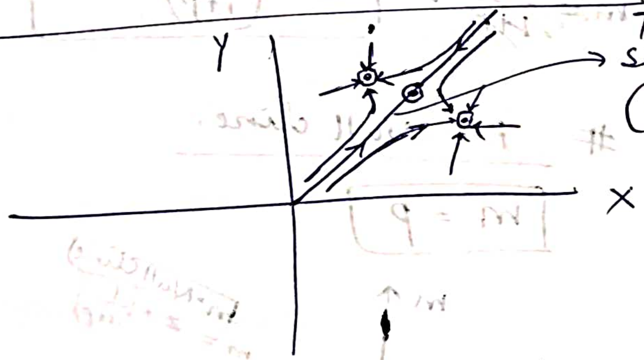
$\Downarrow$

in order to solve $\Rightarrow$ consider it linear.



$\Downarrow$

consider small area.



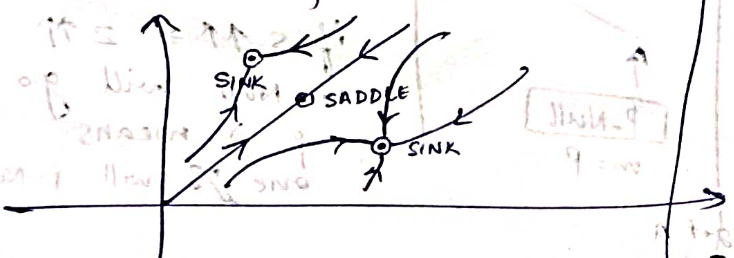
This line called separatrix. (saddle pt.)

say 2 S.S. $\Rightarrow$ both are SINK,

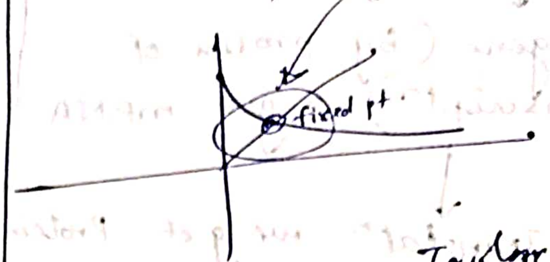
we will check around the S.S only. (Near that fixed pt.)

$\Downarrow$

we will Linearize the problem near the fixed pt.



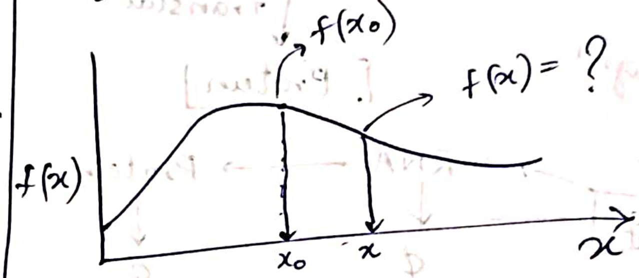
So we will be studying around the fixed pt.



we will use Taylor Series.

$\downarrow$

Linearizatⁿ Near the pt.



$\Rightarrow f(x) = f(x_0) + f'(x_0)(x - x_0)$

we have fⁿ. $F(x, y)$

Need to know fⁿ at around/near x_0 & y_0

means what is $f(x_0, y_0) = ?$

$\Rightarrow f(x) = f(x_0) + f'(x_0)(x - x_0)$

$\Rightarrow \cancel{f(x,y) = f(x,y)}$

$\Rightarrow f(x, y) = f(x_0, y_0) + f_x(x_0, y_0)(x - x_0)$

$+ f_y(x_0, y_0)(y - y_0)$

$+ \cancel{f_{xx}(x_0, y_0)\frac{(x - x_0)^2}{2!}} \ldots$

ignored higher terms.

$$\frac{dx}{dt} = f(x,y)$$

$$\frac{dy}{dt} = g(x,y)$$

Autonomous ∵ still t is not present.

say we have steady state (x_0, y_0)

↓

Need to Linearize system around (x_0, y_0)

∵ x_0 & y_0 are S.S. ⇒ $\frac{dx}{dt} = 0 = f(x_0, ...$

$$\therefore \frac{dx}{dt} = f(x,y) = \cancel{f(x_0,y_0)}^{\,0} + f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0)$$

$$\frac{dy}{dt} = g(x,y) = \cancel{g(x_0,y_0)}^{\,0} + g_x(x_0,y_0)(x-x_0) + g_y(x_0,y_0)(y-y_0)$$

These are constant No. ⇓ similar to coeff Matrix A.

$$\Rightarrow \begin{bmatrix} dx/dt \\ dy/dt \end{bmatrix} = \begin{bmatrix} f_x(x_0,y_0) & f_y(x_0,y_0) \\ g_x(x_0,y_0) & g_y(x_0,y_0) \end{bmatrix} \begin{bmatrix} x-x_0 \\ y-y_0 \end{bmatrix}$$

⇒ Taking $(x-x_0) = m$ & $(y-y_0) = n$

so, $dx/dt = \frac{dm}{dt}$ & $\frac{dy}{dt} = dn/dt$.

partial derivative wrt. x

$$\Rightarrow \begin{bmatrix} dm/dt \\ dn/dt \end{bmatrix} = \underbrace{\begin{bmatrix} f_x & f_y \\ g_x & g_y \end{bmatrix}_{(x_0,y_0)}} \begin{bmatrix} m \\ n \end{bmatrix}$$

called JACOBIAN MATRIX.

$$\underbrace{\vec{\dot{m}} = A \vec{m}}$$

Linear Homogeneous System of ODE.

⇓

⇒ This system's S.S is zero $(0,0)$

⇓

⇒ equivalent to (x_0, y_0) in Real system.

26/08/24.

$$\frac{dx}{dt} = f(x,y)$$
$$\frac{dy}{dt} = g(x,y)$$

① fixed pt. ⟶

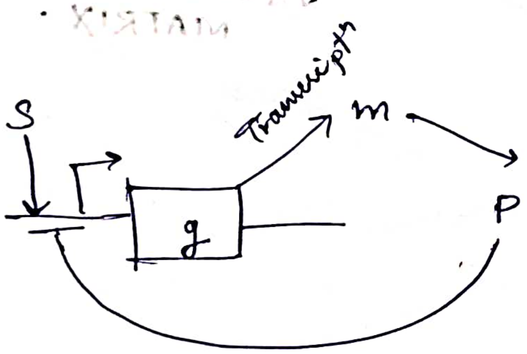
~~Visua~~

a) Algebric Method.

② Linearize around a s.s.

$$J = \begin{vmatrix} f_x & f_y \\ g_x & g_y \end{vmatrix}_{(x^*, y^*)}$$

$$\vec{\dot{m}} = \underbrace{J}_{\text{coeff. Matrix.}} \vec{m}$$



(P) ≡ -ve Regulatⁿ

$$\boxed{\frac{dm}{dt} = \left(\frac{s}{1+s}\right) + \underbrace{\left(\frac{1}{1+p}\right)}_{\text{-ve regulatⁿ}} - m}$$ (1st order decay)

$$\boxed{\frac{dp}{dt} = m - p}$$

These are N. Linear System.
1st step to find s.s. (fixed pt)
(we will use Method of Null Cline)

m - Null Cline

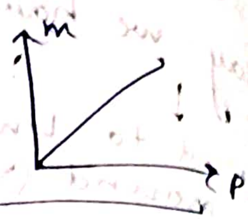
$\Rightarrow \frac{dm}{dt} = 0 \Rightarrow$

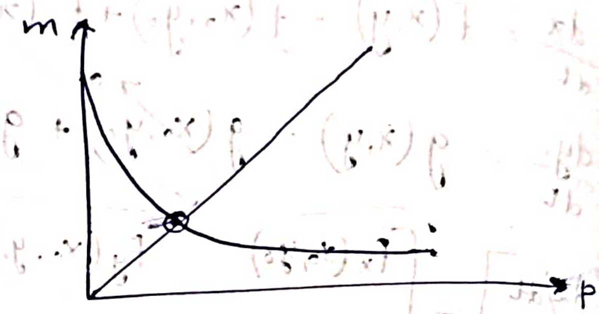
$\dot{m} = \left(\frac{s}{1+s}\right) + \left(\frac{1}{1+p}\right)$

p - Null cline

$\frac{dp}{dt} = 0$

$\Rightarrow m = p$





we will see behaviour of sy[stem] around fixed pt.

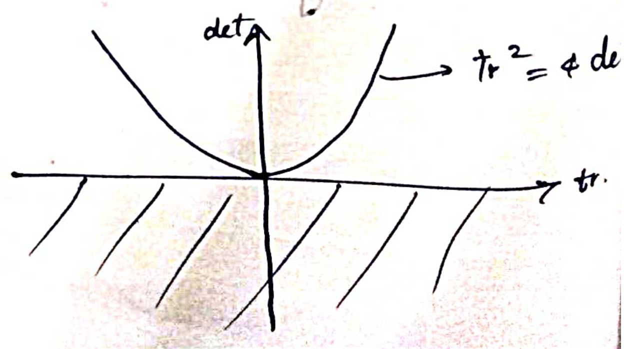
(g) ≡ -ve Auto Regulatⁿ

will do Linearizatⁿ of syste[m]

$$J = \begin{vmatrix} f_x & f_y \\ g_x & g_y \end{vmatrix} = \begin{vmatrix} f_m & f_p \\ g_m & g_p \end{vmatrix}$$

$$J = \begin{vmatrix} -1 & -\frac{1}{(1+p)^2} \\ 1 & -1 \end{vmatrix}$$

Need to check the sign of eigen values.
∵ its 2 D problem, we can use trace - det. plot



Check det

$$\det J = (-1)(-1) - \left[\left(\frac{-1}{(1+p)^2}\right)(1)\right]$$

$$= 1 + \underbrace{\frac{1}{(1+p)^2}}$$

This is +ve Number
∴ can't be zero
& can't be -ve
} ∴ Not saddle

Now find Trace.

$\mathrm{tr}\,J = -1 - 1 = -2$
⇒ means I am in 3rd Quad.
⇒ means its some sort of SINK

Now check if
$\mathrm{tr}^2 > 4\det$?

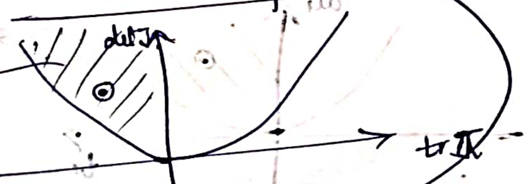
$\mathrm{tr}J^2 = (-2)^2 = 4$

$$4\det J = 4\left(1 + \frac{1}{(1+p)^2}\right)$$

$$= 4 + \frac{4}{(1+p)^2}$$

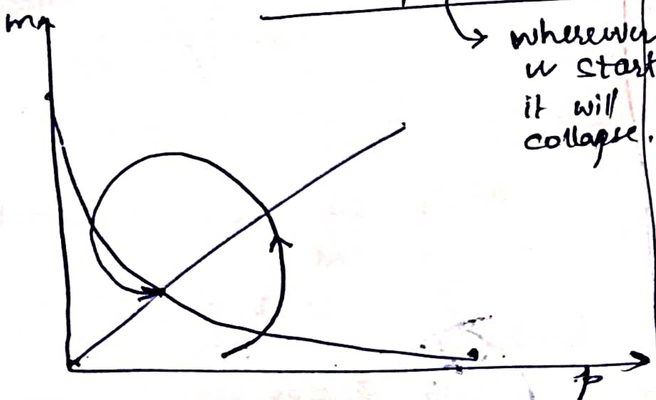
we see, $\underbrace{\mathrm{tr}J^2 < 4\det J}$

means my eigen values are complex No.



means its Spiral Sink.

→ wherever u start it will collapse.

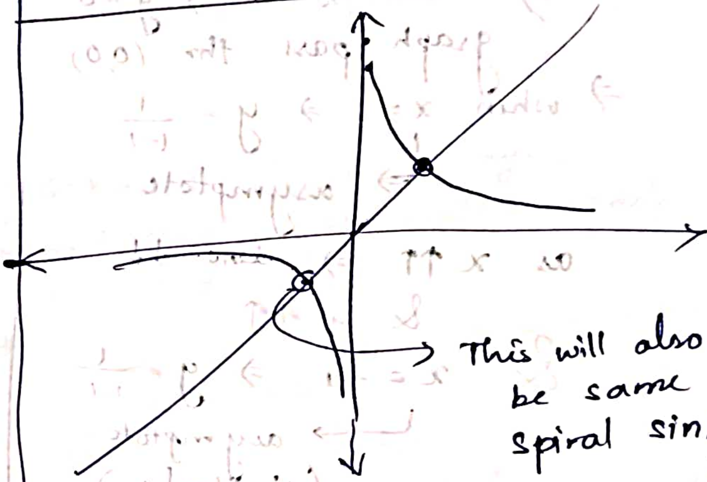


-ve Auto Regulat (-ve feedback)
→ gives stable S.S.
→ ~~gives stable Node~~

Oscillatⁿ

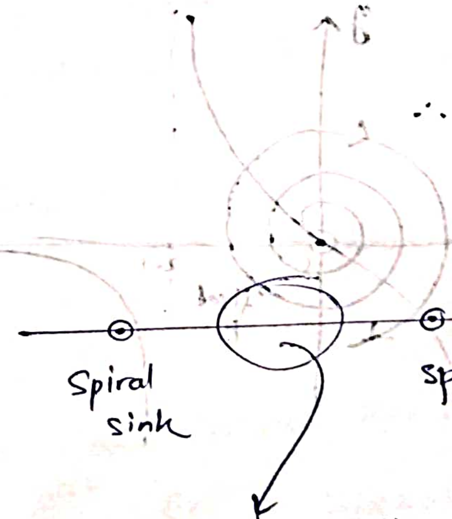
• when we need oscillatⁿ
⇒ we can find on trace J = 0 line.

-ve Regulatⁿ, is favourable as it gives stable S.S.



This will also be same spiral sink.

we have,
$\mathrm{tr}\,J = 4$
$4\det J = 4 + 4/(1+p)^2$ → This is square term ∴ No diff if the p is +ve / -ve.



This position must be unstable.

SINK ≡ valley
UNSTABLE ≡ HILL

⇒ So There are 2 valley ⇒ in Middle there must be Hill

P.T.O.

If Not sink, what it could be ?

$\Rightarrow \dfrac{dx}{dt} = y$

for S.S $\Rightarrow dx/dt = 0 \Rightarrow \boxed{y = 0}$ —— ①

$\dfrac{dy}{dt} = \mu(1-x^2)\,y - x$

say $\mu = 1$

for S.S :

$\hookrightarrow \dfrac{dy}{dt} = (1-x^2)\,y - x = 0$

$\Rightarrow \boxed{y = \dfrac{x}{1-x^2}}$ —— ②

$\Rightarrow$ when $x = 0 \Rightarrow y = 0$

graph : pass thr. $(0,0)$

$\Rightarrow$ when $x = 1 \Rightarrow y = \dfrac{1}{1-1}$

$\hookrightarrow$ asymptote

as $x \uparrow\uparrow \Rightarrow$ deno. $\downarrow\downarrow$

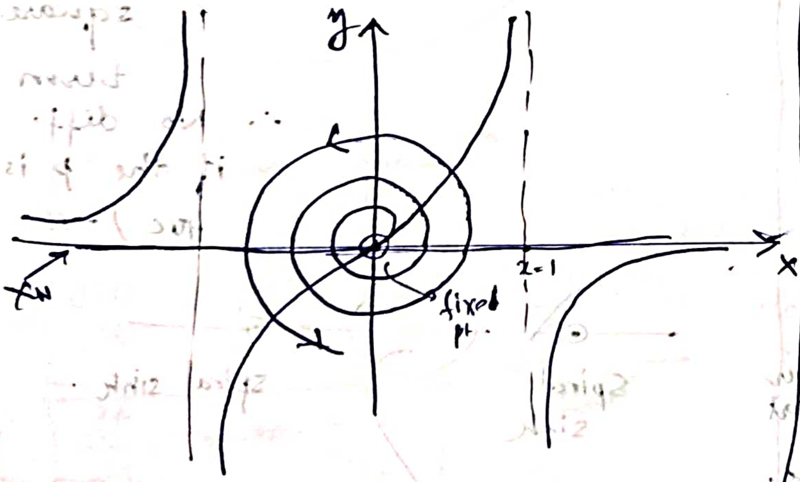
& $y \uparrow\uparrow$

& $x = -1 \Rightarrow y = \dfrac{1}{1-1}$

$\hookrightarrow$ asymptote. (similarly)

$x > 1 \Rightarrow y = \dfrac{1}{1-4}$

& Now draw graph of ① & ②



Now Jacobian,

$\dfrac{dy}{dt} = y - yx^2 - x \;;\; \dfrac{dx}{dt} = y$

$$J = \begin{vmatrix} 0 & 1 \\ -1-2xy & 1-x^2 \end{vmatrix}.$$

Jacobian at SS $= (0,0)$

$$J = \begin{vmatrix} 0 & 1 \\ -1-2(0) & 1-(0) \end{vmatrix}$$

$$J = \begin{vmatrix} 0 & 1 \\ -1 & 1 \end{vmatrix} \;\text{----}\; \begin{pmatrix} \text{at} \\ x=0 \\ y=0 \end{pmatrix}$$

$\det J = 1$

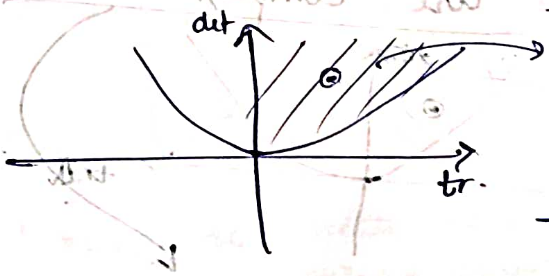
$\operatorname{tr} J = 1 \longrightarrow$ (eigenvalues +ve.)

$\operatorname{tr} J^2 = 1$

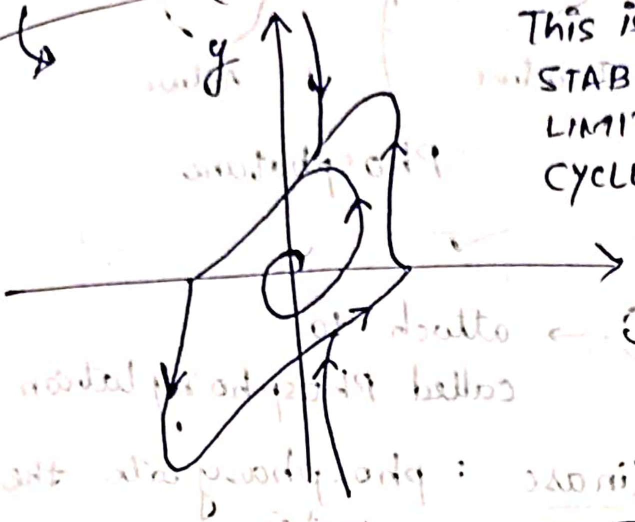
$4 \det J = 4(1) = 4$

$\underbrace{\operatorname{tr} J^2 < 4 \det J}$

means eigenvalues are complex Numbers $\Rightarrow$ Means This fixed point is a spiral source



graph close to this fixed pt $\Rightarrow$ when we zoom near $(0,0)$



This is STABLE LIMIT CYCLE

This is the global Analysis } which can't be obtained from Local analysis

Some textbooks write eq^n as follows :

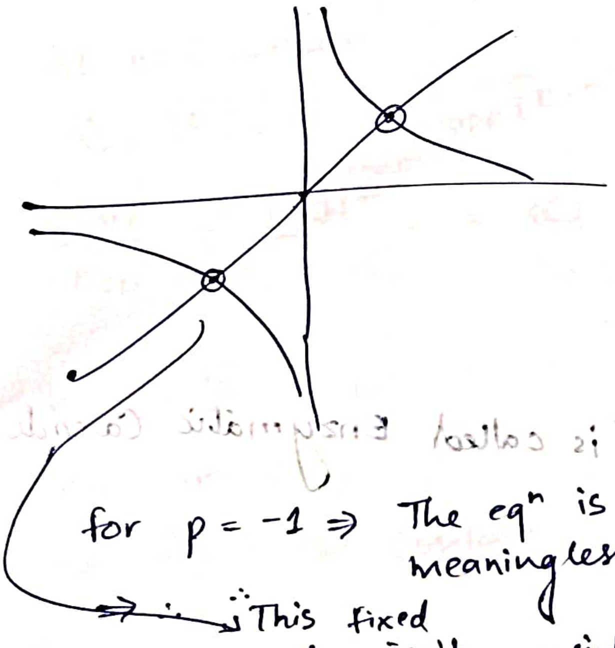
$$\boxed{\frac{d}{dt}\left(\frac{dx}{dt}\right) - \mu\left(1-x^2\right)\frac{dx}{dt} - x = 0}$$

$\hookrightarrow$ Vander Pole Damped Oscillat^n

27/08/24.

Compartment Model :

Primary organs are compartment model



for $p=-1 \Rightarrow$ The eq^n is meaningless.

$\therefore$ This fixed pt. isn't considered

$\therefore$ No Hill in b/w 2 fixed pt.

Phenomenological Model ?

Okham's Razor says,

$\to$ Don't create complicated model than Required.

$\to$ Similar model we will consider in biology.

Glucose $\longrightarrow$ metabolize & creates ATP for (ENERGY) $\to$ in form of ATP.

$\downarrow$

Requires Molecules (Protein)

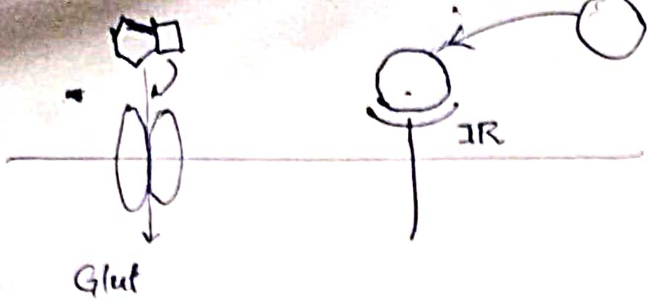
we need Transporter called Glut.

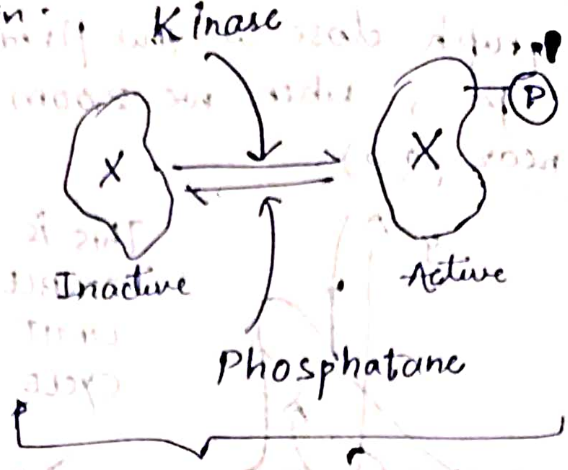
$\hookrightarrow$ say cell kept in glucose. glucose will not enter cell like that just

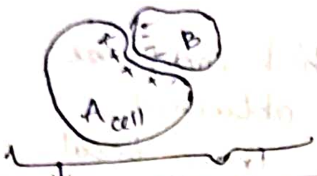
say,

Insulin $\to$ Messanger of cell to tell that there lots of glucose outside cell & thus it 'll produce Transporter.

There are Insulin Receptors (to Recieve Signals.







They are attracted & are having favourable Shape

Complex is formed → C / (A − B)

× Electrostatic Bond is formed.
& Thus Bond can be broken.

$A + B \longrightarrow$ can't form permanent C & is Reversible.

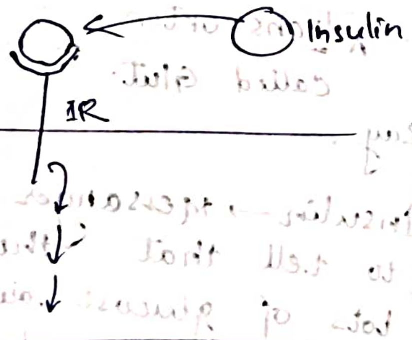
$A + B \rightleftharpoons C$

called Binding

Ⓟ → attach to called Phosphorylation.

Kinase : phosphorylate the protein.

Insulin Receptor (IR) will Trigger cascade of Rxns. as soon as Insulin Binds (fits) to the IR → must be Responsible for product of Glut.

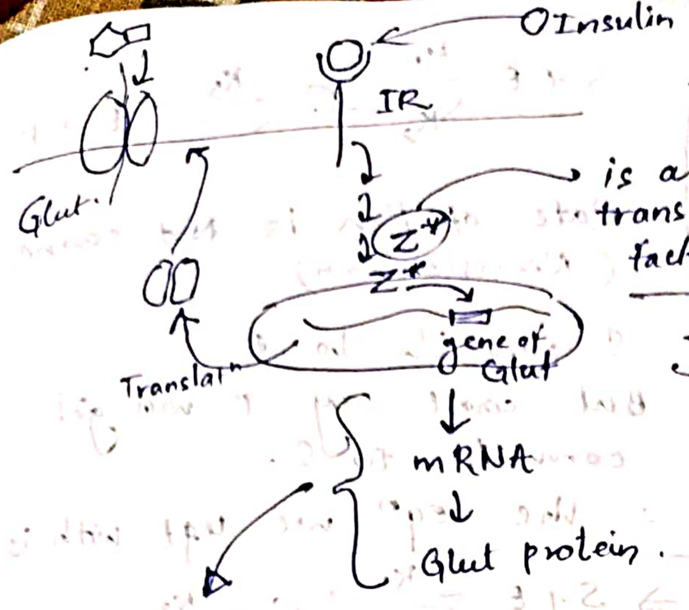


A ≡ Active molecules
A* ≡ Inactive.

$A \underset{C}{\overset{B}{\rightleftharpoons}} A^*$ — B may Helps to get Active

$A^* \rightarrow X \rightleftharpoons X^*$

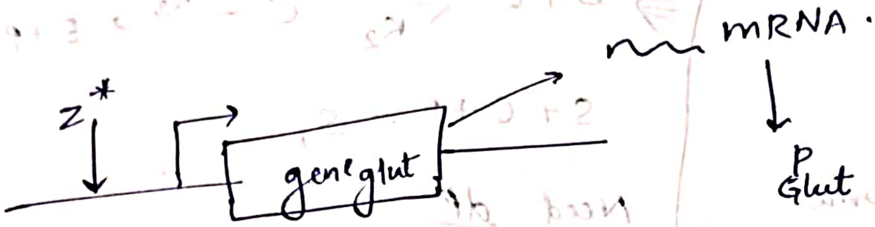
This is called Enzymatic Cascade



→ BINDING..

→ ENZYMATIC RXN

→ TRANSCRIPTION (prod^n of MRNA) & TRANSLATION (prod^n of protein)



Pathways
↳ to transfer informat^n.

① BINDING

$$L + R \underset{K_{OFF}}{\overset{K_{ON}}{\rightleftharpoons}} L{-}R$$

$$\frac{d[L-R]}{dt} = K_{ON}[L][R] - K_{OFF}[L-R]$$

↳ using Law of Mass-Action

$L \equiv$ Ligand

[sq. brackets are not molar concentr. but mass].

Enzyme is called Substrate.

At equilibrium ;

$$K_{ON}[L][R] = K_{OFF}[L-R]$$

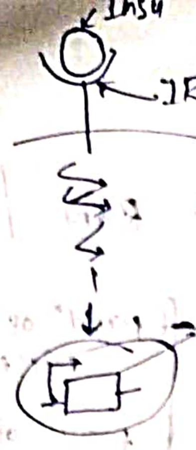
$$\frac{K_{OFF}}{K_{ON}} = \frac{[L][R]}{[L-R]} = K_d$$

↳ Dissociation constant.

K_{ON} unit $\equiv \frac{1}{tM}$

K_{OFF} unit $\equiv \frac{1}{t}$

K_d unit $\equiv$ M (Molar)



Binding

Enzymatic

Transcript^n & Translat^n.

$S \xrightarrow{E} P$

$$\frac{dP}{dt} = K S E$$

P gets degraded by some enzyme E_1 to ϕ.

$$S \xrightarrow{E} P \xrightarrow{E_1} \phi$$

$$\frac{dP}{dt} = K_1 S E - K_2 E_1 P$$

② ENZYMATIC RXN.

$$S \xrightarrow{E} P$$

S is Binding to E (Reversible Rxn) will get S-E complex.

$$S + E \underset{K_2}{\overset{K_1}{\rightleftharpoons}} S\text{-}E \underset{K_4}{\overset{K_3}{\rightleftharpoons}} P\text{-}E \underset{K_6}{\overset{K_5}{\rightleftharpoons}} E + P.$$

E = catalyst.

6 → parameters.

Substrate enzyme complex = S−E

Product enzyme complex = P−E

Considering ~~SE~~ S-E & P-E complex. together we can Reduce ~~into~~ it to C.

$$S + E \underset{K_2}{\overset{K_1}{\rightleftharpoons}} C \overset{K_5}{\rightleftharpoons} E + P$$

Rate of Rxn is Not same (Reversible Rxn).

S converts to P But can't say P will get convert to S.

∴ the eq^n we left with is,

$$\Rightarrow S + E \underset{K_2}{\overset{K_1}{\rightleftharpoons}} C \xrightarrow{K_3} E + P$$

$$S + C + P = S_T$$

Need $\frac{dP}{dt}$

Assumptions to find $\frac{dP}{dt}$

→ some process faster & some are slower.

→ ∴ C Reaches steady state

ie $\frac{dc}{dt} = 0$

↓

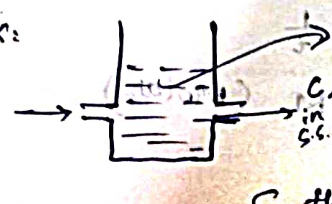
In order to C be at S.S. what must be the criter

↓

Need to have good amount of S.

⇒ concentrat^n of S must be much greater than Total Enzyme available.

$\boxed{S \gg E_T}$ Then ⇒ $\frac{dc}{dt} = 0$

ex: 

C in S.S. {In order to keep the water in S.S. (constant level of water) then constant flo is must. Enough

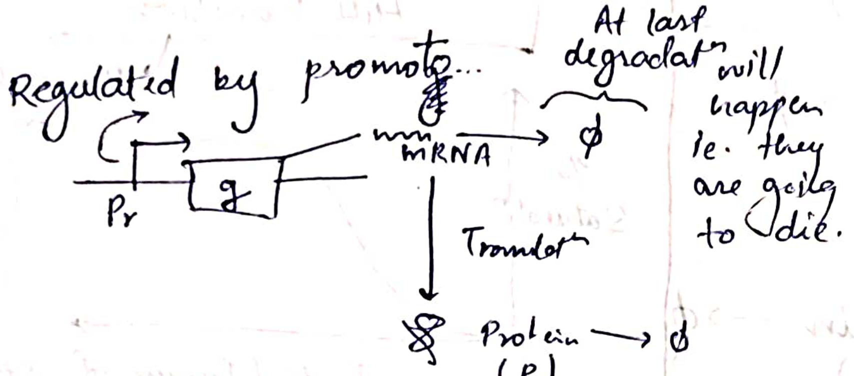
∴ S must be sufficient

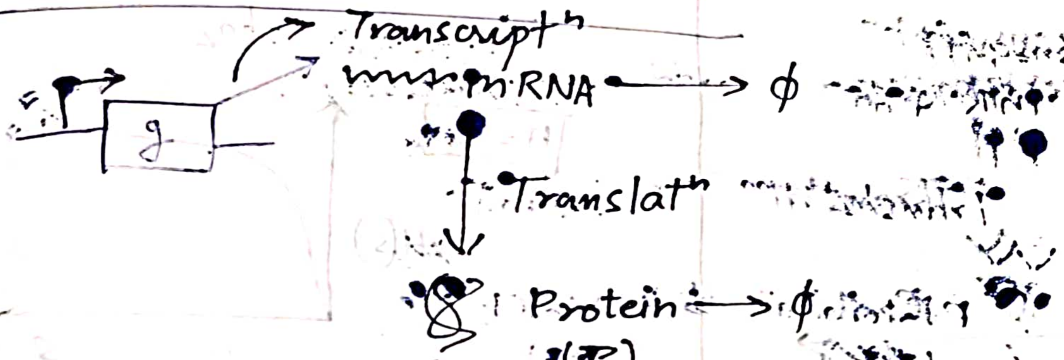
⇒ we get,

$$\frac{dP}{dt} = \frac{k_3 E_T S}{\underbrace{\frac{k_2+k_3}{k_1}}_{K_M} + S}$$

$\boxed{\text{M.M. Kinetic}}$

③ TRANSCRIPTION & TRANSLATION.





Constitutive production?

$$\frac{dP}{dt} = k_1$$

$$\frac{dP}{dt} = k_1 + k_2 S - k_3 P$$

k_1 ↑ Leaky

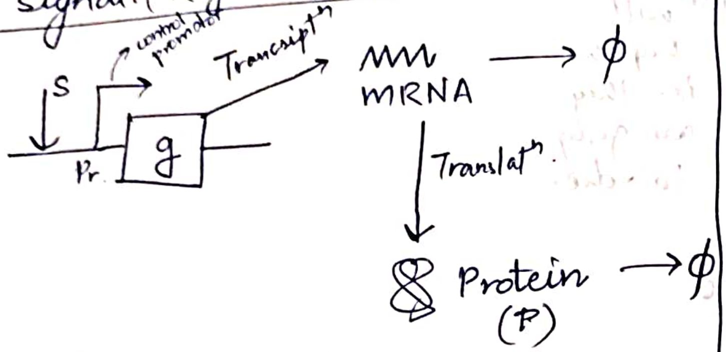
If these type of eq^n doesn't work & need Non-Linearity. what we do?

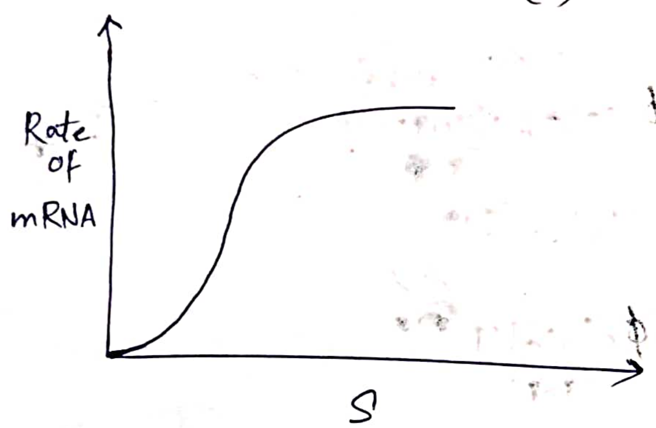
⇓

Say S ~~subst~~ signal comes & trigger the process.

S = molecules / drug.

Signal. (Regulating Signal (Input))





Initially the Rate of Format^n of mRNA is very slow? As we are ↑↑ing S we have ↑↑↑ Rate of mRNA?

As the critical mass of complex ~~present~~ is formed & then suddenly the large format^n starts.

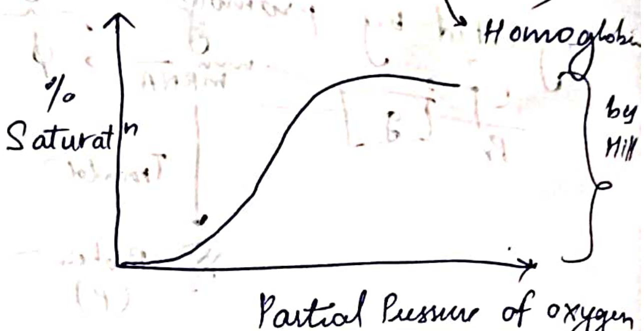
$$\Rightarrow \frac{dm}{dt} = (\text{sigmoid } f^n \text{ of } s)$$

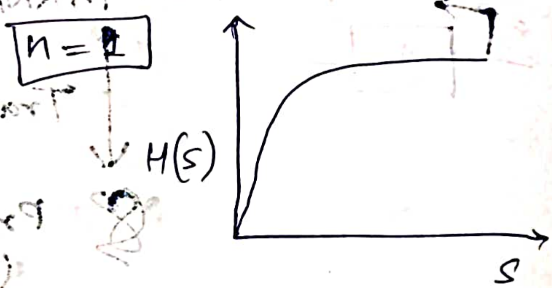
$$\Rightarrow \frac{dm}{dt} = \Sigma(s) - \text{degradat}^n \text{ term of mRNA}$$

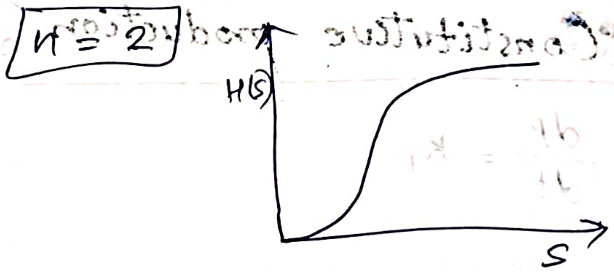
$$\Rightarrow \boxed{\frac{dm}{dt} = k_1\left(\frac{S^n}{H^n + S^n}\right) - k_2 m}$$

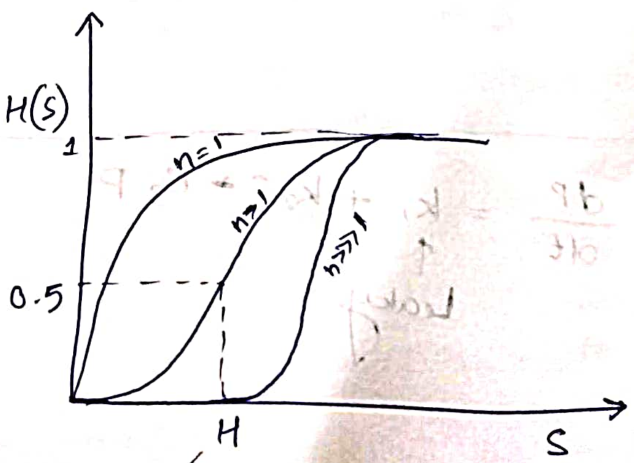
H → Hill constant

Hill Function → Homoglobin by Hill









H → Half Saturation.

$$\frac{dm}{dt} = k_1\left(\frac{s^n}{H^n+S^n}\right) - k_2 m$$

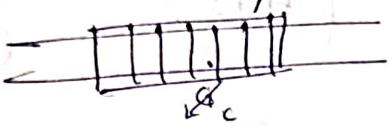
$n \equiv$ Hill coeff

$H =$ Hill constant

Hill function $\equiv$

H represent affinity of control promotor

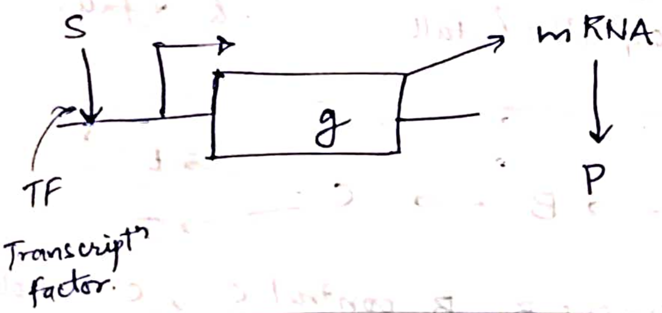
Promoter sequence ?



High affinity ⇒ H will drop

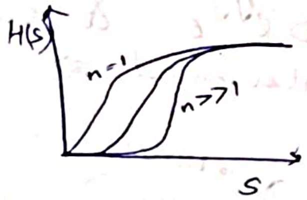
⇓

Little signal (s) will be okay ?



↑ Hill coefficient

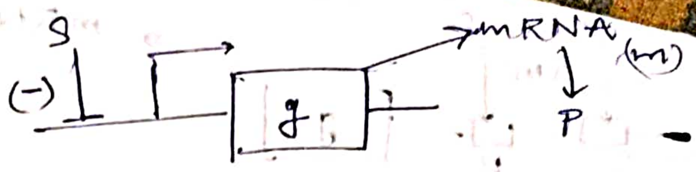
Graph will Δ



Multiple copies of Transcript^n factor in Binding way then, n ↑↑

$\frac{dP}{dt} = k_3 m - k_4 P$

$\frac{dm}{dt} = k_1\left(\frac{s^n}{H^n+S^n}\right) - k_2 m.$

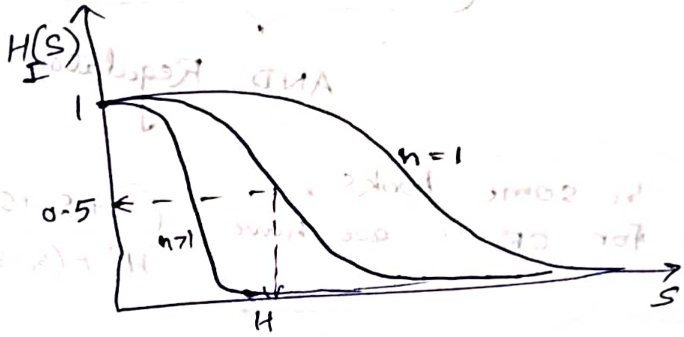


Situat^n when signal inhibits:

⇓

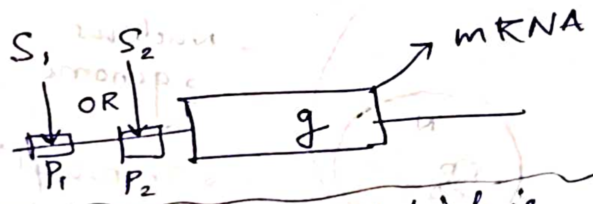
will use Inverse Hill funct^n

$$\boxed{H_I(s) = \frac{H^n}{H^n+S^n}}$$



$S=0 \Rightarrow H_I(s) = 1$

GATES



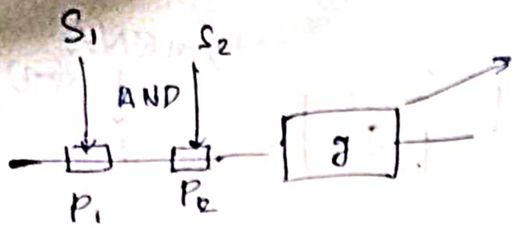
we have gene, which is Regulated by 2 signals.

P_1 & P_2 are promotor.

These signals are independent.

we can say OR signals.

$$\frac{dm}{dt} = \underbrace{k_1\left(\frac{S_1^n}{H_1^n+S_1^n}\right) + k_2\left(\frac{S_2^m}{H_2^m+S_2^m}\right)}_{\text{OR Regulation.}} - k_3 m$$

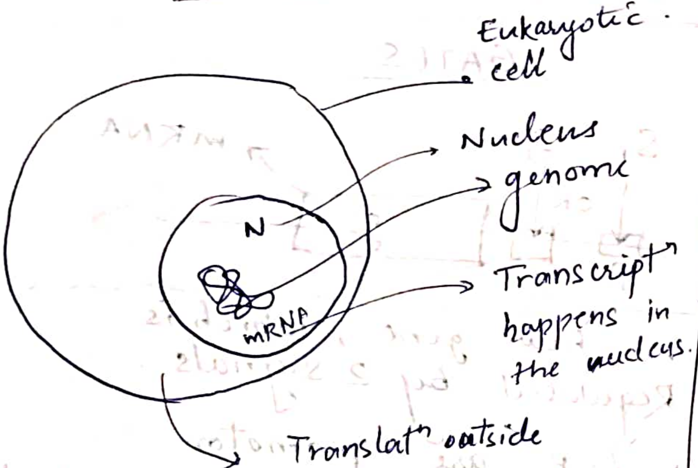


Both Signal are required, then only the production starts.

$$\frac{dm}{dt} = \underbrace{k_1\left(\frac{S_1^n}{H_1^n+S_1^n}\right) \times \left(\frac{S_2^m}{H_2^m+S_2^m}\right)}_{\text{AND Regulator.}} - k_3 m$$

In some books, for OR : we have $\frac{(S_1+S_2+S_3)^n}{H^n+(S_1+S_2+S_3)^n}$

TRANSCRIPTION is faster than TRANSLATION.



By the time Transl^n starts, it may happen than Transcript^n reaches S.S. $\longrightarrow$ ∴ we consider mRNA has reached S.S

ie $dm/dt = 0$

$$\frac{dm}{dt} = k_1\left(\frac{S^n}{H^n+S^n}\right) - k_2 m \quad \text{—— (1)}$$

$$\frac{dP}{dt} = k_3 m - k_4 P \quad \text{—— (2)}$$

$m \longrightarrow$ steady State.

ie. $dm/dt = 0$

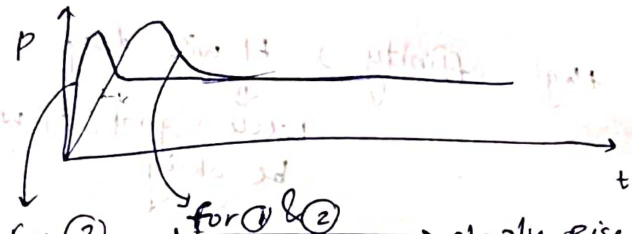
at S.S.

$$m^* = \cancel{\left(\frac{k_1}{k_2}\right)}^{\alpha}\left(\frac{S^n}{H^n+S^n}\right)$$

$$\frac{dP}{dt} = k_3 m^* - k_4 P$$

$$= \cancel{(k_3\alpha)}^{\beta}\left(\frac{S^n}{H^n+S^n}\right) - k_4 P$$

$$\frac{dP}{dt} = \beta\left(\frac{S^n}{H^n+S^n}\right) - k_4 P \quad \text{—— (3)}$$

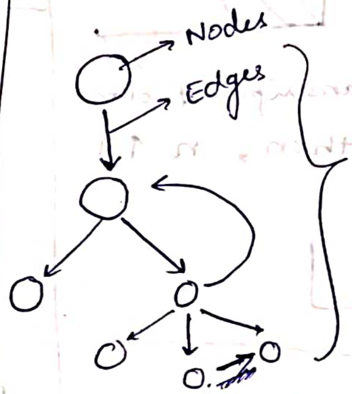


for (3) ↳ sharp Rise & fall.

for (1) & (2) $\longrightarrow$ slowly Rise & fall.

$A \longrightarrow B \longrightarrow C$, $C \longrightarrow E$, $C \longrightarrow D$

A triggers B, B control C, C control D & E & such molecules are interlinked $\Rightarrow$ O = molecule



called Network.

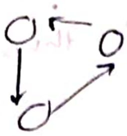
called Directed Graph

made up of Nodes & Edges

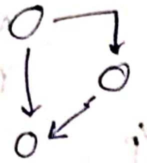
Network Motif :

say, we have 300 number of 3 Nodes System.

+ve feedback (FB)



$n_1 = 300$.



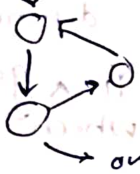
say, there are 2 copies

why only 2 → can be any reason.

Now we will Randomize the Network

say in Random Network its ⇒ 200 ⇒ means

+ve FB ⇒

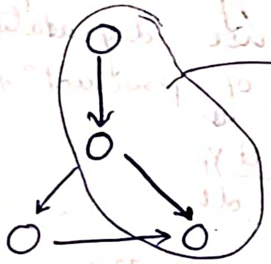


i/p and o/p behaviour?

-ve feedback

give rise to oscillatⁿ

or it stabalize the system.

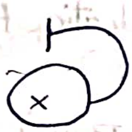


Sub ~~se~~ Network (Sub Group)

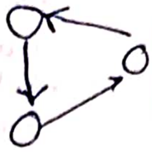
↓

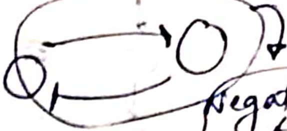
Satistically overs represented Network Motifs.

-ve Autoregulation

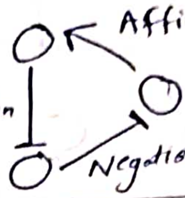


Positive Feedback

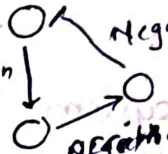




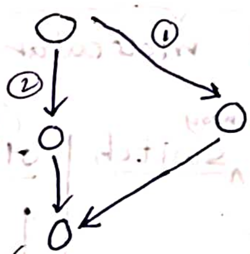
Negatⁿ of Negatⁿ is Affirmaⁿ



Negative Feedback.



Feed Forward → start with 2 different paths & then converge



2 Branches. can go from path 1 or 2

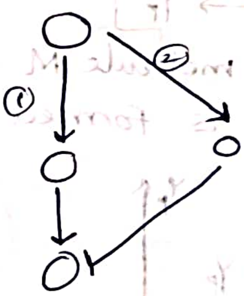
↓

from path 1 they are asking to do some work & similarly from path/channel 2 also.

both path coming & saying, do some work

⇓

this is called: Coherent Feed Forward.



one channel we are saying do some work & other will say don't do work.

⇓

If speed of both channel are same it will not do anything.

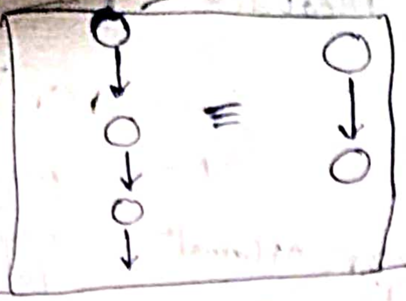
Say, speed of 1 is faster than 2

⇒ at t_1: path 2 will gain ~~spe~~ significance & will say don't do some work

⇒ till time t_1: path 1 will do the work faster

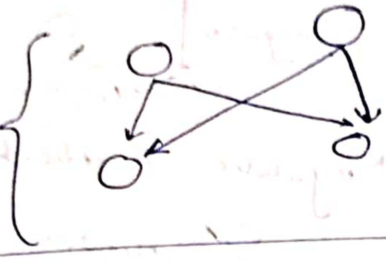
Incoherent Feed Forward.

this can be represented as



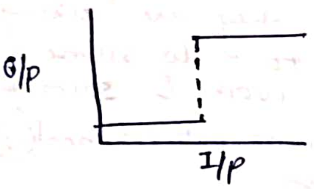
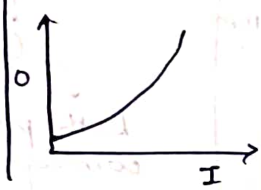
Delay is gained when wholes ↑, but product is same.

Bifan

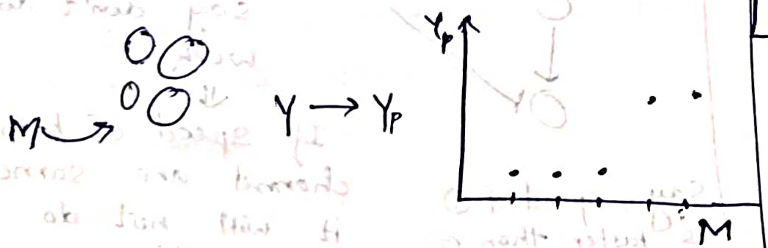


Any Control system, won't work without switches.
Our biological system, also has biological switches

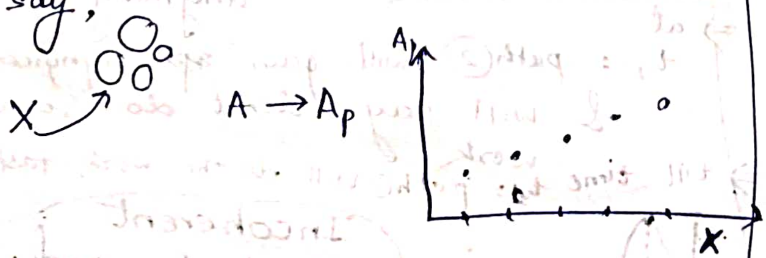
So we need molecular switches

ON or OFF binary switch	other switches (Reostat)
	

Say on some cell, M is doing some work which gives the Y_P $\boxed{Y \rightarrow Y_P}$
treat cells will molecule M, & the product Y_P is formed.

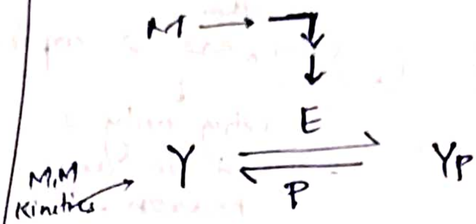


say,



M (sending +ve signal to E & causing phosphotization) enzyme

M ↑↑ses E & thus Y_P ↑↑ses



Also,
$Y_T = Y + Y_P$

Also,
Phosphatase is constant
Also, I will use M.M Kinetics Model :

M.M Kinetics Model :

$$S + E \underset{k_2}{\overset{k_1}{\rightleftharpoons}} C \xrightarrow{k_3} P + E$$

So,
Rate of product^n of Y_P depends on given the amount of E, which depen on ―॥― of M.

In Rate of product^n of Y_P, we Assumed that the Product^n of Y = Degradat^n of Y
∴ we will not use degradation term in Rate of product^n of Y_P
⇓
$\frac{dY_P}{dt}$

$$\boxed{\frac{dY_P}{dt} = \frac{k_{31} E (Y_T - Y_P)}{K_{M1} + (Y_T - Y_P)} - \frac{k_{32} P Y_P}{KM_2 + Y_P}}$$

At steady state :
Y_P^* will be quadratic f^n of (Y_T, E, P, ψ)

$$Y_P^* = G(Y_T, E, P, \psi)$$

→ Goldbar - Khosland

At s.s :

$$\frac{dY_P}{dt} = 0$$

$$\Rightarrow \frac{K_{31} E \left(Y_T - Y_P^*\right)/Y_T}{\underbrace{K_{M1}/Y_T}_{J_1\ (\text{const})} + \dfrac{(Y_T - Y_P^*)}{Y_T}} = \frac{K_{32} P\, Y_P^*/Y_T}{\underbrace{K_{M2}/Y_T}_{J_2} + Y_P^*/Y_T}$$

dividing both Numerator & Denominator by Y_T

$\because Y_T$ is constant in system.

$$y = Y_P^*/Y_T, \quad J_1 = \frac{K_{M1}}{Y_T}, \quad J_2 = \frac{K_{M2}}{Y_T}$$

$\because$ we need to plot Y_P^* vs E graph.

$$\Rightarrow \frac{K_{31} E (1-y)}{J_1 + (1-y)} = \frac{K_{32} P y}{J_2 + y}$$

$\therefore v_1 = K_{31} E$

$v_2 = K_{32} P$

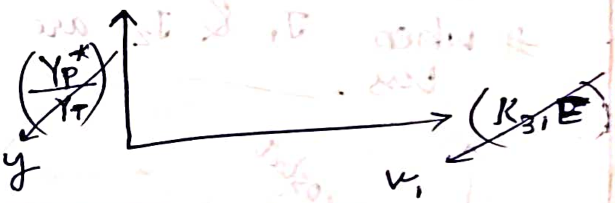
Replacing : $Y_P^* \Rightarrow Y_P^*/Y_T \xrightarrow{} \text{const}$

: $E \Rightarrow K_{31} E \xrightarrow{} \text{const}$

$\Downarrow$

$$\Rightarrow \underbrace{\frac{v_1(1-y)}{J_1 + (1-y)} = \frac{v_2 y}{J_2 + y}}$$

$\therefore$ we will plot



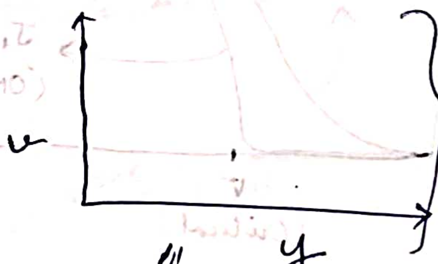
we need some f^n y as a f^n of —

$y = f(\ )$

but let's find, $v_1 = f(\ )$

$\Downarrow$

$$\Rightarrow v_1 = \overbrace{\left(\frac{v_2 y}{J_2 + y}\right)}^{1} \overbrace{\boxed{\left(\frac{J_1 + 1 - y}{1 - y}\right)}}^{2}$$



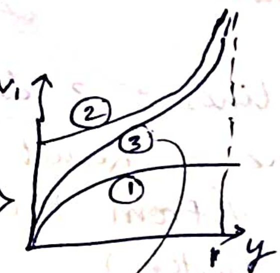
just need to rotate the graph.

in the domain, $y \in (0,1)$

$y = 0 \Rightarrow v_1 = 0$

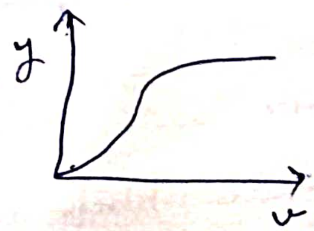
term 1 $\equiv \dfrac{v_2 y}{J_2 + y}$ } this f^n similar to $x/1+x$

term 2 $\equiv \dfrac{J_1 + 1 - y}{1 - y}$ }



$\Downarrow$

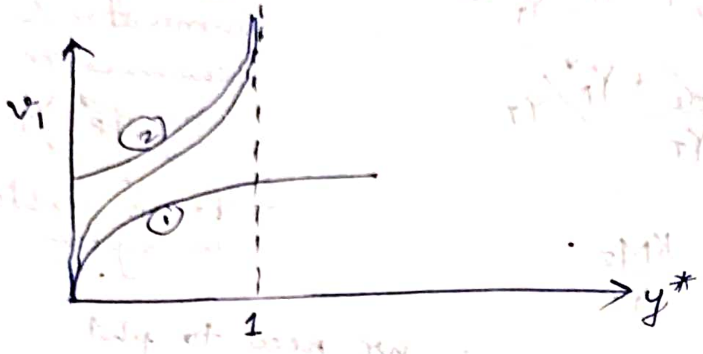
After turning graph.



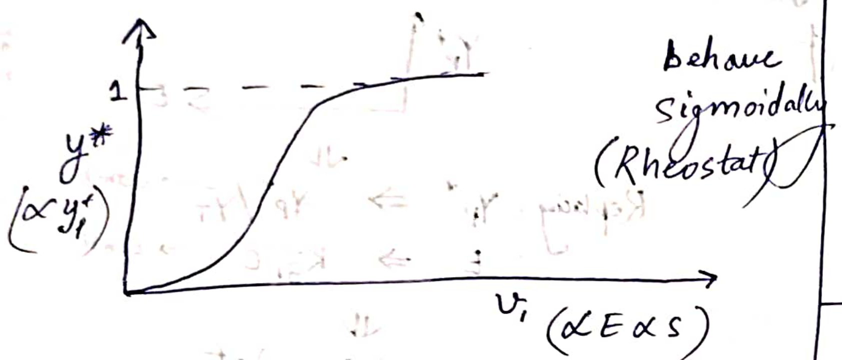
3 graph After multiplying 1 & 2 graph

At s.s.

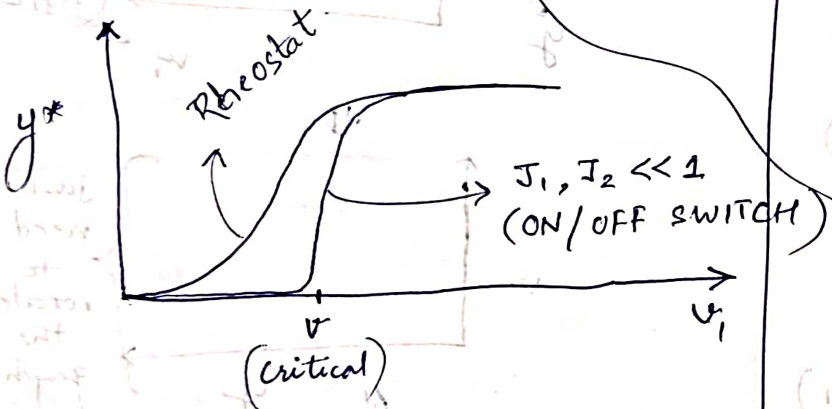
$$v_1 = \underbrace{\left(\frac{v_2 y^*}{J_2 + y^*}\right)}_{①} \underbrace{\left(\frac{J_1 + 1 - y^*}{1 - y^*}\right)}_{②}$$



⇓ Inverting graph ③



when J_1 & J_2 are very less.



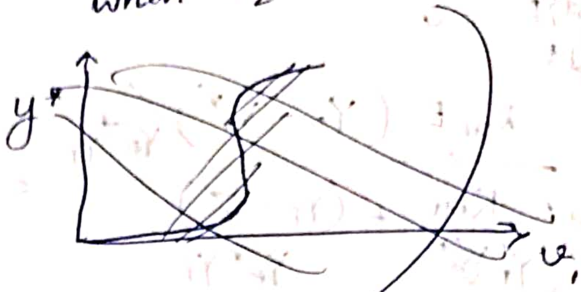
2 diff cell lines ≡ 2 diff protein

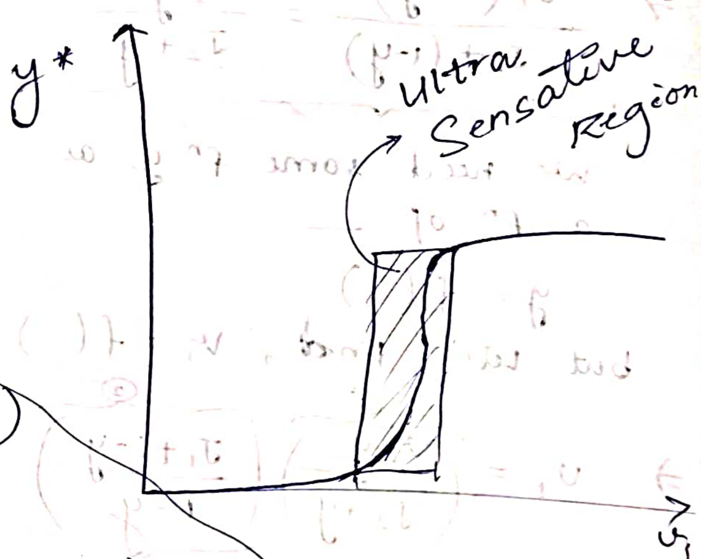
⇒ Thus outcome of Result Δ drastically (from Rheostat to ON/OFF switch)

J_2 : Half Satural point

⇓

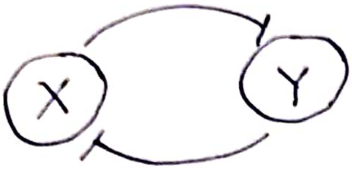
when J_2 is small





will work as ON/OFF Ultra Sensitive switch amount of substrate required to Half saturate the enzyme.

Mutual Reprenium



$\Rightarrow$ X & Y both inhibit the product^n of Y & X at transcript^n level resp.

$$\frac{dX}{dt} = K_1\left(\frac{H_1^n}{H_1^n + Y^n}\right) - K_2 X \quad \text{(1)}$$

Rate of prod^n of X $\uparrow$ as Y $\downarrow$

$$\frac{dY}{dt} = K_3\left(\frac{H_2^m}{H_2^m + X^m}\right) - K_4 Y \quad \text{(2)}$$

(1) & (2) together has 8 parameters.

considering $K_1 = K_3 = \gamma$ & $K_2 = K_4 = 1$ & $H_1 = 1$, $H_2 = 1$ &

considering $n = m = 2$ ($\because$ need sigmoid f^n)

$\Downarrow$

$$\frac{dx}{dt} = \gamma\left(\frac{1}{1+y^2}\right) - X \quad \text{(3)}$$

$$\frac{dY}{dt} = \gamma\left(\frac{1}{1+X^2}\right) - Y \quad \text{(4)}$$

(3) & (4) are our model.

find s.s. $\Rightarrow$

Use Null cline Method:

$\because$ X & Y are molecules

$\therefore$ X & Y > 0

X - Null

$$\frac{dx}{dt} = 0 \Rightarrow X = \left(\frac{\gamma}{1+y^2}\right)$$

Y - Null

$$\frac{dY}{dt} = 0 \Rightarrow Y = \left(\frac{\gamma}{1+X^2}\right)$$

$x = 0 \Rightarrow y = \gamma$

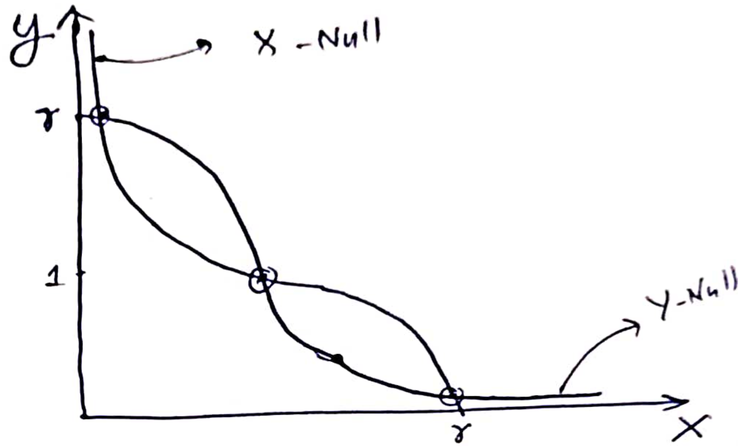
$x \to \infty \Rightarrow y = 0$

$x = 1 \Rightarrow y = \gamma/2$

$\because$ here we have $x^2 \Rightarrow$ will be sigmoid

when x^1 ie $y = \frac{\gamma}{1+x} \Rightarrow$ asympt.

$\therefore$ Y - Null Cline will be sigmoid.



All three can't be stable s.s.

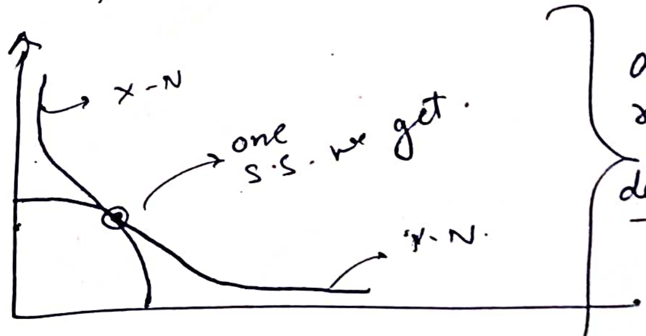
2 possibilities

2 S & 1 Un.S.

2 US. & 1 S.

as we $\downarrow \gamma \Rightarrow$ we get 1 s.s.

No. of possible S.S depend & Δes with γ.



as γ is decreases

$\Downarrow$

called as $\boxed{\text{Bifurcation}}$

1. No. of possible SS Δes with parameters.
2. Stability behavior Δes.
3. Both Δes simultaneously. vector field also Δes