Post Mid-Sem Problem Set-1

1. Two brothers are facing three candies to be shared. They cannot cut a candy into smaller pieces. Each player wants to maximize the number of the candies he gets. Consider an extensive- form game in which the older brother declares the number he wants from the set $\{0, 1, 2, 3\}$ and , knowing this, the younger brother declares the number he wants from the set $\{0, 1, 2, 3\}$. If the sum of the two numbers is not more than 3, then each player receives candies according to the number he declared. If the sum exceeds 3, then both players get 0 candies. Draw the game tree of this game and find all pure strategy subgame perfect equilibria.

2. Two players take turn choosing a number from the st $\{1, 2, 3, 4\}$ with player 1 moving the first. The first player who brings the sum of all the chosen number to 10 or more wins. The pay-off of the winner is 100 and the pay-off of the loser is 0. Show that the backward induction outcome is such that the player who moves second always wins the game.

3. Find the subgame perfect Nash equilibrium of the games given in figure 1 and figure 2. The payoffs are such that first number is for player 1 and second for player 2 and third for player 3.

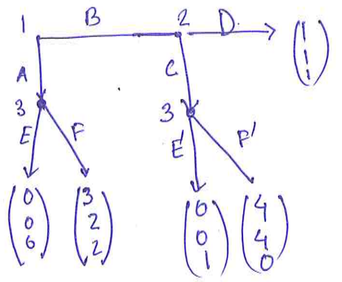



Figure 1

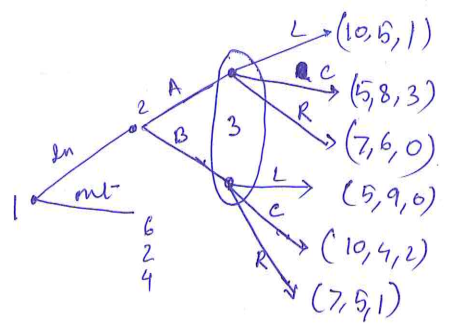



Figure 2

Answer

1) The game tree is given in the first diagram. The subgame perfect Nash equilibria have been shown through the reduced form of the game tree. Younger brother who is second mover or moves in stage II

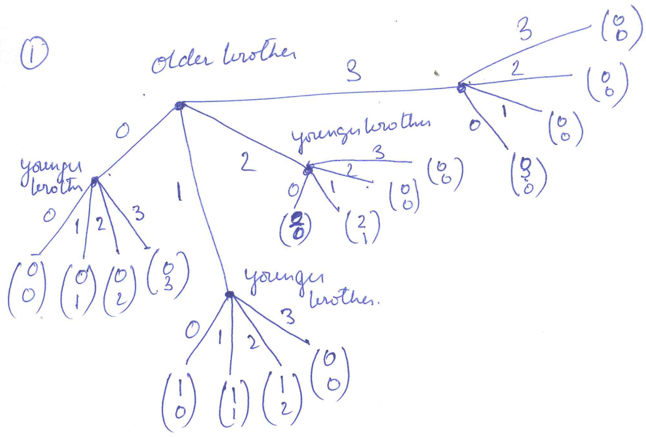
is indifferent among all the actions. So, we have to consider each of them separately. Younger brother can choose any one of these $\{0, 1, 2, 3\}$ numbers as optimal action when older brother has chosen number $\{3\}$. But the optimal action of older brother is to choose $\{2\}$ if younger brother chooses any number greater than $\{0\}$. The optimal action of older brother is to choose $\{3\}$, if younger brother chooses $\{0\}$. Therefore there are number of subgame perfect Nash equilibrium.

2) Suppose player 1 moves first and player 2 moves second. So player 1 moves in odd numbered stage and player 2 moves is even numbered stage. For player 1 to win, the sum of the chosen numbers must be any one of these numbers 6, 7, 8, 9 till the previous even numbered stage. For player 2 to win, the sum of the chosen numbers must be any one of these numbers 6, 7, 8, 9 till the previous odd numbered stage.
Now suppose player 1 chooses number 1 in stage 1, then for player 2 it is optimal to choose 4 in stage 2. In stage 3, whatever number player 1 is choosing, its sum is one of 6, 7, 8, 9. So in stage 4, player 2 wins by choosing appropriately.
Now suppose player 1 chooses number 2 in stage 1, then for player 2 it is optimal to choose 3 in stage 2. In stage 3, whatever number player 1 is choosing, its sum is one of 6, 7, 8, 9. So in stage 4, player 2 wins by choosing appropriately.
Now suppose player 1 chooses number 3 in stage 1, then for player 2 it is optimal to choose 2 in stage 2. In stage 3, whatever number player 1 is choosing, its sum is one of 6, 7, 8, 9. So in stage 4, player 2 wins by choosing appropriately.
Now suppose player 1 chooses number 4 in stage 1, then for player 2 it is optimal to choose 1 in stage 2. In stage 3, whatever number player 1 is choosing, its sum is one of 6, 7, 8, 9. So in stage 4, player 2 wins by choosing appropriately.
We have exhausted all the possibilities. In all the cases player 2 who moves second wins.

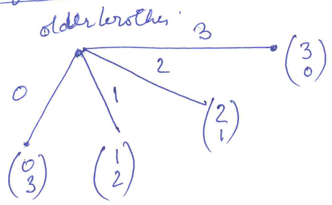
3) The game tree of the first game has been shown below. There is a unique subgame perfect Nash equilibrium. We get it through backward induction. The reduced form of the game is shown below. Suppose player 3 is making decision. It chooses $F$ over $E$ and $E^{'}$ over $F^{'}$. Secondly, player 2 chooses $D$ over $C$ in reduced form. Player 1 chooses $A$ over $B$. This is the subgame perfect Nash equilibrium.

In the second game, the subgame starting from player 2 's node is an imperfect information game. So that subgame is same as simultaneous move game. In the diagram, the simultaneous move game between player 2 and 3 has been shown. The pure strategy Nash equilibrium is that player 3 chooses $C$ and player 2 chooses $A$. Player 1 when making the decision whether to choose *in* or *out* will compare the pay-off in choosing *out* to the Nash equilibrium pay-off when chooses *in*. It is shown in diagram. We get that player 1 comparing its pay-off in these two situations chooses *out*. This is subgame perfect Nash equilibrium.

(1)



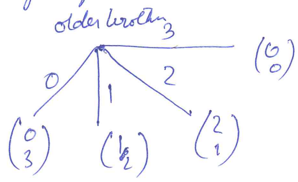

Reduced form:




younger brother is indifferent among $\{0, 1, 2, 3\}$ when older brother plays 3.

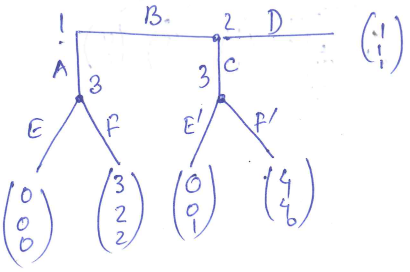
In this case ~~plays~~ older brother chooses 3.

Suppose younger brother chooses any one of $\{1, 2, 3\}$.

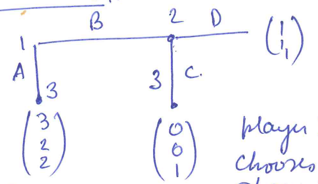


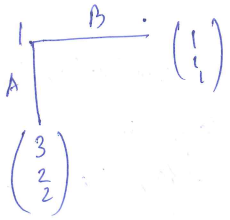

older brother chooses 2.

(3) (a)



Reduced Form

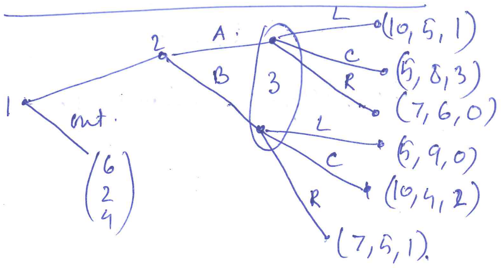


player 2 chooses ~~[illegible]~~ D over C.

Player 1 chooses A over B.

(3) (b)



~~[illegible]~~ game starting from the node of player 2 is imperfect information game.

Player 2.

Player 3

| | A | B |
|---|---|---|
| L | 10,5,1 | 5,9,0 |
| C | 5,8,3 | 10,4,2 |
| R | 7,6,0 | 7,5,1 |

~~Nash~~ .

Pure strategy Nash equilibrium
is C for player 3
and A for player 2.

so

$$1 \xrightarrow{\text{In}} 5, 8, 3$$

$$1 \xrightarrow{\text{out}} 6, 2, 4$$

so player 1 choose 1.