Vehicle Routing Problem Variant

Aditya Jindal(210106004)
Vansh(210106082)

Introduction

The Vehicle Routing Problem is one of the classical optimization problems in logistics; that is, it involves determining the best routes through which vehicles can serve customers in order to minimize some constraints, such as cost, time, or both(1).

This paper aims to solve a variant of the VRP with the main objective of minimizing the total cost of travel and a secondary objective of minimizing the maximum time taken by any bus. Several optimization techniques were applied to the problem, such as Teaching-Learning-Based Optimization (TLBO), Particle Swarm Optimization (PSO), Differential Evolution (DE), and MATLAB's gamultiobj and paretosearch functions.

Objectives

Primary Objective:

The main purpose of this research is to reduce the total travel cost accumulated by the buses in servicing their orders. This includes travel costs from the depot to each customer and between customers within the routing process.

Secondary Objective:

To minimize cost, the objective of this study is also concerned with limiting the maximum travel time of any bus to some predefined value (MaxTime). This double-objective formulation gives a true balance between efficiency and time. It thus addresses an important concern in real-life applications like public transportation and courier services.

Methodology

Execution of Algorithms:

- The optimization techniques were executed over 25 independent runs to assess the robustness and performance of each method.
- Performance was measured based on convergence trends and the quality of the final solutions obtained from each algorithm.

Optimization Techniques Used:

- Teaching-Learning-Based Optimization (TLBO)
- Particle Swarm Optimization (PSO)
- Differential Evolution (DE)
- MATLAB Functions:
    a. gamultiobj (for single-objective optimization)
    b. paretosearch (for multi-objective optimization)

Formulations

Indices and sets:

- $i \in I$ customers;
- $k \in K$ vehicles;

Parameters:

- N: Set of nodes, customers (i) (i.e., cardinality of the set I)
- K: Set of vehicles (k) (i.e., cardinality of the set K)
- C_{ij}: Cost of traveling from node i to node j.
- T_{ij}: Time taken to travel from node i to node j.
- S_i: Cost of traveling from the depot to customer i.
- T_i: Time taken to travel from the depot to customer i.
- M: Maximum allowable time for any vehicle's route.
- U: Penalty cost for exceeding the maximum allowable time.

Variables:

- x_{ik}: Binary variable; $x_{ik} = 1$ if vehicle k serves customer i, 0 *otherwise.*
- p_i : Priority of customer i.
- P_k : Penalty variable; $P_k = 1$ if the total time for vehicle k exceeds M, 0 otherwise.

$$\sum_{k \in K} x_{ik} = 1, \forall i \in N$$

$$\sum_{i \in N} x_{ik} \leq n, \forall k \in K$$

$$\sum_{i \in N} T_i . x_{ik} + \sum_{i \in N} \sum_{j \in N} T_{ij} . x_{ik} . x_{jk} \leq M \ \forall k \in K$$

$$Min \sum_{k \in K} \left(\sum_{i \in N} S_i \, x_{ik} + \sum_{i \in N} \sum_{j \in N} C_{ij} \, x_{ik} x_{jk} \right) + \sum_{k \in K} U \, . \, P_k k$$

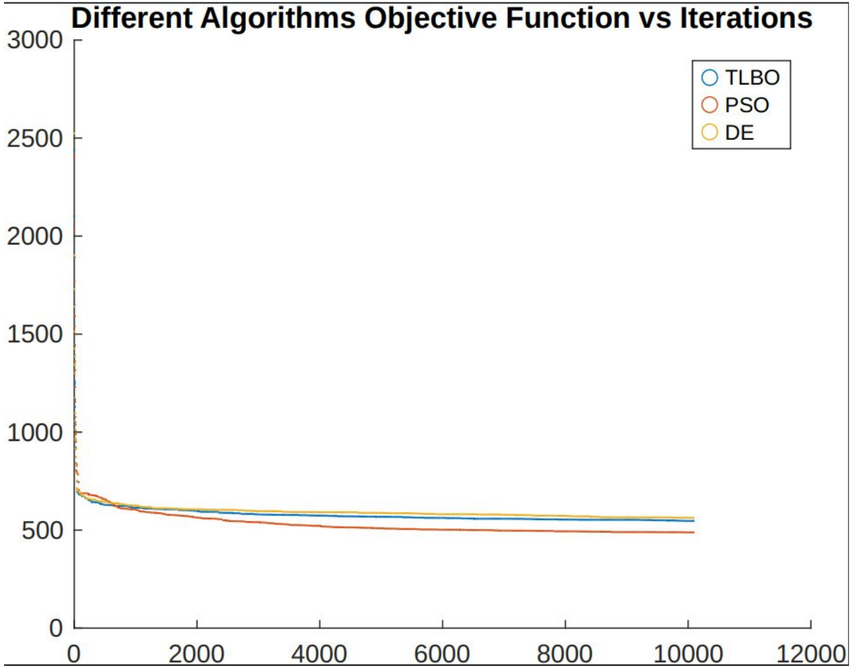
Results - Single-Objective Optimization

TLBO: Converged slower compared to PSO, with higher variability in early iterations.

PSO: Achieved a rapid convergence to near-optimal solutions, stabilizing after approximately 4000 iterations.

DE: Demonstrated a moderate convergence rate, outperforming PSO in later iterations but lagging behind TLBO.

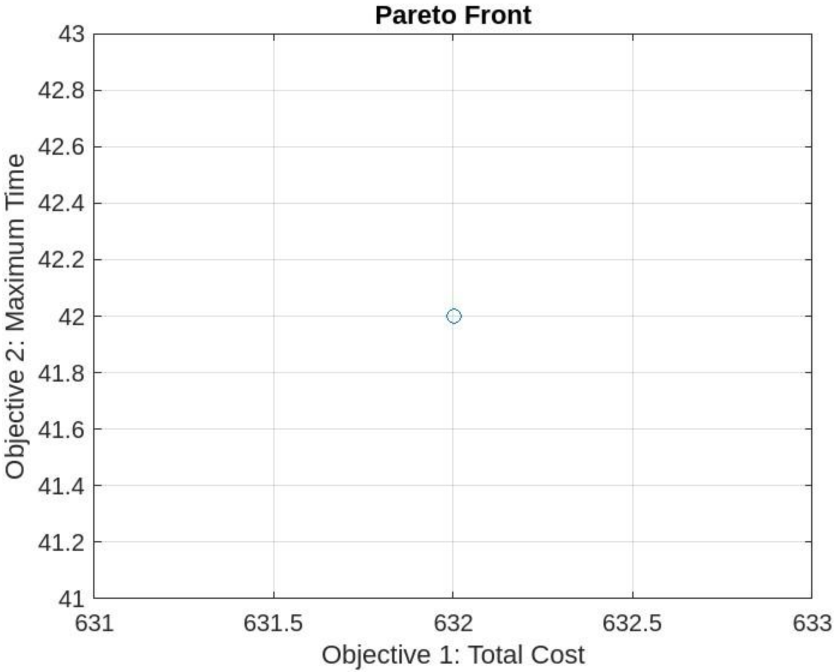
gamultiobj: Provided a benchmark solution for total travel cost, which was competitive with the best-performing metaheuristic.



Results - Multi-Objective Optimization

The Pareto front solutions highlighted the trade-offs between minimizing cost and time. For example, reducing the maximum time taken by any bus often required higher total costs. Solutions ranged from low-cost/high-time combinations to high-cost/low-time configurations, offering flexibility depending on the decision-maker's priorities.

Although the single-objective methods resulted in lower total costs, paretosearch provided a balanced set of solutions that both objectives are solved well. The best compromise solution on the Pareto front obtained a total cost of 632 and a maximum time of 42 units.



Discussion & Conclusion

Execution Time: PSO had the fastest convergence, making it suitable for large-scale problems where computational efficiency is critical. TLBO and DE needed more iterations to converge to similar results, which increases the computational time.

Robustness: In 25 runs, PSO had a consistent near-optimal result with minimal variability.

Scalability: Although computationally intensive for gamultiobj and paretosearch, the MATLAB implementation is scalable for multi-objective optimization.

References

[1] Kris Braekers, Katrien Ramaekers, Inneke Van Nieuwenhuyse, The vehicle routing problem: State of the art classification and review, Computers & Industrial Engineering, Volume 99, 2016, Pages 300- 313, ISSN 0360-8352, https://doi.org/10.1016/j.cie.2015.12.007 (https://www.sciencedirect.com/science/article/pii/S0360835215004775)