Chapter 2

Basic Concepts of Structural Reliability Theory

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Chapter 2

Basic Concepts of Structural Reliability Theory

2.1 Definitions of Structural Reliability

2.1.1 Reliability of Structures

Structural Reliability **is the ability of a structure or structural element to fulfill the specified performance requirements under the prescribed conditions during the prescribed time.**

1. The Prescribed Time

- ***the prescribed time = the design working life***
- ***the design working life ≠ the design reference period***
  - ***Design working life is assumed period for which a structure or a structural element is to be be used for its intended purpose without major repair being necessary.***
  - ***Design reference period is a chosen period of time which is used as a basis for assessing values of variable actions, time-dependent material properties, etc.***

2.1 Definitions of Structural Reliability …2

Table2.1 Design Working Life

Class	Design working life (years)	Examples
1	1 to 5	Temporary
2	25	Replacement structural parts, e.g. gantry girders, bearings
3	50	Buildings and other common structures, other than those listed below
4	100 or more	Monumental buildings, and other special or important structures. Large bridges

2. The Prescribed Conditions

- *the normal design condition*
- *the normal construction condition*
- *the normal operation condition*

The human errors are not considered

3. The Specified Performance Requirements

- ***Ultimate limit state requirement***
  - ***They shall withstand extreme and/or frequently repeated actions occurring during their construction and anticipated use.***
- ***Serviceability limit state requirement***
  - ***They shall perform adequately under all expected actions.***
- ***Structural durability requirement***
  - ***They shall remain fit for use during their design working lives in their environment, given appropriate maintenance.***
- ***Structural integrity requirement***
  - ***They shall not be damaged by events like flood, land slip, fire, explosion, impact, earthquake, tornado, etc.***

2.1.2 Limit States of Structures

1. Definition of Limit State

A *limit state* is a state beyond which a structure or part of it no longer satisfies the design performance requirements.

- ***The concept of a limit state is used to help define failure in the context of structural reliability theory.***
- ***We could say that a structure fails if it cannot perform its intended function, or violation of a limit state.***
- ***A limit state is a boundary between desired and undesired performance of a structure.***
- ***The boundary between desired and undesired performance of a structure is often represented mathematically by a limit state function or performance function.***

2. Types of Limit State — Classification 1

(1) *Ultimate Limit State (ULS)*

Ultimate limit states are mostly related to the loss of load-carrying capacity. Examples of modes of failure in this category include:

- *Exceeding the moment carrying capacity*
- *Formation of a plastic hinge*
- *Crushing of concrete in compression*
- *Shear failure of the web in a steel beam*
- *Loss of the overall stability*
- *Buckling of flange*
- *Buckling of web*
- *Weld rupture*
- *Loss of foundation load-carrying*

(2) *Serviceability Limit State (SLS)*

Serviceability limit states are mostly related to gradual deterioration, user's comfort, or maintenance. They may or may not be directly related to structural integrity. Examples of modes of failure in this category include:

- ***Excess deflection***
- ***Excess vibration***
- ***Permanent deformations***
- ***Cracking***

(3) *Fatigue Limit State (FLS)*

Fatigue limit states are related to loss of strength under repeated loads. Fatigue limit states are related to the accumulation of damage and eventual failure under repeated loads.

A structural component can fail under repeated loads at a level lower than the ultimate load. The failure mechanism involves the formation and propagation of cracks until their rupture. This may result in structural collapse.

2. Types of Limit State — Classification 2

(1) *Irreversible Limit State*

Irreversible limit state is a limit state which will remain permanently exceeded when the actions which caused the excess are removed.

- ***The effect of exceeding an ultimate limit state is almost always irreversible and the first time that this occurs it causes failure.***
- ***In the cases of permanent local damage or permanent unacceptable deformations, exceeding a serviceability limit state is irreversible and the first time that this occurs it causes failure.***

(2) *Reversible Limit State*

Reversible limit state is a limit state which will not be exceeded when the actions which caused the excess are removed.

- ***In the cases of temporary local damage, temporary large deformations and vibrations, exceeding a serviceability limit state is reversible.***

2.1.3 Limit State Functions (Performance Functions)

1. Form of Comprehensive Variables

- *In general, the factors influencing structural reliability can be put into two kinds of comprehensive variables, that is, load effect S (demand) and structural resistance R (supply).*
- *A performance function, or limit state function, can be defined as*

$$Z = g(R, S) = R - S$$

$$Z = R - S > 0 \qquad \textit{for safe state}$$

$$Z = R - S = 0 \qquad \textit{for limit state}$$

$$Z = R - S < 0 \qquad \textit{for failure state}$$

- *The limit state equation is defined as*

$$Z = g(R, S) = R - S = 0$$

Properties of Limit State Functions

- *Z is also named safety margin or margin of safety.*
- *Each limit state function is associated with a particular limit state.*
- *Different limit states may have different limit state functions.*
- *Even for a particular limit state, its limit state functions may be different.*

$$Z = R - S$$

$$Z = \frac{R}{S} - 1$$

$$Z = \ln R - \ln S$$

2. Form of Basic Variables

- *In general, the performance function can be a function of many variables: load components, environment influence, resistance parameters, material properties, dimensions, analysis factors, and so on.*
- *The above random variables are called basic random variables, or basic variables.*
- *The basic variable space is a specified set of basic variables. It is also called state space:* $\bar{X} = (X_1, X_2, \cdots, X_n)$
- *The general form of a performance function:*

$$Z = g(\bar{X}) = g(X_1, X_2, \cdots, X_n)$$

- *The general form of a limit state equation:*

$$Z = g(\bar{X}) = g(X_1, X_2, \cdots, X_n) = 0$$

$g(X_1, X_2, \cdots, X_n) > 0$ *for safe state*

$g(X_1, X_2, \cdots, X_n) = 0$ *for limit state*

$g(X_1, X_2, \cdots, X_n) < 0$ *for failure state*

2.1.4 Measures of Structural Reliability

1. Safety (Survival) Probability of Structures

- *Let the performance function of a structure be* $Z = g(X_1, X_2, \cdots, X_n)$ *,* X_i *be the basic random variables influencing structural functions. Obviously, Z is a random variable. Assume that the PDF of Z is* $f_Z(z)$
- *The probability of the event that the structure is to be safe is defined as safety probability, or survival probability. Mathematically,*

$$P_s = P(Z > 0) = \int_0^{\infty} f_Z(z)dz$$

2. Failure Probability of Structures

- *The probability of the event that the structure is to be unsafe is defined as failure probability. Mathematically,*

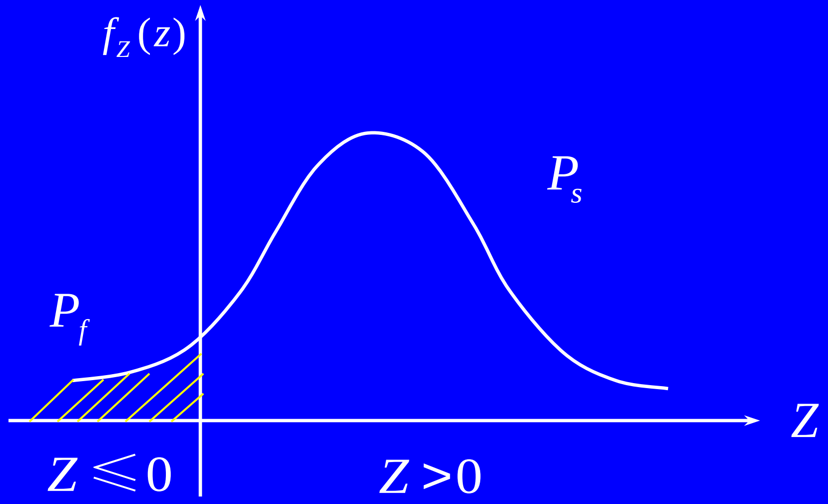
$$P_f = P(Z \leqslant 0) = \int_{-\infty}^{0} f_Z(z)dz$$

Relationship between P_s and P_f

$$P_s + P_f = 1$$

$$P_f = 1 - P_s$$

$$P_s = 1 - P_f$$



3. Reliability Index of Structures

– ***The reliability index of structures is defined as***

$$\beta = -\Phi^{-1}(P_f)$$

where, Φ^{-1} ***is the inverse standardized normal distribution function.***

Chapter 2

Basic Concepts of Structural Reliability Theory

2.2 Failure Probability of Structures

2.2.1 General Basic Variables

1. Assumptions

Consider the performance function

$$Z = g(\bar{X}) = g(X_1, X_2, \cdots, X_n)$$

where, $\bar{X} = (X_1, X_2, \cdots, X_n)$ ***is a n-dimension random vector.***

It is assumed that the joint PDF of $\bar{X}$ ***is*** $f_{\bar{X}}(\bar{x}) = f_{X_1X_2\cdots X_n}(x_1, x_2, \cdots, x_n)$

2. Formula

The failure probability of structures with general basic variables:

$$P_f = P[g(\bar{X}) \leqslant 0] = \int_{\Omega} f_{\bar{X}}(\bar{x})d\bar{x}$$

$$= \int_{\Omega}\cdots\int f_{X_1X_2\cdots X_n}(x_1, x_2, \cdots, x_n)dx_1dx_2\cdots dx_n$$

2.2 Failure Probability of Structures …2

In the above formula, Ω ***is called as failure domain of structures.***

$$\Omega = \{\bar{x} \mid g(\bar{x}) \leqslant 0\} = \{(x_1, x_2, \cdots, x_n) \mid g(x_1, x_2, \cdots, x_n) \leqslant 0\}$$

3. Calculation Method

- ***Analytical Method***
  - ***Precise Analytical Method:*** ***Probability Interference Method (PIM)***
  - ***Approximate Analytical Method***
    - ***FOSM***
    - ***FORM***
    - ***SORM***
- ***Simulation Method***
  - ***Monte Carlo Method (MCS)***
  - ***Latin Hypercube Sampling***
  - ***Important Sampling***
- ***Integral Method***

2.2.2 Two Comprehensive Variables

1. Assumptions

(1) ***S is the total load effect, known:*** $f_S(s)$, $F_S(s)$

(2) ***R is the member resistance, known:*** $f_R(r)$, $F_R(r)$

(3) ***R and S are statistical independent, that is,*** $f_{RS}(r,s) = f_R(r) \cdot f_S(s)$

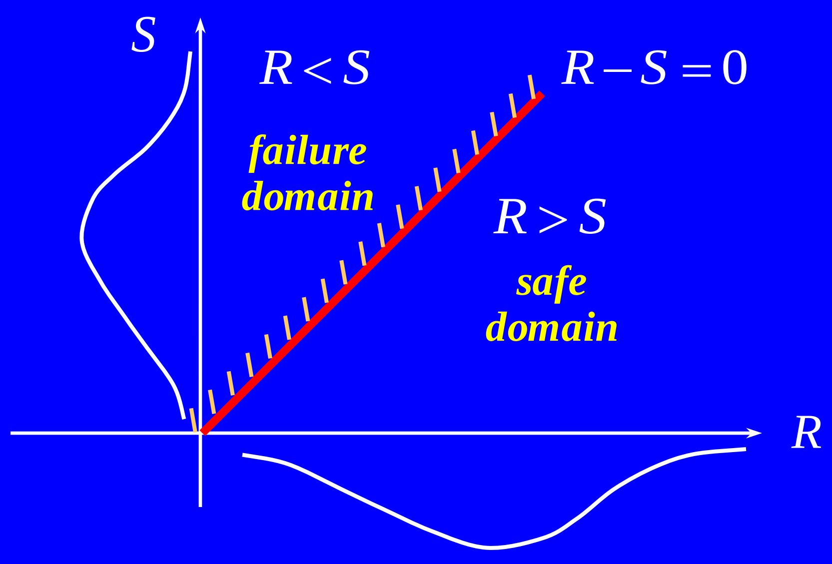
2. Probability Interference Method (PIM)

– ***The limit state function:***

$$Z = R - S = 0$$

– ***The failure domain:***

$$\Omega = \{(r,s) \mid r - s \leqslant 0\}$$

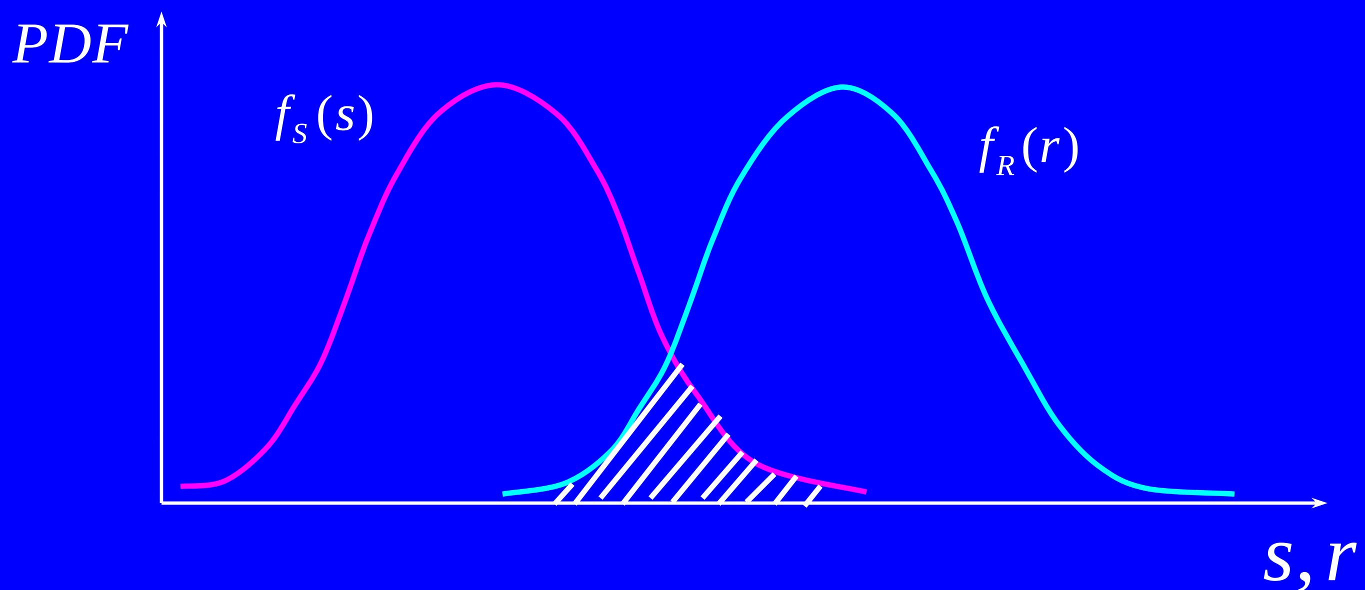


2. Probability Interference Method (PIM) …

– ***The failure probability:***

$$P_f = P(Z \leqslant 0) = P(R - S \leqslant 0)$$

$$= \iint\limits_{r \leqslant s} f_{RS}(r, s)\,drds = \iint\limits_{r \leqslant s} f_R(r) \cdot f_S(s)\,drds$$



2.2 Failure Probability of Structures …5

– ***First integration for r, then for s***

$$P_f = \iint_{r \leqslant s} f_R(r) \cdot f_S(s) dr ds = \int_{-\infty}^{+\infty} \left[\int_{-\infty}^{s} f_R(r) dr \right] \cdot f_S(s) ds$$

$$F_R(s) = P(R \leqslant s)$$

Formula 1 of PIM

$$P_f = \int_{-\infty}^{+\infty} F_R(s) \cdot f_S(s) ds = E\left[F_R(s)\right]$$

– ***First integration for s, then for r***

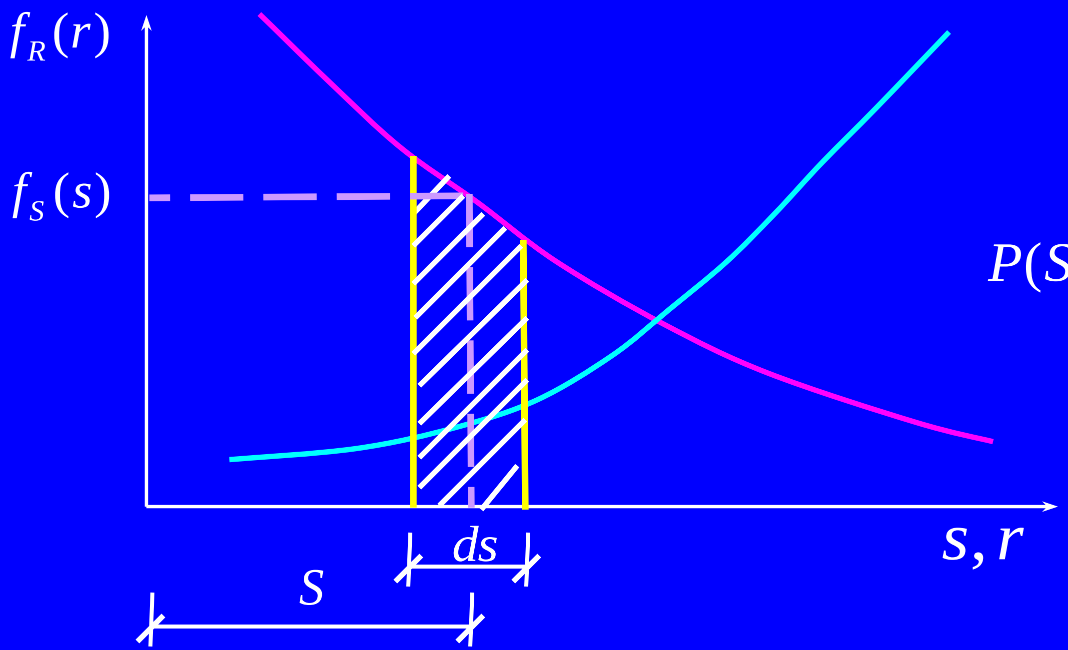
$$P_f = \iint_{r \leqslant s} f_R(r) \cdot f_S(s) dr ds = \int_{-\infty}^{+\infty} \left[\int_{r}^{+\infty} f_S(s) dr \right] \cdot f_R(r) ds$$

$$F_S(r) = P(S \leqslant r) = 1 - P(S > r)$$

Formula 2 of PIM

$$P_f = \int_{-\infty}^{+\infty} \left[1 - F_S(r)\right] \cdot f_R(r) dr = E\left[1 - F_S(r)\right]$$

3. Physical Meaning of Probability Interference



The probability of load effect S being in space ds ***:***

$$P(S-\frac{ds}{2}\leqslant S\leqslant S+\frac{ds}{2})=f_S(s)ds$$

The probability of resistance R being less than a specific load effect s:

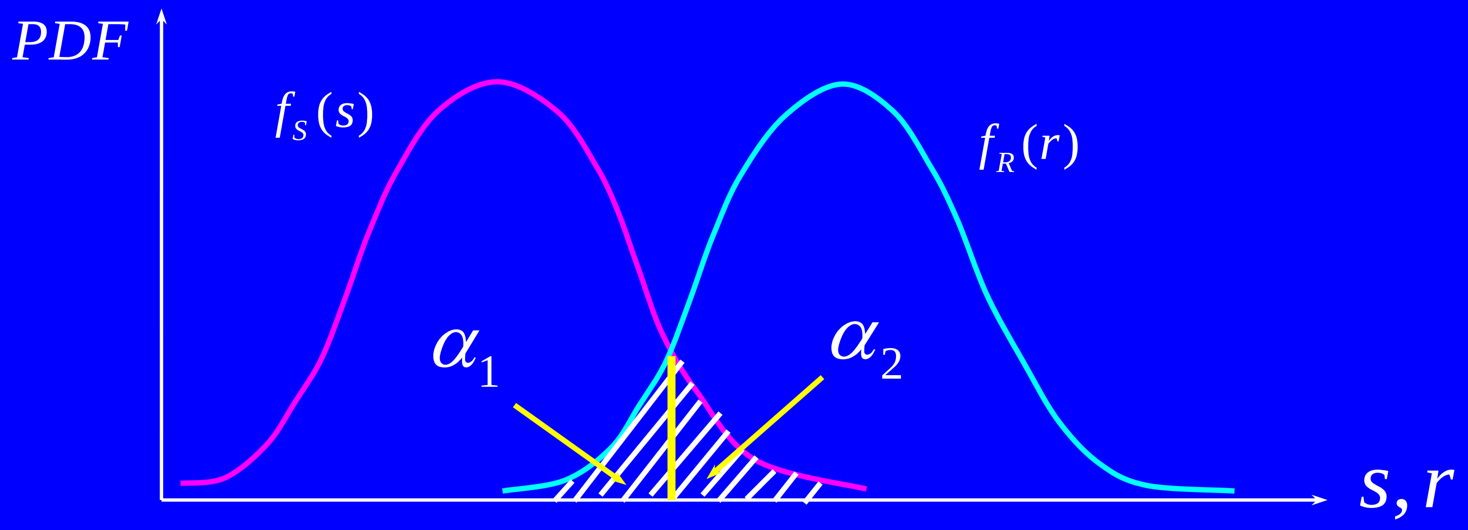
$$P(R<s)=\int_{-\infty}^{s} f_R(r)dr=F_R(s)$$

Since R & S are statistical independent, the probability of R & S simultaneously occurring in space ds ***is:***

$$f_S(s)ds\cdot\int_{-\infty}^{s} f_R(r)dr=F_R(s)f_S(s)ds$$

Attention!

$$P_f \neq \text{interference area}$$



Actually, it has been verified that there exists a relationship as follows:

$$\alpha_1\alpha_2 \leqslant P_f \leqslant \alpha_1 + \alpha_2 - \alpha_1\alpha_2$$

Example 2.1

Consider the performance function of the square section strength of a structural element

$$Z = R - S$$

where R and S are random variables.

The distribution parameters of R & S are shown below:

R is a normal RV

$$\mu_R = 10 kN/cm^2$$

$$\sigma_R = 1 kN/cm^2$$

The PDF of S is

$$f_S(s) = \lambda e^{-\lambda s} (s \geqslant 0)$$

S is a exponent RV

$$\mu_S = 1/\lambda = 5 kN/cm^2$$

$$\sigma_S = 1/\lambda = 5 kN/cm^2$$

Calculate the failure probability of the element.

Solution:

$$F_S(s) = 1 - e^{-\lambda s}$$

$$P_f = \int_0^{+\infty} [1 - F_S(r)] \cdot f_R(r)dr = \int_0^{+\infty} \left[1 - \left(1 - e^{-\lambda r}\right)\right] \cdot f_R(r)dr$$

$$= \int_0^{+\infty} \frac{1}{\sqrt{2\pi}\sigma_R} e^{-\frac{1}{2}\left(\frac{r-\mu_R}{\sigma_R}\right)^2} \cdot e^{-\lambda r} dr$$

$$= \int_0^{+\infty} \frac{1}{\sqrt{2\pi}\sigma_R} \cdot \exp\left(- \frac{\left[r - \left(\mu_R - \lambda\sigma_R^2\right)\right]^2 + 2\lambda\mu_R\sigma_R^2 - \lambda^2\sigma_R^4}{2\sigma_R^2} \right) dr$$

Let $t = \dfrac{r - \left(\mu_R - \lambda\sigma_R^2\right)}{\sigma_R}$ ***, then***

$$P_f = \int_{-\frac{\mu_R - \lambda\sigma_R^2}{\sigma_R}}^{+\infty} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{t^2}{2} - \frac{1}{2}\left(2\lambda\mu_R - \lambda^2\sigma_R^2\right) \right) dt$$

$$P_f = \exp\left(-\frac{1}{2}\left(2\lambda\mu_R - \lambda^2\sigma_R^2\right)\right)\cdot\int_{-\frac{\mu_R-\lambda\sigma_R^2}{\sigma_R}}^{+\infty}\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{t^2}{2}\right)dt$$

$$= \exp\left(-\frac{1}{2}\left(2\lambda\mu_R - \lambda^2\sigma_R^2\right)\right)\cdot\left[1-\Phi\left(-\frac{\mu_R-\lambda\sigma_R^2}{\sigma_R}\right)\right]$$

If we place the practical values of μ_R , σ_R , μ_S , σ_S ***into the above formula, then we will obtain the failure probability:***

$$P_f = e^{-1.98}\cdot\left[1-\Phi\left(-9.8\right)\right] = 0.13807$$

End of Example 2.1

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Basic Concepts of Structural Reliability Theory

2.3 Reliability Index of Structures

2.3.1 R & S are Independent Normal Variables

1. Assumptions

Consider the performance function

$$Z = R - S$$

where, R, S *are normal random variables.*

$$R \sim N(\mu_R, \sigma_R)$$

$$S \sim N(\mu_S, \sigma_S)$$

2. Formula

Since R & S are all normal random variables, then we know that Z is also a normal RV. Therefore, we have

$$\mu_Z = \mu_R - \mu_S$$

$$\sigma_Z = \sqrt{\sigma_R^2 + \sigma_S^2}$$

2.3 Reliability Index of Structures …2

$$f_Z(z) = \frac{1}{\sqrt{2\pi}\sigma_Z} \exp\left[-\frac{1}{2}\left(\frac{z-\mu_Z}{\sigma_Z}\right)^2\right] \qquad (-\infty < z < +\infty)$$

The failure probability P_f ***is:***

$$P_f = P(Z \leqslant 0) = \int_{-\infty}^{0} f_Z(z)dz$$

$$= \int_{-\infty}^{0} \frac{1}{\sqrt{2\pi}\sigma_Z} \exp\left[-\frac{1}{2}\left(\frac{z-\mu_Z}{\sigma_Z}\right)^2\right] dz$$

Let $u = \dfrac{z-\mu_Z}{\sigma_Z}$ ***, then*** $dz = \sigma_Z du$

$$P_f = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{-\frac{\mu_Z}{\sigma_Z}} \exp\left(-\frac{u^2}{2}\right) du = \Phi\left(-\frac{\mu_Z}{\sigma_Z}\right)$$

2.3 Reliability Index of Structures …3

Let $\beta = \dfrac{\mu_Z}{\sigma_Z}$ ***, then*** $P_f = \Phi(-\beta)$ **.** β ***is called Reliability Index.***

Formula of Reliability Index

$$\beta = \frac{\mu_Z}{\sigma_Z} = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}}$$

$$P_f = \Phi(-\beta) \qquad \beta = -\Phi\left(P_f\right)$$

$$P_s = 1 - P_f = 1 - \Phi(-\beta) = \Phi(\beta)$$

MATLAB Programs

```
> format short e
> beta = 1.0:0.5:5.0;
> Pf = normcdf( -beta,0,1)
```

```
> n=1:9
> Pf = 10.^(-n);
> beta = -norminv(Pf)
```

Table2.2 Relationship between β and P_f

β	P_f	β	P_f	β	P_f
1.0	1.59×10^{-1}	2.5	6.21×10^{-3}	4.0	3.17×10^{-5}
1.5	6.68×10^{-2}	3.0	1.35×10^{-3}	4.5	3.40×10^{-6}
2.0	2.28×10^{-2}	3.5	2.33×10^{-4}	5.0	2.87×10^{-7}

$$P_f \left\{ \begin{array}{ll} 10^{-1} & 1.28 \\ 10^{-2} & 2.33 \\ 10^{-3} & 3.09 \\ 10^{-4} & 3.71 \\ 10^{-5} & 4.26 \\ 10^{-6} & 4.75 \\ 10^{-7} & 5.19 \\ 10^{-8} & 5.62 \\ 10^{-9} & 5.99 \end{array} \right\} \beta$$

2.3.2 R & S are Independent Lognormal Variables

1. Assumptions

Consider the performance function

$$Z = \ln R - \ln S$$

where, R, S ***are lognormal random variables.***

$$R \sim LN(\mu_R, \sigma_R)$$

$$S \sim LN(\mu_S, \sigma_S)$$

2. Formula

$$\left.\begin{aligned} \mu_Z &= \mu_{\ln R} - \mu_{\ln S} \\ \sigma_Z &= \sqrt{\sigma_{\ln R}^2 + \sigma_{\ln S}^2} \end{aligned}\right\} \Longrightarrow \beta = \frac{\mu_Z}{\sigma_Z} = \frac{\mu_{\ln R} - \mu_{\ln S}}{\sqrt{\sigma_{\ln R}^2 + \sigma_{\ln S}^2}}$$

2.3 Reliability Index of Structures …6

$$\left.\begin{aligned}
\mu_{\ln R} &= \ln \frac{\mu_R}{\sqrt{1+V_R^2}} \\
\sigma_{\ln R} &= \sqrt{\ln(1+V_R^2)} \\
\mu_{\ln S} &= \ln \frac{\mu_S}{\sqrt{1+V_S^2}} \\
\sigma_{\ln S} &= \sqrt{\ln(1+V_S^2)}
\end{aligned}\right\} \Rightarrow \beta = \frac{\ln\left[\dfrac{\mu_R}{\mu_S}\sqrt{\dfrac{1+V_S^2}{1+V_R^2}}\right]}{\sqrt{\ln\left[\left(1+V_R^2\right)\left(1+V_S^2\right)\right]}}$$

$$\left.\begin{aligned}
V_R &\leqslant 0.3 \\
V_S &\leqslant 0.3
\end{aligned}\right\} \Rightarrow \beta = \frac{\ln \mu_R - \ln \mu_S}{\sqrt{V_R^2 + V_S^2}}$$

Example 2.2

Consider the performance function of a structural element $Z = R - S$ ***, where R and S are normal random variables.***

$$(\mu_R, \sigma_R) = (685.40,\ 64.31) MPa$$

$$(\mu_S, \sigma_S) = (372.89,\ 41.30) MPa$$

Calculate the reliability index of the element.

Solution:

$$\beta = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}} = \frac{685.40 - 372.89}{\sqrt{64.31^2 + 41.30^2}} = 4.09$$

$$P_s = \Phi(\beta) = \Phi(4.09) = 99.99\%$$

Example 2.3

Consider the performance function of a structural element $Z = R - S$, where R and S are lognormal random variables.

$$(\mu_R, \sigma_R) = (135.06, 12.895) MPa$$

$$(\mu_S, \sigma_S) = (58.94, 17.964) MPa$$

Calculate the reliability index of the element.

Solution:

$$V_R = \sigma_R / \mu_R = 0.0955 \quad V_S = \sigma_S / \mu_S = 0.3048$$

$$\beta = \frac{\ln\left[\dfrac{\mu_R}{\mu_S}\sqrt{\dfrac{1+V_S^2}{1+V_R^2}}\right]}{\sqrt{\ln\left[\left(1+V_R^2\right)\left(1+V_S^2\right)\right]}} = 2.777$$

$$P_s = \Phi(\beta) = \Phi(2.777) = 99.72\%$$

If we use the approximate formula, then we will have

$$\beta = \frac{\ln \mu_R - \ln \mu_S}{\sqrt{V_R^2 + V_S^2}} = 2.596$$

$$P_s = \Phi(\beta) = \Phi(2.596) = 99.52\%$$

The relative error of these two methods is:

$$err = \frac{2.777 - 2.596}{2.777} = 6.5\%$$

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2.4 Geometric Meaning of Reliability Index

2.4.1 Reduced Variables

The standard forms of the basic variables R & S can be expressed as:

$$U_R = \frac{R - \mu_R}{\sigma_R}$$

$$U_S = \frac{S - \mu_S}{\sigma_S}$$

The variables U_R and U_S are called reduced variables .

$$R = \mu_R + U_R \sigma_R$$

$$Z = g(R, S) = R - S$$

$$S = \mu_S + U_S \sigma_S$$

2.4 Geometric Meaning of Reliability Index …2

We can find a straight line equation in the space of reduced variables:

$$Z = g(U_R, U_S) = (\mu_R - \mu_S) + \sigma_R U_R - \sigma_S U_S$$

The above equation can be transformed into a normal equation :

$$-\frac{\sigma_R}{\sqrt{\sigma_R^2 + \sigma_S^2}} U_R + \frac{\sigma_S}{\sqrt{\sigma_R^2 + \sigma_S^2}} U_S - \frac{(\mu_R - \mu_S)}{\sqrt{\sigma_R^2 + \sigma_S^2}} = 0$$

$$\left.\begin{aligned} \alpha_R &= \cos\theta_R = -\frac{\sigma_R}{\sqrt{\sigma_R^2 + \sigma_S^2}} \\ \alpha_S &= \cos\theta_S = \frac{\sigma_S}{\sqrt{\sigma_R^2 + \sigma_S^2}} \\ &\frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}} = \beta \end{aligned}\right\} \Longrightarrow \alpha_R U_R + \alpha_S U_S - \beta = 0$$

2.4.2 Geometric Meaning of Reliability Index

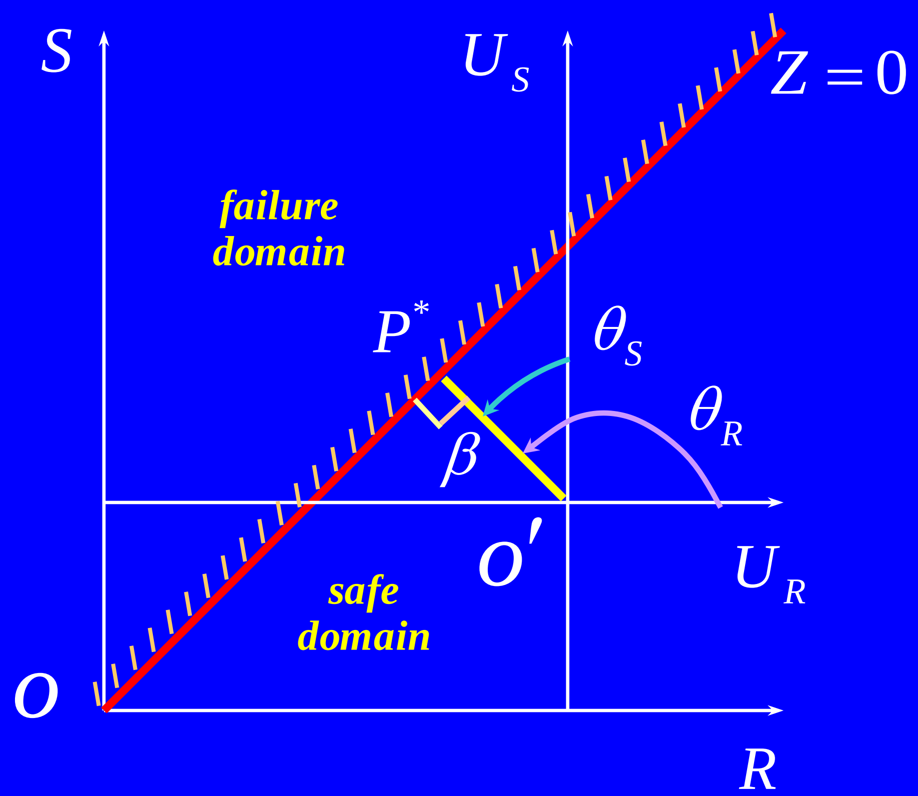
The Reliability Index is the shortest distance from the origin of reduced variables to the limit state equation.

$$O'P^* = \beta$$

- *The definition can be generalized for n variable space.*
- P^* *is called the design point.*
- *the coordinate values of* P^*

$$R^* = \mu_R + \alpha_R \beta \sigma_R$$

$$S^* = \mu_S + \alpha_S \beta \sigma_S$$



Chapter 2

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2.5 Relationship between Reliability Index and Safety Factor

2.5.1 Problems of Safety Factor

– *For traditional design, structural safety is expressed as safety factor:*

$$K = \frac{\mu_R}{\mu_S}$$

– *The design formula of traditional design is*

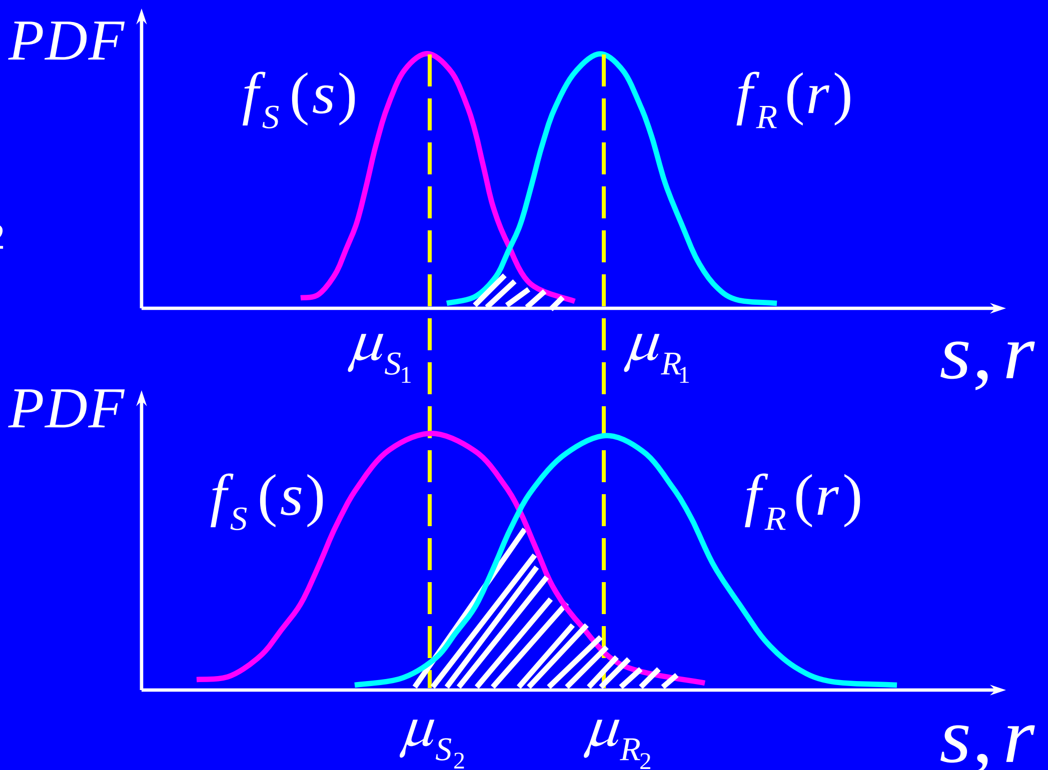
$$\mu_R \geqslant K \mu_S$$

– *Problems of traditional design*

- *The values of K is determined by experience and engineering judgments*
- *K is only related to the mean values of R & S, therefore it cannot reflect the practical failure events of structures.*

2.5 Relationship between Reliability Index and Safety Factor…2

$$\left.\begin{array}{l} K_1 = \dfrac{\mu_{R_1}}{\mu_{S_1}} \\[2ex] K_2 = \dfrac{\mu_{R_2}}{\mu_{S_2}} \end{array}\right\} \Rightarrow K_1 = K_2$$



However, $P_{f\,1} << P_{f\,2}$

- ***P_f is not only related to the center position of PDF of R & S, it is also related to the degree of disperse relative to the means.***
- ***Safety factor K cannot reflect this fact !***
- ***Reliability index β can reflect this fact !***

2.5.2 Relationships between *K* and β

– ***For two independent normal RVs***

$$\beta = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}} = \frac{\dfrac{\mu_R}{\mu_S} - 1}{\sqrt{\left(\dfrac{\mu_R}{\mu_S}\right)^2 V_R^2 + V_S^2}} = \frac{K - 1}{\sqrt{K^2 V_R^2 + V_S^2}}$$

$$K = \frac{1 + \beta\sqrt{V_R^2 + V_S^2 - \beta^2 V_R^2 V_S^2}}{1 - \beta^2 V_R^2}$$

$$\beta \left\{ \begin{array}{l} \textit{Mean values} \\ \textit{Variation coefficients} \\ K \end{array} \right.$$

Homework 2

2.1 Deduce the formula of safety probability P_s of probability interference method.

Required: (1) Give two types of P_s just like P_f .

(2) The key point of your deduction should be shown by figure.

2.2 By using the formula that you deduce in homework 2.1, calculate the safety probability of the performance function shown in Example 2.1.

Known: All statistical information is identical to that in Example 2.1.

End of

Chapter 2