Problem Set 3, HS 239[1]

In Class, 9/10/2024.

1. You have Bernoulli utility function $v = \ln(c)$ Your initial wealth is Rs c_0. You can invest c' of this in an asset that will return to you either double the amount you invested or half the amount you invested, with equal probabilities. The rest, $c - c'$ does not earn any interest rate. Calculate the elasticity of your optimal choice of c' with respect to c_0.

2. In the Markowitz model, what happens to the shape of risk-return locus if ρ is negative? What if $\rho = -1$?What if $\rho = 1$?

3. Mr. X has an initial income of Rs 100: If an accident happens (with probability $\frac{1}{3}$), the income will go down to Rs 70. An insurance company must offer an actuarially fair insurance.

Suppose that utility function of Mr. X is $\ln(W)$. Calculate the premium and indemnity of the insurance contract. How would your answer change if the utility function is $\sqrt{W}$?

4. Continue with the above problem. What happens if the firm charges 1.25 times the fair premium?

5. India and AOT (any other team) are playing a crucial game of cricket in UK (where some form of betting is legal). If India wins, your utility is $\ln(w)$. If AOT wins, your utility is $0.5\ln(w)$. Your initial income is \$1000. You can put a bet of $\$x$ favoring any team. If the other team wins, you lose the money. If the team that is backed by you wins, you will get twice the money (that is, $2x$).

How much bet you will place favoring India (if the amount is negative, your bet favors AOT)? Cricket being a game of uncertainties, each team is predicted to win with a probability 0.5.

[1] These problems are indicative in nature. There is no guarantee that only these and/or similar problems will be asked in the examination or that the exam is "problems only".

[Just to put the matters in perspective, bookies were offering a $1:66$ chance to India before the 1983 world cup final. That means, if you invest 1 Rupee, you lose the rupee if West Indies wins. If India wins, you get back 67 rupees. Cricket World cup final. In 2016, before the EPL, the bookies offered $1:5000$ chance to Leicester City]

6. An individual with endowed income c has a concave utility function $v(c)$.He has contracted a disease which, if not treated, will be fatal with probability $1-p_0$ and will spontaneously cure itself with probability p_0. His "bequest utility"in the event of death is zero everywhere. A physician, who charges Rs z will increase the chance of survival to $p > p_0$. Depict the relationship between p (in horizontal axis) and z in a figure. Interpret its shape.