Quiz 1: Answer Key

HS 239, 2024 Monsoon

1. *A person has the Cobb-Douglas utility* $v = \frac{1}{4}\ln(x_1) + \frac{3}{4}\ln(x_2)$ *. Price of good 1 is 1, that of good 2 is 2 and income is 100.*

a) *Compute the dmand functions for good 1 and 2.*

b) *Assume that price of good 2 falls to 1.Compute the change in consumer surplus.*

(7+3=10)

[**Straight from lectures and problem set 1, noting that** v **is equivalent to** $u = x_1^{25} x_2^{.75}$]

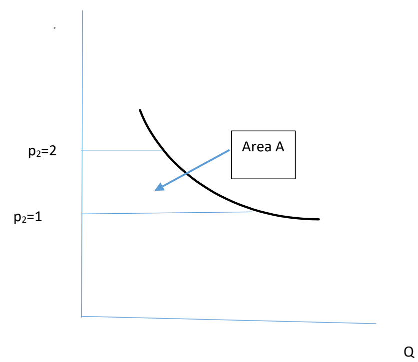
ANS:

a) We need to maximize utility subject to budget constraint $p_1x_1+p_2x_2 = M$. Use the Lagrangian (or any method of your choice) **to show that (mere stating the result will not do, as it is a 7 marks question)** the demand functions are $x_1 = \frac{1}{4} * \frac{M}{p_1}$ and $x_2 = \frac{3}{4} * \frac{M}{p_2}$.

Now $M = 100$ and p_2 falls to 1 from 2. Hence, the required change in CS is the area (on price axis) bounded by the demand curve and the two prices, i.e.

$$\begin{aligned}
\Delta CS &= \int_1^2 \frac{3}{4} * \frac{100.0}{p_2} dp_2 \\
&= 75.0 * [\ln 2.0 - \ln 1] \\
&= 75.0 * \ln(2.0) \\
&= 51.99
\end{aligned}$$

This is clear in the following figure.



Consumer Surplus: Change

2. *Explain why*

a) Indifference curves cannot cross.

b) In perfect competition, the marginal cost curve is the supply curve of a firm.

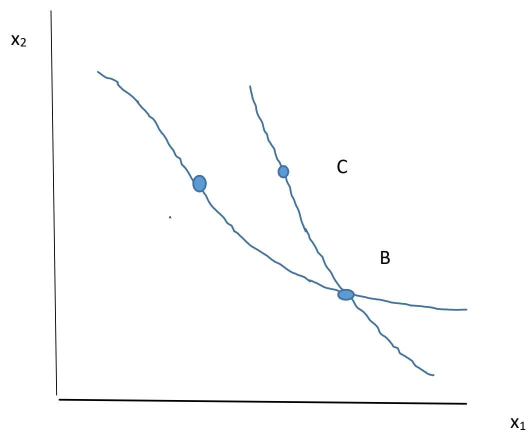
c) In consumers' utility maximization, the Lagrange multiplier is the marginal utility of money.

(5+5+5=10)

[**All of these were discussed in class and/or available in lecture notes/PS1**]

ANS

a) Suppose, Indifference curves cross (see the following figure).



Now $B \sim C$ (same indifference curve) and $B \sim A$. By transitivity of indifference relation, $C \sim A$. But, by construction, C contains more of both commodities compared to bundle A. By monotonicity, C is strictly preferred to A This is a contradiction as the agent, at the same time, strictly prefers C over A and is indifferent between C and A.

b) Under perfect competition, a single producer maximizes his/her profit $pq - c(q)$ taking p as given. The profit maximization exercise leads to

$$p = c'(q) = MC(q)$$

Thus, against each price, the profit maximizing output supplied can be read off from the MC curve. As this uniquely relates price and quantity supplied, the marginal cost curve is the supply curve.

(Note: An "AA" answer should also supply the relevant diagram).

c) **PS 1, Q3, done in class on 21/8.**