

$$-EI \frac{d^2y}{dx^2} = \frac{w_0 L}{2} x - \frac{w_0 x^2}{2}$$

$$\Rightarrow -EI \frac{dy}{dx} = \frac{w_0 L x^2}{4} - \frac{w_0 x^3}{6} + C_1$$

$$\Rightarrow -EI y = \frac{w_0 L x^3}{12} - \frac{w_0 x^4}{24} + C_1 x + C_2$$

$$y(0) = 0 \Rightarrow C_2 = 0$$

$$\text{&} y(L) = 0 \Rightarrow 0 = \frac{w_0 L^4}{12} - \frac{w_0 L^4}{24} + C_1 L$$

$$\Rightarrow C_1 = -\frac{w_0 L^3}{24}$$

$$\Rightarrow y = \frac{L}{EI} \left[\frac{w_0 x^4}{24} - \frac{w_0 x^3 L}{12} + \frac{w_0 L^3 x}{24} \right]$$

$$\Rightarrow \phi(x) = \frac{y(x)}{y(L)} = \frac{\cancel{L} \times w_0}{\cancel{EI}} \left[\frac{x^4}{24} - \frac{2x^3}{12} + \frac{L^3 x}{24} \right] / \left[\cancel{L} \times w_0 \left[\frac{L^4}{384} - \frac{L^4}{96} + \frac{L^4}{48} \right] \right]$$

$$= \frac{16x^4 - 32Lx^3 + 16L^3x}{5L^4}$$

$$\Rightarrow \phi(x) = \frac{16}{5L^4} [x^4 - 2Lx^3 + L^3x]$$

$$\rightarrow m^* = \int_0^L m(x) [\phi(x)]^2 dx$$

$$= \frac{PA \times 256}{25L^8} \int_0^L (x^8 + 4L^2x^6 + L^6x^2 - 4Lx^7 - 4L^4x^4 + 2L^3x^5) dx$$

$$= \frac{256PAL}{25} \left[\frac{1}{9} + \frac{4}{7} + \frac{1}{3} - \frac{4}{8} - \frac{4}{5} + \frac{2}{6} \right]$$

$$\Rightarrow m^* = 0.504 PAL$$

$$\rightarrow K^* = \int_0^L E(x) I(x) [\phi''(x)]^2 dx$$

$$= EI \times \frac{256}{25L^8} \int_0^L (12x^2 - 12Lx)^2 dx$$

$$= EI \times \frac{256}{25L^8} \int_0^L (144x^4 + 144L^2x^2 - 288Lx^3) dx$$

$$= \frac{256 EI}{L^3} \left[\frac{144}{5} + \frac{144}{3} - \frac{288}{4} \right]$$

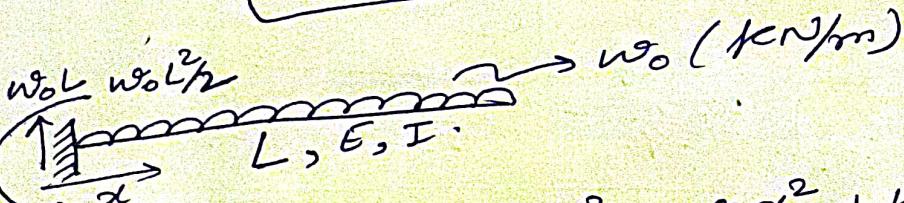
$$\Rightarrow k^* = \frac{49 \cdot 15 EI}{L^3} \approx \frac{48 EI}{L^3}$$

$$\rightarrow F^*(x) = \int_0^L F(x) \phi(x) dx$$

$$= w_0 \times \frac{16}{9 L^4} \int_0^L (x^4 - 2Lx^3 + \frac{2}{3}L^3x) dx$$

$$= \frac{16}{9} w_0 L \left[\frac{1}{5} - \frac{2}{4} + \frac{1}{2} \right]$$

$$\Rightarrow F^*(x) = 0.64 w_0 L$$



$$-EI \frac{d^2y}{dx^2} = -\frac{w_0 L^2}{2} - \frac{w_0 x^2}{2} + w_0 L x$$

$$\Rightarrow -EI \frac{dy}{dx} = -\frac{w_0 L^2 x}{2} - \frac{w_0 x^3}{6} + \frac{w_0 L x^2}{2} + C_1$$

$$\Rightarrow -EI y = -\frac{w_0 L^2 x^2}{4} - \frac{w_0 x^4}{24} + \frac{w_0 L x^3}{6} + C_1 x + C_2$$

$$y'(0) = 0 \Rightarrow C_1 = 0 \quad \& \quad y(0) = 0 \Rightarrow C_2 = 0$$

$$\Rightarrow y = \frac{1}{6I} \left[\frac{x^4}{24} - \frac{Lx^3}{6} + \frac{L^2 x^2}{4} \right]$$

$$\Rightarrow \phi = \frac{y(x)}{y(L)} = \frac{\frac{x^4}{24} - \frac{Lx^3}{6} + \frac{L^2 x^2}{4}}{\frac{L^4}{24} - \frac{L^4}{6} + \frac{L^4}{4}}$$

$$\Rightarrow \phi(x) = \frac{1}{3L^4} [x^4 - 4Lx^3 + 6L^2x^2]$$

$$\rightarrow m^* = \int_0^L m(x) [\phi(x)]^2 dx$$

$$= \frac{PA}{9L^8} \int_0^L (x^8 + 16L^2x^6 + 36L^4x^4 - 8Lx^7 + 12L^2x^6 - 48L^3x^5) dx$$

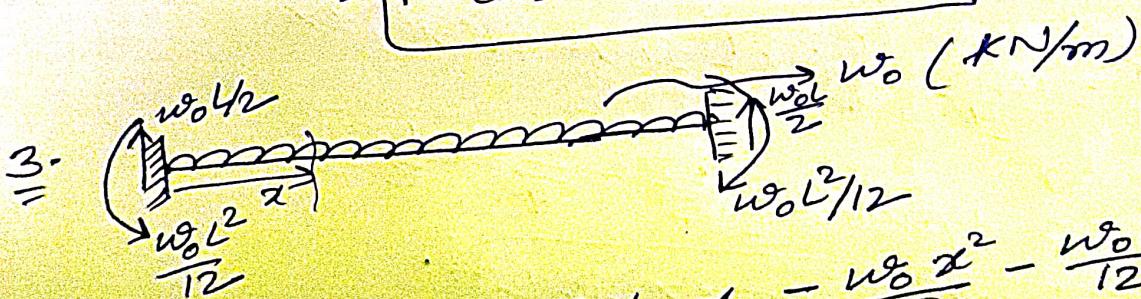
$$= \frac{PAL}{9} \left[\frac{1}{9} + \frac{16}{7} + \frac{36}{5} - \frac{8}{8} + \frac{12}{7} - \frac{48}{6} \right]$$

$$\Rightarrow m^* = 0.257 \text{ PAL} \rightarrow$$

$$\begin{aligned} \rightarrow K^* &= \int_0^L E(x) I(x) [\phi''(x)]^2 dx \\ &= \frac{EI}{9L^8} \int_0^L (12x^2 - 24Lx + 12L^2)^2 dx \\ &= \frac{EI}{9L^8} \int_0^L (144x^4 + 576L^2x^2 + 144L^4 - 576L^3x + 288L^2x^2) dx \\ &= \frac{EI}{9L^3} \left[\frac{144}{5} + \frac{576}{3} + 144 - \frac{576}{4} - \frac{576}{2} + \frac{288}{3} \right] \end{aligned}$$

$$\Rightarrow K^* = \frac{3 \cdot 2 EI}{L^3} \approx \frac{3 EI}{L^3}$$

$$\begin{aligned} \rightarrow F^*(x) &= \int_0^L F(x) \phi(x) dx \\ &= \frac{w_0}{3L^4} \int_0^L (x^4 - 4Lx^3 + 6L^2x^2) dx \\ &= \frac{w_0 L}{3} \left[\frac{1}{5} - \frac{4}{4} + \frac{6}{3} \right] \\ \Rightarrow F^*(x) &= 0.4w_0 L \end{aligned}$$



$$-EI \frac{d^2y}{dx^2} = \frac{w_0 L}{2} x - \frac{w_0 x^2}{2} - \frac{w_0 L^2}{12}$$

$$\begin{aligned} \Rightarrow -EI \frac{dy}{dx} &= \frac{w_0 L x^2}{4} - \frac{w_0 x^3}{6} - \frac{w_0 L^2 x}{12} + C_1 \\ \Rightarrow -EI y &= \frac{w_0 L x^3}{12} - \frac{w_0 x^4}{24} - \frac{w_0 L^2 x^2}{24} + C_2 x + C_2 \end{aligned}$$

$$y'(0) = 0 \Rightarrow C_1 = 0 \quad \& \quad y(0) = 0 \Rightarrow C_2 = 0$$

$$\Rightarrow y = \frac{w_0}{EI} \left(\frac{x^4}{24} - \frac{Lx^2}{12} + \frac{L^2x^2}{24} \right)$$

$$\Rightarrow \phi(x) = \frac{y(x)}{y(y_2)} = \frac{\frac{x^4}{24} - \frac{Lx^3}{12} + \frac{L^2x^2}{24}}{\frac{L^4}{16 \times 24} - \frac{L^4}{96} + \frac{L^4}{96}}$$

$$\Rightarrow \boxed{\phi(x) = \frac{L^6}{L^4} [x^4 - 2Lx^3 + L^2x^2]}$$

$$\rightarrow m^* = PA \times \frac{256}{L^8} \int_0^L (x^8 + 4L^2x^6 + L^4x^4 - 4Lx^7 - 4L^3x^5 + 2L^2x^6) dx$$

$$= PA \times 256L \left[\frac{L}{9} + \frac{4}{7} + \frac{L}{5} - \frac{L}{8} - \frac{4}{6} + \frac{2}{7} \right]$$

$$\Rightarrow \boxed{m^* = 0.406 PA L}$$

$$\rightarrow k^* = EI \times \frac{256}{L^8} \int_0^L (12x^2 - 12Lx + 2L^2)^2 dx$$

$$= EI \times \frac{256}{L^8} \int_0^L (144x^4 + 144L^2x^2 + 4L^4 - 288Lx^3 - 48L^3x + 48x^2L^2) dx$$

$$= \frac{256EI}{L^5} \left[\frac{144}{5} + \frac{144}{3} + 4 - \frac{288}{4} - \frac{48}{2} + \frac{48}{3} \right]$$

$$\Rightarrow \boxed{k^* = \frac{204.80 EI}{L^3}}$$

$$\rightarrow F^* = \frac{16w_0}{L^4} \int_0^L (x^4 - 2Lx^3 + L^2x^2) dx$$

$$= 16w_0 L \left[\frac{L}{5} - \frac{2}{4} + \frac{L}{3} \right]$$

$$\Rightarrow \boxed{F^* = 0.533 w_0 L}$$