Time: 50 min | Quiz: BT612 Systems Biology | Marks: 10

Name: | Roll No.

Instructions: Write the answers briefly, but you must show all the relevant steps in the calculations/derivations. Use conventional mathematical symbols/notations. No marks will be given for a partially correct answer. Marks will be deducted for irrelevant calculations/derivations.

Q1. What is the number of possible steady states for the following system of ODEs? Find these steady states and characterize their stability. In this ODE, r is the last digit of your roll number. If the last digit of your roll number is 0, then $r = 1$. Examples: If your roll number is 240103027, then $r = 7$ and if your roll number is 240103020, then $r = 1$. [4]

$$\frac{dx}{dt} = x^4 + x^3 + rx, \quad \text{for } x \geq 0,\ r \geq 1$$

Q2. Nutrient-dependent microbial growth is modelled using the following system of ODEs. Draw the nullclines and mark the steady states on the nullcline plot. Here, $\mu = (2 + r)$. r is the last digit of your roll number. Examples: If your roll number is 240103027, then $r = 7$ and if your roll number is 240103020, then $r = 0$. [4]

$$\frac{dx}{dt} = \frac{\mu s}{1+s}x - x; \quad \frac{ds}{dt} = (s_o - s) - \frac{\mu s}{1+s}x, \quad \text{for } x \geq 0,\ s \geq 0,\ s_0 > s,\ \mu \geq 2 \text{ and } s_0 > \frac{1}{\mu - 1}$$

Q3. The following reversible reaction is regulated by two enzymes, K and P. Both the enzymes follow the Michaelis–Menten kinetics. Under what conditions would this reaction system work like an ultra-sensitive ON-OFF switch? [2]

$$A \underset{P}{\overset{K}{\rightleftharpoons}} B$$

------------------------------------ Write the answers from here ------------------------------------

Q1. $\frac{dx}{dt} = x^4 + x^3 + rx, \quad x \geq 0, \quad r \geq 1$

At steady state, $dx/dt = 0$.

$\therefore \quad x = 0$ ——— (1)

$x^3 + x^2 + r = 0$ ——— (2)

For $r \geq 1$, Eq (2) does not have any solution such that $x \geq 0$. You can test that either by using the "sign change rule" or by ~~using~~ drawing $f(x) = x^3 + x^2 + r$.

→

Q1.

$\therefore$ For $x \geq 0$, $r \geq 1$, the only possible steady state is $x^* = 0$. | ⊛ Ans

$f(x) = x^4 + x^3 + rx$

$\therefore \frac{d}{dx} f(x) = 4x^3 + 3x^2 + r$

$\therefore$ At $x^* = 0$,

$$\left.\frac{d}{dx} f(x)\right|_{x^*=0} = r > 0 \quad [\text{as } r \geq 1]$$

$\therefore$ The steady state at $x = 0$ is unstable | * Ans

Q2.
$$\frac{dx}{dt} = \frac{\mu s}{1+s} x - x \quad \text{——— (1)}$$

$$\frac{ds}{dt} = (s_0 - s) - \frac{\mu s}{1+s} x \quad \text{——— (2)}$$

$x \geq 0$, $s \geq 0$, $s_0 > s$, $\mu > 2$, $s_0 > \frac{1}{\mu - 1}$

⊛ X-nullclines:

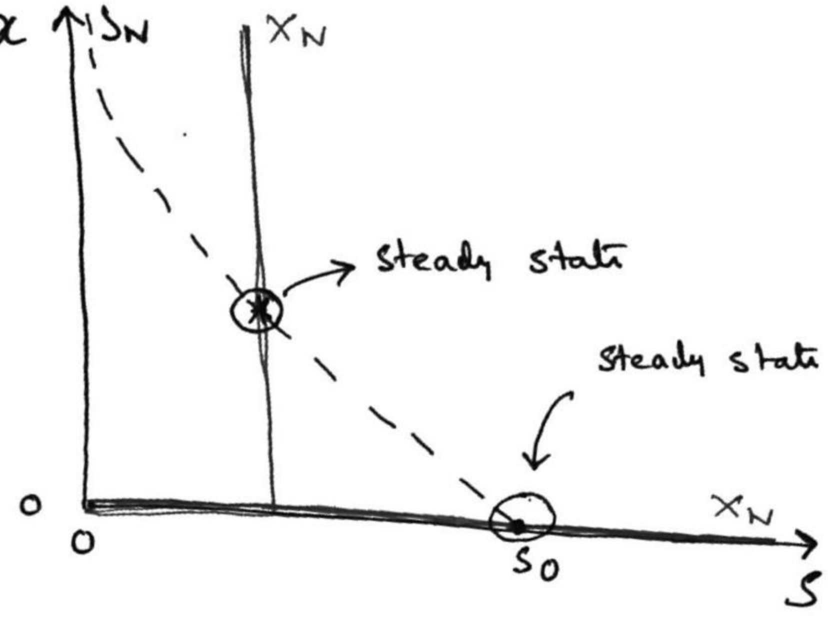
$\frac{dx}{dt} = 0 \Rightarrow x = 0$ and $s = \left(\frac{1}{\mu - 1}\right)$

⊛ S-nullclines:

$\frac{ds}{dt} = 0 \Rightarrow x = \frac{(s_0 - s)(1+s)}{\mu s}$

Note for: S nullcline $\Rightarrow$ $x = 0$ when $s = s_0$. x undefined when $s = 0$. So there is an asymptot.

Q2



Q3.

$$A \underset{P}{\overset{K}{\rightleftharpoons}} B$$

To capture the behavior of this system, we use the following ODE:

$$\frac{dB}{dt} = \frac{k_{31}\, K\, (A_T - B)}{K_{M1} + (A_T - B)} - \frac{K_{32}\, P\, B}{K_{M2} + B}$$

Here, $A_T = A + B$ = constant.

It can be shown that the steady state value of B will have a sharp sigmoid behavior with varying K if $\frac{K_{M1}}{A_T} << 1$ and $\frac{K_{M2}}{A_T} << 1$. This is called saturated regime. Under this condition, this system will work as an ultra-sensitive ON-OFF switch.

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Q1. What is the number of possible steady states for the following system of ODEs? Find these steady states and characterize their stability. In this ODE, p is the last digit of your roll number. If the last digit of your roll number is 0, then $p = 1$. Examples: If your roll number is 240103027, then $p = 7$ and if your roll number is 240103020, then $p = 1$. [4]

$$\frac{dx}{dt} = px + x^2 + x^4, \text{ for } x \geq 0,\ p \geq 1$$

Q2. A bacterial quorum sensing system is modelled using the following system of ODEs. Draw the nullclines of this system and mark the steady states on the nullcline plot. Here, r is the last digit of your roll number. If the last digit of your roll number is 0, then $r = 1$. Examples: If your roll number is 240103027, then $r = 7$ and if your roll number is 240103020, then $r = 1$. [4]

$$\frac{dx}{dt} = \frac{x^2}{1+x^2} - rx,\quad \frac{dy}{dt} = (1-y)y - yx,\ \text{ for } x \geq 0,\ y \geq 0,\ r \geq 1$$

Q3. The following system of ODEs is used to model a network motif. What type of network motif is this? Represent this motif diagrammatically. [2]

$$\frac{dx}{dt} = \frac{k_1}{a+z^3} - d_1 x,\quad \frac{dy}{dt} = k_2 x - d_2 y,\quad \frac{dz}{dt} = k_3 \frac{y}{b+y} - d_3 z,\ \text{ for } x, y, z \geq 0$$

---------- Write answers from here ----------

Q1. $\frac{dx}{dt} = px + x^2 + x^4$, $x \geq 0$, $p \geq 1$

At steady state, $\frac{dx}{dt} = 0$,

$\therefore\ x = 0$ ——— ①

$x^3 + x + p = 0$ ——— ②

Eq ② does not have any solution for $x \geq 0$.

This could be checked using the "sign change rule"

or by plotting $f(x) = x^3 + x + p$.

$\longrightarrow$

Q1.

$$f(x) = px + x^2 + x^4$$ (5)

$$\therefore \frac{d}{dx} f(x) = p + 2x + 4x^3$$

$\therefore$ At steady state $x^* = 0$,

$$\left.\frac{d}{dx} f(x)\right|_{x^*=0} = p > 0 \quad [\text{as } p \geq 1]$$

$\therefore$ For $x \geq 0$, $p \geq 1$, this ODE has only one steady state at $\underline{x = 0}$ and it is $\underline{\text{unstable}}$. Ans.

Q2.

$$\frac{dx}{dt} = \frac{x^2}{1+x^2} - rx \quad \text{——— (1)}$$

$$\frac{dy}{dt} = (1-y)y - yx \quad \text{——— (2)}$$

$x \geq 0$, $y \geq 0$, $r \geq 1$

* X-null clines,

$$\frac{dx}{dt} = 0 \Rightarrow x = 0 \quad \text{——— (3)}$$

$$x = \frac{1 \pm \sqrt{1-4r^2}}{2r} \quad \text{——— (4)}$$

As $r \geq 1$, Eq (4) does not have real solution.

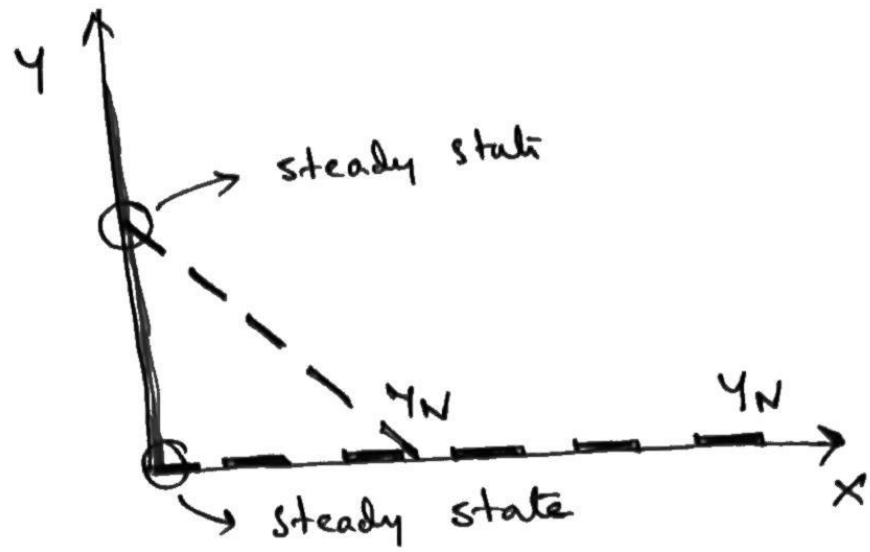
* Y-nullclines,

$$\frac{dy}{dt} = 0 \Rightarrow y = 0,$$

$$y = 1 - x,$$

$\longrightarrow$

Q2.

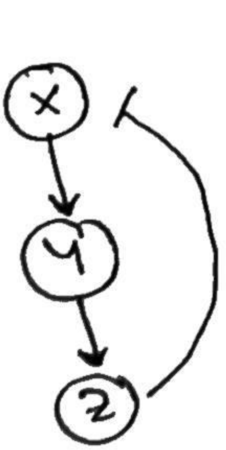


Q3.

$$\frac{dx}{dt} = \underbrace{\frac{k_1}{a+z^3}}_{\text{Inhibition}} - d_1 x$$

$$\frac{dy}{dt} = \underbrace{k_2 x}_{\text{Activation / Induction}} - d_2 y$$

$$\frac{dz}{dt} = k_3 \underbrace{\frac{y}{b+y}}_{\text{Activation / Induction}} - d_3 z$$



Negative Feedback

Marks: 10 | **Quiz: BT612 Systems Biology** | Time: 50 min

Roll No: **Name.**

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$$\frac{dx}{dt} = \frac{x^2}{1+x^2} - rx, \quad \frac{dy}{dt} = (1-y)y - yx, \quad \text{for } x \ge 0,\ y \ge 0,\ r \ge 1$$

* See page 5

Q2. The following system of ODEs is used to model a network motif. What type of network motif is this? Represent this motif diagrammatically. [2]

$$\frac{dx}{dt} = \frac{k_1}{a+z^3} + k_2\frac{y}{b+y} - d_1x, \quad \frac{dy}{dt} = k_3w - d_2y, \quad \frac{dz}{dt} = k_4\frac{w}{b+w} - d_3z, \quad \frac{dw}{dt} = k_w - d_4w, \quad \text{for } x, y, z, w \ge 0$$

Q3. What is the number of possible steady states for the following system of ODEs? Find these steady states and characterize their stability. In this ODE, q is the last digit of your roll number. If the last digit of your roll number is 0, then $q = 1$. Examples: If your roll number is 240103027, then $q = 7$ and if your roll number is 240103020, then $q = 1$. [4]

$$\frac{dx}{dt} = qx + x^3 + x^4, \quad \text{for } x \ge 0,\ q \ge 1$$

* See page 1

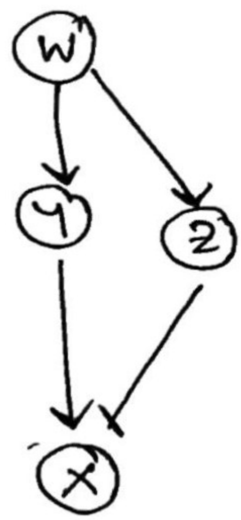
----------------------------------- Write answers from here -----------------------------------

$$\frac{dx}{dt} = \underbrace{\frac{k_1}{a+z^3}}_{\text{Inhibition}} + \underbrace{k_2\frac{y}{b+y}}_{\text{Induction}} - d_1x$$

$$\frac{dy}{dt} = \underbrace{k_3w}_{\text{Induction}} - d_2y$$

$$\frac{dz}{dt} = \underbrace{k_4\frac{w}{b+w}}_{\text{Induction}} - d_3z$$

$$\frac{dw}{dt} = \underbrace{k_w}_{\text{Constant}} - d_4w$$



Incoherent Feed forward.

8

Time: 50 min | **Quiz: BT612 Systems Biology** | Marks: 10

Roll No. **Name:**

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Q1. The following reversible reaction is regulated by two enzymes, K and P. Both the enzymes follow the Michaelis–Menten kinetics. Under what conditions would this reaction system work like an ultra-sensitive ON-OFF switch? [2]

$$X \underset{P}{\overset{K}{\rightleftharpoons}} Y$$

See page 3

Q2. What is the number of possible steady states for the following system of ODEs? Find these steady states and characterize their stability. In this ODE, q is the last digit of your roll number. If the last digit of your roll number is 0, then $q = 1$. Examples: If your roll number is 240103027, then $q = 7$ and if your roll number is 240103020, then $q = 1$. [4]

$$\frac{dx}{dt} = x^4 + x^2 + qx, \quad \text{for } x \geq 0,\ q \geq 1$$

See page 4

Q3. Nutrient-dependent microbial growth is modelled using the following system of ODEs. Draw the nullclines and mark the steady states on the nullcline plot. Here, $\mu = (2 + r)$. r is the last digit of your roll number. Examples: If your roll number is 240103027, then $r = 7$ and if your roll number is 240103020, then $r = 0$. [4]

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See page 2

------------------------------------ Write answers from here ------------------------------------