Problem Set 1, HS 239[1]

In Class, 14/8/2024

1. If utility function is $U = Ax_1^{\alpha}x_2^{1-\alpha}$ and the budget constraint is $p_1x_1 + p_2x_2 = M$, find the demand functions $x_i(p_1, p_2, M)$ and draw the demand curve.Suppose $\alpha = .75$, $M = 100$. What must be the gain in consumer surplus if p_1 falls to 2 from 4?.

2. We define the indirect utility function $v(p_1, p_2, M)$ as the maximum utility given prices and income. What is the form of $v()$ for the above example? What are the signs of $\frac{\partial v}{\partial p_i}$ and $\frac{\partial v}{\partial M}$? Do they make sense?

3. In the consumer optimization procedure as done in class, we introduced a Lagrange multiplier λ. Show that the multiplier is the marginal utility of money.

4. A function $f(x, y)$ is homogeneous of degree r if $f(\theta x, \theta y) = \theta^r f(x, y)$. What would be the degree of homogeneity for a demand function $D(p_x, p_y, M)$?

5. Mr. X has demand curve $q = 100 - 2p$. What is the total willingness to pay for 20 units? What is his consumer surplus if $p = 30$? What is the change in CS if p changes from 20 to 30?

6 Suppose the production function is $Y = K^{.5}L^{.5}$. Cost of capital is 4 and the wage rate is 1. What is the long run marginal cost of production?

7. Cost function for an entrepreneur working under perfect competition is $C = 10+2q+\frac{1}{3}q^3$. If the market price is 6, what is the producer surplus?What is the profit?

8. Suppose that demand for rice is $Q = 20 - 2p$. The price of rice is fixed by the government at Rs 6 per kg. Now suppose the government increases the price to Rs 8 per kg. What is the loss in consumer surplus?

[1]These problems are indicative in nature. There is no guarantee that only these and/or similar problems will be asked in the examination or that the exams are "problems only".

9. Consider the utility function $U = x_1 x_2$. Suppose the *initial situation* is given by $p_1 = p_2 = 1$ and $M = 10$. Now suppose the price of the first good rises to $p_1 = 2.5$.

a) Show that the total effect of consumption of good 1 is -3. That is, reduction of 3 units of consumption.

b) Show that this can be decomposed into a substitution effect of $-\left(5 - \sqrt{10}\right)$ and an income effect of $-\left(\sqrt{10} - 2\right)$.

Draw appropriate diagrams.

10. Ms Y is supposed to have a *lexicographic preference* if for any two bundles (x_0, y_0) and (x_1, y_1), $(x_0, y_0) \succ (x_1, y_1)$ if either $x_0 > x_1$ (irrespective of the amount of y) or if $x_0 = x_1$ and $y_0 > y_1$. She is indifferent between the bundles if $x_0 = x_1$ and $y_0 = y_1$.

a) Show that it is not possible to draw an indifference curve.

b) Does there exist a demand function for x?

11. Suppose Mr X has the following preference over x, y :

$$u(x, y) = x + 2y$$

Given fixed prices and money income M, what are his demand for x?