Post Mid semester exam Problem Set 2

1. Let the market demand be given by the inverse demand curve $P(Q) = 50 - 2Q$,where $Q = q_1 + q_2$. The cost function for each of the two firms in the industry is $C(q_i) = 2q_i$, $i = 1, 2$. Firms are Cournot competitors.

   (a) Derive the best response function of each firm.

   (b) Find the Cournot Nash equilibrium output of the firms and profits.

2. Three oligopolistic firms operate in a market with inverse demand function given by $P(Q) = a - Q$, where $Q = q_1 + q_2 + q_3$ and q_i is the quantity produced by firm i, $i = 1, 2, 3$. Each firm has constant marginal cost of production, c and no fixed cost. The firms choose their quantities as follows: 1) firm 1 chooses $q_1 \geq 0$; 2) firm 2 and 3 observe q_1 and then simultaneously choose q_2 and q_3 respectively. What is the subgame perfect Nash equilibrium?

3. Two firms compete in prices in a market for a homogeneous product. In this market there are N consumers; each buys one unit if the price of the product does not exceed Rs 10, and nothing otherwise. Consumers buy from the firm selling at a lower price. In case both firms charge the same price, assume that $\frac{1}{2}$ consumers buy from each firm. Assume zero cost of production for both firms.

4. Suppose there are two firms, 1 and 2 producing homogeneous product. Firm 1 and 2 compete in prices that is Bertrand Competition. The market demand function is $A - p = Q$, $A > 0$. Firm 1 has zero cost of production. Firm 2 bears fixed a cost of f, $f > 0$. We assume that there are prices such that $\frac{(A-p)p}{2} - f > 0$. Is there any pure strategy Nash equilibrium.

5. The market demand function is $1 - p = Q$ Suppose there are two firms 1 and 2. The capacity of firm 1 is $\frac{1}{3}$ and the capacity of firm 2 is $\frac{1}{4}$. The cost of production is zero till capacity for each firm. The firms cannot produce more than its capacity. Each firm sets a price. Find the pure strategy Nash Equilibrium.

Answer

1. a) There are two firms 1, 2. The inverse market demand function is $50 - 2q = p$, $Q = q_1 + q_2$. Firms 1 and 2 simultaneously chooses output q_1 and q_2 respectively. The payoff of firm 1, profit is $\pi_1(q_1, q_2) = (50-2(q_1+q_2))q_1-2q_1$. The payoff of firm 2,profit is $\pi_2(q_1, q_2) = (50-2(q_1+q_2))q_2-2q_2$. Firm 1 and 2, each maximizes profit taking the output of the other firm as given.
   $\frac{\partial((50 - 2(q_1 + q_2))q_1 - 2q_1)}{\partial q_1} = 0$ at optimum point.
   We get $48 - 2q_2 = 4q_1$, the best response function of firm 1.
   $\frac{\partial((50 - 2(q_1 + q_2))q_2 - 2q_2)}{\partial q_2} = 0$
   We get $48 - 2q_1 = 4q_2$, the best response function of firm 2.

   b) We get the Cournot Nash equilibrium by solving $48 - 2q_2 = 4q_1$ and $48 - 2q_1 = 4q_2$. Output of each firm is $q_1 = q_2 = 8$. The profit $\pi_1 = 128$, $\pi_2 = 128$.

2. The market demand function is $A - Q = p$. There are three firms 1, 2 and 3, each producing homogeneous output $q_i, i = 1, 2, 3$. The cost function of each firm is $cq_i, i = 1, 2, 3$.
   It is a two stage game. In stage I, firm 1 chooses output q_1. In stage II, firm 2 and 3 simultaneously chooses q_2 and q_3 after observing q_1.
   We find the subgame perfect Nash equilibrium using backward induction.
   First we find the output of firm 2 and 3, q_2 and q_3 in stage II, given q_1. The payoff,(profit) of firm 2 is $\pi_2(q_1, q_2, q_3) = (A-(q_1+q_2+q_3))q_2 - cq_2$ and firm 3 is $\pi_3(q_1, q_2, q_3) = (A-(q_1+q_2+q_3))q_3 - cq_3$.

$\frac{\partial \pi_2(q_1,q_2,q_3)}{\partial q_2} = \frac{\partial((A-(q_1+q_2+q_3))q_2-cq_2)}{\partial q_2} = 0$. $2q_2 = A-c-q_1-q_3$, the best response function of firm 2.
$\frac{\partial \pi_3(q_1,q_2,q_3)}{\partial q_3} = \frac{\partial((A-(q_1+q_2+q_3))q_3-cq_3}{\partial q_3} = 0$. $2q_3 = A-c-q_1-q_2$, the best response function of firm 3.
Solving $2q_2 = A-c-q_1-q_3$ and $2q_3 = A-c-q_1-q_2$, we get the best response function of firm 2 and 3 given q_1 of firm 1.

In stage I, firm 1 chooses q_1 that maximizes $\pi_1(q_1,q_2,q_3) = (A-(q_1+q_2+q_3))q_1 - cq_1 = (A-(q_1+\frac{A-c-q_1}{3}+\frac{A-c-q_1}{3}))q_1 - cq_1$.
$\frac{\partial \pi_1(q_1,q_2,q_3)}{\partial q_1} = \frac{\partial((A-(q_1+\frac{A-c-q_1}{2}+\frac{A-c-q_1}{3}))q_1-cq_1)}{\partial q_1} = 0$, at optimum point. Solving the above equation we get $q_1 = \frac{A-c}{2}$. Therefore, $q_2 = \frac{A-c}{6}$ and $q_3 = \frac{A-c}{6}$. This is the subgame perfect Nash equilibrium.

3. There are two firms 1, 2. The demand function faced by firm 1 is

   $D(p_1) = N,$ if $p_1 < p_2$.
   $D(p_1) = \frac{N}{2},$ if $p_1 = p_2$.
   $D(p_1) = 0,$ if $p_1 > p_2$ or $p_1 > 10$.
   We get similar demand function for firm 2 also.
   The payoff (profit) of firm 1 is
   $\pi_1 = p_1 N,$ if $p_1 < p_2$
   $\pi_1 = p_1 \frac{N}{2},$ if $p_1 = p_2$.
   $\pi_1 = 0,$ if $p_1 > p_2$ or $p_1 > 10$.
   We need to find the pure strategy Nash equilibrium. The profit function of firm 1 has been plotted in figure 1. If $p_2 = p^*$ then best response of firm 1 is to set $p_1 = p^* - \epsilon$. In the figure 1, it is clear that by undercutting price the profit of firm 1 increases. So, there will be continuous undercutting of prices by both the firms. $p_1 = p_2 = 0$ is the pure strategy Nash equilibrium.

4. There are two firms 1 and 2. The demand function of firm 1 is,
   $D(p_1) = (A-p_1),$ if $p_1 < p_2$.
   $D(p_1) = \frac{(A-P_1)}{2},$ if $p_1 = p_2$.
   $D(p_1) = 0,$ if $p_1 > p_2$.
   We get similar demand function for firm 2 also.
   The payoff (profit) of firm 1 is
   $\pi_1 = p_1(A-p_1),$ if $p_1 < p_2$
   $\pi_1 = p_1\frac{(A-p_1)}{2},$ if $p_1 = p-2$.
   $\pi_1 = 0,$ if $p_1 > p_2$.
   The payoff (profit) of firm 2 is
   $\pi_2 = p_2(A-p_2) - f,$ if $p_1 > p_2$
   $\pi_2 = p_2\frac{(A-p_2)}{2},$ if $p_1 = p-2$.
   $\pi_2 = 0,$ if $p_1 < p_2$.
   For firm 2, we get $\underline{p}$ and $\bar{p}$ such that $p_2\frac{(A-p_2)}{2} - f < 0,\ for\ p_2 < \underline{p}$, $p_2\frac{(A-p_2)}{2} - f < 0,\ for\ p_2 > \bar{p}$ and $p_2\frac{(A-p_2)}{2} - f \geq 0,\ for\ \underline{p} \leq p_2 \leq \bar{p}$.
   Since revenue function is same for firm 1 and 2 so the monopoly price p^M is same for both firm. Firm 2 will not set any price $p_2 < \underline{p}$. At $p_1 = p_2 = \underline{p}, \pi_1 > 0$. From figure 2, we see that if $p_1 = \underline{p}-\epsilon$, the profit of firm 1 is high. If firm 1 set $p_1 = \underline{p} - \frac{\epsilon}{2}$, $\pi_1(p_1 = \underline{p} - \frac{\epsilon}{2}) > \pi(p_1 = \underline{p} - \epsilon)$ because monopoly price $p^M > \underline{p}$. So firm 1 increases price p_1, if $p_1 < \underline{p}$. As firm 1 keeps on increasing price, $p_1 = \underline{p}$, so again profit falls because demand is shared. This shows that there is no pure strategy Nash equilibrium.

5. Suppose capacity of firm 1 is k_1 and capacity of firm 2 is k_2. We know that, if $k_1 \leq R_1(k_2)$ and $k_2 \leq R_2(k_1)$ then the pure strategy of each firm is to set same price and the price is given by $p = 1-(k_1+k_2)$. The capacity of firm 1 is $\frac{1}{3}$ and firm 2 is $\frac{1}{4}$. First, we find the Cournot reaction function. $\pi_1 = (1-(q_1+q_2))q_1$ and $\pi_2 = (1-(q_1+q_2)q_2$. $\frac{\partial \pi_1}{\partial q_1} = \frac{\partial(1-(q_1+q_2))q_1}{\partial q_1} = 0. \Rightarrow 2q_1 = 1-q_2$.

The reaction function of firm 1.

$\frac{\partial \pi_2}{\partial q_2} = \frac{\partial(1-(q_1+q_2))q_2}{\partial q_2} = 0. \Rightarrow 2q_2 = 1 - q_1$. The reaction function of firm 2.
By solving the reaction function, we get the Cournot output is $q_1^C = \frac{1}{3}$ and $q_2^C = \frac{1}{3}$. The capacity of firm 1 is same as Cournot output and capacity of firm 2 is less than Cournot output. So, in this case the pure strategy Nash equilibrium price is that both firms set the same price and $p = 1 - \frac{1}{3} - \frac{1}{4}$, $p = \frac{5}{12}$.

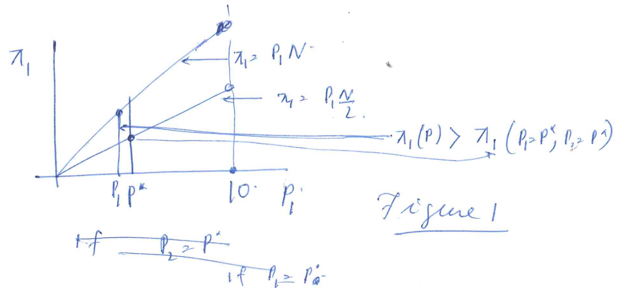

Figure 1

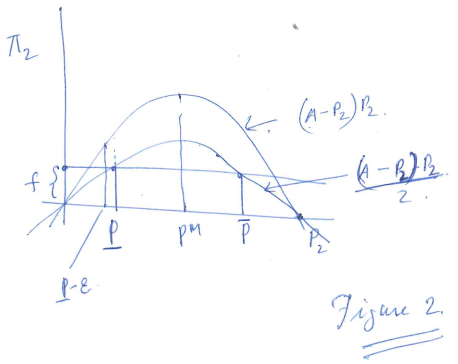

Figure 2