باسمه تعالی

تکلیف شماره ۲ کدگذاری کانال　　　　　　　　　　　　　　　　　　　　　　　　　　　　زمان تحویل:

1- Let $\mathbb{H}$ be the parity-check matrix of an (n, k) linear code C that has both odd- and even-weight codewords. Construct a new linear code C_1 with the following parity-check matrix:

$$\mathbb{H}_1 = \left[\begin{array}{c|ccc} 0 & & & \\ 0 & & & \\ \vdots & & \mathbb{H} & \\ 0 & & & \\ \hline 1 & 1 & 1 \cdots & 1 \end{array}\right].$$

(Note that the last row of $\mathbb{H}_1$ consists of all 1's.)

a. Show that C_1 is an $(n + 1, k)$ linear code. C_1 is called an *extension* of C.

b. Show that every codeword of C_1 has even weight.

c. Show that C_1 can be obtained from C by adding an extra parity-check digit, denoted by v_∞, to the left of each codeword $\mathbf{v}$ as follows: (1) if $\mathbf{v}$ has odd weight, then $v_\infty = 1$, and (2) if $\mathbf{v}$ has even weight, then $v_\infty = 0$. The parity-check digit v_∞ is called an *overall parity-check* digit.

2- Let C be a linear code with both even- and odd-weight codewords. Show that the number of even-weight codewords is equal to the number of odd-weight codewords.

3- Consider an (n, k) linear code C whose generator matrix $\mathbb{G}$ contains no zero column. Arrange all the codewords of C as rows of a 2^k-by-n array.

a. Show that no column of the array contains only zeros.

b. Show that each column of the array consists of 2^{k-1} zeros and 2^{k-1} ones.

c. Show that the set of all codewords with zeros in a particular component position forms a subspace of C. What is the dimension of this subspace?

4- Prove that a linear code is capable of correcting λ or fewer errors and simultaneously detecting $l\,(l > \lambda)$ or fewer errors if its minimum distance $d_{\min} \geq \lambda + l + 1$.

5- Let C_1 be an (n_1, k) linear systematic code with minimum distance d_1 and generator matrix $\mathbb{G}_1 = [\mathbb{P}_1\ \mathbb{I}_k]$. Let C_2 be an (n_2, k) linear systematic code with minimum distance d_2 and generator matrix $\mathbb{G}_2 = [\mathbb{P}_2\ \mathbb{I}_k]$. Consider an $(n_1 + n_2, k)$ linear code with the following parity-check matrix:

$$\mathbb{H} = \left[\begin{array}{c:c} & \mathbb{P}_1^T \\ \mathbb{I}_{n_1+n_2-k} & \mathbb{I}_k \\ & \mathbb{P}_2^T \end{array} \right].$$

Show that this code has a minimum distance of at least $d_1 + d_2$.

6- For any binary (n, k) linear code with minimum distance (or minimum weight) $2t+1$ or greater, show that the number of parity-check digits satisfies the following inequality:

$$n - k \geq \log_2 \left[1 + \binom{n}{1} + \binom{n}{2} + \cdots + \binom{n}{t} \right].$$

The preceding inequality gives an upper bound on the random-error-correcting capability t of an (n, k) linear code. This bound is known as the *Hamming bound* [14]. (*Hint:* For an (n, k) linear code with minimum distance $2t + 1$ or greater, all the n-tuples of weight t or less can be used as coset leaders in a standard array.)