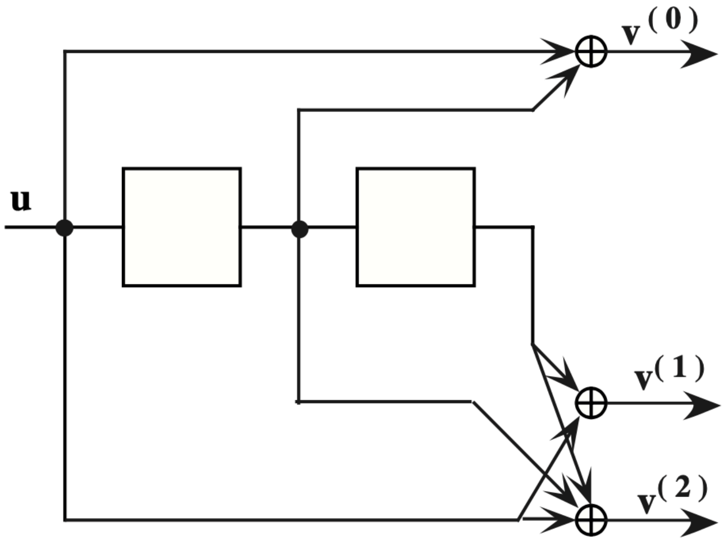
بسم تعالی

حل تکلیف شماره ۵ کدگذاری کانال

1–

(a) The encoder diagram is shown below.



(b) The generator matrix is given by

$$\mathbf{G} = \begin{bmatrix} 111 & 101 & 011 & & & \\ & 111 & 101 & 011 & & \\ & & 111 & 101 & 011 & \\ & & & \ddots & & \ddots \end{bmatrix}.$$

(c) The codeword corresponding to $\mathbf{u} = (11101)$ is given by

$$\mathbf{v} = \mathbf{u} \cdot \mathbf{G} = (111, 010, 001, 110, 100, 101, 011).$$

2–

(a) The generator matrix is given by

$$\mathbf{G} = \begin{bmatrix} 111 & 001 & 010 & 010 & 001 & 011 & & & \\ & 111 & 001 & 010 & 010 & 001 & 011 & & \\ & & 111 & 001 & 010 & 010 & 001 & 011 & \\ & & & \ddots & & & & & \ddots \end{bmatrix}.$$

(b) The parity sequences corresponding to $\mathbf{u} = (1101)$ are given by

$$\begin{aligned} \mathbf{v}^{(1)}(D) &= \mathbf{u}(D) \cdot \mathbf{g}^{(1)}(D) \\ &= (1 + D + D^3)(1 + D^2 + D^3 + D^5) \\ &= 1 + D + D^2 + D^3 + D^4 + D^8, \end{aligned}$$

and

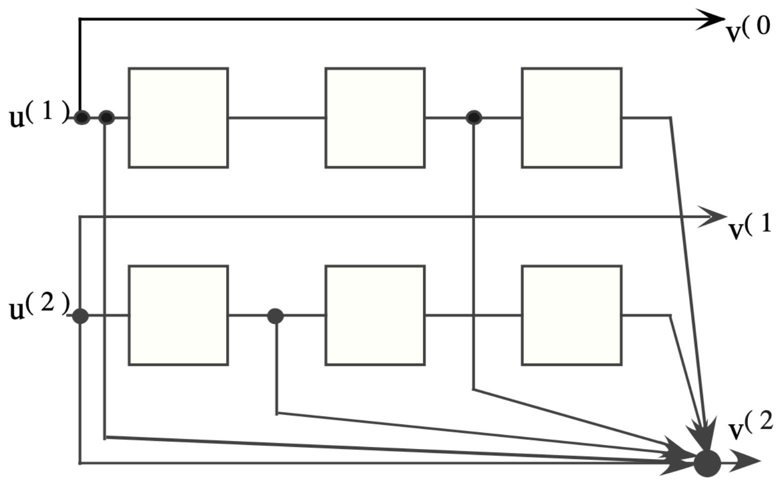
$$\begin{aligned} \mathbf{v}^{(2)}(D) &= \mathbf{u}(D) \cdot \mathbf{g}^{(2)}(D) \\ &= (1 + D + D^3)(1 + D + D^4 + D^5) \\ &= 1 + D^2 + D^3 + D^6 + D^7 + D^8. \end{aligned}$$

Hence,

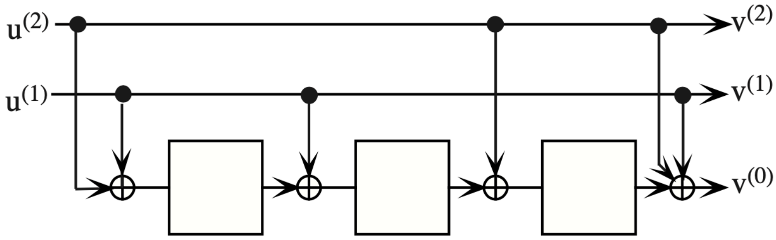
$$\begin{aligned} \mathbf{v}^{(1)} &= (111110001) \\ \mathbf{v}^{(2)} &= (101100111). \end{aligned}$$

3–

(a) The controller canonical form encoder realization, requiring 6 delay elements, is shown below.



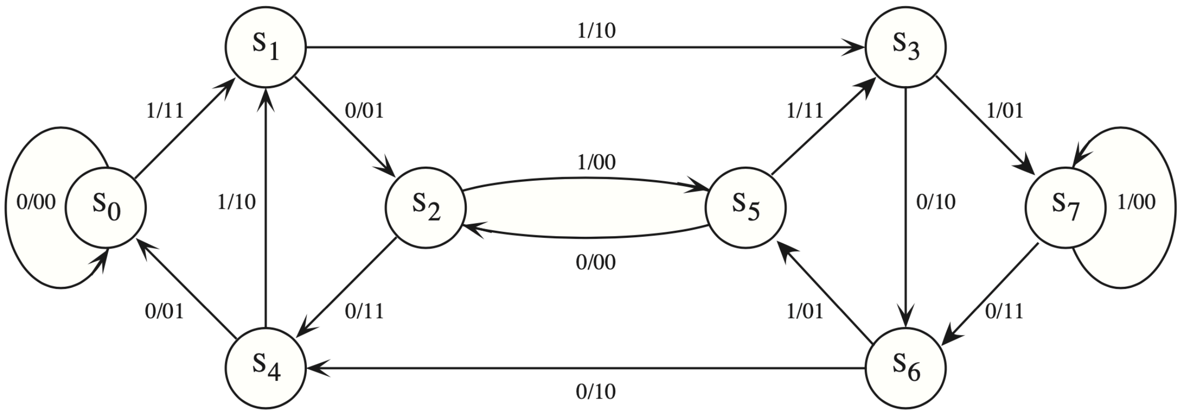
(b) The observer canonical form encoder realization, requiring only 3 delay elements, is shown below.



4–

11.15 (a) The GCD of the generator polynomials is $1 + D^2$ and a feedforward inverse does not exist.

(b) The encoder state diagram is shown below.



(c) The cycles $S_2S_5S_2$ and S_7S_7 both have zero output weight.

(d) The infinite-weight information sequence

$$\mathbf{u}(D) = \frac{1}{1+D^2} = 1 + D^2 + D^4 + D^6 + D^8 + \cdots$$

results in the output sequences

$$\begin{aligned}\mathbf{v}^{(0)}(D) &= \mathbf{u}(D)\left(1+D^2\right) = 1 \\ \mathbf{v}^{(1)}(D) &= \mathbf{u}(D)\left(1+D+D^2+D^3\right) = 1+D,\end{aligned}$$

and hence a codeword of finite weight.

(e) This is a catastrophic encoder realization.

5– Note that

$$\begin{aligned}\sum_{l=0}^{N-1} c_2\left[\log P(r_l|v_l)+c_1\right] &= \sum_{l=0}^{N-1}\left[c_2\log P(r_l|v_l)+c_2c_1\right]\\ &= c_2\sum_{l=0}^{N-1}\log P(r_l|v_l)+Nc_2c_1.\end{aligned}$$

Since

$$\max_{\mathbf{v}}\left\{c_2\sum_{l=0}^{N-1}\log P(r_l|v_l)+Nc_2c_1\right\}=c_2\max_{\mathbf{v}}\left\{\sum_{l=0}^{N-1}\log P(r_l|v_l)\right\}+Nc_2c_1$$

if C_2 is positive, any path that maximizes $\sum_{l=0}^{N-1}\log P(r_l|v_l)$ also maximizes $\sum_{l=0}^{N-1}c_2[\log P(r_l|v_l)+c_1]$.

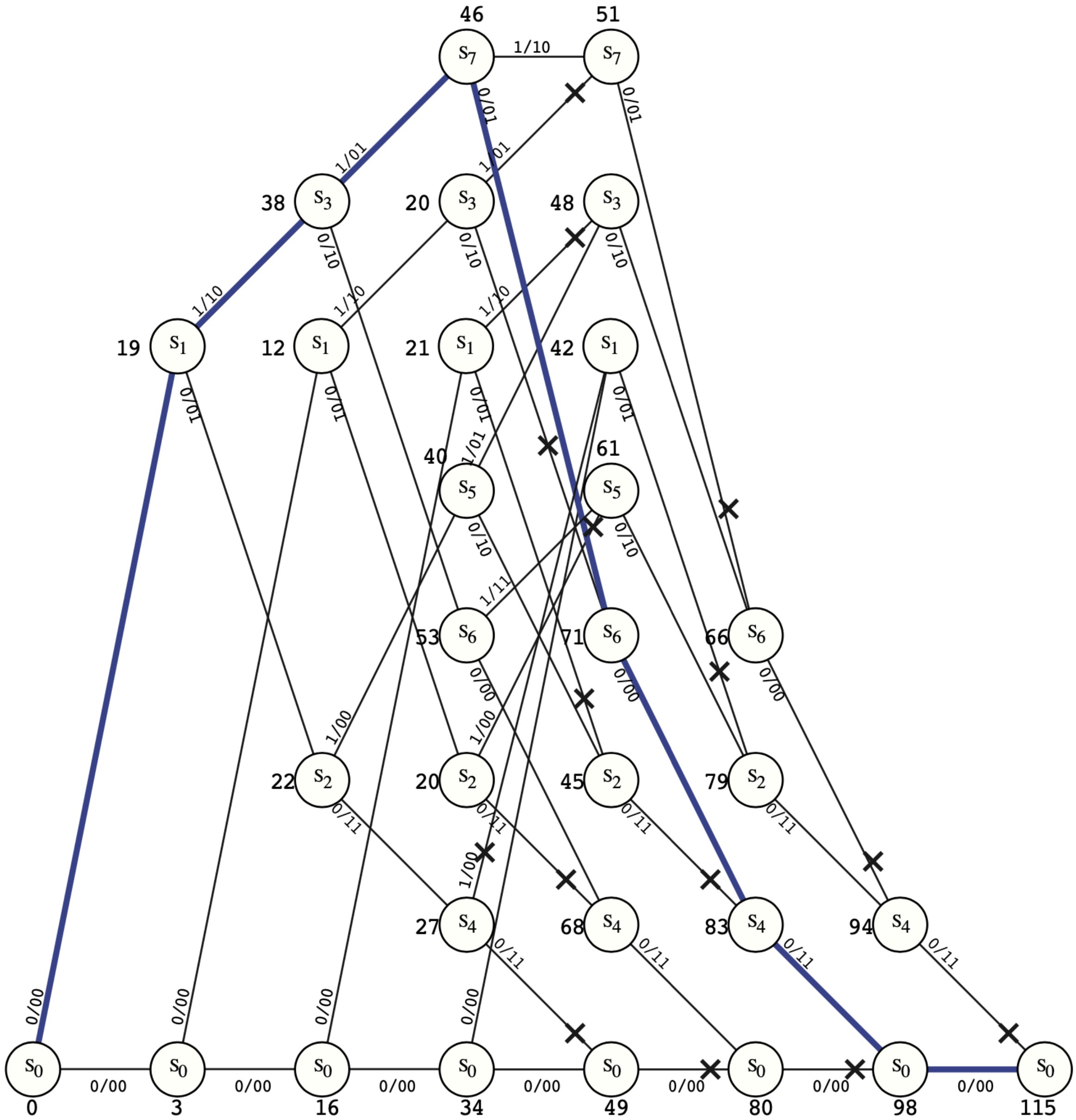
6– (a) Referring to the state diagram of Figure 11.13(a), the trellis diagram for an information sequence of length $h=4$ is shown in the figure below.

(b) After Viterbi decoding the final survivor is

$$\hat{\mathbf{v}}=(11,10,01,00,11,00).$$

This corresponds to the information sequence

$$\hat{\mathbf{u}}=(1110).$$


46
51
s_7
1/10
s_7
0/01
0/01
1/01
1/01
38
s_3
20
s_3
48
s_3
0/10
0/10
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1/10
1/10
19
s_1
12
s_1
21
s_1
42
s_1
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0/01
0/01
40
61
1/01
s_5
s_5
0/10
0/10
1/11
53
s_6
71
s_6
66
s_6
0/00
0/00
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22
s_2
20
s_2
45
s_2
79
s_2
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0/11
1/00
27
s_4
68
s_4
83
s_4
94
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3
16
34
49
80
98
115