باسمه تعالی

تکلیف شماره ۱ کدگذاری کانال زمان تحویل:

1- Let m be a positive integer. If m is not a prime, prove that the set $\{1, 2, \ldots, m-1\}$ is not a group under modulo-m multiplication.

2- Construct the prime field GF(11) with modulo-11 addition and multiplication. Find all the primitive elements and determine the orders of other elements.

3- Let λ be the characteristic of a Galois field GF(q). Let 1 be the unit element of GF(q). Show that the sums

$$1, \quad \sum_{i=1}^{2} 1, \quad \sum_{i=1}^{3} 1, \quad \ldots, \quad \sum_{i=1}^{\lambda-1} 1, \quad \sum_{i=1}^{\lambda} 1 = 0$$

form a subfield of GF(q).

Prove that every finite field has a primitive element.

4- Let S be a subset of the vector space V_n of all n-tuples over GF(2). Prove that S is a subspace if for any $\mathbf{u}$ and $\mathbf{v}$ in S, $\mathbf{u}+\mathbf{v}$ is in S.

5- Given the matrices

$$\mathbf{G} = \begin{bmatrix} 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}, \qquad \mathbf{H} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix},$$

show that the row space of $\mathbf{G}$ is the null space of $\mathbf{H}$, and vice versa.

6- Let S_1 and S_2 be two subspaces of a vector V. Show that the intersection of S_1 and S_2 is also a subspace in V.