باسمه تعالی

تکلیف شماره ۴ کدگذاری کانال　　　　　　　　　　زمان تحویل:

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1- Let $g(X)$ be the generator polynomial of a binary cyclic code of length $n$.

a. Show that if $g(X)$ has $X+1$ as a factor, the code contains no codewords of odd weight.

b. If $n$ is odd and $X+1$ is not a factor of $g(X)$, show that the code contains a codeword consisting of all 1's.

c. Show that the code has a minimum weight of at least 3 if $n$ is the smallest integer such that $g(X)$ divides $X^n+1$.

2- Consider a binary $(n, k)$ cyclic code $C$ generated by $g(X)$. Let

$$g^*(X) = X^{n-k} g(X^{-1})$$

be the reciprocal polynomial of $g(X)$.

a. Show that $g^*(X)$ also generates an $(n, k)$ cyclic code.

b. Let $C^*$ denote the cyclic code generated by $g^*(X)$. Show that $C$ and $C^*$ have the same weight distribution.

(*Hint:* Show that

$$v(X) = v_0 + v_1 X + \cdots + v_{n-2} X^{n-2} + v_{n-1} X^{n-1}$$

is a code polynomial in $C$ if and only if

$$X^{n-1} v(X^{-1}) = v_{n-1} + v_{n-2} X + \cdots + v_1 X^{n-2} + v_0 X^{n-1}$$

3- Consider a cyclic code $C$ of length $n$ that consists of both odd-weight and even-weight codewords. Let $g(X)$ and $A(z)$ be the generator polynomial and weight enumerator for this code. Show that the cyclic code generated by $(X+1)g(X)$ has weight enumerator

$$A_1(z) = \frac{1}{2}[A(z) + A(-z)].$$

4- Consider the $(2^m - 1, 2^m - m - 2)$ cyclic Hamming code $C$ generated by $\mathbb{g}(X) = (X + 1)\mathbb{p}(X)$, where $\mathbb{p}(X)$ is a primitive polynomial of degree $m$. An error pattern of the form

$$\mathbb{e}(X) = X^i + X^{i+1}$$

is called a *double-adjacent-error pattern*. Show that no two double-adjacent-error patterns can be in the same coset of a standard array for $C$. Therefore, the code is capable of correcting all the single-error patterns and all the double-adjacent-error patterns.

5- For a cyclic code, if an error pattern $\mathbb{e}(X)$ is detectable, show that its $i$th cyclic shift $e^{(i)}(X)$ is also detectable.

6- Consider the (15, 5) cyclic code generated by the following polynomial:

$$\mathbb{g}(X) = 1 + X + X^2 + X^4 + X^5 + X^8 + X^{10}.$$

This code has been proved to be capable of correcting any combination of three or fewer errors. Suppose that this code is to be decoded by the simple error-trapping decoding scheme.

a. Show that all the double errors can be trapped.
b. Can all the error patterns of three errors be trapped? If not, how many error patterns of three errors cannot be trapped?
c. Devise a simple error-trapping decoder for this code.