باسمه تعالی

تکلیف شماره ۵ کدگذاری کانال　　　　زمان تحویل:

1-

Consider the (3, 1, 2) nonsystematic feedforward encoder with

$$\mathbf{g}^{(0)} = (1\,1\,0),$$
$$\mathbf{g}^{(1)} = (1\,0\,1),$$
$$\mathbf{g}^{(2)} = (1\,1\,1).$$

a. Draw the encoder block diagram.
b. Find the time-domain generator matrix $\mathbb{G}$.
c. Find the codeword $\mathbf{v}$ corresponding to the information sequence $\mathbf{u} = (1\,1\,1\,0\,1)$.

2-

Consider the (3, 1, 5) systematic feedforward encoder with

$$\mathbf{g}^{(1)} = (1\,0\,1\,1\,0\,1),$$
$$\mathbf{g}^{(2)} = (1\,1\,0\,0\,1\,1).$$

a. Find the time-domain generator matrix $\mathbb{G}$.
b. Find the parity sequences $\mathbf{v}^{(1)}$ and $\mathbf{v}^{(2)}$ corresponding to the information sequence $\mathbf{u} = (1\,1\,0\,1)$.

3-

Consider the (3, 2, 3) systematic feedforward encoder with

$$\mathbf{g}_1^{(2)}(D) = 1 + D^2 + D^3,$$
$$\mathbf{g}_2^{(2)}(D) = 1 + D + D^3.$$

a. Draw the controller canonical form realization of this encoder. How many delay elements are required in this realization?
b. Draw the simpler observer canonical form realization that requires only three delay elements.

4- Consider the $(2, 1, 3)$ nonsystematic feedforward encoder with $\mathbb{G}(D) = [1 + D^2 \quad 1 + D + D^2 + D^3]$.

a. Find the GCD of its generator polynomials.

b. Draw the encoder state diagram.

c. Find a zero-output weight cycle in the state diagram.

d. Find an infinite-weight information sequence that generates a codeword of finite weight.

e. Is this encoder catastrophic or noncatastrophic?

5- Show that the path $\mathbf{v}$ that maximizes $\sum_{l=0}^{N-1} \log P(r_l|v_l)$ also maximizes $\sum_{l=0}^{N-1} c_2[\log P(r_l|v_l) + c_1]$, where c_1 is any real number and c_2 is any positive real number.

6- Consider the $(2, 1, 3)$ encoder of Figure 11.1 with

$$\mathbb{G}(D) = [1 + D^2 + D^3 \quad 1 + D + D^2 + D^3]$$

a. Draw the trellis diagram for an information sequence of length $h = 4$.

b. Assume a codeword is transmitted over the DMC of Problem 12.4. Use the Viterbi algorithm to decode the received sequence $\mathbf{r} = (1_2 1_1, 1_2 0_1, 0_3 0_1, 0_1 1_3, 1_2 0_2, 0_3 1_1, 0_3 0_2)$.